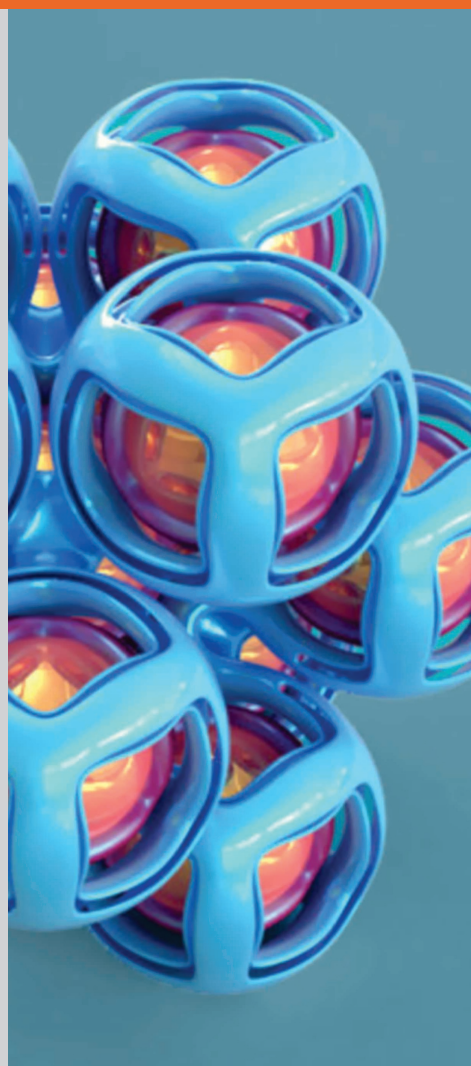
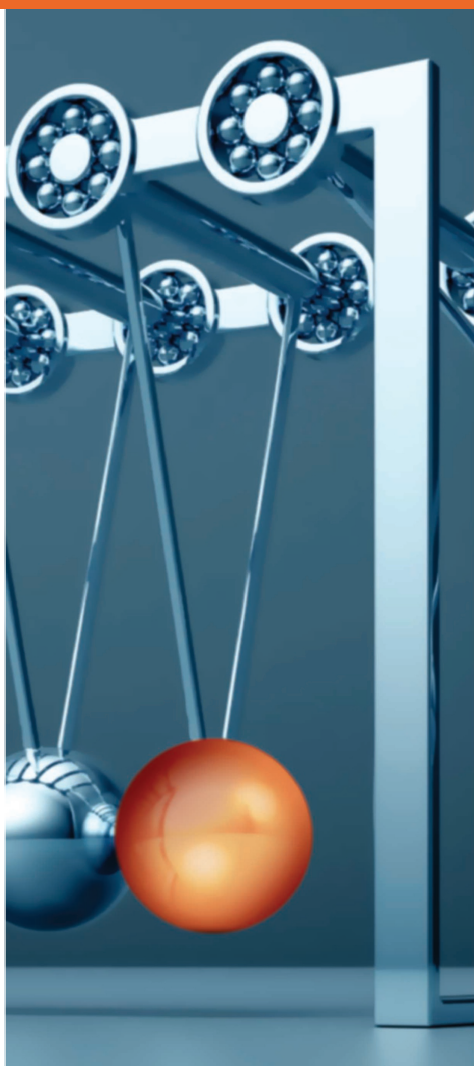


NEET

OBJECTIVE

PHYSICS



DC Pandey

Volume **02**

NEET

OBJECTIVE
PHYSICS

Volume **02**

NEET

OBJECTIVE PHYSICS

Volume **02**

DC Pandey

[B.Tech, M.Tech, Pantnagar, ID 15722]



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Preface

Medical offers the most exciting and fulfilling of careers. As a doctor you can find satisfaction from curing other persons. Although the number of medical colleges imparting quality education and training has significantly increased after independence in the country but due to the simultaneous increase in the number of serious aspirants, the competition is no longer easy for a seat in a prestigious medical college today.

For success, you require an objective approach to your study in the test subjects. This does not mean you 'prepare' yourself for just 'objective questions'. Objective Approach means more than that. It could be defined as that approach through which a student is able to master the concepts of the subject and also the skills required to tackle the questions asked in different formats in entrances such as NEET (National Eligibility cum Entrance Test). These two-volume books, Objective Physics (Vol.1 & 2) are borne out of my experience of teaching physics to medical aspirants, fill the needs of such books in the market.

The plan of the presentation of the subject matter in the books is as follows

- The whole chapter has been divided under logical topic heads to cover the whole syllabi of NEET developing the concepts in an easy going manner, taking the help of suitable examples.
- Important points of the topics have been highlighted in the text under notes, some extra points regarding the topics have been given in Notes to enrich the students.
- The **Solved Examples** given with different concepts of the chapter make the students learn the basic problem solving skills in Physics. It has been ensured that given examples cover all aspects of a concepts comprehensively.
- **Check Point** Exercises given in between the text of all chapter help the readers to remain linked with the text given as they provide them an opportunity to assess themselves while studying the text.
- Exercises at the end of the chapters have been divided into three parts:

Part A- 'Taking it together' has Objective Questions of the concerned chapter. The special point of this exercise is, all the questions have been arranged according to level of difficulty, providing students a systematics practice.

Part B- 'Medical Entrance Special Format Question' this section covers all special type of questions, other than simple MCQs, generally asked in NEET & other Medical Entrances. Here Assertion-Reason, Statement Based and Matching Type Questions have been given.

Part C- 'Medical Entrances Gallery' covering all questions asked in last 11 years' (2021-2011) in NEET & other Medical Entrances.

- The answers / solutions to all the questions given in different exercises have been provided.

At the end I would like to say that suggestions from the respected Teachers & Students for the further improvement of the book will be welcomed open heartedly.

DC Pandey

DEDICATION

This book is dedicated to my honourable grandfather

(Late) Sh. Pitamber Pandey

a Kumaoni poet, and a resident of village Dhaura (Almora), Uttarakhand

Syllabus

UNIT I Electrostatics

Electric charges and their conservation. Coulomb's law-force between two point charges, forces between multiple charges, superposition principle and continuous charge distribution. Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in a uniform electric field. Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

Electric potential, potential difference, electric potential due to a point charge, a dipole and system of charges, equipotential surfaces, electrical potential energy of a system of two point charges and of electric dipoles in an electrostatic field. Conductors and insulators, free charges and bound charges inside a conductor, Dielectrics and electric polarization, capacitors and capacitance, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, energy stored in a capacitor, Van de Graaff generator.

UNIT II Current Electricity

Electric current, flow of electric charges in a metallic conductor, drift velocity and mobility and their relation with electric current, Ohm's law, electrical resistance, V-I characteristics (linear and non-linear), electrical energy and power, electrical resistivity and conductivity. Carbon resistors, colour code for carbon resistors, series and parallel combinations of resistors, temperature dependence of resistance. Internal resistance of a cell, potential difference and emf of a cell, combination of cells in series and in parallel. Kirchhoff's laws and simple applications. Wheatstone bridge, metre bridge. Potentiometer-principle and applications to measure potential difference, and for comparing emf of two cells, measurement of internal resistance of a cell.

UNIT III Magnetic Effects of Current and Magnetism

Concept of magnetic field, Oersted's experiment. Biot-Savart's law and its application to current carrying circular loop. Ampere's law and its applications to infinitely long straight wire, straight and toroidal solenoids. Force on a moving charge in uniform magnetic and electric fields. Cyclotron. Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in a magnetic field, moving coil galvanometer-its current sensitivity and conversion to ammeter and voltmeter. Current loop as a magnetic dipole and its magnetic dipole moment. Magnetic dipole moment of a revolving electron. Magnetic field intensity due to a magnetic dipole (bar magnet) along its axis and perpendicular to its axis. Torque on a magnetic dipole (bar magnet) in a uniform magnetic field, bar magnet as an equivalent solenoid, magnetic field lines, Earth's magnetic field and magnetic elements. Para-, dia-and ferro-magnetic substances with examples. Electromagnetic and factors affecting their strengths. Permanent magnets.

UNIT IV Electromagnetic Induction and Alternating Currents

Electromagnetic induction Faraday's law, induced emf and current, Lenz's Law, Eddy currents. Self and mutual inductance.

Alternating currents, peak and rms value of alternating current/ voltage, reactance and impedance, LC oscillations (qualitative treatment only), LCR series circuit, resonance, power in AC circuits, wattless current. AC generator and transformer.

UNIT V Electromagnetic Waves

Need for displacement current. Electromagnetic waves and their characteristics (qualitative ideas only). Transverse nature of electromagnetic waves. Electromagnetic spectrum (radiowaves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays) including elementary facts about their uses.

UNIT VI Optics

Reflection of light, spherical mirrors, mirror formula. Refraction of light, total internal reflection and its applications optical fibres, refraction at spherical surfaces, lenses, thin lens formula, lens-maker's formula. Magnification, power of a lens, combination of thin lenses in contact combination of a lens and a mirror. Refraction and dispersion of light through a prism. Scattering of light- blue colour of the sky and reddish appearance of the sun at sunrise and sunset.

Optical instruments Human eye, image formation and accommodation, correction of eye defects (myopia and hypermetropia) using lenses. Microscopes and astronomical telescopes (reflecting and refracting) and their magnifying powers. Wave optics: Wave front and Huygens' principle, reflection and refraction of plane wave at a plane surface using wave fronts. Proof of laws of reflection and refraction using Huygens' principle. Interference, Young's double hole experiment and expression for fringe width, coherent sources and sustained interference of light. Diffraction due to a single slit, width of central maximum. Resolving power of microscopes and astronomical telescopes. Polarisation, plane polarised light, Brewster's law, uses of plane polarised light and Polaroids.

UNIT VII Dual Nature of Matter and Radiation

Photoelectric effect, Hertz and Lenard's observations, Einstein's photoelectric equation- particle nature of light. Matter waves- wave nature of particles, de-Brogli relation. Davisson-Germer experiment (experimental details should be omitted, only conclusion should be explained).

UNIT VIII Atoms and Nuclei

Alpha- particle scattering experiments, Rutherford's model of atom, Bohr model, energy levels, hydrogen spectrum. Composition and size of nucleus, atomic masses, isotopes, isobars, isotones. Radioactivity- α , β and γ particles/ rays and their properties decay law.

Mass-energy relation, mass defect, binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

UNIT IX Electronic Devices

Energy bands in solids (qualitative ideas only), conductors, insulators and semiconductors, semiconductor diode- I-V characteristics in forward and reverse bias, diode as a rectifier, I-V characteristics of LED, photodiode, solar cell and Zener diode, Zener diode as a voltage regulator. Junction transistor, transistor action, characteristics of a transistor, transistor as an amplifier (common emitter configuration) and oscillator. Logic gates (OR, AND, NOT, NAND and NOR). Transistor as a switch.

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Electric Charges and Fields

ELECTRIC CHARGES

Electric charge can be defined as an intrinsic property of elementary particles of matter which give rise to electric force between various objects. It is represented by q . The SI unit of electric charge is coulomb (C). A proton has positive charge $(+e)$ and an electron has negative charge $(-e)$, where $e = 1.6 \times 10^{-19}$ C.

Important points regarding electric charge

The following points regarding electric charge are worth noting

- (i) Like charges repel each other and unlike charges attract each other.
- (ii) The property which differentiates two kinds of charge is called the **polarity of charge**. If an object possesses an electric charge, then it is said to be **electrified or charged**. When its net charge is zero, then it is said to be **neutral (just like neutron)**.
- (iii) Charge is a scalar quantity as it has magnitude but no direction. It can be of two types as positive and negative. When some electrons are removed from the atom, it acquires a positive charge and when some electrons are added to the atom, it acquires a negative charge.
- (iv) Charge can be transferred from one body to another.
- (v) Charge is invariant, *i.e.* it does not depend on the velocity of charged particle.
- (vi) A charged particle at rest produces electric field. A charged particle with unaccelerated motion produces both electric and magnetic fields but does not radiate energy. But an accelerated charged particle not only produces an electric and magnetic fields but also radiates energy in the form of electromagnetic waves.
- (vii) $1 \text{ coulomb} = 3 \times 10^9 \text{ esu} = \frac{1}{10} \text{ emu of charge}$, where esu is electrostatic unit of charge. Its CGS unit is stat coulomb.
- (viii) The dimensional formula of charge is $[q] = [AT]$.

Inside

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- Methods of charging

2 Coulomb's law

- Force between multiple charges (Superposition principle)
- Applications of electric force (Coulomb's law)

3 Electric field

- Electric field lines
- Continuous charge distribution
- Electric field of a charged ring

4 Electric dipole

- The field of an electric dipole or dipole field
- Force on dipole
- Torque on an electric dipole
- Work done in rotating a dipole in a uniform electric field

5 Electric flux

- Gauss's law
- Applications of Gauss's law

Properties of an electric charge

If the size of charged bodies is very small as compared to the distance between them, we treat them as **point charges**. In addition to being positive or negative, the charges have the following properties

Additivity of charges

Additivity of charges is the property by virtue of which total charge of a system is obtained simply by adding algebraically all the charges present on the system.

If a system contains two point charges q_1 and q_2 , then the total charge of the system is obtained simply by adding algebraically q_1 and q_2 , i.e. charges add up like real numbers. Proper signs have to be used while adding the charges in a system.

$$q_{\text{net}} = q_1 + q_2$$

Charge is conserved

The total charge of an isolated system is always conserved. It is not possible to create or destroy net charge carried by any isolated system. It can only be transferred from one body to another body. Pair production and pair annihilation are two examples of conservation of charge.

Quantisation of charge

The charge on any body can be expressed as the integral multiple of basic unit of charge, i.e. charge on one electron (e). This phenomenon is called quantisation of electric charge. It can be written as $q = \pm ne$, where $n = 1, 2, 3, \dots$ is any integer (positive or negative) and e is the basic unit of charge.

Charge is said to be quantised because it can only have discrete values rather than any arbitrary value, i.e. free particle can have no charge of any values, i.e. a charged particle can have a charge of $+10e$ or $-6e$ but not a charge of, say $3.57e$.

Note The protons and neutrons are combination of other entities called **quarks**, which have charges of $\pm \frac{1}{3}e$ and $\pm \frac{2}{3}e$. However, isolated quarks have not been observed, but quantum of charge is still e .

Example 1.1 What is the total charge of a system containing five charges $+1, +2, -3, +4$ and -5 in some arbitrary unit?

Sol. As charges are additive in nature, i.e. the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system, i.e.

$$q_{\text{net}} = q_1 + q_2 + q_3 + q_4 + q_5$$

$\therefore$ Total charge $= +1 + 2 - 3 + 4 - 5 = -1$ in the same unit.

Example 1.2 How many electrons are there in one coulomb of negative charge?

Sol. The negative charge is due to the presence of excess electrons. An electron has a charge whose magnitude is

$e = 1.6 \times 10^{-19}$ C, the number of electrons is equal to the charge q divided by the charge e on each electron.

Therefore, the number n of electrons is

$$n = \frac{q}{e} = \frac{1.0}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \text{ electrons}$$

Example 1.3 A sphere of lead of mass 10 g has net charge -2.5×10^{-9} C.

- Find the number of excess electrons on the sphere.
- How many excess electrons are per lead atom? Atomic number of lead is 82 and its atomic mass is 207 g/mol.

Sol. (i) The charge on an electron $= -1.6 \times 10^{-19}$ C

Net charge on sphere $= -2.5 \times 10^{-9}$ C

So, the number of excess electrons

$$= \frac{-2.5 \times 10^{-9} \text{ C}}{-1.6 \times 10^{-19} \text{ C}} = 1.56 \times 10^{10} \text{ electrons}$$

(ii) Atomic number of lead is 82.

Atomic mass of lead is 207 g/mol.

$$\therefore 10 \text{ g of lead will have } \frac{10 \text{ g}}{207 \text{ g/mol}} \times 6.02 \times 10^{23} \text{ atoms/mol}$$

$$= 2.91 \times 10^{22} \text{ atoms}$$

$\therefore$ The number of excess electrons per lead atom

$$= \frac{1.56 \times 10^{10}}{2.91 \times 10^{22}} = 5.36 \times 10^{-13} \text{ electrons}$$

Conductors and insulators

The quantisation of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e .

Conductors The substances through which electric charges can flow easily are called conductors.

Metals, human body and animal bodies, graphite, acids, etc. are examples of conductors.

Insulators The substances through which electric charges cannot flow easily are called insulators.

Most of the non-metals like glass, diamond, porcelain, plastic, nylon, wood, mica, etc. are examples of insulators.

Methods of charging

There are mainly three methods of charging a body, which are given below

1. Charging by rubbing

When two bodies are rubbed together, some electrons from one body gets transferred to the another body. The positive and negative charges appear on the bodies in equal amount simultaneously due to the transfer of electrons.

The body that donates the electrons becomes positively charged while that which receives electrons becomes negatively charged, *e.g.* when a glass rod is rubbed with a silk cloth, the glass rod acquires some positive charge by losing electrons and the silk cloth acquires negative charge of the same amount of gaining electrons as shown in figure.

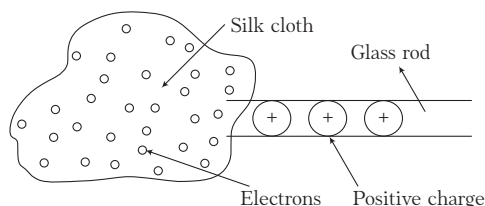


Fig. 1.1 A glass rod rubbed with a silk cloth

Ebonite rod on rubbing with wool becomes negatively charged making the wool positively charged.

2. Charging by contact or conduction

Take two conductors, one is charged and other is uncharged. Bring the conductors in contact with each other. The charge (whether negative or positive) under its

own repulsion will spread over both the conductors. Thus, the conductors will be charged with the same sign. This is called as charging by conduction (through contact).

3. Charging by induction

If a charged body is brought near an uncharged body, then one side of neutral body (closer to charged body) becomes oppositely charged while the other side becomes similarly charged as shown in figure.

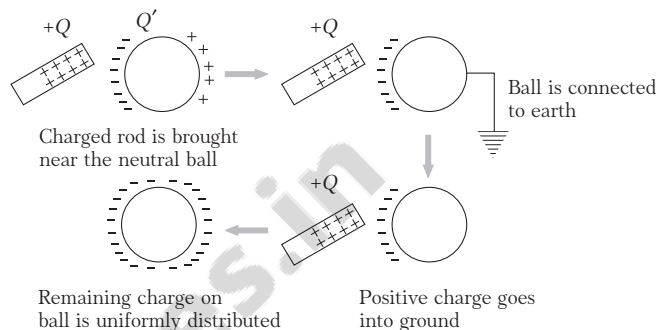


Fig. 1.2 Charging by induction

In this process, charging is done without actual contact of bodies.

CHECK POINT 1.1

- One metallic sphere *A* is given positive charge whereas another identical metallic sphere *B* of exactly same mass as of *A* is given equal amount of negative charge. Then,
 - mass of *A* and mass of *B* still remain equal
 - mass of *A* increases
 - mass of *B* decreases
 - mass of *B* increases
- When 10^{14} electrons are removed from a neutral metal sphere, then the charge on the sphere becomes
 - $16\mu\text{C}$
 - $-16\mu\text{C}$
 - $32\mu\text{C}$
 - $-32\mu\text{C}$
- A conductor has $14.4 \times 10^{-19}\text{C}$ positive charge. The conductor has (charge on electron = $1.6 \times 10^{-19}\text{C}$)
 - 9 electrons in excess
 - 27 electrons in short
 - 27 electrons in excess
 - 9 electrons in short
- Charge on α -particle is
 - $4.8 \times 10^{-19}\text{C}$
 - $1.6 \times 10^{-19}\text{C}$
 - $3.2 \times 10^{-19}\text{C}$
 - $6.4 \times 10^{-19}\text{C}$
- A body has $-80\mu\text{C}$ of charge. Number of additional electrons in it will be
 - 8×10^{-5}
 - 80×10^{-17}
 - 5×10^{14}
 - 1.28×10^{-17}
- Which of the following is correct regarding electric charge?
 - If a body is having positive charge, then there is shortage of electrons.
 - If a body is having negative charge, then there is excess of electrons.
 - Minimum possible charge = $\pm 1.6 \times 10^{-19}\text{C}$.
 - Charge is quantised, *i.e.* $Q = \pm ne$, where $n = 1, 2, 3, 4, \dots$
 - Both (i) and (ii)
 - Both (ii) and (iii)
 - (i), (ii), (iii)
 - All of these
- When a glass rod is rubbed with silk, it
 - gains electrons from silk
 - gives electrons to silk
 - gains protons from silk
 - gives protons to silk
- A comb runs through one's dry hair attracts small bits of paper. This is due to
 - comb is a good conductor
 - paper is a good conductor
 - the atoms in the paper get polarised by the charged comb
 - the comb possesses magnetic properties
- When a positively charged body is earthed electrons from the earth flow into the body. This means the body is
 - uncharged
 - charged positively
 - charged negatively
 - an insulator
- Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is
 - negative and distributed uniformly over the surface of the sphere
 - negative and appears only at the point on the sphere closest to the point charge
 - negative and distributed non-uniformly over the entire surface of the sphere
 - zero

COULOMB'S LAW

Coulomb's law is a quantitative statement about the force between two point charges. It states that "the force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them".

The force is repulsive, if the charges have the same signs and attractive, if the charges have opposite signs.

Suppose two point charges q_1 and q_2 are separated in vacuum by a distance r , then force between two charges is given by

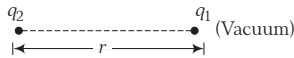


Fig. 1.3 Two charges separated by distance r

$$F_e = \frac{k|q_1 q_2|}{r^2}$$

The constant k is usually put as $k = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 is

called the **permittivity of free space** and has the value $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$. For all practical purposes, we will take $\frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$.

Coulomb's law in vector form

Consider two point charges q_1 and q_2 placed in vacuum at a distance r from each other, repel each other.

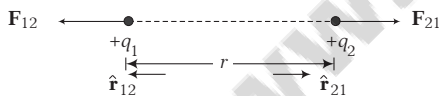


Fig. 1.4 Repulsive Coulombian forces for $q_1 q_2 > 0$

In vector form, Coulomb's law may be expressed as

$$\mathbf{F}_{12} = \text{Force on charge } q_1 \text{ due to } q_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12}$$

where, $\hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{r}$ is a unit vector in the direction from q_1 to q_2 .

Similarly, $\mathbf{F}_{21}$ = force on charge q_2 due to q_1

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{21}$$

where, $\hat{\mathbf{r}}_{21} = \frac{\mathbf{r}_{21}}{r}$ is a unit vector in the direction from q_2 to q_1 .

Similar case arises for attraction between two charges.

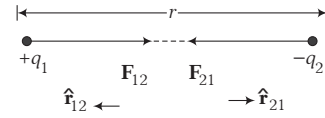


Fig. 1.5 Attractive Coulombian forces for $q_1 q_2 < 0$

Important points related to Coulomb's law

Important points related to Coulomb's law are given below

(i) The electric force is an action reaction pair, i.e. the two charges exert equal and opposite forces on each other. Thus, Coulomb's law obeys Newton's third law. $\mathbf{F}_{12} = -\mathbf{F}_{21}$

(ii) The electric force is conservative in nature.

(iii) Coulomb's law as we have stated above can be used for point charges in vacuum. When a dielectric medium is completely filled in between charges, rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of K (dielectric constant).

$$F'_e = \frac{F_e}{K} = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon} \cdot \frac{q_1 q_2}{r^2} \quad (\text{in medium})$$

Here, $\epsilon = \epsilon_0 K$ is called **permittivity of the medium**.

(iv) The electric force is much stronger than gravitational force, $F_e \gg F_g$. As value of $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$, which is much more than the value of gravitational constant,

$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2 \text{ kg}^{-2}.$$

Example 1.4 A proton and an electron are placed 1.6 cm apart in free space. Find the magnitude and nature of electrostatic force between them.

Sol. Charge on electron = $-1.6 \times 10^{-19} \text{ C}$

Charge on proton = $1.6 \times 10^{-19} \text{ C}$

Using Coulomb's law,

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}, \text{ where } r \text{ is the distance between proton and electron.}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19}) (-1.6 \times 10^{-19})}{(1.6 \times 10^{-2})^2} = -9 \times 10^{-25} \text{ N}$$

Negative sign indicates that the force is attractive in nature.

Example 1.5 The electrostatic force on a small sphere of charge $0.4 \mu\text{C}$ due to another small sphere of charge $-0.8 \mu\text{C}$ in air is 0.2 N . (i) What is the distance between the two spheres? (ii) What is the force on the second sphere due to the first?

Sol. (i) Using the relation, $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$, we get

$$r^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{F}$$

Here, $q_1 = 0.4 \mu\text{C} = 0.4 \times 10^{-6} \text{ C}$,

$q_2 = 0.8 \mu\text{C} = 0.8 \times 10^{-6} \text{ C}$

and $F = 0.2 \text{ N}$

$$\Rightarrow r^2 = 9 \times 10^9 \times \frac{0.4 \times 10^{-6} \times 0.8 \times 10^{-6}}{0.2}$$

$$= 144 \times 10^{-4} \text{ m}^2$$

Therefore, distance between the two spheres,

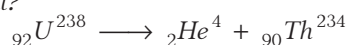
$$r = 12 \times 10^{-2} \text{ m} = 0.12 \text{ m}$$

(ii) Electrostatic forces always, appear in pairs and follows Newton's third law of motion.

$\therefore |\mathbf{F}_{21}| = \text{Force on } q_2 \text{ due to } q_1 = 0.2 \text{ N}$ and is attractive in nature.

Note In the Coulomb's expression for finding force between two charges, do not use sign of charge because this is magnitude of force.

Example 1.6 Nucleus of ${}_{92}\text{U}^{238}$ emits α -particle (${}_2\text{He}^4$). α -particle has atomic number 2 and mass number 4. At any instant, α -particle is at distance of $9 \times 10^{-15} \text{ m}$ from the centre of nucleus of uranium. What is the force on α -particle at this instant?



Sol. Force, $F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$

${}_{92}\text{U}^{238}$ has charge $92e$. When α -particle is emitted, charge on residual nucleus is $92e - 2e = 90e$.

$\therefore q_1 = 90e, q_2 = 2e$, and $r = 9 \times 10^{-15} \text{ m}$

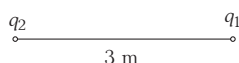
$$\therefore F = 9 \times 10^9 \times \frac{(90e)(2e)}{(9 \times 10^{-15})^2}$$

Force on α -particle,

$$F = \frac{9 \times 10^9 \times 90 \times (1.6 \times 10^{-19})^2 \times 2}{(9 \times 10^{-15})^2} = 512 \text{ N}$$

Example 1.7 Two identical spheres having positive charges are placed 3 m apart repel each other with a force $8 \times 10^{-3} \text{ N}$. Now, charges are connected by a metallic wire and they begin to repel each other with a force of $9 \times 10^{-3} \text{ N}$. Find initial charges on the spheres.

Sol. Let charges are q_1 and q_2 placed 3 m apart from each other.



$$\therefore F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad (\text{Coulomb's law})$$

$$\Rightarrow 8 \times 10^{-3} = \frac{9 \times 10^9 q_1 q_2}{(3)^2} \Rightarrow q_1 q_2 = 8 \times 10^{-12}$$

When the spheres are touched or connected by a metallic wire, charge on each sphere will be $\left(\frac{q_1 + q_2}{2}\right)$.

$$\underbrace{\left(\frac{q_1 + q_2}{2}\right) \left(\frac{q_1 + q_2}{2}\right)}_{3 \text{ m}}$$

According to given condition in the question,

$$\Rightarrow 9 \times 10^{-3} = \frac{9 \times 10^9 \left(\frac{q_1 + q_2}{2}\right)^2}{(3)^2}$$

$$\Rightarrow \left(\frac{q_1 + q_2}{2}\right)^2 = 9 \times 10^{-12} \Rightarrow q_1 + q_2 = 6 \times 10^{-6} \quad \dots(i)$$

$$\text{Now, } (q_1 - q_2)^2 = (q_1 + q_2)^2 - 4q_1 q_2$$

$$= (6 \times 10^{-6})^2 - 4 \times 8 \times 10^{-12} = 4 \times 10^{-12}$$

$$\Rightarrow q_1 - q_2 = 2 \times 10^{-6} \quad \dots(ii)$$

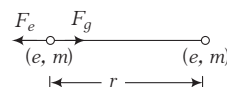
Solving Eqs. (i) and (ii), we get

$$q_1 = 4 \times 10^{-6} \text{ C} = 4 \mu\text{C}, q_2 = 2 \times 10^{-6} \text{ C} = 2 \mu\text{C}$$

Therefore, initial charges on the sphere are $4 \mu\text{C}$ and $2 \mu\text{C}$.

Example 1.8 Two protons are placed at some separation in vacuum. Find the ratio of electric and gravitational force acting between them.

Sol. Let the distance between two protons having charge $+e$ and mass m is placed at a distance r from each other as shown in figure.



$$\text{From Coulomb's law, } F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e \times e}{r^2}$$

$$\text{From gravitation law, } F_g = G \frac{m_1 m_2}{r^2} = G \frac{m \times m}{r^2}$$

where, F_e and F_g are electric and gravitational force respectively. On putting the values, we get

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2}}{\frac{Gm^2}{r^2}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{m^2} \cdot \frac{1}{G}$$

$$= \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(1.67 \times 10^{-27})^2} \times \frac{1}{(6.67 \times 10^{-11})}$$

$$= 1.2 \times 10^{36} \approx 10^{36}$$

Thus, $F_e \gg F_g$

Force between multiple charges (Superposition principle)

According to the principle of superposition, “total force on a given charge due to number of charges is the vector sum of individual forces acting on that charge due to all the charges”. The individual forces are unaffected due to the presence of other charges.

Suppose a system contains n point charges $q_1, q_2, \dots, q_n$. Then, by the principle of superposition, the force on q_1 due to all the other charges is given by

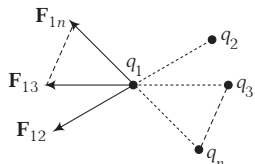
$$\begin{aligned} \mathbf{F}_1 &= \mathbf{F}_{12} + \mathbf{F}_{13} + \dots + \mathbf{F}_{1n} \\ \mathbf{F}_1 &= \frac{q_1}{4\pi\epsilon_0} \left[\frac{q_2 \hat{\mathbf{r}}_{12}}{r_{12}^2} + \frac{q_3 \hat{\mathbf{r}}_{13}}{r_{13}^2} + \dots + \frac{q_n \hat{\mathbf{r}}_{1n}}{r_{1n}^2} \right] \\ &= \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^n \frac{q_i \hat{\mathbf{r}}_{1i}}{r_{1i}^2} \end{aligned}$$


Fig. 1.6 Force between the charges $q_1, q_2, q_3, \dots, q_n$

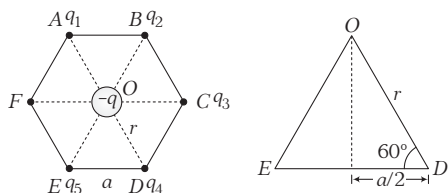
Example 1.9 Equal charges each of $20 \mu\text{C}$ are placed at $x = 0, 2, 4, 8, 16 \text{ cm}$ on X -axis. Find the force experienced by the charge at $x = 2 \text{ cm}$.

Sol. Force on charge at $x = 2 \text{ cm}$ due to charge at $x = 0 \text{ cm}$ and $x = 4 \text{ cm}$ are equal and opposite, so they cancel each other. Net force on charge at $x = 2 \text{ cm}$ is resultant of repulsive forces due to two charges at $x = 8 \text{ cm}$ and $x = 16 \text{ cm}$.

$$\begin{aligned} \therefore F &= \frac{q_1 \times q_2}{4\pi\epsilon_0} \left[\frac{1}{(0.08 - 0.02)^2} + \frac{1}{(0.16 - 0.02)^2} \right] \\ &= 9 \times 10^9 [20 \times 10^{-6}]^2 \left[\frac{1}{(0.06)^2} + \frac{1}{(0.14)^2} \right] = 1.2 \times 10^3 \text{ N} \end{aligned}$$

Example 1.10 Five point charges each of value $+q$ are placed on five vertices of a regular hexagon of side a metre. What is the magnitude of the force on a point charge of value $-q$ coulomb placed at the centre of the hexagon?

Sol. Let the centre of the hexagon be O . When the centre is joined with the vertices of a hexagon, then six triangles are formed. Consider $\triangle ODE$



$$\frac{a/2}{r} = \cos 60^\circ = \frac{1}{2}$$

$$\therefore a = r$$

Given, $q_1 = q_2 = q_3 = q_4 = q_5 = q$

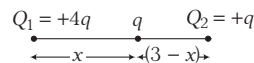
Net force on $-q$ is due to q_3 because forces due to q_1 and q_4 are equal and opposite, so cancel each other. Similarly, forces due to q_2 and q_5 also cancel each other. Hence, the net force on $-q$ is

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{(q)(q)}{r^2} \quad (\text{towards } q_3)$$

$$\text{or} \quad F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2}$$

Example 1.11 Two fixed charges $+4q$ and $+q$ are at a distance 3 m apart. At what point between the charges, a third charge $+q$ must be placed to keep it in equilibrium?

Sol. Remember, if Q_1 and Q_2 are of same nature (means both positive or both negative), then the third charge should be placed between (not necessarily at mid-point) Q_1 and Q_2 on the straight line joining them. But, if Q_1 and Q_2 are of opposite nature, then the third charge will be put outside and close to that charge which is lesser in magnitude.



Here, Q_1 and Q_2 are of same nature that of third charge q , so it will be kept in between at a distance x from Q_1 (as shown in figure). Hence, q will be at a distance $(3 - x)$ from Q_2 . Since, q is in equilibrium, so net force on it must be zero. The forces applied by Q_1 and Q_2 on q are in opposite direction, so as to just balance their magnitudes.

$$\text{Force on } q \text{ by } Q_1 = \frac{kQ_1q}{x^2} \text{ and that by } Q_2 = \frac{kQ_2q}{(3-x)^2}$$

$$\text{Now, } \frac{kQ_1q}{x^2} = \frac{kQ_2q}{(3-x)^2} \text{ or } \frac{Q_1}{x^2} = \frac{Q_2}{(3-x)^2} \text{ or } \frac{4}{x^2} = \frac{1}{(3-x)^2}$$

$$\text{Take the square root, } \frac{2}{x} = \frac{1}{(3-x)}$$

or $6 - 2x = x$ (after cross multiplication) or $x = 2 \text{ m}$. So, q will be placed at a distance 2 m from Q_1 and at 1 m from Q_2 .

Note If $q_1 q_2 > 0$, then

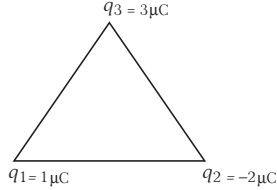
$$x_1 = \frac{r_0}{\sqrt{\frac{q_2}{q_1} + 1}} \quad (x_1 \text{ is distance from } q_1 \text{ between } q_1 \text{ and } q_2)$$

If $q_1 q_2 < 0$, then

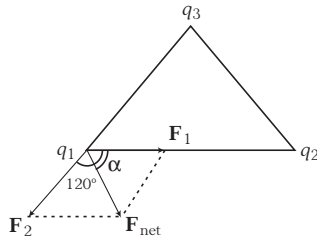
$$x_1 = \frac{r_0}{\sqrt{\frac{q_2}{q_1} - 1}} \quad (x_1 \text{ in this case is not between the charges})$$

Example 1.12 Three charges $q_1 = 1 \mu\text{C}$, $q_2 = -2 \mu\text{C}$ and $q_3 = 3 \mu\text{C}$ are placed on the vertices of an equilateral triangle of side 1.0 m . Find the net electric force acting on charge q_1 .

Sol. Charge q_2 will attract charge q_1 (along the line joining them) and charge q_3 will repel charge q_1 . Therefore, two forces will act on q_1 , one due to q_2 and another due to q_3 . Since, the force is a vector quantity both of these forces (say $\mathbf{F}_1$ and $\mathbf{F}_2$) will be added by vector method. Following are two methods of their addition.



Method I. In the figure,



Magnitude of force between q_1 and q_2 ,

$$|\mathbf{F}_1| = F_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

where, $q_1 = 1 \mu\text{C} = 1 \times 10^{-6} \text{C}$

and $q_2 = 2 \mu\text{C} = 2 \times 10^{-6} \text{C}$.

$$\Rightarrow F_1 = \frac{(9.0 \times 10^9) (1.0 \times 10^{-6}) (2.0 \times 10^{-6})}{(1.0)^2} = 1.8 \times 10^{-2} \text{ N}$$

Similarly, magnitude of force between q_1 and q_3 ,

$$|\mathbf{F}_2| = F_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_3}{r^2}$$

where, $q_3 = 3 \mu\text{C} = 3 \times 10^{-6} \text{C}$.

$$\Rightarrow F_2 = \frac{(9.0 \times 10^9) (1.0 \times 10^{-6}) (3.0 \times 10^{-6})}{(1.0)^2} = 2.7 \times 10^{-2} \text{ N}$$

Now, net force, $|\mathbf{F}_{\text{net}}| = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 120^\circ}$

$$= \left(\sqrt{(1.8)^2 + (2.7)^2 + 2(1.8)(2.7)\left(-\frac{1}{2}\right)} \right) \times 10^{-2} \text{ N}$$

$$= 2.38 \times 10^{-2} \text{ N}$$

$$\text{and } \tan \alpha = \frac{F_2 \sin 120^\circ}{F_1 + F_2 \cos 120^\circ}$$

$$= \frac{(2.7 \times 10^{-2}) (0.87)}{(1.8 \times 10^{-2}) + (2.7 \times 10^{-2}) \left(-\frac{1}{2}\right)}$$

or $\alpha = 79.2^\circ$

Thus, the net electric force on charge q_1 is $2.38 \times 10^{-2} \text{ N}$ at an angle $\alpha = 79.2^\circ$ with a line joining q_1 and q_2 as shown in the figure.

Method II. In this method, let us assume coordinate axes with q_1 at origin as shown in figure. The coordinates of q_1 , q_2 and q_3 in this coordinate system are $(0, 0, 0)$, $(1 \text{ m}, 0, 0)$ and $(0.5 \text{ m}, 0.87 \text{ m}, 0)$, respectively. Now,

$\mathbf{F}_1$ = Force on q_1 due to charge q_2

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3} (\mathbf{r}_1 - \mathbf{r}_2)$$

$$= \frac{(9.0 \times 10^9) (1.0 \times 10^{-6}) (-2.0 \times 10^{-6})}{(1.0)^3} \times [(0 - 1)\hat{i} + (0 - 0)\hat{j} + (0 - 0)\hat{k}]$$

$$= (1.8 \times 10^{-2} \hat{i}) \text{ N}$$

and $\mathbf{F}_2$ = Force on q_1 due to charge q_3

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_3}{|\mathbf{r}_1 - \mathbf{r}_3|^3} (\mathbf{r}_1 - \mathbf{r}_3)$$

$$= \frac{(9.0 \times 10^9) (1.0 \times 10^{-6}) (3.0 \times 10^{-6})}{(1.0)^3} \times [(0 - 0.5)\hat{i} + (0 - 0.87)\hat{j} + (0 - 0)\hat{k}]$$

$$= (-1.35 \hat{i} - 2.349 \hat{j}) \times 10^{-2} \text{ N}$$

Therefore, net force on q_1 is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (0.45 \hat{i} - 2.349 \hat{j}) \times 10^{-2} \text{ N}$$

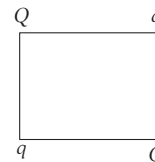
$$|\mathbf{F}| = \sqrt{(0.45)^2 + (2.349)^2} \times 10^{-2} \text{ N} = 2.39 \times 10^{-2} \text{ N}$$

If the net force makes an angle α from the direction of X-axis, then

$$\alpha = \tan^{-1} \left(\frac{-2.349}{0.45} \right) = -79.2^\circ$$

Negative sign of α indicate that the net force is directed below the X-axis.

Example 1.13 Four charges Q , q , Q and q are kept at the four corners of a square as shown below. What is the relation between Q and q , so that the net force on a charge q is zero?

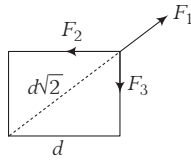


Sol. Here, both the charges q will have same sign either positive or negative. Similarly, both the charges Q will have same sign. Let us make the force on upper right corner q equal to zero.

Lower q will apply a repelling force $\mathbf{F}_1$ on upper q because both the charges have same sign. To balance this force both Q must apply attractive forces $\mathbf{F}_2$ and $\mathbf{F}_3$ of equal magnitude, hence Q and q will have opposite signs. Now, the resultant of $\mathbf{F}_2$ and $\mathbf{F}_3$ will be $F\sqrt{2}$ (Parallelogram law of vector addition), if $|\mathbf{F}_2| = |\mathbf{F}_3| = F$. Also note that $F\sqrt{2}$ will be exactly opposite to $\mathbf{F}_1$.

So, $F_1 = F\sqrt{2}$

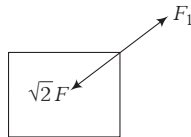
From Coulomb's law,



$$F_1 = \frac{kq^2}{(d\sqrt{2})^2} \text{ and } F = \frac{kQq}{d^2}$$

$$\therefore F_1 = \sqrt{2}F$$

$$\therefore \frac{q^2}{(d\sqrt{2})^2} = \frac{\sqrt{2}Qq}{d^2}$$



$$\therefore Q = \frac{q}{2\sqrt{2}}$$

But as we said Q and q have opposite sign, so $q = -2\sqrt{2}Q$.

Applications of electric force (Coulomb's law)

1. For solving problems related to string

Let us consider two identical balls, having mass m and charge q . These are suspended from a common point by two insulating strings each of length L as shown in figure.

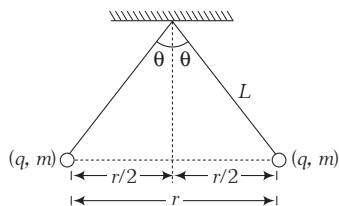


Fig. 1.7 Two identical charged balls are suspended from same point

The balls repel and come into equilibrium at separation r . The ball is in equilibrium under the following forces

(i) Weight of the ball, $w = mg$ and, (ii) Electric force,

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q \cdot q}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2} \quad \dots(i)$$

(iii) **Tension in the string** For this, let us draw FBD of a ball under consideration.

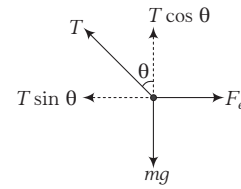


Fig. 1.8 FBD of one of the balls

It is clear from FBD that,

$$T \sin \theta = F_e \quad \dots(ii)$$

$$T \cos \theta = mg \quad \dots(iii)$$

On dividing Eq. (ii) by Eq. (iii), we get

$$\Rightarrow \tan \theta = \frac{F_e}{mg} \quad \dots(iv)$$

$$\text{or } T = \sqrt{(T \sin \theta)^2 + (T \cos \theta)^2} = \sqrt{F_e^2 + (mg)^2} \quad \dots(v)$$

From geometry of the figure,

$$\tan \theta = \frac{r/2}{\sqrt{L^2 - \frac{r^2}{4}}}$$

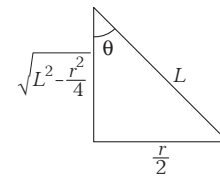


Fig. 1.9 Formation of a triangle through $r/2$ and L

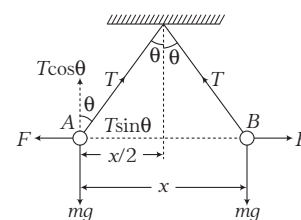
$$\text{If } \theta \text{ is small, } \tan \theta \approx \sin \theta = \frac{r/2}{L}.$$

Now, let us solve some problems related to this to make our concepts more clear.

Example 1.14 Two identical pith balls, each of mass m and charge q are suspended from a common point by two strings of equal length l . Find charge q in terms of given quantities, if angle between the strings is 2θ in equilibrium.

Sol. Let x be the separation between balls in equilibrium.

According to the question, following figure can be drawn



Each ball is in equilibrium under the action of the following forces

(a) Weight of the ball, mg

(b) Repulsive electric force, $F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{x^2}$

(c) Tension in the string, T

Resolving T in the horizontal and vertical direction, since ball is in equilibrium,

$$T \sin \theta = F \quad \dots(i)$$

$$T \cos \theta = mg \quad \dots(ii)$$

By Eqs. (i) and (ii), we get

$$\tan \theta = \frac{F}{mg} = \frac{\frac{1}{4\pi\epsilon_0} \frac{q^2}{x^2}}{mg} \quad \dots(iii)$$

Form figure, $\frac{x}{2} = l \sin \theta \Rightarrow x = 2l \sin \theta$

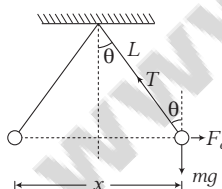
Put value of x in Eq. (iii), we get

$$\begin{aligned} q^2 &= 4\pi\epsilon_0 mgx^2 \tan \theta \\ &= 4\pi\epsilon_0 mg(2l \sin \theta)^2 \tan \theta \\ q &= [16\pi\epsilon_0 mgl^2 \sin^2 \theta \tan \theta]^{1/2} \end{aligned}$$

Example 1.15 Two identical balls, each having a charge q and mass m , are suspended from a common point by two insulating strings each of length L . The balls are held at a separation x and then released. Find

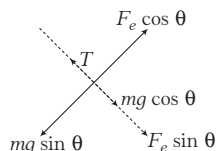
- the electric force on each ball.
- the component of the resultant force on a ball along and perpendicular to string.
- the tension in the string.
- the acceleration of one of the balls. Consider the situation only for the instant just after the release.

Sol. When the separation between the balls is x in equilibrium condition, then according to question, the following figure can be drawn



(i) Electric force between balls, $F_e = \frac{1}{4\pi\epsilon_0} \frac{q^2}{x^2}$.

(ii) Resolving forces along and perpendicular to string.



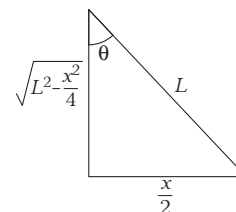
Resultant force on ball along the string,

$$T - (mg \cos \theta + F_e \sin \theta) = 0 \quad [\text{the string is unstretchable}]$$

Similarly, force perpendicular to the string

$$= |F_e \cos \theta - mg \sin \theta|$$

θ can be obtained from geometry.



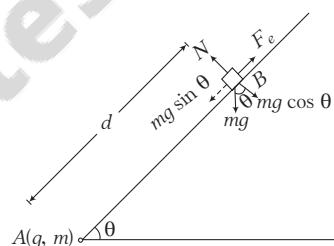
(iii) Since, the net force on the ball along string is zero, hence

$$T = mg \cos \theta + F_e \sin \theta$$

(iv) Acceleration of ball, $a = \frac{|F_e \cos \theta - mg \sin \theta|}{m}$

Example 1.16 A particle A having charge q and mass m is placed at the bottom of a smooth inclined plane of inclination θ . Where should a block B, having same charge and mass, be placed on the incline plane, so that it may remain in equilibrium?

Sol. The following figure can be drawn in accordance with the question



Let block B be placed at distance d . The block B is in equilibrium. So, $mg \cos \theta$ is balanced by normal reaction and $mg \sin \theta$ by repulsive electric force, i.e.

$$F_e = mg \sin \theta$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2} = mg \sin \theta$$

$$\Rightarrow d = q \sqrt{\frac{1}{4\pi\epsilon_0 mg \sin \theta}}$$

2. Lami's theorem

In few problems of electrostatics, Lami's theorem is very useful. According to this theorem, if three concurrent forces F_1 , F_2 and F_3 as shown in figure are in equilibrium or if $F_1 + F_2 + F_3 = 0$, then

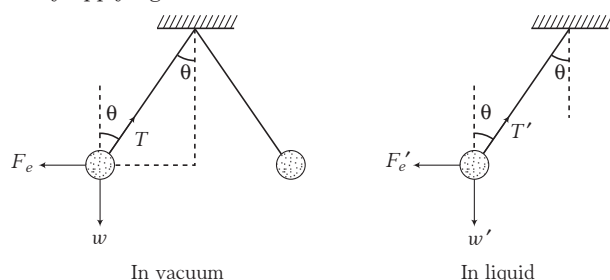
Fig. 1.10 Three forces passing through a point

$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

Example 1.17 Two identical balls each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of the liquid is σ . Find the dielectric constant of the liquid.

Sol. Each ball is in equilibrium under the following three forces
(i) tension, (ii) electric force and
(iii) weight.

So, by applying Lami's theorem,



$$\text{In the liquid, } F'_e = \frac{F_e}{K}$$

where, K = dielectric constant of liquid

and $w' = w - \text{upthrust}$

Applying Lami's theorem in vacuum,

$$\frac{w}{\sin(90^\circ + \theta)} = \frac{F_e}{\sin(180^\circ - \theta)} \quad \text{or} \quad \frac{w}{\cos \theta} = \frac{F_e}{\sin \theta} \quad \dots(i)$$

$$\text{Similarly in liquid, } \frac{w'}{\cos \theta} = \frac{F'_e}{\sin \theta} \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\begin{aligned} \frac{w}{w'} &= \frac{F_e}{F'_e} \quad \text{or} \quad K = \frac{w}{w - \text{upthrust}} \quad \left(\text{as } \frac{F_e}{F'_e} = K \right) \\ &= \frac{V\rho g}{V\rho g - V\sigma g} \quad (\because V = \text{volume of ball}) \end{aligned}$$

$$\text{or} \quad K = \frac{\rho}{\rho - \sigma}$$

CHECK POINT 1.2

- A metallic sphere having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the sphere will be
(a) towards the plate (b) away from the plate
(c) parallel to the plate (d) zero
- Two charges each equal to $2\mu\text{C}$ are 0.5 m apart. If both of them exist inside vacuum, then the force between them is
(a) 1.89 N (b) 2.44 N (c) 0.144 N (d) 3.144 N
- There are two charges $+1\mu\text{C}$ and $+5\mu\text{C}$. The ratio of the forces acting on them will be
(a) 1 : 5 (b) 1 : 1 (c) 5 : 1 (d) 1 : 25
- The force between two charges 0.06 m apart is 5 N. If each charge is moved towards the other, so that new distance becomes 0.04 m, then the force between them will become
(a) 7.20 N (b) 11.25 N (c) 22.50 N (d) 45.00 N
- The charges on two spheres are $+7\mu\text{C}$ and $-5\mu\text{C}$, respectively. They experience a force F . If each of them is given an additional charge of $-2\mu\text{C}$, then the new force attraction will be
(a) F (b) $F/2$ (c) $F/\sqrt{3}$ (d) $2F$
- Two charges of equal magnitudes and at a distance r exert a force F on each other. If the charges are halved and distance between them is doubled, then the new force acting on each charge is
(a) $F/8$ (b) $F/4$ (c) $4F$ (d) $F/16$
- Two charges placed in air repel each other by a force of 10^{-4}N . When oil is introduced between the charges, then the force becomes $2.5 \times 10^{-5}\text{N}$. The dielectric constant of oil is
(a) 2.5 (b) 0.25 (c) 2.0 (d) 4.0
- Two point charges placed at a certain distance r in air exert a force F on each other. Then, the distance r' at which these charges will exert the same force in a medium of dielectric constant K is given by
(a) r (b) r/K
(c) $r/\sqrt{K}$ (d) $r\sqrt{K}$
- F_g and F_e represent gravitational and electrostatic force respectively between electrons situated at a distance of 10 cm. The ratio of F_g/F_e is of the order of
(a) 10^{42} (b) 10^{-21}
(c) 10^{24} (d) 10^{-43}
- Two particles of equal mass m and charge q are placed at a distance of 16 cm. They do not experience any force. The value of $\frac{q}{m}$ is
(a) l (b) $\sqrt{\frac{\pi\epsilon_0}{G}}$ (c) $\sqrt{\frac{G}{4\pi\epsilon_0}}$ (d) $\sqrt{4\pi\epsilon_0 G}$
- A charge q_1 exerts some force on a second charge q_2 . If a third charge q_3 is brought near q_2 , then the force exerted by q_1 on q_2
(a) decreases
(b) increases
(c) remains the same
(d) increases, if q_3 is of same sign as q_1 and decreases, if q_3 is of opposite sign as q_1
- Electric charges of $1\mu\text{C}$, $-1\mu\text{C}$ and $2\mu\text{C}$ are placed in air at the corners A , B and C respectively of an equilateral triangle ABC having length of each side 10 cm. The resultant force on the charge at C is
(a) 0.9 N (b) 1.8 N
(c) 2.7 N (d) 3.6 N

ELECTRIC FIELD

The region surrounding a charge or distribution of charge in which its electrical effects can be observed or experienced is called the **electric field of the charge or distribution of charge**.

Electric field at a point can be defined in terms of either a vector function $\mathbf{E}$ called **electric field strength** or a scalar function V called **electric potential**.

Electric field strength

The electric field strength (often called electric field) at a point is defined as the electrostatic force $\mathbf{F}_e$ per unit positive test charge. Thus, if the electrostatic force experienced by a small test charge q_0 is $\mathbf{F}_e$, then field strength at that point is defined as

$$\mathbf{E} = \lim_{q_0 \rightarrow 0} \frac{\mathbf{F}_e}{q_0}$$

The electric field is a vector quantity and its direction is the same as the direction of the electrostatic force $\mathbf{F}_e$ on a positive test charge.

The SI unit of electric field is N/C. The dimensions for $\mathbf{E}$ is $[\text{MLT}^{-3}\text{A}^{-1}]$. For a positive charge, the electric field will be directed radially outwards from the charge. On the other hand, if the source charge is negative, the electric field vector at each point is directed radially inwards.

Note Suppose there is an electric field strength $\mathbf{E}$ at some point, then the electrostatic force acting on a charge $+q$ is $q\mathbf{E}$ in the direction of $\mathbf{E}$, while on the charge $-q$ it is $q\mathbf{E}$ in the opposite direction of $\mathbf{E}$.

Example 1.18 An electric field of 10^5 N/C points due west at a certain spot. What are the magnitude and direction of the force that acts on a charge of $+2 \mu\text{C}$ and $-5 \mu\text{C}$ at this spot?

Sol. Electrostatic force, $F = qE$

where, q = charge

and E = electric field.

Here, $q_1 = +2 \mu\text{C} = 2 \times 10^{-6} \text{ C}$

and $q_2 = -5 \mu\text{C} = -5 \times 10^{-6} \text{ C}$

$\therefore$ Force on charge q_1 , $F_1 = (2 \times 10^{-6})(10^5) = 0.2 \text{ N}$ (due west)

and force on charge q_2 , F_2
 $= (5 \times 10^{-6})(10^5) = 0.5 \text{ N}$ (due east)

Example 1.19 Calculate the magnitude of an electric field which can just suspend a deuteron of mass $3.2 \times 10^{-27} \text{ kg}$ freely in air.

Sol. Upward force (qE) on the deuteron due to electric field E is equal to weight mg of deuteron, $qE = mg$

$$\therefore E = \frac{mg}{q} = \frac{3.2 \times 10^{-27} \times 9.8}{1.6 \times 10^{-19}} = 19.6 \times 10^{-8} \text{ NC}^{-1}$$

Electric field due to a point charge

The electric field produced by a point charge q can be obtained in general terms from Coulomb's law. The magnitude of the force exerted by the charge q on a test charge q_0 is

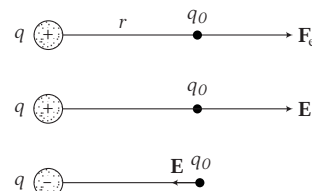


Fig. 1.11 Direction of electric field due to positive and negative charges

$$F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{qq_0}{r^2}$$

Therefore, the intensity of the electric field at this point is

$$\text{given by } E = \frac{F_e}{q_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Note Suppose a charge q is placed at a point whose position vector is $\mathbf{r}_q$ and to obtain the electric field at a point P whose position vector is $\mathbf{r}_p$, then in vector form the electric field is given by

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{|\mathbf{r}_p - \mathbf{r}_q|^3} (\mathbf{r}_p - \mathbf{r}_q)$$

Here,

$$\mathbf{r}_p = x_p \hat{i} + y_p \hat{j} + z_p \hat{k}$$

and

$$\mathbf{r}_q = x_q \hat{i} + y_q \hat{j} + z_q \hat{k}$$

Example 1.20 Find the electric field strength due to a point charge of $5 \mu\text{C}$ at a distance of 80 cm from the charge.

Sol. Given, $q = 5 \mu\text{C} = 5 \times 10^{-6} \text{ C}$

$$r = 80 \text{ cm} = 80 \times 10^{-2} \text{ m}$$

Electric field strength,

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

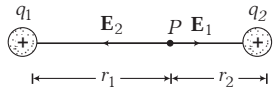
$$\Rightarrow E = 9 \times 10^9 \times \frac{5 \times 10^{-6}}{(80 \times 10^{-2})^2}$$

$$\Rightarrow E = 7.0 \times 10^4 \text{ N/C}$$

Example 1.21 Two point charges $q_1 = 16 \mu\text{C}$ and $q_2 = 4 \mu\text{C}$, are separated in vacuum by a distance of 3.0 m . Find the point on the line between the charges, where the net electric field is zero.

Sol. Between the charges, the two field contributions have opposite directions and the net electric field is zero at a point (say P), where the magnitudes of $\mathbf{E}_1$ and $\mathbf{E}_2$ are equal. However, since, $q_2 < q_1$, point P must be closer to q_2 , in order

that the field of the smaller charge can balance the field of the greater charge.



At P, $E_1 = E_2$ or $\frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{r_2^2}$

$$\therefore \frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}} = \sqrt{\frac{16}{4}} = 2 \quad \dots(i)$$

Also, $r_1 + r_2 = 3.0 \text{ m} \quad \dots(ii)$

Solving these equations, we get

$$r_1 = 2 \text{ m} \quad \text{and} \quad r_2 = 1 \text{ m}$$

Thus, the point P is at a distance of 2 m from q_1 and 1 m from q_2 .

Example 1.22 A charge $q = 1 \mu\text{C}$ is placed at point (1 m, 2 m, 4 m). Find the electric field at point P (0 m, -4 m, 3 m).

Sol. Here, $\mathbf{r}_q = \hat{i} + 2\hat{j} + 4\hat{k}$

and $\mathbf{r}_p = -4\hat{j} + 3\hat{k}$

$$\therefore \mathbf{r}_p - \mathbf{r}_q = -\hat{i} - 6\hat{j} - \hat{k}$$

or $|\mathbf{r}_p - \mathbf{r}_q| = \sqrt{(-1)^2 + (-6)^2 + (-1)^2} = \sqrt{38} \text{ m}$

Now, electric field, $\mathbf{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{|\mathbf{r}_p - \mathbf{r}_q|^3} (\mathbf{r}_p - \mathbf{r}_q)$

Substituting the values, we get

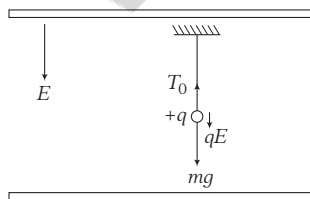
$$\mathbf{E} = \frac{(9.0 \times 10^9)(1.0 \times 10^{-6})}{(38)^{3/2}} (-\hat{i} - 6\hat{j} - \hat{k})$$

$$\mathbf{E} = (-38.42 \hat{i} - 230.52 \hat{j} - 38.42 \hat{k}) \text{ N/C}$$

Example 1.23 A ball having charge q and mass m is suspended from a string of length L between two parallel plates, where a vertical electric field E is established. Find the time period of simple pendulum, if electric field is directed (i) downward and (ii) upward.

Sol. (i) When electric field is downward

For simple pendulum,



$$\text{Time period, } T = 2\pi\sqrt{\frac{L}{a}}$$

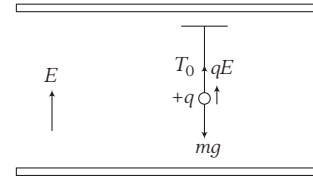
where, a is the effective acceleration.

$$a = \frac{\text{net external force on the ball}}{\text{mass of the ball}}$$

$$\Rightarrow a = \frac{mg + qE}{m} = g + \frac{qE}{m}$$

$$\therefore \text{Time period, } T = 2\pi\sqrt{\frac{L}{g + \frac{qE}{m}}}$$

(ii) When electric field is upward



$$\text{Effective acceleration, } a = \frac{mg - qE}{m} = g - \frac{qE}{m}$$

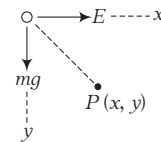
$$\therefore \text{Time period, } T = 2\pi\sqrt{\frac{L}{g - \frac{qE}{m}}}$$

Example 1.24 A ball of mass m having a charge q is released from rest in a region where a horizontal electric field E exists.

(i) Find the resultant force acting on the ball.

(ii) Find the trajectory followed by the ball.

Sol. The forces acting on the ball are weight of the ball in vertically downward direction and the electric force in the horizontal direction.



(i) Resultant force, $F = \sqrt{(mg)^2 + (qE)^2}$

(ii) Let the ball be at point P after time t .

As, $F = ma = qE$

$$\therefore a = \frac{qE}{m}$$

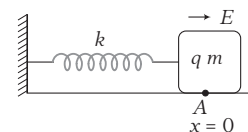
$$\left. \begin{aligned} \text{x-direction, } x &= \frac{1}{2} \frac{qE}{m} t^2 \\ \text{y-direction, } y &= \frac{1}{2} gt^2 \end{aligned} \right\}$$

$$\frac{y}{x} = \frac{g}{qE/m}$$

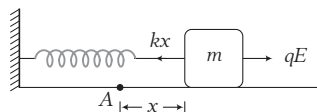
$$\Rightarrow y = \frac{mgx}{qE}$$

Hence, trajectory is a straight line.

Example 1.25 A block of mass m having charge q is attached to a spring of spring constant k . This arrangement is placed in uniform electric field E on smooth horizontal surface as shown in the figure. Initially, spring is unstretched. Find the extension of spring in equilibrium position and maximum extension of spring.



Sol. Method I. Force due to electric field E acting on the charged block results in extension of spring. Let at some instant, extension in spring be x .



Net force on block, $F_{\text{net}} = qE - kx$

In equilibrium, $F_{\text{net}} = qE - kx = 0 \Rightarrow x = x_0 = \frac{qE}{k}$

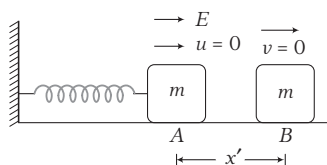
where, k is the spring constant.

In equilibrium, extension of spring, $x_0 = \frac{qE}{k}$

Maximum extension $= 2x_0 = \frac{2qE}{k}$

Method II.

Let the body is displaced from position A to position B. Let the maximum extension produced be x' , then from conservation of energy,



$$\frac{1}{2} kx'^2 = qEx' \Rightarrow x' = \frac{2qE}{k}$$

In fact, the block is executing SHM of time period,

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ and amplitude of oscillation} = \frac{qE}{k}.$$

Electric field due to a system of charges

Consider a system of charges $q_1, q_2, \dots, q_n$ with position vectors $\mathbf{r}_{p1}, \mathbf{r}_{p2}, \mathbf{r}_{p3}, \dots, \mathbf{r}_{pn}$ relative to some origin P as shown in the figure.

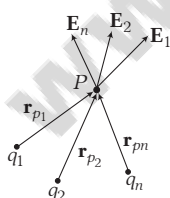


Fig. 1.12 Electric field due to system of charges $q_1, q_2, \dots, q_n$

By the principle of superposition, the electric field $\mathbf{E}$ at point P due to system of charges will be given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \dots + \mathbf{E}_n$$

where, $\mathbf{E}_1, \mathbf{E}_2, \dots, \mathbf{E}_n$ be the electric field at P due to charges $q_1, q_2, \dots, q_n$, respectively.

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{p1}^2} \hat{\mathbf{r}}_{p1} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{p2}^2} \hat{\mathbf{r}}_{p2} + \dots + \frac{1}{4\pi\epsilon_0} \frac{q_n}{r_{pn}^2} \hat{\mathbf{r}}_{pn}$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_{pi}^2} \hat{\mathbf{r}}_{pi}$$

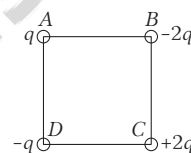
Example 1.26 An infinite number of charges each equal to q are placed along X -axis at $x = 1, x = 2, x = 4, x = 8$ and so on. Find the electric field at the point $x = 0$ due to this set up of charges.

Sol. At the point $x = 0$, the electric field due to all the charges are in the same negative x -direction and hence get added up.

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \frac{q}{8^2} + \dots \right] \\ &= \frac{q}{4\pi\epsilon_0} \left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \right] \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{1 - 1/4} \right) = \frac{q}{3\pi\epsilon_0} \end{aligned}$$

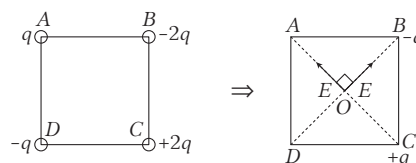
This electric field is along negative X -axis.

Example 1.27 Four charges are placed at the corners of a square of side 10 cm as shown in figure. If q is $1 \mu\text{C}$, then what will be the electric field intensity at the centre of the square?



Sol. Side of square, $a = 0.1 \text{ m}$ and magnitude of charge, $q = 1 \times 10^{-6} \text{ C}$

Half of diagonal of the square $= \frac{a}{\sqrt{2}} = \frac{0.1}{\sqrt{2}} \text{ m}$



Electric field due to charge q ,

$$\begin{aligned} E &= \frac{9 \times 10^9 \times 1 \times 10^{-6}}{\left(\frac{0.1}{\sqrt{2}} \right)^2} \\ &= \frac{9 \times 10^3 \times 2}{0.01} = 18 \times 10^5 \text{ NC}^{-1} \end{aligned}$$

At centre there are two electric field which are perpendicular to each other, so net electric field can be calculated using superposition principle.

$$\begin{aligned} \therefore E_{\text{net}} &= \sqrt{E^2 + E^2} = E\sqrt{2} \\ &= 18 \times 10^5 \times \sqrt{2} = 2.54 \times 10^6 \text{ NC}^{-1} \end{aligned}$$

Electric field lines

An electric field line is an imaginary line or curve drawn through a region of space, so that its tangent at any point is in the direction of the electric field vector at that point.

Electric field lines were introduced by Michael Faraday to visualise electric field.

The electric field lines have the following properties

- (i) The tangent to a field line at any point gives the direction of $\mathbf{E}$ at that point.

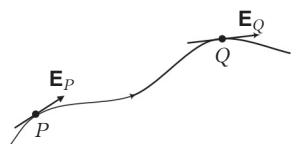


Fig. 1.13 Electric field at points P and Q

In the given figure, electric points P and Q are along the tangents (E_P and E_Q). This is also the path on which a positive test charge will tend to move, if free to do so.

- (ii) Electric field lines always begin from a positive charge and end on a negative charge and do not start or stop in mid-space.
- (iii) The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge. *e.g.* if 100 lines are drawn leaving a $+4\ \mu\text{C}$ charge, then 75 lines would have to end on a $-3\ \mu\text{C}$ charge.

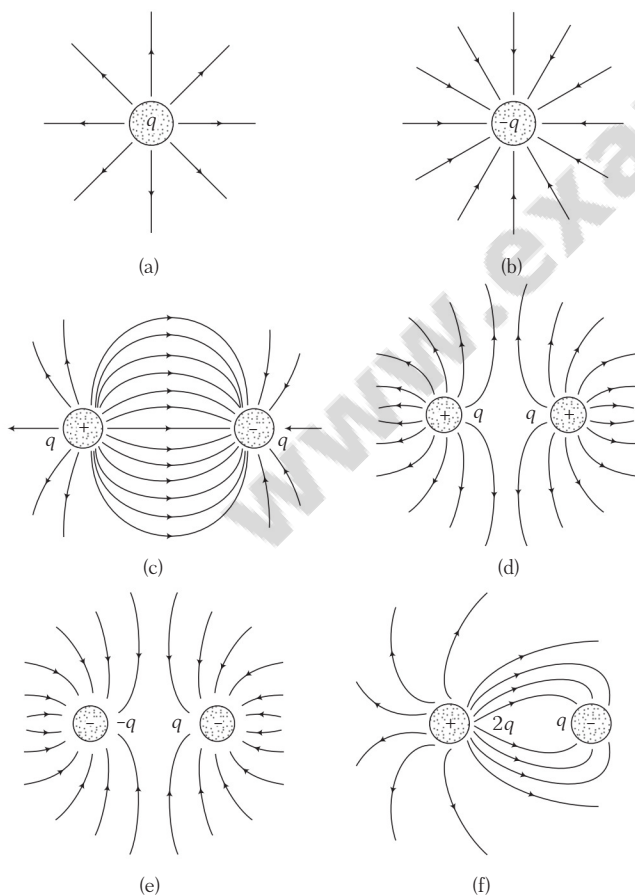


Fig. 1.14 Electric field lines associated with a single as well as combination of charges

- (iv) Two lines can never intersect. If it happens, then two tangents can be drawn at their point of intersection, *i.e.* intensity at that point will have two directions which is not possible.
- (v) In a uniform field, the field lines are straight, parallel and uniformly spaced.
- (vi) The electric field lines can never form closed loops as a field line can never start and end on the same charge.
- (vii) In a region, where there is no electric field, electric field lines are absent. This is why inside a conductor (where, electric field is zero), no electric field lines exist.
- (viii) Electric field lines of force ends or starts normally from the surface of a conductor.
- (ix) The relative closeness of the electric field lines of force in different regions of space indicates the relative strength of the electric field in different regions. In regions, where electric field lines of force are closer, the electric field is stronger, whereas in regions, where line of force are further apart, the field is weaker.

Therefore, in the given figure $|E_A| > |E_B|$.

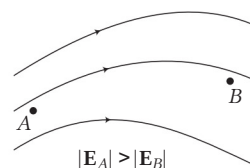


Fig. 1.15 Electric field strength at points A and B

Continuous charge distribution

In most of the cases, we deal with charges having magnitude greater than the charge of an electron. For this, we can imagine that the charge is spread in a region in a continuous manner. Such a charge distribution is known as continuous charge distribution.

Consider a point charge q_0 lying near a region of continuous charge distribution which is made up of large number of small charges dq as shown in figure.

According to Coulomb's law, the force on a point charge q_0 due to small charge dq is

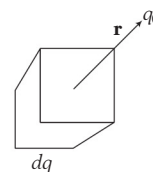


Fig. 1.16 Force on a point charge q_0 due to a continuous charge distribution

$$\mathbf{F} = \frac{q_0}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{\mathbf{r}} \quad \dots(i)$$

where, $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$

There are three types of continuous charge distribution

1. Line charge distribution (λ)

It is a charge distribution along a one-dimensional curve or line, L in space as shown in figure.

Fig. 1.17 Line charge distribution

The charge contained per unit length of the line at any point is called linear charge density. It is denoted by λ .

i.e.
$$\lambda = \frac{dq}{dL}$$

Its SI unit is Cm^{-1} .

Electric field due to the line charge distribution at the location of charge q_0 is

$$\mathbf{E}_L = \frac{1}{4\pi\epsilon_0} \int_L \frac{\lambda}{r^2} dL \hat{\mathbf{r}}$$

2. Surface charge distribution (σ)

It is a charge distribution spread over a two-dimensional surface S in space as shown in figure.

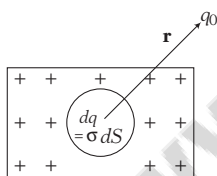


Fig. 1.18 Surface charge distribution

The charge contained per unit area at any point is called surface charge density. It is denoted by σ .

i.e.
$$\sigma = \frac{dq}{dS}$$

Its SI unit is Cm^{-2} .

Electric field due to the surface charge distribution at the location of charge q_0 is

$$\mathbf{E}_S = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma}{r^2} dS \hat{\mathbf{r}}$$

3. Volume charge distribution (ρ)

It is a charge distribution spread over a three-dimensional volume or region V of space as shown in figure.

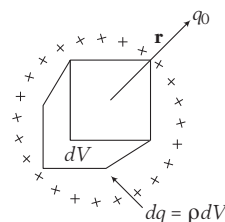


Fig. 1.19 Volume charge distribution

The charge contained per unit volume at any point is called volume charge density. It is denoted by ρ .

i.e.
$$\rho = \frac{dq}{dV}$$

Its SI unit is Coulomb per cubic metre (Cm^{-3}).

Electric field due to the volume charge distribution at the location of charge q_0 is

$$\mathbf{E}_V = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho}{r^2} dV \hat{\mathbf{r}}$$

Example 1.28 What charge would be required to electrify a sphere of radius 25 cm, so as to get a surface charge density of $\frac{3}{\pi} \text{Cm}^{-2}$?

Sol. Given, $r = 25 \text{ cm} = 0.25 \text{ m}$ and $\sigma = \frac{3}{\pi} \text{Cm}^{-2}$

As,
$$\sigma = \frac{q}{4\pi r^2}$$

$$\therefore q = 4\pi r^2 \sigma = 4\pi \times (0.25)^2 \times \frac{3}{\pi} = 0.75 \text{ C}$$

Example 1.29 Sixty four drops of radius 0.02 m and each carrying a charge of $5\mu\text{C}$ are combined to form a bigger drop. Find how the surface charge density of electrification will change, if no charge is lost?

Sol. Volume of each small drop = $\frac{4}{3}\pi(0.02)^3 \text{ m}^3$

Volume of 64 small drops = $\frac{4}{3}\pi(0.02)^3 \times 64 \text{ m}^3$

Let R be the radius of the bigger drop formed, then

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.02)^3 \times 64$$

or
$$R^3 = (0.02)^3 \times 4^3$$

$$\therefore R = 0.02 \times 4 = 0.08 \text{ m}$$

Charge on small drop = $5\mu\text{C} = 5 \times 10^{-6} \text{ C}$

Surface charge density of small drop,

$$\sigma_1 = \frac{q}{4\pi r^2} = \frac{5 \times 10^{-6}}{4\pi(0.02)^2} \text{ Cm}^{-2}$$

Surface charge density of bigger drop,

$$\sigma_2 = \frac{5 \times 10^{-6} \times 64}{4\pi(0.08)^2} \text{ Cm}^{-2}$$

$$\therefore \frac{\sigma_1}{\sigma_2} = \frac{5 \times 10^{-6}}{4\pi(0.02)^2} \times \frac{4\pi(0.08)^2}{5 \times 10^{-6} \times 64} = \frac{1}{4} = 1:4$$

Electric field of a charged ring

Electric field at distance x from the centre of uniformly charged ring of total charge q on its axis is given by

$$E_x = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{qx}{(x^2 + R^2)^{3/2}}$$

Direction of this electric field is along the axis and away from the ring in case of positively charged ring and towards the ring in case of negatively charged ring.

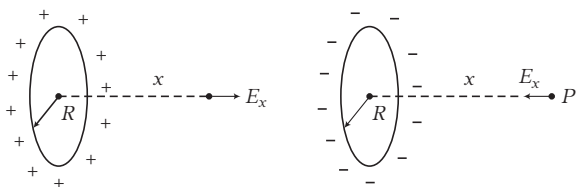


Fig. 1.20 Electric field of positively and negatively charged rings

Special cases

From the above expression, we can see that

- $E_x = 0$ at $x = 0$, i.e. field is zero at the centre of the ring. This would occur because charges on opposite sides of the ring would push a test charge at the centre, in the opposite directions with equal effort and so the forces would add to zero.
- $E_x = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2}$ for $x \gg R$, i.e. when the point P is much farther from the ring, its field is the same as

that of a point charge. To an observer far from the ring, the ring would appear like a point and the electric field reflects this.

- E_x will be maximum, where $\frac{dE_x}{dx} = 0$. Differentiating E_x w.r.t. x and putting it equal to zero, we get $x = \pm \frac{R}{\sqrt{2}}$ and $E_{\max}$ comes out to be, $\frac{2}{3\sqrt{3}} \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \right)$.

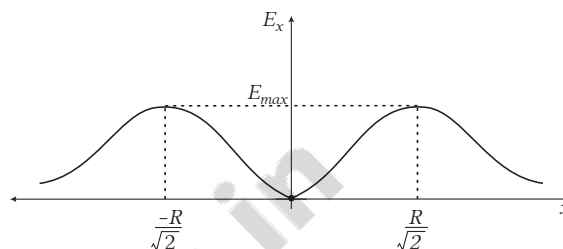


Fig. 1.21 Variation of electric field on the axis of a ring

Example 1.30 A charge of $4 \times 10^{-9} \text{ C}$ is distributed uniformly over the circumference of a conducting ring of radius 0.3 m . Calculate the field intensity at a point on the axis of the ring at 0.4 m from its centre and also at the centre.

Sol. Given, $q = 4 \times 10^{-9} \text{ C}$, $x = 0.4 \text{ m}$ and $R = 0.3 \text{ m}$

Electric field intensity at 0.4 m from its centre,

$$E = \frac{qx}{4\pi\epsilon_0(R^2 + x^2)^{3/2}}$$

$$= \frac{9 \times 10^9 \times 4 \times 10^{-9} \times 0.4}{(0.3^2 + 0.4^2)^{3/2}}$$

$$E = \frac{14.4}{(0.5)^3} = 115.2 \text{ N/C}$$

At the centre of the ring, $x = 0$

$\therefore$ Electric field intensity, $E = 0$

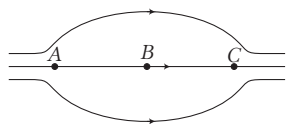
CHECK POINT 1.3

- A charged particle of mass $5 \times 10^{-5} \text{ kg}$ is held stationary in space by placing it in an electric field of strength 10^7 NC^{-1} directed vertically downwards. The charge on the particle is
 - $-20 \times 10^{-5} \mu\text{C}$
 - $-5 \times 10^{-5} \mu\text{C}$
 - $5 \times 10^{-5} \mu\text{C}$
 - $20 \times 10^{-5} \mu\text{C}$
- Electric field strength due to a point charge of $5 \mu\text{C}$ at a distance 80 cm from the charge is
 - $8 \times 10^4 \text{ NC}^{-1}$
 - $7 \times 10^4 \text{ NC}^{-1}$
 - $5 \times 10^4 \text{ NC}^{-1}$
 - $4 \times 10^4 \text{ NC}^{-1}$
- The electric field due to a charge at a distance of 3 m from it, is 500 NC^{-1} . The magnitude of the charge is

$\left[\text{Take, } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2 \right]$

 - $2.5 \mu\text{C}$
 - $2.0 \mu\text{C}$
 - $1.0 \mu\text{C}$
 - $0.5 \mu\text{C}$
- Two charges $+5 \mu\text{C}$ and $+10 \mu\text{C}$ are placed 20 cm apart. The net electric field at the mid-point between the two charges is
 - $4.5 \times 10^6 \text{ NC}^{-1}$ directed towards $+5 \mu\text{C}$
 - $4.5 \times 10^6 \text{ NC}^{-1}$ directed towards $+10 \mu\text{C}$
 - $13.5 \times 10^6 \text{ NC}^{-1}$ directed towards $+5 \mu\text{C}$
 - $13.5 \times 10^6 \text{ NC}^{-1}$ directed towards $+10 \mu\text{C}$
- Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$, respectively. The location of a point on the X -axis at which the net electric field due to these two point charges is zero, is
 - $8L$
 - $4L$
 - $2L$
 - $L/4$

6. A cube of side b has a charge q at each of its vertices. The electric field due to this charge distribution at the centre of this cube will be $\frac{1}{4\pi\epsilon_0}$ times
- (a) q/b^2 (b) $q/2b^2$ (c) $32q/b^2$ (d) zero
7. The figure shows some of the electric field lines corresponding to an electric field. The figure suggests



- (a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$
 (c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$

ELECTRIC DIPOLE

A pair of equal and opposite point charges, that are separated by a short distance is known as electric dipole. Electric dipole occurs in nature in a variety of situations, e.g. in HF, H₂O, HCl etc, the centre of positive charge does not fall exactly over the centre of negative charge. Such molecules are electric dipoles.

Dipole moment

The product of magnitude of one charge and the distance between the charges is called the magnitude of the electric dipole moment p . Suppose the charges of dipole are $-q$ and $+q$ and the small distance between them is $2a$.

Then, the magnitude of the electric dipole moment is given by

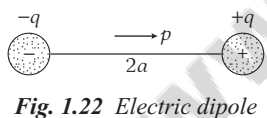


Fig. 1.22 Electric dipole

$$p = (2a) q$$

The electric dipole moment is a vector $\mathbf{p}$ whose direction is along the line joining the two charges pointing from the negative charge to the positive charge.

Its SI unit is Coulomb-metre.

Example 1.31 Charges ± 20 nC are separated by 5 mm. Calculate the magnitude and direction of dipole moment.

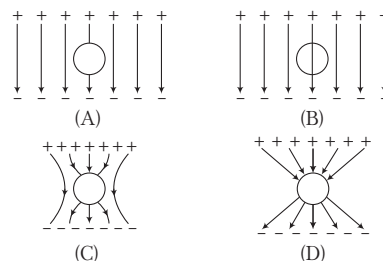
Sol. Given, $q_1 = q_2 = \pm 20$ nC $= \pm 20 \times 10^{-9}$ C

Distance $= 2a = 5$ mm $= 5 \times 10^{-3}$ m

Dipole moment, $p = q(2a)$
 $= 20 \times 10^{-9} \times 5 \times 10^{-3}$
 $= 10^{-10}$ cm

The direction of $\mathbf{p}$ is from negative charge to positive charge.

8. An uncharged sphere of metal is placed in between two charged plates as shown. The lines of force look like



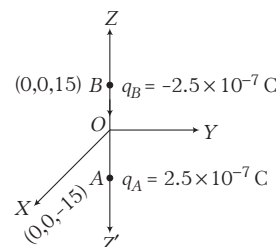
- (a) A (b) B
(c) C (d) D

Example 1.32 A system has two charges, $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A(0, 0, -15 cm) and B(0, 0, +15 cm) respectively. What is the electric dipole moment of the system?

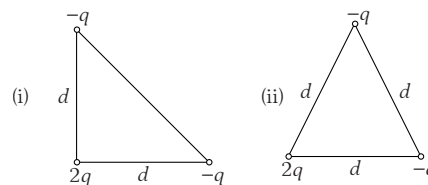
Sol. Electric dipole moment,

$p = \text{magnitude of either charge} \times \text{dipole length}$
 $= q_A \times AB = 2.5 \times 10^{-7} \times 0.30 = 7.5 \times 10^{-8}$ C-m

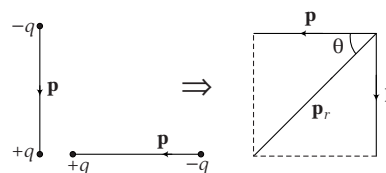
The electric dipole moment is directed from B to A, i.e. from negative charge to positive charge.



Example 1.33 Three charges are placed as shown. Find dipole moment of the arrangements.

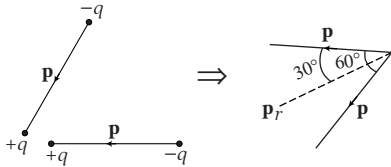


Sol. (i) Here, two dipoles are formed. These are shown in diagram below



Resultant dipole moment, $p_r = \sqrt{2}p = \sqrt{2}qd$
 and $\theta = 45^\circ$

(ii) The two dipoles formed are as shown below



∴ The resultant dipole moment,

$$p_r = 2p \cos 30^\circ = \sqrt{3}p = \sqrt{3}qd \text{ and } \theta = 30^\circ$$

The field of an electric dipole or dipole field

The electric field produced by an electric dipole is called a **dipole field**. The total charge of the electric dipole is zero but dipole field is not zero. It can be found using Coulomb's law and the superposition principle. We will find electric field of an electric dipole at two points as discussed below.

1. Electric field at an axial point of an electric dipole

Let us calculate electric field at the point P at a distance r from the centre of the dipole on the axial line of the dipole on the side of the charge q as shown in figure.

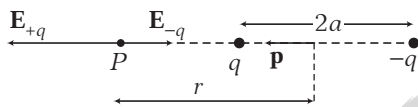


Fig. 1.23 Electric field at an axial point of the dipole

$$E_{-q} = \frac{-q}{4\pi\epsilon_0(r+a)^2}$$

$$E_{+q} = \frac{q}{4\pi\epsilon_0(r-a)^2}$$

The total field at P is $\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q}$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \frac{4ar}{(r^2 - a^2)^2} \quad (\because 2aq = p)$$

$$\therefore \mathbf{E} = \frac{2(2aq)r}{4\pi\epsilon_0(r^2 - a^2)^2}$$

$$= \frac{2r \mathbf{p}}{4\pi\epsilon_0(r^2 - a^2)^2}$$

For short dipole, i.e. for $r \gg a$,

$$\mathbf{E} = \frac{2\mathbf{p}}{4\pi\epsilon_0 r^3}$$

Direction of $\mathbf{E}$ is same as $\mathbf{p}$.

2. Electric field at an equatorial point of an electric dipole

The magnitude of the electric fields due to the two charges $+q$ and $-q$ are given by

$$E_{+q} = \frac{q}{4\pi\epsilon_0(r^2 + a^2)} \text{ and } E_{-q} = \frac{q}{4\pi\epsilon_0(r^2 + a^2)}$$

and they are equal in magnitude. The directions of E_{+q} and E_{-q} are as shown in the figure.

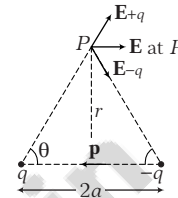


Fig. 1.24 Electric field at an equatorial point of the dipole

The components of electric field normal to the dipole axis cancel away. The components of electric field along the dipole axis add up. The total electric field $\mathbf{E}$ at P is opposite to dipole moment vector $\mathbf{p}$. So, we have

$$\mathbf{E} = -[(E_{+q} + E_{-q}) \cos \theta]$$

$$= \frac{-2q}{4\pi\epsilon_0(r^2 + a^2)} \cdot \frac{a}{(r^2 + a^2)^{1/2}} \left(\because \cos \theta = \frac{a}{(r^2 + a^2)^{1/2}} \right)$$

$$\Rightarrow \mathbf{E} = \frac{-2aq}{4\pi\epsilon_0(r^2 + a^2)^{3/2}} = \frac{-\mathbf{p}}{4\pi\epsilon_0(r^2 + a^2)^{3/2}} \quad [\because 2aq = p]$$

For short dipole, $r \gg a$

$$\therefore \mathbf{E} = \frac{-\mathbf{p}}{4\pi\epsilon_0 r^3}$$

Note The electric field due to short dipole at large distance ($r \gg a$) is proportional to $\frac{1}{r^3}$. If we take the limit, when the dipole size $2a$

approaches zero, the charge q approaches infinity in such a way that the product, $p = q \times 2a$ is finite. Such a dipole is referred to as a point dipole (ideal dipole).

Example 1.34 Two opposite charges each of magnitude $2\mu\text{C}$ are 1 cm apart. Find electric field at a distance of 5 cm from the mid-point on axial line of the dipole. Also, find the field on equatorial line at the same distance from mid-point.

Sol. Electric field (E) on axial line is given by

$$\frac{2pr}{4\pi\epsilon_0(r^2 - a^2)^2}$$

where, p is dipole moment = either charge $\times$ dipole length

Thus, $p = q \cdot 2a = (2 \times 10^{-6}) \times (0.01)$

Also, $r = 5 \times 10^{-2} \text{ m}$

$$\therefore E_a = \frac{9 \times 10^9 \times 2(2 \times 10^{-6} \times 10^{-2}) \times 5 \times 10^{-2}}{[(5 \times 10^{-2})^2 - (0.5 \times 10^{-2})^2]^2}$$

$$= 2.93 \times 10^6 \text{ NC}^{-1}$$

Similarly, electric field (E) on equatorial line is given by

$$E_e = \frac{p}{4\pi\epsilon_0(r^2 + a^2)^{3/2}}$$

The symbols have the same meaning as above,

$$E_e = \frac{9 \times 10^9 \times (2 \times 10^{-6} \times 10^{-2})}{[(5 \times 10^{-2})^2 + (0.5 \times 10^{-2})^2]^{3/2}}$$

$$\therefore E_e = 1.46 \times 10^6 \text{ NC}^{-1}$$

3. Electric field at the position (r, θ)

Due to the positive charge of the dipole, electric field at point P will be in radially outward direction and due to the negative charge it will be radially inward. Now, we have considered the radial component (E_r) and transverse component (E_θ) of the net electric field (E) as shown in figure.

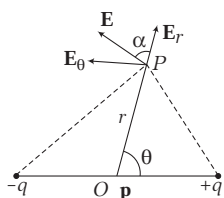


Fig. 1.25 Radial and transverse component of the electric field E of the dipole at point $P(r, \theta)$

$$\therefore E_r = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p \cos \theta}{r^3}$$

$$\text{and } E_\theta = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \sin \theta}{r^3}$$

$$\therefore \text{Net electric field at point } P \text{ is } E = \sqrt{E_r^2 + E_\theta^2}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3} \sqrt{1 + 3 \cos^2 \theta}$$

$$\text{Direction of the electric field, } \tan \alpha = \frac{E_\theta}{E_r} = \frac{1}{2} \tan \theta$$

Example 1.35 What is the magnitude of electric field intensity due to a dipole of moment $2 \times 10^{-8} \text{ C-m}$ at a point distance 1 m from the centre of dipole, when line joining the point to the centre of dipole makes an angle of 60° with dipole axis?

Sol. Given, $p = 2 \times 10^{-8} \text{ C-m}$, $r = 1 \text{ m}$ and $\theta = 60^\circ$

$$\begin{aligned} \therefore \text{Electric field intensity, } E &= \frac{p}{4\pi\epsilon_0 r^3} \sqrt{1 + 3 \cos^2 \theta + 1} \\ &= \frac{2 \times 10^{-8} \times 9 \times 10^9}{(1)^3} \times \sqrt{3(\cos 60^\circ)^2 + 1} \\ &= 238.1 \text{ N/C} \end{aligned}$$

Force on dipole

Suppose an electric dipole of dipole moment $|\mathbf{p}| = 2aq$ is placed in a uniform electric field $\mathbf{E}$ at an angle θ , where θ is the angle between $\mathbf{p}$ and $\mathbf{E}$. A force $\mathbf{F}_1 = q\mathbf{E}$ will act on positive charge and $\mathbf{F}_2 = -q\mathbf{E}$ on negative charge. Since, $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal in magnitude but opposite in direction.

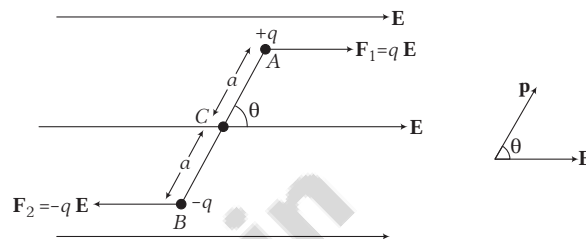


Fig. 1.26 Electric dipole in a uniform electric field

$$\text{Hence, } \mathbf{F}_1 + \mathbf{F}_2 = 0$$

$$\text{or } \mathbf{F}_{\text{net}} = 0$$

Thus, net force on a dipole in uniform electric field is zero. While in a non-uniform electric field, it may or may not be zero.

Torque on an electric dipole

The two equal and opposite forces shown in the above diagram act at different points of the dipole. They form a couple which exerts a **torque**. This torque has a magnitude equal to the magnitude of either force multiplied by the arm of the couple, i.e. perpendicular distance between the two anti-parallel forces.

$$\text{Magnitude of torque} = qE \times 2a \sin \theta = 2qaE \sin \theta$$

$$\tau = pE \sin \theta \quad [\because p = (2a)q]$$

or

$$\tau = \mathbf{p} \times \mathbf{E}$$

Thus, the magnitude of torque is $\tau = pE \sin \theta$. The direction of torque is perpendicular to the plane of paper inwards. Further this torque is zero at $\theta = 0^\circ$ or $\theta = 180^\circ$, i.e. when the dipole is parallel or anti-parallel to $\mathbf{E}$ and maximum at $\theta = 90^\circ$.

Thus, variation of τ with θ is as shown in graph below

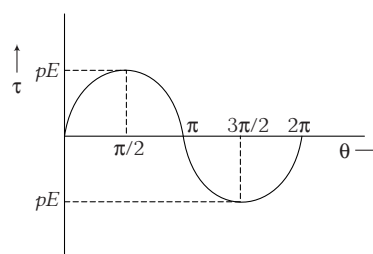


Fig. 1.27 Variation of τ with θ

Example 1.36 An electric dipole with dipole moment $4 \times 10^{-9} \text{ C-m}$ is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4 \text{ NC}^{-1}$. Calculate the magnitude of the torque acting on the dipole.

Sol. Using the formula, $\tau = pE \sin \theta$... (i)

Here, dipole moment, $p = 4 \times 10^{-9} \text{ C-m}$ and $E = 5 \times 10^4 \text{ NC}^{-1}$

Angle between E and p , $\theta = 30^\circ$.

Substituting these values in Eq (i), we get

$$\begin{aligned}\tau &= 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30^\circ \\ &= 20 \times 10^{-5} \times \frac{1}{2} = 10^{-4} \text{ N-m}\end{aligned}$$

Work done in rotating a dipole in a uniform electric field

When an electric dipole is placed in a uniform electric field $\mathbf{E}$, [Fig. (a)] a torque, $\tau = \mathbf{p} \times \mathbf{E}$ acts on it. If we rotate the dipole through a small angle $d\theta$ as shown in Fig. (b), the work done by the torque is

$$dW = \tau d\theta \Rightarrow dW = -pE \sin \theta d\theta$$

The work is negative as the rotation $d\theta$ is opposite to the torque.

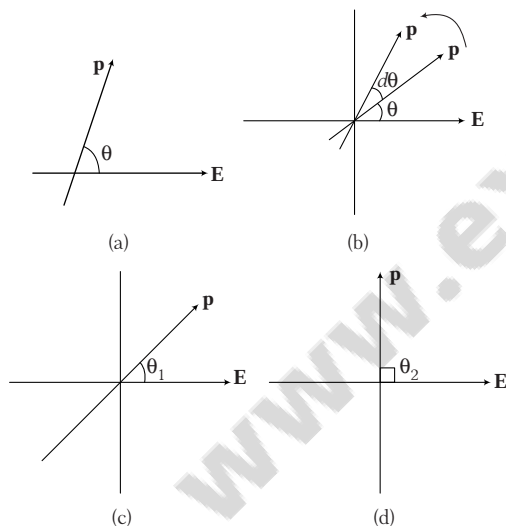


Fig. 1.28 Dipole at different angles with electric field

Total work done by external forces in rotating a dipole from $\theta = \theta_1$ to $\theta = \theta_2$ [Figs. (c) and (d)] will be given by

$$W = \int_{\theta_1}^{\theta_2} pE \sin \theta d\theta$$

$$W_{\text{external force}} = pE (\cos \theta_1 - \cos \theta_2)$$

and work done by electric forces,

$$\begin{aligned}W_{\text{electric force}} &= -W_{\text{external force}} \\ &= pE (\cos \theta_2 - \cos \theta_1)\end{aligned}$$

Taking $\theta_1 = \theta$ and $\theta_2 = 90^\circ$, we have

$$\begin{aligned}W_{\text{electric dipole}} &= p \cdot E (\cos 90^\circ - \cos \theta) = -pE \cos \theta \\ &= -\mathbf{p} \cdot \mathbf{E}\end{aligned}$$

Note If dipole is placed in non-uniform electric field, then magnitude and direction of electric field is different at every point and it will experience both net force and net torque.

Example 1.37 An electric dipole of dipole moment $p = 5 \times 10^{-18} \text{ C-m}$ lying along uniform electric field $E = 4 \times 10^4 \text{ NC}^{-1}$. Calculate the work done in rotating the dipole by 60° .

Sol. It is given that, electric dipole moment,

$$p = 5 \times 10^{-18} \text{ C-m}$$

Electric field strength, $E = 4 \times 10^4 \text{ NC}^{-1}$

When the electric dipole is placed in an electric field $\mathbf{E}$, a torque $\tau = \mathbf{p} \times \mathbf{E}$ acts on it. This torque tries to rotate the dipole through an angle θ .

If the dipole is rotated from an angle θ_1 to θ_2 , then work done by external force is given by

$$W = pE (\cos \theta_1 - \cos \theta_2) \quad \dots (i)$$

Putting $\theta_1 = 0^\circ$, $\theta_2 = 60^\circ$ in the Eq. (i), we get

$$\begin{aligned}W &= pE (\cos 0^\circ - \cos 60^\circ) \\ &= pE (1 - 1/2) = \frac{pE}{2} \\ &= \frac{5 \times 10^{-18} \times 4 \times 10^4}{2} = 10^{-13} \text{ J}\end{aligned}$$

$$\begin{aligned}\Rightarrow W &= 0.1 \times 10^{-12} \text{ J} \\ &= 0.1 \text{ pJ}\end{aligned}$$

CHECK POINT 1.4

- The electric dipole moment of an electron and a proton 4.3 nm apart, is
 (a) $6.8 \times 10^{-28} \text{ C-m}$ (b) $2.56 \times 10^{-29} \text{ C}^2/\text{m}$
 (c) $3.72 \times 10^{-14} \text{ C-m}$ (d) $11 \times 10^{-46} \text{ C}^2/\text{m}$
- If E_a be the electric field strength of a short dipole at a point on its axial line and E_e that on the equatorial line at the same distance, then
 (a) $E_e = 2E_a$ (b) $E_a = 2E_e$
 (c) $E_a = E_e$ (d) None of these
- Electric field at a far away distance r on the axis of a dipole is E_0 . What is the electric field at a distance $2r$ on perpendicular bisector?
 (a) $\frac{E_0}{16}$ (b) $-\frac{E_0}{16}$ (c) $\frac{E_0}{8}$ (d) $-\frac{E_0}{8}$
- The electric field due to an electric dipole at a distance r from its centre in axial position is E . If the dipole is rotated through an angle of 90° about its perpendicular axis, then the magnitude of electric field at the same point will be
 (a) E (b) $E/4$ (c) $E/2$ (d) $2E$

5. When an electric dipole $\mathbf{p}$ is placed in a uniform electric field $\mathbf{E}$, then at what angle between $\mathbf{p}$ and $\mathbf{E}$ the value of torque will be maximum?
 (a) 90° (b) 0°
 (c) 180° (d) 45°
6. An electric dipole of moment $\mathbf{p}$ is placed normal to the lines of force of electric intensity $\mathbf{E}$, then the work done in deflecting it through an angle of 180° is
 (a) pE (b) $+2pE$
 (c) $-2pE$ (d) zero
7. A molecule with a dipole moment p is placed in an electric field of strength E . Initially, the dipole is aligned parallel to the field. If the dipole is to be rotated to anti-parallel to the

field, then the work required to be done by an external agency is

- (a) $-2pE$ (b) $-pE$
 (c) pE (d) $2pE$

8. Two opposite and equal charges of $4 \times 10^{-8} \text{ C}$ are placed $2 \times 10^{-2} \text{ cm}$ away from each other. If this dipole is placed in an external electric field of $4 \times 10^8 \text{ NC}^{-1}$, then the value of maximum torque and the work done in rotating it through 180° will be
 (a) $64 \times 10^{-4} \text{ N-m}$ and $64 \times 10^{-4} \text{ J}$
 (b) $32 \times 10^{-4} \text{ N-m}$ and $32 \times 10^{-4} \text{ J}$
 (c) $64 \times 10^{-4} \text{ N-m}$ and $32 \times 10^{-4} \text{ J}$
 (d) $32 \times 10^{-4} \text{ N-m}$ and $64 \times 10^{-4} \text{ J}$

ELECTRIC FLUX

Electric flux over an area in an electric field is a measure of the number of field lines crossing a surface. It is denoted by ϕ_E . Let $\mathbf{E}$ be electric field at the location of the surface element $d\mathbf{S}$. The electric flux through the entire surface is given by

$$\phi_E = \int_S \mathbf{E} \cdot d\mathbf{S} \Rightarrow \phi_E = \int_S E dS \cos \theta = E \cos \theta \int_S dS$$

Here, θ is smaller angle between $\mathbf{E}$ and $d\mathbf{S}$.

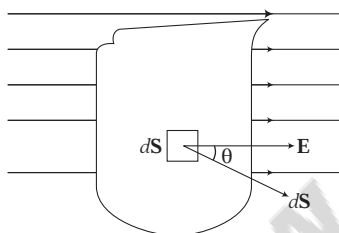


Fig. 1.29 Electric flux over an area dS

For a closed surface, θ is the angle between $\mathbf{E}$ and outward normal to the area element.

Electric flux is a scalar quantity having SI unit V-m or $\text{N-m}^2 \text{ C}^{-1}$.

Note An electric flux can also be defined as the flow of the electric field lines through a surface. When field lines leave or flow out of a closed surface, ϕ_E is positive and when they enter or flow into the surface, ϕ_E is negative.

Area vector

Area is a vector quantity. The direction of a planar area vector is specified by normal to the plane, e.g. in case of closed surface like cube, sphere, etc., direction of area vector $\mathbf{S}$ in outward direction is considered to be positive.

In case of open surface, any normal direction can be considered positive.

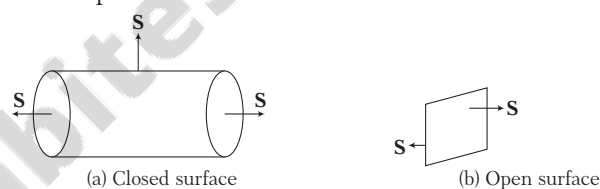


Fig. 1.30 Direction of area vector

The flux of electric field passing through an area is the dot product of electric field vector and area vector.

i.e.
$$\phi = \mathbf{E} \cdot \mathbf{S} = ES \cos \theta$$

Example 1.38 The electric field in a region is given by $\mathbf{E} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$, here, a and b are constants. Find the net flux passing through a square area of side l parallel to yz -plane.

Sol. A square area of side l parallel to yz -plane in vector form can be written as $\mathbf{S} = l^2 \hat{\mathbf{i}}$

Given,
$$\mathbf{E} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$$

$\therefore$ Electric flux passing through the given area will be

$$\begin{aligned} \phi_E &= \mathbf{E} \cdot \mathbf{S} \\ &= (a\hat{\mathbf{i}} + b\hat{\mathbf{j}}) \cdot (l^2 \hat{\mathbf{i}}) = al^2 \end{aligned}$$

Example 1.39 A rectangular surface of sides 10 cm and 15 cm is placed inside a uniform electric field of 25 NC^{-1} , such that normal to the surface makes an angle of 60° with the direction of electric field. Find the flux of electric field through the rectangular surface.

Sol. Here, $E = \text{electric field} = 25 \text{ NC}^{-1}$

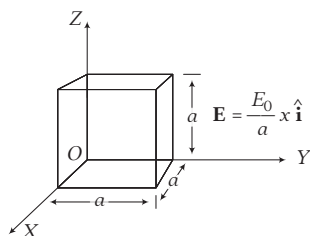
$S = \text{surface area of rectangle}$

$$= l \times b = 0.10 \times 0.15 \text{ m}^2$$

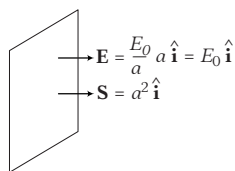
$$\begin{aligned} \text{Flux, } \phi &= ES \cos \theta = (25)(0.15 \times 0.10) (\cos 60^\circ) \\ &= 0.1875 \text{ N-m}^2 \text{ C}^{-1} \end{aligned}$$

Example 1.40 The electric field in a region is given by $\mathbf{E} = \frac{E_0}{a} x \hat{\mathbf{i}}$. Find the electric flux passing through a cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$.

Sol. The given situation is shown below



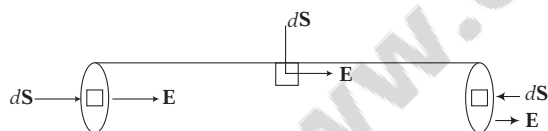
On four faces, electric field and area vector are perpendicular, hence there will be no flux. One face is at origin, i.e. $x = 0 \Rightarrow E = 0$, hence there will be no flux. On the sixth face, $x = a$



$\therefore$ Net electric flux, $\phi = \mathbf{E} \cdot \mathbf{S} = E_0 \hat{\mathbf{i}} \cdot a^2 \hat{\mathbf{i}} = E_0 a^2$

Example 1.41 A cylinder is placed in a uniform electric field $\mathbf{E}$ with axis parallel to the field. Find the total electric flux through the cylinder.

Sol. The direction of $\mathbf{E}$ and $d\mathbf{S}$ on different sections of cylinder are shown below



Flux through the entire cylinder,

$$\begin{aligned} \phi_E &= \int_{\text{Right plane face}} \mathbf{E} \cdot d\mathbf{S} + \int_{\text{Left plane face}} \mathbf{E} \cdot d\mathbf{S} + \int_{\text{curved surface}} \mathbf{E} \cdot d\mathbf{S} \\ \Rightarrow \phi_E &= \int \mathbf{E} \cdot d\mathbf{S} \cos 180^\circ + \int \mathbf{E} \cdot d\mathbf{S} \cos 0^\circ + \int \mathbf{E} \cdot d\mathbf{S} \cos 90^\circ \\ &= -E \int dS + E \int dS + 0 \\ &= -E \times \pi r^2 + E \times \pi r^2 = 0 \end{aligned}$$

Gauss's law

According to Gauss's law, "the net electric flux through any closed surface is equal to the net charge enclosed by it divided by ϵ_0 ". Mathematically, it can be written as

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$$

Gauss's theorem in simplified form can be written as under

$$ES = \frac{q_{\text{in}}}{\epsilon_0} \quad \dots(i)$$

But this form of Gauss's law is applicable only under the following two conditions

- the electric field at every point on the surface is either perpendicular or tangential.
 - magnitude of electric field at every point where it is perpendicular to the surface has a constant value (say E).
- Here, S is the area, where electric field is perpendicular to the surface. Unit of electric flux is $\text{Nm}^2 \text{C}^{-1}$.

Important points regarding Gauss's law in electrostatics

Important points regarding Gauss's law in electrostatics are given below

- Gauss's law is true for any closed surface, no matter what its shape or size.
- The term q on the right side of Gauss's law includes the sum of all charges enclosed by surface. The charges may be located anywhere inside the surface.
- In the situation, when the surface is so taken that there are some charges inside and some outside, the electric field (whose flux appears on the left side of Gauss's law) is due to all the charges, both inside and outside S . However, the term q on the right side of Gauss's law represents only the total charge inside S .
- The surface that we choose for the application of Gauss's law is called the **Gaussian surface**. You may choose any Gaussian surface and apply Gauss's law. However, do not let the Gaussian surface pass through any discrete charge, because electric field is not well defined at the location of discrete charge. However, the Gaussian surface can pass through a continuous charge distribution.
- Gauss's law is often useful towards a much easier calculation of the electrostatic field, when the system has some symmetry. This is achieved by the choice of a suitable Gaussian surface.
- Finally, Gauss's law is based on the inverse square dependence on distance as taken in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law.

Special cases

Some points related to calculation of electric flux in different cases

- (i) If surface contains number of charges, as shown in figure, then q_{in} can be calculated as

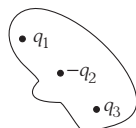


Fig. 1.31 Charges enclosed by a surface

$$q_{\text{in}} = q_1 - q_2 + q_3$$

$$\therefore \phi = \frac{q_{\text{in}}}{\epsilon_0} = \frac{q_1 - q_2 + q_3}{\epsilon_0}$$

- (ii) If a charge q is placed at the centre of a cube, then the flux passing through cube,

$$\phi = \frac{q_{\text{in}}}{\epsilon_0} = \frac{q}{\epsilon_0}$$

The flux passing through each face of cube,

$$\phi' = \frac{\phi}{6} = \frac{q}{6\epsilon_0} \quad (\text{by symmetry})$$

- (iii) To calculate flux passing through open surface, first make surface close in such a manner that point charge comes at the centre and then apply symmetry concept, e.g.

- (a) A charge q is placed at the centre of an imaginary hemispherical surface.



Fig. 1.32 Charge at the centre of a hemisphere

First, make the surface close by placing another hemisphere.

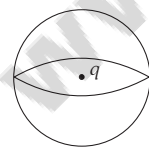


Fig. 1.33 Assume another symmetrical hemisphere to form a sphere

$$\text{Flux through sphere, } \phi = \frac{q_{\text{in}}}{\epsilon_0} = \frac{q}{\epsilon_0}$$

By symmetry, flux through hemisphere,

$$\phi' = \frac{\phi}{2} = \frac{q}{2\epsilon_0}$$

- (b) A charge Q is placed at a distance $a/2$ above the centre of a horizontal square of edge a .

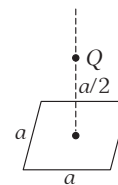


Fig. 1.34 Charge at a distance $a/2$ from square

First, make the surface close by placing five square faces ($a \times a$), so that a cube is formed and charge Q is at centre of a cube.

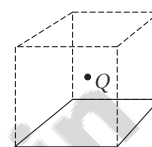


Fig. 1.35 A cube

$$\text{Flux through the cube, } \phi = \frac{q_{\text{in}}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

By symmetry, flux through each square face,

$$\phi' = \frac{\phi}{6} = \frac{Q}{6\epsilon_0}$$

Note In case of closed symmetrical body with charge q at its centre, the electric flux linked with each half will be $\frac{\phi_E}{2} = \frac{q}{2\epsilon_0}$. If the symmetrical closed body has n identical faces with point charge at its centre, flux linked with each face will be $\frac{\phi_E}{n} = \frac{q}{n\epsilon_0}$.

Example 1.42 An uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80.0 \mu\text{C}/\text{m}^2$. (i) Find the charge on the sphere. (ii) What is the total electric flux leaving the surface of the sphere?

Sol. (i) Using the relation $\sigma = \frac{q}{4\pi R^2}$, we get

$$q = 4\pi R^2 \times \sigma = 4 \times \frac{22}{7} \times (1.2)^2 \times 80 \times 10^{-6}$$

$$= 1.45 \times 10^{-3} \text{ C}$$

(ii) Using Gauss's theorem, we get

$$\phi = \frac{q}{\epsilon_0} = \frac{1.45 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$$= 1.64 \times 10^8 \text{ N-m}^2\text{C}^{-1}$$

Example 1.43 A point charge causes an electric flux of $-1.0 \times 10^3 \text{ N-m}^2\text{C}^{-1}$ to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (i) If the radius of the Gaussian surface is doubled, how much flux will pass through the surface? (ii) What is the value of the point charge?

Sol. (i) According to Gauss's law, the electric flux through a Gaussian surface depends upon the charge enclosed inside the surface and not upon its size. Thus, the electric flux will remain unchanged,

$$\text{i.e. } -1.0 \times 10^3 \text{ N-m}^2\text{C}^{-1}.$$

(ii) Using the formula $\phi = \frac{q}{\epsilon_0}$ (Gauss's theorem), we get

$$\Rightarrow q = \epsilon_0 \phi = 8.854 \times 10^{-12} \times (-1.0 \times 10^3) \\ = -8.854 \times 10^{-9} \text{ C} = -8.8 \text{ nC}$$

Example 1.44 A point charge q is placed at the centre of a cube. What is the flux linked

(i) with all the faces of the cube?

(ii) with each face of the cube?

(iii) if charge is not at the centre, then what will be the answers of parts (i) and (ii)?

Sol. (i) According to Gauss's law, $\phi_{\text{total}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{q}{\epsilon_0}$

(ii) The cube is a symmetrical body with 6 faces and the point charge is at its centre, so electric flux linked with each face will be

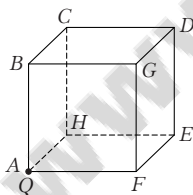
$$\phi_{\text{each face}} = \frac{\phi_{\text{total}}}{6} = \frac{q}{6\epsilon_0}$$

(iii) If charge is not at the centre, the answer of part (i) will remain same while that of part (ii) will change.

Example 1.45 A point charge Q is placed at one corner of a cube. Find flux passing through a cube.

Sol. First, make the surface close by placing three identical cubes at three sides of given cube and four cubes above. Now, charge comes at the centre of 8 cubes.

The flux passing through each cube will be (1/8)th of the flux Q/ϵ_0 . Hence, flux passing through given cube is $Q/8\epsilon_0$.



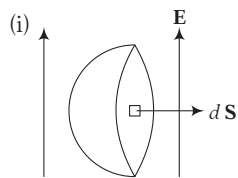
Example 1.46 A hemispherical body of radius R is placed in a uniform electric field E . What is the flux linked with the curved surface, if the field is

(i) parallel to the base

(ii) and perpendicular to the base?

Sol. We know, flux passing through closed surface,

$$\phi = \oint \mathbf{E} \cdot d\mathbf{S} = \frac{q_{\text{in}}}{\epsilon_0}$$



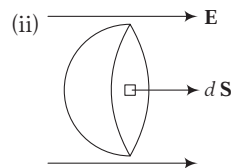
Charge inside hemisphere, $q_{\text{in}} = 0$,

$$\text{i.e. } \oint \mathbf{E} \cdot d\mathbf{S} = 0$$

$$\Rightarrow \phi_{\text{curved}} + \phi_{\text{plane}} = 0$$

$$\Rightarrow \phi_{\text{curved}} + ES \cos 90^\circ = 0$$

$$\Rightarrow \phi_{\text{curved}} = 0$$



$$\text{Also, } \phi_{\text{curved}} + \phi_{\text{plane}} = 0$$

$$\Rightarrow \phi_{\text{curved}} + ES \cos 0^\circ = 0$$

$$\Rightarrow \phi_{\text{curved}} + E\pi R^2 = 0$$

$$\Rightarrow \phi_{\text{curved}} = -E\pi R^2$$

Applications of Gauss's law

To calculate electric field by Gauss's theorem, we will draw a Gaussian surface (either sphere or cylinder, according to the situation) in such a way that electric field is perpendicular at each point of surface and its magnitude is same at every point and then apply Gauss's law. Let us start with some simple cases.

1. Electric field due to an infinitely long straight uniformly charged wire

Consider a long line charge with a linear charge density (charge per unit length), λ . To calculate the electric field at a point, located at a distance r from the line charge, we construct a Gaussian surface, a cylinder of any arbitrary length l of radius r and its axis coinciding with the axis of the line charge. This cylinder has three surfaces. One is curved surface and the two plane parallel surfaces.

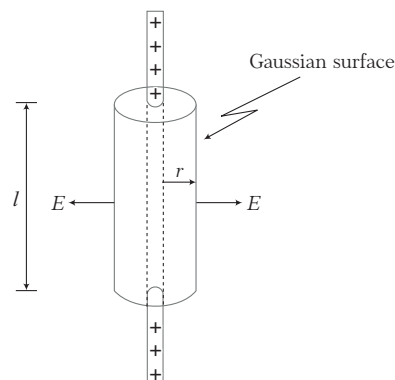


Fig. 1.36 Cylindrical Gaussian surface around a line charge

Field lines at plane parallel surfaces are tangential, so flux passing through these surfaces is zero. The magnitude of electric field is having the same magnitude (say E) at

curved surface and simultaneously the electric field is perpendicular at every point of this surface.

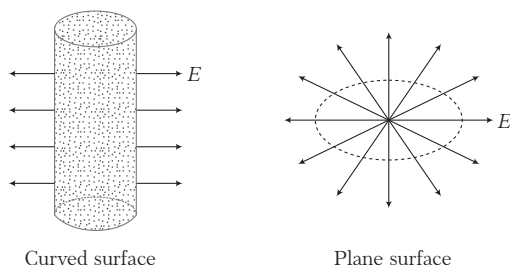


Fig. 1.37 Electric flux through different surfaces

Hence, we can apply the Gauss's law as

$$E S = \frac{q_{\text{in}}}{\epsilon_0}$$

Here, S = area of curved surface $= (2\pi r l)$
and q_{in} = net charge enclosing this cylinder $= \lambda l$.

$$\therefore E (2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

$$\therefore E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\text{i.e. } E \propto \frac{1}{r}$$

So, E - r graph is a rectangular hyperbola as shown in figure.

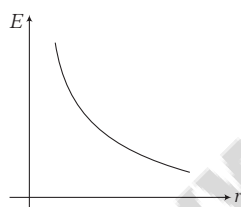
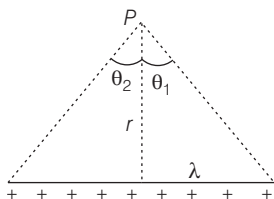


Fig. 1.38 E - r graph for a long charged wire

Note Electric field due to a finite length of straight charged wire,

$$E = \frac{1}{4\pi\epsilon_0} \frac{\lambda}{r} (\sin\theta_1 + \sin\theta_2)$$



Example 1.47 An infinite line charge produces a field of $9 \times 10^4 \text{ NC}^{-1}$ at a distance of 2 cm. Calculate the linear charge density.

Sol. As we know that, electric field due to an infinite line charge is given by the relation, $E = \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{r}$

where, λ is linear charge density and r is the distance of a point where an electric field is produced from the line charge.

$$\text{or } \lambda = 2\pi\epsilon_0 r E$$

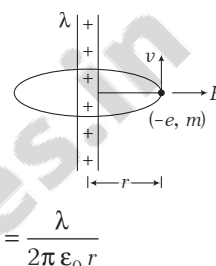
Here, $E = 9 \times 10^4 \text{ NC}^{-1}$ and $r = 2 \text{ cm} = 0.02 \text{ m}$

$\therefore$ Linear charge density,

$$\lambda = \frac{1}{9 \times 10^9} \times \frac{0.02 \times 9 \times 10^4}{2} = 10^{-7} \text{ Cm}^{-1}$$

Example 1.48 A long cylindrical wire carries a positive charge of linear density λ . An electron $(-e, m)$ revolves around it in a circular path under the influence of the attractive electrostatic force. Find the speed of the electron.

Sol. Electric field at perpendicular distance r ,



The electric force on electron, $F = eE$

To move in circular path, necessary centripetal force is provided by electric force.

$$F_c = \frac{mv^2}{r} \Rightarrow \frac{mv^2}{r} = eE = \frac{e\lambda}{2\pi\epsilon_0 r} \Rightarrow v = \sqrt{\frac{e\lambda}{2\pi\epsilon_0 m}}$$

$$\therefore \text{Speed of the electron, } v = \sqrt{\frac{e\lambda}{2\pi\epsilon_0 m}}$$

2. Electric field due to a plane sheet of charge

Consider a flat thin sheet, infinite in size with constant surface charge density σ (charge per unit area).

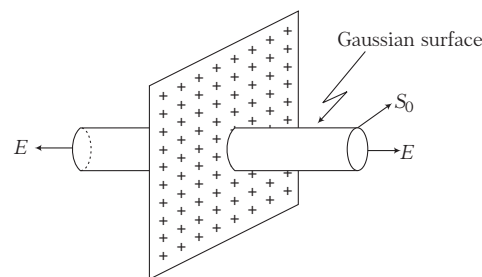


Fig. 1.39 Cylindrical Gaussian surface for a plane charged sheet

Let us draw a Gaussian surface (a cylinder) with one end on one side and other end on the other side and of cross-sectional area S_0 . Field lines will be tangential to the curved surface, so flux passing through this surface is zero. At plane surfaces, electric field has same magnitude and perpendicular to surface. Hence, using

$$ES = \frac{q_{\text{in}}}{\epsilon_0}$$

$$\therefore E(2S_0) = \frac{(\sigma)(S_0)}{\epsilon_0}$$

$$\therefore \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Thus, we see that the magnitude of the field due to plane sheet of charge is independent of the distance from the sheet.

Important point

Suppose two plane sheets having charge densities $+\sigma$ and $-\sigma$ are placed at some separation.

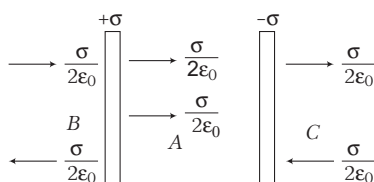


Fig. 1.40 Plane charged sheets placed close to each other

$$\text{Electric field at A, } E_A = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$$\text{Electric field at B, } E_B = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} \Rightarrow E_B = 0$$

$$\text{Similarly, } E_C = 0$$

If two plane sheets having opposite charges are kept close to each other, then electric field exists only between them.

Example 1.49 A large plane sheet of charge having surface charge density $5 \times 10^{-6} \text{ Cm}^{-2}$ lies in XY -plane. Find the electric flux through a circular area of radius 0.1 m , if the normal to the circular area makes an angle of 60° with the Z -axis.

Sol. Given, $\sigma = 5 \times 10^{-6} \text{ Cm}^{-2}$, $r = 0.1 \text{ m}$ and $\theta = 60^\circ$

$$\text{Flux, } \phi = ES \cos \theta = \left(\frac{\sigma}{2\epsilon_0} \right) \pi r^2 \cos \theta$$

$$= \frac{5 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} \times \frac{22}{7} \times (0.1)^2 \cos 60^\circ$$

$$\therefore \phi = 4.44 \times 10^3 \text{ N-m}^2/\text{C}$$

Example 1.50 Two large, thin metal plates are placed parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $1.77 \times 10^{-11} \text{ coulomb per square metre}$. What is electric field

- (i) to the left of the plates, (ii) to the right of the plates
(iii) and in between the plates?

Sol. (i) Electric fields due to both the plates outside them, will be equal in magnitude and opposite in direction, so net field will be zero.

(ii) Electric field outside the plates will be equal in magnitude and opposite in direction so net electric field will be zero.

(iii) In between the plates, the electric fields due to both the plates will be adding up, so net field will be

$$\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \text{ from positive to negative plate.}$$

$$\therefore E = \frac{\sigma}{\epsilon_0} = \frac{1.77 \times 10^{-11}}{8.85 \times 10^{-12}} = 2 \text{ N/C}$$

3. Electric field near a charged conducting surface

When a charge is given to a conducting plate, it distributes itself over the entire outer surface of the plate. The surface charge density σ is uniform and is the same on both surfaces, if plate is of uniform thickness and of infinite size. This is similar to the previous one, the only difference is that, this time charges are on both sides. Hence, applying, $ES = \frac{q_{\text{in}}}{\epsilon_0}$

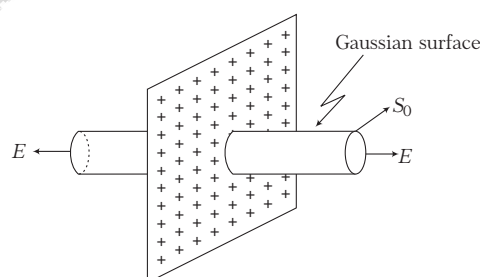


Fig. 1.41 Cylindrical Gaussian surface for a conducting surface

Here, $S = 2S_0$ and $q_{\text{in}} = (\sigma)(2S_0)$

$$\therefore E(2S_0) = \frac{(\sigma)(2S_0)}{\epsilon_0}$$

$$\therefore \boxed{E = \frac{\sigma}{\epsilon_0}}$$

The electric field near a charged conducting surface of any shape is σ/ϵ_0 and perpendicular to the surface.

4. Electric field due to a uniformly charged thin spherical shell

Let O be the centre and R be the radius of a thin, isolated spherical shell or solid conducting sphere carrying a charge $+q$ which is uniformly distributed on the surface. We have to determine electric field intensity due to this shell at points outside the shell, on the surface of the shell and inside the shell.

At external point

We can construct a Gaussian surface (a sphere) of radius $r > R$. At all points of this sphere, the magnitude of electric field is same and its direction is perpendicular to the surface. Thus, we can apply Gauss's theorem,

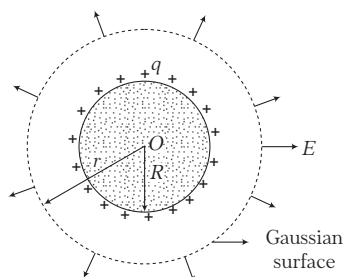


Fig. 1.42 Spherical Gaussian surface around a spherical shell

$$E S = \frac{q_{\text{in}}}{\epsilon_0} \quad \text{or} \quad E (4\pi r^2) = \frac{q}{\epsilon_0}$$

$\therefore$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Hence, the electric field at any external point is the same as, if the total charge is concentrated at centre.

At the surface of sphere, $r = R$

$\therefore$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2}$$

At an internal point

In this case, the Gaussian surface encloses no charge,

i.e. $\phi = E(4\pi r^2) = 0$, $E_{\text{inside}} = 0$

The electric field intensity is zero everywhere inside the charged shell.

The variation of electric field (E) with the distance from the centre (r) is as shown in figure.

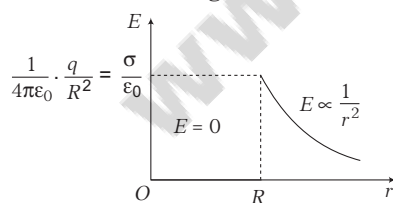


Fig. 1.43 E-r graph for a spherical shell

Note

(i) At the surface, graph is discontinuous.

$$(ii) E_{\text{surface}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} = \frac{q/4\pi R^2}{\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Example 1.51 A thin spherical shell of metal has a radius of 0.25 m and carries charge of 0.2 μC . Calculate the electric intensity at 3.0 m from the centre of the shell.

Sol. The intensity at an external point at a distance r from the

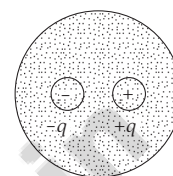
$$\text{centre of the shell is given by } E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Here, $r = 3.0$ m

$$\therefore E = (9.0 \times 10^9) \times \frac{2 \times 10^{-7}}{(3.0)^2} = 200 \text{ N/C}$$

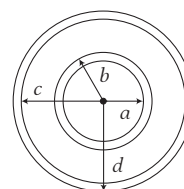
Example 1.52 An electric dipole is placed at the centre of a spherical shell. Find the electric field at an external point of the shell.

Sol. Net charge inside the sphere, $q_{\text{in}} = 0$. Therefore, according to Gauss's law, net flux passing through the sphere is zero. Therefore, electric field at an external point will be zero.

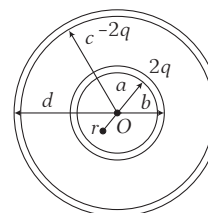


Example 1.53 A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting spherical shell with inner radius c and outer radius d . The inner shell has total charge $+2q$ and the outer shell has charge $+4q$.

- What is the total charge on the (a) inner surface of the small shell, (b) outer surface of the small shell, (c) inner surface of the large shell (d) and outer surface of the large shell?
- Calculate the electric field in terms of q and the distance r from the common centre of two shells for (a) $r < a$, (b) $a < r < b$, (c) $b < r < c$, (d) $c < r < d$ (e) and $r > d$.



Sol. Charge distribution,



- Total charge on inner shell = $2q$
Total charge on outer shell = $4q$
Charge distribution
(a) Charge on inner surface of small shell = 0
(b) Charge on outer surface of small shell = $2q$
(c) Charge on inner surface of large shell = $-2q$
(facing surface have equal and opposite charges)
(d) Charge on outer surface of large shell = $6q$
(total charge on outer shell is $4q$)
- To calculate electric field, draw a sphere with centre O through that point, where electric field is required.

Assume charge to be concentrated at centre and apply formula of point charge.

(a) $r < a$, enclosed charge = 0, $E = 0$

(b) $a < r < b$, enclosed charge = 0, $E = 0$ or electric field inside conductor = 0

(c) $b < r < c$, enclosed charge = $2q$, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{r^2}$

(d) $c < r < d$, enclosed charge = 0, $E = 0$, or electric field inside conductor = 0

(e) $r > d$, enclosed charge = $6q$, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{6q}{r^2}$

5. Electric field due to a non-conducting solid sphere of charge

Suppose positive charge q is uniformly distributed throughout the volume of a non-conducting solid sphere of radius R .

At an internal point

For finding the electric field at a distance r ($< R$) from the centre, let us choose our Gaussian surface a sphere of radius r , concentric with the charge distribution. From symmetry, the magnitude of electric field E has the same value at every point on the Gaussian surface and the direction of $\mathbf{E}$ is radial at every point on the surface. So, applying Gauss's law, we have

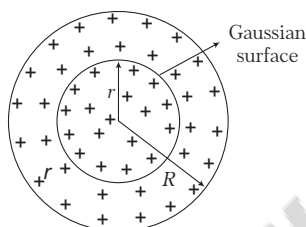


Fig. 1.44 Spherical Gaussian surface inside a solid sphere

$$ES = \frac{q_{\text{in}}}{\epsilon_0} \quad \dots(i)$$

Here, $S = 4\pi r^2$ and $q_{\text{in}} = (\rho) \left(\frac{4}{3} \pi r^3 \right)$

Here, ρ = charge per unit volume = $\frac{q}{(4/3) \pi R^3}$

Substituting these values in Eq. (i), we get

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^3} \cdot r$$

or $E \propto r$

At the centre of sphere, $r = 0$, so $E = 0$

At the surface of sphere, $r = R$, so $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2}$

At an external point

To find the electric field outside the charged sphere, we use a spherical Gaussian surface of radius r ($> R$). This surface encloses the entire charged sphere, so $q_{\text{in}} = q$ and Gauss's law gives,

$$E (4\pi r^2) = \frac{q}{\epsilon_0}$$

or $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$ or $E \propto \frac{1}{r^2}$

Thus, for a uniformly charged solid sphere, we have the following formulae for magnitude of electric field.

$$E_{\text{inside}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^3} \cdot r$$

$$E_{\text{surface}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2}$$

and

$$E_{\text{outside}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

Thus, the electric field at any external point is the same as, if the total charge is concentrated at centre.

The variation of electric field (E) with the distance from the centre of the sphere (r) is shown in figure.

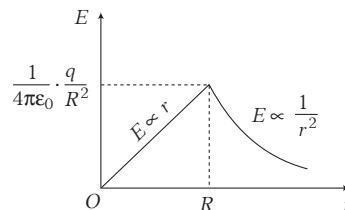


Fig. 1.45 E-r graph for a solid non-conducting sphere

Note If we set $r = R$ in either of the two expressions for E (outside and inside the sphere), we get the same result, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2}$

this is because E is continuous function of r in this case. By contrast, for the charged conducting sphere, the magnitude of electric field is discontinuous at $r = R$ (it jumps from $E = 0$ to $E = \sigma/\epsilon_0$).

Example 1.54 At a point 20 cm from the centre of a uniformly charged dielectric sphere of radius 10 cm, the electric field is 100 V/m. Find the electric field at 3 cm from the centre of the sphere.

Sol. Electric field outside the dielectric sphere, $E_{\text{out}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$

Electric field inside the dielectric sphere, $E_{\text{in}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^3} \cdot r$

$$\therefore E_{\text{in}} = E_{\text{out}} \times \frac{r^2}{R^3}$$

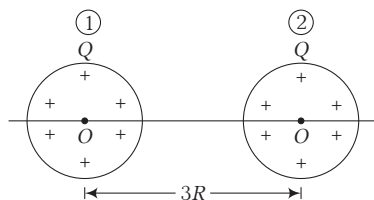
$$\Rightarrow E = 100 \times \frac{3 \times (20)^2}{10^3} = 120 \text{ V/m}$$

Example 1.55 Two non-conducting spheres of radius R have charge Q uniformly distributed on them. The centres of spheres are at $x = 0$ and $x = 3R$. Find the magnitude and direction of the net electric field on the X -axis at

- (i) $x = 0$, (ii) $x = \frac{R}{2}$, (iii) $x = \frac{3R}{2}$ (iv) and $x = 4R$.

Sol. Electric field inside sphere at distance r from centre

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{Qr}{R^3}$$



On the surface or outside, whole charge is assumed to be concentrated at centre.

- (i) At $x = 0$, $E_1 = 0$, $E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q}{(3R)^2}$, towards left

$$E_{\text{net}} = E_2 = \frac{Q}{36\pi\epsilon_0 R^2}, \text{ along } -X\text{-axis}$$

- (ii) At $x = R/2$,

$$E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{QR/2}{R^3} = \frac{Q}{8\pi\epsilon_0 R^2}, \text{ along } +X\text{-axis}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{(5R/2)^2} = \frac{Q}{25\pi\epsilon_0 R^2}, \text{ along } -X\text{-axis}$$

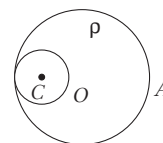
$$E_{\text{net}} = E_1 - E_2 = \frac{17Q}{200\pi\epsilon_0 R^2}, \text{ along } +X\text{-axis}$$

- (iii) At $x = 3R/2$, $E_{\text{net}} = 0$ ($\because E_1 = E_2$)

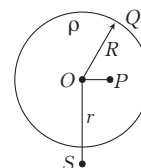
- (iv) At $x = 4R$, $E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q}{(4R)^2}$, $E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2}$

$$E_{\text{net}} = E_1 + E_2 = \frac{17Q}{64\pi\epsilon_0 R^2}, \text{ along } +X\text{-axis}$$

Example 1.56 A non-conducting sphere of radius R has a spherical cavity of radius $R/2$ as shown in figure. The solid part of the sphere has a uniform volume charge density ρ . Find the magnitude and direction of electric field at point (a) O and (b) A .



Sol. For a non-conducting sphere of radius R having volume charge density ρ .

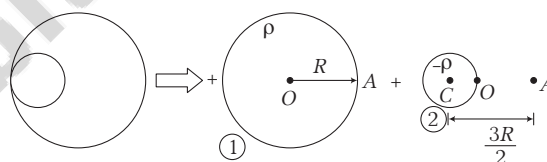


- (i) At P , $r < R$ (inside), $E = \frac{\rho r}{3\epsilon_0}$

- (ii) At S , $r > R$ (outside), $E = \frac{\rho R^3}{3\epsilon_0 r^2}$

- (iii) At Q , $r = R$ (surface), $E = \frac{\rho R}{3\epsilon_0}$

The given sphere can be shown as,



- (a) At O , $E_1 = 0$, $E_2 = \frac{\rho R/2}{3\epsilon_0} = \frac{\rho R}{6\epsilon_0}$, towards left

$$\Rightarrow E_0 = \frac{\rho R}{6\epsilon_0}, \text{ towards left}$$

- (b) At A , $E_1 = \frac{\rho R}{3\epsilon_0}$, towards right

$$E_2 = \frac{\rho(R/2)^3}{3\epsilon_0(3R/2)^2} = \frac{\rho R}{54\epsilon_0}, \text{ towards left}$$

$$\therefore E_A = E_1 - E_2 = \frac{\rho R}{\epsilon_0} \left[\frac{1}{3} - \frac{1}{54} \right] = \frac{17\rho R}{54\epsilon_0}, \text{ towards right}$$

CHECK POINT 1.5

- A surface $S = 10\hat{j}$ is kept in an electric field $\mathbf{E} = 2\hat{i} + 4\hat{j} + 7\hat{k}$. How much electric flux will come out through this surface?
 - 40 unit
 - 50 unit
 - 30 unit
 - 20 unit
- A cube of side a is placed in a uniform electric field $\mathbf{E} = E_0\hat{i} + E_0\hat{j} + E_0\hat{k}$. Total electric flux passing through the cube would be
 - $(\phi_1 + \phi_2)\epsilon_0$
 - $(\phi_2 - \phi_1)\epsilon_0$
 - $(\phi_1 + \phi_2)/\epsilon_0$
 - $(\phi_2 - \phi_1)/\epsilon_0$

- $E_0 a^2$
- $2E_0 a^2$
- $6E_0 a^2$
- None of these

- If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2 , then the electric charge inside the surface will be
 - $(\phi_1 + \phi_2)\epsilon_0$
 - $(\phi_2 - \phi_1)\epsilon_0$
 - $(\phi_1 + \phi_2)/\epsilon_0$
 - $(\phi_2 - \phi_1)/\epsilon_0$

4. Charge of 2 C is placed at the centre of a cube. What is the electric flux passing through one face?

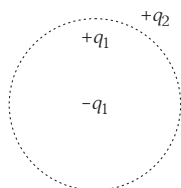
(a) $\frac{1}{3\epsilon_0}$ (b) $\left(\frac{1}{4}\right)\epsilon_0$
 (c) $\frac{2}{\epsilon_0}$ (d) $\frac{3}{\epsilon_0}$

5. The inward and outward electric flux for a closed surface in units of $\text{N}\cdot\text{m}^2\text{C}^{-1}$ are 8×10^3 and 4×10^3 , respectively. Then, the total charge inside the surface is [where, ϵ_0 = permittivity constant]

(a) $4\times 10^3\text{C}$ (b) $-4\times 10^3\text{C}$
 (c) $\frac{(-4\times 10^3)}{\epsilon}\text{C}$ (d) $-4\times 10^3\epsilon_0\text{C}$

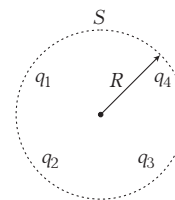
6. If the flux of the electric field through a closed surface is zero, then
 (i) the electric field must be zero everywhere on the surface
 (ii) the electric field may be zero everywhere on the surface
 (iii) the charge inside the surface must be zero
 (iv) the charge in the vicinity of the surfaces must be zero
 (a) (i), (ii) (b) (ii), (iii)
 (c) (ii), (iv) (d) (i), (iii)

7. Consider the charge configuration and spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to



(a) q_2 (b) only the positive charges
 (c) all the charges (d) $+q_1$ and $-q_1$

8. q_1, q_2, q_3 and q_4 are point charges located at points as shown in the figure and S is a spherical Gaussian surface of radius R . Which of the following is true according to the Gauss's law?



(a) $\oint_S (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3) \cdot d\mathbf{A} = \frac{q_1 + q_2 + q_3}{2\epsilon_0}$
 (b) $\oint_S (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3) \cdot d\mathbf{A} = \frac{(q_1 + q_2 + q_3)}{\epsilon_0}$
 (c) $\oint_S (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3) \cdot d\mathbf{A} = \frac{(q_1 + q_2 + q_3 + q_4)}{\epsilon_0}$
 (d) None of the above

9. An infinite line charge produces a field of $18\times 10^4\text{ N/C}$ at 0.02 m. The linear charge density is

(a) $2\times 10^{-7}\text{ C/m}$ (b) 10^{-8} C/m
 (c) 10^7 C/m (d) 10^{-4} C/m

10. A charge of $17.7\times 10^{-4}\text{ C}$ is distributed uniformly over a large sheet of area 200 m^2 . The electric field intensity at a distance 20 cm from it in air will be

(a) $5\times 10^5\text{ N/C}$ (b) $6\times 10^5\text{ N/C}$
 (c) $7\times 10^5\text{ N/C}$ (d) $8\times 10^5\text{ N/C}$

11. From what distance should a 100 eV electron be fired towards a large metal plate having a surface charge density of $-2.0\times 10^{-6}\text{ Cm}^{-2}$, so that it just fails to strike the plate?

(a) 0.50 mm (b) 0.44 mm (c) 0.60 mm (d) 0.77 mm

12. A thin spherical shell of metal has a radius of 0.25 m and carries a charge of $0.2\mu\text{C}$. The electric field intensity at a point on the surface of the shell will be

(a) $2.88\times 10^4\text{ N/C}$ (b) $3.4\times 10^4\text{ N/C}$
 (c) $3.25\times 10^4\text{ N/C}$ (d) $3.88\times 10^4\text{ N/C}$

13. If the electric field near the earth's surface be 300 V/m directed downwards, then the surface density of charge on earth's surface is

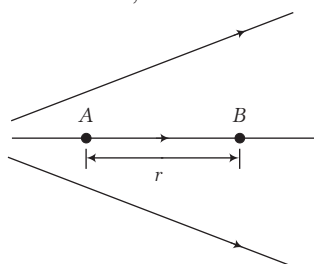
(a) $30\times 10^{-9}\text{ C/m}^2$ (b) $50\times 10^{-9}\text{ C/m}^2$
 (c) $2.6\times 10^{-9}\text{ C/m}^2$ (d) $7.0\times 10^{-9}\text{ C/m}^2$

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1 Figure shows the electric lines of force emerging from a charged body. If the electric field at A and B are E_A and E_B respectively and if the distance between A and B is r , then



- (a) $E_A > E_B$ (b) $E_A < E_B$ (c) $E_A = \frac{E_B}{r}$ (d) $E_A = \frac{E_B}{r^2}$
- 2 The insulation property of air breaks down at $E = 3 \times 10^6$ V/m. The maximum charge that can be given to a sphere of diameter 5 m is approximately (in coulombs)
- (a) 2×10^{-2} (b) 2×10^{-3}
(c) 2×10^{-4} (d) 2×10^{-5}
- 3 The electric field near a conducting surface having a uniform surface charge density σ is given by
- (a) $\frac{\sigma}{\epsilon_0}$ and is parallel to the surface
(b) $\frac{2\sigma}{\epsilon_0}$ and is parallel to the surface
(c) $\frac{\sigma}{\epsilon_0}$ and is normal to the surface
(d) $\frac{2\sigma}{\epsilon_0}$ and is normal to the surface
- 4 A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path(s) shown in figure as
-
- (a) 1 (b) 2
(c) 3 (d) 4
- 5 Two point charges of $20 \mu\text{C}$ and $80 \mu\text{C}$ are 10 cm apart. Where will the electric field strength be zero on the line joining the charges from $20 \mu\text{C}$ charge?
- (a) 0.1 m (b) 0.04 m
(c) 0.033 m (d) 0.33 m
- 6 For a dipole $q = 2 \times 10^{-6}$ C and $d = 0.01$ m. Calculate the maximum torque for this dipole, if $E = 5 \times 10^5 \text{ NC}^{-1}$.
- (a) $1 \times 10^{-3} \text{ N-m}^{-1}$ (b) $10 \times 10^{-3} \text{ N-m}^{-1}$
(c) $10 \times 10^{-3} \text{ N-m}$ (d) $1 \times 10^2 \text{ N-m}^2$
- 7 What is the magnitude of a point charge due to which the electric field 30 cm away has the magnitude of 2 N/C ? [$1/4\pi\epsilon_0 = 9 \times 10^9 \text{ N-m}^2/\text{C}^2$]
- (a) $2 \times 10^{-11} \text{ C}$ (b) $3 \times 10^{-11} \text{ C}$ (c) $5 \times 10^{-11} \text{ C}$ (d) $9 \times 10^{-11} \text{ C}$
- 8 A charge q is lying at mid-point of the line joining the two similar charges Q . The system will be in equilibrium, if the value of q is
- (a) $Q/2$ (b) $-Q/2$ (c) $Q/4$ (d) $-Q/4$
- 9 Two point charges q and $2q$ are placed some distance apart. If the electric field at the location of q be E , then that at the location of $2q$ will be
- (a) $3E$ (b) $E/2$
(c) E (d) None of these
- 10 The electric field at a distance $\frac{3R}{2}$ from the centre of a charged conducting spherical shell of radius R is E . The electric field at a distance $\frac{R}{2}$ from the centre of the sphere is
- (a) zero (b) E (c) $E/2$ (d) $E/3$
- 11 Electric field intensity at a point in between two parallel sheets with like charges of same surface charge densities (σ) is
- (a) $\frac{\sigma}{2\epsilon_0}$ (b) $\frac{\sigma}{\epsilon_0}$ (c) zero (d) $\frac{2\sigma}{\epsilon_0}$
- 12 Two point charges $+2 \text{ C}$ and $+6 \text{ C}$ repel each other with a force of 12 N . If a charge of -4 C is given to each of these charges, the force now is
- (a) 4 N (repulsive) (b) 4 N (attractive)
(c) 12 N (attractive) (d) 8 N (repulsive)

- 13** Three equal charges are placed on the three corners of a square. If the force between q_1 and q_2 is F_{12} and that between q_1 and q_3 is F_{13} , then the ratio of magnitudes (F_{12}/F_{13}) is

(a) $1/2$ (b) 2 (c) $1/\sqrt{2}$ (d) $\sqrt{2}$

- 14** A conductor has been given a charge $-3 \times 10^{-7} \text{ C}$ by transferring electron. Increase in mass (in kg) of the conductor and the number of electrons added to the conductor are respectively

(a) 2×10^{-16} and 2×10^{31} (b) 5×10^{-31} and 5×10^{19}
(c) 3×10^{-19} and 9×10^{16} (d) 2×10^{-18} and 2×10^{12}

- 15** The ratio of electrostatic and gravitational forces acting between electron and proton separated by a distance $5 \times 10^{-11} \text{ m}$, will be (charge on electron $= 1.6 \times 10^{-19} \text{ C}$, mass of electron $= 9.1 \times 10^{-31} \text{ kg}$, mass of proton $= 1.6 \times 10^{-27} \text{ kg}$, $G = 6.7 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$)

(a) 2.36×10^{39} (b) 2.36×10^{40}
(c) 2.34×10^{41} (d) 2.34×10^{42}

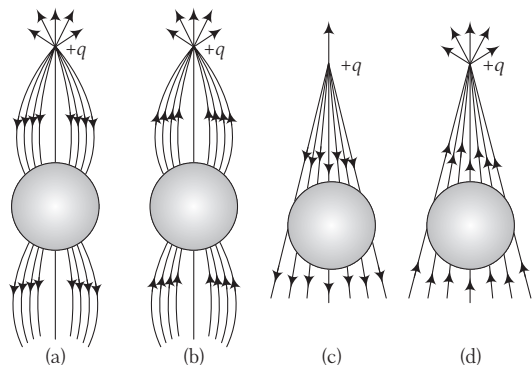
- 16** Two similar small spheres having $+q$ and $-q$ charge are kept at a certain distance. F force acts between the two. If in the middle of two spheres, another similar small sphere having $+q$ charge is kept, then it will experience a force in magnitude and direction as

(a) zero, having no direction (b) $8F$, towards $+q$ charge
(c) $8F$, towards $-q$ charge (d) $4F$, towards $+q$ charge

- 17.** Two small conducting spheres of equal radius have charges $+10 \mu\text{C}$ and $-20 \mu\text{C}$ respectively and placed at a distance R from each other. They experience force F_1 . If they are brought in contact and separated to the same distance, they experience force F_2 . The ratio of F_1 to F_2 is

(a) $1:8$ (b) $-8:1$ (c) $1:2$ (d) $-2:1$

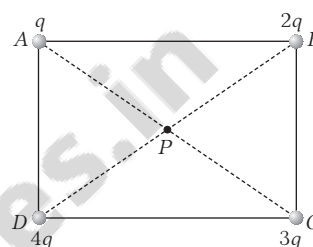
- 18** A positive point charge is brought near an isolated conducting sphere as shown in figure. The electric field is best given by [NCERT Exemplar]



- 19** The centres of two identical small conducting spheres are 1 m apart. They carry charges of opposite kind and attract each other with a force F . When they are connected by a conducting thin wire they repel each other with a force $F/3$. What is the ratio of magnitude of charges carried by the spheres initially?

(a) $1:1$ (b) $2:1$
(c) $3:1$ (d) $4:1$

- 20** q , $2q$, $3q$ and $4q$ charges are placed at the four corners A , B , C and D of a square. The field at the centre P of the square has the direction along



(a) AB (b) CB
(c) AC (d) BD

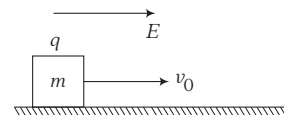
- 21** A ball with charge $-50e$ is placed at the centre of a hollow spherical shell having a charge of $-50e$. What is the charge on the shell's outer surface?

(a) $-50e$ (b) Zero
(c) $-100e$ (d) $+100e$

- 22** Two parallel metal plates having charges $+Q$ and $-Q$ face each other at a certain distance between them. If the plate are now dipped in kerosene oil tank, the electric field between the plates will

(a) became zero (b) increase
(c) decrease (d) remain same

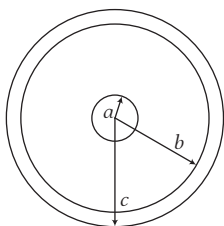
- 23** A charged block is projected on a rough horizontal surface with speed v_0 . The value of coefficient of friction if the kinetic energy of the block remains constant is



(a) $\frac{qE}{mg}$ (b) $\frac{qE}{m}$
(c) qE (d) None of these

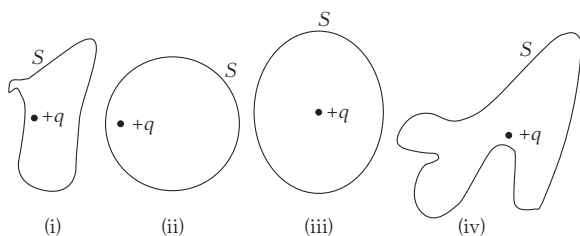
- 24** A solid conducting sphere of radius a has a net positive charge $2Q$. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and has a net charge $-Q$.

The surface charge density on the inner and outer surfaces of the spherical shell will be



- (a) $-\frac{2Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$ (b) $-\frac{Q}{4\pi b^2}, \frac{Q}{4\pi c^2}$
 (c) $0, \frac{Q}{4\pi c^2}$ (d) None of these

25 The electric flux through the surface



- (a) in Fig. (iv) is the largest
 (b) in Fig. (iii) is the least
 (c) in Fig. (ii) is same as Fig. (iii) but is smaller than Fig. (iv)
 (d) is the same for all the figures

26 A mass $m = 20$ g has a charge $q = 3.0$ mC. It moves with a velocity of 20 ms^{-1} and enters a region of electric field of 80 NC^{-1} in the same direction as the velocity of the mass. The velocity of the mass after 3s in this region is
 (a) 80 ms^{-1} (b) 56 ms^{-1} (c) 44 ms^{-1} (d) 40 ms^{-1}

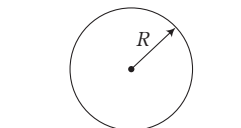
27 Three concentric metallic spherical shells of radii $R, 2R, 3R$ are given charges Q_1, Q_2, Q_3 , respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells $Q_1 : Q_2 : Q_3$ is
 (a) $1 : 2 : 3$ (b) $1 : 3 : 5$
 (c) $1 : 4 : 9$ (d) $1 : 8 : 18$

28 Electric charges $q, q, -2q$ are placed at the corners of an equilateral triangle ABC of side l . The magnitude of electric dipole moment of the system is
 (a) ql (b) $2ql$
 (c) $\sqrt{3}ql$ (d) $4ql$

29 A point charge $+q$ is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is [NCERT Exemplar]
 (a) directed perpendicular to the plane and away from the plane

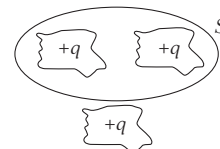
- (b) directed perpendicular to the plane but towards the plane
 (c) directed radially away from the point charge
 (d) directed radially towards the point charge

30 If linear charge density of a wire as shown in the figure is λ , then



- (a) electric field at the centre is $\frac{\lambda}{2\epsilon_0}$
 (b) electric field at the centre of the loop is $\frac{\lambda}{2\pi\epsilon_0 R}$
 (c) electric field at the centre of the loop is $\frac{\lambda}{2\pi\epsilon_0 R} + \frac{\lambda}{2\epsilon_0 R}$
 (d) None of the above

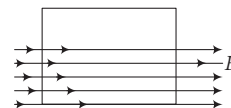
31 Figure shown below is a distribution of charges. The flux of electric field due to these charges through the surface S is



- (a) $3q/\epsilon_0$ (b) $2q/\epsilon_0$
 (c) q/ϵ_0 (d) zero

32 A cylinder of radius R and length L is placed in a uniform electric field E parallel to the cylinder axis. The total flux for the surface of the cylinder is given by
 (a) $2\pi R^2 E$ (b) $\pi R^2 / E$ (c) $(\pi R^2 / \pi R) / E$ (d) zero

33 A square surface of side L metres is in the plane of the paper. A uniform electric field E (volt/m), also in the plane of the paper, is limited only to the lower half of the square surface, (see figure). The electric flux in SI units associated with the surface is



- (a) zero (b) EL^2
 (c) $EL^2 / (2\epsilon_0)$ (d) $EL^2 / 2$

34 Two identical conducting spheres carrying different charges attract each other with a force F when placed in air medium at a distance d apart. The spheres are brought into contact and then taken to their original positions. Now, the two spheres repel each other with a force whose magnitude is equal to that of the initial attractive force.

The ratio between initial charges on the spheres is

- (a) $(3 + \sqrt{8})$ only (b) $-3 + \sqrt{8}$ only
(c) $(3 + \sqrt{8})$ or $(3 - \sqrt{8})$ (d) $+\sqrt{3}$

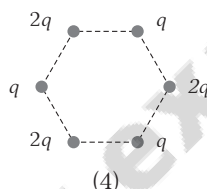
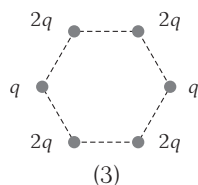
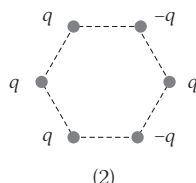
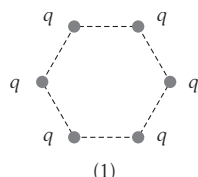
- 35** Under the action of a given coulombic force, the acceleration of an electron is $2.5 \times 10^{22} \text{ ms}^{-2}$. Then, the magnitude of the acceleration of a proton under the action of same force is nearly

- (a) $1.6 \times 10^{-19} \text{ ms}^{-2}$ (b) $9.1 \times 10^{31} \text{ ms}^{-2}$
(c) $1.5 \times 10^{19} \text{ ms}^{-2}$ (d) $1.6 \times 10^{27} \text{ ms}^{-2}$

- 36** A drop of 10^{-6} kg water carries 10^{-6} C charge. What electric field should be applied to balance its weight? (assume, $g = 10 \text{ ms}^{-2}$)

- (a) 10 V/m , upward (b) 10 V/m , downward
(c) 0.1 V/m , downward (d) 0.1 V/m , upward

- 37** Figures below show regular hexagons, with charges at the vertices. In which of the following cases the electric field at the centre is not zero?



- (a) 1 (b) 2 (c) 3 (d) 4

- 38** The electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is $20 \text{ V}\cdot\text{m}$. The flux over a concentric sphere of radius 20 cm will be

- (a) $20 \text{ V}\cdot\text{m}$ (b) $10 \text{ V}\cdot\text{m}$ (c) $40 \text{ V}\cdot\text{m}$ (d) $5 \text{ V}\cdot\text{m}$

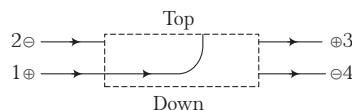
- 39** Two charges of $-4 \mu\text{C}$ and $+4 \mu\text{C}$ are placed at the points $A(1, 0, 4)$ and $B(2, -1, 5)$ located in an electric field $\mathbf{E} = 20 \hat{i} \text{ V/C}\cdot\text{m}$. Then, torque acting on the dipole will be

- (a) $2.31 \times 10^{-4} \text{ N}\cdot\text{m}$ (b) $1.13 \times 10^{-4} \text{ N}\cdot\text{m}$
(c) $8.0 \times 10^{-4} \text{ N}\cdot\text{m}$ (d) $3.04 \times 10^{-4} \text{ N}\cdot\text{m}$

- 40** An infinite line charge produces a field of $7.18 \times 10^8 \text{ N/C}$ at a distance of 2 cm . The linear charge density is

- (a) $7.27 \times 10^{-4} \text{ C/m}$ (b) $7.98 \times 10^{-4} \text{ C/m}$
(c) $7.11 \times 10^{-4} \text{ C/m}$ (d) $7.04 \times 10^{-4} \text{ C/m}$

- 41** The figure shows the path of a positively charged particle 1 through a rectangular region of uniform electric field as shown in the figure. What is the direction of electric field and the direction of particles 2, 3 and 4?



- (a) Top, down, top, down (b) Top, down, down, top
(c) Down, top, top, down (d) Down, top, down, down

- 42** The electric field intensity due to a dipole of length 10 cm and having a charge of $500 \mu\text{C}$, at a point on the axis at a distance 20 cm from one of the charges in air, is

- (a) $6.25 \times 10^7 \text{ N/C}$ (b) $9.28 \times 10^7 \text{ N/C}$
(c) $13.1 \times 10^{10} \text{ N/C}$ (d) $20.5 \times 10^7 \text{ N/C}$

- 43** Two electric dipoles of moment p and $64p$ are placed in opposite direction on a line at a distance of 25 cm . The electric field will be zero at point between the dipoles whose distance from the dipole of moment p is

- (a) 5 cm (b) $\frac{25}{9} \text{ cm}$ (c) 10 cm (d) $\frac{4}{13} \text{ cm}$

- 44** Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F , when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged is brought in contact with B , then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is

- (a) $F/4$ (b) $3F/4$
(c) $F/8$ (d) $3F/8$

- 45** A sample of HCl gas is placed in an electric field of $3 \times 10^4 \text{ NC}^{-1}$. The dipole moment of each HCl molecule is $6 \times 10^{-30} \text{ C}\cdot\text{m}$. The maximum torque that can act on a molecule is

- (a) $2 \times 10^{-34} \text{ C}^2\text{N}^{-1}\text{m}$ (b) $2 \times 10^{-34} \text{ N}\cdot\text{m}$
(c) $18 \times 10^{-26} \text{ N}\cdot\text{m}$ (d) $0.5 \times 10^{34} \text{ C}^{-2}\text{N}^{-1}\text{m}^{-1}$

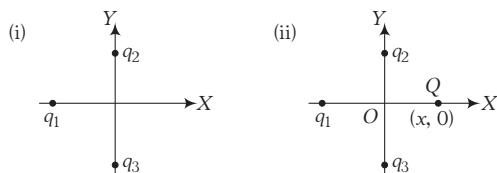
- 46** Two positive ions, each carrying a charge q , are separated by a distance d . If F is the force of repulsion between the ions, then the number of electrons missing from each ion will be (e being the charge on an electron)

- (a) $\frac{4\pi\epsilon_0 F d^2}{e}$ (b) $\sqrt{\frac{4\pi\epsilon_0 F e^2}{d^2}}$
(c) $\sqrt{\frac{4\pi\epsilon_0 F d^2}{e^2}}$ (d) $\frac{4\pi\epsilon_0 F d^2}{e^2}$

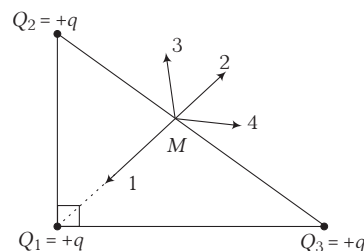
- 47** Among two discs A and B , first has radius 10 cm and charge 10^{-6} C and second has radius 30 cm and charge 10^{-5} C. When they are touched, charges on both are, q_A and q_B respectively, will be
 (a) $q_A = 2.75 \mu\text{C}$, $q_B = 3.15 \mu\text{C}$
 (b) $q_A = 1.09 \mu\text{C}$, $q_B = 1.53 \mu\text{C}$
 (c) $q_A = q_B = 5.5 \mu\text{C}$
 (d) None of the above

- 48** Two point charges $-q$ and $+\frac{q}{2}$ are situated at the origin and at the point $(a, 0, 0)$ respectively. The point along the X -axis where the electric field vanishes is
 (a) $x = \frac{a}{\sqrt{2}}$ (b) $x = \sqrt{2}a$ (c) $x = \frac{\sqrt{2}a}{\sqrt{2}-1}$ (d) $x = \frac{\sqrt{2}a}{\sqrt{2}+1}$

- 49** In figure two positive charges q_2 and q_3 fixed along the Y -axis, exert a net electric force in the $+x$ -direction on a charge q_1 fixed along the X -axis. If a positive charge Q is added at $(x, 0)$, then the force on q_1 [NCERT Exemplar]

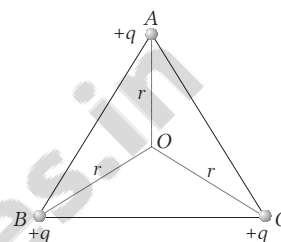


- (a) shall increase along the positive X -axis.
 (b) shall decrease along the positive X -axis.
 (c) shall point along the negative X -axis.
 (d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3
- 50** A hemisphere is uniformly charged positively. The electric field at a point on a diameter away from the centre is directed [NCERT Exemplar]
 (a) perpendicular to the diameter
 (b) parallel to the diameter
 (c) at an angle tilted towards the diameter
 (d) at an angle tilted away from the diameter
- 51** A ring of radius R is uniformly charged. Linear charge density is λ . An imaginary sphere of radius R is drawn with its centre on circumference of ring. Total electric flux passing through the sphere would be
 (a) $\frac{2\pi R\lambda}{\epsilon_0}$ (b) $\frac{\pi R\lambda}{\epsilon_0}$
 (c) zero (d) None of these
- 52** Three point charges as shown are placed at the vertices of an isosceles right angled triangle. Which of the numbered vectors coincides in direction with the electric field at the mid-point M of the hypotenuse?



- (a) 1 (b) 2 (c) 3 (d) 4

- 53** ABC is an equilateral triangle. Charges $+q$ are placed at each corner. The electric intensity at O (say the centroid of the triangle) will be



- (a) $\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$ (b) $\frac{3}{4\pi\epsilon_0} \cdot \frac{q}{r}$ (c) zero (d) $\frac{1}{4\pi\epsilon_0} \cdot \frac{3q}{r^2}$

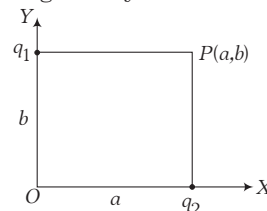
- 54** Equal charges q are placed at the four corners A, B, C and D of a square of length a . The magnitude of the force on the charge at B will be

- (a) $\frac{3q^2}{4\pi\epsilon_0 a^2}$ (b) $\frac{q^2}{4\pi\epsilon_0 a^2}$
 (c) $\left(\frac{1+2\sqrt{2}}{2}\right) \frac{q^2}{4\pi\epsilon_0 a^2}$ (d) $\left(2+\frac{1}{\sqrt{2}}\right) \frac{q^2}{4\pi\epsilon_0 a^2}$

- 55** A small element l is cut from a circular ring of radius a and charge per unit length λ . The net electric field at the centre of ring is

- (a) zero (b) $\frac{-\lambda l}{4\pi\epsilon_0 a^2}$
 (c) infinity (d) $\frac{\lambda}{4\pi\epsilon_0 l}$

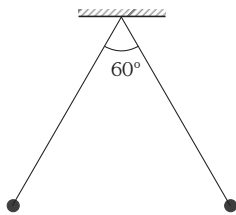
- 56** Two point charges $q_1 = 2 \mu\text{C}$ and $q_2 = 1 \mu\text{C}$ are placed at distances $b = 1$ cm and $a = 2$ cm from the origin of the Y and X -axis as shown in figure. The electric field vector at point $P(a, b)$ will subtend an angle θ with the X -axis given by



- (a) $\tan \theta = 1$ (b) $\tan \theta = 2$
 (c) $\tan \theta = 3$ (d) $\tan \theta = 4$

- 57** Two small spherical balls each carrying a charge $Q = 10\mu\text{C}$ (10 micro-coulomb) are suspended by two insulating threads of equal lengths 1 m each, from a point fixed in the ceiling. It is found that in equilibrium threads are separated by an angle 60° between them, as shown in the figure. What is the tension in the threads ?

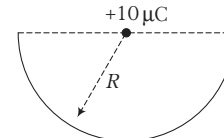
(Given, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm/C}^2$)



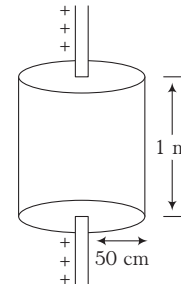
- (a) 18 N (b) 1.8 N
(c) 0.18 N (d) None of these
- 58** An infinite number of charges, each of charge $1\mu\text{C}$, are placed on the X -axis with coordinates $x = 1, 2, 4, 8, \dots, \infty$. If a charge of 1 C is kept at the origin, then what is the net force acting on 1C charge
- (a) 9000 N (b) 12000 N
(c) 24000 N (d) 36000 N
- 59** An electron moving with the speed $5 \times 10^6 \text{ ms}^{-1}$ is shot parallel to the electric field of intensity $1 \times 10^3 \text{ N/C}$. Field is responsible for the retardation of motion of electron. Now, evaluate the distance travelled by the electron before coming to rest for an instant. (Mass of electron $= 9 \times 10^{-31} \text{ kg}$ and charge $= 1.6 \times 10^{-19} \text{ C}$)
- (a) 7 m (b) 0.7 mm
(c) 7 cm (d) 0.7 cm
- 60** An electric dipole coincides on Z -axis and its mid-point is on origin of the coordinates system. The electric field at an axial point at a distance z from origin is $\mathbf{E}_{(z)}$ and electric field at an equatorial point at a distance y from origin is $\mathbf{E}_{(y)}$. Here, $z = y \gg a$, so $\frac{|\mathbf{E}_{(z)}|}{|\mathbf{E}_{(y)}|} = \dots$
- (a) 1 (b) 4
(c) 3 (d) 2
- 61** Three point charges $+q, -2q$ and $+q$ are placed at points $(x = 0, y = a, z = 0)$, $(x = 0, y = 0, z = 0)$ and $(x = a, y = 0, z = 0)$, respectively. The magnitude and direction of the electric dipole moment vector of this charge assembly are
- (a) $\sqrt{2}qa$ along $+y$ -direction

- (b) $\sqrt{2}qa$ along the line joining points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$
(c) qa along the line joining points $(x = 0, y = 0, z = 0)$ and $(x = a, y = a, z = 0)$
(d) $\sqrt{2}qa$ along $+x$ -direction

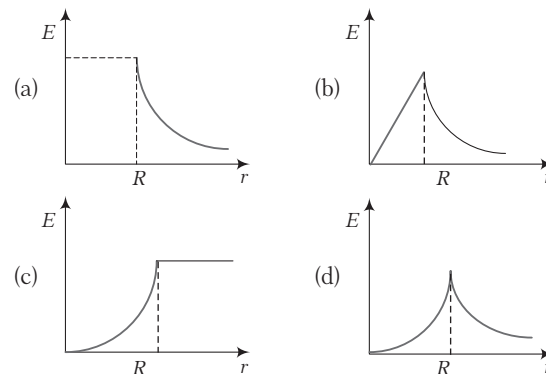
- 62** A charge $10\mu\text{C}$ is placed at the centre of a hemisphere of radius $R = 10 \text{ cm}$ as shown. The electric flux through the hemisphere (in MKS units) is



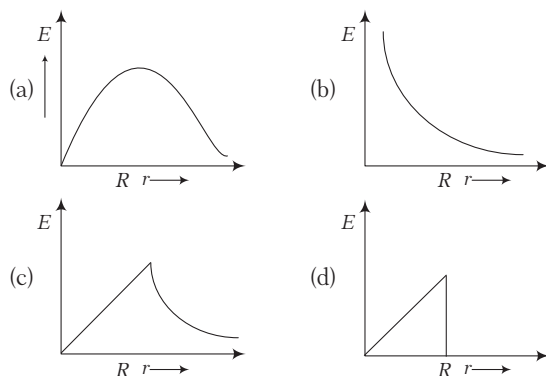
- (a) 20×10^5 (b) 10×10^5
(c) 6×10^5 (d) 2×10^5
- 63** Electric charge is uniformly distributed along a long straight wire of radius 1 mm. The charge per centimetre length of the wire is Q coulomb. Another cylindrical surface of radius 50 cm and length 1 m symmetrically encloses the wire as shown in the figure. The total electric flux passing through the cylindrical surface is



- (a) $\frac{Q}{\epsilon_0}$ (b) $\frac{100Q}{\epsilon_0}$ (c) $\frac{10Q}{(\pi\epsilon_0)}$ (d) $\frac{100Q}{(\pi\epsilon_0)}$
- 64** Which of the following graphs shows the variation of electric field E due to a hollow spherical conductor of radius R as a function of distance from the centre of the sphere?



- 65** In a uniformly charged non-conducting sphere of total charge Q and radius R , the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be



- 66** An electric dipole is situated in an electric field of uniform intensity E whose dipole moment is p and moment of inertia is I . If the dipole is displaced slightly from the equilibrium position, then the angular frequency of its oscillations is

(a) $\left(\frac{pE}{I}\right)^{1/2}$ (b) $\left(\frac{pE}{I}\right)^{3/2}$
 (c) $\left(\frac{I}{pE}\right)^{1/2}$ (d) $\left(\frac{p}{IE}\right)^{1/2}$

- 67** Two point charges $(+Q)$ and $(-2Q)$ are fixed on the X -axis at positions a and $2a$ from origin, respectively. At what positions on the axis, the resultant electric field is zero

(a) Only $x = \sqrt{2}a$ (b) Only $x = -\sqrt{2}a$
 (c) Both $x = \pm \sqrt{2}a$ (d) Only $x = \frac{3a}{2}$

- 68** Charge q_2 of mass m revolves around a stationary charge q_1 in a circular orbit of radius r . The orbital periodic time of q_2 would be

(a) $\left[\frac{4\pi^2 m r^3}{k q_1 q_2}\right]^{1/2}$ (b) $\left[\frac{k q_1 q_2}{4\pi^2 m r^3}\right]^{1/2}$
 (c) $\left[\frac{4\pi^2 m r^4}{k q_1 q_2}\right]^{1/2}$ (d) $\left[\frac{4\pi^2 m r^2}{k q_1 q_2}\right]^{1/2}$

- 69** Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is

(a) $\frac{-Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$
 (c) $\frac{-Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

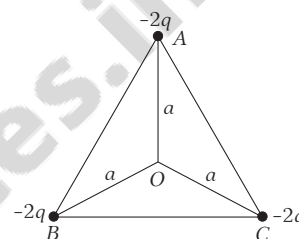
- 70** The distance between the two charges $25\mu\text{C}$ and $36\mu\text{C}$ is 11 cm . At what point on the line joining the two, the intensity will be zero?

- (a) At a distance of 5 cm from $25\mu\text{C}$
 (b) At a distance of 5 cm from $36\mu\text{C}$
 (c) At a distance of 4 cm from $25\mu\text{C}$
 (d) At a distance of 4 cm from $36\mu\text{C}$

- 71** If 10^{10} electrons are acquired by a body every second, the time required for the body to get a total charge of 1 C will be

- (a) 2 h (b) 2 days (c) 2 yr (d) 20 yr

- 72** ABC is an equilateral triangle. Charges $-2q$ are placed at each corner. The electric intensity at O will be

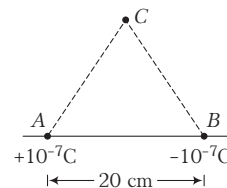


(a) $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ (b) $\frac{1}{4\pi\epsilon_0} \frac{q}{r}$
 (c) zero (d) $\frac{1}{4\pi\epsilon_0} \frac{3q}{r^2}$

- 73** Two equally charged, identical metal spheres A and B repel each other with a force F . The spheres are kept fixed with a distance r between them. A third identical, but uncharged sphere C is brought in contact with A and then placed at the mid-point of the line joining A and B . The magnitude of the net electric force on C is

- (a) F (b) $F/4$ (c) $F/2$ (d) $4F$

- 74** Two point charges $+10^{-7}\text{ C}$ and -10^{-7} C are placed at A and B , 20 cm apart as shown in the figure. Calculate the electric field at C , 20 cm apart from both A and B .



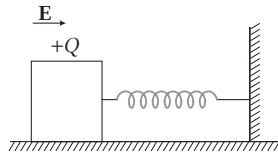
(a) $1.5 \times 10^{-5}\text{ N C}^{-1}$ (b) $2.2 \times 10^4\text{ N C}^{-1}$
 (c) $3.5 \times 10^6\text{ N C}^{-1}$ (d) $3.0 \times 10^5\text{ N C}^{-1}$

- 75** Two copper balls, each weighing 10 g , are kept in air 10 cm apart. If one electron from every 10^6 atoms is transferred from one ball to the other, then

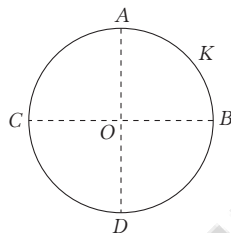
the coulomb force between them is (atomic weight of copper is 63.5)

- (a) 2.0×10^{10} N (b) 2.0×10^4 N
(c) 2.0×10^8 N (d) 2.0×10^6 N

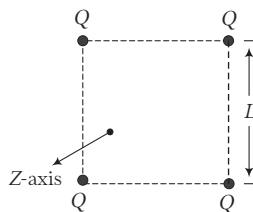
- 76** A wooden block performs SHM on a frictionless surface with frequency ν_0 . The block carries a charge $+Q$ on its surface. If now a uniform electric field $\mathbf{E}$ is switched on as shown, then SHM of the block will be



- (a) of the same frequency and with shifted mean position
(b) of the same frequency and with the same mean position
(c) of changed frequency and with shifted mean position
(d) of changed frequency and with the same mean position
- 77** A thin conducting ring of radius R is given a charge $+Q$. The electric field at the centre O of the ring due to the charge on the part AKB of the ring is E . The electric field at the centre due to the charge on the part $ACDB$ of the ring is

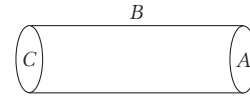


- (a) E along KO (b) $3E$ along OK
(c) $3E$ along KO (d) E along OK
- 78** Four point positive charges of same magnitude (Q) are placed at four corners of a rigid square frame as shown in figure. The plane of the frame is perpendicular to Z -axis. If a negative point charge is placed at a distance z away from the frame ($z \ll L$), then



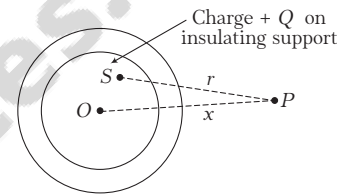
- (a) negative charge oscillates along the Z -axis
(b) it moves away from the frame
(c) it moves slowly towards the frame and stays in the plane of the frame
(d) it passes through the frame only once

- 79** A hollow cylinder has a charge q coulomb within it. If ϕ is the electric flux in units of volt-metre associated with the curved surface B , then the flux linked with the plane surface A in units of volt-metre will be



- (a) $\frac{1}{2} \left(\frac{q}{\epsilon_0} - \phi \right)$ (b) $\frac{q}{2\epsilon_0}$ (c) $\frac{\phi}{3}$ (d) $\frac{q}{\epsilon_0} - \phi$

- 80** The adjacent diagram shows a charge $+Q$ held on an insulating support S and enclosed by a hollow spherical conductor. O represents the centre of the spherical conductor and P is a point such that $OP = x$ and $SP = r$. The electric field at point P will be



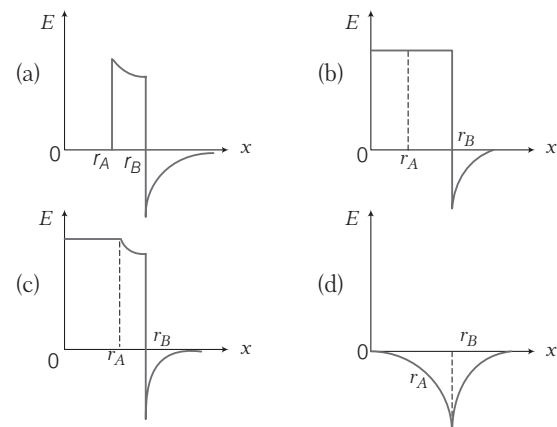
- (a) $\frac{Q}{4\pi\epsilon_0 x^2}$ (b) $\frac{Q}{4\pi\epsilon_0 r^2}$
(c) 0 (d) None of these

- 81** An infinitely long thin straight wire has uniform linear charge density of $\frac{1}{3} \text{ Cm}^{-1}$. Then, the

magnitude of the electric intensity at a point 18 cm away is (given, $\epsilon_0 = 8.8 \times 10^{-12} \text{ C}^2 \text{ Nm}^{-2}$)

- (a) $0.33 \times 10^{11} \text{ NC}^{-1}$ (b) $3 \times 10^{11} \text{ NC}^{-1}$
(c) $0.66 \times 10^{11} \text{ NC}^{-1}$ (d) $1.32 \times 10^{11} \text{ NC}^{-1}$

- 82** Two concentric conducting thin spherical shells A and B having radii r_A and r_B ($r_B > r_A$) are charged to Q_A and $-Q_B$ ($|Q_B| > |Q_A|$). The electrical field along a line, (passing through the centre) is



- 83** Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d ($d \ll l$) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the spheres approach each other with a velocity v . Then, v as a function of distance x between them is,

(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$ (c) $v \propto x^{1/2}$ (d) $v \propto x$

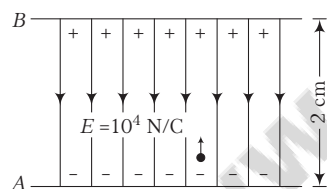
- 84** Charge q is uniformly distributed over a thin half ring of radius R . The electric field at the centre of the ring is

(a) $\frac{q}{2\pi^2\epsilon_0 R^2}$ (b) $\frac{q}{4\pi^2\epsilon_0 R^2}$
(c) $\frac{q}{4\pi\epsilon_0 R^2}$ (d) $\frac{q}{2\pi\epsilon_0 R^2}$

- 85** At what distance along the central axis of a uniformly charged plastic disc of radius R is the magnitude of the electric field equal to one-half the magnitude of the field at the centre of the surface of the disc?

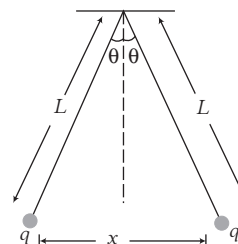
(a) $\frac{R}{\sqrt{2}}$ (b) $\frac{R}{\sqrt{3}}$ (c) $\sqrt{2}R$ (d) $\sqrt{3}R$

- 86** An electron is released from the bottom plate A as shown in the figure ($E = 10^4$ N/C). The velocity of the electron when it reaches plate B will be nearly equal to



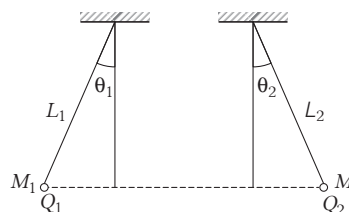
(a) $0.85 \times 10^7 \text{ ms}^{-1}$ (b) $1.0 \times 10^7 \text{ ms}^{-1}$
(c) $1.25 \times 10^7 \text{ ms}^{-1}$ (d) $1.65 \times 10^7 \text{ ms}^{-1}$

- 87** In the given figure, two tiny conducting balls of identical mass m and identical charge q hang from non-conducting threads of equal length L . Assume that θ is so small that $\tan \theta \approx \sin \theta$, then for equilibrium x is equal to



(a) $\left(\frac{q^2 L}{2\pi\epsilon_0 mg} \right)^{1/3}$ (b) $\left(\frac{q L^2}{2\pi\epsilon_0 mg} \right)^{1/3}$
(c) $\left(\frac{q^2 L^2}{4\pi\epsilon_0 mg} \right)^{1/3}$ (d) $\left(\frac{q^2 L}{4\pi\epsilon_0 mg} \right)^{1/3}$

- 88** Two small spheres of masses M_1 and M_2 are suspended by weightless insulating threads of lengths L_1 and L_2 . The spheres carry charges Q_1 and Q_2 , respectively. The spheres are suspended such that they are in level with one another and the threads are inclined to the vertical at angles of θ_1 and θ_2 as shown. Which one of the following conditions is essential, if $\theta_1 = \theta_2$?



(a) $M_1 \neq M_2$ but $Q_1 = Q_2$ (b) $M_1 = M_2$
(c) $Q_1 = Q_2$ (d) $L_1 = L_2$

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) These questions consist of two statements each printed as Assertion and Reason. While answering these questions you are required to choose anyone of the following four responses.

- (a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

- 1 Assertion** Due to two point charges, electric field and potential cannot be zero at two points.

Reason Field is a vector quantity.

- 2 Assertion** In a region, where uniform electric field exists, the net charge within volume of any size is zero.

Reason The electric flux within any closed surface in region of uniform electric field is zero.

- 3 Assertion** Electric lines of forces cross each other.

Reason Electric field at a point gives one direction.

- 4 Assertion** The surface charge densities of two spherical conductors of different radii are equal. Then, the electric field intensities near their surface are also equal.

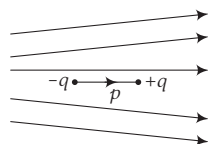
Reason Surface charge density is equal to charge per unit area.

- 5 Assertion** If a dipole is enclosed by a surface, then according to Gauss's law, electric flux linked with it will be zero.

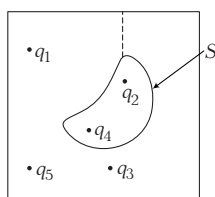
Reason The net charge enclosed by the surface is zero.

Statement based questions

- 1.** Figure shows electric field lines in which an electric dipole p is placed as shown. Which of the following statements is correct? [NCERT Exemplar]



- (a) The dipole will not experience any force
 (b) The dipole will experience a force towards right
 (c) The dipole will experience a force towards left
 (d) The dipole will experience a force upwards
- 2** Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).
 (a) The angular momentum of the charge $-q$ is constant
 (b) The linear momentum of the charge $-q$ is constant
 (c) The angular velocity of the charge $-q$ is constant
 (d) The linear speed of the charge $-q$ is constant
- 3** 'All charge on a conductor must reside on its outer surface'. This statement is true
 (a) in all cases
 (b) for spherical conductors only (Both solid and hollow)
 (c) for hollow spherical conductors only
 (d) for conductors which do not have any sharp points or corners
- 4** Five charges q_1, q_2, q_3, q_4 , and q_5 are fixed at their positions as shown in figure. S is a Gaussian surface. The Gauss's law is given by $\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$. Which of the following statements is correct?



[NCERT Exemplar]

- (a) $\mathbf{E}$ on the LHS of the above equation will have a contribution from q_1, q_5 and q_2, q_3 while q on the RHS will have a contribution from q_2 and q_4 only
 (b) $\mathbf{E}$ on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only
 (c) $\mathbf{E}$ on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only
 (d) Both $\mathbf{E}$ on the LHS and q on the RHS will have contribution from q_2 and q_4 only

- 5** Which of the following statement(s) is/are correct?

I. Two identical balls are charged by q . They are suspended from a common point by two insulating threads of length l each. In equilibrium, the maximum angle between the tension in the threads is 180° . (Ignore gravity).

II. In equilibrium tension in the springs is

$$T = \frac{1}{4\pi\epsilon_0} \frac{q \cdot q}{l^2}$$

- (a) Only I (b) Only II
 (c) Both I and II (d) None of these

Match the columns

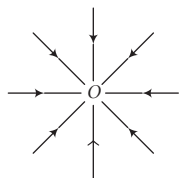
- 1.** Match the following two columns and choose the option from codes given below.

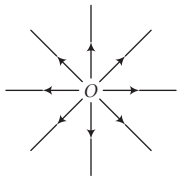
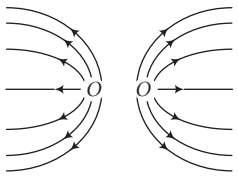
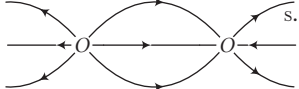
Column I	Column II
A. Electric charge	p. $[M^{-1}L^{-3}T^4A^2]$
B. Electric field strength	q. $[MLT^{-3}A^{-1}]$
C. Absolute permittivity	r. $[MT^{-3}A^{-1}]$
D. Electric dipole	s. None

Codes

- A B C D
 (a) s q p s
 (b) s q r p
 (c) q p s r
 (d) q q p s

- 2.** Match the field lines given in Column I with the charge configuration due to which field lines exist in Column II.

Column I	Column II
A. 	p. A pair of equal and opposite charges

Column I	Column II
B. 	q. A pair of positive charges
C. 	r. A single positive charge
D. 	s. A single negative charge

Codes

A	B	C	D	A	B	C	D
(a) s	q	r	q	(b) p	q	r	s
(c) s	r	q	p	(d) p	s	r	q

3. Four metallic plates are charged as shown in figure. Now, match the following two columns. Then, choose the option from codes given below.

	σ	-2σ	σ
I	II	III	IV

Column I	Column II
A. Electric field in region-I	p. $\frac{\sigma}{\epsilon_0}$
B. Electric field in region-II	q. $-\frac{\sigma}{\epsilon_0}$
C. Electric field in region-III	r. $\frac{\sigma}{2\epsilon_0}$
D. Electric field in region-IV	s. zero

Codes

A	B	C	D	A	B	C	D
(a) p	s	q	r	(b) s	p	q	s
(c) r	q	q	p	(d) s	s	p	q

(C) Medical entrances' gallery

Collection of questions asked in NEET & various medical entrance exams

- 1 A spherical conductor of radius 10 cm has a charge of 3.2×10^{-7} C distributed uniformly. What is the magnitude of electric field at a point 15 cm from the centre of the sphere?

$$\left(\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2 \right) \quad [\text{NEET 2020}]$$

- (a) 1.28×10^5 N/C (b) 1.28×10^6 N/C
(c) 1.28×10^7 N/C (d) 1.28×10^4 N/C

- 2 The electric field at a point on the equatorial plane at a distance r from the centre of a dipole having dipole moment $\mathbf{p}$ is given by ($r \gg$ separation of two charges forming the dipole, ϵ_0 = permittivity of free space) [NEET 2020]

$$\begin{aligned} \text{(a) } \mathbf{E} &= \frac{\mathbf{p}}{4\pi\epsilon_0 r^3} & \text{(b) } \mathbf{E} &= \frac{2\mathbf{p}}{4\pi\epsilon_0 r^3} \\ \text{(c) } \mathbf{E} &= -\frac{\mathbf{p}}{4\pi\epsilon_0 r^2} & \text{(d) } \mathbf{E} &= -\frac{\mathbf{p}}{4\pi\epsilon_0 r^3} \end{aligned}$$

- 3 The acceleration of an electron due to the mutual attraction between the electron and a proton when they are $1.6 \text{ \AA}$ apart is,

$$\left(\text{take, } \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}, m_e \approx 9 \times 10^{-31} \text{ kg}, e = 1.6 \times 10^{-19} \right)$$

[NEET 2020]

- (a) 10^{24} m/s^2 (b) 10^{23} m/s^2
(c) 10^{22} m/s^2 (d) 10^{25} m/s^2

- 4 Two point charges A and B , having charges $+Q$ and $-Q$ respectively, are placed at certain distance apart and force acting between them is F . If 25% charge of A is transferred to B , then force between the charges becomes [NEET 2019]

$$\text{(a) } \frac{9F}{16} \quad \text{(b) } \frac{16F}{9} \quad \text{(c) } \frac{4F}{3} \quad \text{(d) } F$$

- 5 Two parallel infinite line charges with linear charge densities $+\lambda \text{ C/m}$ and $-\lambda \text{ C/m}$ are placed at a distance of $2R$ in free space. What is the electric field mid-way between the two line charges? [NEET 2019]

$$\begin{aligned} \text{(a) } \frac{2\lambda}{\pi\epsilon_0 R} \text{ N/C} & \quad \text{(b) } \frac{\lambda}{\pi\epsilon_0 R} \text{ N/C} \\ \text{(c) } \frac{\lambda}{2\pi\epsilon_0 R} \text{ N/C} & \quad \text{(d) } \text{Zero} \end{aligned}$$

- 6** A hollow metal sphere of radius R is uniformly charged.

The electric field due to the sphere at a distance r from the centre

[NEET 2019]

- (a) zero as r increases for $r < R$, decreases as r increases for $r > R$
 (b) zero as r increases for $r < R$, increases as r increases for $r > R$
 (c) decreases as r increases for $r < R$ and for $r > R$
 (d) increases as r increases for $r < R$ and for $r > R$

- 7** Two metal spheres, one of radius R and the other of radius $2R$ respectively have the same surface charge density σ . They are brought in contact and separated. What will be the new surface charge densities on them?

[NEET Odisha 2019]

- (a) $\sigma_P = \frac{5}{6}\sigma$, $\sigma_Q = \frac{5}{2}\sigma$ (b) $\sigma_P = \frac{5}{2}\sigma$, $\sigma_Q = \frac{5}{6}\sigma$
 (c) $\sigma_P = \frac{5}{2}\sigma$, $\sigma_Q = \frac{5}{3}\sigma$ (d) $\sigma_P = \frac{5}{3}\sigma$, $\sigma_Q = \frac{5}{6}\sigma$

- 8** A sphere encloses an electric dipole with charge $\pm 3 \times 10^{-6}$ C. What is the total electric flux across the sphere?

[NEET Odisha 2019]

- (a) -3×10^{-6} N-m²/C (b) Zero
 (c) 3×10^6 N-m²/C (d) 6×10^{-6} N-m²/C

- 9** An electron falls from rest through a vertical distance h in a uniform and vertically upward directed electric field E . The direction of electric field is now reversed, keeping its magnitude the same. A proton is allowed to fall from rest in it through the same vertical distance h . The time of fall of the electron, in comparison to the time of fall of the proton is

[NEET 2018]

- (a) 10 times greater (b) 5 times greater
 (c) smaller (d) equal

- 10** Positive charge Q is distributed uniformly over a circular ring of radius R . A point particle having a mass (m) and a negative charge $-q$ is placed on its axis at a distance x from the centre. Assuming $x < R$, find the time period of oscillation of the particle, if it is released from there [neglect gravity].

[AIIMS 2018]

- (a) $\left[\frac{16\pi^3 \epsilon_0 R^3 m}{Qq} \right]^{1/2}$ (b) $\left[\frac{8\pi^2 \epsilon_0 R^3}{q} \right]^{1/2}$
 (c) $\left[\frac{2\pi^3 \epsilon_0 R^3}{3q} \right]^{1/2}$ (d) None of these

- 11** An electric dipole consists of two opposite charges each $0.05 \mu\text{C}$ separated by 30 mm. The dipole is placed in an uniform external electric field of 10^6 NC^{-1} . The maximum torque exerted by the field on the dipole is

[AIIMS 2018]

- (a) $6 \times 10^{-3} \text{ Nm}$ (b) $3 \times 10^{-3} \text{ Nm}$
 (c) $15 \times 10^{-3} \text{ Nm}$ (d) $1.5 \times 10^{-3} \text{ Nm}$

- 12** If point charges $Q_1 = 2 \times 10^{-7} \text{ C}$ and $Q_2 = 3 \times 10^{-7} \text{ C}$ are at 30 cm separation, then find electrostatic force between them.

[JIPMER 2018]

- (a) $2 \times 10^{-3} \text{ N}$ (b) $6 \times 10^{-3} \text{ N}$ (c) $5 \times 10^{-3} \text{ N}$ (d) $1 \times 10^{-3} \text{ N}$

- 13** Suppose the charge of a proton and an electron differ slightly. One of them is $-e$ and the other is $(e + \Delta e)$. If the net of electrostatic force and gravitational force between two hydrogen atoms placed at a distance d (much greater than atomic size) apart is zero, then Δe is of the order (Take, mass of hydrogen, $m_h = 1.67 \times 10^{-27} \text{ kg}$)

[NEET 2017]

- (a) 10^{-20} C (b) 10^{-23} C
 (c) 10^{-37} C (d) 10^{-47} C

- 14** A certain charge Q is divided into two parts q and $Q - q$. How the charge Q and q must be related, so that when q and $(Q - q)$ is placed at a certain distance apart experience maximum electrostatic repulsion?

[JIPMER 2017]

- (a) $Q = 2q$ (b) $Q = 3q$ (c) $Q = 4q$ (d) $Q = 4q + c$

- 15** Two identical conducting balls A and B have positive charges q_1 and q_2 respectively but $q_1 \neq q_2$. The balls are brought together so that they touch each other and then kept in their original positions. The force between them is

[JIPMER 2017]

- (a) less than that before the balls touched
 (b) greater than that before the balls touched
 (c) same as that before the balls touched
 (d) zero

- 16** A positively charged ball hangs from a silk thread. We put a positive test charge q_0 at a point and measure F / q_0 , then it can be predicted that the electric field strength E

[JIPMER 2017]

- (a) $> F / q_0$ (b) $= \frac{F}{q}$
 (c) $< F / q_0$ (d) Cannot be estimated

- 17** An electric dipole is placed at an angle of 30° with an electric field intensity $2 \times 10^5 \text{ N/C}$. It experiences a torque equal to 4 N-m. The charge on the dipole, if the dipole length is 2 cm is

[NEET 2016]

- (a) 8 mC (b) 2 mC
 (c) 5 mC (d) $7 \mu\text{C}$

- 18** The electric field in a certain region is acting radially outward and is given by $E = Ar$. A charge contained in a sphere of radius a centred at the origin of the field will be given by

[CBSE AIPMT 2015]

- (a) $4\pi\epsilon_0 Aa^2$ (b) $A\epsilon_0 a^2$
 (c) $4\pi\epsilon_0 Aa^3$ (d) $\epsilon_0 Aa^3$

- 19** An electron of mass M_e , initially at rest, moves through a certain distance in a uniform electric field in time t_1 . A proton of mass M_p also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2 / t_1 is nearly equal to [AIIMS 2015]

(a) 1 (b) $\sqrt{\frac{M_p}{M_e}}$ (c) $\sqrt{\frac{M_e}{M_p}}$ (d) 1836

- 20** A total charge of $5 \mu\text{C}$ is distributed uniformly on the surface of the thin walled hemispherical cup. If the electric field strength at the centre of the hemisphere is $9 \times 10^8 \text{ NC}^{-1}$, then the radius of the cup is

(Take, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N-m}^2\text{C}^{-2}$) [EAMCET 2015]

(a) 5 mm (b) 10 mm (c) 5 cm (d) 10 cm

- 21** Two small spherical shells A and B are given positive charges of 9 C and 4 C respectively and placed such that their centres are separated by 10 m. If P is a point in between them, where the electric field intensity is zero, then the distance of the point P from the centre of A is [Kerala CEE 2015]

(a) 5 m (b) 6 m (c) 7 m (d) 8 m
(e) 4 m

- 22** A point charge q is situated at a distance r on axis from one end of a thin conducting rod of length L having a charge Q [uniformly distributed along its length]. The magnitude of electric force between the two is [Guj. CET 2015]

(a) $\frac{kQq}{r^2}$ (b) $\frac{2kQ}{r(r+L)}$ (c) $\frac{kQq}{r(r-L)}$ (d) $\frac{kQq}{r(r+L)}$

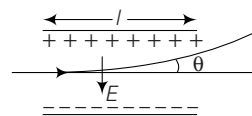
- 23** When 10^{19} electrons are removed from a neutral metal plate through some process, then the charge on it becomes [Guj. CET 2015]

(a) +1.6 C (b) -1.6 C (c) 10^{19} C (d) 10^{-19} C

- 24** A charge Q is uniformly distributed over a large plastic plate. The electric field at point P close to centre of plate is 10 Vm^{-1} . If the plastic plate is replaced by copper plate of the same geometrical dimension and carrying the same charge Q , then the electric field at that point will be [CG PMT 2015]

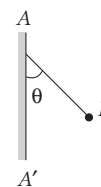
(a) zero (b) 5 Vm^{-1} (c) 10 Vm^{-1} (d) 20 Vm^{-1}

- 25** A uniform electric field is created between two parallel charged plates as shown below. An electron enters the field symmetrically between the plates with a speed of v_0 . The length of each plate is l . Find the angle of deviation of path of the electron as it comes out of the field. [CG PMT 2015]



(a) $\theta = \tan^{-1} \frac{El}{mv_0^2}$ (b) $\theta = \tan^{-1} \left(\frac{eEl}{mv_0^2} \right)$
(c) $\theta = \tan^{-1} \left(\frac{eEl}{mv_0} \right)$ (d) $\theta = \tan^{-1} \left(\frac{eE}{mv_0^2} \right)$

- 26** The line AA' is on charged infinite conducting plane which is perpendicular to the plane of the paper. The plane has a surface density of charge σ and B is ball of mass m with a like charge of magnitude q . B is connected by string from a point on the line AA' . The tangent of angle (θ) formed between the line AA' and the string is [WB JEE 2015]



(a) $\frac{q\sigma}{2\epsilon_0 mg}$ (b) $\frac{q\sigma}{4\pi\epsilon_0 mg}$
(c) $\frac{q\sigma}{2\pi\epsilon_0 mg}$ (d) $\frac{q\sigma}{\epsilon_0 mg}$

- 27** The angle between the dipole moment and electric field at any point on the equatorial plane is [KCET 2015]

(a) 180° (b) 0° (c) 45° (d) 90°

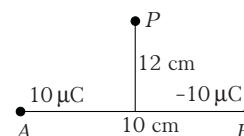
- 28** Pick out the statement which is incorrect? [KCET 2015]

(a) A negative test charge experiences a force opposite to the direction of the field.
(b) The tangent drawn to a line of force represents the direction of electric field.
(c) Field lines never intersect.
(d) The electric field lines form closed loop.

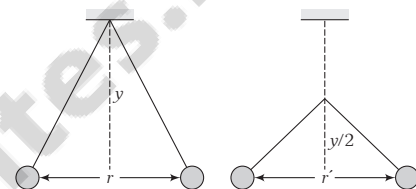
- 29** A Gaussian surface in the cylinder of cross-section πa^2 and length L is immersed in a uniform electric field $\mathbf{E}$ with the cylinder axis parallel to the field. The flux ϕ of the electric field through the closed surface is [EAMCET 2015]

(a) $2\pi a^2 \mathbf{E}$ (b) $\pi a^2 \mathbf{E} L$ (c) $\pi a^2 (2+L) \mathbf{E}$ (d) zero

- 30** Two charges of $10 \mu\text{C}$ and $-10 \mu\text{C}$ are placed at points A and B separated by a distance of 10 cm. Find the electric field at a point P on the perpendicular bisector of AB at a distance of 12 cm from its middle point. [UK PMT 2015]



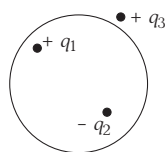
(a) $16.4 \times 10^6 \text{ NC}^{-1}$ (b) $28.4 \times 10^6 \text{ NC}^{-1}$
(c) $8.2 \times 10^6 \text{ NC}^{-1}$ (d) $4.1 \times 10^6 \text{ NC}^{-1}$

- 31** If the electric field lines is flowing along axis of a cylinder, then the flux of this field through the cylindrical surface with the axis parallel to the field is [where, r = radius of cylinder] [UP CPMT 2015]
 (a) $\frac{\sigma}{2\pi r\epsilon_0}$ (b) $\frac{\sigma}{\epsilon_0}$ (c) zero (d) $\frac{\sigma}{2\epsilon_0}$
- 32** An inclined plane of length 5.60 m making an angle of 45° with the horizontal is placed in a uniform electric field $E = 100 \text{ Vm}^{-1}$. A particle of mass 1 kg and charge 10^{-2} C is allowed to slide down from rest position from maximum height of slope. If the coefficient of friction is 0.1, then the time taken by the particle to reach the bottom is [Guj. CET 2015]
 (a) 1 s (b) 1.41 s
 (c) 2 s (d) None of these
- 33** Two charged spheres separated at a distance d exert a force F on each other. If they are immersed in a liquid of dielectric constant $K = 2$, then the force (if all conditions are same) is [UK PMT 2014]
 (a) $F/2$ (b) F
 (c) $2F$ (d) $4F$
- 34** If a charge on the body is 1 nC, then how many electrons are present on the body? [KCET 2014]
 (a) 1.6×10^{19} (b) 6.25×10^9
 (c) 6.25×10^{27} (d) 6.25×10^{28}
- 35** Electric field at a point of distance r from a uniformly charged wire of infinite length having linear charge density λ is directly proportional to [Kerala CEE 2014]
 (a) r^{-1} (b) r
 (c) r^2 (d) r^{-2}
- 36** Two equal and opposite charges of masses m_1 and m_2 are accelerated in a uniform electric field through the same distance. What is the ratio of their accelerations, if their ratio of masses is $\frac{m_1}{m_2} = 0.5$? [KCET 2014]
 (a) $\frac{a_1}{a_2} = 0.5$ (b) $\frac{a_1}{a_2} = 1$
 (c) $\frac{a_1}{a_2} = 2$ (d) $\frac{a_1}{a_2} = 3$
- 37** An electric dipole of dipole moment $\mathbf{p}$ is placed in a uniform external electric field $\mathbf{E}$. Then, the [Kerala CEE 2014]
 (a) torque experienced by the dipole is $\mathbf{E} \times \mathbf{p}$
 (b) torque is zero, if $\mathbf{p}$ is perpendicular to $\mathbf{E}$
 (c) torque is maximum, if $\mathbf{p}$ is perpendicular to $\mathbf{E}$
 (d) potential energy is maximum, if $\mathbf{p}$ is parallel to $\mathbf{E}$
 (e) potential energy is maximum, if $\mathbf{p}$ is perpendicular to $\mathbf{E}$
- 38** An electric dipole placed in a non-uniform electric field experiences [UK PMT 2014]
 (a) Both a torque and a net force
 (b) Only a force but no torque
 (c) Only a torque but no net force
 (d) No torque and no net force
- 39** What is the nature of Gaussian surface involved in Gauss's law of electrostatics? [KCET 2014]
 (a) Scalar (b) Electrical
 (c) Magnetic (d) Vector
- 40** Two pith balls carrying equal charges are suspended from a common point by strings of equal length, the equilibrium separation between them is r . Now, the strings are rigidly clamped at half the height. The equilibrium separation between the balls now becomes [NEET 2013]
- 
- (a) $\left(\frac{1}{\sqrt{2}}\right)^2$ (b) $\left(\frac{r}{\sqrt{2}}\right)$ (c) $\left(\frac{2r}{\sqrt{3}}\right)$ (d) $\left(\frac{2r}{3}\right)$
- 41** An electric charge does not have which of the following properties? [J&K CET 2013]
 (a) Total charge conservation
 (b) Quantisation of charge
 (c) Two types of charge
 (d) Circular line of force
- 42** The force of repulsion between two electrons at a certain distance is F . The force between two protons separated by the same distance is ($m_p = 1836m_e$) [Kerala CET 2013]
 (a) $2F$ (b) F (c) $1836F$ (d) $\frac{F}{1836}$
- 43** Equal charge q each are placed at the vertices A and B of an equilateral triangle ABC of side a . The magnitude of electric intensity at the point C is [UP CPMT 2012]
 (a) $\frac{q}{4\pi\epsilon_0 a^2}$ (b) $\frac{\sqrt{2} q}{4\pi\epsilon_0 a^2}$ (c) $\frac{\sqrt{3} q}{4\pi\epsilon_0 a^2}$ (d) $\frac{2q}{4\pi\epsilon_0 a^2}$
- 44** If two charges $+4e$ and $+e$ are at a distance x apart, then at what distance charge q must be placed from $+e$, so that it is in equilibrium? [BCECE (Mains) 2012]
 (a) $\frac{x}{2}$ (b) $\frac{x}{3}$
 (c) $\frac{x}{6}$ (d) $\frac{2x}{3}$

- 45** If a mass of 20 g having charge 3.0 mC moving with velocity 20 ms^{-1} enters a region of electric field of 80 NC^{-1} in the same direction as the velocity of mass, then the velocity of mass after 3 s in the region will be [BCECE 2012]

(a) 40 ms^{-1} (b) 44 ms^{-1}
(c) 56 ms^{-1} (d) 80 ms^{-1}

- 46** The given figure shows a spherical Gaussian surface and a charge distribution. When calculating the flux of electric field through the Gaussian surface, then the electric field will be due to [AMU 2012]



(a) $+q_3$ alone (b) $+q_1$ and $+q_3$
(c) $+q_1$, $+q_3$ and $-q_2$ (d) $+q_1$ and $-q_2$

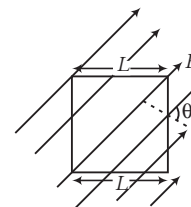
- 47** If the electric field is given by $(5\hat{i} + 4\hat{j} + 9\hat{k})$, then the electric flux through a surface of area 20 unit lying in the yz -plane will be [AFMC 2012]

(a) 100 unit (b) 80 unit
(c) 180 unit (d) 20 unit

- 48** A charge Q is enclosed by a Gaussian spherical surface of radius R . If the radius is doubled, then the outward electric flux will [CBSE AIPMT 2011]

(a) be reduced to half (b) remain the same
(c) be doubled (d) increase four times

- 49** A square surface of side L metre in the plane of the paper is placed in a uniform electric field E (volt/metre) acting along the same plane at an angle θ with the horizontal side of the square as shown in figure. The electric flux linked to the surface in unit of Nm^2C^{-1} is [CBSE AIPMT 2011]



(a) EL^2 (b) $EL^2 \cos \theta$
(c) $EL^2 \sin \theta$ (d) zero

- 50** The electric field at a point due to an electric dipole, on an axis inclined at an angle $\theta (< 90^\circ)$ to the dipole axis, is perpendicular to the dipole axis, if the angle θ is [KCET 2011]

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}(1/2)$
(c) $\tan^{-1}(\sqrt{2})$ (d) $\tan^{-1}(1/\sqrt{2})$

- 51** A soap bubble is given negative charge, then its radius will be [DCE 2011]

(a) increase (b) decrease
(c) remain changed (d) fluctuate

- 52** An electric dipole is placed in a uniform electric field with the dipole axis making an angle θ with the direction of the electric field. The orientation of the dipole for stable equilibrium is [J&K CET 2011]

(a) $\pi/6$ (b) $\pi/3$
(c) 0 (d) $\pi/2$

- 53** There exists an electric field of 1 N/C along y -direction. The flux passing through the square of 1 m placed in xy -plane inside the electric field is [J&K CET 2011]

(a) $1.0 \text{ Nm}^2\text{C}^{-1}$ (b) $10.0 \text{ Nm}^2\text{C}^{-1}$
(c) $2.0 \text{ Nm}^2\text{C}^{-1}$ (d) zero

- 54** The total electric flux emanating from a closed surface enclosing an α -particle is (e = electronic charge) [Kerala CEE 2011]

(a) $2e/\epsilon_0$ (b) e/ϵ_0
(c) $e\epsilon_0$ (d) $\epsilon_0 e/4$
(e) $4e/\epsilon_0$

ANSWERS

CHECK POINT 1.1

1. (d)	2. (a)	3. (d)	4. (c)	5. (c)	6. (d)	7. (b)	8. (c)	9. (a)	10. (d)
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CHECK POINT 1.2

1. (a)	2. (c)	3. (b)	4. (b)	5. (a)	6. (d)	7. (d)	8. (c)	9. (d)	10. (d)
11. (c)	12. (b)								

CHECK POINT 1.3

1. (b)	2. (b)	3. (d)	4. (a)	5. (c)	6. (d)	7. (c)	8. (c)
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CHECK POINT 1.4

1. (a)	2. (b)	3. (a)	4. (c)	5. (a)	6. (d)	7. (d)	8. (d)
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CHECK POINT 1.5

1. (a)	2. (d)	3. (b)	4. (a)	5. (d)	6. (b)	7. (c)	8. (d)	9. (a)	10. (a)
11. (b)	12. (a)	13. (c)							

(A) Taking it together

1. (a)	2. (b)	3. (c)	4. (d)	5. (c)	6. (c)	7. (a)	8. (d)	9. (b)	10. (a)
11. (c)	12. (b)	13. (b)	14. (d)	15. (a)	16. (c)	17. (b)	18. (a)	19. (c)	20. (b)
21. (c)	22. (c)	23. (a)	24. (a)	25. (d)	26. (b)	27. (b)	28. (c)	29. (a)	30. (d)
31. (b)	32. (d)	33. (a)	34. (c)	35. (c)	36. (a)	37. (b)	38. (a)	39. (b)	40. (b)
41. (a)	42. (a)	43. (a)	44. (d)	45. (c)	46. (c)	47. (c)	48. (c)	49. (a)	50. (a)
51. (d)	52. (b)	53. (c)	54. (c)	55. (b)	56. (b)	57. (b)	58. (b)	59. (c)	60. (d)
61. (b)	62. (c)	63. (b)	64. (a)	65. (c)	66. (a)	67. (b)	68. (a)	69. (b)	70. (a)
71. (d)	72. (c)	73. (a)	74. (b)	75. (c)	76. (a)	77. (d)	78. (a)	79. (a)	80. (a)
81. (a)	82. (a)	83. (a)	84. (a)	85. (b)	86. (a)	87. (a)	88. (b)		

(B) Medical entrance special format questions

• Assertion and reason

1. (b)	2. (a)	3. (d)	4. (b)	5. (a)
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• Statement based questions

1. (c)	2. (a)	3. (a)	4. (b)	5. (a)
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• Match the columns

1. (a)	2. (c)	3. (b)
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(C) Medical entrances' gallery

1. (a)	2. (a)	3. (c)	4. (a)	5. (b)	6. (a)	7. (d)	8. (b)	9. (c)	10. (a)
11. (d)	12. (b)	13. (c)	14. (a)	15. (b)	16. (a)	17. (b)	18. (c)	19. (b)	20. (a)
21. (b)	22. (d)	23. (a)	24. (c)	25. (b)	26. (a)	27. (a)	28. (d)	29. (d)	30. (d)
31. (c)	32. (b)	33. (a)	34. (b)	35. (a)	36. (c)	37. (c)	38. (a)	39. (d)	40. (b)
41. (d)	42. (b)	43. (c)	44. (b)	45. (c)	46. (c)	47. (a)	48. (b)	49. (d)	50. (c)
51. (a)	52. (c)	53. (d)	54. (a)						

Hints & Explanations

• CHECK POINT 1.1

1 (d) Negative charge means excess of electron which increases the mass of sphere B.

2 (a) Charge on the sphere due to removed electrons
 $q = +ne = 10^{14} \times 1.6 \times 10^{-19}$

$$\therefore q = 1.6 \times 10^{-5} \text{C} = 16 \mu\text{C}$$

3 (d) A conductor has positive charge. So, there is a deficiency of electrons.

$$\therefore \text{Number of electrons} = \frac{14.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 9$$

4 (c) Charge on α -particle, $q = ne$

$$\therefore q = +2e = 2 \times 1.6 \times 10^{-19} \text{C} = 3.2 \times 10^{-19} \text{C}$$

5 (c) As, $q = ne$ or $n = \frac{q}{e}$

$$\therefore \text{Number of extra electrons, } n = \frac{80 \times 10^{-6}}{1.6 \times 10^{-19}} = 5 \times 10^{14}$$

7 (b) When we rub glass rod with silk, excess electrons are transferred from glass to silk. So, glass rod becomes positively charged and silk becomes negatively charged.

9 (a) When positively charged body connected to earth, then electrons flow from earth to body and body becomes neutral or uncharged.

• CHECK POINT 1.2

2 (c) Force, $F = 9 \times 10^9 \cdot \frac{q^2}{r^2}$

$$\Rightarrow F = 9 \times 10^9 \cdot \frac{(2 \times 10^{-6})^2}{(0.5)^2} = 0.144 \text{ N}$$

3 (b) Force, $F_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{(1 \times 10^{-6})(5 \times 10^{-6})}{r^2}$

[if distance between them is r]

$$\text{Also, force, } F_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{(5 \times 10^{-6})(1 \times 10^{-6})}{r^2}$$

$$\therefore \frac{F_1}{F_2} = \frac{1}{1}$$



But direction of F_1 and F_2 are different.

4 (b) According to Coulomb's law,

$$F \propto \frac{1}{r^2} \Rightarrow \frac{F_1}{F_2} = \left(\frac{r_2}{r_1}\right)^2$$

$$\therefore \frac{5}{F_2} = \left(\frac{0.04}{0.06}\right)^2$$

$\therefore$ Force between two charges, $F_2 = 11.25 \text{ N}$

$$5 (a) F = \frac{1}{4\pi\epsilon_0} \cdot \frac{(+7 \times 10^{-6})(-5 \times 10^{-6})}{r^2} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{35 \times 10^{12}}{r^2} \text{ N}$$

$$F' = \frac{1}{4\pi\epsilon_0} \cdot \frac{(+5 \times 10^{-6})(-7 \times 10^{-6})}{r^2} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{35 \times 10^{12}}{r^2} \text{ N}$$

$$\therefore F' = F$$

$$6 (d) F = K \cdot \frac{q^2}{r^2}$$

If q is halved, r is doubled, then

$$\Rightarrow F' = K \cdot \frac{(q/2)^2}{(2r)^2}$$

$$\Rightarrow F' = K \cdot \frac{q^2}{16r^2}$$

$$\Rightarrow F' = \frac{F}{16}$$

The new force acting on each charge is $\frac{F}{16}$.

7 (d) By using, $K = \frac{F_a}{F_m}$

$$\Rightarrow K = \frac{10^{-4}}{2.5 \times 10^{-5}} = 4$$

8 (c) We have, $F = F'$ or $\frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r'^2} K \Rightarrow r' = \frac{r}{\sqrt{K}}$

9 (d) Gravitational force, $F_g = \frac{G(m_e)(m_e)}{r^2}$

$$\text{Also, electrostatic force, } F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{(e)(e)}{r^2}$$

$$\therefore \frac{F_g}{F_e} = \frac{G(m_e)^2}{\left(\frac{1}{4\pi\epsilon_0}\right)e^2} = \frac{6.67 \times 10^{-11} \times (9.1 \times 10^{-31})^2}{9 \times 10^9 \times (1.6 \times 10^{-19})^2} = 2.39 \times 10^{-43}$$

So, ratio of F_g/F_e is of order 10^{-43} .

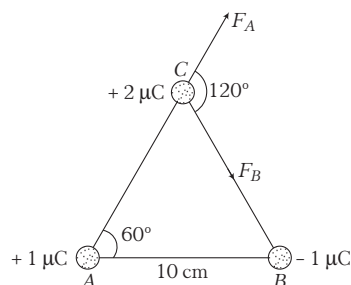
10. (d) They will not experience any force, if $|\mathbf{F}_G| = |\mathbf{F}_e|$

$$\Rightarrow \frac{Gm^2}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{r^2}$$

$$\Rightarrow \frac{q^2}{m^2} = 4\pi\epsilon_0 G$$

$$\Rightarrow q/m = \sqrt{4\pi\epsilon_0 G}$$

12. (b) Let F_A = Force on charge at C due to charge at A



$$\therefore F_A = 9 \times 10^9 \times \frac{10^{-6} \times 2 \times 10^{-6}}{(10 \times 10^{-2})^2} = 1.8 \text{ N}$$

Similarly, F_B = Force on charge at point C due to charge at B

$$= 9 \times 10^9 \times \frac{10^{-6} \times 2 \times 10^{-6}}{(10 \times 10^{-2})^2} = 1.8 \text{ N}$$

$\therefore$ Net force on C ,

$$F_{\text{net}} = \sqrt{(F_A)^2 + (F_B)^2 + 2F_A F_B \cos 120^\circ}$$

$$= \sqrt{(1.8)^2 + (1.8)^2 + 2(1.8)(1.8)(-1/2)} = 1.8 \text{ N}$$

• CHECK POINT 1.3

1. (b) $QE = mg \Rightarrow Q = \frac{mg}{E} = \frac{5 \times 10^{-5} \times 10}{10^7} = 5 \times 10^{-5} \mu\text{C}$

Since, electric field is acting downward, so for balancing charge must be negative.

2. (b) Electric field, $E = 9 \times 10^9 \cdot \frac{Q}{r^2}$

$$= 9 \times 10^9 \times \frac{5 \times 10^{-6}}{(0.8)^2} \approx 7 \times 10^4 \text{ N/C}$$

3 (d) Electric field,

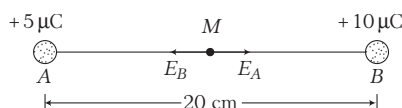
$$E = 9 \times 10^9 \times \frac{Q}{r^2}$$

$$\Rightarrow 500 = 9 \times 10^9 \times \frac{Q}{(3)^2}$$

$$\Rightarrow Q = 0.5 \mu\text{C}$$

4 (a) E_A = Electric field at mid-point M due to $+5 \mu\text{C}$ charge

$$= 9 \times 10^9 \times \frac{5 \times 10^{-6}}{(0.1)^2} = 45 \times 10^5 \text{ N/C}$$



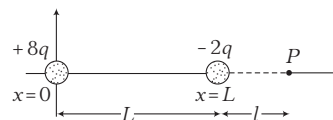
E_B = Electric field at M due to $+10 \mu\text{C}$ charge

$$= 9 \times 10^9 \times \frac{10 \times 10^{-6}}{(0.1)^2} = 90 \times 10^5 \text{ N/C}$$

Net electric field at $M = |\mathbf{E}_B| - |\mathbf{E}_A| = 45 \times 10^5 \text{ N/C}$

$= 4.5 \times 10^6 \text{ N/C}$, in the direction of E_B , i.e. towards $+5 \mu\text{C}$ charge.

- 5 (c) The net field will be zero at a point outside the charges and near the charge which is smaller in magnitude.



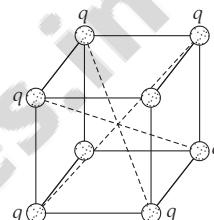
Suppose electric field is zero at P as shown in the figure.

Hence, at P ,

$$\frac{k 8q}{(L+l)^2} = \frac{k \cdot (2q)}{l^2} \Rightarrow l = L$$

So, distance of P from origin is, $L + L = 2L$

- 6 (d) Due to symmetry of charges, electric field intensity is equal and opposite due to charges. So, they cancel out. Hence, net electric field due to charge distribution at centre of cube is zero.



- 7 (c) At point A and C , electric field lines are dense and equally spaced, so $E_A = E_C$.

While at B , they are far apart.

$$\therefore E_A = E_C > E_B$$

- 8 (c) Electric lines of force never intersect the conductor. They are perpendicular and slightly curved near the surface of conductor.

• CHECK POINT 1.4

1. (a) Dipole moment, $p = q \times 2a = 1.6 \times 10^{-19} \times 4.3 \times 10^{-9}$

$$= 6.8 \times 10^{-28} \text{ C-m}$$

2. (b) On equatorial line electric field is given by

$$E_{\text{equatorial}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$$

On axial line,

$$E_{\text{axial}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}$$

$$\therefore E_{\text{axial}} = 2E_{\text{equatorial}}$$

or $E_a = 2E_e$

3 (a) Axial electric field, $E_{\text{axis}} = \frac{2kp}{r^3}$ [along P]

Equatorial electric field, $E_{\perp} = \frac{kp}{(2r)^3}$ [opposite to P]

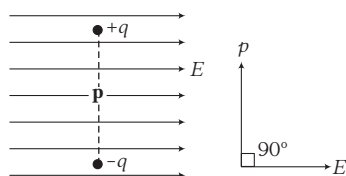
$$\Rightarrow E_{\perp} = \frac{E_{\text{axis}}}{16} = \frac{E_0}{16}$$

- 4 (c) If dipole is rotated through an angle of 90° about its perpendicular axis, then given point comes on equatorial line. So, field becomes half of previous value, i.e. $E/2$.

6 (d) Here,

$$\theta_1 = 90^\circ$$

$$\theta_2 = 90^\circ + 180^\circ = 270^\circ$$



$$\therefore \text{Work done} = \int_{\theta_1=90^\circ}^{\theta_2=270^\circ} pE \sin \theta d\theta$$

$$= [-pE \cos \theta]_{90^\circ}^{270^\circ} = 0$$

7 (d) Work done in rotating the dipole,

$$W = pE(\cos \theta_1 - \cos \theta_2)$$

$$= pE(\cos 0^\circ - \cos 180^\circ)$$

$$= pE[1 - (-1)] = 2pE$$

8 (d) Maximum torque is given by

$$\tau_{\max} = pE \quad [\because \sin 90^\circ = 1]$$

$$= (q \times 2a)E = (4 \times 10^{-8} \times 2 \times 10^{-4}) \times 4 \times 10^8$$

$$= 32 \times 10^{-4} \text{ N-m}$$

If $\theta = 180^\circ$, then

work done, $W = pE(1 - \cos 180^\circ)$

$$= pE[1 - (-1)]$$

$$W = 2pE = 2 \times 32 \times 10^{-4}$$

$$= 64 \times 10^{-4} \text{ J}$$

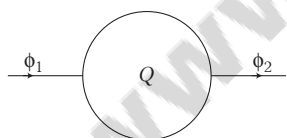
CHECK POINT 1.5

1 (a) Electric flux through the surface,

$$\phi = \mathbf{E} \cdot \mathbf{S} = (2\hat{i} + 4\hat{j} + 7\hat{k}) \cdot (10\hat{j}) = 40 \text{ unit}$$

2 (d) Net electric flux passing from a closed surface in uniform electric field is always zero.

3 (b) From Gauss's law,



$$\text{Net flux} = \frac{\text{Total charge enclosed}}{\epsilon_0}$$

$$= \frac{1}{\epsilon_0} \times Q$$

$$\therefore Q = \epsilon_0(\phi_2 - \phi_1)$$

4 (a) Flux from one face = $\frac{1}{6}$ (total flux)

$$= \frac{1}{6} \left(\frac{q}{\epsilon_0} \right) = \frac{1}{3\epsilon_0} \quad [\because q = 2\text{ C}]$$

5 (d) By Gauss's law, $\phi = \frac{1}{\epsilon_0} [Q_{\text{enclosed}}]$

$$\Rightarrow Q_{\text{enclosed}} = \phi \epsilon_0 = [-8 \times 10^3 + 4 \times 10^3] \epsilon_0$$

$$= -4 \times 10^3 \epsilon_0 \text{ C}$$

8 (d) By using Gauss's law, $\int \mathbf{E} \cdot d\mathbf{A} = \int (\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4) \cdot d\mathbf{A}$

$$= \frac{1}{\epsilon_0} [Q_{\text{enclosed}}] = \frac{(q_1 + q_2 + q_3 + q_4)}{\epsilon_0}$$

9 (a) Electric field, $E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2\lambda}{4\pi\epsilon_0 r}$

or $\lambda = \frac{E \times 4\pi\epsilon_0 r}{2} = 18 \times 10^4 \times \frac{1}{9 \times 10^9} \times \frac{0.02}{2}$

$$= 2 \times 10^{-7} \text{ C/m}$$

10. (a) Surface charge density, $\sigma = \frac{q}{A} = \frac{17.7 \times 10^{-4} \text{ C}}{200}$

The electric field outside the sheet is given by

$$E = \frac{\sigma}{2\epsilon_0} = \frac{17.7 \times 10^{-4}}{2 \times 8.85 \times 10^{-12} \times 200} = 5 \times 10^5 \text{ N/C}$$

11. (b) Electric field, $E = \sigma/\epsilon_0$

$$F = Fe = \frac{\sigma e}{\epsilon_0} = \frac{(2.0 \times 10^{-6})(1.6 \times 10^{-19})}{8.85 \times 10^{-12}}$$

The work done by the electron against this force in travelling a distance x metre, $W = Fx$

Also, $W = K = 100 \text{ eV} = 1.6 \times 10^{-19} \times 100 \text{ J}$

$$\therefore \frac{(2.0 \times 10^{-6})(1.6 \times 10^{-19})x}{8.85 \times 10^{-12}} = 100 \times (1.6 \times 10^{-19})$$

$$x = 0.44 \text{ mm}$$

12. (a) Electric field, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} = (9 \times 10^9) \times \frac{0.2 \times 10^{-6}}{(0.25)^2}$

$$= 2.88 \times 10^4 \text{ N/C}$$

13. (c) We have, $E = \sigma/\epsilon_0$

$$\Rightarrow \sigma = E\epsilon_0 = 300 \times (8.85 \times 10^{-12}) = 2.6 \times 10^{-9} \text{ C/m}^2$$

(A) Taking it together

1. (a) Electric lines are closer at A. So, the strength of electric field at A is more than B, i.e. $E_A > E_B$.

2. (b) Electric field on the surface of a conducting sphere is

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

$$\therefore \text{Charge, } q = Er^2 \cdot 4\pi\epsilon_0 = \frac{3 \times 10^6 \times (2.5)^2}{9 \times 10^9} \approx 2 \times 10^{-3} \text{ C}$$

4 (d) Electric line of force are perpendicular to the surface of a conductor. Inside the sphere, no lines are present.

Hence, option (d) is correct.

5 (c) Let neutral point be obtained at a distance x from $20 \mu\text{C}$ charge. Hence, at neutral point

$$\frac{1}{4\pi\epsilon_0} \frac{20 \times 10^{-6}}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{80 \times 10^{-6}}{(0.10 - x)^2}$$

$$\frac{20}{x^2} = \frac{80}{(0.10 - x)^2}$$

$$x = + 0.033 \text{ m}$$

6 (c) Torque, $\tau_{\max} = pE = q(d)E = 2 \times 10^{-6} \times 0.01 \times 5 \times 10^5$
 $= 10 \times 10^{-3} \text{ N-m}$

7 (a) Electric field due to a point charge, $E = \frac{q}{4\pi\epsilon_0 r^2}$
 $\therefore q = E \times 4\pi\epsilon_0 r^2 = 2 \times \frac{1}{9 \times 10^9} \times \left(\frac{30}{100}\right)^2 = 2 \times 10^{-11} \text{ C}$

8 (d) For the system to be in equilibrium, net force on charge Q should be zero. So, Q and q should be unlike in nature.

$\therefore \frac{k \cdot Qq}{r^2} + \frac{kQ \cdot Q}{(2r)^2} = 0$ or charge, $q = \frac{-Q}{4}$

9 (b) As, electric field at q is E , so force on charge q

$F = qE$... (i)

Let the electric field at $2q$ is E' , so force on $2q$ will be

$F' = 2qE'$... (ii)

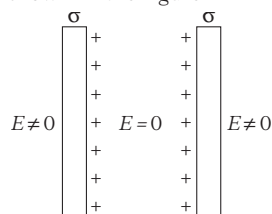
But, according to Coulomb's law,

$F' = F \Rightarrow 2qE' = qE$ [from Eqs. (i) and (ii)]

$\Rightarrow E' = E/2$

10 (a) Electric field inside shell is zero.

11 (c) Situation is shown in the figure



Electric field between two parallel sheets $= \frac{2}{2\epsilon_0} (\sigma - \sigma) = 0$

12 (b) Force, $F = k \frac{q_1 q_2}{r^2} \Rightarrow 12 = k \frac{2 \times 6}{r^2}$... (i)

When a charge of -4C is given to each of these charges, then

$q_1 = -2\text{C}, q_2 = 2\text{C}$

and $F' = k \frac{(-2)(2)}{r^2}$... (ii)

On dividing Eq. (ii) by Eq. (i), we get

$\frac{F'}{12} = \frac{-4}{12}$

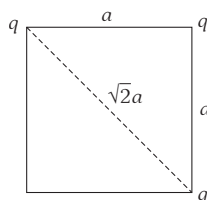
$F' = -4\text{N}$

Here, negative sign indicates that force is attractive.

13 (b) Force, $F_{12} = \frac{k(q^2)}{a^2}$

Force, $F_{13} = \frac{k(q^2)}{(\sqrt{2}a)^2}$

$\therefore \text{Ratio, } \frac{F_{12}}{F_{13}} = \frac{2}{1}$



14 (d) Here, $q = -3 \times 10^{-7} \text{ C}$

Number of electrons transferred to the conductor is

$n = \frac{q}{e} = \frac{-3 \times 10^{-7} \text{ C}}{-1.6 \times 10^{-19} \text{ C}} \approx 2 \times 10^{12}$

Mass of one electron, $m_e = 9.1 \times 10^{-31} \text{ kg}$

Increase mass of the conductor $= m_e \times n$

$= 9.1 \times 10^{-31} \times 2 \times 10^{12}$

$= 18.2 \times 10^{-19} \text{ kg} \approx 2 \times 10^{-18} \text{ kg}$

15 (a) Gravitational force,

$F_G = \frac{Gm_e m_p}{r^2}$

$F_G = \frac{6.7 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-27}}{(5 \times 10^{-11})^2}$

$= 3.9 \times 10^{-47} \text{ N}$

Electrostatic force, $F_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$

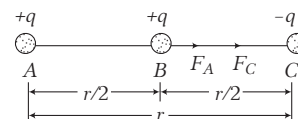
$F_e = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(5 \times 10^{-11})^2}$

$= 9.22 \times 10^{-8} \text{ N}$

So, $\frac{F_e}{F_G} = \frac{9.22 \times 10^{-8}}{3.9 \times 10^{-47}}$

$= 2.36 \times 10^{39}$

16 (c) Situation is shown in figure below



Force between A and C,

$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{r^2}$

When sphere B is kept at the mid-point of line joining A and C, then net force on B is

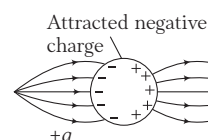
$F_{\text{net}} = F_A + F_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{(r/2)^2} + \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{(r/2)^2}$

$= 8 \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{r^2} = 8F$

17 (b) We have, $q'_1 = q'_2 = \left(\frac{q_1 + q_2}{2}\right) = \left(\frac{10 - 20}{2}\right) = -5\mu\text{C}$

$\therefore \frac{F_1}{F_2} = \frac{q_1 q_2}{q'_1 q'_2} = \frac{(10)(-20)}{(-5)(-5)} = -\frac{8}{1} \quad (\because F \propto q_1 q_2)$

18 (a) When a positive point charge is brought near an isolated conducting sphere without touching the sphere, then the free electrons in the sphere are attracted towards the positive charge making left side negatively charged. This leaves an excess of positive charge on the rear (right) surface of sphere as shown below



Electric field lines start from positive charge and end at negative charge (in this case from positive point charge to negative charge created inside the sphere).

Also, electric field line emerges from a positive charge, in case of single charge and ends at infinity.

Here, all these conditions are fulfilled in figure (a).

- 19 (c) From Coulomb's law,

$$q_1 q_2 \propto F \quad [\because r = 1 \text{ m}] \dots (i)$$

$$\left(\frac{q_1 - q_2}{2} \right) \left(\frac{q_1 - q_2}{2} \right) \propto \frac{F}{3} \quad \dots (ii)$$

Dividing Eq. (i) by Eq (ii), we get

$$\frac{q_1}{q_2} = \frac{3}{1}$$

- 20 (b) $\mathbf{E}_q + \mathbf{E}_{3q}$ is along PA

$$\mathbf{E}_{2q} + \mathbf{E}_{4q} \text{ is along } PB$$

$\therefore \mathbf{E}_{\text{net}}$ is along CB .

- 21 (c) Let the induced charge on inner surface and outer surface is q_1 and q_2 respectively, then $q_1 + q_2 = -50e$.

Here, charge q_1 induced due to ball is $+50e$.

$$\text{So, } +50e + q_2 = -50e$$

$$\Rightarrow q_2 = -100e$$

- 22 (c) Electric field in vacuum, $E_0 = \frac{\sigma}{\epsilon_0}$

$$\text{In medium, } E_2 = \frac{\sigma}{\epsilon_0 K}$$

If $K > 1$, then $E_2 < E_0$.

i.e. if the plates are dipped in kerosene oil tank, the electric field between the plates will decrease.

- 23 (a) Force on a charged particle, $qE = \mu mg$

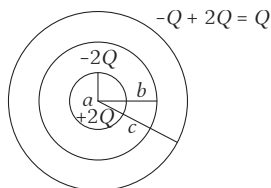
where, μ is coefficient of friction

$$\therefore \mu = \frac{qE}{mg}$$

- 24 (a) Surface charge density (σ) = $\frac{\text{Charge}}{\text{Surface area}}$

So, surface charge density on the inner surface

$$\sigma_{\text{inner}} = \frac{-2Q}{4\pi b^2}$$



and surface charge density on the outer surface, $\sigma_{\text{outer}} = \frac{Q}{4\pi c^2}$

- 25 (d) Gauss's law of electrostatic states that, the total electric flux through a closed surface is equal to the charge enclosed divided by the permittivity, i.e. $\phi = \frac{Q}{\epsilon_0}$. Thus, electric flux

through a surface does not depend on the shape, size or area of a surface but it depends on the total charge enclosed by the surface.

So, here in this question, all the figures have same electric flux as all of them has single positive charge.

- 26 (b) When charged particle enters in a uniform electric field, then force on charged particle, $F = qE$

$$\text{Also, } F = ma$$

$$\therefore ma = qE$$

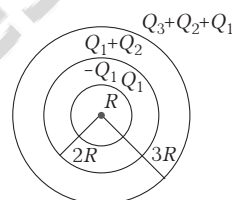
$$\text{or acceleration, } a = \frac{qE}{m} = \frac{3 \times 10^{-3} \times 80}{20 \times 10^{-3}} = 12 \text{ ms}^{-2}$$

So, from equations of motion

$$v = u + at = 20 + 12 \times 3 = 56 \text{ ms}^{-1}$$

- 27 (b) On the outer surfaces of the shell surface charge densities are equal

$$\frac{Q_3 + Q_2 + Q_1}{4\pi(3R)^2} = \frac{Q_2 + Q_1}{4\pi(2R)^2} = \frac{Q_1}{4\pi R^2}$$



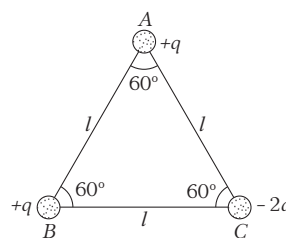
$$\frac{Q_3 + Q_2 + Q_1}{9} = \frac{Q_2 + Q_1}{4} \text{ and } \frac{Q_2 + Q_1}{4} = \frac{Q_1}{1}$$

$$\Rightarrow Q_2 = 3Q_1$$

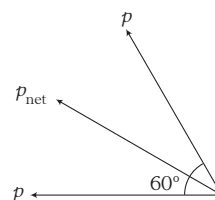
$$\therefore \frac{Q_3 + Q_2 + Q_1}{9} = \frac{Q_1}{1} \Rightarrow Q_3 = 5Q_1$$

Hence, $Q_1 : Q_2 : Q_3 = 1 : 3 : 5$

- 28 (c) The situation is shown below.



The direction of dipole moment is shown in figure.



Net electric dipole moment,

$$p_{\text{net}} = \sqrt{p^2 + p^2 + 2pp \cos 60^\circ} = \sqrt{3}p$$

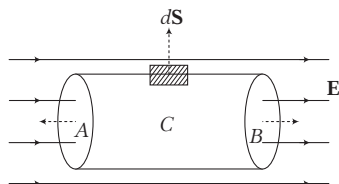
$$= \sqrt{3}ql \quad [\because p = ql]$$

- 29** (a) When a positive point charge brought near an isolated conducting plane, then some negative charge develops on the surface of the plane towards the charge and an equal positive charge develops on opposite side to the plane. So, the field at point p on other side is directed perpendicular to plane and away from it.

- 30** (d) Electric field at centre due to circular portion is zero because electric field due to each charged element at centre will be cancel out by the electric field due the charged element of its just opposite side.

- 31** (b) Flux of electric field, $\phi = \frac{1}{\epsilon_0} \times Q_{in} = \frac{1}{\epsilon_0} (2q)$

- 32** (d) Flux through surface A , $\phi_A = E \times \pi R^2$ and B , $\phi_B = -E \times \pi R^2$



Flux through curved surface C ,

$$\phi_C = \int \mathbf{E} \cdot d\mathbf{S} = \int E dS \cos 90^\circ = 0$$

$\therefore$ Total flux through cylinder = $\phi_A + \phi_B + \phi_C = 0$

- 33** (a) Electric flux, $\phi_E = \int \mathbf{E} \cdot d\mathbf{S}$

$$= \int E dS \cos \theta = \int E dS \cos 90^\circ = 0$$

- 34** (c) $F_1 = \frac{kQ_1Q_2}{d^2}$ and $F_2 = \frac{k\left(\frac{Q_1-Q_2}{2}\right)^2}{d^2}$

According to question, $F_1 = F_2$

$$Q_1Q_2 = \frac{(Q_1-Q_2)^2}{4}$$

$$\Rightarrow 4Q_1Q_2 = Q_1^2 + Q_2^2 - 2Q_1Q_2$$

$$0 = Q_1^2 + Q_2^2 - 6Q_1Q_2$$

$$\Rightarrow \frac{Q_1}{Q_2} = 3 \pm \sqrt{8}$$

- 35** (c) The acceleration of the electron due to given coulombic force, F is $a_e = \frac{F}{m_e}$... (i)

where, m_e is the mass of the electron.

The acceleration of the proton due to same force F is

$$a_p = \frac{F}{m_p} \quad \dots (ii)$$

where, m_p is the mass of the proton.

On dividing Eq. (ii) by Eq. (i), we get $\frac{a_p}{a_e} = \frac{m_e}{m_p}$

$$a_p = \frac{a_e m_e}{m_p} = \frac{(2.5 \times 10^{22} \text{ ms}^{-2})(9.1 \times 10^{-31} \text{ kg})}{(1.67 \times 10^{-27} \text{ kg})}$$

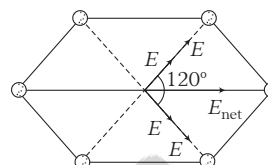
$$= 13.6 \times 10^{18} \text{ ms}^{-2} \approx 1.5 \times 10^{19} \text{ ms}^{-2}$$

- 36** (a) By using, $QE = mg$

$$\Rightarrow E = \frac{mg}{Q} = \frac{10^{-6} \times 10}{10^{-6}} = 10 \text{ V/m, upward because}$$

charge is positive.

- 37** (b) In Figs. (1), (3) and (4), net electric field is zero because electric field at a point due to positive charge acts away from the charge and due to negative charge, it acts towards the charge. So for Fig. (2),



Here, net electric field in Fig. (2) is

$$= \sqrt{(2E)^2 + (2E)^2 + (2E)(2E) \cdot 2 \cos 120^\circ} = 2E$$

which is not zero.

- 38** (a) According to Gauss's law, total flux coming out of a closed surface enclosing charge q is given by $\phi = \int \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$.

From this expression, it is clear that total flux linked with a closed surface only depends on the enclosed charge and independent of the shape and size of the surface.

$$\phi = \int \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0} = 20 \text{ Vm} \quad [\text{given}]$$

This $\frac{q}{\epsilon_0}$ is constant as long as the enclosed charge is constant.

The flux over a concentric sphere of radius 20 cm = 20 V-m.

- 39** (b) Given, $A(1, 0, 4)$ and $B(2, -1, 5)$

$$\therefore \mathbf{AB} = [(2-1)\hat{i} + (-1-0)\hat{j} + (5-4)\hat{k}]$$

$$\mathbf{AB} = [\hat{i} - \hat{j} + \hat{k}]$$

Torque, $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} = q\mathbf{AB} \times \mathbf{E}$

$$\boldsymbol{\tau} = 4 \times 10^{-6} (\hat{i} - \hat{j} + \hat{k}) \times 20 \hat{i}$$

$$\boldsymbol{\tau} = 8 \times 10^{-5} (\hat{k} + \hat{j})$$

Magnitude of torque,

$$\tau = 8 \times 10^{-5} \sqrt{1^2 + 1^2}$$

$$\tau = 1.13 \times 10^{-4} \text{ N-m}$$

- 40.** (b) Given, $E = 7.18 \times 10^8 \text{ N/C}$

and $r = 2 \text{ cm} = 2 \times 10^{-2}$

Electric field is given by

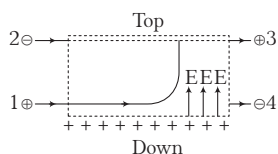
$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\lambda = 2\pi\epsilon_0 r E$$

$$\lambda = \frac{4\pi\epsilon_0 r E}{2} = \frac{2 \times 10^{-2} \times 7.18 \times 10^8}{2 \times 9 \times 10^9}$$

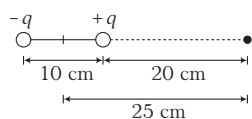
$$= 7.98 \times 10^{-4} \text{ C/m}$$

- 41 (a) The figure shows, the path of a positive charged particle (1) through a rectangular region of uniform electric field.



Since, positive charged particle moves in a parabolic path in electric field, it means the direction of electric field is upward. The direction of particle (2) which is negative is downward. The direction of deflection of particle (3) which is positive is upward and direction of deflection of particle (4) is downward.

- 42 (a) Here, $p = (500 \times 10^{-6}) \times (10 \times 10^{-2}) = 5 \times 10^{-5}$ C-m



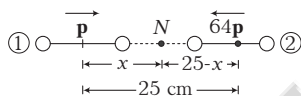
$$r = 25 \text{ cm} = 0.25 \text{ m}, a = 5 \text{ cm} = 0.05 \text{ m}$$

$$\text{Electric field intensity, } E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{(r^2 - l^2)^2}$$

$$E = \frac{9 \times 10^9 \times 2 \times 5 \times 10^{-5} \times 0.25}{\{(0.25)^2 - (0.05)^2\}^2}$$

$$= 6.25 \times 10^7 \text{ N/C}$$

- 43 (a) Suppose neutral point N lies at a distance x from dipole of moment p or at a distance $(25 - x)$ from dipole of moment $64p$.



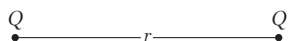
At point N electric field due to dipole ①
= Electric field due to dipole ②

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{x^3} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2(64p)}{(25-x)^3}$$

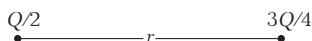
$$\Rightarrow \frac{1}{x^3} = \frac{64}{(25-x)^3}$$

$$\Rightarrow x = 5 \text{ cm}$$

- 44 (d) Initially force, $F = k \frac{Q^2}{r^2}$



Finally, when a third spherical conductor comes in contact alternately with B and C then removed, the charges on B and C become $Q/2$ and $3Q/4$, respectively.



$$\text{Now, force, } F' = k \frac{(Q/2)(3Q/4)}{r^2} = \frac{3}{8} F$$

- 45 (c) The torque on a dipole moment is $\tau = \mathbf{p} \times \mathbf{E}$. The torque has maximum value when $\mathbf{p}$ and $\mathbf{E}$ are perpendicular to each other, so that $\sin \theta$ is maximum, i.e. $\sin \theta = 1$.

$$\tau = (3 \times 10^4) (6 \times 10^{-30})$$

$$= 18 \times 10^{-26} \text{ N-m}$$

- 46 (c) Two positive ions, each carrying a charge q are kept at a distance d , then it is found that force of repulsion between them is $F = \frac{kqq}{d^2} = \frac{1}{4\pi\epsilon_0} \frac{qq}{d^2}$

$$\text{where, } q = ne$$

$$\therefore F = \frac{1}{4\pi\epsilon_0} \frac{n^2 e^2}{d^2}$$

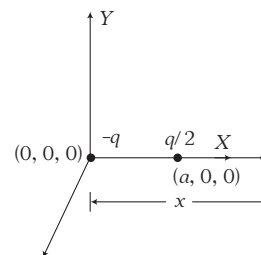
$$\text{Number of electrons, } n = \sqrt{\frac{4\pi\epsilon_0 F d^2}{e^2}}$$

- 47 (c) When both discs A and B are touched, charge flows from higher value (higher potential) to lower value (lower potential) till it equalises on the two discs.

$$\text{Given, } q_1 = 10^{-6} \text{ C, } q_2 = 10^{-5} \text{ C}$$

$$\therefore q = \frac{q_1 + q_2}{2} = \frac{10^{-6} + 10^{-5}}{2} = 5.5 \mu\text{C}$$

- 48 (c) Suppose the field vanishes at a distance x , we have

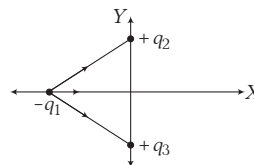


$$\frac{kq}{x^2} = \frac{kq/2}{(x-a)^2}$$

$$\text{or } 2(x-a)^2 = x^2 \text{ or } \sqrt{2}(x-a) = x$$

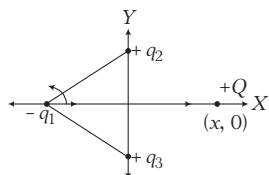
$$\text{or } (\sqrt{2} - 1)x = \sqrt{2}a \text{ or } x = \frac{\sqrt{2}a}{\sqrt{2} - 1}$$

- 49 (a) The net force on q_1 by q_2 and q_3 is along the $+x$ -direction, so nature of force between q_1, q_2 and q_1, q_3 is attractive. This can be represented by the figure given alongside.



The attractive force between these charges states that q_1 is a negative charge (since, q_2 and q_3 are positive).

Thus, nature of force between q_1 and newly introduced charge Q (positive) is attractive and net force on q_1 by q_2, q_3 and Q are along the same direction as given in the diagram below.



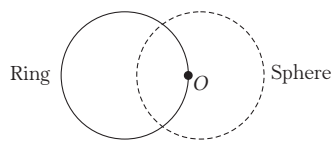
The figure given above clearly shows that the force on q_1 shall increase along the positive X -axis due to the positive charge Q .

- 50 (a)** When the point is situated, on a diameter away from the centre of a uniformly positive charged hemisphere, then the electric field is perpendicular to the diameter. The component of electric intensity parallel to the diameter cancel out.

- 51 (d)** As imaginary sphere of radius R is drawn with its centre on circumference of ring.

So, $q_{\text{in}} < \pi R \lambda$.

where, λ is linear charge density



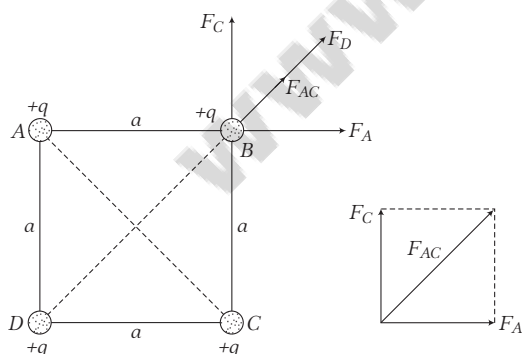
$$\therefore \phi < \frac{\pi R \lambda}{\epsilon_0}$$

- 52 (b)** Electric field due to Q_2 and Q_3 cancel each other.

So, the vector numbered 2 coincides in direction with electric field due to Q_1 at mid-point M of the hypotenuse.

- 53 (c)** Three vectors of equal magnitude are inclined at 120° with the adjacent vector. So, net electric intensity will be zero.

- 54 (c)** The given situation is shown below.



$$\text{Here, } F_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2} \text{ and } F_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2}$$

Net force on B, $F_{\text{net}} = F_{AC} + F_D$

$$= \sqrt{F_A^2 + F_C^2} + F_D$$

$$= \sqrt{\left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2}\right)^2 + \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2}\right)^2} + \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{(a\sqrt{2})^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2} \left(\sqrt{2} + \frac{1}{2}\right) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{a^2} \left(\frac{1+2\sqrt{2}}{2}\right)$$

- 55 (b)** If ring is complete, then net field at centre is zero. As small portion is cut, then field opposite to this portion is not cancelled out.

Charge of small portion $= \lambda \cdot l$

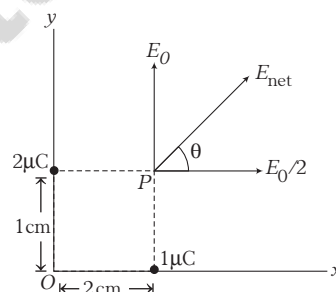
$$\therefore \text{Electric field due to this portion} = \frac{\lambda l}{4\pi\epsilon_0 a^2}$$

Let E_R is electric field of remaining portion.

$$\therefore E_R + \frac{\lambda l}{4\pi\epsilon_0 a^2} = 0$$

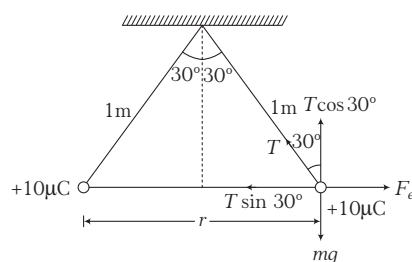
$$\Rightarrow E_R = \frac{-\lambda l}{4\pi\epsilon_0 a^2}$$

- 56 (b)** $E \propto \frac{q}{r^2} \Rightarrow \tan \theta = \frac{E_0}{E_0/2} = 2$



- 57 (b)** In the following figure, in equilibrium,

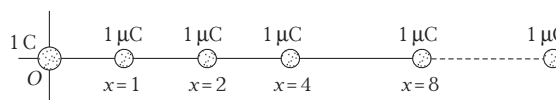
$$F_e = T \sin 30^\circ \text{ and } r = 1 \text{ m}$$



$$\Rightarrow 9 \times 10^9 \cdot \frac{Q^2}{r^2} = T \times \frac{1}{2} \Rightarrow 9 \times 10^9 \cdot \frac{(10 \times 10^{-6})^2}{1^2} = T \times \frac{1}{2}$$

$\therefore$ Tension in the threads, $T = 1.8 \text{ N}$

- 58 (b)** The schematic diagram of distribution of charges on X -axis is shown in figure below.



Total force acting on 1 C charge is given by

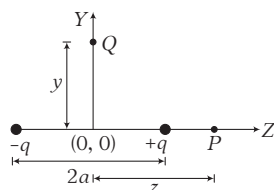
$$F = \frac{1}{4\pi\epsilon_0} \left[\frac{1 \times 1 \times 10^{-6}}{(1)^2} + \frac{1 \times 1 \times 10^{-6}}{(2)^2} + \frac{1 \times 1 \times 10^{-6}}{(4)^2} + \frac{1 \times 1 \times 10^{-6}}{(8)^2} + \dots \infty \right]$$

$$= \frac{10^{-6}}{4\pi\epsilon_0} \left(\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots \infty \right) = 9 \times 10^9 \times 10^{-6} \left(\frac{1}{1 - (1/4)} \right)$$

$$= 9 \times 10^9 \times 10^{-6} \times \frac{4}{3} = 9 \times \frac{4}{3} \times 10^3 = 12000 \text{ N}$$

- 59 (c) Electric force, $qE = ma \Rightarrow a = \frac{qE}{m}$
- $$\therefore a = \frac{1.6 \times 10^{-19} \times 1 \times 10^3}{9 \times 10^{-31}} = \frac{1.6}{9} \times 10^{15}$$
- $$\therefore u = 5 \times 10^6 \text{ ms}^{-1} \text{ and } v = 0$$
- $$\therefore \text{From } v^2 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a}$$
- $$\therefore \text{Distance, } s = \frac{(5 \times 10^6)^2 \times 9}{2 \times 1.6 \times 10^{15}} = 7 \text{ cm (approx.)}$$

- 60 (d) The magnitude of electric field at an axial point P at a distance z from the origin is given by



$$|E_{(z)}| = \frac{4qaz}{4\pi\epsilon_0(z^2 - a^2)^2} = \frac{2pz}{4\pi\epsilon_0(z^2 - a^2)^2}$$

where, $p = 2qa$ is the electric dipole moment

$$\text{For } z \gg a, |E_{(z)}| = \frac{2p}{4\pi\epsilon_0 z^3}$$

The magnitude of electric field at an equatorial point Q at a distance y from the origin is given by

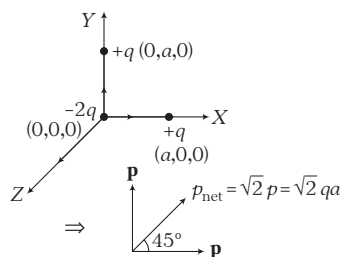
$$|E_{(y)}| = \frac{1}{4\pi\epsilon_0} \frac{2qa}{(y^2 + a^2)^{3/2}} = \frac{p}{4\pi\epsilon_0(y^2 + a^2)^{3/2}}$$

$$\text{For } y \gg a, |E_{(y)}| = \frac{p}{4\pi\epsilon_0 y^3}$$

For $z = y \gg a$,

$$\therefore \frac{|E_{(z)}|}{|E_{(y)}|} = 2$$

- 61 (b)



- 62 (c) According to Gauss's theorem, electric flux through the sphere $= \frac{q}{\epsilon_0}$.

$$\therefore \text{Electric flux through the hemisphere} = \frac{1}{2} \frac{q}{\epsilon_0}$$

$$= \frac{10 \times 10^{-6}}{2 \times 8.854 \times 10^{-12}} = 0.56 \times 10^6 \text{ N-m}^2\text{C}^{-1}$$

$$\approx 0.6 \times 10^6 \text{ N-m}^2\text{C}^{-1} = 6 \times 10^5 \text{ N-m}^2\text{C}^{-1}$$

- 63 (b) Charge enclosed by cylindrical surface (length 100 cm) is $Q_{\text{encl}} = 100Q$. By applying Gauss's law,

$$\phi = \frac{1}{\epsilon_0} (Q_{\text{encl}}) = \frac{1}{\epsilon_0} (100Q)$$

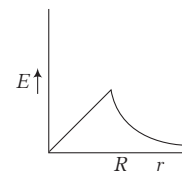
$\therefore$ The lines are parallel to the surface.

- 64 (a) Electric field due to a hollow spherical conductor is given by following equations $E = 0$, for $r < R$... (i)

$$\text{and } E = \frac{Q}{4\pi\epsilon_0 r^2}, \text{ for } r \geq R \quad \dots \text{(ii)}$$

i.e. inside the conductor field will be zero and outside the conductor will vary according to $E \propto \frac{1}{r^2}$.

- 65 (c) The field increases linearly from centre inside the sphere (from $r = 0$ to $r = R$) become maximum at the surface of the sphere and decrease rapidly with distance $\left(\propto \frac{1}{r^2} \right)$ outside the sphere. So,



the graph will be as shown.

- 66 (a) When dipole is given a small angular displacement θ about its equilibrium position, then the restoring torque will be

$$\tau = -pE \sin \theta = -pE \theta \quad (\text{as } \sin \theta = \theta)$$

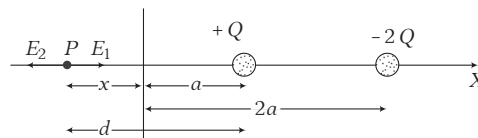
$$\text{or } I \frac{d^2\theta}{dt^2} = -pE \theta \quad \left(\text{as } \tau = I\alpha = I \frac{d^2\theta}{dt^2} \right)$$

$$\text{As, } \frac{d^2\theta}{dt^2} = -\omega^2 \theta \Rightarrow \omega^2 = \frac{pE}{I} \Rightarrow \omega = \sqrt{\frac{pE}{I}}$$

- 67 (b) Suppose electric field is zero at a point P lies at a distance d from the charge $+Q$.

$$\text{At } P, \frac{KQ}{d^2} = \frac{K(2Q)}{(a+d)^2}$$

$$\Rightarrow \frac{1}{d^2} = \frac{2}{(a+d)^2} \Rightarrow d = \frac{a}{(\sqrt{2}-1)}$$



Since, $d > a$, i.e. point P must lie on negative X -axis as shown at a distance x from origin,

$$\text{hence, } x = d - a = \frac{a}{(\sqrt{2}-1)} - a = \sqrt{2}a.$$

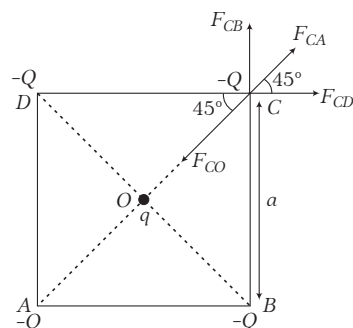
Actually, P lies on negative X -axis, so $x = -\sqrt{2}a$

$$68 \text{ (a)} \quad \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = mr\omega^2 = \frac{4\pi^2 mr}{T^2} \quad \left(\because \omega = \frac{2\pi}{T} \right)$$

$$\Rightarrow T^2 = \frac{(4\pi\epsilon_0)r^2(4\pi^2 mr)}{q_1 q_2}$$

$$\therefore \text{Time period, } T = \left[\frac{4\pi^2 mr^3}{kq_1 q_2} \right]^{1/2}$$

- 69 (b) The system is in equilibrium means the force experienced by each charge is zero. It is clear that charge placed at centre would be in equilibrium for any value of q , so we are considering the equilibrium of charge placed at any corner.



$$F_{CD} + F_{CA} \cos 45^\circ + F_{CO} \cos 45^\circ = 0$$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{(-Q)(-Q)}{a^2} + \frac{1}{4\pi\epsilon_0} \frac{(-Q)(-Q)}{(\sqrt{2}a)^2} \times \frac{1}{\sqrt{2}} + \frac{1}{4\pi\epsilon_0} \frac{(-Q)q}{(\sqrt{2}a/2)^2} \times \frac{1}{\sqrt{2}} = 0$$

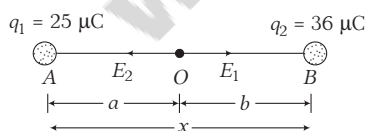
$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{a^2} + \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{2a^2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{4\pi\epsilon_0} \cdot \frac{2Qq}{a^2} \cdot \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow Q + \frac{Q}{2\sqrt{2}} - \sqrt{2}q = 0 \Rightarrow 2\sqrt{2}Q + Q - 4q = 0$$

$$\Rightarrow 4q = (2\sqrt{2} + 1)Q$$

Therefore, $q = \frac{Q}{4}(2\sqrt{2} + 1)$

- 70 (a) Let electric field is zero at point O in the figure.



$$\therefore E_1 = E_2$$

$$\therefore E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{a^2}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{b^2}$$

$$\text{Also, } x = a + b \text{ or } 11 = a + b$$

$$\therefore b = 11 - a$$

$$\text{Now, } \frac{1}{4\pi\epsilon_0} \frac{q_1}{a^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{(11-a)^2}$$

$$\therefore \frac{q_1}{q_2} = \frac{a^2}{(11-a)^2}$$

$$\text{or } \sqrt{\frac{q_1}{q_2}} = \frac{a}{11-a} \text{ or } \sqrt{\frac{25}{36}} = \frac{a}{11-a}$$

$$\text{or } \frac{5}{6} = \frac{a}{11-a} \text{ or } 6a = 55 - 5a$$

$$\therefore a = 5 \text{ cm}$$

So, intensity will be zero at a distance of 5 cm from $25 \mu\text{C}$.

- 71 (d) 1 electron has a charge of $1.6 \times 10^{-19} \text{ C}$.

10^{10} electrons would have a charge of

$$q = ne = 1.6 \times 10^{-19} \times 10^{10} = 1.6 \times 10^{-9} \text{ C}$$

Thus, in 1s charge accumulated = $1.6 \times 10^{-9} \text{ C}$

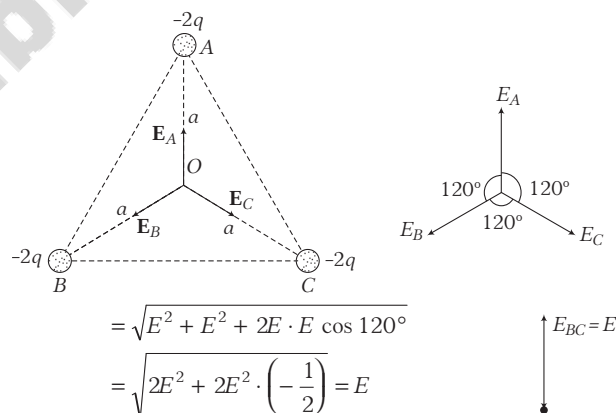
So, time taken to accumulate 1 C

$$= \frac{1}{1.6 \times 10^{-9}} = 0.625 \times 10^9$$

$$= 6.25 \times 10^8 \text{ s} = 173611 \text{ h}$$

$$= 7233 \text{ days} \approx 20 \text{ yr}$$

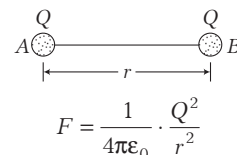
- 72 (c) The resultant of E_B and E_C is



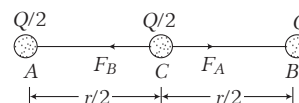
Now, situation is shown in figure.

Here, E_A and E_{BC} are equal and opposite, so, they cancel out. So, resultant electric field at O due to E_A , E_B and E_C is zero.

- 73 (a) Case I



Case II



When sphere C is touched to A, then equal charge $Q/2$ distributes on A and C.

$$\therefore F_A = \frac{1}{4\pi\epsilon_0} \frac{(Q/2)^2}{(r/2)^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{r^2}$$

$$F_B = \frac{1}{4\pi\epsilon_0} \cdot \frac{(Q)(Q/2)}{(r/2)^2} = 2 \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{r^2}$$

$$\therefore \text{Net force on } C, F_{\text{net}} = F_B - F_A = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q^2}{r^2} = F$$

74 (b) The electric field at C due to charge $+10^{-7}\text{C}$ at A is

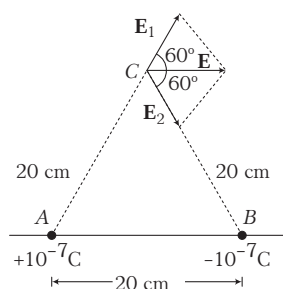
$$\mathbf{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{10^{-7}}{(0.2)^2} \text{ along } AC$$

The electric field at C due to charge -10^{-7}C at B is

$$\mathbf{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{10^{-7}}{(0.2)^2} \text{ along } CB$$

$$\text{As, } |\mathbf{E}_1| = |\mathbf{E}_2|$$

By symmetry, the vertical components will cancel out and horizontal components will add.



$\therefore$ The resultant electric field at C is

$$\begin{aligned} E &= 2E_1 \cos 60^\circ = 2 \times \frac{1}{4\pi\epsilon_0} \frac{10^{-7}}{(0.2)^2} \times \frac{1}{2} \\ &= \frac{9 \times 10^9 \times 10^{-7}}{(0.2)^2} = 2.2 \times 10^4 \text{ N/C} \end{aligned}$$

75 (c) Number of electrons,

$$n = \frac{6 \times 10^{23}}{63.5} \times 10 \times \frac{1}{10^6} = \frac{6 \times 10^{18}}{63.5}$$

As

$$q = ne$$

$$q = \frac{6 \times 10^{18} \times 1.6 \times 10^{-19}}{63.5}$$

or

$$q = 1.5 \times 10^{-2} \text{ C}$$

$\therefore$

$$\begin{aligned} F &= \frac{9 \times 10^9 \times 1.5 \times 10^{-2} \times 1.5 \times 10^{-2}}{\left(\frac{10}{100}\right)^2} \\ &= 2.0 \times 10^8 \text{ N} \end{aligned}$$

76 (a) The frequency will be same, $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ but due to the constant force *i.e.* qE , the equilibrium position gets shifted by $\frac{qE}{k}$ in forward direction. So, option (a) is correct.

77 (d) Electric field at the centre of charged circular ring is zero. Hence, electric field at O due to the part $ACDB$ is equal in magnitude and opposite in direction that due to the part AKB , *i.e.* E along OK .

78 (a) On negative charge, the resultant force acts as a restoring force and proportional to displacement. When it reaches in the plane xy , then the resultant force is zero and the mass moves down along Z -axis due to inertia. Thus, oscillation is set along Z -axis.

79 (a) Total flux, $\phi_{\text{total}} = \phi_A + \phi_B + \phi_C = \frac{q}{\epsilon_0}$

$$\therefore \phi_B = \phi \text{ and } \phi_A = \phi_C = \phi' \quad [\text{assumed}]$$

$$\therefore 2\phi' + \phi = \frac{q}{\epsilon_0}$$

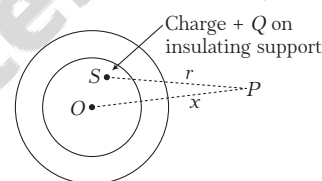
$$\text{The flux through the plane surface } A, \phi' = \frac{1}{2} \left(\frac{q}{\epsilon_0} - \phi \right)$$

80 (a) According to Gauss's theorem,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} Q_{\text{enclosed}}$$

$$E \cdot 4\pi x^2 = \frac{Q}{\epsilon_0}$$

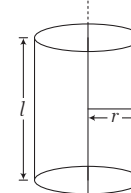
$$\text{Electric field at the point } P, E = \frac{Q}{4\pi\epsilon_0 x^2}$$



81 (a) Charge density of long wire, $\lambda = \frac{1}{3} \text{ C-m}^{-1}$

From Gauss's theorem, $\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$

$$\Rightarrow E \oint dS = \frac{q}{\epsilon_0} \text{ or } E 2\pi r l = \frac{q}{\epsilon_0}$$

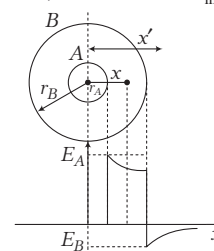


$$\Rightarrow E = \frac{q}{2\pi\epsilon_0 r l} = \frac{q/l}{2\pi\epsilon_0 r} \Rightarrow \frac{\lambda \times 2}{2\pi\epsilon_0 r \times 2} = \frac{\lambda \times 2}{4\pi\epsilon_0 r}$$

The magnitude of the electric intensity,

$$\begin{aligned} E &= 9 \times 10^9 \times \frac{1}{3} \times 2 \times \frac{1}{18 \times 10^{-2}} \\ &= 0.33 \times 10^{11} \text{ NC}^{-1} \end{aligned}$$

82 (a) Inside the shell A , electric field $E_{\text{in}} = 0$



At the surface of shell A,

$$E_A = \frac{kQ_A}{r_A^2} \quad [\text{a fixed positive value}]$$

Between the shell A and B, at a distance x from the common centre,

$$E = \frac{k \cdot Q_A}{x^2} \quad [\text{as } x \text{ increases, } E \text{ decreases}]$$

At the surface of shell B,

$$E_B = \frac{k(Q_A - Q_B)}{r_B^2} \quad [\text{a fixed negative value because } |Q_A| < |Q_B|]$$

Outside the both shell, at a distance x' from the common centre,

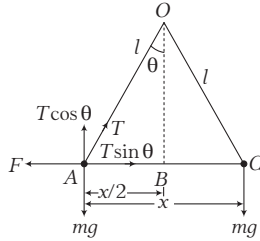
$$E_{\text{out}} = \frac{k(Q_A - Q_B)}{x'^2} \quad [\text{as } x' \text{ increase negative value of } E_{\text{out}} \text{ decrease and it becomes zero at } x = \infty]$$

83 (a) $\therefore T \sin \theta = F$

Dividing the two equations, we get

$$= \frac{q}{4\pi\epsilon_0 x^2} \text{ and } T \cos \theta = mg$$

$$\therefore \tan \theta = \frac{1}{4\pi\epsilon_0} \frac{q^2}{x^2 mg}$$



$$\text{or } \frac{x}{2l} = \frac{q^2}{4\pi\epsilon_0 x^2 mg} \quad \left(\because \tan \theta \approx \sin \theta = \frac{x}{2l} \right)$$

$$\text{or } \frac{x}{2l} \propto \frac{q^2}{x^2} \text{ or } q^2 \propto x^3$$

$$\Rightarrow q \propto x^{\frac{3}{2}} \Rightarrow \frac{dq}{dt} \propto \frac{3}{2} x^{\frac{1}{2}} \frac{dx}{dt} \quad \left(\frac{dq}{dt} = \text{constant} \right)$$

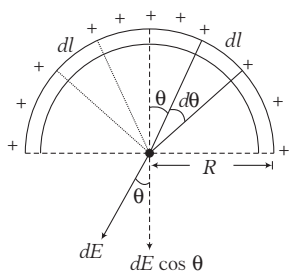
$$\therefore v \propto x^{\frac{1}{2}} \quad \left(\because v = \frac{dx}{dt} \right)$$

84 (a) From figure, $dl = R d\theta$,

Charge on dl , $dq = \lambda R d\theta$

Electric field at centre due to dl is

$$dE = k \frac{\lambda R d\theta}{R^2} \quad \left(\because \lambda = \frac{q}{\pi R} \right)$$



We need to consider only the component $dE \cos \theta$, as the component $dE \sin \theta$ will cancel out because of the field at C due to the symmetrical element dl' .

Total field at centre, $|E| = 2 \int_0^{\pi/2} dE \cos \theta$

$$= \frac{2k\lambda}{R} \int_0^{\pi/2} \cos \theta d\theta$$

$$= \frac{2k\lambda}{R} = \frac{q}{2\pi^2 \epsilon_0 R^2}$$

85 (b) At a point on the axis of uniformly charged disc at a distance x from the centre of the disc, the magnitude of the electric field is,

$$E = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

At centre, $E_c = \frac{\sigma}{2\epsilon_0}$

Given that, $\frac{E}{E_c} = \frac{1}{2}$

Then, $1 - \frac{x}{\sqrt{x^2 + R^2}} = \frac{1}{2}$

or $\frac{x}{\sqrt{x^2 + R^2}} = \frac{1}{2}$

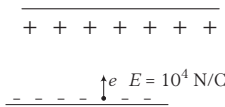
On squaring both sides, we get

$$x^2 = \frac{x^2}{4} + \frac{R^2}{4}$$

Thus, $x^2 = \frac{R^2}{3} \Rightarrow x = \frac{R}{\sqrt{3}}$

86 (a) The force acting on the electron = eE .

Acceleration of the electron = $\frac{eE}{m}$



Here, $s = 2 \times 10^{-2} \text{ m}$, $u = 0$, $v = ?$

$$\therefore v^2 - u^2 = 2as$$

$$\Rightarrow v^2 = 2 \left(\frac{e}{m} \right) E \times 2 \times 10^{-2} \text{ m}$$

Also, $\frac{e}{m} = 1.76 \times 10^{11} \text{ C/kg}$

$$\begin{aligned} \therefore v^2 &= 2 \times 1.76 \times 10^{11} \times 10^4 \times 2 \times 10^{-2} \\ &= 7.04 \times 10^{13} \\ &= 70.4 \times 10^{12} \end{aligned}$$

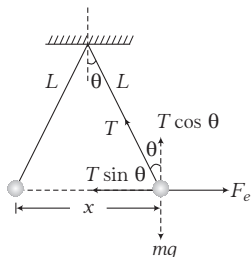
The velocity of the electron when it reaches plate B,

$$v \approx 0.85 \times 10^7 \text{ m/s}$$

87 (a) In equilibrium,

$$F_e = T \sin \theta \quad \dots(i)$$

$$\text{and} \quad mg = T \cos \theta \quad \dots(ii)$$



$$\tan \theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg}$$

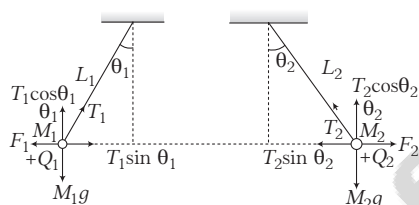
$$\text{Also,} \quad \tan \theta \approx \sin \theta = \frac{x/2}{L}$$

$$\text{Hence,} \quad \frac{x}{2L} = \frac{q^2}{4\pi\epsilon_0 x^2 \times mg}$$

$$\Rightarrow \quad x^3 = \frac{2q^2 L}{4\pi\epsilon_0 mg} \Rightarrow x = \left(\frac{q^2 L}{2\pi\epsilon_0 mg} \right)^{1/3}$$

88 (b) The three forces acting on each sphere are

- (i) tension
- (ii) weight
- (iii) electrostatic force of repulsion



For sphere 1, in equilibrium, from figure,

$$T_1 \cos \theta_1 = M_1 g \text{ and } T_1 \sin \theta_1 = F_1$$

$$\therefore \quad \tan \theta_1 = \frac{F_1}{M_1 g}$$

For sphere 2, in equilibrium, from figure,

$$T_2 \cos \theta_2 = M_2 g \text{ and } T_2 \sin \theta_2 = F_2$$

$$\therefore \quad \tan \theta_2 = \frac{F_2}{M_2 g}$$

Force of repulsion between two charges are same.

$$\therefore \quad F_1 = F_2$$

$$\text{Here, } \theta_1 = \theta_2 \text{ only, if } \frac{F_1}{M_1 g} = \frac{F_2}{M_2 g} \Rightarrow M_1 = M_2$$

(B) Medical entrance special format questions

• Assertion and reason

2. (a) Flux, $\phi_{\text{net}} = \frac{q_{\text{in}}}{\epsilon_0}$

If a closed body is placed in an electric field

(either uniform or non-uniform) total flux linked with it will be zero,

$$\text{i.e.} \quad \phi_{\text{net}} = 0 \Rightarrow q_{\text{in}} = 0$$

3 (d) If electric lines of force cross each other, then the electric field at the point of intersection will have two directions simultaneously which is not possible physically.

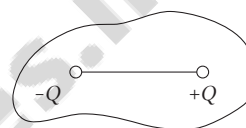
4 (b) As $\sigma_1 = \sigma_2$ (given)

$$\therefore \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}, \text{ or } \frac{q_1}{q_2} = \frac{r_1^2}{r_2^2} \text{ [let } r_1 \text{ and } r_2 \text{ be two different radii]}$$

Then, the ratio of electric field intensities near the surface of spherical conductor,

$$\frac{E_1}{E_2} = \frac{q_1}{4\pi\epsilon_0 r_1^2} \times \frac{4\pi\epsilon_0 r_2^2}{q_2} = \frac{q_1}{q_2} \times \frac{r_2^2}{r_1^2} = 1, \text{ i.e. } E_1 = E_2$$

5 (a) If a dipole is enclosed by a surface as shown in figure,



$$\text{then } Q_{\text{enclosed}} = Q - Q = 0$$

$$\therefore \quad \phi = 0 \quad (\text{from Gauss's law})$$

• Statement based questions

1. (c) The space between the electric field lines is increasing here from left to right and its characteristics states that, strength of electric field decreases with increase in the space between electric field lines. As a result, force on charges also decreases from left to right.

Thus, the force on charge $-q$ is greater than the force on charge $+q$ and in turn dipole will experience a force towards left.

2. (a) Torque about Q of charge $-q$ is zero. So, angular momentum of the charge $-q$ is constant but distance between charges is changing. So, force is changing and, hence speed and velocity are changing.

4 (b) According to Gauss's law, the term q on the right side of

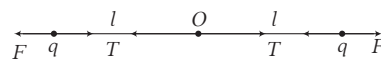
$$\text{the equation } \int_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0} \text{ includes the sum of all charges}$$

enclosed by the surface.

The charges may be located anywhere inside the surface, if the surface is so chosen that there are some charges inside and some outside, then the electric field on the left side of equation is due to all the charges, both inside and outside of S .

So, $\mathbf{E}$ on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

5 (a) In equilibrium, figure can be drawn as



$$T = \frac{1}{4\pi\epsilon_0} \frac{q \cdot q}{(2l)^2}$$

• Match the columns

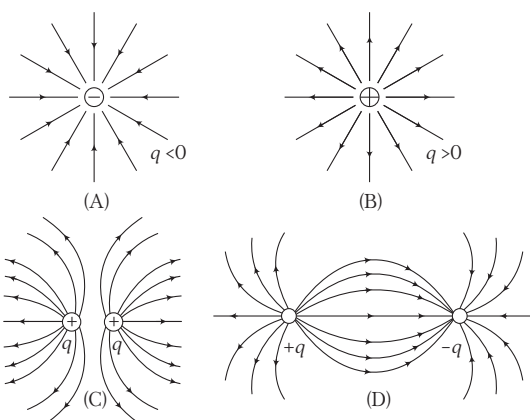
- 1 (a) Electric charge = [AT]

Electric field strength = [MLT⁻³A⁻¹]

Absolute permittivity = [M⁻¹L⁻³T⁴A²]

Electric dipole = [M⁰L¹T¹A¹]

- 2 (c) Figures show the field lines around some simple charge configurations. The field lines of a single positive charge are radially outward while those of a single negative charge are radially inward. The field lines around a system of two positive charges (q, q) give a pictorial description of their mutual repulsion. While that of a pair of equal and opposite charges shows attraction.



- 3 (b) $E_I = \frac{(\sigma) + (-2\sigma + \sigma)}{2\epsilon_0} = 0$; $E_{II} = \frac{\sigma - (-2\sigma + \sigma)}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$
 $E_{III} = \frac{(\sigma - 2\sigma) - \sigma}{2\epsilon_0} = -\frac{\sigma}{\epsilon_0}$; $E_{IV} = \frac{(\sigma - 2\sigma + \sigma)}{2\epsilon_0} = 0$

(C) Medical entrances' gallery

1. (a) Given, radius, $r = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$

Charge, $q = 3.2 \times 10^{-7} \text{ C}$

Electric field, $E = ?$

Electric field at a point ($x = 15 \text{ cm}$) from the centre of the sphere is

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{x^2} = 9 \times 10^9 \times \frac{3.2 \times 10^{-7}}{(15 \times 10^{-2})^2}$$

$$= 1.28 \times 10^5 \text{ N/C}$$

- 2 (a) Electric field due to electric dipole on equatorial plane at a distance r from the centre of dipole is given as

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\mathbf{p}}{r^3}$$

- 3 (c) Force due to mutual attraction between the electron and proton. (when, $r = 1.6 \text{ \AA} = 1.6 \times 10^{-10} \text{ m}$) is given as

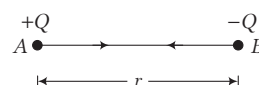
$$F = 9 \times 10^9 \times \frac{e^2}{r^2}$$

$$= 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{(1.6 \times 10^{-10})^2} = 9 \times 10^{-9} \text{ N}$$

$\therefore$ Acceleration of electron

$$= \frac{F}{m_e} = \frac{9 \times 10^{-9}}{9 \times 10^{-31}} = 10^{22} \text{ m/s}^2$$

- 4 (a) The force between two point charges A and B , having charges $+Q$ and $-Q$ respectively is given by



$$F = \frac{k Q_A Q_B}{r^2} = \frac{k Q (-Q)}{r^2} = -\frac{k Q^2}{r^2} \quad \dots(i)$$

where, $k = \text{constant} = \frac{1}{4\pi\epsilon_0}$

and r = distance between two charges A and B .

When 25% charge of A is transferred to B , then new amount of charge on A and B respectively becomes,

$$Q'_A = \frac{75}{100} (Q_A) = \frac{3}{4} Q$$

$$Q'_B = \left(\frac{25}{100} Q_A + Q_B \right) = \left(\frac{1}{4} Q - Q \right) = -\frac{3}{4} Q$$

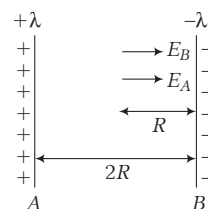
So, the new force between the two charges now becomes

$$F' = \frac{k Q'_A Q'_B}{r^2} = \frac{k \left(\frac{3Q}{4} \right) \left(-\frac{3Q}{4} \right)}{r^2}$$

$$= \frac{-9kQ^2}{16r^2} = \frac{9}{16} F \quad [\text{from Eq. (i)}]$$

Thus, the new force between the charges is 9/16 times the initial force between the charges.

- 5 (b) Consider two infinite line charges with linear charge densities $+\lambda \text{ C/m}$ and $-\lambda \text{ C/m}$ respectively, which are lying in y -direction as shown in the figure below



Then, the electric field due to line A at the mid-way between the two line charges, i.e. at R is

$$|E_A| = \frac{\lambda}{2\pi\epsilon_0 R} \text{ N/C} \quad \dots(i)$$

which lies along positive x -axis (outward), i.e. from A to B .

Similarly, the electric field due to line B at the mid-way between the two line charges, i.e. at R is

$$|E_B| = \frac{\lambda}{2\pi\epsilon_0 R} \text{ N/C} \quad \dots(ii)$$

Due to negative charge on B , E_B also lies along positive x -axis (inward), i.e. from A to B .

So, the resultant electric field at R is given as

$$|E_R| = |E_A| + |E_B|$$

Substituting the values from Eqs. (i) and (ii), we get

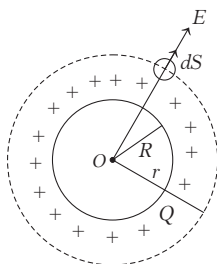
$$\begin{aligned} |E_R| &= \frac{\lambda}{2\pi\epsilon_0 R} + \frac{\lambda}{2\pi\epsilon_0 R} \\ &= \frac{\lambda}{\pi\epsilon_0 R} \text{ N/C} \end{aligned}$$

which also lies along the positive x -axis, i.e. from A to B .

- 6** (a) As the hollow sphere is uniformly charged, so the net charge will appear on the surface of the sphere.

(i) The electric field at a point outside the hollow sphere is

$$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \quad [\text{from Gauss' law}]$$



$$\Rightarrow E (4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\Rightarrow E \propto \frac{1}{r^2}$$

(ii) The electric field at the surface ($r = R$),

$$E = \frac{Q}{4\pi\epsilon_0 R^2}$$

(iii) The electric field inside hollow sphere ($r < R$) is

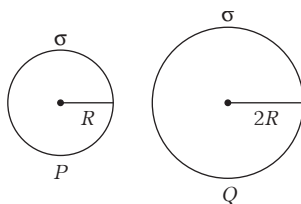
$$E = \frac{Q}{4\pi\epsilon_0 r^2} = 0 \quad [\because Q_{\text{inside}} = 0]$$

- 7** (d) The surface charge density of a closed metal sphere having charge Q is given by

$$\sigma = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{A}$$

or $Q = \sigma A$

Thus, the charges on sphere P and Q having same charge density as shown in the figure below is given by



$$Q_P = \sigma \times 4\pi R^2 = 4\pi\sigma R^2 \quad \dots(i)$$

$$\text{and} \quad Q_Q = \sigma \times 4\pi (2R)^2 = 16\pi\sigma R^2 \quad \dots(ii)$$

When they are brought in contact with each other, the total charge will be

$$\begin{aligned} Q_t &= Q_P + Q_Q \\ &= 4\pi\sigma R^2 + 16\pi\sigma R^2 \quad [\text{from Eqs. (i) and (ii)}] \\ &= 20\pi\sigma R^2 \quad \dots(iii) \end{aligned}$$

When two metallic charged spheres are connected to each other, then charge is flowing the sphere of higher potential to the sphere of lower potential till they may attain at common potential.

$$\begin{aligned} \frac{Q_P}{4\pi\epsilon_0 R} &= \frac{Q_Q}{4\pi\epsilon_0 2R} \\ \Rightarrow \frac{Q_P}{R} &= \frac{Q_Q}{2R} \Rightarrow \frac{Q_P}{Q_Q} = \frac{1}{2} \end{aligned}$$

So, the new charges on the sphere P and Q after separation will be distributed as

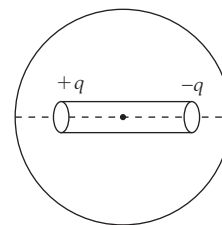
$$\Rightarrow Q'_P = \frac{1}{3} Q_t \text{ and } Q'_Q = \frac{2}{3} Q_t$$

After separation, the new surface charge densities on P and Q will be

$$\begin{aligned} \sigma_P &= \frac{Q'_P}{\text{Area}} = \frac{1}{3} \frac{Q_t}{\text{Area}} \\ &= \frac{1}{3} \frac{20\pi\sigma R^2}{4\pi R^2} = \frac{5}{3} \sigma \end{aligned}$$

$$\begin{aligned} \text{and} \quad \sigma_Q &= \frac{Q'_Q}{\text{Area}} = \frac{2}{3} \frac{Q_t}{\text{Area}} = \frac{2}{3} \times \frac{20\pi\sigma R^2}{4\pi (2R)^2} \\ &= \frac{2}{3} \times \frac{5}{4} \sigma = \frac{5}{6} \sigma \end{aligned}$$

- 8** (b) When a sphere encloses a charged dipole,



Here, $q = \pm 3 \times 10^{-6} \text{ C}$

Thus, according to Gauss's law, the net electric flux across the closed surface is equal to the net charge enclosed by it divided by ϵ_0 , i.e.

$$\phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{+3 \times 10^{-6} - 3 \times 10^{-6}}{\epsilon_0} = 0$$

Hence, electric flux across the sphere is zero.

- 9** (c) Force on a charged particle in the presence of an electric field is given as

$$F = qE \quad \dots(i)$$

where, q is the charge on the charged particle and E is the electric field.

From Newton's second law of motion, force on a particle with mass m is given as

$$F = ma \quad \dots(ii)$$

where, a is the acceleration.

From Eqs. (i) and (ii), we get

$$F = ma = qE$$

$$\Rightarrow a = \frac{qE}{m} \quad \dots(iii)$$

Now, consider that a particle falls from rest through a vertical distance h . Therefore, $u = 0$ and the second equation of motion becomes

$$h = ut + \frac{1}{2}at^2$$

$$\text{or } h = 0 \times t + \frac{1}{2}at^2 = \frac{1}{2} \times \frac{qE}{m} t^2 \quad [\text{from Eq. (iii)}]$$

$$\Rightarrow t^2 = \frac{2hm}{qE} \quad \text{or } t = \sqrt{\frac{2hm}{qE}}$$

Since, the particles given in the question are electron and proton and the quantity $\sqrt{2h/qE}$ (here, $q_p = q_e = e$) for both of them is constant. Thus, we can write

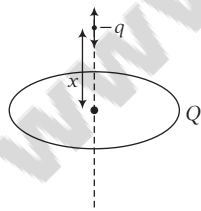
$$t = k\sqrt{m}$$

$$\text{where, } k = \sqrt{\frac{2h}{qE}} \quad \text{or } t \propto \sqrt{m}$$

As, mass of proton (m_p) $\gg$ mass of electron (m_e).

Thus, the time of fall of an electrons would be smaller than the time of fall of a protons.

- 10** (a) When the negative charge is shifted at a distance x from the centre of the ring along its axis, then force acting on the point charge due to the ring is



$$F = qE \quad (\text{towards centre})$$

$$= q \cdot \frac{kQx}{(R^2 + x^2)^{3/2}}$$

If $R \gg x$, then $R^2 + x^2 \simeq R^2$

$$\text{and } F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qqx}{R^3} \quad (\text{towards centre})$$

Since force on charged particle is acting in opposite direction of electric field, hence acceleration of charged particle towards the centre of ring is given by

$$\Rightarrow a = -\frac{F}{m} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Qqx}{mR^3}$$

Since, restoring force $F_E \propto -x$, therefore motion of charged particle will be SHM.

Acceleration on charged particle is also given by

$$a = -\omega^2 x$$

$$\frac{-1}{4\pi\epsilon_0} \cdot \frac{Qqx}{mR^3} = -\omega^2 x$$

$$\omega = \sqrt{\frac{Qq}{4\pi\epsilon_0 mR^3}}$$

$$\text{Time period of SHM, } T = \frac{2\pi}{\omega} = \left[\frac{16\pi^3 \epsilon_0 R^3 m}{Qq} \right]^{1/2}$$

- 11** (d) Given, $q = 0.05 \mu\text{C} = 5 \times 10^{-8} \text{C}$,

$$2a = 30 \text{ mm} = 0.03 \text{ m}$$

$$\text{and } E = 10^6 \text{ NC}^{-1}$$

Torque acting on an electric dipole placed in an uniform electric field,

$$\tau = pE \sin\theta$$

For maximum torque, $\theta = 90^\circ$

$$\begin{aligned} \therefore \tau_{\text{max}} &= pE = E(q \times 2a) \\ &= 10^6 \times 5 \times 10^{-8} \times 0.03 \\ &= 1.5 \times 10^{-3} \text{ N-m} \end{aligned}$$

- 12** (b) $\therefore$ Electrostatic force, $F = k \frac{Q_1 Q_2}{r^2}$

$$= 9 \times 10^9 \times \frac{2 \times 10^{-7} \times 3 \times 10^{-7}}{(30 \times 10^{-2})^2} = 6 \times 10^{-3} \text{ N}$$

- 13** (c) Net charge on one H-atom,

$$q = -e + e + \Delta e = \Delta e$$

Net electrostatic repulsive force between two H-atoms,

$$F_r = \frac{kq^2}{d^2} = \frac{k(\Delta e)^2}{d^2}$$

Similarly, net gravitational attractive force between two H-atoms,

$$F_G = \frac{Gm_h^2}{d^2}$$

It is given that, $F_r - F_G = 0$

$$\Rightarrow \frac{k(\Delta e)^2}{d^2} - \frac{Gm_h^2}{d^2} = 0$$

$$\Rightarrow (\Delta e)^2 = \frac{Gm_h^2}{k}$$

$$(\Delta e)^2 = \frac{(6.67 \times 10^{-11})(1.67 \times 10^{-27})^2}{9 \times 10^9}$$

$$\Rightarrow \Delta e = 1.437 \times 10^{-37} \text{ C}$$

$\therefore$ Order is Δe is 10^{-37} C .

- 14** (a) The electrostatic force of repulsion between the charge q and $(Q - q)$ at separation r is given by

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q(Q - q)}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ - q^2}{r^2}$$

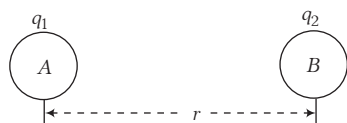
For maximum electrostatic force of repulsion, then $\frac{\partial F}{\partial q} = 0$

$$\text{i.e. } \frac{1}{4\pi\epsilon_0} \cdot \frac{(Q - 2q)}{r^2} = 0$$

As, $1/4\pi\epsilon_0 r^2$ is constant, therefore

$$Q - 2q = 0 \text{ or } Q = 2q$$

- 15 (b)** According to Coulomb's law, the force of repulsion between two conducting balls is given by $F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$



When the charged balls A and B are brought in contact, each sphere will attain equal charge q'

$$q' = \frac{q_1 + q_2}{2}$$

Now, the force of repulsion between them at the same distance r is

$$F' = \frac{q' \times q'}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0} \left[\frac{\left(\frac{q_1 + q_2}{2}\right) \left(\frac{q_1 + q_2}{2}\right)}{r^2} \right] = \frac{\left(\frac{q_1 + q_2}{2}\right)^2}{4\pi\epsilon_0 r^2}$$

$$\text{As, } \left(\frac{q_1 + q_2}{2}\right)^2 > q_1 q_2$$

$$\therefore F' > F$$

- 16 (a)** Due to presence of test charge q_0 in front of positively charged ball, there would be a redistribution of charge on the ball. In the redistribution of charge, there will be less charge on front half surface and more charge on the back half surface. As a result, the net force F between ball and charge will decrease, i.e. the electric field is decreased. Thus, actual electric field will be greater than F/q_0 .

- 17 (b)** Torque on an electric dipole in an electric field,

$$\tau = \mathbf{p} \times \mathbf{E} \Rightarrow |\tau| = pE \sin \theta$$

where, θ is angle between $\mathbf{E}$ and $\mathbf{p}$

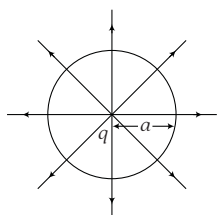
$$\Rightarrow 4 = p \times 2 \times 10^5 \times \sin 30^\circ \Rightarrow p = 4 \times 10^{-5}$$

$$\therefore q2l = 4 \times 10^{-5}$$

$$\text{where, } 2l = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$\therefore \text{The charge on dipole, } q = \frac{4 \times 10^{-5}}{2 \times 10^{-2}} = 2 \times 10^{-3} \text{ C} = 2 \text{ mC}$$

- 18 (c)** Given, $E = Ar$... (i)



Here, $r = a$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2}$$

From Eq. (i), we get

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2} = Aa$$

$\therefore$ Charge at the origin of the field, $q = 4\pi\epsilon_0 Aa^3$

- 19 (b)** $s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$ or $t = \sqrt{\frac{2s}{a}}$ ($\because u = 0$)

$$\text{As } s \text{ is same, } t \propto \frac{1}{\sqrt{a}}$$

$$\Rightarrow \frac{t_2}{t_1} = \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{F_e/M_e}{F_p/M_p}} = \sqrt{\frac{M_p}{M_e}}$$

- 20 (a)** The field at centre of hemispherical cup is given by

$$E = \frac{\sigma}{4\epsilon_0} \text{ and } \sigma = \frac{Q}{S}$$

$$\therefore E = \frac{Q}{S \times 4\epsilon_0}$$

On substituting values, we get

$$\Rightarrow 9 \times 10^8 = \frac{Q}{2\pi R^2 \times 4\epsilon_0}$$

$$\Rightarrow 9 \times 10^8 = \frac{5 \times 10^{-6}}{4\pi \epsilon_0 \times 2R^2}$$

$$\Rightarrow 9 \times 10^8 = \frac{5 \times 10^{-6} \times 9 \times 10^9}{2R^2}$$

$$\Rightarrow 2R^2 = 5 \times 10^{-6} \times 10 = 5 \times 10^{-5}$$

$$R = \sqrt{\frac{5}{2} \times 10^{-5}} = \sqrt{2.5 \times 10^{-5}}$$

$$\Rightarrow R = \sqrt{25 \times 10^{-6}} = 5 \times 10^{-3} \text{ m} = 5 \text{ mm}$$

- 21 (b)** At the point P electric field intensity is zero.

So, the electric field intensity due to both charges are equal and opposite.

Let distance of q_1 charge from point P is r .

So, distance of q_2 charge from point P is $(10 - r)$.

$$\frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{(10 - r)^2}$$

$$\Rightarrow \frac{q_1}{r^2} = \frac{q_2}{(10 - r)^2}$$

$$\Rightarrow \frac{9}{r^2} = \frac{4}{(10 - r)^2} \quad (\because q_1 = 9 \text{ C}, q_2 = 4 \text{ C})$$

$$\Rightarrow \left(\frac{3}{r}\right)^2 = \left(\frac{2}{10 - r}\right)^2$$

$$\Rightarrow \frac{3}{r} = \frac{2}{10 - r}$$

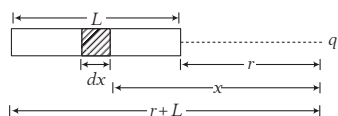
$$\Rightarrow 3(10 - r) = 2r$$

$$\Rightarrow 30 - 3r = 2r$$

$$\Rightarrow 5r = 30 \Rightarrow r = 6 \text{ m}$$

- 22 (d)** As a point charge q is situated at a distance r on axis from one end of a thin conducting rod of length L having charge Q as shown in figure.

Total charge = Q



Consider an elementary charge dq having length dx at a distance x from the charge q , then

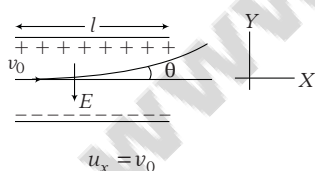
$$dq = \rho dx = \frac{Q}{L} dx$$

Now, force between dq and q

$$dF = \frac{1}{4\pi\epsilon_0} \frac{q dq}{x^2} = k \frac{q Q dx}{L x^2}$$

$$\begin{aligned} \therefore \text{Total force, } F &= \int dF = \frac{k Q q}{L} \int_r^{r+L} x^{-2} dx \\ &= \frac{k Q q}{L} \left[-\frac{1}{x} \right]_r^{r+L} \\ &= \frac{-k Q q}{L} \left[\frac{1}{r+L} - \frac{1}{r} \right] \\ &= \frac{-k Q q}{L} \left[\frac{r - r - L}{(r+L)r} \right] = \frac{k Q q}{r(r+L)} \end{aligned}$$

- 23 (a)** As, 10^{19} is removed from the neutral metal.
So, loss of charge = $1.6 \times 10^{-19} \times 10^{19} = 1.6 \text{ C}$
So, charge on metal = $+1.6 \text{ C}$
- 24 (c)** Electric field at centre of plastic sheet is the same as at close to the centre of the copper plate.
- 25 (b)** Initial velocity of the electron along x-direction,



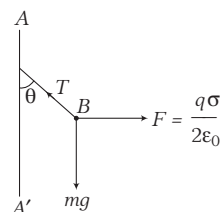
As, applied electric field is vertical,

$$\begin{aligned} v_x &= u_x = v_0 \\ \Rightarrow u_y &= 0 \text{ and } v_y = u_y + a_y t \\ \text{or } v_y &= 0 + \frac{eE}{m} \times \frac{l}{v_0} \quad [\because l = v_0 t] \\ \text{or } v_y &= \frac{eEl}{mv_0} \\ \text{We can write, } \tan \theta &= \frac{v_y}{v_x} \\ \Rightarrow \tan \theta &= \frac{eEl}{mv_0} \times \frac{1}{v_0} = \frac{eEl}{mv_0^2} \\ \Rightarrow \theta &= \tan^{-1} \left(\frac{eEl}{mv_0^2} \right) \end{aligned}$$

- 26 (a)** The diagram is as follows

The electric field due to charged infinite conducting sheet is

$$E = \frac{\sigma}{2\epsilon_0}$$



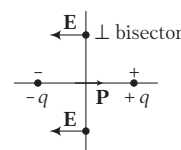
Now, force (electric force) on the charged ball is

$$F = qE = \frac{q\sigma}{2\epsilon_0}$$

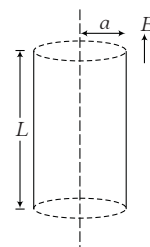
The resultant of electric force and mg balance the tension produced in the string.

$$\text{So, } \tan \theta = \frac{F}{mg} = \frac{q\sigma / 2\epsilon_0}{mg} = \frac{q\sigma}{2\epsilon_0 mg}$$

- 27 (a)** The angle between the dipole moment and electric field at any point on the equatorial plane is 180° .



- 28 (d)** Electric field lines does not form closed loop as it originates from the positive charge and terminate at negative charge. So, the option (d) is incorrect.
- 29 (d)** Gaussian surface is given below,



Net flux crossing through surface of cylinder is given by

$$\begin{aligned} \phi &= \text{Flux through upper disc} + \text{Flux through lower disc} \\ &\quad + \text{Flux through curved surface} \\ &= E(\pi a^2) + (-E\pi a^2) + 0 \\ &= \pi a^2 E - \pi a^2 E = 0 \end{aligned}$$

- 30 (d)** At equatorial, electric field,

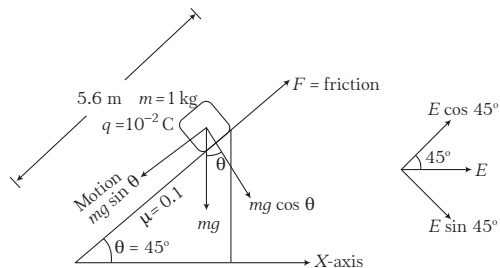
$$\begin{aligned} E &= \frac{9 \times 10^9 \times 2ql}{(r^2 + l^2)^{3/2}} \quad (\because l = 5 \text{ cm}) \\ E &= \frac{9 \times 10^9 \times 2 \times 5 \times 10^{-2} \times 10 \times 10^{-6}}{[(0.12)^2 + (0.05)^2]^{3/2}} \end{aligned}$$

Electric field, $E \approx 4.1 \times 10^6 \text{ N/C}$

- 31 (c)** Assuming the small area ΔS on the cylindrical surface. The normal to this area will be perpendicular to the axis of the cylinder, but the electric field is parallel to axis.

$$\text{Hence, } \Delta\phi = \mathbf{E} \cdot \Delta\mathbf{S} \cos \theta = E \cdot \Delta S \cos \frac{\pi}{2} = 0$$

- 32 (b)** The figure of the above situation is shown below



The electric field, $E = 100 \text{ V m}^{-1}$

For the downward motion of the particle of mass 1 kg.

$$mg \sin 45^\circ - qE \cos 45^\circ - \mu (mg \cos 45^\circ + qE \sin 45^\circ) = ma$$

$$\Rightarrow 1 \times 10 \times \frac{1}{\sqrt{2}} - \frac{1}{100} \times 100 \times \frac{1}{\sqrt{2}} - \frac{1}{10}$$

$$\left(1 \times 10 \times \frac{1}{\sqrt{2}} + \frac{1}{100} \times 100 \times \frac{1}{\sqrt{2}} \right) = 1 \times a$$

$$\Rightarrow \frac{10}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{10} \left(\frac{10}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) = a$$

$$\Rightarrow \frac{10}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{10\sqrt{2}} = a$$

$$\Rightarrow a = \frac{79}{10\sqrt{2}} = \frac{7.9}{\sqrt{2}}$$

$$\Rightarrow a \approx 5.6 \text{ ms}^{-2}$$

Now, time taken by the particle to cover 56 m distance along

$$\text{the incline plane is } t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 5.6}{5.6}} = \sqrt{2} = 1.41 \text{ s}$$

- 33 (a)** Consider the situation shown in the diagram.

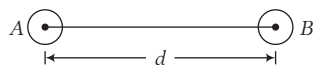


Let charges on sphere A and B are q_1 and q_2 respectively. Force between the charges placed in air by Coulomb's law (spheres can be assumed as point charges placed at their centres)

$$F_{AB} = k_1 \frac{q_1 q_2}{d^2} \quad \dots(i)$$

where,

$$k_1 = \frac{1}{4\pi\epsilon_0}$$



When the spheres are immersed in a liquid, force between

$$\text{the charges is } F'_{AB} = \frac{q_1 q_2}{4\pi\epsilon_r d^2} = \frac{q_1 q_2}{4\pi\epsilon_0 K} \times \frac{1}{d^2}$$

$$= \frac{1}{(4\pi\epsilon_0) \times 2} \times \frac{q_1 q_2}{d^2} = \frac{k_1}{2} \times \frac{q_1 q_2}{d^2} \quad \dots(ii)$$

where, K is dielectric constant of the liquid = 2

$$\text{From Eqs. (i) and (ii), we get } \frac{F_{AB}}{F'_{AB}} = \frac{k_1}{k_1/2} = 2$$

According to question, $F_{AB} = F$

$$\Rightarrow \frac{F}{F'_{AB}} = 2 \Rightarrow F'_{AB} = \frac{F}{2}$$

- 34 (b)** Given, charge on the body,

$$q = 1 \text{ nC} = 1 \times 10^{-9} \text{ C} \quad (\because 1 \text{ nC} = 10^{-9} \text{ C})$$

Charge on the electron $e = 1.6 \times 10^{-19}$

From the property of quantisation of charge, $q = ne$

$$\begin{aligned} \Rightarrow \text{Number of charges, } n &= \frac{q}{e} = \frac{1 \times 10^{-9}}{1.6 \times 10^{-19}} \\ &= 0.625 \times 10^{-9} \times 10^{19} \\ &= 6.25 \times 10^9 \end{aligned}$$

- 35 (a)** We know that, electric field at distance r from an infinitely long linear charge is given by

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \Rightarrow \lambda = 2\pi\epsilon_0 \cdot rE \text{ or } E \propto r^{-1}$$

- 36 (c)** We know that, force experienced by charge in electric field

$$F_1 = q_1 E = m_1 a_1 \quad \dots(i)$$

$$F_2 = q_2 E = m_2 a_2 \quad \dots(ii)$$

As,

$$q_1 = q_2 \Rightarrow F_1 = F_2$$

From Eqs. (i) and (ii), we get

$$\frac{a_1}{a_2} = \frac{F_1/m_1}{F_2/m_2} = \frac{F \cdot m_2}{F \cdot m_1}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{m_2}{m_1} = \frac{1}{0.5} = \frac{10}{5} = 2 \quad \left(\because \frac{m_1}{m_2} = 0.5 \right)$$

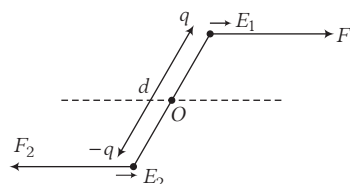
- 37 (c)** Torque on dipole form, $\tau = \mathbf{p} \times \mathbf{E} \sin \theta$

So, torque is maximum if $\mathbf{p}$ is perpendicular to $\mathbf{E}$.

Potential energy, $U = -\mathbf{p} \cdot \mathbf{E} = -pE \cos \theta$

Hence, when $\theta = 180^\circ$, i.e. $\mathbf{p}$ is anti-parallel to $\mathbf{E}$, then potential energy will be maximum.

- 38 (a)** Consider the diagram, where an electric dipole is placed in non-uniform electric field.



Electric field at the sight of q is E_1

and electric field at the sight of $-q$ is E_2

F_1 = force on the charge $q = qE_1$

F_2 = force on the charge $-q = -qE_2$

Net force on the dipole, $F = F_1 + F_2 = q(E_1 - E_2)$

$\because |E_1| \neq |E_2|$, so, $F_1 \neq F_2$ or $|F| \neq 0$

τ_1 = torque on the dipole due to $E_1 = p \times E_1$ [clockwise]

τ_2 = torque on the dipole due to $E_2 = p \times E_1$
[clockwise]

where, $p = qd$ is dipole moment of the dipole,

$$\tau_{\text{net}} = \tau_1 + \tau_2 = p \times E_1 + p \times E_2 \neq 0$$

Hence, both torque (τ_{net}) and force (F) act on the dipole.

- 39** (d) The Gauss's law of electrostatics gives a relation between electric flux through any closed hypothetical surface (called a Gaussian surface) and the charge enclosed by the surface.

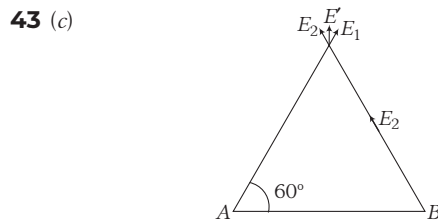
So, the nature of Gaussian surface is vector.

40 (b) $\tan \theta = \frac{F}{mg} \Rightarrow \frac{r/2}{y} = \frac{kq^2}{r^2 mg} \Rightarrow y \propto r^3$

Therefore, $\left(\frac{r'}{r}\right)^3 = \frac{y/2}{y} \Rightarrow r' = r \left(\frac{1}{2}\right)^{1/3}$

- 41** (d) Electric lines of force forms open loops while magnetic lines of force forms closed loops.

- 42** (b) In same medium, the force between two protons separated by same distance will F , as it is independent of the masses but depends on the charge of the particle.



Here, $E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2}$

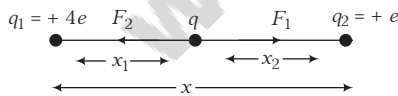
$$\Rightarrow E' = \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos \theta}$$

Given, $\theta = 60^\circ$

$$E' = \sqrt{3E_1^2}$$

$$\Rightarrow E' = \frac{q\sqrt{3}}{4\pi\epsilon_0 a^2}$$

- 44** (b) For equilibrium of q ,



$$|F_1| = |F_2|$$

$$\therefore \frac{1}{4\pi\epsilon_0} \frac{qq_1}{x_1^2} = \frac{1}{4\pi\epsilon_0} \frac{qq_2}{x_2^2}$$

$$x_2 = \frac{x}{\sqrt{\frac{q_1}{q_2} + 1}} = \frac{x}{\sqrt{\frac{4e}{e} + 1}} = \frac{x}{3}$$

- 45** (c) As the mass is moving in the electric field, then

$$ma = qE$$

$$\Rightarrow a = \frac{qE}{m} = \frac{3 \times 10^{-3} \times 80}{20 \times 10^{-3}} = 12 \text{ ms}^{-2}$$

By using, $v = u + at$

$$v = 20 + 12 \times 3 = 56 \text{ ms}^{-1}$$

- 46** (c) The electric field is due to all charges present whether inside or outside the given surface. So option (c) is correct.

- 47** (a) Electric flux is equal to the scalar product of an area vector and electric field $\mathbf{E}$. As the surface is lying in yz -plane,

$$\therefore \phi = \mathbf{E} \cdot d\mathbf{A} = (5\hat{i} + 4\hat{j} + 9\hat{k}) \cdot (20\hat{i}) = 100 \text{ unit}$$

- 48** (b) Total flux = $\frac{\text{Net enclosed charge}}{\epsilon_0}$

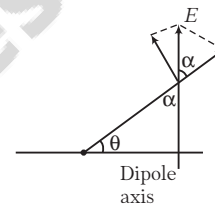
Hence, we can say that the electric flux depends only on net charge enclosed by surface. So, flux will remain the same.

- 49** (d) Flux of electric field $\mathbf{E}$ through any area A is defined as $\phi = EA \cos \theta$

The lines are parallel to the surface.

$\Rightarrow$ Angle between E and $A = 90^\circ$, hence $\phi = 0$.

- 50** (c) Here, $\alpha + \theta = 90^\circ$



If the net field from the inclined axis makes angle θ , then

$$\Rightarrow \tan \alpha = \frac{1}{2} \tan \theta \Rightarrow \tan \theta = 2 \tan \alpha = 2 \tan (90^\circ - \theta)$$

$$\Rightarrow \tan^2 \theta = 2 \Rightarrow \tan \theta = \sqrt{2}$$

$$\Rightarrow \theta = \tan^{-1}(\sqrt{2})$$

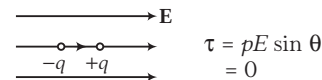
- 51** (a) If σ is the surface tension and r is the radius of soap bubble, then $p_{\text{excess}} = 4\sigma / r$

When the bubble is charged, $p_{\text{excess}} = p_{\text{electrostatics}} + \frac{4\sigma}{r}$

After electrification, surface tension decreases.

This decrease the pressure and increase the radius $\left(p \propto \frac{1}{r}\right)$.

- 52** (c) For stable equilibrium, the angle θ should be zero.



- 53** (d) The flux passing through the square of 1 m placed in xy -plane inside the electric field is zero because number of field lines entering are equal to number of field lines (flux) leaving the surface. So, net flux will be zero.

- 54** (a) As, we know that, charge on α -particle is double to that on electron.

$$\therefore \text{By Gauss' theorem, } |\phi_E| = \left| \frac{q}{\epsilon_0} \right|$$

$$\Rightarrow |\phi_E| = \left| \frac{-2e}{\epsilon_0} \right| = \frac{2e}{\epsilon_0}$$

CHAPTER 02

Electrostatic Potential and Capacitance

The electric field around a charge can be described in two ways by an electric field ($\mathbf{E}$) and in the form of electrostatic or electric potential (V).

The electric field ($\mathbf{E}$) is a vector quantity and we have already discussed about it in detail in the previous chapter.

In this chapter, we will study about the electric potential and how these quantities are interrelated to each other. We will also study about capacitor, *i.e.* a device used for storing electric energy.

ELECTRIC POTENTIAL

The electric potential at any point in the region of electric field is defined as the amount of work done in bringing a unit positive test charge from infinity to that point along any arbitrary path. It is a scalar quantity and is denoted by V .

$$\text{Electric potential (V)} = \frac{\text{Work done (W)}}{\text{Charge (} q_0 \text{)}}$$

According to the nature of charge, electric potential can be positive (due to positive charge), negative (due to negative charge) or zero.

Unit and dimensional formula of electric potential

Its SI unit is volt (V) and $1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$

and CGS unit is stat volt (esu), where $1 \text{ volt} = \frac{1}{300} \text{ stat volt}$

Its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

Example 2.1 How much work will be done in bringing a charge of $400 \mu C$ from infinity to some point P in the region of electric field? Given that the electric potential at point P is $20V$.

Sol. Electric potential at any point can be written as,

$$\text{Electric potential (V)} = \frac{\text{work done (W)}}{\text{charge (} q_0 \text{)}} \quad \dots (i)$$

Inside

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- Electrostatic potential due to a system of charges
- Electric potential due to a continuous charge distribution
- Electric potential due to electric dipole

2 Equipotential surfaces

- Variation of electric potential on the axis of a charged ring
- Potential due to charged sphere
- Motion of charged particle in electric field

3 Electric potential energy

- Potential energy of a system of charges

4 Electrostatics of conductors

- Dielectrics and polarisation
- Capacitors and capacitance
- Parallel plate capacitor

5 Combination of capacitors

- Special method to solve combination of capacitors
- Kirchhoff's law for capacitor circuits
- Energy stored in charged capacitor
- Common potential
- van de Graaff generator

Given, $V_P = 20 \text{ V}$ and $q_0 = 400 \mu\text{C} = 400 \times 10^{-6} \text{ C}$

On putting the values in Eq. (i), we get

$$20 = \frac{W}{400 \times 10^{-6}}$$

$$\therefore W = 20 \times 400 \times 10^{-6} = 8 \times 10^{-3} \text{ J}$$

Example 2.2 Find the work done by some external force in moving a charge $q = 2 \mu\text{C}$ from infinity to a point, where electric potential is 10^4 V .

Sol. Given, charge, $q = 2 \mu\text{C} = 2 \times 10^{-6} \text{ C}$ and electric potential, $V = 10^4 \text{ V}$

Using the relation, $W = qV$

$$\text{We have, } W = (2 \times 10^{-6})(10^4) = 2 \times 10^{-2} \text{ J}$$

Electric potential difference

The electric potential difference between two points A and B is equal to the work done by the external force in moving a unit positive charge against the electrostatic force from point B to A along any path between these two points.

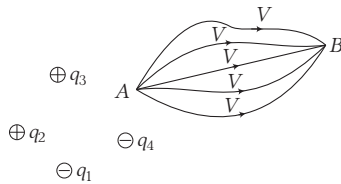


Fig 2.1 Electric potential between points A and B

If V_A and V_B be the electric potential at point A and B respectively, then $\Delta V = V_A - V_B$

or

$$\Delta V = \frac{W_{AB}}{q}$$

The SI unit of potential difference is volt (V).

The dimensional formula for electric potential difference is given by $[\text{ML}^2\text{T}^{-3}\text{A}^{-1}]$.

Note Following three formulae are very useful in the problems related to work done in electric field.

$$(W_{a-b})_{\text{electric force}} = q_0 (V_a - V_b)$$

$$(W_{a-b})_{\text{external force}} = q_0 (V_b - V_a) = - (W_{a-b})_{\text{electric force}}$$

$$(W_{\infty-a})_{\text{external force}} = q_0 V_a$$

Here, q_0 , V_a and V_b are to be substituted with sign.

Example 2.3 The electric potential at point A is 20 V and at B is -40 V . Find the work done by an external force and electrostatic force in moving an electron slowly from B to A .

Sol. Here, the test charge is an electron, i.e.

$$q_0 = -1.6 \times 10^{-19} \text{ C}$$

$$V_A = 20 \text{ V}$$

$$\text{and } V_B = -40 \text{ V}$$

Work done by external force,

$$\begin{aligned} (W_{B-A})_{\text{external force}} &= q_0 (V_A - V_B) \\ &= (-1.6 \times 10^{-19}) [(20) - (-40)] \\ &= -9.6 \times 10^{-18} \text{ J} \end{aligned}$$

Work done by electric force,

$$\begin{aligned} (W_{B-A})_{\text{electric force}} &= - (W_{B-A})_{\text{external force}} \\ &= -(-9.6 \times 10^{-18} \text{ J}) \\ &= 9.6 \times 10^{-18} \text{ J} \end{aligned}$$

Note Here, we can see that an electron (a negative charge) moves from B (lower potential) to A (higher potential) and the work done by electric force is positive. Therefore, we may conclude that whenever a negative charge moves from a lower potential to higher potential, work done by the electric force is positive or when a positive charge moves from lower potential to higher potential, the work done by the electric force is negative.

Electrostatic potential due to a point charge

Let P be the point at a distance r from the origin O at which the electric potential due to point charge $+q$ is required.

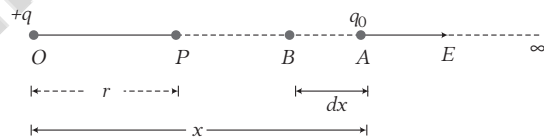


Fig. 2.2 Electrostatic potential at point due to a point charge

The electric potential at a point P is the amount of work done in carrying a unit positive charge from ∞ to point P .

Suppose a test charge q_0 is placed at point A at distance x from O .

The electrostatic force acting on charge q_0 is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_0}{x^2}, \text{ along } OA \quad \dots(i)$$

Small work done in moving the charge through a small distance dx from A to B is given by

$$dW = \mathbf{F} \cdot d\mathbf{x} = F dx \cos 180^\circ = -F dx \quad (\because \cos 180^\circ = -1)$$

Total work done in moving a positive test charge q_0 from ∞ to the point P is given by

$$\begin{aligned} W &= \int_{\infty}^r -F dx = \int_{\infty}^r -\frac{1}{4\pi\epsilon_0} \frac{qq_0}{x^2} dx \\ &= -\frac{qq_0}{4\pi\epsilon_0} \int_{\infty}^r x^{-2} dx = -\frac{qq_0}{4\pi\epsilon_0} \left[\frac{-1}{x} \right]_{\infty}^r \\ &= \frac{qq_0}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{\infty} \right] \quad \left(\because \int x^{-2} dx = -\frac{1}{x} \right) \end{aligned}$$

$$W = \frac{qq_0}{4\pi\epsilon_0 r} \quad \dots(iii)$$

From the definition of electric potential,

$$\therefore \boxed{V = \frac{W}{q_0} = \frac{q}{4\pi\epsilon_0 r}} \quad \dots(iv)$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Here, we assume that electrostatic potential is zero at infinity. Eq. (iv) shows that at equal distances from a point charge q , value of V is same.

Hence, electrostatic potential due to a single charge is spherically symmetric.

Figure given below shows the variation of electrostatic potential with distance, i.e. $V \propto \frac{1}{r}$.

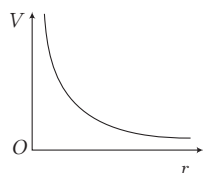


Fig. 2.3 Variation of V w.r.t. r

Example 2.4 Determine the potential at a point 0.50 m (i) from a $+20 \mu\text{C}$ point charge and (ii) from a $-20 \mu\text{C}$ point charge.

Sol. The potential due to a point charge is given by $V = \frac{kq}{r}$.

(i) Potential due to a positive $20 \mu\text{C}$ charge,

$$V = \frac{kq}{r} = (9.0 \times 10^9) \left(\frac{20 \times 10^{-6}}{0.50} \right) \\ = 3.6 \times 10^5 \text{ V}$$

(ii) Potential due to a negative $-20 \mu\text{C}$ charge,

$$V = (9.0 \times 10^9) \left(\frac{-20 \times 10^{-6}}{0.50} \right) \\ = -3.6 \times 10^5 \text{ V}$$

Example 2.5 What is the electrostatic potential at the surface of a silver nucleus of diameter 12.4 fermi? [Atomic number (Z) for silver is 47]

Sol. Given, radius of silver nucleus, $r = \frac{12.4}{2} = 6.2 \text{ fm}$
 $= 6.2 \times 10^{-15} \text{ m}$
 (1 fermi = 10^{-15} m)

and $Z = 47$

$\therefore$ Charge, $q = Ze = 47 \times 1.6 \times 10^{-19} \text{ C}$

$\therefore$ Electrostatic potential,

$$V = \frac{q}{4\pi\epsilon_0 r} = \frac{9 \times 10^9 \times 47 \times 1.6 \times 10^{-19}}{6.2 \times 10^{-15}} = 1.09 \times 10^7 \text{ V}$$

Electrostatic potential due to a system of charges

Let there be a number of point charges $q_1, q_2, q_3, \dots, q_n$ at distances $r_1, r_2, r_3, \dots, r_n$ respectively from the point P , where electric potential is to be calculated.

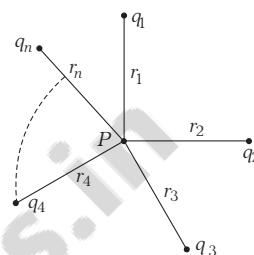


Fig. 2.4 A system of charges

Potential at P due to charge q_1 ,

$$V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{1P}}$$

Similarly,

$$V_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{2P}}$$

$$V_3 = \frac{1}{4\pi\epsilon_0} \frac{q_3}{r_{3P}}$$

$$\vdots$$

$$V_n = \frac{1}{4\pi\epsilon_0} \frac{q_n}{r_{nP}}$$

Using superposition principle, we obtain resultant potential at P due to total charge configuration as the algebraic sum of the potentials due to individual charges.

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \frac{q_3}{r_{3P}} + \dots + \frac{q_n}{r_{nP}} \right)$$

$\Rightarrow$

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_{iP}}}$$

The net potential at a point due to multiple charges is equal to the algebraic sum of the potentials due to individual charges at that particular point.

Mathematically, it is expressed as $V_{\text{net}} = \sum_{i=1}^n V_i$

Note If $r_1, r_2, r_3, \dots, r_n$ are position vectors of the charges $q_1, q_2, q_3, \dots, q_n$ respectively, then electrostatic potential at point P whose position vector is $\mathbf{r}_0$ would be $V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|}$

Example 2.6 Three point charges $q_1 = 1 \mu\text{C}$, $q_2 = -2 \mu\text{C}$ and $q_3 = 3 \mu\text{C}$ are placed at $(1 \text{ m}, 0, 0)$, $(0, 2 \text{ m}, 0)$ and $(0, 0, 3 \text{ m})$, respectively. Find the electric potential at origin.

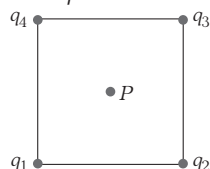
Sol. The net electric potential at origin is

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} \right]$$

Substituting the values, we have

$$V = (9.0 \times 10^9) \left(\frac{1}{1.0} - \frac{2}{2.0} + \frac{3}{3.0} \right) \times 10^{-6} = 9.0 \times 10^3 \text{ V}$$

Example 2.7 In the given figure, there are four point charges placed at the vertices of a square of side, $a = 1.4 \text{ m}$. If $q_1 = +18 \text{ nC}$, $q_2 = -24 \text{ nC}$, $q_3 = +35 \text{ nC}$ and $q_4 = +16 \text{ nC}$, then find the electric potential at the centre P of the square. Assume the potential to be zero at infinity.



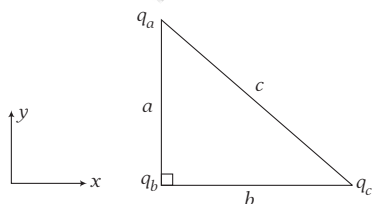
Sol. The distance of the point P from each charge is

$$r = \frac{a}{\sqrt{2}} = \frac{1.4 \text{ m}}{1.4} = 1 \text{ m}$$

Potential, $V = V_1 + V_2 + V_3 + V_4$

$$\begin{aligned} &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \frac{q_4}{r_4} \right] \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 + q_2 + q_3 + q_4}{r} \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{(18 - 24 + 35 + 16) \times 10^{-9}}{1} \\ &= (9 \times 10^9) \times (45 \times 10^{-9}) \text{ V} = 405 \text{ V} \end{aligned}$$

Example 2.8 Suppose that three point charges, q_a , q_b and q_c are arranged at the vertices of a right-angled triangle, as shown. What is the absolute electric potential at the position of the third charge, if $q_a = -6.0 \mu\text{C}$, $q_b = +4.0 \mu\text{C}$, $q_c = +2.0 \mu\text{C}$, $a = 4.0 \text{ m}$ and $b = 3.0 \text{ m}$?



Sol. The electric potential at P (the position of the third charge) due to the presence of the first charge is

$$V_a = k_e \frac{q_a}{c} = (9 \times 10^9) \frac{-6 \times 10^{-6}}{\sqrt{4^2 + 3^2}} = -1.08 \times 10^4 \text{ V}$$

Likewise, the electric potential due to the presence of the second charge is

$$V_b = k_e \frac{q_b}{b} = (9 \times 10^9) \frac{4 \times 10^{-6}}{3} = 1.20 \times 10^4 \text{ V}$$

The net potential of the third charge V_c is simply the algebraic sum of the potentials due to the others two charges taken in isolation. Thus, $V_c = V_a + V_b = 1.20 \times 10^3 \text{ V}$

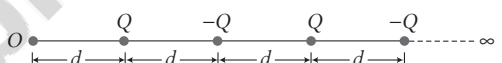
Example 2.9

(i) Infinite charges each of Q are placed, at $x = d, 2d, 4d, \dots, \infty$. Find electric potential at origin O .

(ii) A charge $+Q$ is placed at each of the points $x = d, x = 3d, x = 5d, \dots, \infty$ on the X -axis, and a charge $-Q$ is placed at each of the points, $x = 2d, x = 4d, x = 6d, \dots, \infty$. Find the electric potential at the origin O .

Sol. (i) 

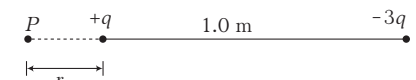
$$\begin{aligned} \text{Electric potential, } V_O &= \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{d} + \frac{Q}{2d} + \frac{Q}{4d} + \dots \infty \right] \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{d} \left[1 + \frac{1}{2} + \frac{1}{4} + \dots \infty \right] \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{d} \left[\frac{1}{1 - 1/2} \right] = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{d} \end{aligned}$$

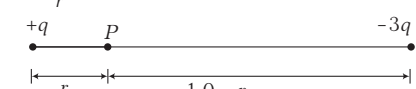
(ii) 

$$\begin{aligned} \text{Electric potential, } V_O &= \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{d} - \frac{Q}{2d} + \frac{Q}{3d} - \frac{Q}{4d} + \dots \infty \right] \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{d} \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \infty \right] \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{d} \log_e 2 \end{aligned}$$

Example 2.10 Find out the points on the line joining two charges $+q$ and $-3q$ (kept at a distance of 1.0 m), where electric potential is zero.

Sol. Let P be the point on the axis either to the left or to the right of charge $+q$ at a distance r where potential is zero. Hence,



or 

$$V_P = \frac{q}{4\pi\epsilon_0 r} - \frac{3q}{4\pi\epsilon_0 (1+r)} = 0$$

Solving this, we get $r = 0.5 \text{ m}$

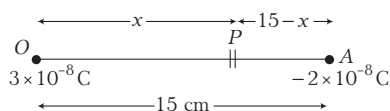
Further, $V_P = \frac{q}{4\pi\epsilon_0 r} - \frac{3q}{4\pi\epsilon_0 (1-r)} = 0$

which gives, $r = 0.25 \text{ m}$

Thus, the potential will be zero at point P on the axis which is either 0.5 m to the left or 0.25 m to the right of charge $+q$.

Example 2.11 Two charges $3 \times 10^{-8} \text{ C}$ and $-2 \times 10^{-8} \text{ C}$ are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? (Take the potential at infinity to be zero)

Sol. Let us take the origin O at the location of the positive charge. The line joining the two charges is taken to be the X -axis and the negative charge is taken to be on the right side of the origin.

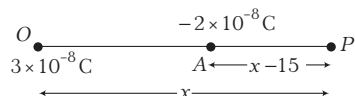


Let P be the required point on the X -axis, where the potential is zero. If x is the x -coordinate of P , obviously x must be positive. If x lies between O and A , we have

$$\frac{1}{4\pi\epsilon_0} \left[\frac{3 \times 10^{-8}}{x \times 10^{-2}} - \frac{2 \times 10^{-8}}{(15 - x) \times 10^{-2}} \right] = 0$$

where, x is in cm, i.e. $\frac{3}{x} - \frac{2}{15 - x} = 0$

which gives, $x = 9 \text{ cm}$



If x lies on the extended line OA , the required condition is

$$\frac{3}{x} - \frac{2}{x - 15} = 0$$

which gives, $x = 45 \text{ cm}$

Note that the formula for potential used in the calculation requires choosing potential to be zero at infinity.

Electric potential due to a continuous charge distribution

We can imagine that a continuous charge distribution consists of a number of small charge elements located at position $\mathbf{r}_i$. If $\mathbf{r}$ is the position vector of point P , then the electric potential at point P due to the continuous charge

distribution can be written as $V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{|\mathbf{r} - \mathbf{r}_i|}$

When the charge is distributed continuously in a volume V , then $dq = \rho dV$, where ρ is **volume charge density**. The potential at point P due to the **volume charge distribution** will be

$$V_V = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho dV}{|\mathbf{r} - \mathbf{r}_i|}$$

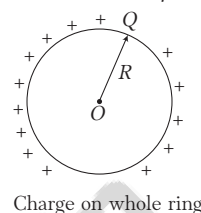
When the charge is distributed continuously over an area S , then $dq = \sigma dS$, where σ is **surface charge density**.

$$\therefore V_S = \frac{1}{4\pi\epsilon_0} \int_S \frac{\sigma dS}{|\mathbf{r} - \mathbf{r}_i|}$$

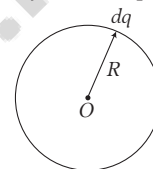
When the charge is distributed uniformly along a line L , then $dq = \lambda dL$, where λ is **line charge density**.

$$\therefore V_L = \frac{1}{4\pi\epsilon_0} \int_L \frac{\lambda dL}{|\mathbf{r} - \mathbf{r}_i|}$$

Example 2.12 A charge Q is distributed uniformly on a ring of radius R as shown in the following diagrams. Find the electric potential at the centre O of the ring.



Sol. Consider a small charge dq . Electric potential at O due to dq ,



$$dV = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{R}$$

$$V_0 = \sum dV = \frac{1}{4\pi\epsilon_0 R} \sum dq = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R}$$

Electric potential due to electric dipole

Here, we are going to determine the potential due to an electric dipole.

Let AB be an electric dipole of length $2a$ and let P be any point, where $OP = r$.

Let θ be the angle between r and the dipole axis.

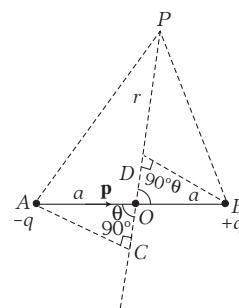


Fig. 2.5 Electric dipole

Here, $AB = 2a$, $AO = OB = a$ and $OP = r$

$$\text{In } \triangle OAC, \quad \cos \theta = \frac{OC}{OA} = \frac{OC}{a}$$

$$\therefore OC = a \cos \theta$$

Also,

$$OD = a \cos \theta$$

$$\text{If } r \gg a, PA = PC = OP + OC = r + a \cos \theta$$

$$PB = PD = OP - OD = r - a \cos \theta$$

V is the potential due to electric dipole,

$$V = \left(\frac{1}{4\pi\epsilon_0} \right) \left[\frac{q}{PB} - \frac{q}{PA} \right]$$

$$V = \left(\frac{1}{4\pi\epsilon_0} \right) q \left[\frac{1}{(r - a \cos \theta)} - \frac{1}{(r + a \cos \theta)} \right]$$

$$V = \left(\frac{1}{4\pi\epsilon_0} \right) \left[\frac{2a q \cos \theta}{r^2 - a^2 \cos^2 \theta} \right]$$

$$V = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{p \cos \theta}{(r^2 - a^2 \cos^2 \theta)} \quad (\because p = 2aq)$$

If $r \gg a$, $a^2 \cos^2 \theta$ can be neglected in comparison to r^2 ,

then the resultant potential at point P ,

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

where, p is dipole moment.

Special Cases

- (i) When the point P lies on the axial line of the dipole on the side of positive charge, $\theta = 0^\circ \Rightarrow \cos \theta = 1^\circ$

$$\therefore V = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{p}{(r^2 - a^2)}$$

If r is very large as compared to $2a$, i.e. $r \gg 2a$, then a^2 can be neglected in comparison to r^2 , then potential at the point P ,

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2}$$

- (ii) When P lies on other side, $\theta = 180^\circ$

$$\Rightarrow \cos \theta = -1$$

$$\therefore V = - \left(\frac{1}{4\pi\epsilon_0} \right) \frac{p}{(r^2 - a^2)}$$

$$\text{If } r \gg a, \text{ then } V = - \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2}$$

- (iii) When the point P lies on equatorial line, $\theta = 90^\circ$

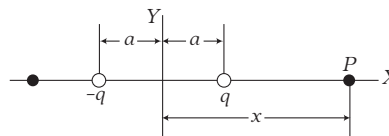
$$\Rightarrow \cos \theta = 0$$

$$\therefore V = 0$$

- (iv) In general, $V = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{\mathbf{p} \cdot \mathbf{r}}{r^3}$, for a short dipole, at a

point whose position vector with respect to dipole is $\mathbf{r}$.

Example 2.13 An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$ as shown in figure. The dipole is along the X -axis and is centred at the origin.



(i) Calculate the electric potential at point P .

(ii) Calculate V at a point far from the dipole.

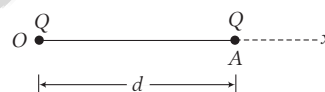
Sol. (i) For a point in figure,

$$V = k_e \sum \frac{q_i}{r_i} = k_e \left(\frac{q}{x-a} - \frac{q}{x+a} \right) = \frac{2k_e qa}{x^2 - a^2}$$

- (ii) If point P is far from the dipole, such that $x \gg a$, then a^2 can be neglected in the terms, $x^2 - a^2$ and V

$$\text{becomes } V = \frac{2k_e qa}{x^2} \quad (x \gg a)$$

Example 2.14 Sketch the variation of electric potential on the x -axis with respect to x for $x = -\infty$ to $x = +\infty$ in the following cases.



Sol.

Electric potential at P ,

$$V_P = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{x} + \frac{Q}{d-x} \right] = \frac{Q}{4\pi\epsilon_0} \cdot \frac{d}{x(d-x)}$$

V_P is minimum, if $x(d-x)$ is maximum.

$x(d-x)$ is maximum,

$$\text{if } \frac{d}{dx} x(d-x) = 0 \Rightarrow d - 2x = 0 \Rightarrow x = d/2$$

At mid-point of OA , electric potential is minimum.

At O : $x \rightarrow 0, V \rightarrow \infty$

At A : $x \rightarrow d, V \rightarrow \infty$

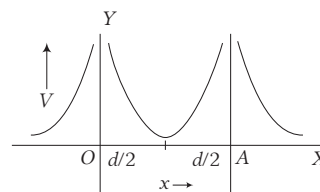
From O to A , electric potential decreases reaching to minimum value and then increases.

Left of O : At distance x from O ,

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{x} + \frac{Q}{d+x} \right]$$

$$x \rightarrow 0, V \rightarrow \infty$$

$$x \rightarrow -\infty, V \rightarrow 0$$



Right of A : At distance x from O, $V = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{x} + \frac{Q}{x-d} \right]$

$$x \rightarrow d, V \rightarrow \infty$$

$$x \rightarrow \infty, V \rightarrow 0$$

Relation between electric field and electric potential

Let us first consider the case when electric potential V is known and we want to calculate $\mathbf{E}$. The relation is as under,

In case of cartesian coordinates

$$\mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}$$

$$\text{Here, } E_x = -\frac{\partial V}{\partial x} = -(\text{partial derivative of } V \text{ w.r.t. } x)$$

$$E_y = -\frac{\partial V}{\partial y} = -(\text{partial derivative of } V \text{ w.r.t. } y)$$

$$E_z = -\frac{\partial V}{\partial z} = -(\text{partial derivative of } V \text{ w.r.t. } z)$$

$$\therefore \mathbf{E} = -\left[\frac{\partial V}{\partial x} \hat{\mathbf{i}} + \frac{\partial V}{\partial y} \hat{\mathbf{j}} + \frac{\partial V}{\partial z} \hat{\mathbf{k}} \right]$$

Sometimes it is also written as, $\mathbf{E} = -\text{gradient } V$

$$= -\text{grad } V = -\nabla V = -\frac{dV}{dr}$$

$$\boxed{\mathbf{E} = -\frac{dV}{dr}}$$

Here, negative sign shows that the potential decreases in the direction of electric field.

Potential gradient is a vector quantity.

Example 2.15 The electric potential in a region is represented as $V = 2x + 3y - z$

Obtain expression for electric field strength.

Sol. As we know, $\mathbf{E} = -\left[\frac{\partial V}{\partial x} \hat{\mathbf{i}} + \frac{\partial V}{\partial y} \hat{\mathbf{j}} + \frac{\partial V}{\partial z} \hat{\mathbf{k}} \right]$

$$\text{So, } \frac{\partial V}{\partial x} = \frac{\partial}{\partial x} (2x + 3y - z) = 2$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y} (2x + 3y - z) = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z} (2x + 3y - z) = -1$$

$$\text{Electric field, } \mathbf{E} = -2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

Example 2.16 Find V_{ab} in an electric field,

$$\mathbf{E} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \frac{N}{C}$$

$$\text{where, } \mathbf{r}_a = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) m$$

$$\text{and } \mathbf{r}_b = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) m$$

Sol. Here, the given field is uniform (constant). So using,

$$dV = -\mathbf{E} \cdot d\mathbf{r}$$

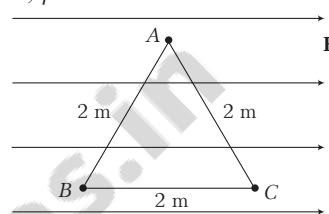
$$\text{or } V_{ab} = V_a - V_b = -\int_{r_b}^{r_a} \mathbf{E} \cdot d\mathbf{r}$$

$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$$

$$= -\int_{(2,1,-2)}^{(1,-2,1)} (2dx + 3dy + 4dz)$$

$$= -[2x + 3y + 4z]_{(2,1,-2)}^{(1,-2,1)} = -1 \text{ V}$$

Example 2.17 In uniform electric field, $\mathbf{E} = 10 \text{ NC}^{-1}$ as shown in figure, find



$$(i) V_A - V_B$$

$$(ii) V_B - V_C$$

Sol. (i) $V_B > V_A$, so $V_A - V_B$ will be negative.

$$\text{Further, } d_{AB} = 2 \cos 60^\circ = 1 \text{ m}$$

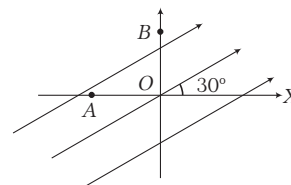
$$\therefore V_A - V_B = -Ed_{AB} = (-10)(1) = -10 \text{ V}$$

(ii) $V_B > V_C$, so $V_B - V_C$ will be positive.

$$\text{Further, } d_{BC} = 2.0 \text{ m}$$

$$\therefore V_B - V_C = (10)(2) = 20 \text{ V}$$

Example 2.18 A uniform electric field of 100 Vm^{-1} is directed at 30° with the positive X-axis as shown in figure. Find the potential difference, V_{BA} if $OA = 2 \text{ m}$ and $OB = 4 \text{ m}$.



Sol. This problem can be solved by both the methods as discussed below.

Method 1 Electric field in vector form can be written as

$$\mathbf{E} = (100 \cos 30^\circ \hat{\mathbf{i}} + 100 \sin 30^\circ \hat{\mathbf{j}}) \text{ Vm}^{-1}$$

$$= (50\sqrt{3} \hat{\mathbf{i}} + 50\hat{\mathbf{j}}) \text{ Vm}^{-1}$$

$$A = (-2 \text{ m}, 0, 0)$$

$$\text{and } B = (0, 4 \text{ m}, 0)$$

$$\therefore V_{BA} = V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{r}$$

$$= -\int_{(-2 \text{ m}, 0, 0)}^{(0, 4 \text{ m}, 0)} (50\sqrt{3} \hat{\mathbf{i}} + 50\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}})$$

$$= -[50\sqrt{3}x + 50y]_{(-2 \text{ m}, 0, 0)}^{(0, 4 \text{ m}, 0)} = -100(2 + \sqrt{3}) \text{ V}$$

Method 2 We can also use, $V = Ed$

With the view that, $V_A > V_B$ or $V_B - V_A$ will be negative.

Here, $d_{AB} = OA \cos 30^\circ + OB \sin 30^\circ$

$$= 2 \times \frac{\sqrt{3}}{2} + 4 \times \frac{1}{2} = (\sqrt{3} + 2)$$

$$\therefore V_B - V_A = -Ed_{AB} = -100(2 + \sqrt{3})$$

Example 2.19 Two points A and B are 2 cm apart and a uniform electric field E acts along the straight line AB directed from A to B with $E = 200 \text{ NC}^{-1}$. A particle of charge $+10^{-6} \text{ C}$ is taken from A to B along AB. Calculate

- the force on the charge,
- the potential difference, $V_A - V_B$ and
- the work done on the charge by $\mathbf{E}$.

Sol. (i) Electrostatic force on the charge,

$$F = qE = (10^{-6})(200) = 2 \times 10^{-4} \text{ N}$$

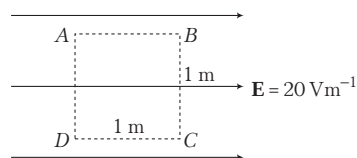
(ii) In uniform electric field,

$$V = Ed$$

$$\therefore \text{Potential difference, } V_A - V_B = 200 \times 2 \times 10^{-2} = 4 \text{ V}$$

$$\begin{aligned} \text{(iii) } W &= Fs \cos \theta = (2 \times 10^{-4})(2 \times 10^{-2}) \cos 0^\circ \\ &= 4 \times 10^{-6} \text{ J} \end{aligned}$$

Example 2.20 In the uniform electric field shown in figure, find



$$\text{(i) } V_A - V_D$$

$$\text{(ii) } V_A - V_C$$

$$\text{(iii) } V_B - V_D$$

$$\text{(iv) } V_C - V_D$$

CHECK POINT 2.1

- Find the work done by some external force in moving a charge $q = 4 \mu\text{C}$ from infinity to a point, where electric potential is 10^4 V .
 (a) $4 \times 10^{-2} \text{ J}$ (b) $2 \times 10^{-2} \text{ J}$
 (c) $8 \times 10^{-2} \text{ J}$ (d) $1 \times 10^{-2} \text{ J}$
- Equal charges are given to two spheres of different radii. The potential will
 (a) be more on the smaller sphere
 (b) be more on the bigger sphere
 (c) be equal on both the spheres
 (d) depend on the nature of the materials of the spheres
- The electric potential at a point in free space due to a charge Q coulomb is $Q \times 10^{11} \text{ volts}$. The electric field at the point is
 (a) $4\pi\epsilon_0 Q \times 10^{20} \text{ Vm}^{-1}$ (b) $12\pi\epsilon_0 Q \times 10^{22} \text{ Vm}^{-1}$
 (c) $4\pi\epsilon_0 Q \times 10^{22} \text{ Vm}^{-1}$ (d) $12\pi\epsilon_0 Q \times 10^{20} \text{ Vm}^{-1}$

Sol. Using the relation, $V = Ed$ (in uniform $\mathbf{E}$)

where, V = potential difference between the two points

E = magnitude of $\mathbf{E}$

and d = projection of line (joining two points) along $\mathbf{E}$.

$$\text{(i) } V_A - V_D = 0 \text{ as } d = 0$$

$$\text{(ii) } V_A - V_C = + (20)(1) = 20 \text{ V}$$

$$\text{(iii) } V_B - V_D = - (20)(1) = -20 \text{ V}$$

$$\text{(iv) } V_C - V_D = - (20)(1) = -20 \text{ V}$$

Example 2.21 An electric field $\mathbf{E} = (20\hat{i} + 30\hat{j}) \text{ N/C}$ exists in the space. If the potential at the origin is taken to be zero, find the potential at $(2\text{m}, 2\text{m})$.

Sol. We have, $E = \frac{-dV}{dr}$, so it can be written in vector form as

$$dV = -\mathbf{E} \cdot d\mathbf{r}$$

Note, we can write, $\mathbf{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$$\text{and } d\mathbf{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\text{Therefore, } \mathbf{E} \cdot d\mathbf{r} = E_x \cdot dx + E_y \cdot dy + E_z \cdot dz$$

In the given question, the z -component of E or the point is not given. So, we can write

$$\mathbf{E} \cdot d\mathbf{r} = E_x \cdot dx + E_y \cdot dy$$

$$\text{Now, } dV = -\mathbf{E} \cdot d\mathbf{r} = - (20\hat{i} + 30\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$\text{or } dV = -20 dx - 30 dy$$

Now, we will have to integrate it within limits.

Given, $V = 0$ when $x = 0$ and $y = 0$ (lower limit) and we have to calculate V . When $x = 2$ and $y = 2$ (upper limit)

$$\text{Therefore, } \int_0^V dV = -20 \cdot \int_{x=0}^{x=2} dx - 30 \int_{y=0}^{y=2} dy$$

$$\text{or } [V]_0^V = -20 [x]_0^2 - 30 [y]_0^2$$

$$\text{or } V - 0 = -20(2 - 0) - 30(2 - 0)$$

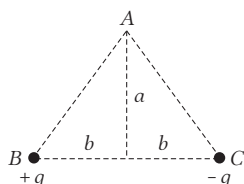
$$\therefore \text{Potential, } V = -40 - 60 = -100 \text{ V}$$

- In an hydrogen atom, the electron revolves around the nucleus in an orbit of radius $0.53 \times 10^{-10} \text{ m}$. Then, the electrical potential produced by the nucleus at the position of the electron is
 (a) -13.6 V (b) -27.2 V (c) 27.2 V (d) 13.6 V
- Three charges $2q, -q, -q$ are located at the vertices of an equilateral triangle. At the centre of the triangle,
 (a) the field is zero but potential is non-zero
 (b) the field is non-zero but potential is zero
 (c) Both field and potential are zero
 (d) Both field and potential are non-zero
- In a region of constant potential,
 (a) the electric field is uniform
 (b) the electric field is zero
 (c) there can be no charge inside the region
 (d) the electric field shall necessarily change if a charge is placed outside the region

7. The work done in bringing a 20 C charge from point A to point B for distance 0.2 m is 2 J. The potential difference between the two points will be (in volt)

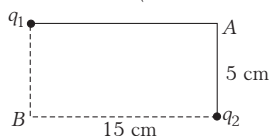
(a) 0.2 (b) 8
(c) 0.1 (d) 0.4

8. As shown in the figure, charges $+q$ and $-q$ are placed at the vertices B and C of an isosceles triangle. The potential at the vertex A is



(a) $\frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{\sqrt{a^2 + b^2}}$ (b) zero
(c) $\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\sqrt{a^2 + b^2}}$ (d) $\frac{1}{4\pi\epsilon_0} \cdot \frac{(-q)}{\sqrt{a^2 + b^2}}$

9. In the rectangle shown below, the two corners have charges $q_1 = -5 \mu\text{C}$ and $q_2 = +2.0 \mu\text{C}$. The work done in moving a charge $+3.0 \mu\text{C}$ from B to A is (take, $k = 10^{10} \text{ N-m}^2/\text{C}^2$)



(a) 2.8 J (b) 3.5 J
(c) 4.5 J (d) 5.5 J

10. Two point charges $-q$ and $+q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$, respectively. The potential at a point $(0, 0, z)$, where $z > a$ is

(a) $\frac{qa}{4\pi\epsilon_0 z^2}$ (b) $\frac{q}{4\pi\epsilon_0 a}$ (c) $\frac{2qa}{4\pi\epsilon_0 (z^2 - a^2)}$ (d) $\frac{2qa}{4\pi\epsilon_0 (z^2 + a^2)}$

11. Two plates are 2 cm apart and a potential difference of 10 V is applied between them, then the electric field between the plates is

(a) 20 NC^{-1} (b) 500 NC^{-1} (c) 5 NC^{-1} (d) 250 NC^{-1}

12. At a certain distance from a point charge the electric field is 500 Vm^{-1} and the potential is 3000 V. What is this distance?

(a) 6 m (b) 12 m (c) 36 m (d) 144 m

13. Two charges of $4 \mu\text{C}$ each are placed at the corners A and B of an equilateral triangle of side length 0.2 m in air. The electric potential at C is $\left[\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N-m}^2/\text{C}^2 \right]$

(a) $9 \times 10^4 \text{ V}$ (b) $18 \times 10^4 \text{ V}$
(c) $36 \times 10^4 \text{ V}$ (d) $36 \times 10^{-4} \text{ V}$

14. The electric potential V at any point $O(x, y, z)$ all in metres) in space is given by $V = 4x^2 \text{ V}$. The electric field at the point $(1 \text{ m}, 0, 2 \text{ m})$ in volt metre^{-1} is

(a) 8, along negative X-axis (b) 8, along positive X-axis
(c) 16, along negative X-axis (d) 16, along positive Z-axis

15. The electric potential V is given as a function of distance x (metre) by $V = (5x^2 + 10x - 9) \text{ V}$. Value of electric field at $x = 1$ is

(a) -20 Vm^{-1} (b) 6 Vm^{-1} (c) 11 Vm^{-1} (d) -23 Vm^{-1}

EQUIPOTENTIAL SURFACES

Any surface over which the electric potential is same everywhere is called an **equipotential surface**. For a single charge q , the potential is given by $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

This shows that if r is constant, then V will be constant. Thus, equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge.

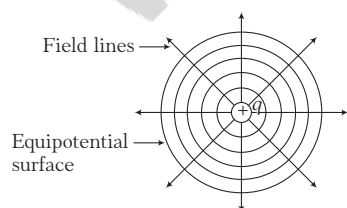


Fig. 2.6 Concentric spherical surfaces having charge q at the centre

Properties of a equipotential surfaces

- (i) Potential difference between any two points on an equipotential surface is zero.

- (ii) Two equipotential surfaces can never intersect each other. If two equipotential surfaces intersect each other, then at the point of intersection of these, there will be two directions of electric field which is impossible.

- (iii) As, the work done by electric force is zero when a test charge is moved along the equipotential surface, it follows that $\mathbf{E}$ must be perpendicular to the surface at every point, so that the electric force $q_0 \mathbf{E}$ will always be perpendicular to the displacement of a charge moving on the surface. Thus, field lines and equipotential surfaces are always mutually perpendicular.

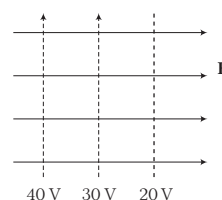


Fig. 2.7

Equipotential surface for charge distribution

- (i) The equipotential surfaces are a family of concentric spheres for a point charge or a sphere of charge as shown in the following figures.

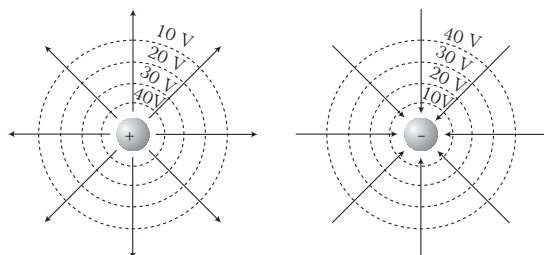


Fig. 2.8 Equipotential surfaces

- (ii) When potential is applied between two charged plates, electric field ($\mathbf{E}$) is setup between them and this $\mathbf{E}$ is normal to the equipotential surface as shown below

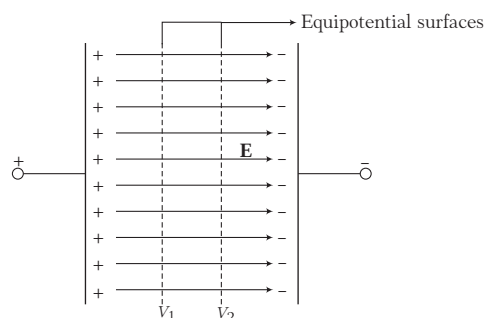


Fig. 2.9 Equipotential surfaces between two charged plates

- (iii) The equipotential surfaces are a family of concentric cylinders for a line of charge or cylinder of charge. Equipotential surface for line charge is shown below

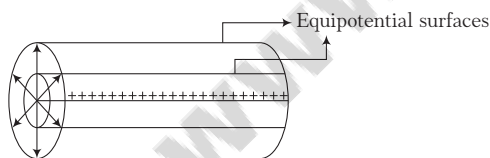


Fig. 2.10 Equipotential surfaces for a line charge distribution

- (iv) Equipotential surfaces for two positive charges of equal magnitude are shown below

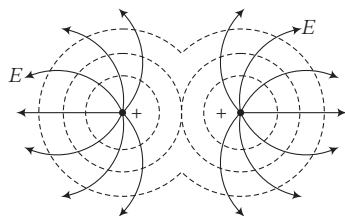


Fig. 2.11 Equipotential surfaces for like charges

- (v) Equipotential surfaces for two equal and opposite charges can be drawn as

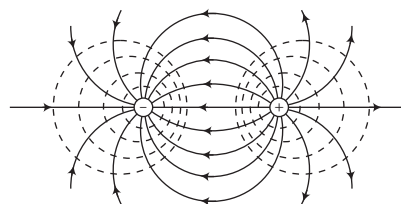


Fig. 2.12 Equipotential surfaces for unlike charges

Note While drawing the equipotential surfaces, we should keep in mind the two main points

- These are perpendicular to electric field lines at all places.
- Electric field lines always flow from higher potential to lower potential.

Example 2.22 Equipotential spheres are drawn round a point charge. As we move away from the charge, will the spacing between two spheres having a constant potential difference decrease, increase or remain constant.

Sol. In the given figure, we have $V_1 > V_2$

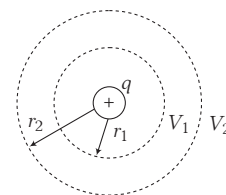
$$\therefore V_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1}$$

$$\text{and } V_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_2}$$

$$\begin{aligned} \text{Now, } V_1 - V_2 &= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{r_2 - r_1}{r_1 r_2} \right) \end{aligned}$$

$$\therefore (r_2 - r_1) = \frac{(4\pi\epsilon_0)(V_1 - V_2)}{q} (r_1 r_2)$$

For a constant potential difference ($V_1 - V_2$), $r_2 - r_1 \propto r_1 r_2$ i.e., the spacing between two spheres ($r_2 - r_1$) increases as we move away from the charge, because the product $r_1 r_2$ will increase.



Variation of electric potential on the axis of a charged ring

Let a charge q is uniformly distributed over the circumference of a ring as shown in Fig. (a) and is non-uniformly distributed in Fig (b).

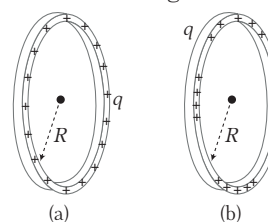


Fig. 2.13 Variation of electric potential

The electric potential at the centre of the ring in both the cases is given by

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$$

where, R is radius of ring.

The electric potential at point P which is at a distance r from the centre of ring is given by

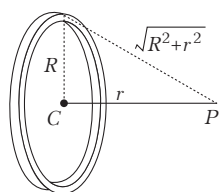


Fig. 2.14 Electric potential at P at a distance r from centre of ring

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{\sqrt{R^2 + r^2}}$$

From these expressions, we can see that electric potential is maximum at the centre and decreases as we move away from the centre on the axis.

This potential varies with the distance r as shown in figure below.

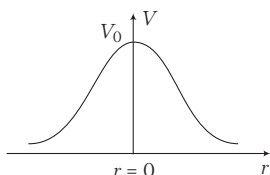
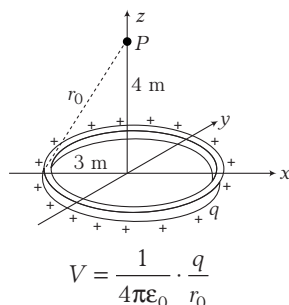


Fig. 2.15 Variation of potential V with distance r

In the figure, $V_0 = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$

Example 2.23 A charge $q = 10 \mu\text{C}$ is distributed uniformly over the circumference of a ring of radius 3 m placed on xy -plane with its centre at origin. Find the electric potential at a point $P(0, 0, 4 \text{ m})$.

Sol. The electric potential at point P would be



Here, r_0 = distance of point P from the circumference of ring
 $= \sqrt{(3)^2 + (4)^2} = 5 \text{ m}$

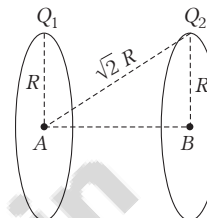
and $q = 10 \mu\text{C} = 10^{-5} \text{ C}$

Substituting the values, we have

$$\text{Electric potential, } V = \frac{(9.0 \times 10^9) (10^{-5})}{(5.0)} = 1.8 \times 10^4 \text{ V}$$

Example 2.24 Two identical thin rings, each of radius R , are coaxially placed at a distance R . If Q_1 and Q_2 are respectively, the charges uniformly spread on the two rings, find the work done in moving a charge q from centre of ring having charge Q_1 to the other ring.

Sol.



$$V_A = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R} \right]$$

$$V_B = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R} \right]$$

$$\begin{aligned} V_B - V_A &= \frac{1}{4\pi\epsilon_0 R} \left[Q_1 \left(\frac{1}{\sqrt{2}} - 1 \right) + Q_2 \left(1 - \frac{1}{\sqrt{2}} \right) \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{(Q_2 - Q_1)(\sqrt{2} - 1)}{\sqrt{2}R} \end{aligned}$$

Work done in moving a charge q from A to B ,

$$W_{A \rightarrow B} = q(V_B - V_A) = \frac{1}{4\pi\epsilon_0} \cdot \frac{(\sqrt{2} - 1)(Q_2 - Q_1)q}{\sqrt{2}R}$$

Potential due to charged sphere

Electric potential due to conducting and non-conducting charged sphere are given below

Electric potential due to a conducting charged sphere

Let us consider a uniformly charged spherical shell. The radius of shell is R and its total charge is q , which is uniformly distributed over the surface. We can construct a Gaussian surface (a sphere) of radius $r > R$.

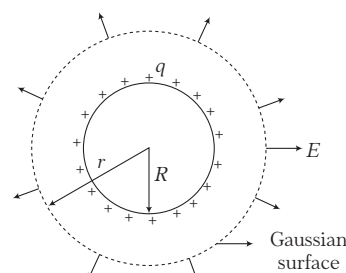


Fig. 2.16

At external points, the potential at any point is the same when the whole charge is assumed to be concentrated at the centre. At the surface of the sphere, $r = R$

$$\therefore \text{Potential, } V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

The electric field inside the shell is zero. This implies that potential is constant inside the shell and therefore equals to its value at the surface. Thus, we can write,

$$V_{\text{inside}} = V_{\text{surface}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

and
$$V_{\text{outside}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

The potential (V) varies with the distance from the centre (r) as shown in Fig. 2.17.

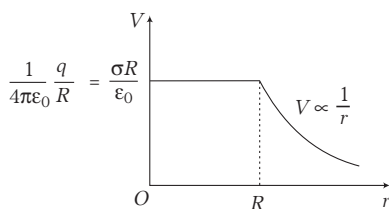


Fig. 2.17 Potential variation for conducting charged sphere

Example 2.25 A spherical drop of water carrying a charge of $3 \times 10^{-19} \text{ C}$ has a potential of 500 V at its surface. What is the radius of the drop? If two drops of the same charge and the same radius combine to form a single spherical drop, what is the potential at the surface of the new drop?

Sol. The potential V is given by $V = \frac{q}{4\pi\epsilon_0 r}$

Here, $q = 3 \times 10^{-19} \text{ C}$ and $V = 500 \text{ V}$

$$\text{Hence, } r = \frac{q}{4\pi\epsilon_0 V} = \frac{9 \times 10^9 \times 3 \times 10^{-19}}{500} = 5.4 \times 10^{-12} \text{ m}$$

Volume of one drop is $\frac{4}{3} \pi r^3$.

Total volume of both drops is $\frac{8}{3} \pi r^3$.

Let r' be the radius of the new drop formed, equating the volumes, we have $\frac{4}{3} \pi r'^3 = \frac{8}{3} \pi r^3$. This gives, $r' = 2^{1/3} r$.

Charge on the new drop $= 2q = 6 \times 10^{-19} \text{ C}$

$$\text{New potential, } V' = \frac{2q}{4\pi\epsilon_0 r'} = \frac{2q}{4\pi\epsilon_0 2^{1/3} r}$$

Hence, by putting values, we get

$$V' = \frac{1000}{2^{1/3}} = 794 \text{ V}$$

Electric potential due to a non-conducting charged sphere

For a uniformly charged non-conducting solid sphere, we have the following formulae for potential,

$$V_{\text{outside}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$V_{\text{surface}} = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$$

$$V_{\text{inside}} = \frac{1}{4\pi\epsilon_0} \frac{q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{r^2}{R^2} \right]$$

At the centre potential will be

$$V_C = \frac{3}{2} \left[\frac{1}{4\pi\epsilon_0} \frac{q}{R} \right],$$

which is equal to $\frac{3}{2}$ times the potential at surface. This can

be obtained by putting $r = 0$ in the formula of V_{inside} .

The variation of potential (V) with distance from the centre is as shown

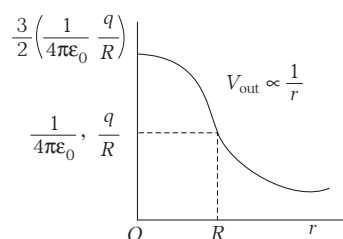


Fig. 2.18 Potential variation for non-conducting charged sphere

Electric potential due to a group of concentric shells

Consider a pair of two uniformly charged concentric shells having radii a and b ($a < b$) and carrying charges q_1 and q_2 , respectively. We will calculate potential at three different points A , B and C . A lies inside the inner shell, B lies in the space between the two shells and C lies outside the outer shell.

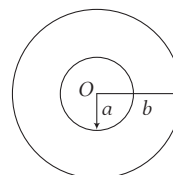


Fig. 2.19

Potential at point C

$OC = r$, where $r > b$.

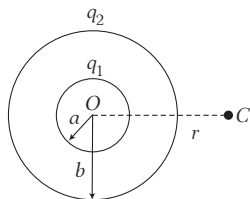


Fig. 2.20

So, potential at C is $V_C = \frac{q_1 + q_2}{4\pi\epsilon_0 r}$
(due to inner and outer shell)

Potential at point B

Let $OB = r$, where $a < r < b$.

As the point B, lies inside the outer shell,

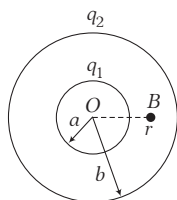


Fig. 2.21

Potential at B due to outer shell is $V_2 = \frac{q_2}{4\pi\epsilon_0 b}$.

The point B lies outside the inner shell, the potential at B,

due to inner shell is $V_1 = \frac{q_1}{4\pi\epsilon_0 r}$.

So, potential at B is $V_B = \frac{q_1}{4\pi\epsilon_0 r} + \frac{q_2}{4\pi\epsilon_0 b}$

Potential at point A

Let $OA = r$, such that $r < a$. The point A lies inside the outer

shell. So, potential at A, due to outer shell is $V_2 = \frac{q_2}{4\pi\epsilon_0 b}$.

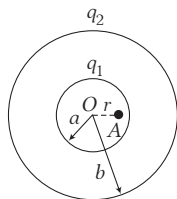


Fig. 2.22

Similarly, potential at A, due to inner shell is $V_1 = \frac{q_1}{4\pi\epsilon_0 a}$.

So, potential at A is $V_A = \frac{q_1}{4\pi\epsilon_0 a} + \frac{q_2}{4\pi\epsilon_0 b}$.

Example 2.26 A charge Q is distributed over two concentric hollow spheres of radii r and R ($> r$) such that the surface densities are equal. Find the potential at any point inside the smaller sphere.

Sol. Let q and q' be the charges on the inner and outer sphere.

As surface charge densities are equal.

$$\therefore \frac{q}{4\pi r^2} = \frac{q'}{4\pi R^2} \text{ or } qR^2 = q' r^2$$

Also, $q + q' = Q$. This gives $q = Q - q'$

Solving the two equations, we get $q = \frac{Qr^2}{R^2 + r^2}$, $q' = \frac{QR^2}{R^2 + r^2}$

Now, potential at the centre is given by

$$V_C = \frac{q}{4\pi\epsilon_0 r} + \frac{q'}{4\pi\epsilon_0 R} = \frac{Q(r+R)}{4\pi\epsilon_0 (R^2 + r^2)}$$

This is equal to potential at any point inside the smaller sphere.

Motion of charged particle in electric field

Consider a charged particle having charge q and mass m is initially at rest in an electric field of strength E . The particle will experience an electric force which causes its motion.

The force experienced by the charged particle is F , where

$$F = qE$$

$\therefore$ Acceleration produced by this force is $a = \frac{F}{m} = \frac{qE}{m}$... (i)

Suppose at point A particle is at rest and after some time t , it reaches the point B and attains velocity v .

$\therefore$

$$v = at$$

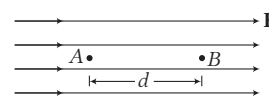


Fig. 2.23

If potential difference between A and B be ΔV and the

distance between them is d , then $v = \frac{qEt}{m} = \sqrt{\frac{2q\Delta V}{m}}$... (ii)

As, momentum, $p = mv$

$$\therefore p = m \left(\frac{qEt}{m} \right) = qEt \quad [\text{from Eq. (ii)}]$$

or

$$p = m \times \sqrt{\frac{2q\Delta V}{m}} = \sqrt{2mq\Delta V}$$

Kinetic energy of a charged particle

Kinetic energy gained by the particle in time t ,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} m \left(\frac{qEt}{m} \right)^2 \quad [\text{from Eq. (ii)}]$$

$$= \frac{q^2 E^2 t^2}{2m} \text{ or } K = \frac{1}{2} m \times \frac{2q\Delta V}{m} = q\Delta V$$

Example 2.27 A bullet of mass 2 g is moving with a speed of 10 m/s. If the bullet has a charge of 2 μC , through what potential it be accelerated starting from rest, to acquire the same speed?

Sol. Use the relation, $qV = \frac{1}{2}mv^2$

Here, $q = 2 \times 10^{-6} \text{ C}$, $m = 2 \times 10^{-3} \text{ kg}$ and $v = 10 \text{ ms}^{-1}$

$$\text{Therefore, potential, } V = \frac{mv^2}{2q} = \frac{2 \times 10^{-3} \times (10)^2}{2 \times 2 \times 10^{-6}} = \frac{10^{-1}}{2 \times 10^{-6}} \\ = 5 \times 10^4 \text{ V} = 50 \text{ kV}$$

Work done by a charged particle

According to work energy theorem, gain in kinetic energy = work done in displacement of charge, i.e. $W = q\Delta V$

where, ΔV = potential difference between the two positions of charge q , i.e. $\Delta V = \mathbf{E} \cdot \Delta \mathbf{r} = E\Delta r \cos \theta$, where θ is the angle between direction of electric field and direction of motion of charge.

If charge q is given a displacement $\mathbf{r} = (x\hat{i} + y\hat{j} + z\hat{k})$ in an electric field $\mathbf{E} = (E_x\hat{i} + E_y\hat{j} + E_z\hat{k})$, then the work done is

$$W = q(\mathbf{E} \cdot \mathbf{r}) = q(E_x x + E_y y + E_z z)$$

Work done in displacing a charge in an electric field is path independent because electric force in field is conservative.

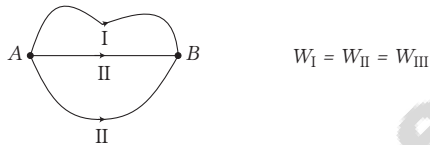


Fig. 2.24 Different path followed by charged particle

When charged particle enters perpendicularly in an electric field, it describes a parabolic path as shown

- (i) **Equation of trajectory** Throughout the motion, particle has uniform velocity along X-axis and horizontal displacement (x) is given by the equation, $x = ut$. Since, the motion of the particle is accelerated along Y-axis.

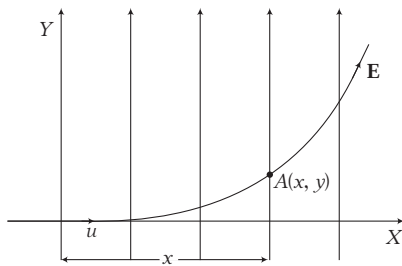


Fig. 2.25 Trajectory of charged particle

So,
$$y = \frac{1}{2} \left(\frac{qE}{m} \right) \left(\frac{x}{u} \right)^2 \quad \left(\because t = \frac{x}{u} \right)$$

This is the equation of a parabola, because

$$y \propto x^2$$

- (ii) **Velocity at any instant** At any instant t , $v_x = u$ and

$$v_y = \frac{qEt}{m}. \text{ So, } v = |v| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{q^2 E^2 t^2}{m^2}}$$

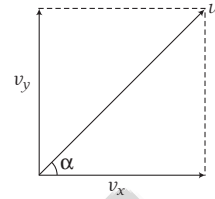


Fig. 2.26 Resultant velocity

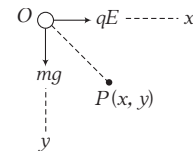
If α is the angle made by v with X-axis, then

$$\tan \alpha = \frac{v_y}{v_x} = \frac{qEt}{mu}$$

Example 2.28 A ball of mass m having a charge q is released from rest in a region where a horizontal electric field E exists.

- (i) Find the resultant force acting on the ball.
(ii) Find the trajectory followed by the ball.

Sol. The forces acting on the ball are weight of the ball in vertically downward direction and the electric force in the horizontal direction.



- (i) Resultant force, $F = \sqrt{(mg)^2 + (qE)^2}$.

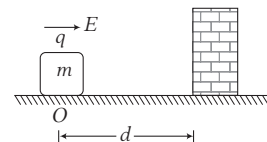
- (ii) Let the ball be at point P after time t , then the trajectory followed by ball can be written as

$$\text{In } x\text{-direction, } x = \frac{1}{2} \frac{qE}{m} t^2$$

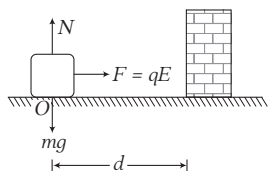
$$\text{In } y\text{-direction, } y = \frac{1}{2} g t^2 \Rightarrow \frac{y}{x} = \frac{g}{qE/m} \Rightarrow y = \frac{mgx}{qE}$$

Trajectory is a straight line.

Example 2.29 A block having charge q and mass m is resting on a smooth horizontal surface at a distance d from the wall as shown. Discuss the motion of the block when a uniform electric field E is applied horizontally towards the wall assuming that collision of the block with the wall is completely elastic.



Sol. The figure can be shown as



Acceleration of the block can be given as $a = \frac{qE}{m}$.

Time taken by the block to reach the wall at distance

$d \left(= \frac{1}{2} at^2 \right)$ is

$$t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2d}{qE/m}} = \sqrt{\frac{2md}{qE}}$$

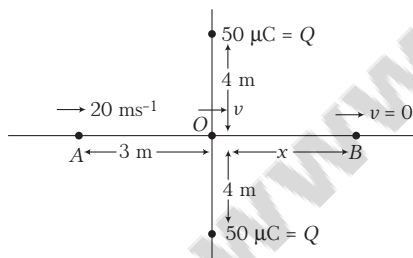
After completely elastic collision, the speed of the block is reversed and then it moves under retardation and comes to rest at O . Again it is accelerated towards the wall and so on. In fact, it is executing oscillatory motion of the time period

$$T = 2t = 2\sqrt{\frac{2md}{qE}}$$

As, force qE is constant, the motion of block is not simple harmonic.

Example 2.30 Two point charges each $50 \mu\text{C}$ are fixed on Y -axis at $y = +4 \text{ m}$ and $y = -4 \text{ m}$. Another charged particle having charge $-50 \mu\text{C}$ and mass 20 g is moving along the positive X -axis. When it is at $x = -3 \text{ m}$, its speed is 20 ms^{-1} . Find the speed of charged particle when it reaches origin. Also, find distance of charged particle from origin, when its kinetic energy becomes zero.

Sol.



CHECK POINT 2.2

1. The work done to move a charge along an equipotential from A to B

(a) cannot be defined as $-\int_A^B \mathbf{E} \cdot d\mathbf{l}$

(b) must be defined as $-\int_A^B \mathbf{E} \cdot d\mathbf{l}$

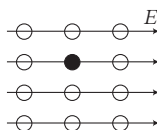
(c) is zero

(d) can have a non-zero value

2. There is a uniform electric field of intensity E which is as shown. How many labelled points have the same electric potential as the fully shaded point?

(a) 2
(c) 8

(b) 3
(d) 11



Let the speed of particle at origin be v . Applying energy conservation between A and O , we get

$$K_A + U_A = K_O + U_O$$

$$K_A + (-Q)V_A = K_O + (-Q)V_O$$

$$\Rightarrow \frac{1}{2} mv_A^2 + (-Q) \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{\sqrt{3^2 + 4^2}} = \frac{1}{2} mv_O^2$$

$$+ (-Q) \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{4}$$

$$\Rightarrow \frac{1}{2} m (v_O^2 - v_A^2) = \frac{2Q^2}{4\pi\epsilon_0} \left(\frac{1}{4} - \frac{1}{5} \right)$$

$$= 2 (50 \times 10^{-6})^2 \times 9 \times 10^9 \times \frac{1}{20}$$

$$\frac{1}{2} \times 20 \times 10^{-3} (v_O^2 - 20^2) = 2.25$$

$$\Rightarrow v_O^2 - 400 = 225 \Rightarrow v_O^2 = 625 \Rightarrow v_O = 25 \text{ ms}^{-1}$$

Applying energy conservation between A and B , we have

$$K_A + U_A = K_B + U_B$$

$$K_A + (-Q)V_A = 0 + (-Q)V_B$$

$$\frac{1}{2} mv_A^2 + (-Q) \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{5} = (-Q) \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q}{\sqrt{4^2 + x^2}}$$

$$\frac{1}{2} mv_A^2 = \frac{1}{4\pi\epsilon_0} 2Q^2 \left[\frac{1}{5} - \frac{1}{\sqrt{4^2 + x^2}} \right]$$

$$\Rightarrow \frac{1}{2} \times 20 \times 10^{-3} \times (20)^2 = 9 \times 10^9 \times 2 \times$$

$$(50 \times 10^{-6})^2 \left[\frac{1}{5} - \frac{1}{\sqrt{4^2 + x^2}} \right]$$

$$\Rightarrow 4 = 45 \left(\frac{1}{5} - \frac{1}{\sqrt{16^2 + x^2}} \right) \Rightarrow \frac{1}{\sqrt{16^2 + x^2}} = \frac{1}{5} - \frac{4}{45} = \frac{1}{9}$$

Therefore, $x = \sqrt{65} \text{ m}$

3. The electric potential at the surface of an atomic nucleus ($Z = 50$) of radius $9.0 \times 10^{-15} \text{ m}$

(a) 80 V

(b) $8 \times 10^6 \text{ V}$

(c) 9 V

(d) $9 \times 10^5 \text{ V}$

4. A hollow metal sphere of radius 5 cm is charged, so that the potential on its surface is 10 V. The potential at the centre of the sphere is

(a) 0 V

(b) 10 V

(c) same as at point 5 cm away from the surface

(d) same as at point 25 cm away from the surface

5. Two charged spheres of radii R_1 and R_2 having equal surface charge density. The ratio of their potential is
 (a) R_1/R_2 (b) R_2/R_1 (c) $(R_1/R_2)^2$ (d) $(R_2/R_1)^2$
6. The radii of two concentric spherical conducting shells are r_1 and r_2 ($r_2 > r_1$). The charge on the outer shell is q . The charge on the inner shell which is connected to the earth is
 (a) $q\left(\frac{r_2}{r_1}\right)$ (b) $q^2\left(\frac{r_1}{r_2}\right)$ (c) $-q(r_1/r_2)$ (d) $q^2\left(\frac{r_2}{r_1}\right)$
7. If a charged spherical conductor of radius 10 cm has potential V at a point distant 5 cm from its centre, then the potential at a point distant 15 cm from the centre will be
 (a) $\frac{1}{3}V$ (b) $\frac{2}{3}V$ (c) $\frac{3}{2}V$ (d) $3V$
8. A hollow conducting sphere of radius R has a charge $(+q)$ on its surface. What is the electric potential within the sphere at a distance $r = R/3$ from its centre
 (a) Zero (b) $\frac{1}{4\pi\epsilon_0} \frac{Q}{r}$ (c) $\frac{1}{4\pi\epsilon_0} \frac{Q}{R}$ (d) $\frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
9. A spherical conductor of radius 2 m is charged to a potential of 120 V. It is now placed inside another hollow spherical conductor of radius 6 m. Calculate the potential to which the bigger sphere would be raised.
 (a) 20 V (b) 60 V (c) 80 V (d) 40 V
10. Three concentric spherical shells have radii a, b and c ($a < b < c$) and have surface charge densities $\sigma, -\sigma$ and σ , respectively. If V_A, V_B and V_C denote the potentials of the three shell, ... for $c = a + b$, we have
 (a) $V_C = V_A \neq V_B$ (b) $V_C = V_B \neq V_A$
 (c) $V_C \neq V_B \neq V_A$ (d) $V_C = V_B = V_A$
11. The electrostatic potential of a uniformly charged thin spherical shell of charge Q and radius R at a distance r from the centre is
 (a) $\frac{Q}{4\pi\epsilon_0 r}$ for points outside and $\frac{Q}{4\pi\epsilon_0 R}$ for points inside the shell
 (b) $\frac{Q}{4\pi\epsilon_0 r}$ for both points inside and outside the shell
 (c) zero for points outside and $\frac{Q}{4\pi\epsilon_0 r}$ for points inside the shell
 (d) zero for both points inside and outside the shell
12. The diameter of a hollow metallic sphere is 60 cm and the sphere carries a charge of $500 \mu\text{C}$. The potential at a distance of 100 cm from the centre of the sphere will be
 (a) $6 \times 10^7 \text{ V}$ (b) $7 \times 10^6 \text{ V}$
 (c) $4.5 \times 10^6 \text{ V}$ (d) $5 \times 10^6 \text{ V}$
13. Obtain the energy in joule acquired by an electron beam when accelerated through a potential difference of 2000 V. How much speed will the electron gain?
 (a) $\frac{8}{3} \times 10^7 \text{ m/s}$ (b) $\frac{7}{3} \times 10^7 \text{ m/s}$
 (c) $\frac{5}{3} \times 10^7 \text{ m/s}$ (d) $\frac{2}{3} \times 10^7 \text{ m/s}$
14. A particle A has charge $+16q$ and a particle B has charge $+4q$ with each of them having the same mass m . When allowed to fall from rest through the same electric potential difference, the ratio of their speed $\frac{V_A}{V_B}$ will become
 (a) 2:1 (b) 1:2 (c) 1:4 (d) 4:1
15. What potential difference must be applied to produce an electric field that can accelerate an electron to one-tenth the velocity of light?
 (a) 1352 V (b) 2511 V
 (c) 2531 V (d) 3521 V

ELECTRIC POTENTIAL ENERGY

When a charged particle moves in an electric field, the field exerts a force (electric force) that can do work on the charge particle. This work can be expressed in terms of electric potential energy.

It is represented by U . Electric potential can be written as potential energy per unit charge, i.e. $V = \frac{W}{q} = \frac{U}{q}$

Electric potential energy is defined only in a conservative field.

Relation between work and energy

If the force $\mathbf{F}$ is conservative, the work done by $\mathbf{F}$ can always be expressed in terms of a **potential energy** U . When the charge particle moves from a point where the potential energy is U_a to a point where it is U_b , the change in potential energy is, $\Delta U = U_b - U_a$.

This is related by the work $W_{a \rightarrow b}$ as

$$W_{a \rightarrow b} = U_a - U_b = -(U_b - U_a) = -\Delta U \quad \dots(i)$$

Here, $W_{a \rightarrow b}$ is the work done in displacing the charge particle from a to b by the **conservative force** (here electrostatic). Moreover, we can see from Eq. (i) that if $W_{a \rightarrow b}$ is positive, ΔU is negative, i.e. the potential energy decreases. So, whenever the work done by a conservative force is positive, the potential energy of the system decreases and *vice versa*.

That's what happens when a charge particle is thrown upwards, the work done by gravity is negative, and the potential energy increases.

Electron volt

One electron volt (1eV) is the energy equal to the work done in moving a single elementary charge particle, such as the electron or the proton through a potential difference of exactly one volt (1 V).

$$\begin{aligned} 1 \text{ eV} &= e (1 \text{ V}) = (1.6 \times 10^{-19} \text{ C}) (1 \text{ J/C}) \\ &= 1.6 \times 10^{-19} \text{ J} \end{aligned}$$

Example 2.31 A spherical oil drop of radius 10^{-4} cm has a total charge equivalent to 40 electrons. Calculate the energy that would be required to place an additional electron on the drop.

Sol. Initial charge on the oil drop, $q = 40 \times 1.6 \times 10^{-19}$
 $= 64 \times 10^{-19}$ C

$$\begin{aligned} \text{Potential of the oil drop} &= \frac{q}{4\pi\epsilon_0 r} \\ &= \frac{9 \times 10^9 \times 64 \times 10^{-19}}{10^{-6}} = 576 \times 10^{-4} \text{ V} \end{aligned}$$

Now, energy required = work done in bringing an electron from infinity to a point of potential.

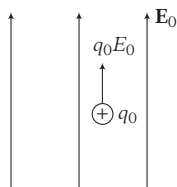
$$\text{As, potential} = \frac{\text{work}}{\text{charge}}$$

$$\begin{aligned} \therefore \text{Work} &= \text{Potential} \times \text{Charge} = 576 \times 10^{-4} \times 1.6 \times 10^{-19} \\ &= 921.6 \times 10^{-23} \text{ J} \end{aligned}$$

Example 2.32 A uniform electric field E_0 is directed along positive y-direction. Find the change in electric potential energy of a positive test charge q_0 when it is displaced in the field from $y_i = a$ to $y_f = 2a$ along the Y-axis.

Sol. Electrostatic force on the test charge,

$$F_e = q_0 E_0 \quad (\text{along positive y-direction})$$



$$\therefore W_{i-f} = -\Delta U \text{ or } \Delta U = -W_{i-f} = -[q_0 E_0 (2a - a)] = -q_0 E_0 a$$

Note Here, work done by electrostatic force is positive. Hence, the potential energy is decreasing.

Example 2.33 Find the change in electric potential energy, ΔU as a charge of 2.20×10^{-6} C, moves from a point A to point B, given that the change in electric potential between these points is $\Delta V = V_B - V_A = 24.0$ V.

Sol. As, $\Delta V = \frac{\Delta U}{q_0}$, where ΔU is change in potential energy

$$\therefore \Delta U = q_0 \Delta V = (2.20 \times 10^{-6}) (24.0) = 5.28 \times 10^{-5} \text{ J}$$

Example 2.34 A charge is moved in an electric field of a fixed charge distribution from point A to another point B slowly. The work done by external agent in doing so, is 100 J. What is the work done by the electric field of the charge distribution as the charge moves from A to B?

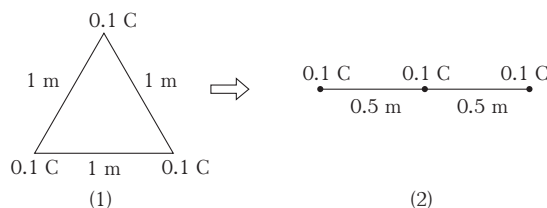
Sol. Work done, $W_{\text{ext}} = \Delta U = U_B - U_A = 100$ J

As, $\mathbf{F}_{\text{ext}} = -\mathbf{F}_E$ for the charge to move slowly, so

$$W_E = -W_{\text{ext}} = -100 \text{ J}$$

Example 2.35 Three point charges of 0.1 C each are placed at the corners of an equilateral triangle with side 1 m. If this system is supplied energy at the rate of 1 kW, how much time will be required to move one of the charges to the mid-point of the line joining the two others?

Sol.



$$\begin{aligned} U_1 &= \frac{1}{4\pi\epsilon_0} \left[\frac{0.1 \times 0.1}{1} \times 3 \right] \\ &= 9 \times 10^9 \times 3 \times 10^{-2} = 27 \times 10^7 \text{ J} \end{aligned}$$

$$\begin{aligned} U_2 &= \frac{1}{4\pi\epsilon_0} \left[\frac{0.1 \times 0.1}{0.5} \times 2 + \frac{0.1 \times 0.1}{1} \right] \\ &= 9 \times 10^9 \times 0.05 = 45 \times 10^7 \text{ J} \end{aligned}$$

$$W_{1 \rightarrow 2} = U_2 - U_1 = 18 \times 10^7 \text{ J}$$

Rate at which energy is supplied is given by

$$P = \frac{W}{t}$$

$$\therefore t = \frac{W}{P} = \frac{18 \times 10^7}{10^3} = 18 \times 10^4 \text{ s} = 50 \text{ h}$$

Potential energy of a system of charges

The potential energy for a system of two point charges and for a collection of more than two charges are given below.

1. Potential energy for a system of two point charges

Potential energy of the system of two charges q_1 and q_2 will be the work done to bring them from infinity to point A and point B, respectively.

Work done to bring charge q_1 from infinity to point A,

$$W_1 = 0$$

Work done to bring charge q_2 from infinity to point B,

$$W_2 = q_2 (V_B - V_A)$$

$$\text{As, } W_2 = q_2 \times \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r}$$

Now, potential energy of the system of charges q_1 and q_2 ,

$$U = W_1 + W_2$$

$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$$

Example 2.36 In a hydrogen atom, the electron and proton are bound together at a distance of about $0.53\text{\AA}$. Estimate the potential energy of the system in eV, assuming zero potential energy at infinite separation between the electron and the proton.

Sol. Charge on electron, $q_1 = -1.6 \times 10^{-19}\text{ C}$

Charge on proton, $q_2 = 1.6 \times 10^{-19}\text{ C}$

Separation between electron and proton,

$$r = 0.53\text{\AA} = 0.53 \times 10^{-10}\text{ m}$$

The change in energy when electron-proton system is formed is

$$\begin{aligned}\Delta U &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r} \\ &= (9 \times 10^9) \times \frac{(-1.6 \times 10^{-19})(1.6 \times 10^{-19})}{0.53 \times 10^{-10}} \\ &= -4.35 \times 10^{-18}\text{ J} \\ &= -\frac{4.35 \times 10^{-18}}{1.6 \times 10^{-19}} = -27.2\text{ eV}\end{aligned}$$

At infinite separation, the potential energy is zero, therefore the energy when the system is formed, is

$$U = 0 + \Delta U = -27.2\text{ eV}$$

Example 2.37 A point charge $q_1 = -5.8\text{ }\mu\text{C}$ is held stationary at the origin. A second point charge $q_2 = +4.3\text{ }\mu\text{C}$ moves from the point $(0.26\text{ m}, 0, 0)$ to $(0.38\text{ m}, 0, 0)$. How much work is done by the electric force on q_2 ?

Sol. Work done by the electrostatic forces $= U_i - U_f$

$$\begin{aligned}&= \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{r_i} - \frac{1}{r_f} \right) = \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{r_f - r_i}{r_i r_f} \right) \\ &= \frac{(-5.8 \times 10^{-6})(4.3 \times 10^{-6})(9 \times 10^9)(0.38 - 0.26)}{(0.38)(0.26)} \\ &= -0.272\text{ J}\end{aligned}$$

Example 2.38 What minimum work must be done by an external force to bring a charge $q = 3.0\text{ }\mu\text{C}$ from a great distance away (take, $r = \infty$) to a point 0.50 m from a charge $Q = 20.0\text{ }\mu\text{C}$?

Sol. The change in potential energy equal to the positive of the work required by an external force $W = \Delta U = q(V_B - V_A)$. We get the potentials V_B and V_A using $V = \frac{kQ}{r}$

The work required is equal to the change in potential energy,

$$W = q(V_B - V_A) = q \left(\frac{kQ}{r_B} - \frac{kQ}{r_A} \right)$$

where, $r_B = 0.500\text{ m}$ and $r_A = \infty$.

Work done,

$$\begin{aligned}W &= (3.00 \times 10^{-6}\text{ C}) \frac{(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.00 \times 10^{-5}\text{ C})}{(0.50\text{ m})} \\ &= 1.08\text{ J}\end{aligned}$$

Example 2.39 A point charge $q_1 = 9.1\text{ }\mu\text{C}$ is held fixed at origin. A second point charge $q_2 = -0.42\text{ }\mu\text{C}$ and a mass $3.2 \times 10^{-4}\text{ kg}$ is placed on the X-axis, 0.96 m from the origin. The second point charge is released at rest. What is its speed when it is 0.24 m from the origin?

Sol. From conservation of mechanical energy, we have decrease in gravitational potential energy = increase in kinetic energy

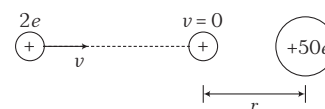
$$\begin{aligned}\text{or } \frac{1}{2}mv^2 &= U_i - U_f = \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{r_i} - \frac{1}{r_f} \right) \\ &= \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{r_f - r_i}{r_i r_f} \right)\end{aligned}$$

Speed of the second charge,

$$\begin{aligned}v &= \sqrt{\frac{q_1 q_2}{2\pi\epsilon_0 m} \left(\frac{r_f - r_i}{r_i r_f} \right)} \\ &= \sqrt{\frac{(9.1 \times 10^{-6})(-0.42 \times 10^{-6}) \times 2 \times 9 \times 10^9}{3.2 \times 10^{-4}} \left(\frac{0.24 - 0.96}{(0.24)(0.96)} \right)} \\ &= 26\text{ ms}^{-1}\end{aligned}$$

Example 2.40 An α -particle with kinetic energy 10 MeV is leading towards a stationary tin nucleus of atomic number 50. Calculate the distance of closest approach.

Sol. Due to repulsion by the tin nucleus, the kinetic energy of the α -particle gradually decreases at the expense of electrostatic potential energy.



$\therefore$ Decrease in kinetic energy = Increase in potential energy

$$\text{or } \frac{1}{2}mv^2 = U_f - U_i$$

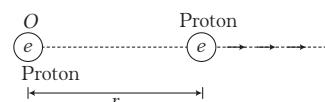
$$\text{or } \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r} - 0$$

$$\therefore r = \frac{1}{4\pi\epsilon_0} \cdot \frac{(2e)(50e)}{(\text{KE})}$$

Substituting the values, we get

$$\begin{aligned}\text{Distance, } r &= \frac{(9 \times 10^9)(2 \times 1.6 \times 10^{-19})(1.6 \times 10^{-19} \times 50)}{10 \times 10^6 \times 1.6 \times 10^{-19}} \\ &= 14.4 \times 10^{-15}\text{ m}\end{aligned}$$

Example 2.41 A proton is fixed at origin. Another proton is released from rest, from a point at a distance r from origin. Taking charge of origin as e and mass as m , find the speed of the proton (i) at a distance $2r$ from origin, (ii) at large distance from origin.



Sol. The proton moves away under electrostatic repulsion.

As there is no external force, $W_{\text{ext}} = 0$

$$\Rightarrow \Delta KE + \Delta PE = 0$$

$$\Rightarrow \left(\frac{1}{2} mv^2 \right) + (U_f - U_i) = 0$$

$$(i) \text{ We have, } U_f = \frac{e^2}{4\pi\epsilon_0(2r)}, \text{ and } U_i = \frac{e^2}{4\pi\epsilon_0(r)}$$

$$\therefore \frac{1}{2} mv^2 = \frac{e^2}{4\pi\epsilon_0(2r)}$$

$$\text{or speed of the proton, } v = \sqrt{\frac{e^2}{4\pi\epsilon_0 r m}}$$

$$(ii) \text{ We have, } U_f = 0, U_i = \frac{e^2}{4\pi\epsilon_0 r}$$

$$\therefore \frac{1}{2} mv^2 = \frac{e^2}{4\pi\epsilon_0 r}$$

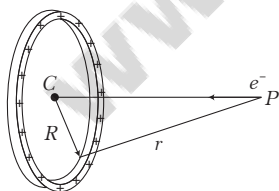
$$\text{Speed of the proton, } v = \sqrt{\frac{2e^2}{4\pi\epsilon_0 r m}}$$

Example 2.42 A uniformly charged thin ring has radius 10.0 cm and total charge + 12.0 nC. An electron is placed on the ring's axis at a distance 25.0 cm from the centre of the ring and is constrained to stay on the axis of the ring. When the electron is released from rest, then

(i) describe the subsequent motion of the electron.

(ii) find the speed of the electron when it reaches the centre of the ring.

Sol. (i) The electron will be attracted towards the centre C of the ring. At C net force is zero, but on reaching C, electron has some kinetic energy and due to inertia it crosses C, but on the other side it is further attracted towards C. Hence, motion of electron is oscillatory about point C.



(ii) As the electron approaches C, its speed (hence, kinetic energy) increases due to force of attraction towards the centre C. This increase in kinetic energy is equal to change in electrostatic potential energy. Thus,

$$\frac{1}{2} mv^2 = U_i - U_f$$

$$\Rightarrow \frac{1}{2} mv^2 = U_P - U_C = (-e)[V_P - V_C] \quad \dots(i)$$

Here, V is the potential due to ring.

$$V_P = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \quad (\text{where, } q = \text{charge on ring})$$

$$= \frac{(9 \times 10^9)(12 \times 10^{-9})}{(\sqrt{(10)^2 + (25)^2}) \times 10^{-2}} = 401 \text{ V}$$

$$V_C = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R} = \frac{(9 \times 10^9)(12 \times 10^{-9})}{10 \times 10^{-2}} = 1080 \text{ V}$$

Substituting the proper values in Eq. (i), we get

$$\frac{1}{2} \times 9.1 \times 10^{-31} \times v^2 = (-1.6 \times 10^{-19})(401 - 1080)$$

$\therefore$ Speed of the electron, $v = 15.45 \times 10^6 \text{ ms}^{-1}$

2. Potential energy for a collection of more than two charges

The potential energy of a system of n charges is given by

$$U = \frac{K}{2} \sum_{\substack{i,j \\ i \neq j}}^n \frac{q_i q_j}{r_{ij}} \quad \left(\text{here, } K = \frac{1}{4\pi\epsilon_0} \right)$$

The factor of 1/2 is applied only with the summation sign because on expanding the summation, each pair is counted twice. It is represented by U .

For example, electric potential energy of four point charges q_1, q_2, q_3 and q_4 would be given by

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_4 q_3}{r_{43}} + \frac{q_4 q_2}{r_{42}} + \frac{q_4 q_1}{r_{41}} + \frac{q_3 q_2}{r_{32}} + \frac{q_3 q_1}{r_{31}} + \frac{q_2 q_1}{r_{21}} \right]$$

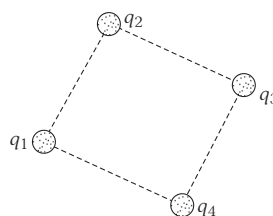
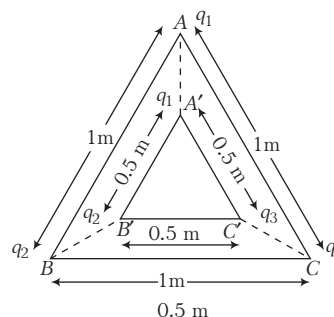


Fig. 2.27 System of four charges

Here, all the charges are to be substituted with sign.

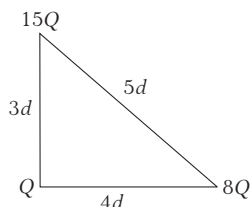
Example 2.43 Three point charges of 1 C, 2 C and 3 C are placed at the corners of an equilateral triangle of side 1 m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side 0.5 m.

Sol. Work done = $U_f - U_i$



$$\begin{aligned}
 &= \frac{1}{4\pi\epsilon_0} \left(\frac{1}{r_f} - \frac{1}{r_i} \right) [q_3q_2 + q_3q_1 + q_2q_1] \\
 &= 9 \times 10^9 \left(\frac{1}{0.5} - \frac{1}{1} \right) [(3)(2) + (3)(1) + (2)(1)] \\
 &= 99 \times 10^9 \text{ J}
 \end{aligned}$$

Example 2.44 Find the electric potential energy of the system of charges.



Sol. Number of pairs = 3

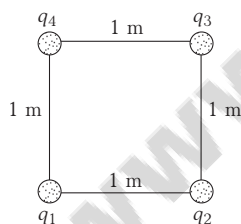
The electric potential energy for a system of charges is given by

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1q_2}{r} + \frac{q_2q_3}{r} + \frac{q_1q_3}{r} \right]$$

On putting the values, we get

$$\begin{aligned}
 U &= \frac{1}{4\pi\epsilon_0} \cdot \left[\frac{15Q \cdot Q}{3d} + \frac{Q \cdot 8Q}{4d} + \frac{15Q \cdot 8Q}{5d} \right] \\
 &= \frac{1}{4\pi\epsilon_0} \cdot \frac{31Q^2}{d}
 \end{aligned}$$

Example 2.45 Four charges $q_1 = 1 \mu\text{C}$, $q_2 = 2 \mu\text{C}$, $q_3 = -3 \mu\text{C}$ and $q_4 = 4 \mu\text{C}$ are kept on the vertices of a square of side 1 m. Find the electric potential energy of this system of charges.



Sol. We have, $r_{41} = r_{43} = r_{32} = r_{21} = 1 \text{ m}$

$$\text{and } r_{42} = r_{31} = \sqrt{(1)^2 + (1)^2} = \sqrt{2} \text{ m}$$

Potential energy of a system of three charges,

$$\begin{aligned}
 U &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_4q_3}{r_{43}} + \frac{q_4q_2}{r_{42}} + \frac{q_4q_1}{r_{41}} + \frac{q_3q_2}{r_{32}} + \frac{q_3q_1}{r_{31}} + \frac{q_2q_1}{r_{21}} \right] \\
 U &= (9.0 \times 10^9)(10^{-6})(10^{-6}) \\
 &\quad \left[\frac{(4)(-3)}{1} + \frac{(4)(2)}{\sqrt{2}} + \frac{(4)(1)}{1} + \frac{(-3)(2)}{1} + \frac{(-3)(1)}{\sqrt{2}} + \frac{(2)(1)}{1} \right] \\
 &= (9.0 \times 10^{-3}) \left[-12 + \frac{5}{\sqrt{2}} \right] = -7.62 \times 10^{-2} \text{ J}
 \end{aligned}$$

Note Here, negative sign of U implies that positive work has been done by electrostatic forces in assembling these charges at respective distances from infinity.

Example 2.46 Two point charges are located on the X -axis, $q_1 = -1 \mu\text{C}$ at $x = 0$ and $q_2 = +1 \mu\text{C}$ at $x = 1 \text{ m}$.

(i) Find the work that must be done by an external force to bring a third point charge $q_3 = +1 \mu\text{C}$ from infinity to $x = 2 \text{ m}$.

(ii) Find the total potential energy of the system of three charges.

Sol. (i) The work that must be done on q_3 by an external force is equal to the difference of potential energy ΔU when the charge is at $x = 2 \text{ m}$ and the potential energy when it is at infinity.

$$\begin{aligned}
 \therefore W = \Delta U = U_f - U_i &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f} + \frac{q_2q_1}{(r_{21})_f} \right] \\
 &\quad - \frac{1}{4\pi\epsilon_0} \left[\frac{q_3q_2}{(r_{32})_i} + \frac{q_3q_1}{(r_{31})_i} + \frac{q_2q_1}{(r_{21})_i} \right]
 \end{aligned}$$

$$\text{Here, } (r_{21})_i = (r_{21})_f \text{ and } (r_{32})_i = (r_{31})_i = \infty$$

$$\therefore W = \frac{1}{4\pi\epsilon_0} \left[\frac{q_3q_2}{(r_{32})_f} + \frac{q_3q_1}{(r_{31})_f} \right]$$

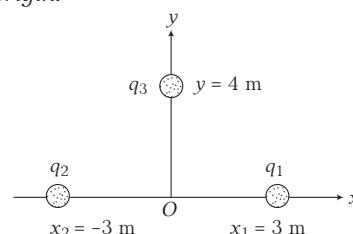
Substituting the values, we get

$$\begin{aligned}
 W &= (9.0 \times 10^9) (10^{-12}) \left[\frac{(1)(1)}{(1.0)} + \frac{(1)(-1)}{(2.0)} \right] \\
 &= 4.5 \times 10^{-3} \text{ J}
 \end{aligned}$$

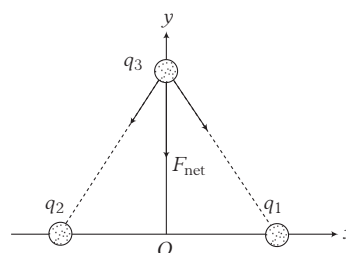
(ii) The total potential energy of the three charges is given by

$$\begin{aligned}
 U &= \frac{1}{4\pi\epsilon_0} \left(\frac{q_3q_2}{r_{32}} + \frac{q_3q_1}{r_{31}} + \frac{q_2q_1}{r_{21}} \right) \\
 &= (9.0 \times 10^9) \left[\frac{(1)(1)}{(1.0)} + \frac{(1)(-1)}{(2.0)} + \frac{(1)(-1)}{(1.0)} \right] (10^{-12}) \\
 &= -4.5 \times 10^{-3} \text{ J}
 \end{aligned}$$

Example 2.47 Two point charges $q_1 = q_2 = 2 \mu\text{C}$ are fixed at $x_1 = +3 \text{ m}$ and $x_2 = -3 \text{ m}$ as shown in figure. A third particle of mass 1 g and charge $q_3 = -4 \mu\text{C}$ are released from rest at $y = 4.0 \text{ m}$. Find the speed of the particle as it reaches the origin.



Sol. Here, the charge q_3 is attracted towards q_1 and q_2 both. So, the net force on q_3 is towards origin.



By this force, charge is accelerated towards origin, but this acceleration is not constant.

So, to obtain the speed of particle at origin by kinematics, we have to find first the acceleration at same intermediate position and then will have to integrate it with proper limits.

On the other hand, it is easy to use principle of conservation of energy as the forces are conservative.

Let v be the speed of particle at origin. From conservation of mechanical energy,

$$U_i + K_i = U_f + K_f$$

or
$$\frac{1}{4\pi\epsilon_0} \left[\frac{q_3 q_2}{(r_{32})_i} + \frac{q_3 q_1}{(r_{31})_i} + \frac{q_2 q_1}{(r_{21})_i} \right] + 0$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q_3 q_2}{(r_{32})_f} + \frac{q_3 q_1}{(r_{31})_f} + \frac{q_2 q_1}{(r_{21})_f} \right] + \frac{1}{2} m v^2$$

Here, $(r_{21})_i = (r_{21})_f$

Substituting the proper values, we have

$$(9.0 \times 10^9) \left[\frac{(-4)(2)}{(5.0)} + \frac{(-4)(2)}{(5.0)} \right] \times 10^{-12}$$

$$= (9.0 \times 10^9) \left[\frac{(-4)(2)}{(3.0)} + \frac{(-4)(2)}{(3.0)} \right] \times 10^{-12} + \frac{1}{2} \times 10^{-3} \times v^2$$

$$\therefore (9 \times 10^{-3}) \left(-\frac{16}{5} \right) = (9 \times 10^{-3}) \left(-\frac{16}{3} \right) + \frac{1}{2} \times 10^{-3} \times v^2$$

$$(9 \times 10^{-3}) (16) \left(\frac{2}{15} \right) = \frac{1}{2} \times 10^{-3} \times v^2$$

$$\therefore v = 6.2 \text{ ms}^{-1}$$

Potential energy in an external field

Here, mainly two situations are possible.

Potential energy of a single charge in external field

Potential energy of a single charge q at a point with position vector r in an external field $= q \cdot V(r)$

where, $V(r)$ is the potential at a point (i.e. position vector r) due to external electric field E .

Potential energy of a system of two charges in an external field

Consider two charges q_1 and q_2 kept in an external field E at A and B .

Then, potential energy of a system is given by

$$U = q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

where, r_1 and r_2 are the position vector of point A and B respectively,

$V(r_1)$ is potential at r_1 due to the external field

and $V(r_2)$ is potential at r_2 due to the external field.

Potential energy of a dipole in a uniform electric field

The work done in rotating the dipole through a small angle $d\theta$.

$$dW = \tau d\theta = -pE \sin\theta d\theta$$

Suppose initially dipole is kept in a uniform electric field at angle θ_1 . Now, to turn it through an angle θ_2 (with the field). Then, work done

$$W = \int_{\theta_1}^{\theta_2} pE \sin\theta d\theta$$

$$W = -pE [\cos\theta_2 - \cos\theta_1]$$

If $\theta_1 = 0^\circ$ and $\theta_2 = \theta$, i.e. initially dipole is kept along the field and then it turns through θ , so work done,

$$W = pE(1 - \cos\theta)$$

Potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction.

If the dipole is rotated by an angle $\theta_1 = 90^\circ$ to $\theta_2 = \theta$, then potential energy is given by

$$U = pE(\cos 90^\circ - \cos\theta)$$

$$= -pE \cos\theta = -\mathbf{p} \cdot \mathbf{E}$$

Example 2.48 When an electric dipole is placed in a uniform electric field making angle θ with electric field, it experiences a torque τ . Calculate the minimum work done in changing the orientation to 2θ .

Sol.

$$\tau = pE \sin\theta \Rightarrow pE = \frac{\tau}{\sin\theta}$$

$$W = \Delta U = -pE \cos\theta_2 + pE \cos\theta_1$$

$$\Rightarrow W = pE [\cos\theta - \cos 2\theta]$$

$$\Rightarrow W = \frac{\tau}{\sin\theta} [\cos\theta - \cos 2\theta]$$

Equilibrium of dipole

(i) Work done, when $\theta = 0^\circ$

$$U = -pE \cos 0^\circ = -pE$$

Thus, the potential energy of a dipole is minimum when its dipole moment is parallel to the external field. This is the position of stable equilibrium.

(ii) When $\theta = 180^\circ$

$$U = -pE \cos 180^\circ = +pE$$

Thus, the potential energy of a dipole is maximum when its dipole moment is antiparallel to the external field. This is the position of unstable equilibrium.

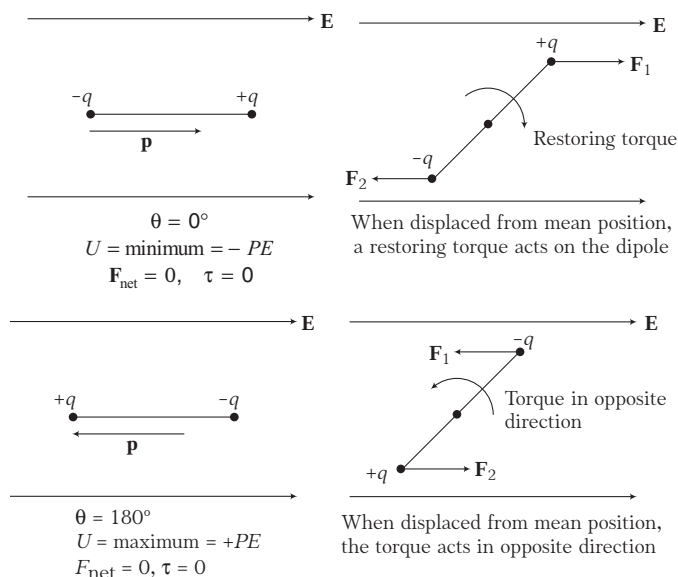


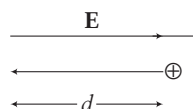
Fig. 2.28

Example 2.49 Two point charges $+2e$ and $-2e$ are situated at a distance of $2.4 \text{ \AA}$ from each other and constitute an electric dipole. This dipole is placed in a uniform electric field of $4.0 \times 10^5 \text{ Vm}^{-1}$. Calculate

- electric dipole moment,
- potential energy of the dipole in equilibrium position
- and work done in rotating the dipole through 180° from the equilibrium position.

CHECK POINT 2.3

- When a positive charge q is taken from lower potential to a higher potential point, then its potential energy will
 - decrease
 - increase
 - remain unchanged
 - become zero
- When one electron is taken towards the other electron, then the electric potential energy of the system
 - decreases
 - increases
 - remains unchanged
 - becomes zero
- Two positive point charges of $12 \mu\text{C}$ and $8 \mu\text{C}$ are 10 cm apart. The work done in bringing them 4 cm is
 - 5.8 J
 - 5.8 eV
 - 13 J
 - 13 eV
- A proton moves a distance d in a uniform electric field $\mathbf{E}$ as shown in the figure. Does the electric field do a positive or negative work on the proton? Does the electric potential energy of the proton increase or decrease?



- Negative, increase
 - Positive, decrease
 - Negative, decrease
 - Positive, increase
- The electrostatic potential energy between proton and electron separated by a distance $1 \text{ \AA}$ is
 - 13.6 eV
 - 27.2 eV
 - -14.4 eV
 - 1.44 eV

Sol. (i) The electric dipole moment is $p = q \times 2l$ where, q is the charge of one end of the dipole and $2l$ is the distance between the two charges.

Here, $q = 2e = 3.2 \times 10^{-19} \text{ C}$ and $2l = 2.4 \text{ \AA}$

$$= 2.4 \times 10^{-10} \text{ m}$$

$$\therefore p = (3.2 \times 10^{-19}) \times (2.4 \times 10^{-10})$$

$$= 7.68 \times 10^{-29} \text{ C-m}$$

(ii) The potential energy of a dipole in an electric field E is

$$U = -pE \cos \theta$$

where, θ is the angle between the axis of the dipole and the field. In equilibrium position,

$$\theta = 0^\circ, \text{ and so } U_0 = -pE$$

Here, $p = 7.68 \times 10^{-29} \text{ C-m}$ and $E = 4.0 \times 10^5 \text{ Vm}^{-1}$

$$\therefore U_0 = -(7.68 \times 10^{-29}) \times (4.0 \times 10^5)$$

$$= -3.07 \times 10^{-23} \text{ J}$$

(iii) The work done (increase in energy) in rotating the dipole through angle θ from the direction of the electric field E is given by $W = pE(1 - \cos \theta)$

If $\theta = 180^\circ$ or $\cos \theta = -1$,

then $W = 2pE$

$$= 2 \times (7.68 \times 10^{-29}) \times (4.0 \times 10^5)$$

$$= 6.14 \times 10^{-23} \text{ J}$$

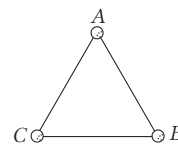
6. Identify the wrong statement.

- The electrical potential energy of a system of two protons shall increase if the separation between the two is decreased.
- The electrical potential energy of a proton-electron system will increase if the separation between the two is decreased.
- The electrical potential energy of a proton-electron system will increase if the separation between the two is increased.
- The electrical potential energy of a system of two electrons shall increase if the separation between the two is decreased.

7. Two positive point charges of 12 and 5 microcoulombs, are placed 10 cm apart in air. The work needed to bring them 4 cm closer is

- 2.4 J
- 3.6 J
- 4.8 J
- 6.0 J

8. Three identical charges each of $2 \mu\text{C}$ are placed at the vertices of a triangle ABC as shown in the figure.



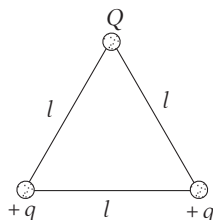
If $AB + AC = 12 \text{ cm}$ and $AB \cdot AC = 32 \text{ cm}^2$, the potential energy of the charge at A is

- 1.53 J
- 5.31 J
- 3.15 J
- 1.35 J

9. If three charges are placed at the vertices of equilateral triangle of charge q each, what is the net potential energy, if the side of equilateral triangle is l cm?

(a) $\frac{1}{4\pi\epsilon_0} \frac{q^2}{l}$ (b) $\frac{1}{4\pi\epsilon_0} \frac{2q^2}{l}$ (c) $\frac{1}{4\pi\epsilon_0} \frac{3q^2}{l}$ (d) $\frac{1}{4\pi\epsilon_0} \frac{4q^2}{l}$

10. Three charges Q , $+q$ and $+q$ are placed at the vertices of an equilateral triangle of side l as shown in the figure. If the net electrostatic energy of the system is zero, then Q is equal to

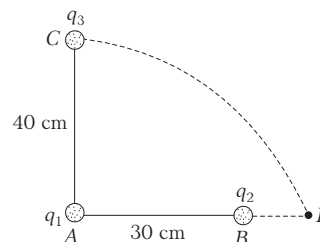


(a) $-\frac{q}{2}$ (b) $-q$ (c) $+q$ (d) zero

11. If identical charges $(-q)$ are placed at each corner of a cube of side b , then electric potential energy of charge $(+q)$ which is placed at centre of the cube will be

(a) $\frac{8\sqrt{2}q^2}{4\pi\epsilon_0 b}$ (b) $\frac{-8\sqrt{2}q^2}{\pi\epsilon_0 b}$ (c) $\frac{-4\sqrt{2}q^2}{\pi\epsilon_0 b}$ (d) $\frac{-4q^2}{\sqrt{3}\pi\epsilon_0 b}$

12. Two charges q_1 and q_2 are placed 30 cm apart as shown in the figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D . The change in the potential energy of the system is $\frac{q_3}{4\pi\epsilon_0} k$, where k is



(a) $8q_2$ (b) $8q_1$
(c) $6q_2$ (d) $6q_1$

13. For dipole $q = 2 \times 10^{-6}$ C and $d = 0.01$ m, calculate the maximum torque for this dipole if $E = 5 \times 10^5$ N/C.

(a) 1×10^{-3} N/m (b) 10×10^{-2} N/m
(c) 10×10^{-3} N/m (d) 1×10^2 N/m

14. A molecule with a dipole moment p is placed in a electric field of strength E . Initially, the dipole is aligned parallel to the field. If the dipole is to be rotated to be anti-parallel to the field, then the work required to be done by an external agency is

(a) $-2pE$ (b) $-pE$
(c) pE (d) $2pE$

15. Three point charges of 1C, 2C and 3C are placed at the corners of an equilateral triangle of side 100 cm. The work done to move these charges to the corners of a similar equilateral triangle of side 50 cm, will be

(a) 9.9×10^{10} J (b) 9.9×10^9 J
(c) 5.2×10^{10} J (d) 5.9×10^9 J

ELECTROSTATICS OF CONDUCTORS

Whenever a conductor is placed in an external electric field, the free electrons in it experience a force due to it and start moving opposite to the field. This movement makes one side of conductor positively charged and the other as negatively charged. This creates an electric field in the conductor in a direction opposite to external electric field (called induced field). The movement of electrons continues till the net electric field (resultant of external electric field and induced field) becomes zero. This is a state of **electrostatic equilibrium**.

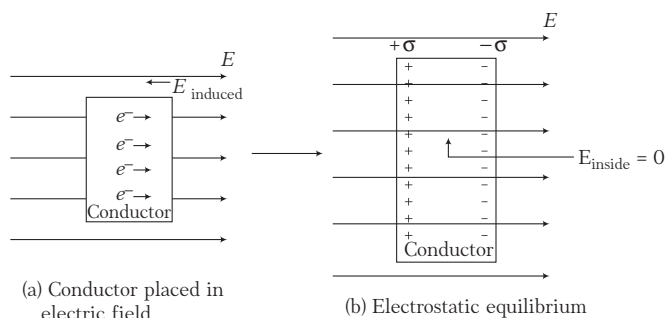


Fig. 2.29

A conductor in electrostatic equilibrium has the following properties

- Under static conditions, electric field inside a conductor is zero.
- Just outside a conductor, $\mathbf{E}$ is normal to its surface. If it were not, then it would have a component along the conductor's surface, that would cause the motion of charges. That contradicts the state of electrostatic equilibrium.
- The whole body of conductor is equipotential, as $E_{\text{inside}} = 0$.
- At any point inside that body of the conductor, the electric field due to charges appearing on the surface of conductor is equal and opposite to the external field.

Some phenomena related to electrostatics of conductors are given below.

(i) Electrostatic shielding

Suppose we have a very sensitive electronic instrument that we want to protect from external electric fields that might cause wrong measurements. We surround the

instrument with a conducting box or we keep the instrument inside the cavity of a conductor.

By doing this, charge in the conductor is so distributed that the net electric field inside the cavity becomes zero and the instrument is protected from the external fields. This is called electrostatic shielding.

It is electrostatic shielding that protects a person from lightning strikes if he is in a car.

Except for spherical surfaces, the charge is not distributed uniformly on the surface of a conductor. At the sharp points or edges, the surface charge density (σ) is very high and hence the electric field becomes very strong.

The air around such sharp points may become ionised producing the corona discharge in which the charge jumps from the conductor to air because of the dielectric breakdown of air.

(ii) Electrostatic pressure

Electrostatic pressure is defined as the force per unit area on the surface of a conductor due to its own charge. If σ is the surface charge density, then

$$\text{Electrostatic pressure} = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2} \epsilon_0 E^2$$

$$\text{where, } E = \frac{\sigma}{\epsilon_0} \quad (\text{electric field near conductor})$$

where, σ = surface charge density.

(iii) Effect of metallic slab between two charged plates

Figure shows a metallic slab between two charged plates. The field $\mathbf{E}$ due to the charged plates is directed towards right and the field $\mathbf{E}$ due to the induced charge in the slab is directed toward left, and hence the net field inside the slab becomes zero.

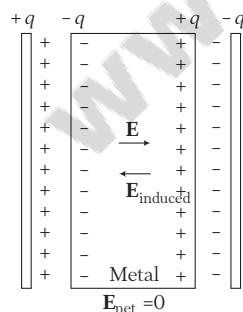


Fig. 2.30

(iv) Earthing a conductor

Earth is a good conductor of electricity. For most practical purpose, its potential is assumed to be zero. A conductor is said to be earthed or grounded whenever it is connected to the earth. In that case, its potential becomes zero.

The charge will flow from conductor to earth or from earth to conductor to make its potential zero. The symbol used for earthing is



Fig. 2.31 A symbol for earthing

Let us discuss some of the examples related to this concept.

(a) Fig. (a) shows a conducting shell with charge q .

Fig. (b) the conducting shell with charge q has been surrounded by another larger conducting shell which is uncharged. Now, charges $-q$ and $+q$ are induced on its inner and outer surfaces but net charge in the outer shell is still zero.

In the Fig. (c), the outer shell of the Fig. (b) is earthed, The free charge on the outer surface goes to the earth but the inner charge remains bounded to the charge on the inner shell, so that the potential of the outer shell connected to the earth becomes zero.

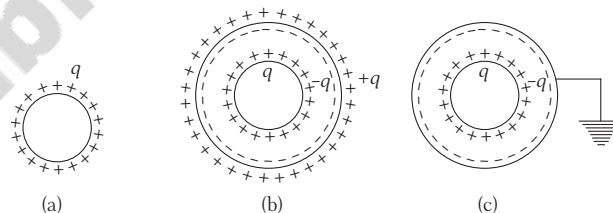


Fig. 2.32

(b) Fig. (a), shows two concentric conducting shells.

Some charge q_1 is given to the outer shell. No charge is developed on the inner shell.

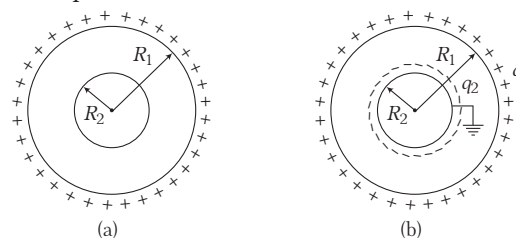


Fig. 2.33

In the Fig. (b), the inner shell is earthed and hence some charge q_2 is developed on it, so that its potential becomes zero.

On the surface of the inner shell, the net potential is

$$\frac{1}{4\pi\epsilon_0} \left(\frac{q_2}{R_2} + \frac{q_1}{R_1} \right) = 0$$

or

$$q_2 = -q_1 \frac{R_2}{R_1}$$

(c) Fig. (a) is an uncharged metallic solid sphere of radius R .

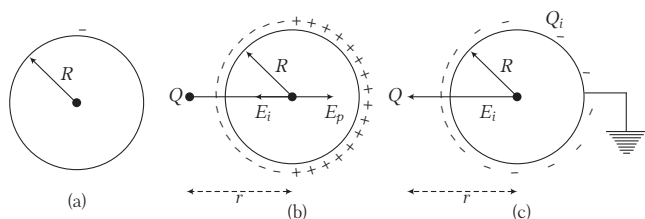


Fig. 2.34

In Fig. (b), a positive point charge Q has been placed at a distance r from the centre of the sphere. The point charge Q exerts force on the electrons in the sphere and hence the free electrons redistribute themselves, so that the left half is negatively charged and the right half is positively charged. The charge distribution is non-uniform.

At the centre of the sphere, the field due to the point charge Q is $E_p = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$, toward right, since

the field inside the conductor is zero. The field due to the induced charge on the sphere is equal in magnitude of E_p but opposite in direction.

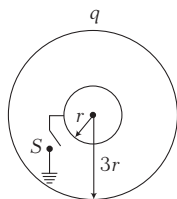
i.e. $\boxed{\mathbf{E}_i = -\mathbf{E}_p}$

The potential at any point of the sphere = potential at its centre = $\frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

In the Fig. (c), the sphere is earthed, so $V = 0$, hence $\frac{1}{4\pi\epsilon_0} \left(\frac{Q}{r} - \frac{Q_i}{R} \right) = 0$ or, $Q_i = \frac{QR}{r}$

Note that, the induced charge Q_i is non-uniformly distributed on the surface of the sphere.

Example 2.50 Figure shows two conducting thin concentric shells of radii r and $3r$. The outer shell carries charge q . Inner shell is neutral. Find the charge that will flow from inner shell to earth after the switch S is closed.



Sol. Let q' be the charge on inner shell when it is earthed.

$$\therefore \frac{1}{4\pi\epsilon_0} \left[\frac{q'}{r} + \frac{q}{3r} \right] = 0$$

$$\therefore q' = -\frac{q}{3}$$

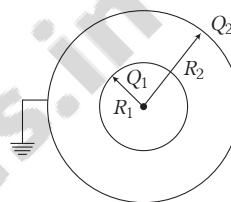
i.e., $+\frac{q}{3}$ charge will flow from inner shell to earth.

Example 2.51 A sphere of 4 cm radius is suspended within a hollow sphere of 6 cm radius. The inner sphere is charged to a potential of 3 esu when the outer sphere is earthed, find the charge on the inner sphere.

Sol. First of all understand that esu means electrostatic unit

(CGS unit) and in this system $\frac{1}{4\pi\epsilon_0}$ is replaced by 1. So,

potential due to Q esu of charge will be Q/R in esu. Now, the diagram for the above question is as follows



Let the inner sphere and outer sphere have charges Q_1 and Q_2 on them, respectively. Their radii being R_1 and R_2 such that ($R_1 < R_2$). Given, $R_1 = 4$ cm and $R_2 = 6$ cm. We have two unknowns Q_1 and Q_2 . So, we will form two equations by equating to the potential of both the spheres. Consider the outer sphere. It is grounded, so the potential of this sphere is zero.

or $\frac{Q_1}{R_1} + \frac{Q_2}{R_2} = 0 \quad \dots(i)$

Note, the contribution of Q_2 in potential will be its surface potential but for Q_1 , the outer sphere is outside it. So, Q_1 will be treated at the centre.

Hence, from Eq. (i),

$$Q_1 + Q_2 = 0 \Rightarrow Q_2 = -Q_1$$

Now, write the expression for the potential of inner sphere and make it equal to 3 esu. For the inner sphere both the charges will contribute their respective surface potentials.

Therefore, $\frac{Q_1}{R_1} + \frac{Q_2}{R_2} = 3$

Substituting $Q_2 = -Q_1$, we get

$$\frac{Q_1}{R_1} - \frac{Q_1}{R_2} = 3, \text{ therefore } Q_1 \left[\frac{1}{4} - \frac{1}{6} \right] = 3 \text{ or } Q_1 = 36 \text{ esu}$$

Dielectrics and polarisation

Dielectrics are insulating (non-conducting) materials that can produce electric effect without conduction. Movement of free charges is not possible in a dielectric. So, they behave differently. When a dielectric material is kept in an electric field, then the external field induces dipole moment. Due to which, net charge on the surface of the

dielectric appears and as a result electric field is produced that opposes the external field. Induced field is less in magnitude than the external field, so field inside the dielectric gets reduced.

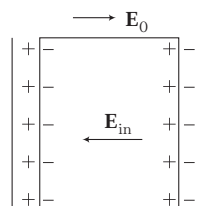


Fig. 2.35 Polarised dielectric slab

$$\mathbf{E} = |\mathbf{E}_0| - |\mathbf{E}_{in}|$$

where, $\mathbf{E}$ = resultant electric field in the dielectric,

$\mathbf{E}_0$ = external electric field between two plates

and $\mathbf{E}_{in}$ = electric field inside the dielectric.

Types of dielectrics

There are two types of dielectrics

(i) Polar dielectrics

A polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). Such molecules have permanent dipole moment, e.g., HCl, H_2O . So, a polar dielectric is one which is having a net dipole moment zero in the absence of electric field due to the random orientation of polar molecules as shown in figure.

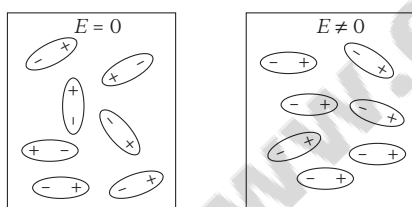


Fig. 2.36 Behaviour of a polar dielectric in external field

In the presence of electric field polar molecules tend to line up in the direction of electric field, and the substance has finite dipole moment.

(ii) Non-polar dielectrics

In a non-polar molecule, the centres of positive and negative charges coincide. The molecule thus has no permanent (or intrinsic) dipole moment, e.g., oxygen (O_2) and hydrogen (H_2) molecules. In an external electric field, the positive and negative charges of a non-polar molecules are displaced in opposite direction. This occurs till the point where external force is balanced by the restoring force (due to internal field in the molecule). Thus, dipole moment is induced in non-polar molecules and dielectric is said to be **polarised**. The induced dipole moments of different

molecules add up giving a net dipole moment of the dielectric in the presence of the external field.

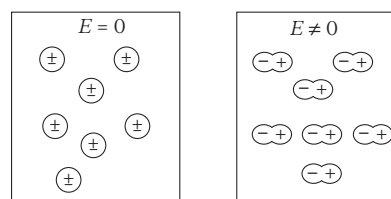


Fig. 2.37 Behaviour of a non-polar dielectric in external field

Dielectric constant (K)

The ratio of the strength of the applied electric field to the strength of the reduced value of the electric field on placing the dielectric between the two charged plates called the dielectric constant of the dielectric medium.

It is also known as **relative permittivity** or **specific inductive capacity** (SIC) and is denoted by K (or ϵ_r).

Therefore, dielectric constant of a dielectric medium is given by

$$K = \frac{E_0}{E}$$

The value of K is always greater than 1.

For water, value of $K = 80$.

Polarisation (P)

It is a process of inducing equal and opposite charges on the two faces of the dielectrics on the application of electric field. In other words, the induced dipole moment developed per unit volume in a dielectric slab on placing it, in an electric field is called **polarisation**. It is denoted by P . If p is induced dipole moment acquired by an atom of the dielectric and N is the number of atoms per unit volume, then polarisation is given by

$$P = Np$$

The induced dipole moment acquired by the atom is found to be directly proportional to the reduced value of electric field and is given by

$$P = \alpha \epsilon_0 E_0$$

where, α is constant of proportionality and is called **atomic polarisability**.

Electric susceptibility (χ)

The polarisation density of dielectric slab is directly proportional to the reduced value of the electric field and may be expressed as

$$P = \chi \epsilon_0 E_0$$

where, χ is a constant of proportionality and is called electric susceptibility of the dielectric slab. It is a dimensionless constant.

Note

1. For vacuum, $\chi = 0$
2. $K = 1 + \chi$

Dielectric strength

The maximum electric field that a dielectric can withstand without breakdown is called its dielectric strength.

Its SI unit is Vm^{-1} and its practical unit is $\text{kV}(\text{mm})^{-1}$.

For air it is about $3 \times 10^6 \text{ Vm}^{-1}$.

Capacitors and capacitance

A capacitor is a device which is used to store electric charge or potential energy. It is made up of two isolated conductors carrying equal and opposite charges placed at small separation.

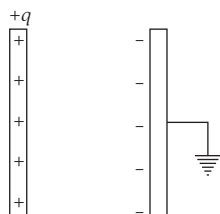


Fig. 2.38

The space between the conductors may be vacuum or an insulating material. Capacitor is represented as

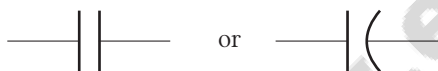


Fig. 2.39

Capacitance of a conductor

When a charge q is given to a conductor, it spreads over the outer surface of the conductor. The whole conductor comes to the same potential (say V). This potential V is directly proportional to the charge q , i.e.

$$V \propto q$$

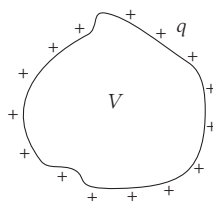


Fig. 2.40

$$q = CV$$

or

$$C = \frac{q}{V}$$

This C is constant of proportionality and is called the **capacitance** of the conductor.

The SI unit of capacitance is called **farad** (F). One farad is equal to coulomb per volt (1 C/V).

$\therefore$ 1 farad (1F) = 1 coulomb/volt (1C/V)

Its dimensional formula is $[\text{ML}^{-2}\text{T}^4\text{A}^2]$.

Capacitance C depends on the shape, size and separation of the system of two conductors and is independent of the charge given to the body.

Example 2.52 A capacitor of $0.75 \mu\text{F}$ is charged to a voltage of 16 V. What is the magnitude of the charge on each plate of the capacitor?

Sol. Using $q = CV$, we get

$$q = CV = (0.75 \times 10^{-6} \text{ F})(16 \text{ V}) = 1.2 \times 10^{-5} \text{ C}$$

Capacitance of an isolated spherical conductor

When a charge q is given to a spherical conductor of radius R , the potential on it is

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$$

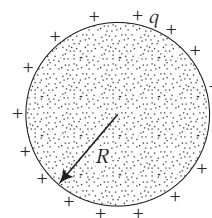


Fig. 2.41 Spherical conductor

From this expression, we find that

$$\frac{q}{V} = 4\pi\epsilon_0 R = C$$

Thus, capacitance of the spherical conductor is

$$C = 4\pi\epsilon_0 R$$

From this expression, we can draw the following conclusions

- (i) $C \propto R$ or C depends on R only. Which we have already stated that C depends on the dimensions of the conductor. Moreover if two conductors have radii R_1 and R_2 , then

$$\frac{C_1}{C_2} = \frac{R_1}{R_2}$$

- (ii) Earth is also a spherical conductor of radius $R = 6.4 \times 10^6$ m. The capacity of earth is therefore,

$$C = \left(\frac{1}{9 \times 10^9} \right) (6.4 \times 10^6) \\ \approx 711 \times 10^{-6} \text{ F or } C = 711 \mu\text{F}$$

From here, we can see that farad is a large unit. As capacity of such a huge conductor is only $711 \mu\text{F}$.

Capacity of a spherical conductor enclosed by an earthed concentric spherical shell

Let a system two concentric conducting spheres of radii a and b , where a less than b . Inner sphere is given charge q while outer sphere is earthed. Potential difference between the spheres is given by

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) \quad \dots(i)$$

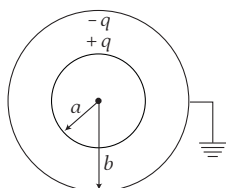


Fig. 2.42

Hence, the capacitance of this system will be

$$C = \frac{q}{V}$$

or
$$C = 4\pi\epsilon_0 \left(\frac{ab}{b-a} \right) \quad [\text{from Eq. (i)}]$$

Example 2.53 A sphere of radius 0.03 m is suspended within a hollow sphere of radius 0.05 m. If the inner sphere is charged to a potential of 1500 V and outer sphere is earthed, find the capacitance and the charge on the inner sphere.

Sol. Here, $a = 0.03$ m, $b = 0.05$ m and $V = 1500$ V

The capacitance of the air-filled spherical capacitor is

$$C = \frac{4\pi\epsilon_0 ab}{(b-a)} = \frac{0.03 \times 0.05}{9 \times 10^9 \times (0.05 - 0.03)} \\ = 8.33 \times 10^{-12} \text{ F} = 8.33 \text{ pF}$$

Charge,
$$q = CV = 8.33 \times 10^{-12} \times 1500 \\ = 1.25 \times 10^{-8} \text{ C}$$

Example 2.54 The thickness of air layer between the two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacitance as the sphere of 1.2 m diameter. Find the radii of its surfaces.

Sol. Here,
$$\frac{4\pi\epsilon_0 ab}{b-a} = 4\pi\epsilon_0 R \quad \text{or} \quad \frac{ab}{b-a} = R$$

Now, $b - a = 2$ cm and $R = \frac{1.2}{2}$ m = 60 cm

$\therefore \frac{ab}{2} = 60$

or $ab = 120$

$(b+a)^2 = (b-a)^2 + 4ab = 2^2 + 4 \times 120 = 484$

or $b+a = 22$

or $2+a+a = 22 \quad (\because b-a = 2 \text{ cm})$

$\therefore a = 10$ cm and $b = 12$ cm

Cylindrical capacitor

Cylindrical capacitor consists of two co-axial cylinders of radii a and b ($a < b$), inner cylinder is given charge $+q$ while outer cylinder is earthed.

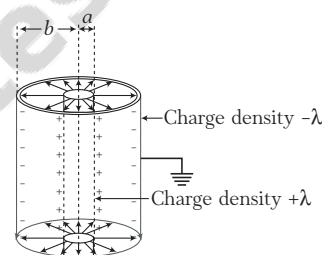


Fig. 2.43

Common length of the cylinders is l , then capacity of a cylindrical capacitor is given by

$$C = \frac{2\pi\epsilon_0 l}{\log_e (b/a)}$$

Parallel plate capacitor

A parallel plate air capacitor consists of two parallel metallic plates separated by a small distance.

Suppose the area of each plate is A and the separation between the two plates is d . Also assume that the space between the plates contains vacuum. Suppose, the charge density on each of these surfaces has a magnitude $\sigma = q/A$.

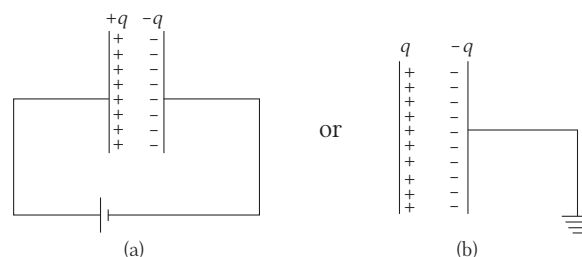


Fig. 2.44

We know that the intensity of electric field at a point between two plane parallel sheets of equal and opposite charges is given by $E = \frac{\sigma}{\epsilon_0}$.

The potential difference between the plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d}$$

$$\therefore V = Ed = \left(\frac{\sigma}{\epsilon_0} \right) d = \frac{qd}{A\epsilon_0}$$

$\therefore$ The capacitance of the parallel plate capacitor is given by

$$C = \frac{q}{V} = \frac{A\epsilon_0}{d} \quad \text{or} \quad C = \frac{\epsilon_0 A}{d}$$

The force between plates of capacitor is attractive and can be expressed as

$$F = qE = q \left(\frac{\sigma}{2\epsilon_0} \right)$$

$$\text{or} \quad F = \frac{1}{2} \epsilon_0 A E^2 \quad (\because q = \sigma A = \epsilon_0 E A)$$

If the space between the plates be filled with some dielectric medium of dielectric constant K , then the electric field between the plates is increased to K -times and given as,

$$C = \frac{K\epsilon_0 A}{d} \text{ farad}$$

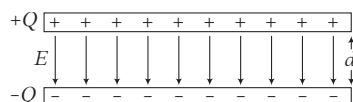
Note

(i) Instead of two plates, if there are n similar plates at equal distances from each other and the alternate plates are connected together, then the capacitance of the arrangement is given by

$$C = \frac{(n-1) \epsilon_0 A}{d}$$

(ii) From the above relation, it is clear that the capacitance depends only on geometrical factors (A and d).

Example 2.55 A parallel plate capacitor is constructed with plates of area 0.0280 m^2 and separation 0.550 mm . Find the magnitude of the charge on each plate of this capacitor when the potential difference between the plates is 20.1 V .



Sol. Using the formula,

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12})(0.0280)}{0.550 \times 10^{-3}}$$

We obtain the capacitance of the parallel plate capacitor,

$$C = 4.51 \times 10^{-10} \text{ F}$$

Capacitance of a capacitor partially filled with dielectric

Suppose, the space between parallel plates of capacitor is partially filled with a dielectric (dielectric constant = K) of thickness t ($< d$). Then, the filling thickness between the plates is t in the dielectric and $(d - t)$ in vacuum (or air)

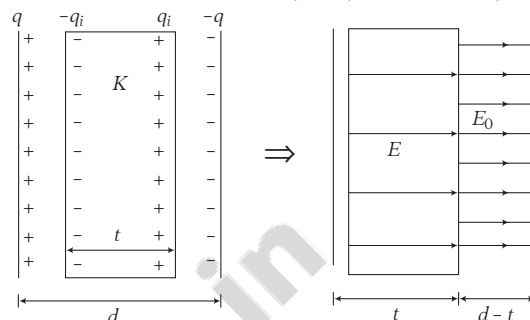


Fig. 2.45

If, $E_0 = \frac{q}{\epsilon_0 A}$ is the electric field in the region, where

dielectric is absent, then electric field inside the dielectric will be $E = E_0/K$. The potential difference between the plates of the capacitor is

$$V = Et + E_0(d - t) = \frac{E_0}{K}t + E_0(d - t) \\ = E_0 \left(d - t + \frac{t}{K} \right) = \frac{q}{A\epsilon_0} \left(d - t + \frac{t}{K} \right)$$

Now, as per the definition of capacitance,

$$C = \frac{q}{V} = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$$

or

$$C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$$

Special cases

(i) If the slab completely fills the space between the plates, then $t = d$ and therefore

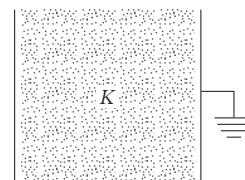


Fig. 2.46

$$C = \frac{\epsilon_0 A}{d/K} = \frac{K\epsilon_0 A}{d} \quad \text{or} \quad \frac{C}{C_0} = K$$

where, $C_0 = \frac{\epsilon_0 A}{d}$ is capacitance without the dielectric.

- (ii) If a conducting slab (*i.e.* $K = \infty$) is placed between the plates, then $C = \frac{\epsilon_0 A}{d - t + t/\infty} = \frac{\epsilon_0 A}{d - t}$

This can also be understood from the following figure

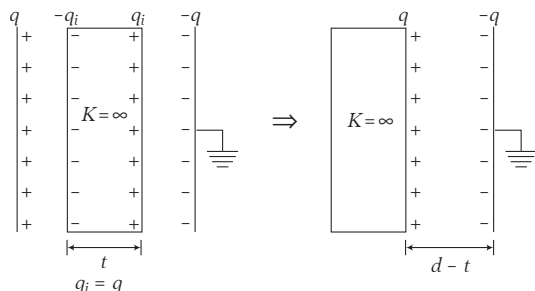


Fig. 2.47

- (iii) If the space between the plates is completely filled with a conductor, then $t = d$ and $K = \infty$.

Then,
$$C = \frac{\epsilon_0 A}{d - d + \frac{d}{\infty}} = \infty$$

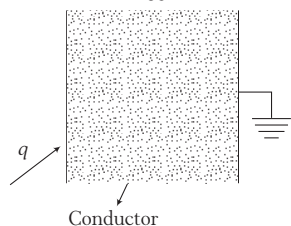


Fig. 2.48

- (iv) If more than one dielectric slabs are placed between the capacitor, then

$$C = \frac{\epsilon_0 A}{d - t_1 - t_2 - \dots - t_n + \frac{t_1}{K_1} + \frac{t_2}{K_2} + \dots + \frac{t_n}{K_n}}$$

Example 2.56 A dielectric slab of thickness 1.0 cm and dielectric constant 5 is placed between the plates of a parallel plate capacitor of plates area 0.01 m^2 and separation 2.0 cm. Calculate the change in capacity on introduction of dielectric. What would be the change, if the dielectric slab was conducting?

Sol. Given, $t = 1.0 \text{ cm} = 10^{-2} \text{ m}$, $K = 5$, $A = 0.01 \text{ m}^2 = 10^{-2} \text{ m}^2$ and $d = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$

Capacity with air in between the plates,

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 10^{-2}}{2 \times 10^{-2}} = 4.425 \times 10^{-12} \text{ F}$$

Capacity with dielectric slab in between the plates,

$$C = \frac{\epsilon_0 A}{\left[d - \left(1 - \frac{1}{K} \right) t \right]} = \frac{8.85 \times 10^{-12} \times 10^{-2}}{2 \times 10^{-2} - 10^{-2} \left(1 - \frac{1}{5} \right)} = 7.375 \times 10^{-12} \text{ F}$$

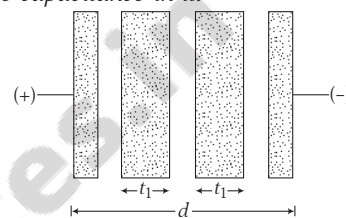
Capacity with conducting slab in between the plates,

$$C' = \frac{\epsilon_0 A}{d - t} = \frac{8.85 \times 10^{-12} \times 10^{-2}}{2 \times 10^{-2} - 1 \times 10^{-2}} = 8.85 \times 10^{-12} \text{ F}$$

$$\begin{aligned} \text{Increase in capacity on introduction of dielectric} \\ = C - C_0 = 7.375 \times 10^{-12} - 4.425 \times 10^{-12} \\ = 2.95 \times 10^{-12} \text{ F} \end{aligned}$$

$$\begin{aligned} \text{Increase in capacity on introduction of conducting slab} \\ = C' - C_0 = 8.85 \times 10^{-12} - 4.425 \times 10^{-12} \\ = 4.425 \times 10^{-12} \text{ F} \end{aligned}$$

Example 2.57 An air-cored capacitor of plate area A and separation d has a capacity C . Two dielectric slabs are inserted between its plates in the manners as shown. Calculate the capacitance in it.



Sol. Let the charges on the plates are Q and $-Q$.

$$\text{Electric field in free space is } E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

$$\text{Electric field in first slab is } E_1 = \frac{E_0}{K_1} = \frac{Q}{A\epsilon_0 K_1}$$

$$\text{Electric field in second slab is } E_2 = \frac{E_0}{K_2} = \frac{Q}{A\epsilon_0 K_2}$$

The potential difference between the plates is

$$\begin{aligned} V &= E_0(d - t_1 - t_2) + E_1 t_1 + E_2 t_2 \\ \Rightarrow V &= E_0 \left(d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2} \right) \left(\text{as } E_1 = \frac{E_0}{K_1}, E_2 = \frac{E_0}{K_2} \right) \end{aligned}$$

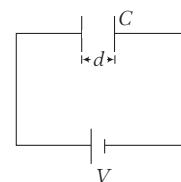
$$\therefore V = \frac{Q}{A\epsilon_0} \left(d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2} \right)$$

$$\therefore C = \frac{\epsilon_0 A}{d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}}$$

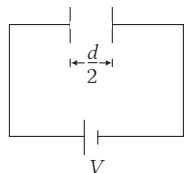
Example 2.58 A parallel plate capacitor has plate area A and separation d between the plates. The capacitor is connected to a battery of emf V . (i) Find the charge on the capacitor. (ii) The plate separation is decreased to $d/2$. Find the extra charge given by the battery to the positive plate.

Sol. (i) Capacitance, $C = \frac{A\epsilon_0}{d}$

$$\therefore \text{Charge, } Q = CV = \frac{A\epsilon_0 V}{d}$$



$$(ii) C' = \frac{A\epsilon_0}{d/2} = \frac{2A\epsilon_0}{d} \Rightarrow Q' = C'V = \frac{2A\epsilon_0 V}{d}$$

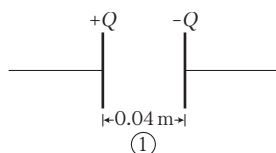


$$\text{Extra charge given by battery, } \Delta Q = Q' - Q = \frac{A\epsilon_0 V}{d}$$

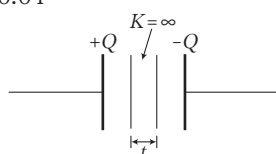
Example 2.59 The distance between the plates of a parallel plate capacitor is 0.04 m. A field of 5000 V/m is established between the plates and an uncharged metal plate of thickness 0.01 m is inserted into the condenser, parallel to its plate. Find the potential difference between the plates (i) before the introduction of the metal plate and (ii) after its introduction. What would be the potential difference, if a plate of dielectric constant $K = 2$ is introduced in place of metal plate?

Sol. (i) Potential difference across capacitor,

$$V = Ed = 5000 \times 0.04 = 200 \text{ V}$$



$$(ii) C = \frac{A\epsilon_0}{d} = \frac{A\epsilon_0}{0.04}$$

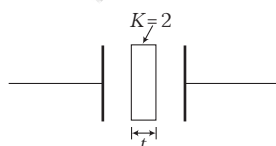


$$C' = \frac{A\epsilon_0}{d - t + \frac{t}{K}} = \frac{A\epsilon_0}{0.04 - 0.01 + \frac{0.01}{\infty}} = \frac{A\epsilon_0}{0.03}$$

Since, charge remains same,

$$Q = CV = C'V' \Rightarrow \frac{A\epsilon_0}{0.04} \times 200 = \frac{A\epsilon_0}{0.03} V'$$

$\therefore$ Potential difference, $V' = 150 \text{ V}$



$$C'' = \frac{A\epsilon_0}{d - t + \frac{t}{K}} = \frac{A\epsilon_0}{0.04 - 0.01 + \frac{0.01}{2}} = \frac{A\epsilon_0}{0.035}$$

$$Q = CV = C''V''$$

$$\frac{A\epsilon_0}{0.04} \times 200 = \frac{A\epsilon_0}{0.035} V''$$

$\therefore$ Potential, $V'' = 175 \text{ V}$

Effect of dielectric on various parameters

A parallel plate capacitor is charged by a battery which is then disconnected. A dielectric slab is then inserted to fill the space between the plates.

Let q_0 , C_0 , V_0 and E_0 be the charge, capacitance, potential difference and electric field respectively, before the dielectric slab is inserted.

Now, we will discuss two cases

Effect of dielectric when the battery is kept disconnected from the capacitor

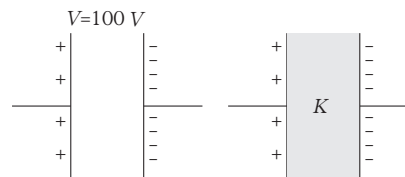
$$\text{As } q_0 = C_0 V_0, E_0 = \frac{V_0}{d}$$

- The charge on the plates of capacitor remains q_0 .
- The electric field gets reduced, i.e. $E = \frac{E_0}{K}$.
- The potential difference gets reduced, i.e. $V = \frac{V_0}{K}$.
- The capacitance increases, i.e. $C = KC_0$.
- The energy decreases, i.e. $U = \frac{U_0}{K}$.

Effect of dielectric when battery remains connected across the capacitor

- The charge now becomes Kq_0 , i.e. $q = Kq_0$.
- The electric field remains constant, i.e. $E = E_0$.
- The potential difference remains constant at V_0 , i.e. $V = V_0$.
- The capacitance increases from C_0 to C , i.e. $C = KC_0$.
- The energy increases, i.e. $U = KU_0$.

Example 2.60 An isolated $16 \mu\text{F}$ parallel plate air capacitor has a potential difference of 100 V. A dielectric slab having relative permittivity (i.e. dielectric constant) = 5 is introduced to fill the space between the two plates completely. Calculate



(i) the new capacitance of the capacitor.

(ii) the new potential difference between the two plates of capacitor.

Sol. (i) The new capacity of the capacitor, $C = KC_0 = 5 \times 16 = 80 \mu\text{F}$

(ii) Since, the capacitor is isolated, therefore the charge on the capacitor remains the same, thus its capacity is increased on expense of its potential drop.

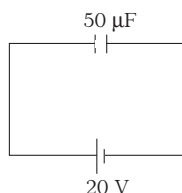
Therefore, the new potential difference,

$$V = \frac{V_0}{K} = \frac{100}{5} = 20 \text{ V}$$

Example 2.61 A capacitor ($C = 50 \mu\text{F}$) is charged to a potential difference of 20 V. The charging battery is disconnected and the capacitor is connected to another cell of emf 10 V with the positive plate of capacitor joined with the positive terminal of cell.

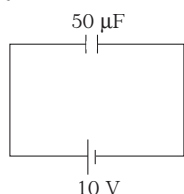
- (i) Find charge flown through 10 V cell.
 (ii) Is work done by the cell or is it done on the cell? Find its magnitude.

Sol. Circuit for 20 V battery,



Charge on capacitor, $Q_1 = 50 \times 20 = 1000 \mu\text{C}$

Circuit for 10 V battery,



Charge on capacitor, $Q_2 = 50 \times 10 = 500 \mu\text{C}$, $Q_2 < Q_1$

- (i) Charge flown through cell of emf 10 V,

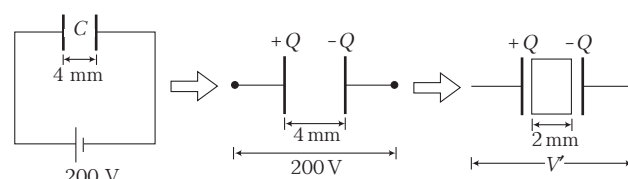
$$\Delta Q = Q_1 - Q_2 = 500 \mu\text{C}$$

- (ii) Since, $Q_2 < Q_1$, work is done on the cell (W is negative)

$$W = \Delta QV = 500 \times 10 \times 10^{-6} = 5 \times 10^{-3} \text{ J}$$

Example 2.62 A parallel plate capacitor ($C = 50 \mu\text{F}$, $d = 4 \text{ mm}$) is charged to 200 V and then charging battery is removed. Now, a dielectric slab ($K = 4$) of thickness 2 mm is placed between the plates. Find new potential difference across capacitor.

Sol.



$$C = \frac{A\epsilon_0}{d}$$

$$\Rightarrow A\epsilon_0 = Cd = 50 \times 10^{-6} \times 4 \times 10^{-3}$$

$$C' = \frac{A\epsilon_0}{d - t + \frac{t}{K}} = \frac{Cd}{d - t + \frac{t}{K}}$$

$$= \frac{Cd}{(5/2) \times 10^{-3}}$$

$$= \frac{50 \times 10^{-6} \times 4 \times 10^{-3}}{(5/2) \times 10^{-3}}$$

$$= 80 \times 10^{-6} \text{ F}$$

Since, charge remains same,

$$Q = CV = C'V'$$

$$50 \times 200 = 80V'$$

$\Rightarrow$ Potential difference across capacitor,

$$V' = 125 \text{ V}$$

CHECK POINT 2.4

1. Identify the false statement.

- Electric field is zero inside the conductor and just outside, it is normal to the surface.
- Electric field is zero in the cavity of a hollow charged conductor.
- A polar dielectric is one which is having a net dipole moment zero in the absence of electric field.
- H_2 , N_2 , O_2 , CO_2 and CH_4 are examples of polar dielectric.

2. Eight drops of mercury of equal radii possessing equal charges combine to form a big drop. Then, the capacitance of bigger drop compared to each individual small drop is

- 8 times
- 4 times
- 2 times
- 32 times

3. The capacity of a spherical conductor is

- $\frac{R}{4\pi\epsilon_0}$
- $\frac{4\pi\epsilon_0}{R}$
- $4\pi\epsilon_0 R$
- $4\pi\epsilon_0 R^2$

4. The earth has volume V and surface area A , then capacitance would be

- $4\pi\epsilon_0 \frac{A}{V}$
- $2\pi\epsilon_0 \frac{A}{V}$
- $12\pi\epsilon_0 \frac{V}{A}$
- $12\pi\epsilon_0 \frac{A}{V}$

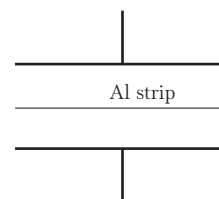
5. If the circumference of a sphere is 2 m, then capacitance of sphere in water would be

- 2700 pF
- 2760 pF
- 2780 pF
- 2800 pF

6. The capacity of parallel plate condenser depends on

- the type of metal used
- the thickness of plates
- the potential applied across on the plates
- the separation between the plates

7. As shown in the figure, a very thin sheet of aluminium is placed in between the plates of the condenser. Then, the capacity will

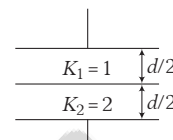


- increase
- decrease
- remains unchanged
- may increase or decrease

8. The potentials of the two plates of capacitor are + 10 V and - 10 V. The charge on one of the plates is 40 C. The capacitance of the capacitor is

- 2 F
- 4 F
- 0.5 F
- 0.25 F

9. The capacitance of a parallel plate capacitor is $12\ \mu\text{F}$. If the distance between the plates is doubled and area is halved, then new capacitance will be
(a) $8\ \mu\text{F}$ (b) $6\ \mu\text{F}$ (c) $4\ \mu\text{F}$ (d) $3\ \mu\text{F}$
10. A parallel plate condenser has a capacitance $50\ \mu\text{F}$ in air and $110\ \mu\text{F}$ when immersed in an oil. The dielectric constant K of the oil is
(a) 0.45 (b) 0.55 (c) 1.10 (d) 2.20
11. A $500\ \mu\text{F}$ capacitor is charged at a steady rate of $100\ \mu\text{C/s}$. The potential difference across the capacitor will be $10\ \text{V}$ after an interval of
(a) 5 s (b) 25 s
(c) 20 s (d) 50 s
12. There is an air filled $1\ \text{pF}$ parallel plate capacitor. When the plate separation is doubled and the space is filled with wax, the capacitance increases to $2\ \text{pF}$. The dielectric constant of wax is
(a) 2 (b) 4 (c) 6 (d) 8
13. A parallel plate capacitor with air between the plates has a capacitance of $9\ \text{pF}$. The separation between its plates is d . The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $K_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now
(a) $45\ \text{pF}$ (b) $40.5\ \text{pF}$
(c) $20.25\ \text{pF}$ (d) $1.8\ \text{pF}$
14. Two parallel plate of area A are separated by two different dielectrics as shown in figure. The net capacitance is



- (a) $\frac{4\epsilon_0 A}{3d}$ (b) $\frac{3\epsilon_0 A}{R}$ (c) $\frac{2\epsilon_0 A}{d}$ (d) $\frac{\epsilon_0 A}{d}$

COMBINATION OF CAPACITORS

1. Series grouping

In a series connection, charge on each capacitor remains same and equals to the main charge supplied by the battery but potential difference across each capacitors is different such that $V = V_1 + V_2 + V_3$

where V_1, V_2, V_3 are the potential difference across capacitor C_1, C_2 and C_3 , respectively.

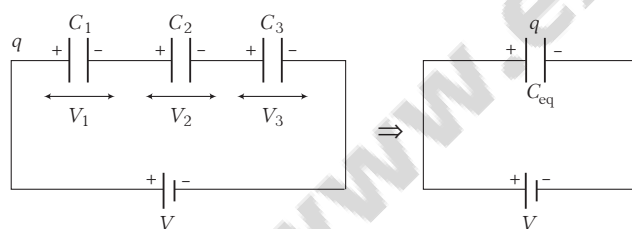


Fig. 2.49

The arrangement shown above is series connection. Here, the equivalent capacitance is

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Following points are important in case of series combination of capacitors

- (i) In series combination, potential difference and potential energy distributes in the inverse ratio of capacitance, i.e. $V \propto \frac{1}{C}$ and $U \propto \frac{1}{C}$.

- (ii) If two capacitors having capacitances C_1 and C_2 are connected in series, then $C = \frac{C_1 C_2}{C_1 + C_2}$

$$V_1 = \left(\frac{C_2}{C_1 + C_2} \right) V$$

and

$$V_2 = \left(\frac{C_1}{C_1 + C_2} \right) V$$

- (iii) If n capacitors of equal capacity C are connected in series, with supply voltage V , then their equivalent capacitance is $\frac{C}{n}$ and potential difference across each capacitor is $V' = \frac{V}{n}$.

- (iv) If n identical plates are arranged as shown in figure, then they constitute $(n - 1)$ capacitors in series. Each capacitor has capacitance $\frac{\epsilon_0 A}{d}$

and $C_{\text{eq}} = \frac{\epsilon_0 A}{(n - 1)d}$

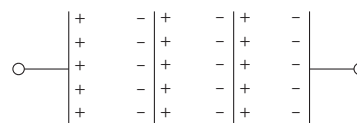


Fig. 2.50

In this situation, except two extreme plates, each plate is common to adjacent capacitors.

- (v) Below are some of the examples of combination of capacitors arranged in series.

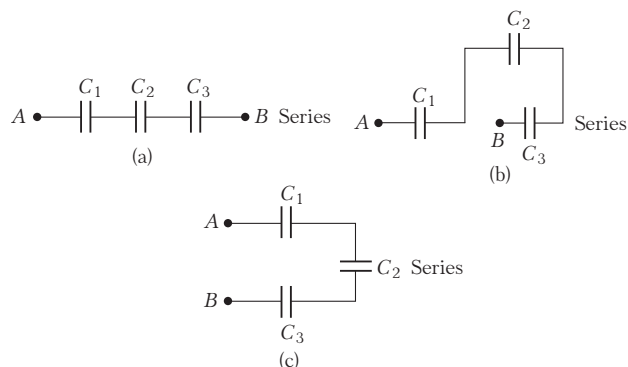
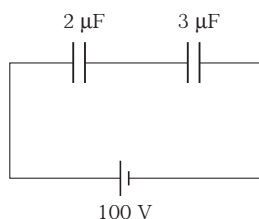


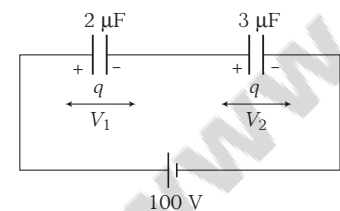
Fig. 2.51

Example 2.63 In the circuit shown in figure, find

- the equivalent capacitance.
- the charge stored in each capacitor and
- the potential difference across each capacitor.



Sol. (i) The equivalent capacitance,



$$C = \frac{C_1 C_2}{C_1 + C_2}$$

$$\text{or } C = \frac{(2)(3)}{2+3} = 1.2 \mu\text{F}$$

- (ii) The charge q , stored in each capacitor is

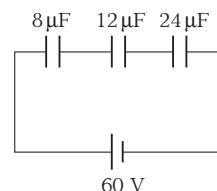
$$q = CV = (1.2 \times 10^{-6})(100) \text{ C} = 120 \mu\text{C}$$

- (iii) In series combination, $V \propto \frac{1}{C}$

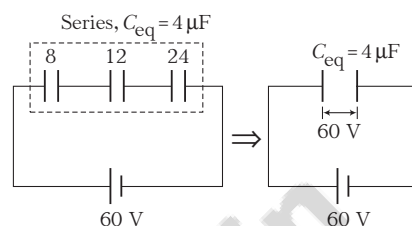
$$\therefore V_1 = \left(\frac{C_2}{C_1 + C_2} \right) V = \left(\frac{3}{2+3} \right) (100) = 60 \text{ V}$$

$$\text{and } V_2 = V - V_1 = 100 - 60 = 40 \text{ V}$$

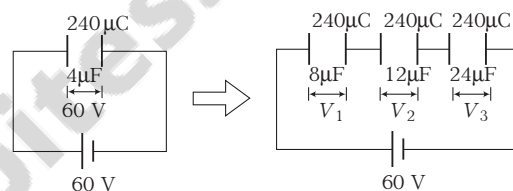
Example 2.64 Find the potential difference and charge on each capacitor.



Sol.



Charge on each capacitor, $Q = C_{\text{eq}} V = 4 \times 60 = 240 \mu\text{C}$
Now, the individual potential can be shown as



In series, charge on individual capacitors and equivalent capacitor is same and equal to $240 \mu\text{C}$.

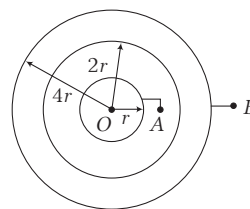
$$V_1 = \frac{240}{8} = 30 \text{ V}$$

$$V_2 = \frac{240}{12} = 20 \text{ V}$$

$$V_3 = \frac{240}{24} = 10 \text{ V}$$

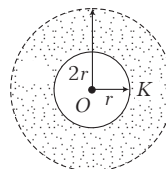
Example 2.65 Find the equivalent capacitance between A and B.

(i)

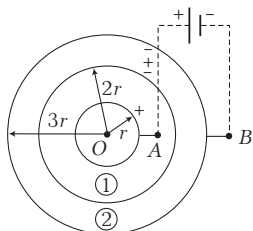


Three conducting concentric shells of radii r , $2r$ and $4r$.

(ii)



An isolated ball-shaped conductor of radius r surrounded by an adjacent concentric layer of dielectric (K) and outer radius $2r$.

Sol. (i)

The capacitance of the two capacitors can be calculated as

$$\text{Capacitor 1, } C_1 = \frac{4\pi\epsilon_0 r \cdot 2r}{(2r - r)} = 8\pi\epsilon_0 r$$

$$\text{Capacitor 2, } C_2 = \frac{4\pi\epsilon_0 2r \cdot 3r}{(3r - 2r)} = 24\pi\epsilon_0 r$$

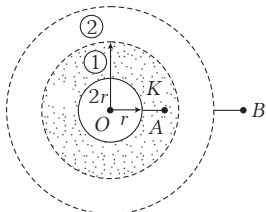
Connect a battery between A and B, and check polarity, +, -, +, -.

Hence, C_1 and C_2 are in series.

$\therefore$ Equivalent capacitance between A and B,

$$C = \frac{C_1 C_2}{C_1 + C_2} = 6\pi\epsilon_0 r$$

(ii)



The capacitance of two capacitors can be calculated as

$$\text{Capacitor 1, } C_1 = \frac{4\pi\epsilon_0 K \cdot r \cdot 2r}{2r - r} = 8\pi\epsilon_0 K r$$

Capacitor 2, (Isolated sphere of radius $2r$ and outer radius infinite)

$$C_2 = 4\pi\epsilon_0 2r = 8\pi\epsilon_0 r$$

C_1 and C_2 in series, as shown in (ii),

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8\pi\epsilon_0 K r}{K + 1}$$

2. Parallel grouping

In a parallel connection, potential difference across each capacitor remains same and equal to the applied potential difference but charge gets distributed, i.e. $q = q_1 + q_2 + q_3$ where, q_1 , q_2 and q_3 are the charges across capacitor C_1 , C_2 and C_3 , respectively and q is the net charge flowing across the circuit.

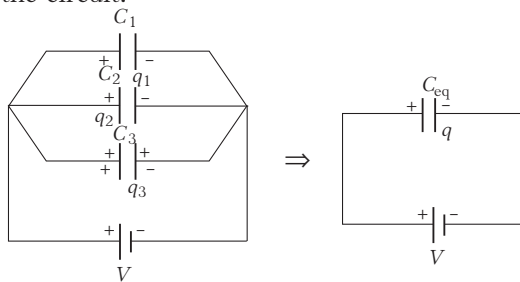


Fig. 2.52

The arrangement shown in figure is called a parallel connection. Here, the equivalent capacitance is equal to

$$C_{eq} = C_1 + C_2 + C_3$$

Following points are important in case of parallel combination of capacitors

- (i) In parallel combination, charge and energy distributes in the ratio of their capacitance, i.e. $q \propto C$ and $U \propto C$. If two capacitors having capacitance C_1 and C_2 respectively are connected in parallel, then

$$C_{eq} = C_1 + C_2$$

$$q_1 = \left(\frac{C_1}{C_1 + C_2} \right) q \text{ and } q_2 = \left(\frac{C_2}{C_1 + C_2} \right) q$$

- (ii) If n identical capacitors are connected in parallel, then equivalent capacitance is $C_{eq} = nC$ and charge on each capacitor is $q' = \frac{q}{n}$.

- (iii) If n identical plates are arranged such that even numbered of plates are connected together and odd number of plates are connected together as shown in figure, then $(n-1)$ capacitors are formed and will be in parallel.

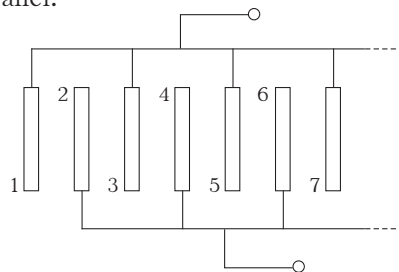


Fig. 2.53

The capacitance of each capacitor is $\frac{\epsilon_0 A}{d}$ and the equivalent capacitance, $C_{eq} = \frac{(n-1) \epsilon_0 A}{d}$.

- (iv) If there are n identical capacitors, then $\frac{C_P}{C_S} = n^2$

where, C_P and C_S are the equivalent capacitances when n capacitors are connected in parallel and series, respectively. Below are some of the examples of combination of capacitors arranged in parallel.

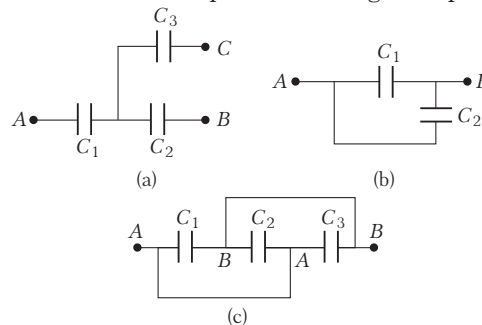
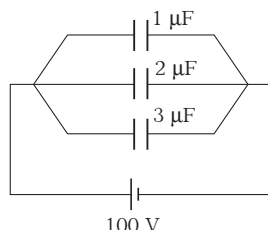


Fig. 2.54

Example 2.66 In the circuit shown in figure, find

- (i) the equivalent capacitance and
(ii) the charge stored in each capacitor.



Sol. (i) The capacitors are in parallel. Hence, the equivalent capacitance is,

$$C = C_1 + C_2 + C_3 \\ = (1 + 2 + 3) = 6 \mu\text{F}$$

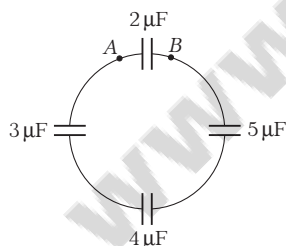
(ii) Total charge drawn from the battery,

$$q = CV = 6 \times 100 \mu\text{C} \\ = 600 \mu\text{C}$$

This charge will be distributed in the ratio of their capacities. Hence,

$$q_1 : q_2 : q_3 = C_1 : C_2 : C_3 = 1 : 2 : 3 \\ \therefore q_1 = \left(\frac{1}{1 + 2 + 3} \right) \times 600 = 100 \mu\text{C} \\ q_2 = \left(\frac{2}{1 + 2 + 3} \right) \times 600 = 200 \mu\text{C} \\ \text{and } q_3 = \left(\frac{3}{1 + 2 + 3} \right) \times 600 = 300 \mu\text{C}$$

Example 2.67 For the following arrangement, find the equivalent capacitance between A and B.



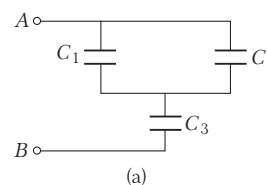
Sol. Here, the 3 μF, 4 μF and 5 μF capacitors are in series.

$$\therefore C_{\text{eq}} = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} \right)^{-1} \\ = \frac{60}{12 + 20 + 15} = \frac{60}{47} \mu\text{F}$$

Now, 2 μF and $\frac{60}{47} \mu\text{F}$ are in parallel, so we have

$$C_{\text{eq}} = C_1 + C_2 = \frac{154}{47} \mu\text{F}$$

Example 2.68



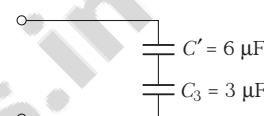
(i) Find the equivalent capacitance of the combination shown in the Fig. (a), when $C_1 = 2.0 \mu\text{F}$, $C_2 = 4.0 \mu\text{F}$ and $C_3 = 3.0 \mu\text{F}$.

(ii) The input terminals A and B in Fig. (a) are connected to a battery of 12 V. Find the potential and the charge of each capacitor.

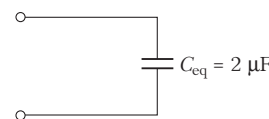
Sol. (i) C_1 and C_2 are in parallel, hence their equivalent capacitance is

Fig. (b) shows the combination of C' and C_3 in series.

$$C' = C_1 + C_2 = 2.0 + 4.0 = 6.0 \mu\text{F}$$



(b) C_1 and C_2 are replaced by C'



(c) C' and C_3 are replaced by C_{eq}

The final equivalent capacitance, shown in Fig. (c) is given by

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C'} + \frac{1}{C_3} = \frac{1}{6.0} + \frac{1}{3.0} = \frac{1}{2.0}$$

or $C_{\text{eq}} = 2.0 \mu\text{F}$

(ii) To find the charge and potential difference, we retrace the path to the original in Fig. (a).

The charge supplied by the battery is

$$q = C_{\text{eq}} V, \text{ when the inputs are joined to a } V \text{ volt battery.} \\ = (2.0)(12.0) = 24.0 \mu\text{C}$$

The charge on each capacitor in series in Fig. (b) is

$$q_3 = q' = q = 24.0 \mu\text{C}$$

$$\text{So, } V_3 = \frac{q_3}{C_3} = \frac{24.0 \mu\text{C}}{3.0 \mu\text{F}} = 8 \text{ V}$$

The potential difference across C' in Fig. (b) is

$$V' = \frac{q'}{C'} = \frac{24.0 \mu\text{C}}{6.0 \mu\text{F}} = 4 \text{ V}$$

The same potential difference V' appears across C_1 and C_2 in Fig. (a) and hence the charges on them are

$$q_1 = C_1 V_1 = (2.0 \mu\text{F})(4.0 \text{ V}) = 8.0 \mu\text{C}$$

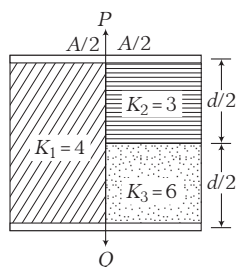
$$\text{and } q_2 = C_2 V_2 = (4.0 \mu\text{F})(4.0 \text{ V}) = 16.0 \mu\text{C}$$

Thus, we have $V_1 = V_2 = 4.0 \text{ V}$, $V_3 = 8 \text{ V}$

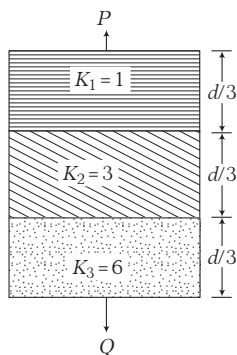
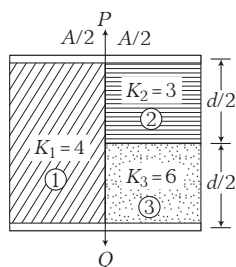
$$\text{and } q_1 = 8.0 \mu\text{C}, q_2 = 16.0 \mu\text{C}, q_3 = 24.0 \mu\text{C}$$

Example 2.69 Find the equivalent capacitance between P and Q . There, A is area of each plate and d is separation between plates.

(i)



(ii)

**Sol.** (i)

$$\text{Capacitance, } C_1 = \frac{K_1 \frac{A}{2} \epsilon_0}{d} = \frac{4 \frac{A}{2} \epsilon_0}{d} = \frac{2A\epsilon_0}{d}$$

$$\left(\because C = \frac{KA\epsilon_0}{d} \right)$$

$$\text{Capacitance, } C_2 = \frac{K_2 \frac{A}{2} \epsilon_0}{\frac{d}{2}} = \frac{3A\epsilon_0}{d}$$

$$\text{Capacitance, } C_3 = \frac{K_3 \frac{A}{2} \epsilon_0}{\frac{d}{2}} = \frac{6A\epsilon_0}{d}$$

C_2 and C_3 are in series,

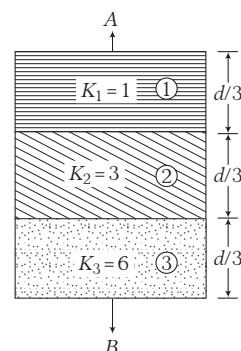
$$C' = \frac{C_2 C_3}{C_2 + C_3} = \frac{2A\epsilon_0}{d}$$

and C' and C_1 are in parallel.

Hence, equivalent capacitance,

$$C_{\text{eq}} = C_1 + C' = \frac{2A\epsilon_0}{d} + \frac{2A\epsilon_0}{d} = \frac{4A\epsilon_0}{d}$$

$$(ii) \text{ Capacitance, } C_1 = \frac{K_1 A \epsilon_0}{d/3} = \frac{3A\epsilon_0}{d}$$



$$\text{Capacitance, } C_2 = \frac{K_2 A \epsilon_0}{d/3} = \frac{9A\epsilon_0}{d}$$

$$\text{Capacitance, } C_3 = \frac{K_3 A \epsilon_0}{d/3} = \frac{18A\epsilon_0}{d}$$

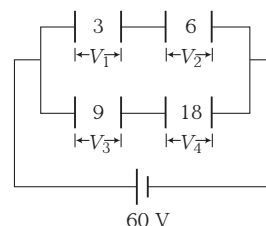
C_1 , C_2 and C_3 are in series,

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Putting the values of C_1 , C_2 and C_3 , we get

$$\text{Equivalent capacitance, } C_{\text{eq}} = \frac{2A\epsilon_0}{d}$$

Example 2.70 Find the potential difference and charge on each capacitances. All capacitances are in μF .



Sol. Here, two branches are in parallel. Potential difference across each branch is 60 V. There are two capacitors in series, use direct formulae as explained earlier.

$$V_1 = \left(\frac{6}{3+6} \right) \times 60 = 40 \text{ V}$$

$$\text{and } V_2 = 60 - V_1 = 20 \text{ V}$$

$$\text{Similarly, } V_3 = \left(\frac{18}{9+18} \right) \times 60 = 40 \text{ V}$$

$$\text{and } V_4 = 60 - V_3 = 20 \text{ V}$$

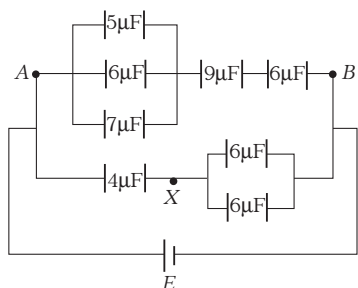
$$\text{Charge on } 3\mu\text{F capacitor, } q_1 = 3 \times 40 = 120 \mu\text{C}$$

$$\text{Charge on } 6\mu\text{F capacitor, } q_2 = 6 \times 20 = 120 \mu\text{C}$$

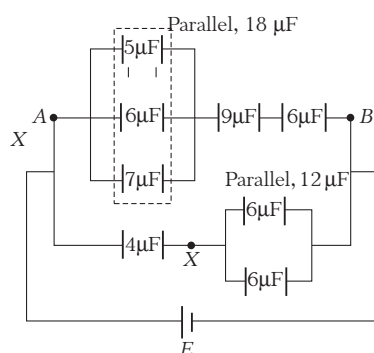
$$\text{Charge on } 9\mu\text{F capacitor, } q_3 = 9 \times 40 = 360 \mu\text{C}$$

$$\text{and charge on } 18\mu\text{F capacitor, } q_4 = 18 \times 20 = 360 \mu\text{C}$$

Example 2.71 If charge on $5\ \mu\text{F}$ capacitor is $50\ \mu\text{C}$, then find the potential difference on $4\ \mu\text{F}$ and emf of battery.

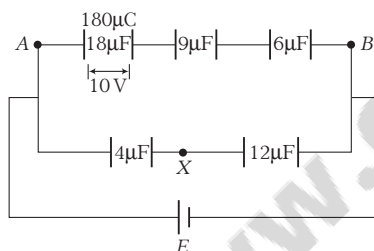


Sol. The equivalent circuit can be drawn as

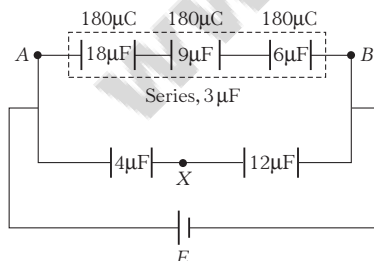


Step I Charge on $5\ \mu\text{F}$ capacitor is $50\ \mu\text{C}$, hence potential difference across it is $10\ \text{V}$. ($\because V = Q/C$)

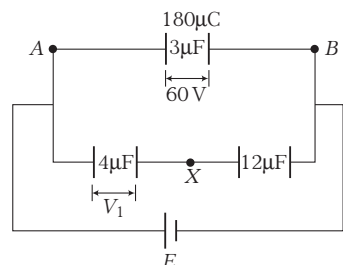
Step II



Step III



Step IV

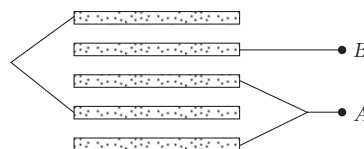


$$\text{emf of battery} = \frac{180}{3} = 60\ \text{V}$$

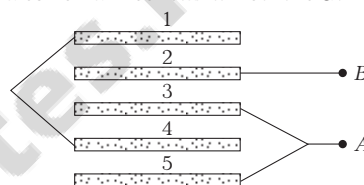
$$\therefore V_1 = \left(\frac{12}{4+12} \right) \times 60 = 45\ \text{V}$$

Potential difference on $4\ \mu\text{F}$ capacitor is $45\ \text{V}$.

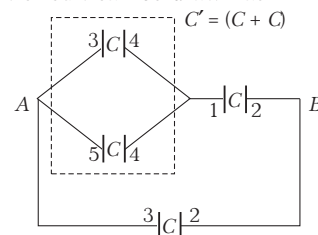
Example 2.72 In the following figure, area of each plate is A and d is separation between adjacent plates. Find the capacitance of system between points A and B .



Sol. $\because$ Area and distance are constant for each capacitor, so capacitance also remain constant. Let it is C .

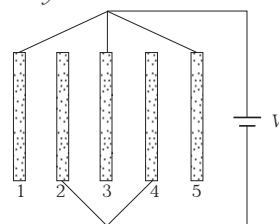


The equivalent circuit can be drawn as

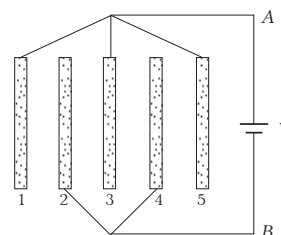


$$\text{Hence, equivalent capacitance is } C_{AB} = \frac{(C+C) \times C}{(C+C)+C} + C = \frac{5C}{3}$$

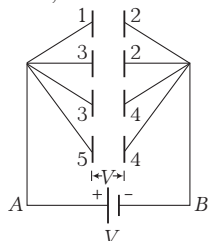
Example 2.73 Five identical capacitor plates each of area A are arranged, such that adjacent plates are at a distance d apart. The plates are connected to a source of emf V as shown below. What is the magnitude and nature of charge on plates 1 and 3, respectively?



Sol.



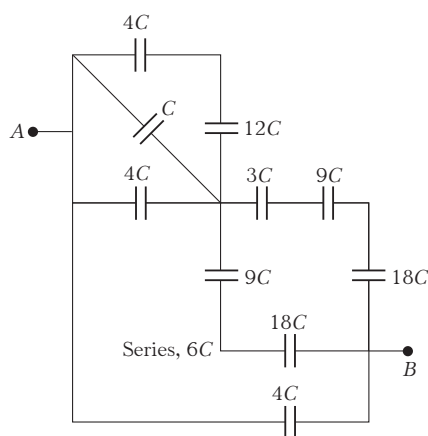
First, rearrange the plates,



$$\text{Charge on plate 1, } q = CV = \frac{A\epsilon_0 V}{d}$$

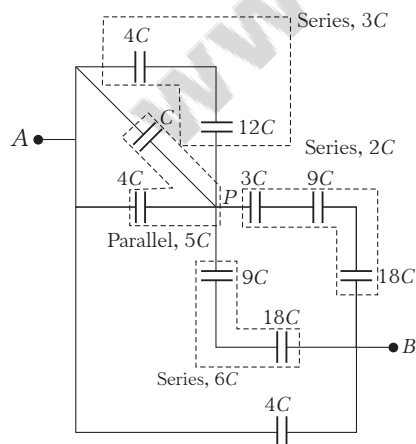
$$\text{Charge on plate 3, } q' = CV + CV = 2CV = \frac{2A\epsilon_0 V}{d}$$

Example 2.74 Find the equivalent capacitance between A and B.



Sol. In series, capacitance, $C_1 = \frac{4C \times 12C}{4C + 12C} = 3C$

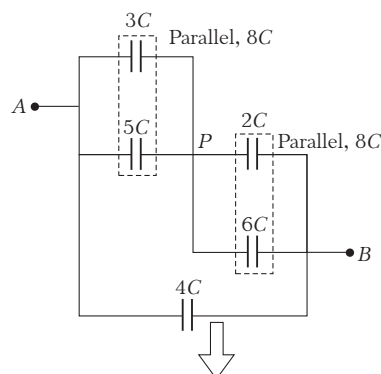
$$\text{In series, } \frac{1}{C_2} = \frac{1}{3C} + \frac{1}{9C} + \frac{1}{18C} = \frac{9}{18C} \Rightarrow C_2 = 2C$$



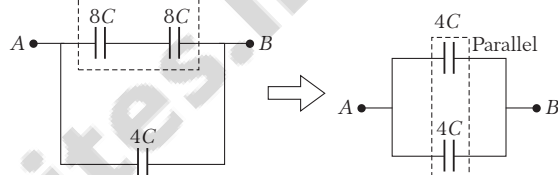
$$\text{In parallel, } C_3 = C + 4C = 5C$$

$$\text{In series, capacitance, } C_4 = \frac{9C \times 18C}{9C + 18C} = 6C$$

Now, the circuit becomes,



$$\text{Series, } \frac{8C}{2} = 4C$$



The equivalent capacitance between A and B,

$$C_{eq} = 4C + 4C = 8C$$

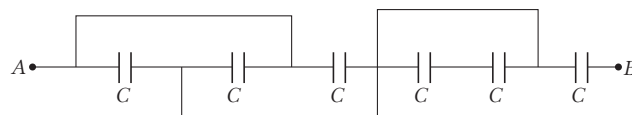
Special method to solve combination of capacitors

1. Method of same potential

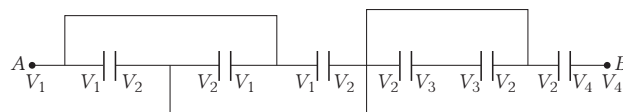
Give any arbitrary potentials ($V_1, V_2, \dots$, etc.) to all terminals of capacitors. But notice that the points connected directly by a conducting wire will have the same potential.

The capacitors having the same potential difference (PD) are in parallel. Make a table corresponding to the figure. Now corresponding to this table a simplified figure can be formed and from this figure C_{eq} can be calculated.

Example 2.75 Find equivalent capacitance between points A and B shown in figure.



Sol. By, giving arbitrary potentials to terminals, we get the following arrangement of potentials.

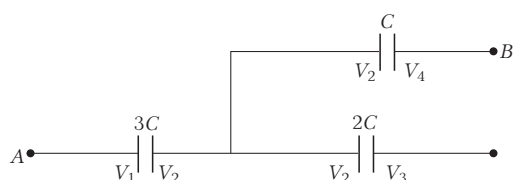


Three capacitors have potential difference, $V_1 - V_2$. So, they are in parallel. Their equivalent capacitance is $3C$.

Two capacitors have potential difference, $V_2 - V_3$. So, their equivalent capacitance is $2C$ and lastly there is one capacitor across which potential difference is $V_2 - V_4$. So, let us make a table corresponding to this information.

Potential difference	Capacitance
$V_1 - V_2$	$3C$
$V_2 - V_3$	$2C$
$V_2 - V_4$	C

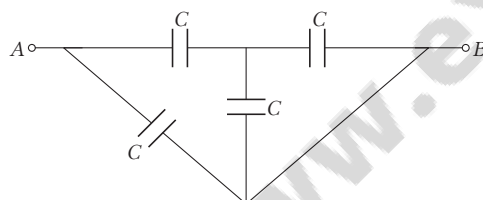
Now corresponding to this table, we make a simple figure as shown in figure.



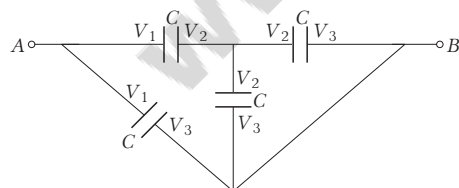
As we have to find the equivalent capacitance between points A and B, across which potential difference is $V_1 - V_4$. From the simplified figure, we can see that the capacitor of capacitance $2C$ is out of the circuit points A and B as shown. Now, $3C$ and C are in series and their equivalent capacitance is

$$C_{eq} = \frac{(3C)(C)}{3C + C} = \frac{3}{4}C$$

Example 2.76 Find equivalent capacitance between points A and B.



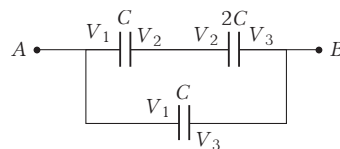
Sol. The circuit can be drawn as



Two capacitors have potential difference $V_2 - V_3$, so their equivalent capacitance is $2C$. Let us make a corresponding table based on the potential each capacitor is having.

Potential difference	Capacitance
$V_1 - V_2$	C
$V_2 - V_3$	$2C$
$V_1 - V_3$	C

Now corresponding to this table, we make a simple figure as shown below.

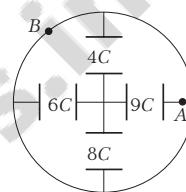


From the figure we can see that C and $2C$ are connected in series.

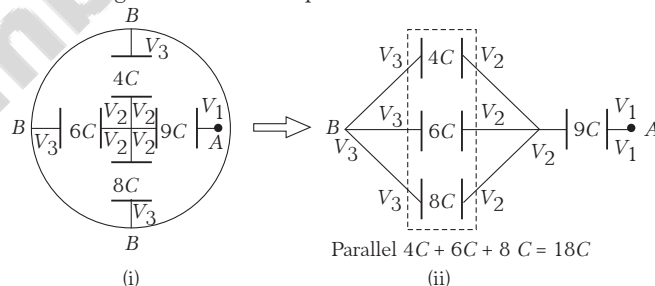
$$\therefore C' = \frac{(C)(2C)}{C + 2C} = \frac{2}{3}C$$

This combination is connected in parallel with C . So, net capacitance of the given circuit is $C_{net} = C + \frac{2}{3}C = \frac{5}{3}C$

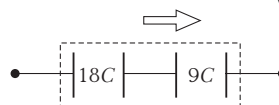
Example 2.77 Find the equivalent capacitance between A and B.



Sol. As, we have solved in above examples, we can solve here also using the same technique.



$$\text{Parallel } 4C + 6C + 8C = 18C$$



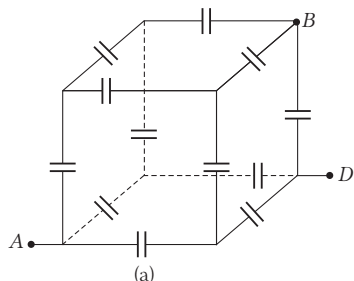
$$\text{Series capacitance} = \frac{9C \times 18C}{9C + 18C} = 6C$$

$$\Rightarrow C_{eq} = 6C$$

2. Connection removal method

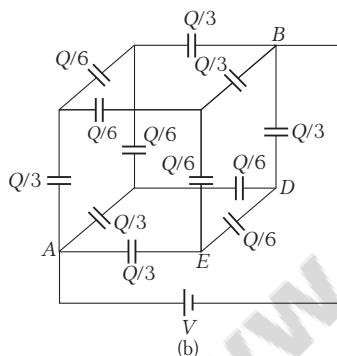
This method is useful, when the circuit diagram is symmetric except for the fact that the input and output are reversed. That is the flow of current is a mirror image between input and output above a particular axis. In such cases, some junctions are unnecessarily made. Even if, we remove that junction, there is no difference in the remaining circuit or current distribution. But after removing the junction, the problem becomes very simple. The following example illustrates the theory.

Example 2.78 Figure shows a combination of twelve capacitors, each of capacitance C , forming a cube. Find the equivalent capacitance of the combination (i) between the diagonally opposite corners A and B of the cube (ii) and between the diagonally opposite corners A and D of a face of the cube.



Sol. (i) Suppose the charge supplied by the battery is Q . This will be equally divided on the three capacitors connected to A , because on looking from A to B , three sides of the cube have identical properties. Hence, each capacitor connected to A has charge $\frac{Q}{3}$. Similarly, each capacitor connected to B also has charge $\frac{Q}{3}$. In the

Fig. (b), the charges shown are the charge on the capacitors (i.e. charges on their positive plates)

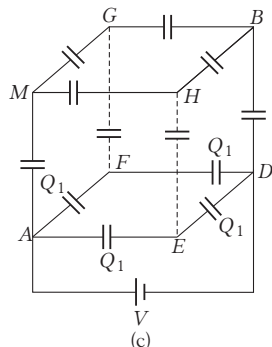


$$\text{Now, } V = (V_A - V_E) + (V_E - V_D) + (V_D - V_B)$$

$$V = \frac{Q/3}{C} + \frac{Q/6}{C} + \frac{Q/3}{C} = \frac{5Q}{6C}$$

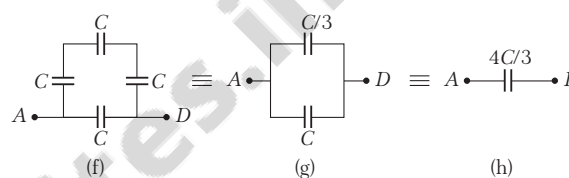
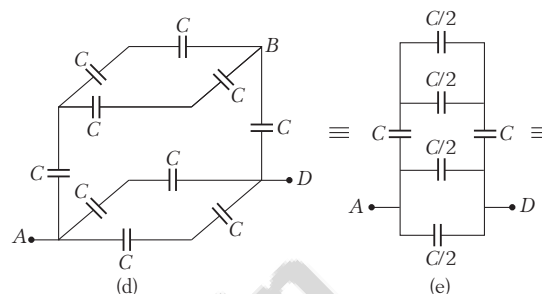
$$\therefore \text{Equivalent capacitance, } C_{eq} = \frac{Q}{V} = \frac{6}{5} C$$

(ii)



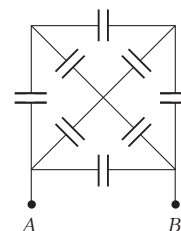
On looking from A to D into the circuit, and from D to A into the circuit, we find symmetry. Hence, the charge

on each of the four capacitors of the face $AEDF$ is same (say Q_1). It means there is no charge on the capacitors between F and G , and between E and H . Hence to find the equivalent capacitance, the combination may be taken without these two capacitors, which has been shown in the Fig. (d).

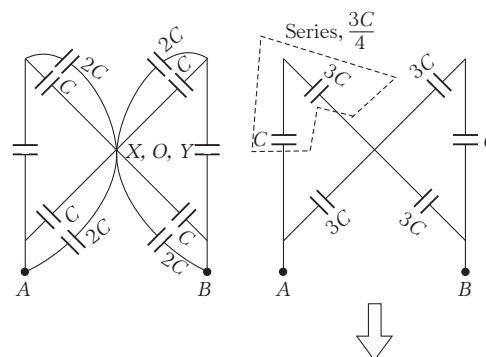
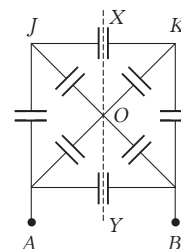


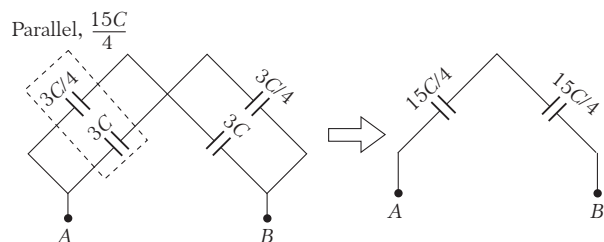
$\therefore$ Equivalent capacitance between A and D is $\frac{4C}{3}$.

Example 2.79 Find the equivalent capacitance between A and B . All the capacitors have capacitance C .



Sol.





Equivalent capacitance between A and B,

$$\frac{1}{C_{eq}} = \frac{1}{15C/4} + \frac{1}{15C/4}$$

$$\Rightarrow C_{eq} = \frac{15C}{8}$$

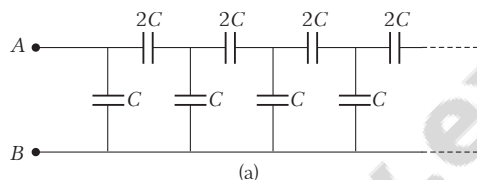
3. Infinite series problems

This consists of an infinite series of identical loops. To find C_{eq} of such a series first we consider by ourself a value (say x) of C_{eq} .

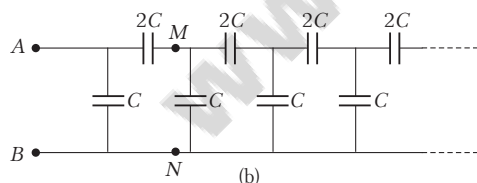
Then, we break the chain in such a manner that only one loop is left with us and in place of the remaining portion we connect a capacitor x .

Then, we find the C_{eq} and put it equal to x . With this we get a quadratic equation in x . By solving this equation we can find the desired value of x .

Example 2.80 Find the equivalent capacitance of the infinite ladder shown in the figure (a) between the points A and B.



Sol. If we look at the infinite ladder between M and N, the arrangement obtained is exactly same, as we have between A and B.



Now, suppose that the equivalent capacitance between A and B is x .

Then the equivalent capacitance between M and N is also x . Hence,

$$A \bullet \begin{array}{c} 2C \\ \parallel \\ C \end{array} \begin{array}{c} \parallel \\ x \end{array} B \bullet \Rightarrow C \parallel \begin{array}{c} 2Cx \\ 2C+x \end{array} \Rightarrow C + \frac{2Cx}{2C+x}$$

Thus,
$$x = C + \frac{2Cx}{2C+x} = \frac{2C^2 + 3Cx}{2C+x}$$

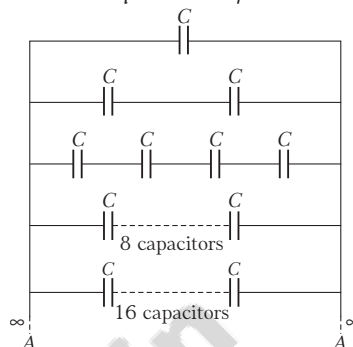
or
$$x^2 - Cx - 2C^2 = 0$$

$$\Rightarrow (x+C)(x-2C) = 0$$

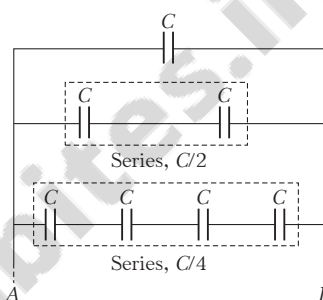
$$\therefore x \neq -C$$

$$\therefore x = 2C$$

Example 2.81 Find the equivalent capacitance between A and B.



Sol.



In second branch, the capacitances are connected in series having net capacitance $\frac{C}{2}$, in third branch, the capacitances are connected in series having net capacitance $\frac{C}{4}$,

$\therefore C, \frac{C}{2}, \frac{C}{4}, \dots, \infty$ are in parallel, therefore

$$C_{eq} = C + \frac{C}{2} + \frac{C}{4} + \dots \infty$$

$$= C \left(1 + \frac{1}{2} + \frac{1}{2^2} + \dots \infty \right) = C \left(\frac{1}{1 - \frac{1}{2}} \right) = 2C$$

4. Wheatstone's bridge circuits

Wheatstone's bridge consists of five capacitors, or a number of capacitors which can be reduced to five as shown in the following arrangement

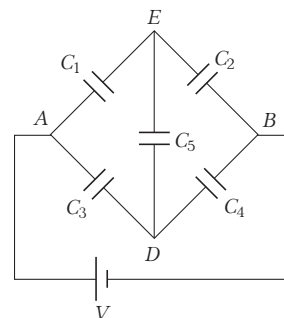


Fig. 2.55 Wheatstone's bridge

If $\frac{C_1}{C_2} = \frac{C_3}{C_4}$, bridge is said to be balanced and in that case

$$V_E = V_D \quad \text{or} \quad V_E - V_D = 0 \quad \text{or} \quad V_{ED} = 0$$

i.e., no charge is stored in C_5 . Hence, it can be removed from the circuit.

Some of the different forms of the Wheatstone bridge are given below.

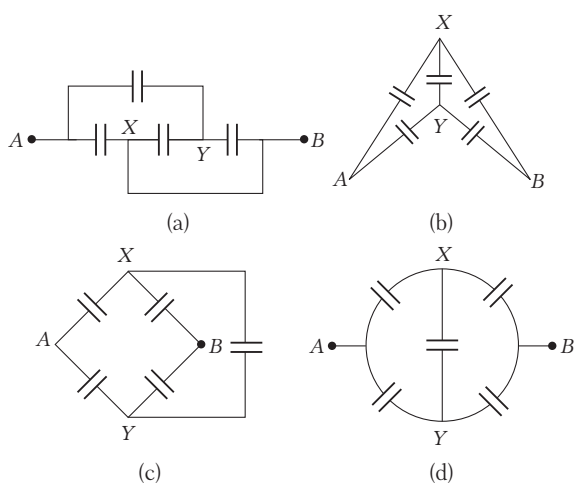
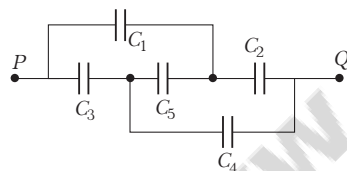
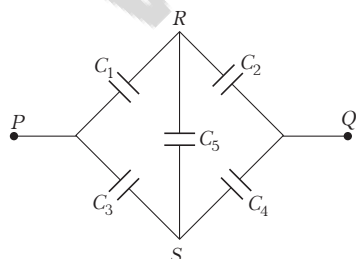


Fig. 2.56

Example 2.82 In the network of capacitors given ahead. Find the effective capacitance between the points P and Q. (Take, $C_1 = C_2 = C_3 = C_4 = 4\mu\text{F}$)



Sol. The given network consists of two closed capacitor circuits one containing C_1, C_3 and C_5 , and the other containing C_2, C_4, C_5 . Thus, C_5 is common in both. Hence, the network can be replaced as a Wheatstone bridge arrangement



C_1 and C_2 are in series between the points P and Q. Similarly, C_3 and C_4 are also in series. Suppose the equivalent capacitance of C_1 and C_2 is C' and that of C_3 and C_4 is C'' . Then, we have

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{4 \times 4}{4 + 4} = 2\mu\text{F}$$

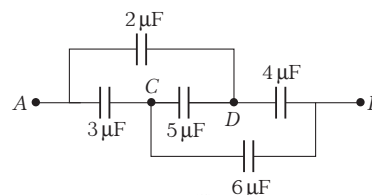
Similarly, $C'' = 2\mu\text{F}$

Now, capacitors C' and C'' are in parallel between P and Q. Hence, the equivalent capacitance is given by

$$C = C' + C'' = 2 + 2 = 4\mu\text{F}$$

Example 2.83

(i) Find the equivalent capacitance of the combination between A and B in the figure.



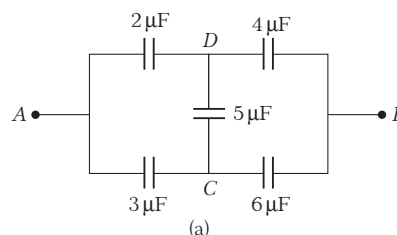
(ii) If the points A and B are maintained at 15 V and 0 V respectively, then find the charges on $3\mu\text{F}$, $4\mu\text{F}$ and $5\mu\text{F}$ capacitors.

(iii) What is the potential of the point C?

Sol. (i) The simplified form of the given combination has been shown in the Fig. (a). This forms Wheatstone's bridge.

$$\text{Here, } \frac{C_1}{C_2} = \frac{2\mu\text{F}}{4\mu\text{F}} = \frac{1}{2}$$

$$\frac{C_3}{C_4} = \frac{3\mu\text{F}}{6\mu\text{F}} = \frac{1}{2}$$



$$\text{Thus, } \frac{C_1}{C_2} = \frac{C_3}{C_4} \quad (\text{balancing condition})$$

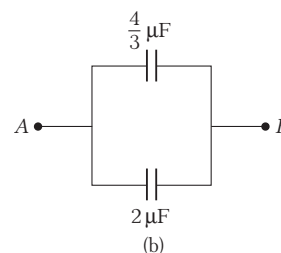
$$\text{Hence, } C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

$$= \left(\frac{2 \times 4}{2 + 4} + \frac{3 \times 6}{3 + 6} \right) = \left(\frac{4}{3} + 2 \right) = \frac{10}{3} \mu\text{F}$$

(ii) Potential difference, $V_A - V_B = 15\text{ V}$

$\therefore$ The charge on $\frac{4}{3}\mu\text{F}$ capacitor in Fig. (b) is

$$q_1 = \left(\frac{4}{3} \right) (15) = 20\mu\text{C}$$



Hence, the charge on $4\ \mu\text{F}$ capacitor in Fig. (a) is also

$$q_1 = 20\ \mu\text{C}$$

The charge on $2\ \mu\text{F}$ capacitor in Fig. (b) is

$$q_2 = (2)(15) = 30\ \mu\text{C}$$

So, the charge on the $3\ \mu\text{F}$ capacitor in Fig. (a) is also

$$q_2 = 30\ \mu\text{C}$$

Due to balanced Wheatstone's bridge, the charge on capacitor $5\ \mu\text{F}$ is zero.

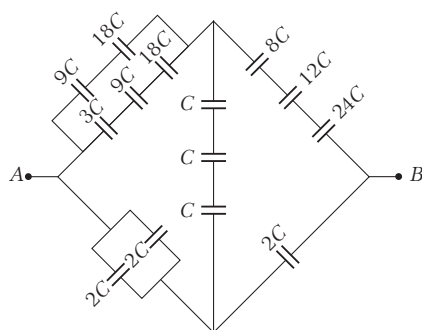
Hence, charges are $30\ \mu\text{C}$ on $3\ \mu\text{F}$ capacitor, $20\ \mu\text{C}$ on $4\ \mu\text{F}$ capacitor and zero on $5\ \mu\text{F}$ capacitor.

(iii) The potential difference across $3\ \mu\text{F}$ capacitor of Fig. (a) is

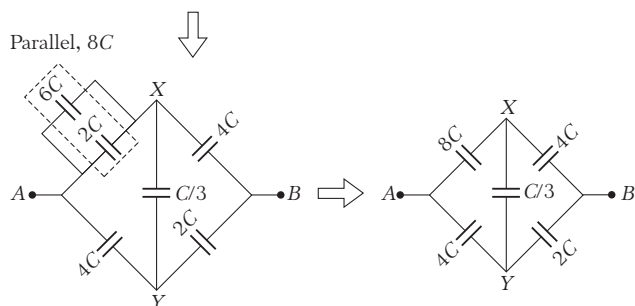
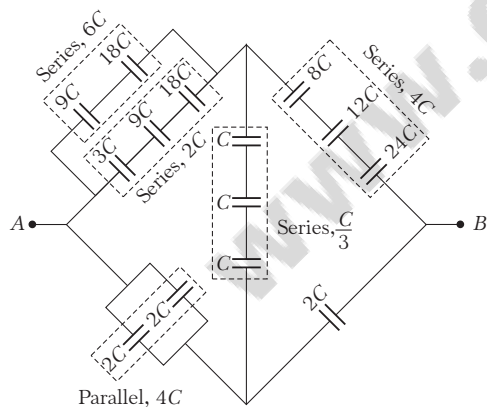
$$V_A - V_C = \frac{30}{3} = 10\ \text{V}$$

$$\begin{aligned} \text{or} \quad 15\ \text{V} - V_C &= 10\ \text{V} \\ \Rightarrow V_C &= 5\ \text{V} \end{aligned}$$

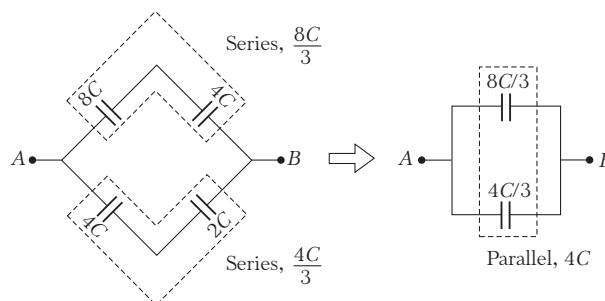
Example 2.84 Find the equivalent capacitance between A and B.



Sol.



It is a balanced Wheatstone bridge, so remove capacitor between X and Y.



The equivalent capacitance between A and B = $4C$

Kirchhoff's Law for capacitor circuits

Kirchhoff's law can be applied to a circuit containing resistances, capacitors and batteries. Its two laws are as follows

First law

This is basically the law of conservation of charge.

Following two points are important regarding the first law

- In case of a battery, both terminals of the battery supply equal amount of charge.
- In an isolated system (not connected to either of the terminals of a battery or to the earth), net charge remains constant.

For example, in the figure shown, the positive terminal of the battery supplies a positive charge ($q_1 + q_2$). Similarly, the negative terminal supplies a negative charge of magnitude ($q_3 + q_4$).

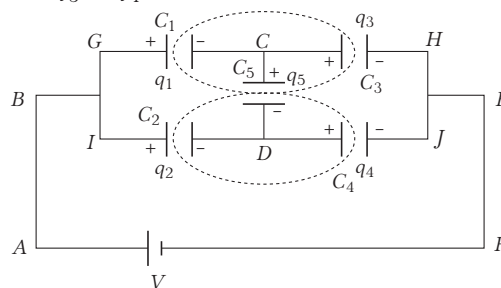


Fig. 2.57

Hence,

$$q_1 + q_2 = q_3 + q_4$$

Further, the plates enclosed by the dotted lines form an isolated system, as they are neither connected to a battery terminal nor to the earth. Initially, no charge was present on these plates. Hence, after charging net charge on these plates should be zero. Therefore,

$$q_3 + q_5 - q_1 = 0 \quad \text{and} \quad q_4 - q_2 - q_5 = 0$$

These are the three equations which can be obtained from the first law.

Second law

In a capacitor, potential drops when one moves from positive plate to the negative plate by q/C and in a battery it drops by an amount equal to the emf of the battery. Applying second law in loop $ABGHEFA$, we have

$$-\frac{q_1}{C_1} - \frac{q_3}{C_3} + V = 0$$

Similarly, the second law in loop $GCDIG$ gives the equation,

$$-\frac{q_1}{C_1} - \frac{q_5}{C_5} + \frac{q_2}{C_2} = 0$$

Use following sign convention while solving the problems on Kirchhoff's law,

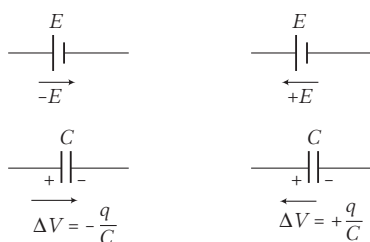
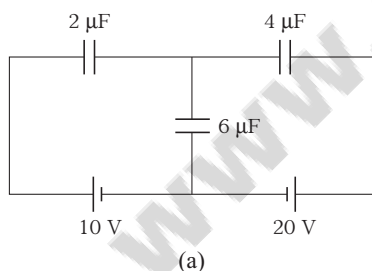


Fig. 2.58

Arrow shows the direction of current or flow of charges.

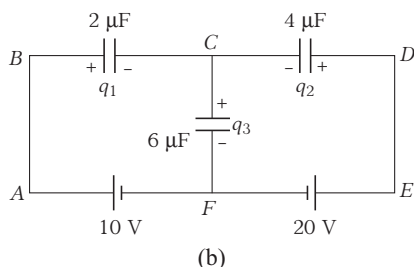
Note When an arrangement of capacitors cannot be simplified by any of the method discussed in the previous section, then we need to apply the Kirchhoff's law to solve the circuit.

Example 2.85 Find the charges on the three capacitors shown in figure.



(a)

Sol. Let the charges on three capacitors be as shown in figure.



(b)

Charge supplied by 10 V battery is q_1 and that from 20 V battery is q_2 . Then, $q_1 + q_2 = q_3$... (i)

This relation can also be obtained in a different manner. The charges on the three plates which are in contact add to zero.

Because these plates taken together form an isolated system which cannot receive charges from the batteries. Thus,

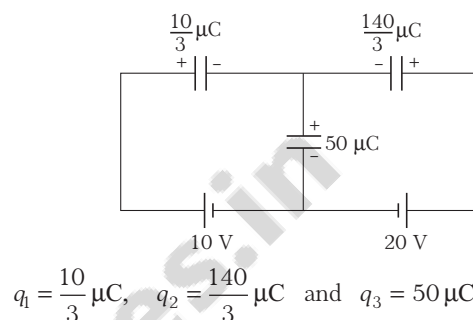
$$q_3 - q_1 - q_2 = 0 \text{ or } q_3 = q_1 + q_2$$

Applying second law in loops $BCFAB$ and $CDEFC$, we have

$$-\frac{q_1}{2} - \frac{q_3}{6} + 10 = 0 \text{ or } q_3 + 3q_1 = 60 \quad \dots(ii)$$

$$\text{and } \frac{q_2}{4} - 20 + \frac{q_3}{6} = 0 \text{ or } 3q_2 + 2q_3 = 240 \quad \dots(iii)$$

Solving the above three equations, we get



$$q_1 = \frac{10}{3} \mu\text{C}, \quad q_2 = \frac{140}{3} \mu\text{C} \text{ and } q_3 = 50 \mu\text{C}$$

Energy stored in charged capacitor

The total amount of work done in charging the capacitor is stored up in the capacitor in the form of electric potential energy. Suppose, at time t , a charge q is present on the capacitor and V is the potential of the capacitor. If dq is the amount of charge that is brought against the forces of the field done to the charge already present on the capacitor, the additional work needed will be

$$dW = (dq) V = \left(\frac{q}{C} \right) \cdot dq \quad (\text{as } V = q/C)$$

$\therefore$ Total work done to charge a capacitor to a value q_0 , can be obtained by integrating additional work from limits 0 to q_0 , we get

$$W = \frac{q_0^2}{2C}$$

$\therefore$ Energy stored by a charged capacitor,

$$U = W = \frac{q_0^2}{2C} = \frac{1}{2} CV_0^2 = \frac{1}{2} q_0 V_0$$

Thus, if a capacitor is given a charge q , then the potential energy stored in it is,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} qV$$

The above relation shows that the charged capacitor is the electrical analog of a stretched spring whose elastic

potential energy is $\frac{1}{2} kx^2$. The charge q is analogous to the

elongation x and the reciprocal of capacitance, i.e. $\frac{1}{C}$ to the force constant k .

Total energy stored in series combination or parallel combination of capacitors is equal to the sum of energies stored in individual capacitors, i.e. $U = U_1 + U_2 + U_3 + \dots$

Energy density between the plates

The energy stored per unit volume of space in a capacitor is called energy density, i.e.

$$\text{Energy density, } u = \frac{\text{Energy stored}}{\text{Volume of capacitor}} = \frac{U}{Ad} \quad \dots(i)$$

Charge on either plate of capacitor is $Q = \sigma A = \epsilon_0 EA$

Energy stored in the capacitor is

$$U = \frac{Q^2}{2C} = \frac{(\epsilon_0 EA)^2}{2 \cdot \epsilon_0 A/d} = \frac{1}{2} \epsilon_0 E^2 \cdot Ad \quad \dots(ii)$$

By, using Eqs. (i) and (ii), we get

$$\therefore u = \frac{1}{2} \epsilon_0 E^2$$

Note Change in energy on introducing a dielectric slab.

- When a dielectric slab is inserted between the plates of a charged capacitor, with battery connected to its plates. Then, the capacitance becomes K (dielectric constant) times and energy stored in the capacitor becomes KU_0 .
- When a dielectric slab is inserted between the plates of a charged capacitor and battery is disconnected. Then, the charge on the plates remains unchanged and energy stored in the capacitor becomes $\frac{U_0}{K}$, i.e. energy decreases.

Example 2.86 Calculate energy stored in a capacitor of $5\mu\text{F}$ when it is charged to a potential of 250 V ?

Sol. Given, $C = 5\mu\text{F} = 5 \times 10^{-6}\text{ F}$ and $V = 250\text{ V}$

Energy stored in a capacitor,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times (5 \times 10^{-6}) \times (250)^2 = 0.156\text{ J}$$

Example 2.87 The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm . The capacitor is charged to 400 V . How much electrostatic energy is stored in it? How much when it is filled with a dielectric medium $K = 3$ and then charged? If it is first charged as an air capacitor and then filled with the dielectric.

Sol. Capacitance of the parallel plate capacitor,

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 90 \times 10^{-4}}{2.5 \times 10^{-3}} = 3.19 \times 10^{-11}\text{ F}$$

The energy stored in the air-capacitor,

$$\begin{aligned} U_0 &= \frac{1}{2} C_0 V_0^2 = \frac{1}{2} \times (3.19 \times 10^{-11}) \times (400)^2 \\ &= 2.55 \times 10^{-6}\text{ J} \end{aligned}$$

The capacitance of the dielectric filled capacitor is

$$C = KC_0$$

$$\begin{aligned} \text{Energy, } U &= \frac{1}{2} CV_0^2 = \frac{1}{2} KC_0 V_0^2 \\ &= KU_0 = 3 \times (2.55 \times 10^{-6}) = 7.65 \times 10^{-6}\text{ J} \end{aligned}$$

If the capacitor is first charged and, then filled with the dielectric, then the charge remains constant but the potential difference between the plates decreases. The potential difference is $V = \frac{V_0}{K}$.

The new energy of capacitor,

$$\begin{aligned} U &= \frac{1}{2} CV^2 = \frac{1}{2} (KC_0) (V_0/K)^2 \\ &= \frac{1}{K} \left(\frac{1}{2} C_0 V_0^2 \right) = \frac{U_0}{K} \\ &= \frac{2.55 \times 10^{-6}\text{ J}}{3} = 8.5 \times 10^{-7}\text{ J} \end{aligned}$$

Example 2.88 Two capacitors of capacitance $C_1 = 2\mu\text{F}$ and $C_2 = 8\mu\text{F}$ are connected in series and the resulting combination is connected across 300 V . Calculate the charge, potential difference and energy stored in the capacitors separately.

Sol. If C is the equivalent capacitance, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$$

$$\therefore C = \frac{8}{5} = 1.6\mu\text{F}$$

$$\text{Charge, } q = CV = 1.6 \times 10^{-6} \times 300 = 4.8 \times 10^{-4}\text{ C}$$

$$\text{Potential across capacitance, } C_1 = V_1 = \frac{q}{C_1} = 240\text{ V}$$

$$\text{Potential across capacitance, } C_2 = V_2 = \frac{q}{C_2} = 60\text{ V}$$

Energy stored in capacitance C_1 is

$$\begin{aligned} U_1 &= \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (240)^2 \\ &= 5.76 \times 10^{-2}\text{ J} \end{aligned}$$

Energy stored in capacitance C_2 is

$$\begin{aligned} U_2 &= \frac{1}{2} C_2 V_2^2 \\ &= \frac{1}{2} \times 8 \times 10^{-6} \times (60)^2 \\ &= 1.44 \times 10^{-2}\text{ J} \end{aligned}$$

Example 2.89 The capacitance of a variable radio capacitor can be changed from 50 pF to 200 pF by turning the dial from 0° to 180° . With the dial set at 180° , the capacitor is connected to a 400 V battery. After charging, the capacitor is disconnected from the battery and dial is turned at 0° .

- What is the potential difference across the capacitor when dial reads 0° ?

(ii) How much work is required to turn the dial, if friction is neglected?

Sol. When dial reads 180° , $C = 200$ pF

When dial reads 0° , $C' = 50$ pF

(i) Since, charge remains same,

$$Q = CV = C'V' \Rightarrow 200 \times 400 = 50 \times V$$

$\therefore$ The potential difference across the capacitor,
 $V = 1600$ V

(ii) Charge, $Q = CV = 200 \times 10^{-12} \times 400 = 8 \times 10^{-8}$ C

$$\begin{aligned} W_{1 \rightarrow 2} &= U' - U = \frac{Q^2}{2C'} - \frac{Q^2}{2C} \\ W &= \frac{Q^2}{2} \left(\frac{1}{C'} - \frac{1}{C} \right) \\ &= \frac{(8 \times 10^{-8})^2}{2} \left[\frac{1}{50} - \frac{1}{200} \right] \times \frac{1}{10^{-12}} \\ &= 48 \times 10^{-6} \text{ J} = 48 \mu\text{J} \end{aligned}$$

Common potential (redistribution of charge)

When two charged conductors or capacitors are joined together by a conducting wire and having different potentials, then charge flows from capacitor at higher potential to the capacitor at lower potential. This flow of charge continues till their potential becomes equal, this equal potential is called **common potential**.

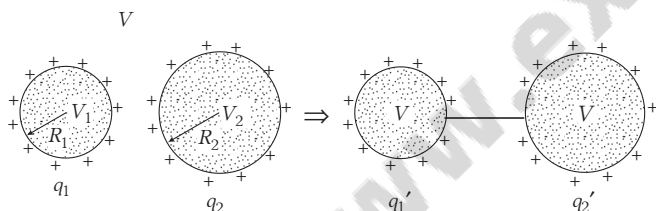


Fig. 2.59

Suppose two conductors of capacities C_1 and C_2 have charges q_1 and q_2 respectively and raised to potential V_1 and V_2 respectively are joined together by a conducting wire, then charge redistributes in these conductors in the ratio of their capacities. Charge redistributes till potential of both the conductors become equal. Thus, let q_1' and q_2' be the final charges on them, then

$$q_1' = C_1 V \text{ and } q_2' = C_2 V$$

or
$$\frac{q_1'}{q_2'} = \frac{C_1}{C_2}$$

and if they are spherical conductors, then $\frac{C_1}{C_2} = \frac{R_1}{R_2}$

$$\therefore \frac{q_1'}{q_2'} = \frac{C_1}{C_2} = \frac{R_1}{R_2}$$

where, R_1 and R_2 are radius of capacitors.

Since, the total charge is $(q_1 + q_2)$. Therefore,

$$q_1' = \left(\frac{C_1}{C_1 + C_2} \right) (q_1 + q_2)$$

and
$$q_2' = \left(\frac{C_2}{C_1 + C_2} \right) (q_1 + q_2)$$

Common potential, $V = \frac{\text{Total charge}}{\text{Total capacity}}$

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Note If the two capacitors are connected in such a way that their dissimilar plates are connected together, then their common potential is

$$V = \frac{C_1 V_1 - C_2 V_2}{C_1 + C_2}$$

Loss of energy during redistribution of charge

The initial charge on the capacitor C_1 is q and after joining it becomes q' . The transferred charge from the capacitor C_1 to the capacitor C_2 is $q - q'$.

Now,
$$q - q' = C_1 V_1 - C_1 V = C_1 \left(V_1 - \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right)$$

$$q - q' = \frac{C_1 C_2 (V_1 - V_2)}{C_1 + C_2}$$

This is the quantity of the transferred charge.

The loss of energy due to redistribution of charge is given by

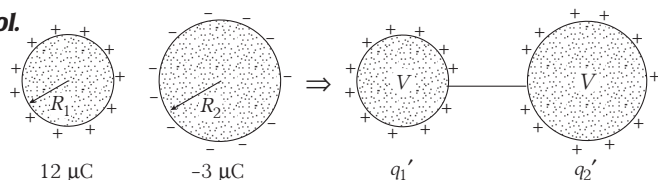
$$\Delta U = \frac{C_1 C_2}{2 (C_1 + C_2)} (V_1 - V_2)^2$$

Now as C_1 , C_2 and $(V_1 - V_2)^2$ are always positive. $U_i > U_f$, i.e. there is a decrease in energy. Hence, energy is always lost in redistribution of charge. Further,

$$\Delta U = 0, \quad \text{if } V_1 = V_2$$

this is because no flow of charge takes place when both the conductors are at same potential.

Example 2.90 Two isolated spherical conductors have radii 5 cm and 10 cm, respectively. They have charges of $12 \mu\text{C}$ and $-3 \mu\text{C}$. Find the charges after they are connected by a conducting wire. Also find the common potential after redistribution.

Sol.

Net charge = $(12 - 3) \mu\text{C} = 9 \mu\text{C}$

Charge is distributed in the ratio of their capacities (or radii in case of spherical conductors), so

$$\frac{q_1'}{q_2'} = \frac{R_1}{R_2} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore q_1' = \left(\frac{1}{1+2} \right) (9) = 3 \mu\text{C}$$

$$\text{and } q_2' = \left(\frac{2}{1+2} \right) (9) = 6 \mu\text{C}$$

Common potential,

$$\begin{aligned} V &= \frac{q_1 + q_2}{C_1 + C_2} = \frac{(9 \times 10^{-6})}{4\pi\epsilon_0(R_1 + R_2)} \\ &= \frac{(9 \times 10^{-6})(9 \times 10^9)}{(15 \times 10^{-2})} \\ &= 5.4 \times 10^5 \text{ V} \end{aligned}$$

Example 2.91 Two parallel plates capacitors A and B having capacitance of $1 \mu\text{F}$ and $5 \mu\text{F}$ are charged separately to the same potential of 100V . Now, the positive plate of A is connected to the negative plate of B and the negative plate of A to the positive plate of B. Find the final charges on each capacitor.

Sol. As, for a capacitor, $q = CV$, so initially the charge on each capacitor,

$$q_1 = C_1 V_1 = (1 \times 10^{-6}) \times 100 = 100 \mu\text{C}$$

$$\text{and } q_2 = C_2 V_2 = (5 \times 10^{-6}) \times 100 = 500 \mu\text{C}$$

Now when two capacitors are joined to each other such that positive plate of one is connected with the negative of the other, by conservation of charge

$$\begin{aligned} \Rightarrow q &= q_1 + q_2 \\ |q| &= |q_1 - q_2| \\ &= (500 - 100) \mu\text{C} \\ &= 400 \mu\text{C} \end{aligned}$$

So, common potential,

$$V = \frac{(q_1 + q_2)}{(C_1 + C_2)} = \frac{400 \times 10^{-6}}{(1 + 5) \times 10^{-6}} = \frac{200}{3} \text{ V}$$

and hence after sharing, charge on each capacitor

$$\begin{aligned} q_1' &= C_1 V = (1 \times 10^{-6}) \times \frac{200}{3} = \frac{200}{3} \mu\text{C} \\ q_2' &= C_2 V = (5 \times 10^{-6}) \times \frac{200}{3} = \frac{1000}{3} \mu\text{C} \end{aligned}$$

Example 2.92 A capacitor of capacitance $5 \mu\text{F}$ is charged to potential 20 V and then isolated. Now, an uncharged capacitor is connected in parallel to it. If the charge distributes equally on these capacitors, find total energy stored in capacitors.

Sol. Let C be unknown capacitance,

$$Q_1' = Q_2' \Rightarrow C_1 V = CV$$

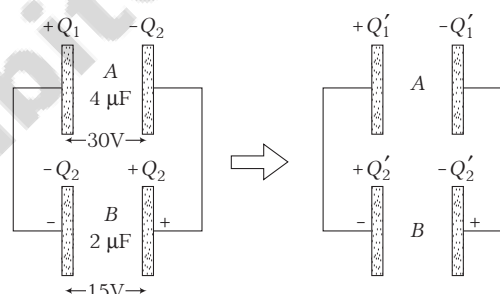
$$5V = CV \Rightarrow C = 5 \mu\text{F}$$

$$\text{Common potential, } V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{5 \times 20 + 5 \times 0}{5 + 5} = 10 \text{ V}$$

Total energy stored in capacitors,

$$\begin{aligned} U_f &= \frac{1}{2} (C_1 + C_2) V^2 = \frac{1}{2} (5 + 5) (10)^2 \times 10^{-6} \text{ J} \\ &= 500 \times 10^{-6} = 500 \mu\text{J} \end{aligned}$$

Example 2.93 A capacitor A of capacitance $4 \mu\text{F}$ is charged to 30 V and another capacitor B of capacitance $2 \mu\text{F}$ is charged to 15 V . Now, the positive plate of A is connected to the negative plate of B and negative plate of A to the positive plate of B. Find the final charge of each capacitor and loss of electrostatic energy in the process.

Sol.

Here, charge on A, $Q_1 = C_1 V_1 = 4 \times 30 = 120 \mu\text{C}$

Charge on B, $Q_2 = C_2 V_2 = 2 \times 15 = 30 \mu\text{C}$

When positive plate is connected to negative plate, we find net charge as, $|Q_1 - Q_2| = 120 - 30 = 90 \mu\text{C}$

The charge $90 \mu\text{C}$ will be redistributed in such a manner that capacitors acquire same potential.

Common potential,

$$\begin{aligned} V &= \frac{|C_1 V_1 - C_2 V_2|}{C_1 + C_2} \\ &= \frac{4 \times 30 - 2 \times 15}{4 + 2} = 15 \text{ V} \end{aligned}$$

Hence, $Q_1' = C_1 V = 4 \times 15 = 60 \mu\text{C}$

and $Q_2' = C_2 V = 2 \times 15 = 30 \mu\text{C}$

Loss of energy,

$$\begin{aligned} \Delta U &= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 + V_2)^2 \\ &= \frac{1}{2} \cdot \frac{4 \times 2}{4 + 2} (30 - 15)^2 \times 10^{-6} \text{ J} = 150 \mu\text{J} \end{aligned}$$

Van de Graaff generator

Principle It is based on the phenomenon of corona discharge, that the charge given to a hollow conductor get transferred to the outer surface to be distributed uniformly.

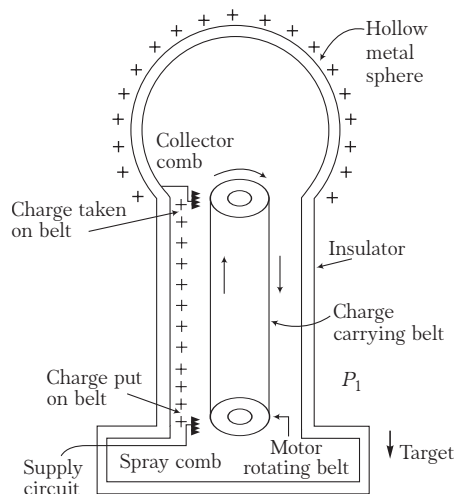


Fig. 2.60 Van de Graaff generator

Use It is a device which is used for generating high electric potential of the order of ten million volts. Such high electric potentials are needed for accelerating charged particles to very high speeds.

If we place a charge anywhere in a conductor, the charge will move to the outside surface, and the field inside the conductor will be zero.

Theory Robert van de Graaff took advantage of this concept in 1931 to build an generator, an apparatus, that produces highly energetic charged particles.

Such particles are useful for microscopic probes of matter such as cancer treatments. Van de Graaff used a device similar in concept to the apparatus shown schematically in figure.

An insulated belt (or chain) continuously bridge charge to the inside of a hollow conductor, which then moves to the outside surface of the conductor. The electric potential on the spherical conducting surface increases as

charge flows to its surface $V = \frac{q}{4\pi\epsilon_0 R}$. An ion source

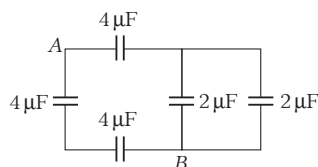
produces charged atoms whose sign are such as to be repelled from the region of high potential and thus accelerated.

Such devices are called van de Graaff accelerators and the beams they or other accelerators produce play an important role in modern technology. For example, such beams are used to make microcircuits.

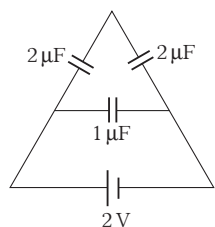
CHECK POINT 2.5

- Three capacitors each of capacitance C and of breakdown voltage V are joined in series. The capacitance and breakdown voltage of the combination will be
 - $\frac{C}{3}, \frac{V}{3}$
 - $3C, \frac{V}{3}$
 - $\frac{C}{3}, 3V$
 - $3C, 3V$
- Three condensers each of capacitance 2 F are put in series. The resultant capacitance is
 - 6 F
 - $\frac{3}{2}\text{ F}$
 - $\frac{2}{3}\text{ F}$
 - 5 F
- Two capacitors of capacitance $2\text{ }\mu\text{F}$ and $3\text{ }\mu\text{F}$ are joined in series. Outer plate of first capacitor is at 1000 V and outer plate of second capacitor is earthed (grounded). Now the potential on inner plate of each capacitor will be
 - 700 V
 - 200 V
 - 600 V
 - 400 V
- A series combination of three capacitors of capacities $1\text{ }\mu\text{F}$, $2\text{ }\mu\text{F}$ and $8\text{ }\mu\text{F}$ is connected to a battery of emf 13 V . The potential difference across the plates of $2\text{ }\mu\text{F}$ capacitor will be
 - 1 V
 - 8 V
 - 4 V
 - $\frac{13}{3}\text{ V}$
- Four capacitors of equal capacitance have an equivalent capacitance C_1 when connected in series and an equivalent capacitance C_2 when connected in parallel. The ratio $\frac{C_1}{C_2}$ is
 - $\frac{1}{4}$
 - $\frac{1}{16}$
 - $\frac{1}{8}$
 - $\frac{1}{12}$
- Three capacitors each of capacity $4\text{ }\mu\text{F}$ are to be connected in such a way that the effective capacitance is $6\text{ }\mu\text{F}$. This can be done by
 - connecting them in parallel
 - connecting two in series and one in parallel
 - connecting two in parallel and one in series
 - connecting all of them in series
- In the figure shown, the effective capacitance between the points A and B , if each has capacitance C , is
 - $2C$
 - $C/5$
 - $5C$
 - $C/2$

8. In the circuit as shown in the figure, the effective capacitance between A and B is

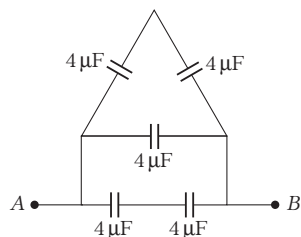


- (a) $3 \mu\text{F}$ (b) $2 \mu\text{F}$ (c) $4 \mu\text{F}$ (d) $8 \mu\text{F}$
9. The charge on anyone of the $2 \mu\text{F}$ capacitors and $1 \mu\text{F}$ capacitor will be given respectively (in μC) as



- (a) 1, 2 (b) 2, 1 (c) 1, 1 (d) 2, 2

10. Equivalent capacitance between A and B is



- (a) $8 \mu\text{F}$ (b) $6 \mu\text{F}$
(c) $26 \mu\text{F}$ (d) $\frac{10}{3} \mu\text{F}$

11. The energy stored in a capacitor of capacitance $100 \mu\text{F}$ is 50 J . Its potential difference is
(a) 50 V (b) 500 V
(c) 1000 V (d) 1500 V
12. A series combination of n_1 capacitors, each of value C_1 , is charged by a source of potential difference 4 V . When another parallel combination of n_2 capacitors, each of value C_2 , is charged by a source of potential difference V , it has the same (total) energy stored in it, as the first combination has. The value of C_2 , in terms of C_1 is, then
(a) $\frac{16C_1}{n_1 n_2}$ (b) $\frac{2C_1}{n_1 n_2}$
(c) $16 \frac{n_2}{n_1} C_1$ (d) $2 \frac{n_2}{n_1} C_1$
13. If the charge on a capacitor is increased by 2 C , then the energy stored in it increases by 21% . The original charge on the capacitor is
(a) 10 C (b) 20 C
(c) 30 C (d) 40 C
14. A capacitor of capacitance value $1 \mu\text{F}$ is charged to 30 V and the battery is then disconnected. If it is connected across a $2 \mu\text{F}$ capacitor, then the energy lost by the system is
(a) $300 \mu\text{J}$ (b) $450 \mu\text{J}$ (c) $225 \mu\text{J}$ (d) $150 \mu\text{J}$
15. A parallel plate capacitor is charged to a potential difference of 50 V . It is then discharged through a resistance for 2 s and its potential drops by 10 V . Calculate the fraction of energy stored in the capacitance
(a) 0.14 (b) 0.25 (c) 0.50 (d) 0.64

Chapter Exercises

(A) Taking it together

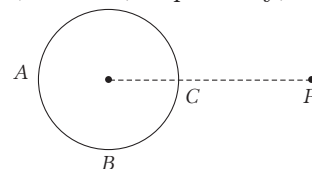
Assorted questions of the chapter for advanced level practice

- 1 Angle between equipotential surface and lines of force is
(a) zero (b) 180°
(c) 90° (d) 45°
- 2 From a point charge there is a fixed point A. At that point there is an electric field of 500 V/m and potential of 3000 V. Then, the distance of point A from the point charge is
(a) 6 m (b) 12 m
(c) 36 m (d) 144 m
- 3 A charge of 5 C is given a displacement of 0.5 m. The work done in the process is 10 J. The potential difference between the two points will be
(a) 2 V (b) 0.25 V
(c) 1 V (d) 25 V
- 4 An electron enters in higher potential region V_2 from lower potential region V_1 , then its velocity will
(a) increase
(b) change in direction but not in magnitude
(c) not change in direction of field
(d) change in direction perpendicular to field
- 5 When the separation between two charges is increased, the electric potential energy of the charges
(a) increases (b) decreases
(c) remains the same (d) may increase or decrease
- 6 If a positive charge is shifted from a low potential region to a high potential region, then the electric potential energy
(a) increases (b) decreases
(c) remains the same (d) may increase or decrease
- 7 The work done in carrying a charge of $5\text{ }\mu\text{C}$ from a point A to a point B in an electric field is 10 mJ. Then, potential difference ($V_B - V_A$) is
(a) +2 kV (b) -2 kV
(c) +200 kV (d) -200 kV
- 8 The capacitance of a capacitor does not depend on the
(a) medium between the plates
(b) size of the plates
(c) charges on the plates
(d) separation between the plates
- 9 In a charged capacitor, the energy resides in
(a) the positive charges
(b) Both the positive and negative charges
(c) the field between the plates
(d) around the edge of the capacitor plates
- 10 Which of the following is not true?
(a) For a point charge, the electrostatic potential varies as $1/r$
(b) For a dipole, the potential depends on the position vector and dipole moment vector
(c) The electric dipole potential varies as $1/r$ at large distance
(d) For a point charge, the electrostatic field varies as $1/r^2$
- 11 The energy stored in a condenser is in the form of
(a) kinetic energy (b) potential energy
(c) elastic energy (d) magnetic energy
- 12 The potential energy of a charged parallel plate capacitor is U_0 . If a slab of dielectric constant K is inserted between the plates, then the new potential energy will be
(a) $\frac{U_0}{K}$ (b) $U_0 K^2$ (c) $\frac{U_0}{K^2}$ (d) U_0^2
- 13 A charge Q is placed at the origin. The electric potential due to this charge at a given point in space is V . The work done by an external force in bringing another charge q from infinity to the point is
(a) $\frac{V}{q}$ (b) Vq (c) $V + q$ (d) V
- 14 The force between the plates of a parallel plate capacitor of capacitance C and distance of separation of the plates d with a potential difference V between the plates, is
(a) $\frac{CV^2}{2d}$ (b) $\frac{C^2V^2}{2d^2}$ (c) $\frac{C^2V^2}{d^2}$ (d) $\frac{V^2d}{C}$
- 15 A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge [NCERT Exemplar]
(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

- 16** Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately [NCERT Exemplar]
 (a) spheres (b) planes
 (c) paraboloids (d) ellipsoids
- 17** An electron enters in high potential region V_2 from lower potential region V_1 , then its velocity
 (a) will increase
 (b) will change in direction but not in magnitude
 (c) No change in direction of field
 (d) No change in direction perpendicular to field
- 18** The capacitance of a metallic sphere is $1\mu\text{F}$, if its radius is nearly
 (a) 9 km (b) 10 m
 (c) 1.11 m (d) 1.11 cm
- 19** The unit of electric field is not equivalent to
 (a) N/C (b) J/C
 (c) V/m (d) J/C-m
- 20** Electric potential at a point x from the centre inside a conducting sphere of radius R and carrying charge Q is
 (a) $\frac{1}{4\pi\epsilon_0} \frac{Q}{R}$ (b) $\frac{1}{4\pi\epsilon_0} \frac{Q}{x}$ (c) $\frac{1}{4\pi\epsilon_0} xQ$ (d) zero
- 21** If a charged spherical conductor of radius 5 cm has potential V at a point distant 5 cm from its centre, then the potential at a point distant 30 cm from the centre will be
 (a) $\frac{1}{3}V$ (b) $\frac{1}{6}V$ (c) $\frac{3}{2}V$ (d) $3V$
- 22** Two plates are at potentials -10 V and $+30\text{ V}$. If the separation between the plates be 2 cm. The electric field between them is
 (a) 2000 V/m (b) 1000 V/m
 (c) 500 V/m (d) 3000 V/m
- 23** The potential at a point due to an electric dipole will be maximum and minimum when the angles between the axis of the dipole and the line joining the point to the dipole are respectively
 (a) 90° and 180° (b) 0° and 90°
 (c) 90° and 0° (d) 0° and 180°
- 24** An electric dipole when placed in a uniform electric field E will have minimum potential energy if the dipole moment makes the following angle with E
 (a) π (b) $\pi/2$
 (c) zero (d) $3\pi/2$
- 25** How much kinetic energy will be gained by an α -particle in going from a point at 70 V to another point at 50 V ?
 (a) 40 eV (b) 40 keV (c) 40 MeV (d) 0 eV
- 26** A charged particle of mass m and charge q is released from rest in an electric field of constant magnitude E . The KE of the particle after time t is
 (a) $\frac{Eq^2m}{2t^2}$ (b) $\frac{2E^2t^2}{mq}$ (c) $\frac{E^2q^2t^2}{2m}$ (d) $\frac{Eqm}{2t}$
- 27** Two positive charges $12\mu\text{C}$ and $8\mu\text{C}$ are 10 cm apart. The work done in bringing them 4 cm closer is
 (a) 5.8 J (b) 5.8 eV
 (c) 13 J (d) 13 eV
- 28** The capacitance of the earth, viewed as a spherical conductor of radius 6408 km is
 (a) $980\mu\text{F}$ (b) $1424\mu\text{F}$
 (c) $712\mu\text{F}$ (d) $356\mu\text{F}$
- 29** A thin metal plate P is inserted between the plates of a parallel plate capacitor of capacitance C in such a way that its edges touch the two plates. The capacitance now becomes
 (a) $C/2$ (b) $2C$ (c) zero (d) ∞
- 30** A capacitor of capacity C has charge Q and stored energy is W . If the charge is increased to $2Q$, then the stored energy will be
 (a) $2W$ (b) $W/2$ (c) $4W$ (d) $W/4$
- 31** A $2\mu\text{F}$ capacitor is charged to 100 V and then its plates are connected by a conducting wire. The heat produced is
 (a) 1 J (b) 0.1 J (c) 0.01 J (d) 0.001 J
- 32** If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to
 (a) nCV^2 (b) $\frac{1}{2}nCV^2$ (c) $\frac{CV^2}{n}$ (d) $\frac{1}{2n}CV^2$
- 33** A variable condenser is permanently connected to a 100 V battery. If the capacity is changed from $2\mu\text{F}$ to $10\mu\text{F}$, then change in energy is equal to
 (a) $2 \times 10^{-2}\text{ J}$ (b) $2.5 \times 10^{-2}\text{ J}$
 (c) $3.5 \times 10^{-2}\text{ J}$ (d) $4 \times 10^{-2}\text{ J}$
- 34** Two condensers of capacity $0.3\mu\text{F}$ and $0.6\mu\text{F}$ respectively are connected in series. The combination is connected across a potential of 6 V . The ratio of energies stored by the condensers will be
 (a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{4}$ (d) 4
- 35** A capacitor of capacity C_1 , is charged by connecting it across a battery of emf V_0 . The battery is then removed and the capacitor is connected in parallel with an uncharged capacitor of capacity C_2 . The potential difference across this combination is

- (a) $\frac{C_2}{C_1 + C_2} V_0$ (b) $\frac{C_1}{C_1 + C_2} V_0$
 (c) $\frac{C_1 + C_2}{C_2} V_0$ (d) $\frac{C_1 + C_2}{C_1} V_0$
- 36** A capacitor is charged by using a battery which is then disconnected. A dielectric slab is then inserted between the plates which results in
 (a) reduction of charges on the plates and increase of potential difference across the plates
 (b) increase in the potential difference across the plates, reduction in stored energy, but no change in the charge on the plates
 (c) decrease in the potential difference across the plates, reduction in stored energy, but no change in the charge on the plates
 (d) None of the above
- 37** A parallel plate air capacitor is charged to a potential difference of V . After disconnecting the battery, distance between the plates of the capacitor is increased using an insulating handle. As a result, the potential difference between the plates
 (a) decreases (b) increases
 (c) becomes zero (d) does not change
- 38** Two concentric metallic spherical shells are given equal amount of positive charges. Then,
 (a) the outer sphere is always at a higher potential
 (b) the inner sphere is always at a higher potential
 (c) Both the spheres are at the same potential
 (d) no prediction can be made about their potentials unless the actual value of charges and radii are known
- 39** Dielectric constant of pure water is 81. Its permittivity will be
 (a) 7.16×10^{-10} MKS units (b) 8.86×10^{-12} MKS units
 (c) 1.02×10^{13} MKS units (d) Cannot be calculated
- 40** Two spherical conductors each of capacity C are charged to potential V and $-V$. These are then connected by means of a fine wire. The loss of energy is
 (a) zero (b) $\frac{1}{2} CV^2$
 (c) CV^2 (d) $2 CV^2$
- 41** Two spheres A and B of radius 4 cm and 6 cm are given charges of $80 \mu\text{C}$ and $40 \mu\text{C}$, respectively. If they are connected by a fine wire, then the amount of charge flowing from one to the other is
 (a) $20 \mu\text{C}$ from A to B (b) $20 \mu\text{C}$ from B to A
 (c) $32 \mu\text{C}$ from B to A (d) $32 \mu\text{C}$ from A to B
- 42** The electric potential difference between two parallel plates is 2000 V. If the plates are separated by 2 mm, then what is the magnitude of electrostatic force on a charge of 4×10^{-6} C located midway between the plates?
 (a) 4 N (b) 6 N
 (c) 8 N (d) 1.5×10^{-6} N
- 43** Two conducting spheres A and B of radii 4 cm and 2 cm carry charges of 18×10^{-8} statcoulomb and 9×10^{-8} statcoulomb, respectively, of positive electricity. When they are put in electrostatic contact, then the charge will
 (a) not flow at all (b) flow from A to B
 (c) flow from B to A (d) disappear
- 44** Two insulated charged spheres of radii R_1 and R_2 having charges Q_1 and Q_2 are respectively, connected to each other. There is
 (a) an increase in the energy of the system
 (b) no change in the energy of the system
 (c) always decrease in energy
 (d) a decrease in energy of the system unless $Q_1 R_2 = Q_2 R_1$
- 45** A small sphere is charged to a potential of 50 V and a big hollow sphere is charged to a potential of 100 V. Electricity will flow from the smaller sphere to the bigger one when
 (a) the smaller one is placed inside the bigger one and connected by a wire
 (b) bigger one placed by the side of the smaller one and connected by a wire
 (c) Both are correct
 (d) Both are wrong
- 46** Two identical charges are placed at the two corners of an equilateral triangle. The potential energy of the system is U . The work done in bringing an identical charge from infinity to the third vertex is
 (a) U (b) $2U$
 (c) $3U$ (d) $4U$
- 47** Four electric charges $+q$, $+q$, $-q$ and $-q$ are placed at the corners of a square of side $2L$. The electric potential at point A , midway between the two charges $+q$ and $+q$, is
 (a) $\frac{1}{4\pi\epsilon_0} \frac{2q}{L} \left(1 + \frac{1}{\sqrt{5}}\right)$
 (b) $\frac{1}{4\pi\epsilon_0} \frac{2q}{L} \left(1 - \frac{1}{\sqrt{5}}\right)$
 (c) zero
 (d) $\frac{1}{4\pi\epsilon_0} \frac{2q}{L} (1 + \sqrt{5})$
- 48** A hollow metal sphere of radius 10 cm is charged such that the potential on its surface becomes 80 V. The potential at the centre of the sphere is
 (a) 80 V (b) 800 V
 (c) 8 V (d) zero

- 49** A parallel plate capacitor has a uniform electric field E (V/m) in the space between the plates. If the distance between the plates is d (m) and area of each plate is A (m²), then the energy (joule) stored in the capacitor is
 (a) $\frac{1}{2} \epsilon_0 E^2$ (b) $\epsilon_0 E A d$
 (c) $\frac{1}{2} \epsilon_0 E^2 A d$ (d) $E^2 A d / \epsilon_0$
- 50** Charges $5 \mu\text{C}$ and $10 \mu\text{C}$ are placed 1 m apart. Work done to bring these charges at a distance 0.5 m from each other is ($k = 9 \times 10^9$ SI units)
 (a) 9×10^4 J (b) 18×10^4 J
 (c) 45×10^{-2} J (d) 9×10^{-1} J
- 51** A particle of mass 2×10^{-3} kg, charge 4×10^{-3} C enters in an electric field of 5 V/m, then its kinetic energy after 10 s is
 (a) 0.1 J (b) 1 J (c) 10 J (d) 100 J
- 52** The ionisation potential of mercury is 10.39 V. How far an electron must travel in an electric field of 1.5×10^6 V/m to gain sufficient energy to ionise mercury?
 (a) $\frac{10.39}{1.6 \times 10^{-19}}$ m (b) $\frac{10.39}{2 \times 1.6 \times 10^{-19}}$ m
 (c) $10.39 \times 1.6 \times 10^{-19}$ m (d) $\frac{10.39}{1.5 \times 10^6}$ m
- 53** 0.2 F capacitor is charge to 600 V by a battery. On removing the battery, it is connected with another parallel plate condenser of 1F. The potential decreases to
 (a) 100 V (b) 120 V (c) 300 V (d) 600 V
- 54** Work done in placing a charge of 8×10^{-18} C on a condenser of capacity 100 μF is
 (a) 16×10^{-32} J (b) 31×10^{-26} J
 (c) 4×10^{-10} J (d) 32×10^{-32} J
- 55** In a parallel plate capacitor the separation between the plates is 3 mm with air between them. Now a 1 mm thick layer of a material of dielectric constant 2 is introduced between the plates due to which the capacity increases. In order to bring its capacity to the original value the separation between the plates must be made
 (a) 1.5 mm (b) 2.5 mm
 (c) 3.5 mm (d) 4.5 mm
- 56** The electric potential at any point x , y and z in metres is given by $V = 3x^2$. The electric field at a point (2, 0, 1) is
 (a) 12 Vm^{-1} (b) -6 Vm^{-1}
 (c) 6 Vm^{-1} (d) -12 Vm^{-1}
- 57** An electron of mass m and charge e is accelerated from rest through a potential difference V in vacuum. The final speed of the electron will be
 (a) $V \sqrt{\frac{e}{m}}$ (b) $\sqrt{\frac{eV}{m}}$ (c) $\sqrt{\frac{2eV}{m}}$ (d) $\frac{2eV}{m}$
- 58** If an electron moves from rest from a point at which potential is 50 V to another point at which potential is 70 V, then its kinetic energy in the final state will be
 (a) 3.2×10^{-10} J (b) 3.2×10^{-18} J
 (c) 1 N (d) 1 dyne
- 59** The work done in bringing a 20 C charge from point A to point B for distance 0.2 m is 2 J. The potential difference between the two points will be (in volt)
 (a) 0.2 (b) 8 (c) 0.1 (d) 0.4
- 60** If 4×10^{20} eV energy is required to move a charge of 0.25 C between two points, then what will be the potential difference between them?
 (a) 178 V (b) 256 V
 (c) 356 V (d) None of these
- 61** Kinetic energy of an electron accelerated in a potential difference of 100 V is
 (a) 1.6×10^{-17} J (b) 1.6×10^{21} J
 (c) 1.6×10^{-29} J (d) 1.6×10^{-34} J
- 62** A hollow conducting sphere is placed in an electric field produced by a point charge placed at P as shown in figure. Let V_A , V_B and V_C be the potentials at points A, B and C, respectively, then

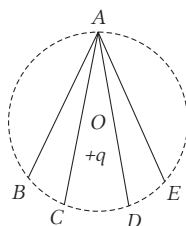


- (a) $V_C > V_B$ (b) $V_B > V_C$ (c) $V_A > V_B$ (d) $V_A = V_C$
- 63** Two unlike charges of magnitude q are separated by a distance $2d$. The potential at a point midway between them is
 (a) zero (b) $\frac{1}{4\pi\epsilon_0}$ (c) $\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{d}$ (d) $\frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{d}$
- 64** Two spheres A and B of radius a and b respectively are at same electric potential. The ratio of the surface charge densities of A and B is
 (a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $\frac{a^2}{b^2}$ (d) $\frac{b^2}{a^2}$
- 65** A capacitor of 2 μF charged to 50V is connected in parallel with another capacitor of 1 μF charged to 20V. The common potential and loss of energy will be
 (a) 40 V, 300 μJ (b) 50 V, 400 μJ
 (c) 40 V, 600 μJ (d) 50 V, 700 μJ

- 66** In the electric field of a point charge q , a certain charge is carried from point A to B, C, D and E .

Then, the work done

- (a) is least along the path AB
 (b) is least along the path AD
 (c) is zero along all the paths AB, AC, AD and AE
 (d) is least along AE



- 67** A uniform electric field having a magnitude E_0 and direction along the positive X -axis exists. If the potential V is zero at $x = 0$, then its value at $X = +x$ will be

- (a) $+xE_0$ (b) $-xE_0$ (c) $+x^2E_0$ (d) $-x^2E_0$

- 68** In a uniform electric field a charge of 3 C experiences a force of 3000 N. The potential difference between two points 1 cm apart along the electric lines of force will be

- (a) 10 V (b) 100 V (c) 30 V (d) 300 V

- 69** A particle A has charge $+q$ and a particle B has charge $+4q$ with each of them having the same mass m . When allowed to fall from rest through the same electric potential difference, the ratio of their speed

$\frac{v_A}{v_B}$ will become

- (a) 2:1 (b) 1:2 (c) 1:4 (d) 4:1

- 70** Three particles, each having a charge of $10\ \mu\text{C}$ are placed at the corners of an equilateral triangle of side 10 cm. The electrostatic potential energy of the

system is (given $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$)

- (a) zero (b) infinite (c) 27 J (d) 100 J

- 71** A mass $m = 20\ \text{g}$ has a charge $q = 3.0\ \text{mC}$. It moves with a velocity of 20 m/s and enters a region of electric field of 80 N/C in the same direction as the velocity of the mass. The velocity of the mass after 3 s in this region is

- (a) 80 m/s (b) 56 m/s
 (c) 44 m/s (d) 40 m/s

- 72** Four identical charges $+50\ \mu\text{C}$ each are placed, one at each corner of a square of side 2 m. How much external energy is required to bring another charge of $+50\ \mu\text{C}$ from infinity to the centre of the square?

- (a) 64 J (b) 41 J (c) 16 J (d) 10 J

- 73** Two equal charges q are placed at a distance of $2a$ and a third charge $-2q$ is placed at the mid-point. The potential energy of the system is

- (a) $\frac{q^2}{8\pi\epsilon_0 a}$ (b) $\frac{6q^2}{8\pi\epsilon_0 a}$ (c) $-\frac{7q^2}{8\pi\epsilon_0 a}$ (d) $-\frac{9q^2}{8\pi\epsilon_0 a}$

- 74** An alpha particle is accelerated through a potential difference of $10^6\ \text{V}$. Its kinetic energy will be

- (a) 1 MeV (b) 2 MeV (c) 4 MeV (d) 8 MeV

- 75** The ratio of momenta of an electron and an α -particle which are accelerated from rest by a potential difference of 100 V is

- (a) 1 (b) $\sqrt{\frac{2m_e}{m_\alpha}}$ (c) $\sqrt{\frac{m_e}{m_\alpha}}$ (d) $\sqrt{\frac{m_e}{2m_\alpha}}$

- 76** Two particles of masses m and $2m$ with charges $2q$ and q are placed in a uniform electric field E and allowed to move for same time. Find the ratio of their kinetic energies

- (a) 8:1 (b) 4:1 (c) 2:1 (d) 16:1

- 77** A spherical condenser has inner and outer spheres of radii a and b , respectively. The space between the two is filled with air. The difference between the capacities of two condensers formed when outer sphere is earthed and when inner sphere is earthed will be

- (a) zero (b) $4\pi\epsilon_0 a$ (c) $4\pi\epsilon_0 b$ (d) $4\pi\epsilon_0 a \left(\frac{b}{b-a} \right)$

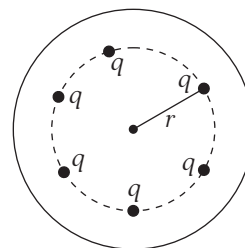
- 78** Three charges are placed at the vertices of an equilateral triangle of side 10 cm. Assume $q_1 = 1\ \mu\text{C}$, $q_2 = -2\ \mu\text{C}$ and $q_3 = 4\ \mu\text{C}$. Work done in separating the charges to infinity is

- (a) $-4.5\ \text{J}$ (b) $4.5\ \text{J}$
 (c) 45 J (d) None of these

- 79** At a distance of 1 m from a fixed charge of 1 mC, a particle of mass 2 g and charge $1\ \mu\text{C}$ is held stationary. Both the charges are placed on a smooth horizontal surface. If the particle is made free to move, then its speed at a distance of 10 m from the fixed charge will be

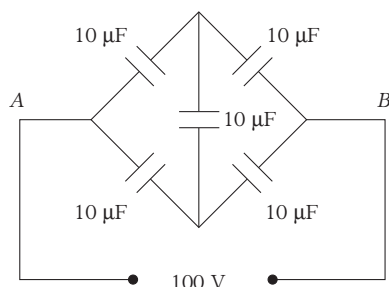
- (a) $10\ \text{ms}^{-1}$ (b) $20\ \text{ms}^{-1}$ (c) $60\ \text{ms}^{-1}$ (d) $90\ \text{ms}^{-1}$

- 80** A point charge q is surrounded by six identical charges at distance r shown in the figure. How much work is done by the force of electrostatic repulsion, when the point charge at the centre is removed to infinity?

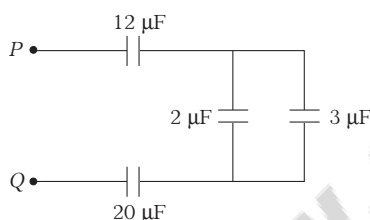


- (a) $6q/4\pi\epsilon_0 r$ (b) $6q^2/4\pi\epsilon_0 r$
 (c) $36q^2/4\pi\epsilon_0 r$ (d) Zero

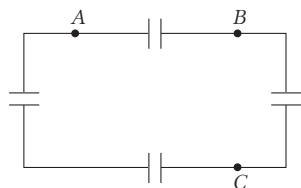
- 81** Five capacitors of $10\ \mu\text{F}$ capacity each are connected to a DC potential of $100\ \text{V}$ as shown in the adjoining figure. The equivalent capacitance between the points A and B will be equal to



- (a) $40\ \mu\text{F}$ (b) $20\ \mu\text{F}$
(c) $30\ \mu\text{F}$ (d) $10\ \mu\text{F}$
- 82** Three capacitors of capacitances $3\ \mu\text{F}$, $9\ \mu\text{F}$ and $18\ \mu\text{F}$ are connected once in series and another time in parallel. The ratio of equivalent capacitance in the two cases $\left(\frac{C_S}{C_P}\right)$ will be
- (a) 1:15 (b) 15:1
(c) 1:1 (d) 1:3
- 83** In the circuit diagram shown in the adjoining figure, the resultant capacitance between points P and Q is

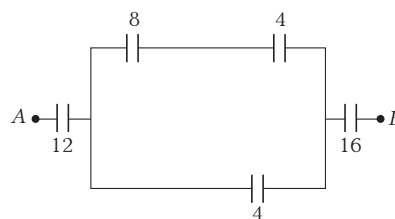


- (a) $47\ \mu\text{F}$ (b) $3\ \mu\text{F}$
(c) $60\ \mu\text{F}$ (d) $10\ \mu\text{F}$
- 84** Four capacitors each of capacity $3\ \mu\text{F}$ are connected as shown in the adjoining figure. The ratio of equivalent capacitance between A and B and between A and C will be



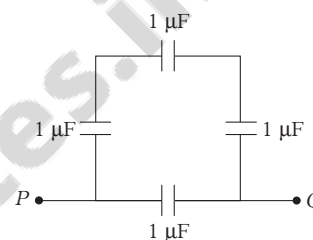
- (a) 4 : 3 (b) 3 : 4
(c) 2 : 3 (d) 3 : 2

- 85** What is the equivalent capacitance between A and B in the given figure (all are in micro farad)?



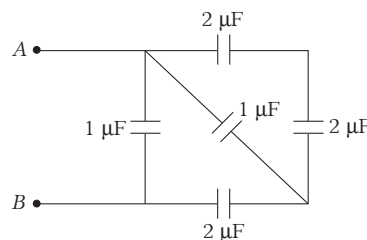
- (a) $\frac{13}{18}\ \text{F}$ (b) $\frac{48}{13}\ \text{F}$ (c) $\frac{1}{31}\ \text{F}$ (d) $\frac{240}{71}\ \text{F}$

- 86** Four capacitors are connected as shown. The equivalent capacitance between the points P and Q is



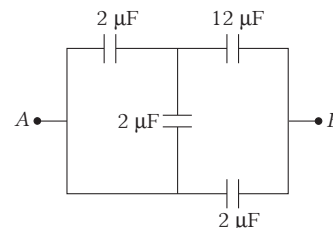
- (a) $4\ \mu\text{F}$ (b) $\frac{1}{4}\ \mu\text{F}$ (c) $\frac{3}{4}\ \mu\text{F}$ (d) $\frac{4}{3}\ \mu\text{F}$

- 87** The total capacity of the system of capacitors shown in the adjoining figure between the points A and B is



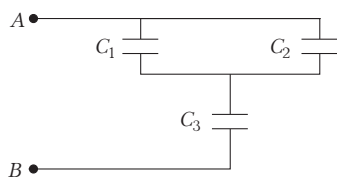
- (a) $1\ \mu\text{F}$ (b) $2\ \mu\text{F}$ (c) $3\ \mu\text{F}$ (d) $4\ \mu\text{F}$

- 88** Four capacitors are connected in a circuit as shown in the figure. The effective capacitance between points A and B will be

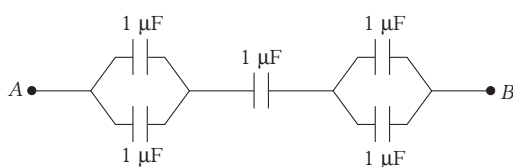


- (a) $\frac{28}{9}\ \mu\text{F}$ (b) $4\ \mu\text{F}$ (c) $5\ \mu\text{F}$ (d) $18\ \mu\text{F}$

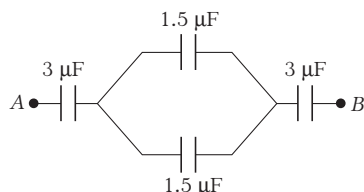
- 89** In the given network capacitance, $C_1 = 10 \mu\text{F}$, $C_2 = 5 \mu\text{F}$ and $C_3 = 4 \mu\text{F}$. What is the resultant capacitance between A and B (approximately)



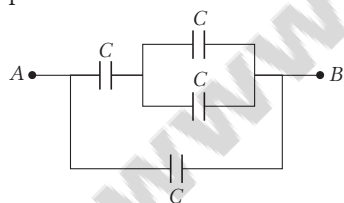
- (a) $2.2 \mu\text{F}$ (b) $3.2 \mu\text{F}$ (c) $1.2 \mu\text{F}$ (d) $4.7 \mu\text{F}$
- 90** The equivalent capacitance between points A and B is



- (a) $2 \mu\text{F}$ (b) $3 \mu\text{F}$ (c) $5 \mu\text{F}$ (d) $0.5 \mu\text{F}$
- 91** The capacitance between the points A and B in the given circuit will be



- (a) $1 \mu\text{F}$ (b) $2 \mu\text{F}$ (c) $3 \mu\text{F}$ (d) $4 \mu\text{F}$
- 92** Four equal capacitors, each of capacity C , are arranged as shown. The effective capacitance between points A and B is



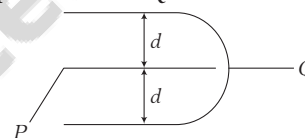
- (a) $\frac{5}{8} C$ (b) $\frac{3}{5} C$ (c) $\frac{5}{3} C$ (d) C
- 93** There are seven identical capacitors. The equivalent capacitance when they are connected in series is C . The equivalent capacitance when they are connected in parallel is
- (a) $C/49$ (b) $C/7$ (c) $7C$ (d) $49C$

- 94** The capacitance of a parallel plate capacitor is $16 \mu\text{F}$. When a glass slab is placed between the plates, the potential difference reduces to $1/8$ th of the original value. What is the dielectric constant of glass?
- (a) 4 (b) 8 (c) 16 (d) 32

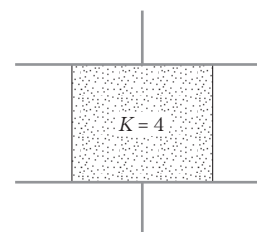
- 95** A parallel plate condenser with air between the plates possesses the capacity of 10^{-12} F . Now, the plates are removed apart, so that the separation is twice the original value. The space between the plates is filled with a material of dielectric constant 4.0. Then new value of the capacity is (in farad)
- (a) 4×10^{-12} (b) 3×10^{-12}
(c) 2×10^{-12} (d) 0.5×10^{-12}

- 96** Three condensers each of capacity C microfarad are connected in series. An exactly similar set is connected in parallel to the first one. The effective capacity of the combination is $4 \mu\text{F}$. Then, the value of C in microfarad is
- (a) 8 (b) 6
(c) 4 (d) 2

- 97** Three plates of common surface area A are connected as shown. The effective capacitance between points P and Q will be



- (a) $\frac{\epsilon_0 A}{d}$ (b) $\frac{3\epsilon_0 A}{d}$ (c) $\frac{3}{2} \frac{\epsilon_0 A}{d}$ (d) $\frac{2\epsilon_0 A}{d}$
- 98** Eight drops of mercury of equal radii combine to form a big drop. The capacitance of the bigger drop as compared to each smaller drop is
- (a) 2 times (b) 8 times (c) 4 times (d) 16 times
- 99** Consider a parallel plate capacitor of capacity $10 \mu\text{F}$ with air filled in the gap between the plates. Now, one-half of the space between the plates is filled with a dielectric of dielectric constant 4, as shown in the figure. The capacity of the capacitor changes to



- (a) $25 \mu\text{F}$ (b) $20 \mu\text{F}$ (c) $40 \mu\text{F}$ (d) $5 \mu\text{F}$
- 100** A capacitor of capacity C is connected with a battery of potential V . The distance between its plates is reduced to half, assuming that the battery remains the same. Then, the new energy given by the battery will be
- (a) $CV^2/4$ (b) $CV^2/2$
(c) $3CV^2/4$ (d) CV^2

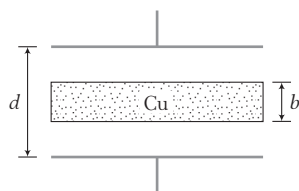
- 101** A parallel plate capacitor has plate separation d and capacitance $25 \mu\text{F}$. If a metallic foil of thickness $\frac{2}{7}d$ is introduced between the plates, the capacitance would become

(a) $25 \mu\text{F}$ (b) $35 \mu\text{F}$ (c) $\frac{125}{7} \mu\text{F}$ (d) $\frac{175}{2} \mu\text{F}$

- 102** The capacity and the energy stored in a charged parallel plate condenser with air between its plates are respectively, C_0 and W_0 . If the air is replaced by glass (dielectric constant = 5) between the plates, the capacity of the plates and the energy stored in it will respectively be

(a) $5C_0, 5W_0$ (b) $5C_0, \frac{W_0}{5}$ (c) $\frac{C_0}{5}, 5W_0$ (d) $\frac{C_0}{5}, \frac{W_0}{5}$

- 103** A slab of copper of thickness b is inserted in between the plates of parallel plate capacitor as shown in figure. The separation of the plates is d . If $b = d/2$, then the ratio of capacities of the capacitor after and before inserting the slab will be

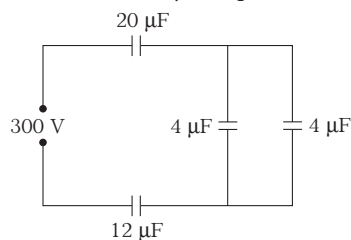


(a) $\sqrt{2} : 1$ (b) $2 : 1$
(c) $1 : 1$ (d) $1 : \sqrt{2}$

- 104** A charged capacitor when filled with a dielectric $K = 3$ has charge Q_0 , voltage V_0 and field E_0 . If the dielectric is replaced with another one having $K = 9$, the new values of charge, voltage and electric field will be respectively

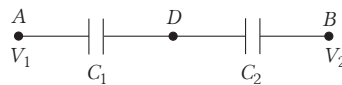
(a) $3Q_0, 3V_0, 3E_0$ (b) $Q_0, 3V_0, 3E_0$
(c) $Q_0, \frac{V_0}{3}, 3E_0$ (d) $Q_0, \frac{V_0}{3}, \frac{E_0}{3}$

- 105** In the adjoining figure, four capacitors are shown with their respective capacities and the potential difference is applied. The charge and the potential difference across the $4 \mu\text{F}$ capacitor will be



(a) $600 \mu\text{C}; 150 \text{ V}$ (b) $300 \mu\text{C}; 75 \text{ V}$
(c) $800 \mu\text{C}; 200 \text{ V}$ (d) $580 \mu\text{C}; 145 \text{ V}$

- 106** Two condensers C_1 and C_2 in a circuit are joined as shown in figure. The potential of point A is V_1 and that of B is V_2 . The potential of point D will be

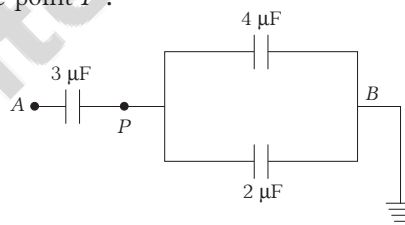


(a) $\frac{1}{2}(V_1 + V_2)$ (b) $\frac{C_2V_1 + C_1V_2}{C_1 + C_2}$
(c) $\frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ (d) $\frac{C_2V_1 - C_1V_2}{C_1 + C_2}$

- 107** Three capacitors of $2 \mu\text{F}$, $3 \mu\text{F}$ and $6 \mu\text{F}$ are joined in series and the combination is charged by means of a 24 V battery. The potential difference between the plates of the $6 \mu\text{F}$ capacitor is

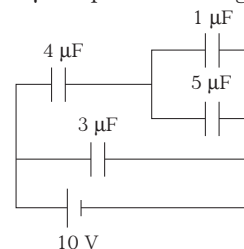
(a) 4 V (b) 6 V (c) 8 V (d) 10 V

- 108** In the figure a potential of $+1200 \text{ V}$ is given to point A and point B is earthed, what is the potential at the point P?



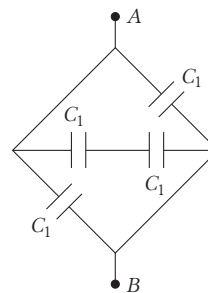
(a) 100 V (b) 200 V (c) 400 V (d) 800 V

- 109** The charge on $4 \mu\text{F}$ capacitor in the given circuit is (in μC)



(a) 12 (b) 24 (c) 36 (d) 32

- 110** Four identical capacitors are connected as shown in diagram. When a battery of 6 V is connected between A and B, then the charge stored is found to be $1.5 \mu\text{C}$. The value of C_1 is



(a) $2.5 \mu\text{F}$ (b) $15 \mu\text{F}$ (c) $1.5 \mu\text{F}$ (d) $0.1 \mu\text{F}$

- 111** A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As one goes from 0 to $3d$
- the magnitude of the electric field remains the same
 - the direction of the electric field remains the same
 - the electric potential decrease continuously
 - the electric potential increases at first, then decreases and again increases

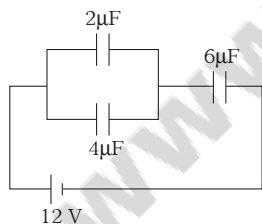
- 112** A $2\ \mu\text{F}$ condenser is charged upto 200 V and then battery is removed. On combining this with another uncharged condenser in parallel, the potential differences between two plates are found to be 40 V. The capacity of second condenser is

- $2\ \mu\text{F}$
- $4\ \mu\text{F}$
- $8\ \mu\text{F}$
- $16\ \mu\text{F}$

- 113** Consider two conductors. One of them has a capacity of 2 units and the capacity of the other is unknown. They are charged until their potentials are 4 and 5 units, respectively. The two conductors are now connected by a wire when their common potential is found to be 4.6 units. Then, the unknown capacity has the value (in the same units as above)

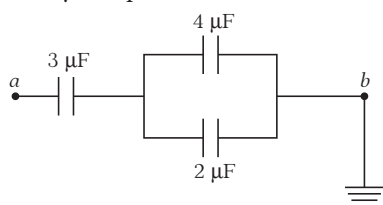
- 6
- 5
- 4
- 3

- 114** Two capacitors $2\ \mu\text{F}$ and $4\ \mu\text{F}$ are connected in parallel. A third capacitor of $6\ \mu\text{F}$ capacity is connected in series. The combination is connected across a 12 V battery. The voltage across a $2\ \mu\text{F}$ capacitor is



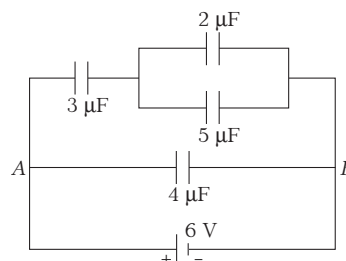
- 2 V
- 6 V
- 8 V
- 1 V

- 115** In the given circuit, if point b is connected to earth and a potential of 1200 V is given to a point a , the charge on $4\ \mu\text{F}$ capacitor is



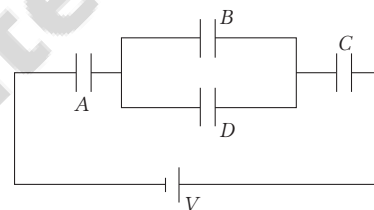
- $800\ \mu\text{C}$
- $1600\ \mu\text{C}$
- $2400\ \mu\text{C}$
- $3000\ \mu\text{C}$

- 116** A circuit is shown in the given figure. Find out the charge on the condenser having capacity $5\ \mu\text{F}$.



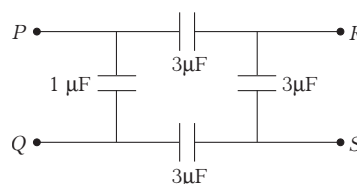
- $4.5\ \mu\text{C}$
- $9\ \mu\text{C}$
- $7\ \mu\text{C}$
- $30\ \mu\text{C}$

- 117** A potential of $V = 3000\ \text{V}$ is applied to a combination of four initially uncharged capacitors as shown in the figure. Capacitors A , B , C and D have capacitances $C_A = 6.0\ \mu\text{F}$, $C_B = 5.2\ \mu\text{F}$, $C_C = 1.5\ \mu\text{F}$ and $C_D = 3.8\ \mu\text{F}$, respectively. If the battery is disconnected, then potential difference across capacitor B is (approximately)



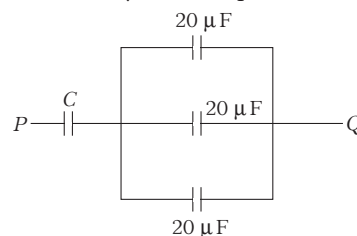
- 3000 V
- zero
- 530 V
- 350 V

- 118** Four capacitors are arranged as shown in below figure. All are initially uncharged. A 30 V battery is placed across terminal PQ to charge the capacitors and is then removed. The voltage across the terminals RS is then (in volt)



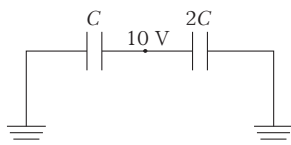
- 10
- 20
- 30
- 40

- 119** If the equivalent capacitance between points P and Q of the combination of the capacitors shown in figure below is $30\ \mu\text{F}$, the capacitor C is



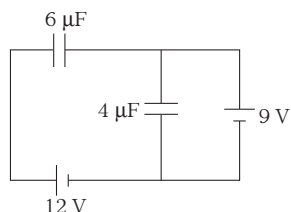
- $60\ \mu\text{F}$
- $30\ \mu\text{F}$
- $10\ \mu\text{F}$
- $5\ \mu\text{F}$

- 120** In the circuit shown in figure $C = 6 \mu\text{F}$. The charge stored in the capacitor of capacity C is



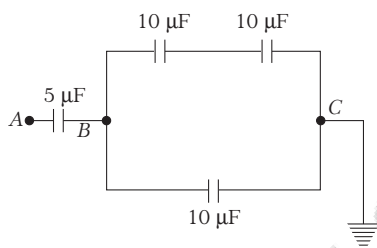
- (a) zero (b) $90 \mu\text{C}$ (c) $40 \mu\text{C}$ (d) $60 \mu\text{C}$

- 121** In the circuit shown in figure. Charge stored in $6 \mu\text{F}$ capacitor will be



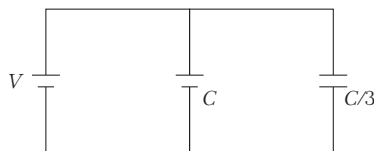
- (a) $18 \mu\text{C}$ (b) $54 \mu\text{C}$ (c) $36 \mu\text{C}$ (d) $72 \mu\text{C}$

- 122** In the given circuit, if point C is connected to the earth and a potential of $+2000 \text{ V}$ is given to the point A , the potential at B is



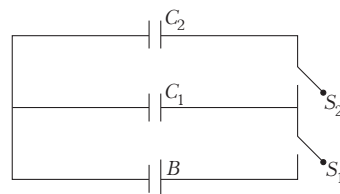
- (a) 1500 V (b) 1000 V
(c) 500 V (d) 400 V

- 123** Two condensers, one of capacity C and the other capacity $\frac{C}{3}$, are connected to a V volt battery as shown. The work done in charging fully both the condensers is



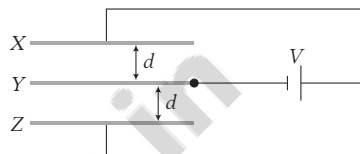
- (a) $2CV^2$ (b) $(1/4)CV^2$ (c) $(2/3)CV^2$ (d) $\frac{1}{2}CV^2$

- 124** In the circuit shown here $C_1 = 6 \mu\text{F}$, $C_2 = 3 \mu\text{F}$ and battery $B = 20 \text{ V}$. The switch S_1 is first closed. It is then opened and afterwards S_2 is closed. What is the charge finally on C_2 ?



- (a) $120 \mu\text{C}$ (b) $80 \mu\text{C}$
(c) $40 \mu\text{C}$ (d) $20 \mu\text{C}$

- 125** Consider the arrangement of three plates X , Y and Z each of area A and separation d . The energy stored when the plates are fully charged is



- (a) $\frac{\epsilon_0 A V^2}{2d}$ (b) $\frac{\epsilon_0 A V^2}{d}$ (c) $\frac{2\epsilon_0 A V^2}{d}$ (d) $\frac{3\epsilon_0 A V^2}{2d}$

- 126** Point charges $+4q$, $-q$ and $+4q$ are kept on the X -axis at points $x = 0$, $x = a$ and $x = 2a$, respectively. Then,

- (a) Only $-q$ is in stable equilibrium
(b) None of the charges are in equilibrium
(c) All the charges are in unstable equilibrium
(d) All the charges are in stable equilibrium

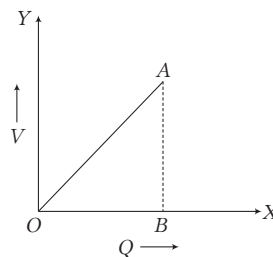
- 127** Two spherical conductors of radii 4 cm and 5 cm are charged to the same potential. If σ_1 and σ_2 be respective value of surface density of charge on both the conductors, then the ratio of σ_1/σ_2 will be

- (a) $\frac{16}{25}$ (b) $\frac{25}{10}$ (c) $\frac{4}{5}$ (d) $\frac{5}{4}$

- 128** A hollow charged metal sphere has radius r . If the potential difference between its surface and a point at a distance $3r$ from the centre is V , then electric field intensity at a distance $3r$ is

- (a) $\frac{V}{2r}$ (b) $\frac{V}{3r}$ (c) $\frac{V}{6r}$ (d) $\frac{V}{4r}$

- 129** Charge Q on a capacitor varies with voltage V as shown in the figure, where Q is taken along the X -axis and V along the Y -axis. The area of triangle OAB represents

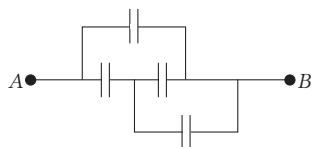


- (a) capacitance
- (b) capacitive reactance
- (c) electric field between the plates
- (d) energy stored in the capacitor

130 How many $1\mu\text{F}$ capacitors must be connected in parallel to store a charge of 1 C with a potential of 110 V across the capacitors?

- (a) 990 (b) 900 (c) 9090 (d) 909

131 In the figure below, the capacitance of each capacitor is $3\mu\text{F}$. The effective capacitance between points A and B is



- (a) $\frac{3}{4}\mu\text{F}$ (b) $3\mu\text{F}$ (c) $6\mu\text{F}$ (d) $5\mu\text{F}$

132 A $500\mu\text{F}$ capacitor is charged at the steady rate of $100\mu\text{C/s}$. How long will it take to raise the potential difference between the plates of the capacitor to 10 V ?

- (a) 5 s (b) 10 s (c) 50 s (d) 100 s

133 A ball of mass 1 g and charge 10^{-8} C moves from a point A , where potential is 600 V to the point B where potential is zero. Velocity of the ball at the point B is 20 cms^{-1} . The velocity of the ball at the point A will be

- (a) 22.8 cms^{-1} (b) 228 cms^{-1}
(c) 16.8 ms^{-1} (d) 168 ms^{-1}

134 Three capacitors of capacitances $1\mu\text{F}$, $2\mu\text{F}$ and $4\mu\text{F}$ are connected first in a series combination, and then in parallel combination. The ratio of their equivalent capacitances will be

- (a) $2:49$ (b) $49:2$ (c) $4:49$ (d) $49:4$

135 An electron moving with the speed $5 \times 10^6\text{ m/s}$ is shot parallel to the electric field of intensity $1 \times 10^3\text{ N/C}$. Field is responsible for the retardation of motion of electron. Now evaluate the distance travelled by the electron before coming to rest for an instant (mass of $e = 9 \times 10^{-31}\text{ kg}$, charge $= 1.6 \times 10^{-19}\text{ C}$).

- (a) 7 m (b) 0.7 mm
(c) 7 cm (d) 0.7 cm

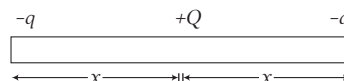
136 The electric potential at a point (x, y, z) is given by

$$V = -x^2y - xz^3 + 4$$

The electric field $\mathbf{E}$ at that point is

- (a) $\mathbf{E} = \hat{i}(2xy + z^3) + \hat{j}x^2 + \hat{k}3xz^2$
(b) $\mathbf{E} = \hat{i}2xy + \hat{j}(x^2 + y^2) + \hat{k}(3xz - y^2)$
(c) $\mathbf{E} = \hat{i}z^3 + \hat{j}xyz + \hat{k}z^2$
(d) $\mathbf{E} = \hat{i}(2xy - z^3) + \hat{j}xy^2 + \hat{k}3z^2x$

137 Three charges $-q, +Q$ and $-q$ are placed in a straight line as shown.



If the total potential energy of the system is zero, then the ratio $\frac{q}{Q}$ is

- (a) 2 (b) 5.5 (c) 4 (d) 1.5

138 The mutual electrostatic potential energy between two protons which are at a distance of $9 \times 10^{-15}\text{ m}$, in ${}_{92}\text{U}^{235}$ nucleus is

- (a) $1.56 \times 10^{-14}\text{ J}$ (b) $5.5 \times 10^{-14}\text{ J}$
(c) $2.56 \times 10^{-14}\text{ J}$ (d) $4.56 \times 10^{-14}\text{ J}$

139 Three capacitor of capacitance C (μF) are connected in parallel to which a capacitor of capacitance C is connected in series. Effective capacitance is 3.75 , then capacity of each capacitor is

- (a) $4\mu\text{F}$ (b) $5\mu\text{F}$ (c) $6\mu\text{F}$ (d) $8\mu\text{F}$

140 Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B . [NCERT Exemplar]

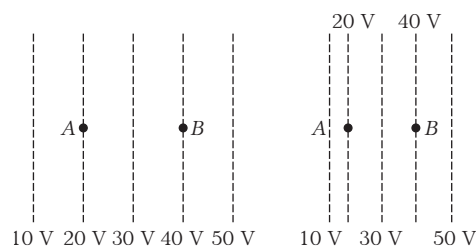


Fig. (i)

Fig. (ii)

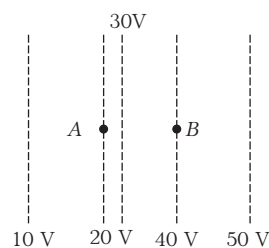


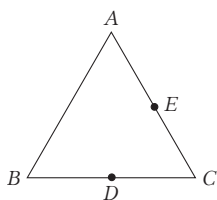
Fig. (iii)

- (a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in Fig.(i)

- 141** Two conducting spheres of radii 3 cm and 1 cm are separated by a distance of 10 cm in free space. If the spheres are charged to same potential of 10 V each, then the force of repulsion between them is

(a) $\left(\frac{1}{3}\right) \times 10^{-9}$ N (b) $\left(\frac{2}{9}\right) \times 10^{-9}$ N
 (c) $\left(\frac{1}{9}\right) \times 10^{-9}$ N (d) $\left(\frac{4}{9}\right) \times 10^{-9}$ N

- 142** Three charges, each $+q$, are placed at the corners of an isosceles triangle ABC of sides BC and AC , $2a$. D and E are the mid-points of BC and CA . The work done in taking a charge Q from D to E is



(a) zero (b) $\frac{3qQ}{4\pi\epsilon_0 a}$ (c) $\frac{3qQ}{8\pi\epsilon_0 a}$ (d) $\frac{qQ}{4\pi\epsilon_0 a}$

- 143** An electric charge 10^{-3} μC is placed at the origin (0, 0) of xy -coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and $(2, 0)$, respectively. The potential difference between the points A and B will be
- (a) 9 V (b) zero
 (c) 2 V (d) 3.5 V

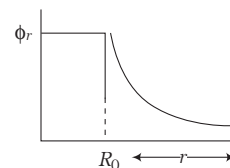
- 144** Two identical thin rings each of radius 10 cm carrying charges 10 C and 5 C are coaxially placed at a distance 10 cm apart. The work done in moving a charge q from the centre of the first ring to that of the second is

(a) $\frac{q}{8\pi\epsilon_0} \cdot \left[\frac{\sqrt{2}+1}{\sqrt{2}} \right]$ (b) $\frac{q}{8\pi\epsilon_0} \cdot \left[\frac{\sqrt{2}-1}{\sqrt{2}} \right]$
 (c) $\frac{q}{4\pi\epsilon_0} \cdot \left[\frac{\sqrt{2}+1}{\sqrt{2}} \right]$ (d) $\frac{q}{4\pi\epsilon_0} \cdot \left[\frac{\sqrt{2}-1}{\sqrt{2}} \right]$

- 145** Two equal charges q of opposite sign separated by a distance $2a$ constitute an electric dipole of dipole moment p . If P is a point at a distance r from the centre of the dipole and the line joining the centre of the dipole to this point makes an angle θ with the axis of the dipole, then the potential at P is given by ($r \gg 2a$) (where, $p = 2qa$).

(a) $V = \frac{p \sin \theta}{4\pi\epsilon_0 r^2}$ (b) $V = \frac{p \cos \theta}{4\pi\epsilon_0 r}$
 (c) $V = \frac{p \sin \theta}{4\pi\epsilon_0 r}$ (d) $V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$

- 146** The electrostatic potential ϕ_r of a spherical symmetrical system kept at origin, is shown in the adjacent figure, and given as

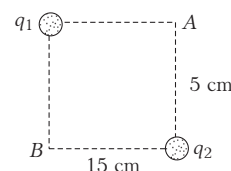


$$\phi_r = \frac{q}{4\pi\epsilon_0 r} \quad (r \geq R_0)$$

$$\phi_r = \frac{q}{4\pi\epsilon_0 R_0} \quad (r \leq R_0)$$

Which of the following option is incorrect?

- (a) For spherical region $r \leq R_0$ total electrostatic energy stored is zero.
 (b) Within $r = 2R_0$, total charge is $q/2$.
 (c) There will be no charge anywhere except at $r = R$.
 (d) Electric field is discontinuous at $r = R_0$.
- 147** Electric charges of $+10 \mu\text{C}$, $+5 \mu\text{C}$, $-3 \mu\text{C}$ and $+8 \mu\text{C}$ are placed at the corners of a square of side $\sqrt{2}$ m, the potential at the centre of the square is
- (a) 1.8 V (b) 1.8×10^6 V
 (c) 1.8×10^5 V (d) 1.8×10^4 V
- 148** The displacement of a charge Q in the electric field $\mathbf{E} = e_1 \hat{i} + e_2 \hat{j} + e_3 \hat{k}$ is $\mathbf{r} = a \hat{i} + b \hat{j}$. The work done is
- (a) $Q(ae_1 + be_2)$ (b) $Q\sqrt{(ae_1)^2 + (be_2)^2}$
 (c) $Q(e_1 + e_2)\sqrt{a^2 + b^2}$ (d) $Q(\sqrt{e_1^2 + e_2^2})(a + b)$
- 149** Two electric charges $12 \mu\text{C}$ and $-6 \mu\text{C}$ are placed 20 cm apart in air. There will be a point P on the line joining these charges and outside the region between them, at which the electric potential is zero. The distance of P from $-6 \mu\text{C}$ charge is
- (a) 0.10 m (b) 0.15 m
 (c) 0.20 m (d) 0.25 m
- 150** In the rectangle, shown below, the two corners have charges $q_1 = -5 \mu\text{C}$ and $q_2 = +2.0 \mu\text{C}$. The work done in moving a charge $+6.0 \mu\text{C}$ from B to A is (take, $1/4 \pi\epsilon_0 = 10^{10} \text{ N-m}^2/\text{C}^2$)



(a) 2.8 J (b) 3.5 J (c) 4.5 J (d) 5.6 J

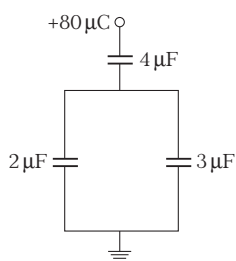
- 151** Electric potential at any point is $V = -5x + 3y + \sqrt{15}z$, then the magnitude of the electric field is

(a) $3\sqrt{2}$ (b) $4\sqrt{2}$ (c) $5\sqrt{2}$ (d) 7

- 152** A thin spherical conducting shell of radius R has a charge q . Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P , a distance $R/2$ from the centre of the shell is

(a) $\frac{(q+Q)}{4\pi\epsilon_0} \frac{2}{R}$ (b) $\frac{2Q}{4\pi\epsilon_0 R}$
 (c) $\frac{2Q}{4\pi\epsilon_0 R} - \frac{2q}{4\pi\epsilon_0 R}$ (d) $\frac{2Q}{4\pi\epsilon_0 R} + \frac{q}{4\pi\epsilon_0 R}$

- 153** In the given circuit, a charge of $+80 \mu\text{C}$ is given to the upper plate of the $4 \mu\text{F}$ capacitor. Then, in the steady state, the charge on the upper plate of the $3 \mu\text{F}$ capacitor is



(a) $+32 \mu\text{C}$ (b) $+40 \mu\text{C}$
 (c) $+48 \mu\text{C}$ (d) $+80 \mu\text{C}$

- 154** Capacitance of a parallel plate capacitor becomes $4/3$ times its original value, if a dielectric slab of thickness $t = d/2$ is inserted between the plates (d is the separation between the plates). The dielectric constant of the slab is
- (a) 8 (b) 4
 (c) 6 (d) 2

- 155** Point charge $q_1 = 2 \mu\text{C}$ and $q_2 = -1 \mu\text{C}$ are kept at points $x = 0$ and $x = 6$, respectively. Electrical potential will be zero at points
- (a) $x = 2$ and $x = 9$ (b) $x = 1$ and $x = 5$
 (c) $x = 4$ and $x = 12$ (d) $x = -2$ and $x = 2$

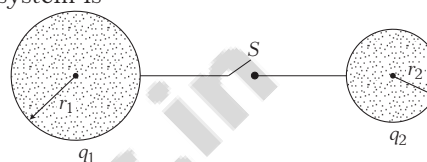
- 156** Eight small drops, each of radius r and having same charge q are combined to form a big drop. The ratio between the potentials of the bigger drop and the smaller drop is
- (a) 8:1 (b) 4:1 (c) 2:1 (d) 1:8

- 157** Eight oil drops of same size are charged to a potential of 50 V each. These oil drops are merged into one single large drop. What will be the potential of the large drop?
- (a) 50 V (b) 100 V (c) 200 V (d) 400 V

- 158** Three charges Q , $+q$ and $+q$ are placed at the vertices of an equilateral triangle. If the net electrostatic energy of the system is zero, then Q is equal to

(a) $(-q/2)$ (b) $-q$
 (c) $+q$ (d) zero

- 159** Consider a system composed of two metallic spheres of radii r_1 and r_2 connected by a thin wire and switch S as shown in the figure. Initially S is in open position, and the spheres carry charges q_1 and q_2 , respectively. If the switch is closed, the potential of the system is



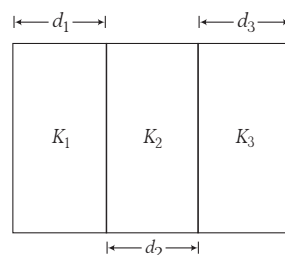
(a) $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_1 r_2}$ (b) $\frac{1}{4\pi\epsilon_0} \left(\frac{q_1 + q_2}{r_1 + r_2} \right)$
 (c) $\frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right)$ (d) $\frac{1}{4\pi\epsilon_0} \left(\frac{q_1 + q_2}{\sqrt{r_1 r_2}} \right)$

- 160** Separation between the plates of a parallel plate capacitor is d and the area of each plate is A . When a slab of material of dielectric constant K and thickness t ($t < d$) is introduced between the plates, its capacitance becomes

(a) $\frac{\epsilon_0 A}{d + t \left(1 - \frac{1}{K} \right)}$ (b) $\frac{\epsilon_0 A}{d + t \left(1 + \frac{1}{K} \right)}$
 (c) $\frac{\epsilon_0 A}{d - t \left(1 - \frac{1}{K} \right)}$ (d) $\frac{\epsilon_0 A}{d - t \left(1 + \frac{1}{K} \right)}$

- 161** The distance between the circular plates of a parallel plate condenser 40 mm in diameter, in order to have same capacity as a sphere of radius 1 m is
- (a) 0.01 mm (b) 0.1 mm
 (c) 1.0 mm (d) 10 mm

- 162** The expression for the capacity of the capacitor formed by compound dielectric placed between the plates of a parallel plate capacitor as shown in figure, will be (area of plate $= A$)



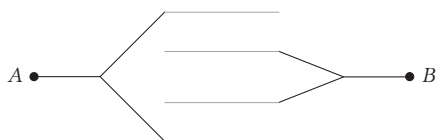
- (a) $\frac{\epsilon_0 A}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3}\right)}$ (b) $\frac{\epsilon_0 A}{\left(\frac{d_1 + d_2 + d_3}{K_1 + K_2 + K_3}\right)}$
 (c) $\frac{\epsilon_0 A (K_1 K_2 K_3)}{d_1 d_2 d_3}$ (d) $\epsilon_0 \left(\frac{AK_1}{d_1} + \frac{AK_2}{d_2} + \frac{AK_3}{d_3}\right)$

163 The equivalent capacitance between A and B will be



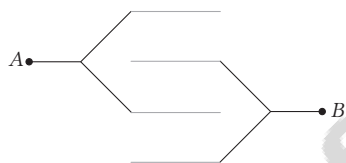
- (a) $2C$ (b) $\frac{C}{2}$ (c) $3C$ (d) $\frac{2}{C}$

164 Four plates of equal area A are separated by equal distances d and are arranged as shown in the figure. The equivalent capacity is



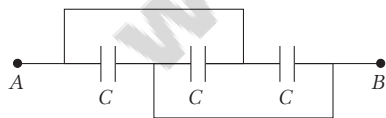
- (a) $\frac{2\epsilon_0 A}{d}$ (b) $\frac{3\epsilon_0 A}{d}$ (c) $\frac{4\epsilon_0 A}{d}$ (d) $\frac{\epsilon_0 A}{d}$

165 Four plates of the same area of cross-section are joined as shown in the figure. The distance between each plate is d . The equivalent capacity across A and B will be



- (a) $\frac{2\epsilon_0 A}{d}$ (b) $\frac{3\epsilon_0 A}{d}$ (c) $\frac{3\epsilon_0 A}{2d}$ (d) $\frac{\epsilon_0 A}{d}$

166 Three equal capacitors, each with capacitance C are connected as shown in figure. Then, the equivalent capacitance between point A and B is

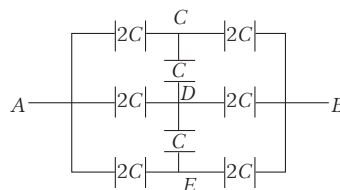


- (a) C (b) $3C$ (c) $\frac{C}{3}$ (d) $\frac{3C}{2}$

167 If a slab of insulating material 4×10^{-3} m thick is introduced between the plates of a parallel plate capacitor, the separation between plates has to be increased by 3.5×10^{-3} m to restore the capacity to original value. The dielectric constant of the material will be

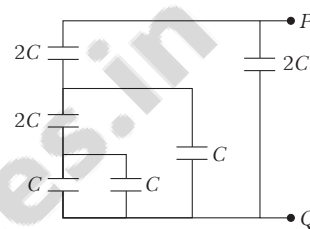
- (a) 6 (b) 8 (c) 10 (d) 12

168 In this figure, the equivalent capacitance between A and B will be



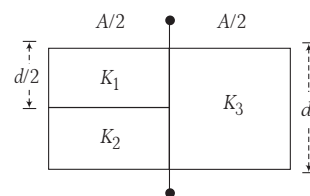
- (a) $\frac{C}{2}$ (b) $\frac{C}{3}$ (c) $3C$ (d) $2C$

169 The resultant capacitance of given circuit between points P and Q is



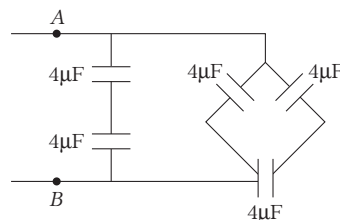
- (a) $3C$ (b) $2C$ (c) C (d) $\frac{C}{3}$

170 In the figure, a capacitor is filled with dielectrics. The resultant capacitance is



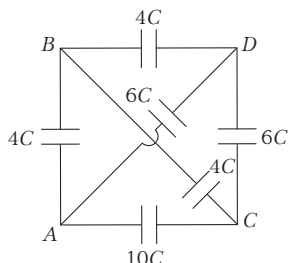
- (a) $\frac{2\epsilon_0 A}{d} \left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \right]$
 (b) $\frac{\epsilon_0 A}{d} \left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \right]$
 (c) $\frac{2\epsilon_0 A}{d} [K_1 + K_2 + K_3]$
 (d) None of the above

171 The equivalent capacitance between points A and B in the circuit will be



- (a) $\frac{10}{3} \mu\text{F}$ (b) $4 \mu\text{F}$
 (c) $6 \mu\text{F}$ (d) $8 \mu\text{F}$

- 172** The equivalent capacitance between the points A and C is given by



- (a) $\frac{10}{3} C$ (b) $15 C$ (c) $\frac{3}{10} C$ (d) $20 C$
- 173** Potential difference between two points ($V_A - V_B$) in an electric field $\mathbf{E} = (2\hat{i} - 4\hat{j}) \text{ N/C}$, where $A = (0, 0)$ and $B = (3 \text{ m}, 4 \text{ m})$ is
- (a) 10 V (b) -10 V (c) 16 V (d) -16 V

- 174** A and B are two thin concentric hollow conductors having radii a and $2a$ and charge $2Q$ and Q , respectively. If potential of outer sphere is 5 V , then potential of inner sphere is

- (a) 20 V (b) 10 V (c) $\frac{25}{3} \text{ V}$ (d) $\frac{50}{3} \text{ V}$

- 175** A spherical conductor of radius 2 m is charged to a potential of 120 V . It is now placed inside another hollow spherical conductor of radius 6 m . Calculate the potential of bigger sphere, if the smaller sphere is made to touch the bigger sphere.

- (a) 120 V (b) 60 V (c) 80 V (d) 40 V

- 176** In Millikan's oil drop experiment, an oil drop carrying a charge Q is held stationary by a potential difference 2400 V between the plates. To keep a drop of half the radius stationary, the potential difference had to be made 600 V . What is the charge on the second drop?

- (a) $\frac{Q}{4}$ (b) $\frac{Q}{2}$ (c) Q (d) $\frac{3Q}{2}$

- 177** There are four concentric shells A, B, C and D of radii $a, 2a, 3a$ and $4a$, respectively. Shells B and D are given charges $+q$ and $-q$, respectively. Shell C is now earthed. The potential difference $V_A - V_C$ is

(take, $K = \frac{1}{4\pi\epsilon_0}$)

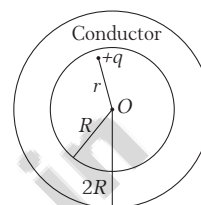
- (a) $\frac{Kq}{2a}$ (b) $\frac{Kq}{3a}$ (c) $\frac{Kq}{4a}$ (d) $\frac{Kq}{6a}$

- 178** A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V .

If the shell is now given a charge of $-3Q$, then the new potential difference between the same two surface is

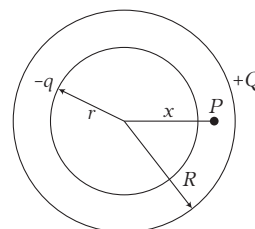
- (a) V (b) $2V$
(c) $4V$ (d) $-2V$

- 179** A point charge q is placed at a distance r from the centre O of an uncharged spherical shell of inner radius R and outer radius $2R$. The distance $r < R$. The electric potential at the centre of the shell will be



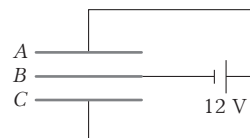
- (a) $\frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{2R} \right)$ (b) $\frac{q}{4\pi\epsilon_0 r}$
(c) $\frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} + \frac{1}{2R} \right)$ (d) $\frac{q}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{1}{R} \right)$

- 180** A hollow sphere of radius r is put inside another hollow sphere of radius R . The charges on the two are $+Q$ and $-q$ as shown in the figure. A point P is located at a distance x from the common centre such that $r < x < R$. The potential at the point P is



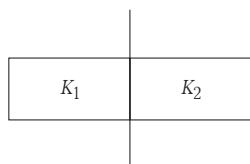
- (a) $\frac{1}{4\pi\epsilon_0} \left(\frac{Q-q}{x} \right)$ (b) $\frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} - \frac{q}{r} \right)$
(c) $\frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} - \frac{q}{x} \right)$ (d) $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} - \frac{Q}{x} \right)$

- 181** Three plates A, B, C each of area 50 cm^2 have separation 3 mm between A and B and 3 mm between B and C . The energy stored when the plates are fully charged is



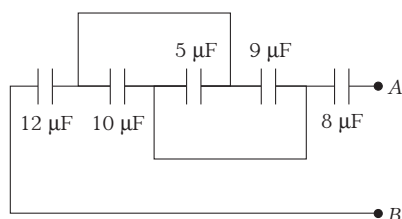
- (a) $1.6 \times 10^{-9} \text{ J}$ (b) $2.1 \times 10^{-9} \text{ J}$
(c) $5 \times 10^{-9} \text{ J}$ (d) $7 \times 10^{-9} \text{ J}$

- 182** A parallel plate capacitor with air as medium between the plates has a capacitance of $10\ \mu\text{F}$. The area of capacitor is divided into two equal halves and filled with two media as shown in the figure having dielectric constant $K_1 = 2$ and $K_2 = 4$. The capacitance of the system will now be



- (a) $10\ \mu\text{F}$ (b) $20\ \mu\text{F}$ (c) $30\ \mu\text{F}$ (d) $40\ \mu\text{F}$

- 183** The capacities and connection of five capacitors are shown in the adjoining figure. The potential difference between the points A and B is $60\ \text{V}$. Then, the equivalent capacity between A and B and the charge on $5\ \mu\text{F}$ capacitance will be respectively



- (a) $44\ \mu\text{F}$, $300\ \mu\text{C}$ (b) $16\ \mu\text{F}$, $150\ \mu\text{C}$
(c) $15\ \mu\text{F}$, $200\ \mu\text{C}$ (d) $4\ \mu\text{F}$, $50\ \mu\text{C}$

- 184** A charge $+Q$ is uniformly distributed over a thin ring of the radius R . The velocity of an electron at the moment when it passes through the centre O of the ring, if the electron was initially far away on the axis of the ring is ($m = \text{mass of electron}$, $K = \frac{1}{4\pi\epsilon_0}$)

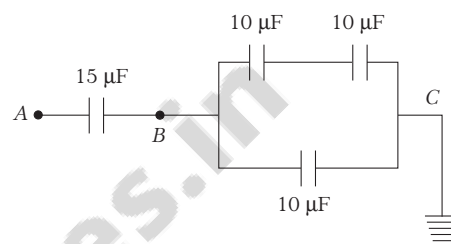
- (a) $\sqrt{\left(\frac{2KQe}{mR}\right)}$ (b) $\sqrt{\left(\frac{KQe}{m}\right)}$
(c) $\sqrt{\left(\frac{Kme}{QR}\right)}$ (d) $\sqrt{\left(\frac{KQe}{mR}\right)}$

- 185** A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V . Another capacitor of capacitance $2C$ is connected to another battery and is charged to potential difference $2V$. The charging batteries are now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

- (a) zero (b) $\frac{25CV^2}{6}$ (c) $\frac{3CV^2}{2}$ (d) $\frac{9CV^2}{2}$

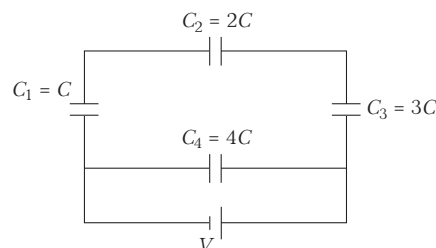
- 186** Condenser A has a capacity of $15\ \mu\text{F}$ when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity of $1\ \mu\text{F}$ with air between the plates. Both are charged separately by a battery of $100\ \text{V}$. After charging, both are connected in parallel without the battery and the dielectric medium being removed. The common potential now is
(a) $400\ \text{V}$ (b) $800\ \text{V}$ (c) $1200\ \text{V}$ (d) $1600\ \text{V}$

- 187** In the given circuit if point C is connected to the earth and a potential of $+2000\ \text{V}$ is given to the point A, the potential at B is



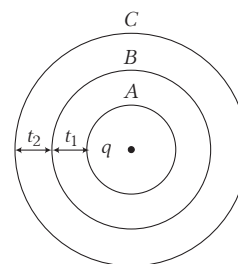
- (a) $1500\ \text{V}$ (b) $1000\ \text{V}$
(c) $500\ \text{V}$ (d) $400\ \text{V}$

- 188** A network of four capacitors of capacity equal to $C_1 = C$, $C_2 = 2C$, $C_3 = 3C$ and $C_4 = 4C$ are connected to a battery as shown in the figure. The ratio of the charges on C_2 and C_4 is



- (a) $\frac{22}{3}$ (b) $\frac{3}{22}$
(c) $\frac{7}{4}$ (d) $\frac{4}{7}$

- 189** Figure shows three spherical and equipotential surfaces A, B and C around a point charge q . The potential difference $V_A - V_B = V_B - V_C$. If t_1 and t_2 be the distances between them, then

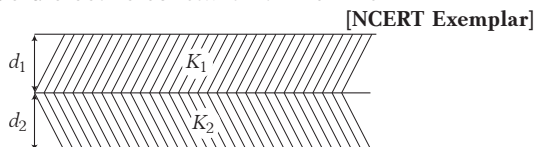


- (a) $t_1 = t_2$ (b) $t_1 > t_2$
(c) $t_1 < t_2$ (d) $t_1 \leq t_2$

- 190** A charged oil drop of mass 2.5×10^{-7} kg is in space between the two plates, each of area 2×10^{-2} m² of a parallel plate capacitor. When the upper plate has a charge of 5×10^{-7} C and the lower plate has an equal negative charge, then the oil remains stationary. The charge of the oil drop is (take, $g = 10$ m/s²)

(a) 9×10^{-1} C (b) 9×10^{-6} C
(c) 8.85×10^{-13} C (d) 1.8×10^{-14} C

- 191** A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness d ($= d_1 + d_2$) and effective dielectric constant K . The K is



[NCERT Exemplar]

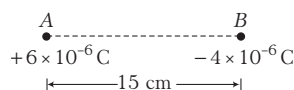
(a) $\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}$ (b) $\frac{K_1 d_1 + K_2 d_2}{K_1 + K_2}$
(c) $\frac{K_1 K_2 (d_1 + d_2)}{K_1 d_2 + K_2 d_1}$ (d) $\frac{2K_1 K_2}{K_1 + K_2}$

- 192** A number of condensers, each of the capacitance $1 \mu\text{F}$ and each one of which gets punctured if a potential difference just exceeding 500 V is applied, are provided. An arrangement suitable for giving capacitance of $2 \mu\text{F}$ across which 3000 V may be applied requires at least
- (a) 6 component capacitors (b) 12 component capacitors
(c) 72 component capacitors (d) 2 component capacitors

- 193** A series combination of n_1 capacitors, each of value C_1 , is charged by a source of potential difference $4V$. When another parallel combination of n_2 capacitors, each of value C_2 , is charged by a source of potential difference V , it has the same (total) energy stored in it, as the first combination has. The value of C_2 , in terms of C_1 is, then

(a) $\frac{2C_1}{n_1 n_2}$ (b) $16 \frac{n_1}{n_2} C_1$ (c) $2 \frac{n_2}{n_1} C_1$ (d) $\frac{16C_1}{n_1 n_2}$

- 194** Two charges $+6 \mu\text{C}$ and $-4 \mu\text{C}$ are placed 15 cm apart as shown. At what distances from A to its right, the electrostatic potential is zero (distances in cm)?



(a) 4, 9, 60 (b) 9, 45, infinity
(c) 20, 45, infinity (d) 9, 15, 45

- 195** Assume that an electric field $\mathbf{E} = 30x^2 \hat{i}$ exists in space. Then, the potential difference $V_A - V_o$, where V_o is the potential at the origin and V_A is the potential at $x = 2$ m, is

(a) 120 V (b) -120 V (c) -80 V (d) 80 V

- 196** An electron initially at rest falls a distance of 1.5 cm in a uniform electric field of magnitude 2×10^4 N/C. The time taken by the electron to fall this distance is
- (a) 1.3×10^2 s (b) 2.1×10^{-2} s
(c) 1.6×10^{-10} s (d) 2.9×10^{-9} s

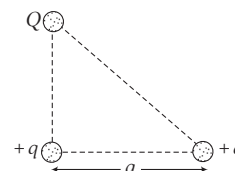
- 197** The potential at a point x (measured in μm) due to some charges situated on the X -axis is given by

$$V(x) = \frac{20}{(x^2 - 4)} \text{ V. The electric field } E \text{ at } x = 4 \mu\text{m} \text{ is}$$

given by

(a) $5/3$ V/ μm and in the negative x -direction
(b) $5/3$ V/ μm and in the positive x -direction
(c) $10/9$ V/ μm and in the negative x -direction
(d) $10/9$ V/ μm and in the positive x -direction

- 198** Three charges Q , $+q$ and $+q$ are placed at the vertices of a right angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero, if Q is equal to

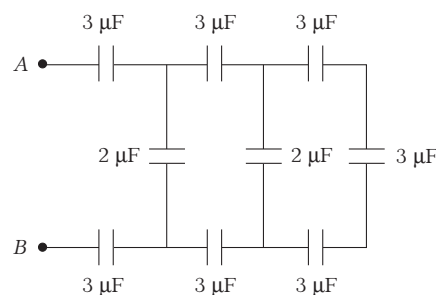


(a) $\frac{-q}{1 + \sqrt{2}}$ (b) $\frac{-2q}{2 + \sqrt{2}}$ (c) $-2q$ (d) $+q$

- 199** Four point charges each $+q$ is placed on the circumference of a circle of diameter $2d$ in such a way that they form a square. The potential at the centre (in CGS unit) is

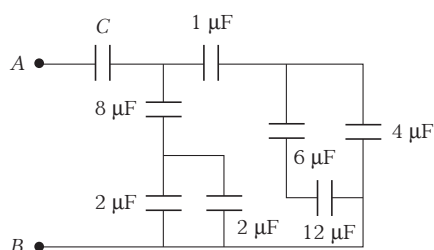
(a) zero (b) $\frac{4q}{d}$ (c) $\frac{4d}{q}$ (d) $\frac{q}{4d}$

- 200** The resultant capacitance between A and B in the following figure is equal to

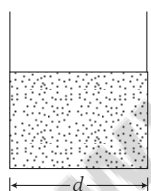


(a) $1 \mu\text{F}$ (b) $3 \mu\text{F}$ (c) $2 \mu\text{F}$ (d) $1.5 \mu\text{F}$

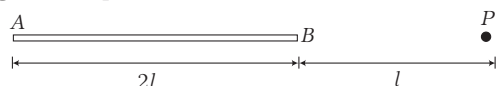
- 201** In the following circuit, the resultant capacitance between A and B is $1 \mu\text{F}$. The value of C is



- (a) $\frac{32}{11} \mu\text{F}$ (b) $\frac{11}{32} \mu\text{F}$
 (c) $\frac{23}{32} \mu\text{F}$ (d) $\frac{32}{23} \mu\text{F}$
- 202** A small conducting sphere of radius r is lying concentrically inside a bigger hollow conducting sphere of radius R . The bigger and smaller spheres are charged with Q and q ($Q > q$) and are insulated from each other. The potential difference between the spheres will be
- (a) $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} - \frac{Q}{R} \right)$ (b) $\frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} + \frac{q}{r} \right)$
 (c) $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} - \frac{q}{R} \right)$ (d) $\frac{1}{4\pi\epsilon_0} \left(\frac{q}{R} - \frac{Q}{r} \right)$
- 203** A parallel plate air capacitor has a capacitance C . When it is half filled with a dielectric of dielectric constant 5, then the percentage increase in the capacitance will be

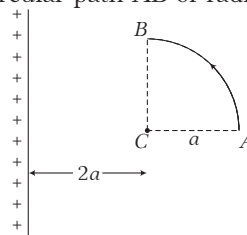


- (a) 400% (b) 66.6%
 (c) 33.3% (d) 200%
- 204** Charge Q is uniformly distributed on a dielectric rod AB of length $2l$. The potential at P shown in the figure is equal to

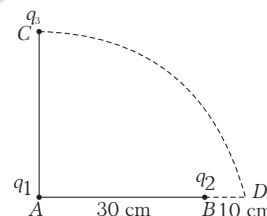


- (a) $\frac{Q}{4\pi\epsilon_0(2l)}$ (b) $\frac{Q}{4\pi\epsilon_0(l)} \ln(2)$
 (c) $\frac{Q}{4\pi\epsilon_0(2l)} \ln(3)$ (d) None of these
- 205** The arc AB with the centre C and the infinitely long wire having linear charge density λ are lying in the same plane. The minimum amount of work to be

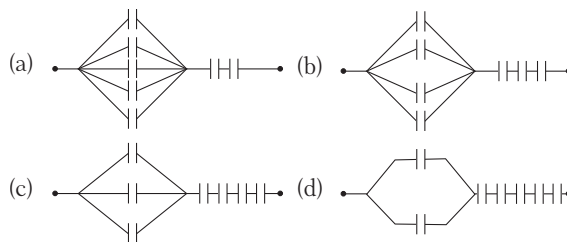
done to move a point charge q_0 from point A to B through a circular path AB of radius a is equal to



- (a) $\frac{q_0\lambda}{4\pi\epsilon_0} \ln\left(\frac{2}{3}\right)$ (b) $\frac{q_0\lambda}{2\pi\epsilon_0} \ln\left(\frac{3}{2}\right)$
 (c) $\frac{q_0\lambda}{2\pi\epsilon_0} \ln\left(\frac{2}{3}\right)$ (d) None of these
- 206** Two charges q_1 and q_2 are placed 30 cm apart, as shown in figure. A third charge q_3 is moved along the arc of a circle of radius 40 cm from C to D . The change in the potential energy of the system is $\frac{q_3}{4\pi\epsilon_0} K$, where K is

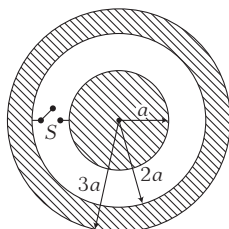


- (a) $8q_2$ (b) $8q_1$ (c) $6q_2$ (d) $6q_1$
- 207** Three identical metallic uncharged spheres A , B and C of radius a are kept on the corners of an equilateral triangle of side d ($d \gg a$). A fourth sphere (radius a) which has charge Q touches A and is then removed to a position far away. B is earthed and then the earth connection is removed. Sphere C is then earthed, the charge on sphere C is
- (a) $\frac{Qa}{2d} \left(\frac{2d-a}{2d} \right)$ (b) $\frac{Qa}{2d} \left(\frac{2d-a}{d} \right)$
 (c) $\frac{Qa}{2d} \left(\frac{a-d}{d} \right)$ (d) $\frac{2Qa}{d} \left(\frac{d-a}{2d} \right)$
- 208** Seven capacitors each of capacity $2 \mu\text{F}$ are to be so connected to have a total capacity $(10/11) \mu\text{F}$. Which will be the necessary figure as shown?



- 209** A solid conducting sphere of radius a having a charge q is surrounded by a concentric conducting spherical shell of inner radius $2a$ and outer radius $3a$ as shown in figure. Find the amount of heat produced when switch is closed.

(Take, $K = \frac{1}{4\pi\epsilon_0}$)

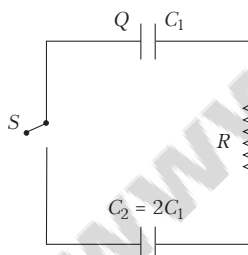


- (a) $\frac{Kq^2}{2a}$ (b) $\frac{Kq^2}{3a}$ (c) $\frac{Kq^2}{4a}$ (d) $\frac{Kq^2}{6a}$

- 210** Three identical charges are placed at corners of an equilateral triangle of side l . If force between any two charges is F , then the work required to double the dimensions of triangle is

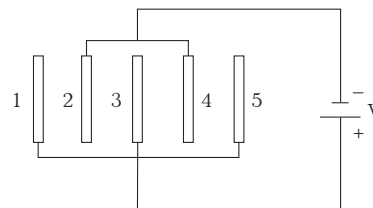
- (a) $-3Fl$ (b) $3Fl$ (c) $\left(-\frac{3}{2}\right)Fl$ (d) $\left(\frac{3}{2}\right)Fl$

- 211** Two capacitors C_1 and $C_2 = 2C_1$ are connected in a circuit with a switch between them as shown in the figure. Initially the switch is open and C_1 holds charge Q . The switch is closed. At steady state, the charge on each capacitor will be



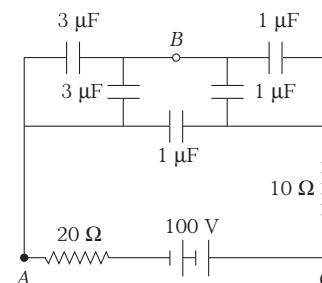
- (a) $Q, 2Q$ (b) $Q/3, 2Q/3$
(c) $3Q/2, 3Q$ (d) $2Q/3, 4Q/3$

- 212** Five identical plates each of area A are joined as shown in the figure. The distance between the plates is d . The plates are connected to a potential difference of V volt. The charge on plates 1 and 4 will be



- (a) $-\frac{\epsilon_0 AV}{d}, \frac{2\epsilon_0 AV}{d}$
(b) $\frac{\epsilon_0 AV}{d}, \frac{2\epsilon_0 AV}{d}$
(c) $\frac{\epsilon_0 AV}{d}, -\frac{2\epsilon_0 AV}{d}$
(d) $-\frac{\epsilon_0 AV}{d}, -\frac{2\epsilon_0 AV}{d}$

- 213** In the figure shown, what is the potential difference between the points A and B and between B and C respectively in steady state?



- (a) $V_{AB} = V_{BC} = 100$ V
(b) $V_{AB} = 75$ V, $V_{BC} = 25$ V
(c) $V_{AB} = 25$ V, $V_{BC} = 75$ V
(d) $V_{AB} = V_{BC} = 50$ V

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1 Assertion Conductor having equal positive charge and volume, must also have same potential.

Reason Potential depends on charge and geometry of conductor.

2 Assertion When two positive point charges move away from each other, then their electrostatic potential energy decreases.

Reason Change in potential energy between two points is equal to the work done by electrostatic forces.

3 Assertion A charged capacitor is disconnected from a battery. Now, if its plates are separated further, the potential energy will fall.

Reason Energy stored in a capacitor is equal to the work done in charging it.

4 Assertion When a capacitor is charged by a battery, half of the energy supplied by the battery is stored in the capacitor and rest half is lost.

Reason If resistance in the circuit is zero, then there will be no loss of energy.

5 Assertion If the distance between parallel plates of a capacitor is halved and dielectric constant is made three times, then the capacitance becomes 6 times.

Reason Capacity of the capacitor depends upon the nature of the material between the plates.

Statement based questions

1 An electric dipole of moment $\mathbf{p}$ is placed in a uniform electric field $\mathbf{E}$. Then,

- (i) the torque on the dipole is $\mathbf{p} \times \mathbf{E}$.
- (ii) the potential energy of the system is $\mathbf{p} \cdot \mathbf{E}$.
- (iii) the resultant force on the dipole is zero.

Choose the correct option, based on above statements.

- (a) (i), (ii) and (iii) are correct
- (b) (i) and (iii) are correct and (ii) is wrong
- (c) Only (i) is correct
- (d) (i) and (ii) are correct and (iii) is wrong

2 Identify the false statement.

- (a) Inside a charged or neutral conductor electrostatic field is zero.
- (b) The electrostatic field at the surface of the charged conductor must be tangential to the surface at any point.
- (c) There is no net charge at any point inside the conductor.
- (d) Electrostatic potential is constant throughout the volume of the conductor.

3 The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard.

S_1 : At any point inside the sphere, electric intensity is zero.

S_2 : At any point inside the sphere, the electrostatic potential is 100 V.

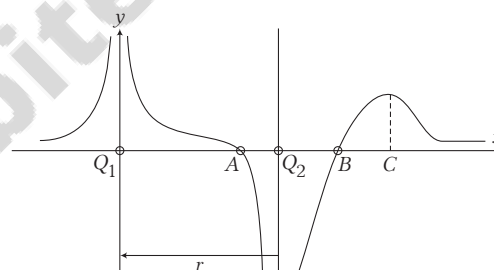
Which of the following is a correct statement?

- (a) S_1 is true but S_2 is false. [NCERT Exemplar]
- (b) Both S_1 and S_2 are false.
- (c) S_1 is true, S_2 is also true and S_1 is the cause of S_2 .
- (d) S_1 is true, S_2 is also true but the statements are independent.

4 A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinders are initially electrically neutral, then choose the correct statement.

- (a) A potential difference appears between the two cylinders when a charge density is given to the inner cylinder.
- (b) A potential difference appears between the two cylinders when a charge density is given to the outer cylinder.
- (c) No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders.
- (d) No potential difference appears between the two cylinders when same charge density is given to both the cylinders.

5 The curve represents the distribution of potential along the straight line joining the two charges Q_1 and Q_2 (separated by a distance r), then which of the following statements are correct?



- 1. $|Q_1| > |Q_2|$
 - 2. Q_1 is positive in nature.
 - 3. A and B are equilibrium points.
 - 4. C is a point of unstable equilibrium.
- (a) 1 and 2
 - (b) 1, 2 and 3
 - (c) 1, 2 and 4
 - (d) 1, 2, 3 and 4

Match the columns

1 Two charges $+q$ and $-q$ are placed at distance r .

Match the following two columns when distance between them is changed to r' and choose the correct code.

Column I	Column II
A. $r' = 2r$	(p) Potential energy will become half
B. $r' = \frac{r}{2}$	(q) Force between them will become $\frac{1}{4}$ th
C. $r' = 4r$	(r) Potential energy will become four times
D. $r' = \frac{r}{4}$	(s) None

Codes

	A	B	C	D
(a)	p,q	s	s	r
(b)	p	s	r	s
(c)	p,q	q	s	s
(d)	r,r	p	q	r

- 2** A capacitor is connected with a battery. With battery remains connected some changes are done in capacitor/battery, which are given in Column I. Corresponding to it match the two columns and choose the correct code.

Column I	Column II
A. Distance between capacitor plates is halved	(p) Capacity of capacitor will become two times
B. A metallic slab completely filled between the plates	(q) Charge stored in capacitor will become two times
C. A dielectric slab of dielectric constant $K = 2$ is completely filled between the plates	(r) Energy stored in capacitor will become two times
	(s) Capacitance become infinite

Codes

	A	B	C		A	B	C
(a)	p,q,s	s	q	(b)	q,r	s	p,q
(c)	p,q,r	s	p,q,r	(d)	r,p,q	q	s,q,r

- 3** A parallel plate capacitor is charged by a battery which is then disconnected. A dielectric slab is then inserted to fill the space between the plates. Match the changes that could occur with Column II and choose the correct code.

Column I	Column II
A. Charge on the capacitor plates	(p) Decrease by a factor of K
B. Intensity of electric field	(q) Increase by a factor of K
C. Energy stored	(r) Remains same
D. Capacitance	(s) None

Codes

	A	B	C	D			A	B	C	D
(a)	p	s	q	r	(b)	r	p	p	q	
(c)	r	q	s	p	(d)	p	r	q	s	

- 4** The area of parallel plates of an air-filled capacitor is 0.20 m^2 and the distance between them is 0.01 m . The potential difference across the plates is 3000 V . When a 0.01 m thick dielectric sheet is introduced between the plates, then the potential difference decreases to 1000 V . Now, match the two columns (all in SI units) and choose the correct code.

Column I	Column II
A. Capacitance of air-filled capacitor	(p) 5.31×10^{-7}
B. Charge on each plate	(q) 5.31×10^{-10}
C. Dielectric constant of the material	(r) 3
D. Capacitance after the dielectric sheet is introduced	(s) 1.77×10^{-10}

Codes

	A	B	C	D			A	B	C	D
(a)	q	s	p	r	(b)	s	r	p	q	
(c)	s	p	r	q	(d)	q	r	p	s	

(C) Medical entrances' gallery

Collection of questions asked in NEET and various medical entrance exams

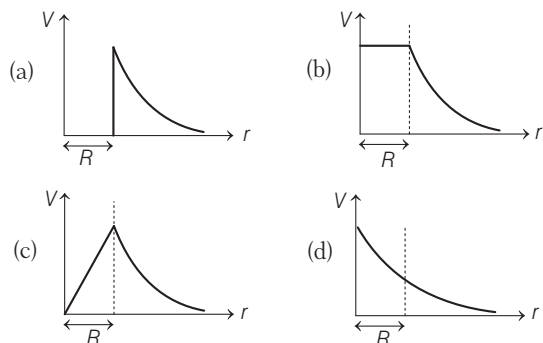
- 1** In a certain region of space with volume 0.2 m^3 , the electric potential is found to be 5 V throughout. The magnitude of electric field in this region is [NEET 2020]
 (a) 0.5 N/C (b) 1 N/C (c) 5 N/C (d) zero
- 2** The capacitance of a parallel plate capacitor with air as medium is $6 \mu\text{F}$. With the introduction of a dielectric medium, the capacitance becomes $30 \mu\text{F}$. The permittivity of the medium is (take, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$) [NEET 2020]
 (a) $1.77 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$ (b) $0.44 \times 10^{-10} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$
 (c) $5.00 \text{ C}^2\text{N}^{-1}\text{m}^{-2}$ (d) $0.44 \times 10^{-13} \text{ C}^2\text{N}^{-1}\text{m}^{-2}$

- 3** A short electric dipole has a dipole moment of $16 \times 10^{-9} \text{ C-m}$. The electric potential due to the dipole at a point at a distance of 0.6 m from the centre of the dipole, situated on a line making an angle of 60° with the dipole axis is

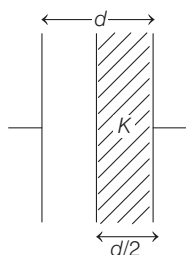
$$\left(\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 / \text{C}^2 \right)$$

[NEET 2020]

- (a) 200 V (b) 400 V
 (c) zero (d) 50 V
- 4** The variation of electrostatic potential with radial distance r from the centre of a positively charged metallic thin shell of radius R is given by the graph [NEET 2020]

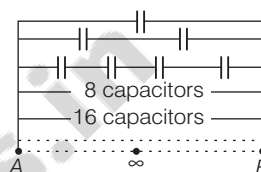


5. A parallel plate capacitor having cross-sectional area A and separation d has air in between the plates. Now, an insulating slab of same area but thickness $d/2$ is inserted between the plates as shown in figure having dielectric constant $K(=4)$. The ratio of new capacitance to its original capacitance will be [NEET 2020]

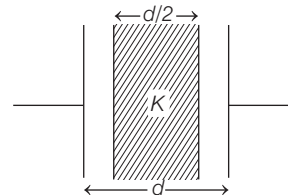


- (a) 2 : 1 (b) 8 : 5 (c) 6 : 5 (d) 4 : 1
6. A capacitor of capacitance 15 nF having dielectric slab of $\epsilon_r = 2.5$, dielectric strength 30 MV/m and potential difference $= 30\text{ V}$. Calculate the area of the plate. [AIIMS 2019]
- (a) $6.7 \times 10^{-4}\text{ m}^2$ (b) $4.2 \times 10^{-4}\text{ m}^2$
 (c) $8.0 \times 10^{-4}\text{ m}^2$ (d) $9.85 \times 10^{-4}\text{ m}^2$
7. Potential difference is given as $V(x) = -x^2y$ volt. Find electric field at a point $(1, 2)$. [JIPMER 2019]
- (a) $\hat{i} + 4\hat{j}\text{ Vm}^{-1}$ (b) $-4\hat{i} - \hat{j}\text{ Vm}^{-1}$
 (c) $4\hat{i} + \hat{j}\text{ Vm}^{-1}$ (d) $4\hat{i} - \hat{j}\text{ Vm}^{-1}$
8. In a parallel plate capacitor, the capacity increases, if
 (a) area of the plate is decreased [MHT CET 2019]
 (b) distance between the plates increases
 (c) area of the plate is increased
 (d) dielectric constant decrease
9. A parallel plate capacitor is charged. If the plates are pulled apart [DCE 2019]
 (a) the capacitance increases
 (b) the potential difference increases
 (c) the total charge increases
 (d) the charge and potential difference remains the same

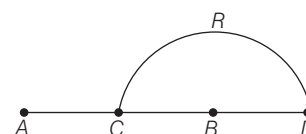
10. The electrostatic force between the metal plates of an isolated parallel plate capacitor C having a charge Q and area A is [NEET 2018]
 (a) proportional to square root of the distance between the plates
 (b) linearly proportional to the distance between the plates
 (c) independent of the distance between the plates
 (d) inversely proportional to the distance between the plates
11. An infinite number of identical capacitor each of capacitance $1\text{ }\mu\text{F}$ are connected as shown in the figure. Then, the equivalent capacitance between A and B is [AIIMS 2018]



- (a) $1\text{ }\mu\text{F}$ (b) $\frac{1}{2}\text{ }\mu\text{F}$
 (c) $2\text{ }\mu\text{F}$ (d) ∞
12. Find the capacitance as shown in the figure. [JIPMER 2018]

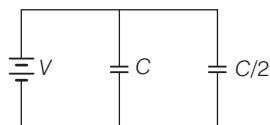


- (a) $2KA\epsilon_0 / (K+1)d$ (b) $\frac{2KA\epsilon_0}{d}$
 (c) $(K+1)A\epsilon_0/2d$ (d) $\frac{2KA\epsilon_0}{(K^2+1)d}$
13. If a capacitor having capacitance 2 F and plate separation of 0.5 cm , will have area [JIPMER 2018]
 (a) 1130 cm^2 (b) 1130 km^2
 (c) 1130 m^2 (d) None of these
14. Charges $+q$ and $-q$ are placed at points A and B respectively, which are a distance $2L$ apart, C is the mid-point between A and B . The work done in moving a charge $+Q$ along the semi-circle CRD is [AIIMS 2017]



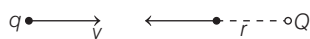
- (a) $\frac{qQ}{4\pi\epsilon_0 L}$ (b) $\frac{qQ}{2\pi\epsilon_0 L}$
 (c) $\frac{qQ}{6\pi\epsilon_0 L}$ (d) $\frac{-qQ}{6\pi\epsilon_0 L}$

- 15** Two capacitors C and $C/2$ are connected to a battery of V volts, as shown below [AIIMS 2017]

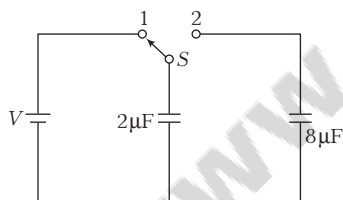


The work done in charging both the capacitor fully is

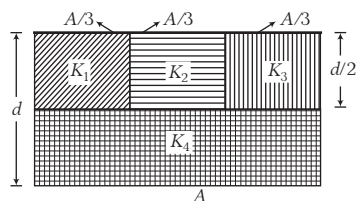
- (a) $2CV^2$ (b) $(1/2)CV^2$ (c) $(3/4)CV^2$ (d) $(1/4)CV^2$
- 16** A charged particle q is shot with speed v towards another fixed charged particle Q . It approaches Q upto a closest distance r and then returns. If q were given a speed $2v$, the closest distance of approach would be [JIPMER 2017]



- (a) r (b) $2r$ (c) $r/2$ (d) $r/4$
- 17** A parallel plate capacitor of capacitance 100 pF is to be constructed by using paper sheets of 1 mm thickness as dielectric. If the dielectric constant of paper is 4, the number of circular metal foils of diameter 2 cm each required for the purpose is [VITEEE 2017]
- (a) 40 (b) 20 (c) 30 (d) 10
- 18** A capacitor of $2\mu\text{F}$ is charged as shown in the figure. When the switch S is turned to position 2, the percentage of its stored energy dissipated is [NEET 2016]



- (a) 20% (b) 75% (c) 80% (d) 0%
- 19** A parallel plate capacitor of area A , plate separation d and capacitance C is filled with four dielectric materials having dielectric constants K_1, K_2, K_3 and K_4 as shown in the figure below. If a single dielectric material is to be used to have the same capacitance C in this capacitor, then its dielectric constant K is given by [NEET 2016]

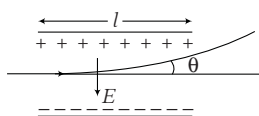


- (a) $K = K_1 + K_2 + K_3 + 3K_4$
 (b) $K = \frac{2}{3}(K_1 + K_2 + K_3) + 2K_4$
 (c) $\frac{2}{K} = \frac{3}{K_1 + K_2 + K_3} + \frac{1}{K_4}$
 (d) $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \frac{3}{2K_4}$
- 20** A parallel plate air capacitor of capacitance C is connected to a cell of emf V and then disconnected from it. A dielectric slab of dielectric constant K , which can just fill the air gap of the capacitor, is now inserted in it. Which of the following is incorrect? [CBSE AIPMT 2015]
- (a) The potential difference between the plates decreases K times
 (b) The energy stored in the capacitor decreases K times
 (c) The change in energy stored is $\frac{1}{2}CV^2\left(\frac{1}{K} - 1\right)$
 (d) The charge on the capacitor is not conserved
- 21** An electron of mass M_e , initially at rest, moves through a certain distance in a uniform electric field in time t_1 . A proton of mass M_p also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio t_2/t_1 is nearly equal to [AIIMS 2015]
- (a) 1 (b) $\sqrt{\frac{M_p}{M_e}}$
 (c) $\sqrt{\frac{M_e}{M_p}}$ (d) $\frac{M_e}{M_p}$
- 22** An isolated sphere has a capacitance of 50 pF. What would be the radius of the sphere? [UK PMT 2015]
- (a) 90 cm (b) 45 cm
 (c) 11.50 cm (d) 5.75 cm
- 23** A parallel plate capacitor has 91 plates, all are identical and arranged with same spacing between them. If the capacitance between adjacent plates is 3 pF. What will be the resultant capacitance? [EAMCET 2015]
- (a) 273 pF (b) 30 pF
 (c) 94 pF (d) 270 pF
- 24** A particle of mass 1.96×10^{-15} kg is kept in equilibrium between two horizontal metal plates having potential difference of 400 V separated apart by 0.02 m. Then, the charge on the particle is (e = electronic charge) [Kerala CEE 2015]
- (a) $3e$ (b) $6e$
 (c) $2e$ (d) $5e$
 (e) $4e$

- 25** The distance of the closest approach of an alpha particle fired at a nucleus with kinetic energy K is r_0 . The distance of the closest approach when the α particle is fired at the same nucleus with kinetic energy $2K$ will be [Guj. CET 2015]

(a) $4r_0$ (b) $\frac{r_0}{2}$ (c) $\frac{r_0}{4}$ (d) $2r_0$

- 26** A uniform electric field is created between two parallel charged plates as shown below. An electron enters the field symmetrically between the plates with a speed of v_0 . The length of each plate is l . Find the angle of deviation of path of the electron as it comes out of the field. [CG PMT 2015]



(a) $\theta = \tan^{-1} \frac{El}{mv_0^2}$ (b) $\theta = \tan^{-1} \left(\frac{eEl}{mv_0^2} \right)$
 (c) $\theta = \tan^{-1} \left(\frac{eEl}{mv_0} \right)$ (d) $\theta = \tan^{-1} \left(\frac{eE}{mv_0^2} \right)$

- 27** A battery charges a parallel plate capacitor separated by distance (d) so that the energy (V_0) is stored in the system. A slab of dielectric constant (K) and thickness (d) is then introduced between the plates of capacitor. The new energy of the system is given by [CG PMT 2015]

(a) KV_0 (b) $K^2 V_0$
 (c) $\frac{V_0}{K}$ (d) $\frac{V_0}{K^2}$

- 28** A capacitor of capacitance $100 \mu\text{F}$ is charged by connecting it to a battery of voltage 12 V with internal resistance 2Ω . The time after which 99% of the maximum charge is stored on the capacitor is [UP CPMT 2015]

(a) 0.92 ms (b) 0.72 ms (c) 0.34 ms (d) 0.54 ms

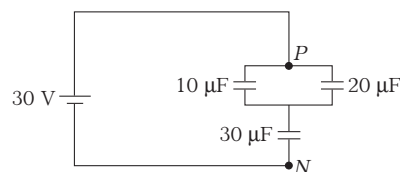
- 29** A parallel plate capacitor is charged and then isolated. The effect of increasing the plate separation on charge, potential and capacitance respectively are [KCET 2015]

(a) constant, decreases, increases
 (b) constant, decreases, decreases
 (c) constant, increases, decreases
 (d) increases, decreases, decreases

- 30** A spherical shell of radius 10 cm is carrying a charge q . If the electric potential at distances 5 cm , 10 cm and 15 cm from the centre of the spherical shell is V_1 , V_2 and V_3 respectively, then [KCET 2015]

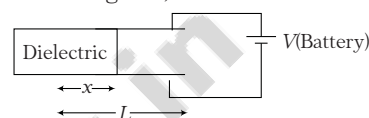
(a) $V_1 = V_2 > V_3$ (b) $V_1 > V_2 > V_3$
 (c) $V_1 = V_2 < V_3$ (d) $V_1 < V_2 < V_3$

- 31** Calculate the charge on equivalent capacitance of the combination shown in figure between the points P and N . [UK PMT 2015]



(a) $450 \mu\text{C}$ (b) $225 \mu\text{C}$ (c) $350 \mu\text{C}$ (d) $900 \mu\text{C}$

- 32** Consider the diagram,



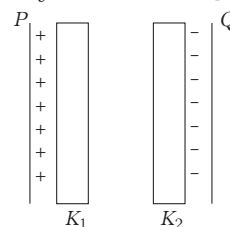
A parallel plate capacitor has the plate width t and length L while the separation between the plates is d . The capacitor is connected to a battery of voltage rating V . A dielectric which carefully occupy, the space between the plates of the capacitor is slowly inserted between the plates. When length x of the dielectric slab is introduced into the capacitor, then energy stored in the system is [UP CPMT 2015]

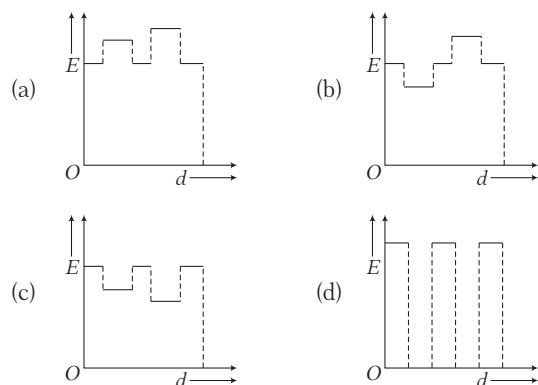
(a) $\frac{\epsilon_0 t V^2}{2d} L$ (b) $\frac{\epsilon_0 t V^2}{2d} L [x + 1]$
 (c) $\frac{\epsilon_0 t V^2}{2d} [L + x (K - 1)]$ (d) $\frac{\epsilon_0 t^2 V^2}{2d^2} [L^2 + x + 1]$

- 33** A conducting sphere of radius R is given a charge Q . The electric potential and the electric field at the centre of the sphere respectively are [CBSE AIPMT 2014]

(a) zero and $\frac{Q}{4\pi\epsilon_0 R^2}$ (b) $\frac{Q}{4\pi\epsilon_0 R}$ and zero
 (c) $\frac{Q}{4\pi\epsilon_0 R}$ and $\frac{Q}{4\pi\epsilon_0 R^2}$ (d) Both are zero

- 34** Two thin dielectric slabs of dielectric constants K_1 and K_2 ($K_1 < K_2$) are inserted between the plates of a parallel plate capacitor, as shown in the figure alongside. The variation of electric field E between the plates with distance d as measured from plate P is correctly shown by [CBSE AIPMT 2014]





- 35** Two charges of equal magnitude q are placed in air at a distance $2a$ apart and third charge $-2q$ is placed at mid-point. The potential energy of the system is (where, ϵ_0 = permittivity of free space) [MHT CET 2014]

(a) $-\frac{q^2}{8\pi\epsilon_0 a}$ (b) $-\frac{3q^2}{8\pi\epsilon_0 a}$ (c) $-\frac{5q^2}{8\pi\epsilon_0 a}$ (d) $-\frac{7q^2}{8\pi\epsilon_0 a}$

- 36** Consider two concentric spherical metal shells of radii r_1 and r_2 ($r_2 > r_1$). If the outer shell has a charge q and the inner one is grounded, then the charge on the inner shell is [WB JEE 2014]

(a) $\frac{-r_2}{r_1} q$ (b) zero (c) $\frac{-r_1}{r_2} q$ (d) $-q$

- 37** What is the electric potential at a distance of 9 cm from 3 nC? [KCET 2014]

(a) 270 V (b) 3 V
(c) 300 V (d) 30 V

- 38** Three capacitors $3 \mu\text{F}$, $6 \mu\text{F}$ and $6 \mu\text{F}$ are connected in series to a source of 120 V. The potential difference in volt, across the $3 \mu\text{F}$ capacitor will be [WB JEE 2014]

(a) 24 V (b) 30 V
(c) 40 V (d) 60 V

- 39** The capacitance of two concentric spherical shells of radii R_1 and R_2 ($R_2 > R_1$) is [EAMCET 2014]

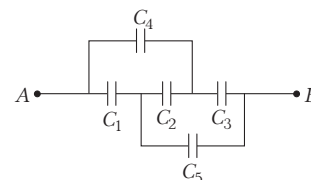
(a) $4\pi\epsilon_0 R_2$ (b) $4\pi\epsilon_0 \frac{(R_2 - R_1)}{R_1 R_2}$
(c) $4\pi\epsilon_0 \frac{R_1 R_2}{(R_2 - R_1)}$ (d) $4\pi\epsilon_0 R_1$

- 40** Two capacitors of 10 pF and 20 pF are connected to 200 V and 100 V sources, respectively. If they are connected by the wire, then what is the common potential of the capacitors? [KCET 2014]

(a) 133.3 V (b) 150 V
(c) 300 V (d) 400 V

- 41** In the given figure, the capacitors C_1, C_3, C_4 and C_5 have a capacitance $4 \mu\text{F}$ each. If the capacitor C_2 has

a capacitance $10 \mu\text{F}$, then effective capacitance between A and B will be [UK PMT 2014]

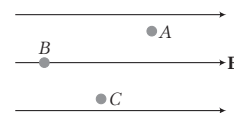


- (a) $2 \mu\text{F}$ (b) $4 \mu\text{F}$ (c) $6 \mu\text{F}$ (d) $8 \mu\text{F}$

- 42** Two concentric spheres kept in air have radii R and r . They have similar charge and equal surface charge density σ . The electrical potential at their common centre is (where, ϵ_0 = permittivity of free space) [MHT CET 2014]

(a) $\frac{\sigma(R+r)}{\epsilon_0}$ (b) $\frac{\sigma(R-r)}{\epsilon_0}$
(c) $\frac{\sigma(R+r)}{2\epsilon_0}$ (d) $\frac{\sigma(R+r)}{4\epsilon_0}$

- 43** A, B and C are three points in a uniform electric field. The electric potential is [NEET 2013]



- (a) maximum at A
(b) maximum at B
(c) maximum at C
(d) same at all the three points A, B and C

- 44** A hollow sphere of radius 0.1 m has a charge of $5 \times 10^{-8} \text{ C}$. The potential at a distance of 5 cm from the centre of the sphere is

$\left(\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2} \right)$ [J&K CET 2013]

- (a) 4000 V (b) 4500 V
(c) 5000 V (d) 6000 V

- 45** Two identical capacitors are first connected in series and then in parallel. The ratio of equivalent capacitance is [J&K CET 2013]

- (a) 1:1 (b) 1:2
(c) 1:3 (d) 1:4

- 46** Two capacitors having capacitances C_1 and C_2 are charged with 120 V and 200 V batteries, respectively. When they are connected in parallel now, it is found that the potential on each one of them is zero. Then, [EAMCET 2013]

- (a) $5 C_1 = 3 C_2$ (b) $8 C_1 = 5 C_2$
(c) $9 C_1 = 5 C_2$ (d) $3 C_1 = 5 C_2$

- 47** A small oil drop of mass 10^{-6} kg is hanging in at rest between two plates separated by 1 mm having a potential difference of 500 V. The charge on the drop is ($g = 10 \text{ ms}^{-2}$) [Karnataka CET 2013]

(a) 2×10^{-9} C (b) 2×10^{-11} C
(c) 2×10^{-6} C (d) 2×10^{-9} C

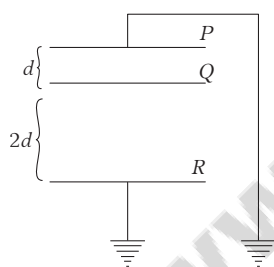
- 48** Two metal spheres of radii 0.01 m and 0.02 m are given a charge of 15 mC and 45 mC, respectively. They are then connected by a wire. The final charge on the first sphere is $\dots \times 10^{-3}$ C. [Karnataka CET 2013]

(a) 40 (b) 30
(c) 20 (d) 10

- 49** The concentric spheres of radii R and r have positive charges q_1 and q_2 with equal surface charge densities. What is the electric potential at their common centre? [Karnataka CET 2013]

(a) $\frac{\sigma}{\epsilon_0} (R + r)$ (b) $\frac{\sigma}{\epsilon_0} (R - r)$
(c) $\frac{\sigma}{\epsilon_0} \left(\frac{1}{R} + \frac{1}{r} \right)$ (d) $\frac{\sigma}{\epsilon_0} \left(\frac{1}{R} \right)$

- 50** See the diagram, area of each plate is 2.0 m^2 and $d = 2 \times 10^{-3}$ m. A charge of 8.85×10^{-8} C is given to Q . Then, the potential of Q becomes [Karnataka CET 2013]



(a) 13 V (b) 10 V
(c) 6.67 V (d) 8.825 V

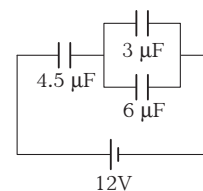
- 51** A soap bubble is charged to a potential 12 V. If its radius is doubled, then the potential of the bubble becomes [Kerala CET 2013]

(a) 12 V (b) 24 V
(c) 3 V (d) 6 V
(e) 9 V

- 52** A sphere of 4 cm radius is suspended within a hollow sphere of 6 cm radius. If the inner sphere is charged to a potential 3 esu while the outer sphere is earthed, then the charge on the inner sphere will be [UP CPMT 2013]

(a) $\frac{1}{4}$ esu (b) 30 esu
(c) 36 esu (d) 54 esu

- 53** In the adjoining figure, the potential difference across the $4.5 \mu\text{F}$ capacitor is [UP CPMT 2013]



(a) 4 V (b) 6 V
(c) 8 V (d) 4.5 V

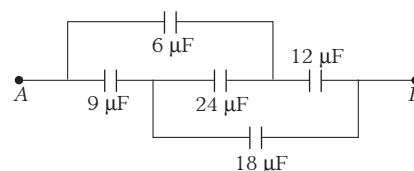
- 54** Four point charges $-Q$, $-q$, $2q$ and $2Q$ are placed, one at each corner of the square. The relation between Q and q for which the potential at the centre of the square is zero, is [CBSE AIPMT 2012]

(a) $Q = -q$ (b) $Q = -\frac{1}{q}$
(c) $Q = q$ (d) $Q = \frac{1}{q}$

- 55** A spherical drop of capacitance $1 \mu\text{F}$ is broken into eight drops of equal radius. Then, the capacitance of each small drop is [AIIMS 2012]

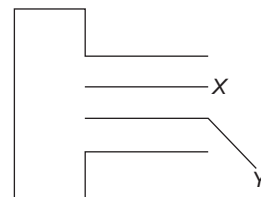
(a) $\frac{1}{2} \mu\text{F}$ (b) $\frac{1}{4} \mu\text{F}$ (c) $\frac{1}{8} \mu\text{F}$ (d) $8 \mu\text{F}$

- 56** The equivalent capacitance between points A and B will be [BCECE (Mains) 2012]



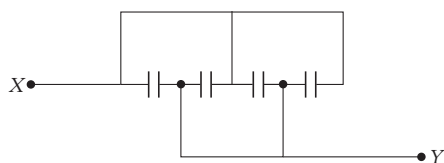
(a) $10 \mu\text{F}$ (b) $15 \mu\text{F}$ (c) $10.8 \mu\text{F}$ (d) $69 \mu\text{F}$

- 57** Four metallic plates each with a surface area of one side A are placed at a distance d from each other as shown in figure. Then, the capacitance of the system between X and Y is [BCECE (Mains) 2012]



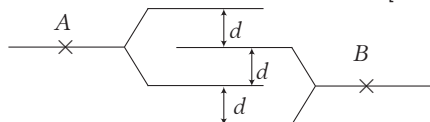
(a) $\frac{2\epsilon_0 A}{d}$ (b) $\frac{2\epsilon_0 A}{3d}$ (c) $\frac{3\epsilon_0 A}{d}$ (d) $\frac{3\epsilon_0 A}{2d}$

- 58** Four capacitors each of capacity $8 \mu\text{F}$ are connected with each other as shown in figure. The equivalent capacitance between points X and Y will be [BHU 2012]



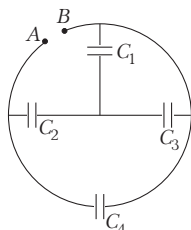
- (a) $2 \mu\text{F}$ (b) $8 \mu\text{F}$ (c) $16 \mu\text{F}$ (d) $32 \mu\text{F}$

- 59** The equivalent capacity between points A and B in figure will be, while capacitance of each capacitor is $3 \mu\text{F}$. [UP CPMT 2012]



- (a) $2 \mu\text{F}$ (b) $4 \mu\text{F}$ (c) $7 \mu\text{F}$ (d) $9 \mu\text{F}$

- 60** In the arrangement of capacitors shown in figure, each capacitor is of $9 \mu\text{F}$, then the equivalent capacitance between the points A and B is [UP CPMT 2012]



- (a) $9 \mu\text{F}$ (b) $18 \mu\text{F}$ (c) $4.5 \mu\text{F}$ (d) $15 \mu\text{F}$

- 61** A capacitor having capacity of $2 \mu\text{F}$ is charged to 200 V and then the plates of the capacitor are connected to a resistance wire. The heat produced (in joule) will be [Manipal 2012]

- (a) 2×10^{-2} (b) 4×10^{-2}
(c) 4×10^4 (d) 4×10^{10}

- 62** The potential of a large liquid drop when eight liquid drops are combined is 20 V. Then, the potential of each single drop was [Manipal 2012]

- (a) 10 V (b) 7.5 V (c) 5 V (d) 2.5 V

- 63** The electric field in a certain region is given by $\mathbf{E} = 5\hat{i} - 3\hat{j}$ kV/m. The potential difference $V_B - V_A$ between points A and B, having coordinates (4, 0, 3) m and (10, 3, 0) m respectively, is equal to [AMU 2012]

- (a) 21 kV (b) -21 kV
(c) 39 kV (d) -39 kV

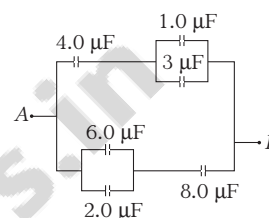
- 64** In a parallel plate capacitor with plate area A and charge Q, the force on one plate because of the charge on the other is equal to [AMU 2012]

- (a) $\frac{Q^2}{\epsilon_0 A^2}$ (b) $\frac{Q^2}{2\epsilon_0 A^2}$ (c) $\frac{Q^2}{\epsilon_0 A}$ (d) $\frac{Q^2}{2\epsilon_0 A}$

- 65** Two capacitors $3 \mu\text{F}$ and $4 \mu\text{F}$, are individually charged across a 6 V battery. After being disconnected from the battery, they are connected together with the negative plate of one attached to the positive plate of the other. What is the final total energy stored? [AMU 2012]

- (a) 1.26×10^{-4} J (b) 2.57×10^{-4} J
(c) 1.26×10^{-6} J (d) 2.57×10^{-6} J

- 66** The equivalent capacitance between A and B for the combination of capacitors shown in figure, where all capacitances are in microfarad is [AFMC 2012]



- (a) $6.0 \mu\text{F}$ (b) $4.0 \mu\text{F}$
(c) $2.0 \mu\text{F}$ (d) $3.0 \mu\text{F}$

- 67** It is possible to have a positively charged body at [AIIMS 2011]

- (a) zero potential (b) negative potential
(c) positive potential (d) All of these

- 68** If an electron is brought towards another electron, then the electric potential energy of the system [AIIMS 2011]

- (a) increases (b) decreases
(c) become zero (d) remaining the same

- 69** Electric potential of earth is taken to be zero because earth is a good [AIIMS 2011]

- (a) insulator (b) conductor
(c) semiconductor (d) dielectric

- 70** When a capacitor is connected to a battery, [AIIMS 2011]

- (a) a current flows in the circuit for sometime, then decreases to zero
(b) no current flows in the circuit at all
(c) an alternating current flows in the circuit
(d) None of the above

- 71** Which of the following is not true? [EAMCET 2011]

- (a) For a point charge, the electrostatic potential varies as $1/r$
(b) For a dipole, the potential depends on the position vector and dipole moment vector
(c) The electric dipole potential varies as $1/r$ at large distance
(d) For a point charge, the electrostatic field varies as $1/r^2$

ANSWERS

CHECK POINT 2.1

1. (a)	2. (a)	3. (c)	4. (c)	5. (b)	6. (b)	7. (c)	8. (b)	9. (a)	10. (c)
11. (b)	12. (a)	13. (c)	14. (a)	15. (a)					

CHECK POINT 2.2

1. (c)	2. (b)	3. (b)	4. (b)	5. (a)	6. (c)	7. (b)	8. (c)	9. (d)	10. (a)
11. (a)	12. (c)	13. (a)	14. (a)	15. (c)					

CHECK POINT 2.3

1. (b)	2. (b)	3. (c)	4. (a)	5. (c)	6. (c)	7. (b)	8. (d)	9. (c)	10. (a)
11. (d)	12. (a)	13. (c)	14. (d)	15. (a)					

CHECK POINT 2.4

1. (d)	2. (c)	3. (c)	4. (c)	5. (d)	6. (d)	7. (c)	8. (a)	9. (d)	10. (d)
11. (d)	12. (b)	13. (b)	14. (a)						

CHECK POINT 2.5

1. (c)	2. (c)	3. (d)	4. (c)	5. (b)	6. (b)	7. (a)	8. (c)	9. (d)	10. (a)
11. (c)	12. (a)	13. (b)	14. (a)	15. (d)					

(A) Taking it together

1. (c)	2. (a)	3. (a)	4. (a)	5. (d)	6. (a)	7. (a)	8. (c)	9. (c)	10. (c)
11. (b)	12. (a)	13. (b)	14. (a)	15. (c)	16. (a)	17. (a)	18. (a)	19. (b)	20. (a)
21. (b)	22. (a)	23. (d)	24. (c)	25. (a)	26. (c)	27. (a)	28. (c)	29. (d)	30. (c)
31. (c)	32. (b)	33. (d)	34. (b)	35. (b)	36. (c)	37. (b)	38. (b)	39. (a)	40. (c)
41. (d)	42. (a)	43. (a)	44. (d)	45. (a)	46. (b)	47. (b)	48. (a)	49. (c)	50. (d)
51. (c)	52. (d)	53. (a)	54. (d)	55. (c)	56. (d)	57. (c)	58. (b)	59. (c)	60. (b)
61. (a)	62. (d)	63. (a)	64. (b)	65. (a)	66. (c)	67. (b)	68. (a)	69. (b)	70. (c)
71. (b)	72. (a)	73. (c)	74. (b)	75. (d)	76. (d)	77. (c)	78. (d)	79. (d)	80. (b)
81. (d)	82. (a)	83. (b)	84. (a)	85. (d)	86. (d)	87. (b)	88. (c)	89. (b)	90. (d)
91. (a)	92. (c)	93. (d)	94. (b)	95. (c)	96. (b)	97. (d)	98. (a)	99. (a)	100. (d)
101. (b)	102. (b)	103. (b)	104. (d)	105. (d)	106. (c)	107. (a)	108. (c)	109. (b)	110. (d)
111. (b)	112. (c)	113. (d)	114. (b)	115. (b)	116. (b)	117. (d)	118. (a)	119. (a)	120. (d)
121. (a)	122. (c)	123. (c)	124. (c)	125. (b)	126. (c)	127. (d)	128. (c)	129. (d)	130. (c)
131. (d)	132. (c)	133. (a)	134. (c)	135. (c)	136. (a)	137. (c)	138. (c)	139. (b)	140. (c)
141. (a)	142. (a)	143. (b)	144. (b)	145. (d)	146. (b)	147. (c)	148. (a)	149. (c)	150. (d)
151. (d)	152. (d)	153. (c)	154. (d)	155. (c)	156. (b)	157. (c)	158. (a)	159. (b)	160. (c)
161. (b)	162. (a)	163. (b)	164. (a)	165. (b)	166. (b)	167. (b)	168. (c)	169. (a)	170. (d)
171. (d)	172. (b)	173. (b)	174. (c)	175. (d)	176. (b)	177. (d)	178. (a)	179. (a)	180. (c)
181. (b)	182. (c)	183. (d)	184. (a)	185. (c)	186. (b)	187. (b)	188. (b)	189. (c)	190. (c)
191. (c)	192. (c)	193. (d)	194. (b)	195. (c)	196. (d)	197. (d)	198. (b)	199. (b)	200. (a)
201. (d)	202. (c)	203. (d)	204. (c)	205. (b)	206. (a)	207. (c)	208. (a)	209. (c)	210. (c)
211. (b)	212. (c)	213. (c)							

Hints & Explanations

CHECK POINT 2.1

1 (a) Using the relation, $W = qV_a$

where, V_a is the electric potential at point A.

We have, $W = (4 \times 10^{-6})(10^4) = 4 \times 10^{-2} \text{ J}$

2 (a) $\therefore V = \frac{kq}{r}$, i.e. $V \propto \frac{1}{r}$

$\therefore$ Potential on smaller sphere will be more.

3 (c) Electric potential at a distance r due to point charge Q is

$$V = \frac{kQ}{r} \text{ and electric field at same point is } E = \frac{kQ}{r^2}$$

$$\Rightarrow E = \frac{kQ}{(kQ/V)^2} = \frac{V^2}{kQ} = \frac{(Q \times 10^{11})^2}{kQ} = 4\pi\epsilon_0 Q \times 10^{22} \text{ V/m}$$

$$\left(\because k = \frac{1}{4\pi\epsilon_0} \right)$$

4 (c) The electrical potential produced by the nucleus at the position of the electron,

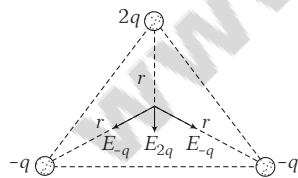
$$\begin{aligned} V &= 9 \times 10^9 \times \frac{q}{r} \\ &= 9 \times 10^9 \times \frac{(+1.6 \times 10^{-19})}{0.53 \times 10^{-10}} = 27.2 \text{ V} \end{aligned}$$

5 (b) Obviously, from charge configuration, at the centre electric field is non-zero. Potential at the centre due to charge

$$2q, V_{2q} = \frac{2q}{r} \text{ and potential due to } -q \text{ charge, } V_{-q} = -\frac{q}{r}$$

(where, r = distance of centre point)

$\therefore$ Total potential, $V = V_{2q} + V_{-q} + V_{-q} = 0$



6 (b) The electric field intensity E and electric potential V are related as $E = -\frac{dV}{dr}$ and for $V = \text{constant}$, $\frac{dV}{dr} = 0$

This imply that electric field intensity, $E = 0$.

7 (c) Potential difference, $\Delta V = \frac{W}{q} = \frac{2}{20} = 0.1 \text{ V}$

8 (b) Potential at A = Potential due to $(+q)$ charge + Potential due to $(-q)$ charge

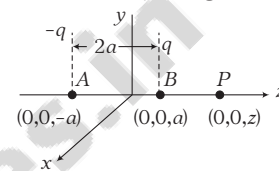
$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\sqrt{a^2 + b^2}} + \frac{1}{4\pi\epsilon_0} \cdot \frac{(-q)}{\sqrt{a^2 + b^2}} = 0$$

$$9 (a) V_A = 10^{10} \left[\frac{-5 \times 10^{-6}}{15 \times 10^{-2}} + \frac{2 \times 10^{-6}}{5 \times 10^{-2}} \right] = \frac{1}{15} \times 10^6 \text{ V}$$

$$V_B = 10^{10} \left[\frac{2 \times 10^{-6}}{15 \times 10^{-2}} - \frac{5 \times 10^{-6}}{5 \times 10^{-2}} \right] = -\frac{13}{15} \times 10^6 \text{ V}$$

$$\therefore W = q(V_A - V_B) = 3 \times 10^{-6} \left[\frac{1}{15} \times 10^6 - \left(-\frac{13}{15} \times 10^6 \right) \right] = 2.8 \text{ J}$$

10 (c) Potential at P due to $(+q)$ charge



$$V_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(z-a)}$$

$$\text{Potential at } P \text{ due to } (-q) \text{ charge, } V_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{-q}{(z+a)}$$

Total potential at P due to (AB) electric dipole,

$$\begin{aligned} V &= V_1 + V_2 \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(z-a)} - \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(z+a)} \\ &= \frac{2qa}{4\pi\epsilon_0(z^2 - a^2)} \end{aligned}$$

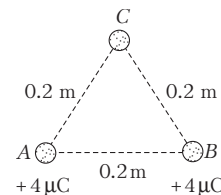
$$11 (b) \text{ Electric field, } E = \frac{V}{d} = \frac{10}{2 \times 10^{-2}} = 500 \text{ N/C}$$

12 (a) We have, $V = E \times d$

$$\Rightarrow \text{Distance, } d = \frac{V}{E} = \frac{3000}{500} = 6 \text{ m}$$

13 (c) Potential at C, $V = 2 \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} \right)$

$$= \left(9 \times 10^9 \times \frac{4 \times 10^{-6}}{0.2} \right) \times 2 = 36 \times 10^4 \text{ V}$$



14 (a) The electric potential $V(x, y, z) = 4x^2 \text{ V}$

$$\text{Now, } \mathbf{E} = - \left(\hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \right)$$

$$\text{Now, } \frac{\partial V}{\partial x} = 8x, \frac{\partial V}{\partial y} = 0 \text{ and } \frac{\partial V}{\partial z} = 0$$

Hence, $\mathbf{E} = -8x\hat{\mathbf{i}}$ V/m. So, at point (1m, 0, 2m)

$$\mathbf{E} = -8\hat{\mathbf{i}} \text{ V/m or } 8, \text{ along negative } X\text{-axis.}$$

- 15 (a) We have, electric field,

$$E = -\frac{dV}{dx} = -\frac{d}{dx}(5x^2 + 10x - 9) = -10x - 10$$

$$\therefore (E)_{x=1} = -10 \times 1 - 10 = -20 \text{ V/m}$$

• CHECK POINT 2.2

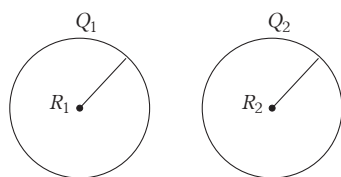
- 1 (c) Work done in displacing a charge particle is given by $W_{AB} = q(V_B - V_A)$ and the line integral of electrical field from point A to B gives potential difference $V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$

For equipotential surface, $V_B - V_A = 0$ and hence $W = 0$.

$$\begin{aligned} 3 (b) \text{ Potential, } V &= \frac{Kq}{r} = \frac{KZe}{r} = \frac{9 \times 10^9 \times 50 \times 16 \times 10^{-19}}{9 \times 10^{-15}} \\ &= 8 \times 10^6 \text{ V} \end{aligned}$$

- 4 (b) Since, potential inside the hollow sphere is same as that on the surface.

- 5 (a) Let Q_1 and Q_2 are the charges on sphere of radii R_1 and R_2 , respectively.



Surface charge density, $\sigma = \frac{\text{charge}}{\text{area}}$

According to given problem, $\sigma_1 = \sigma_2$

$$\frac{Q_1}{4\pi R_1^2} = \frac{Q_2}{4\pi R_2^2}$$

$$\therefore \frac{Q_1}{Q_2} = \frac{R_1^2}{R_2^2} \quad \dots(i)$$

In case of a charged sphere,

$$V_s = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

$$\therefore V_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R_1}, V_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{R_2}$$

$$\begin{aligned} \Rightarrow \frac{V_1}{V_2} &= \frac{Q_1}{R_1} \times \frac{R_2}{Q_2} = \frac{Q_1}{Q_2} \times \frac{R_2}{R_1} \\ &= \left(\frac{R_1}{R_2}\right)^2 \times \left(\frac{R_2}{R_1}\right) = \frac{R_1}{R_2} \quad [\text{using Eq. (i)}] \end{aligned}$$

- 6 (c) Let q' be the charge on the inner shell. Then, the potential of the inner is

$V =$ potential due to its own q' + potential due to the outer charge

$$= \frac{1}{4\pi\epsilon} \left(\frac{q}{r_1} + \frac{q}{r_2} \right)$$

But $V = 0$, because the inner shell is earthed.

$$\therefore \frac{q_1}{r_1} + \frac{q_2}{r_2} = 0 \quad q' = -q \left(\frac{r_1}{r_2} \right)$$

- 7 (b) Potential inside the sphere will be same as that on its surface, i.e. $V = V_{\text{surface}} = \frac{q}{10}$ stat volt, $V_{\text{out}} = \frac{q}{15}$ stat volt

$$\therefore \frac{V_{\text{out}}}{V} = \frac{2}{3} \Rightarrow V_{\text{out}} = \frac{2}{3}V$$

- 8 (c) Inside a conducting body, potential is same everywhere and equals to the potential at its surface.

- 9 (d) If charge acquired by the smaller sphere is q , then its potential, $120 = \frac{kq}{2}$... (i)

Whole charge comes to outer sphere.

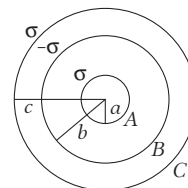
Also, potential of the outer sphere,

$$V = \frac{kq}{6} \quad \dots(ii)$$

From Eq. (i) and (ii), we get

$$V = 40 \text{ V}$$

- 10 (a) $V_A = \frac{\sigma}{\epsilon_0} (a - b + c)$



$$V_B = \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{c} - b + c \right)$$

$$\Rightarrow V_C = \frac{\sigma}{\epsilon_0} \left(\frac{a^2}{c} - \frac{b^2}{c} + c \right)$$

On putting $c = a + b \Rightarrow V_A = V_C \neq V_B$

- 12 (c) Potential, $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} = 9 \times 10^9 \times \frac{500 \times 10^{-6}}{1.0}$
 $= 4.5 \times 10^6 \text{ V}$

- 13 (a) The kinetic energy gained by a charge of q coulomb through a potential difference of V volt is

$$K = qV \text{ J}$$

$$K = (1.6 \times 10^{-19})(2000) = 3.2 \times 10^{-16} \text{ J}$$

We have, $K = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2K}{m}} = \frac{2 \times 3.2 \times 10^{-16}}{9 \times 10^{-31}} = \frac{8}{3} \times 10^7 \text{ m/s}$$

- 14 (a) Speed of the particle, $v = \sqrt{\frac{2QV}{m}}$

$$\Rightarrow v \propto \sqrt{Q} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{Q_A}{Q_B}} = \sqrt{\frac{16q}{4q}} = \frac{2}{1}$$

- 15 (c) The kinetic energy of the electron,

$$K = eV = \frac{1}{2}mv^2$$

$$\Rightarrow V = \frac{1}{2} \frac{mv^2}{e}$$

$$\text{Here, } v = \frac{c}{10} = \frac{3 \times 10^8}{10} = 3 \times 10^7 \text{ m/s}$$

$$V = \frac{1}{2} \frac{(9.0 \times 10^{-31})(3.0 \times 10^7)^2}{(1.6 \times 10^{-19})}$$

$$\text{Potential, } V = 2531 \text{ V}$$

• CHECK POINT 2.3

- 1 (b) $\Delta \text{PE} = \text{Work done by external agent}$
 $= (V_f q - V_i q), V_f > V_i \Rightarrow \Delta \text{PE} > 0$, i.e. PE will increase.

- 2 (b) Potential energy of the system will be given by

$$= \frac{(-e)(-e)}{4\pi\epsilon_0 r} = \frac{e^2}{4\pi\epsilon_0 r}$$

As, r decreases, potential energy increases.

- 3 (c) Work done, $W = U_f - U_i = 9 \times 10^9 \times Q_1 Q_2 \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$

$$\Rightarrow W = 9 \times 10^9 \times 12 \times 10^{-6} \times 8 \times 10^{-6} \left[\frac{1}{4 \times 10^{-2}} - \frac{1}{10 \times 10^{-2}} \right]$$

$$= 12.96 \text{ J} \approx 13 \text{ J}$$

- 4 (a) Since, the proton is moving against the direction of electric field, so work is done by the proton against electric field. It implies that electric field does negative work on the proton. Again, proton is moving in electric field from low potential region to high potential region, hence its potential energy increases.

- 5 (c) Electric potential energy, $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d}$

$$\therefore U = \frac{(9 \times 10^9) \times (1.6 \times 10^{-19}) \times (-1.6 \times 10^{-19})}{10^{-10}} \text{ J}$$

$$= -9 \times 10^9 \times 1.6 \times 10^{-19} \text{ eV} = -14.4 \text{ eV}$$

Note In the solution given all the values are positive. It is important to mention the sign.

- 6 (c) Potential energy as well as force are positive, if there is repulsion between the particles and negative, if there is attraction.

We take only the magnitude of values when discussing decrease or increase of energy.

$$\text{As, } U = \frac{Q_1 Q_2}{4\pi\epsilon_0 r}$$

Plus or minus i.e., whether both are of the same sign or different, if r decreases, the value increase. Therefore, option (c) is wrong.

- 7 (b) Potential energy of charges Q_1 and Q_2 at 10 cm apart,

$$U_i = \frac{1}{4\pi\epsilon_0} \frac{12 \times 10^{-6} \times 5 \times 10^{-6}}{0.1}$$

$$= \frac{9 \times 10^9 \times 60 \times 10^{-12}}{0.1} = 54 \times 10^{-1} = 5.4 \text{ J}$$

Potential energy of charge Q_1 and Q_2 at 6 cm apart,

$$U_2 = \frac{9 \times 10^9 \times 60 \times 10^{-12}}{0.06} = 9 \text{ J}$$

$$\therefore \text{Work done} = (9 - 5.4) \text{ J} = 3.6 \text{ J}$$

- 8 (d) We have, $AB + AC = 12 \text{ cm}$... (i)

$$AB \cdot AC = 32 \text{ cm}^2$$

$$\therefore AB - AC = \sqrt{(AB + AC)^2 - 4AB \cdot AC}$$

$$AB - AC = 4 \text{ cm}$$

From Eqs. (i) and (ii), we get

$$AB = 8 \text{ cm}, AC = 4 \text{ cm}$$

Potential energy at point A,

$$U_A = \frac{1}{4\pi\epsilon_0} \left[\frac{q_A q_B}{AB} + \frac{q_A q_C}{AC} \right]$$

$$= \frac{9 \times 10^9 \times 4 \times 10^{-12}}{10^{-2}} \left(\frac{1}{8} + \frac{1}{4} \right) = 1.35 \text{ J}$$

- 9 (c) Potential energy, $U = \frac{1}{4\pi\epsilon_0} \left[\frac{Q_1 Q_2}{r_1} + \frac{Q_2 Q_3}{r_2} + \frac{Q_1 Q_3}{r_3} \right]$

$$\text{Net potential energy, } U_{\text{net}} = \frac{3}{4\pi\epsilon_0} \cdot \frac{q^2}{l}$$

- 10 (a) Potential energy of the system,

$$U = K \frac{Qq}{l} + \frac{Kq^2}{l} + \frac{KqQ}{l} = 0$$

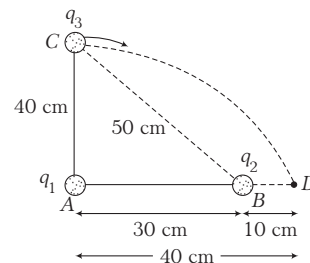
$$\Rightarrow \frac{Kq}{l} (Q + q + Q) = 0 \Rightarrow Q = -\frac{q}{2}$$

- 11 (d) Length of the diagonal of a cube having each side b is $\sqrt{3}b$. So, distance of centre of cube from each vertex is $\frac{\sqrt{3}b}{2}$.

Hence, potential energy of the given system of charge is

$$U = 8 \times \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{(-q)(q)}{\frac{\sqrt{3}b}{2}} \right] = \frac{-4q^2}{\sqrt{3}\pi\epsilon_0 b}$$

- 12 (a) Change in potential energy $(\Delta U) = U_f - U_i$



$\Rightarrow$ Potential energy,

$$\Delta U = \frac{1}{4\pi\epsilon_0} \left[\left(\frac{q_1 q_3}{0.4} + \frac{q_2 q_3}{0.1} \right) - \left(\frac{q_1 q_3}{0.4} + \frac{q_2 q_3}{0.5} \right) \right]$$

$$\Rightarrow \Delta U = \frac{1}{4\pi\epsilon_0} [8q_2 q_3] = \frac{q_3}{4\pi\epsilon_0} (8q_2)$$

$$\therefore k = 8q_2$$

13 (c) Torque, $\tau_{\max} = pE = q(2l)E = 2 \times 10^{-6} \times 0.01 \times 5 \times 10^5$
 $= 10 \times 10^{-3} \text{ N-m}$

14 (d) Work done, $W = pE(1 - \cos \theta) = pE(1 - \cos 180^\circ)$

$$W = pE[1 - (-1)] = 2pE$$

15 (a) Given, $q_1 = 1\text{C}$, $q_2 = 2\text{C}$, $q_3 = 3\text{C}$ and $r_1 = 100\text{ cm} = 1\text{m}$

$$\begin{aligned} \text{Initial PE of system, } U_1 &= \frac{1}{4\pi\epsilon_0 r_1} (q_1 q_2 + q_2 q_3 + q_3 q_1) \\ &= \frac{9 \times 10^9}{1} (1 \times 2 + 2 \times 3 + 3 \times 1) \\ &= 99 \times 10^9 \text{ J} \end{aligned}$$

When $r_2 = 50\text{ cm} = 0.5\text{ m}$

$$\begin{aligned} \text{Final PE of system, } U_2 &= \frac{1}{4\pi\epsilon_0 r_2} (q_1 q_2 + q_2 q_3 + q_3 q_1) \\ &= \frac{9 \times 10^9}{0.5} (1 \times 2 + 2 \times 3 + 3 \times 1) = 2 \times 99 \times 10^9 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Work done, } W &= U_2 - U_1 = 2 \times 99 \times 10^9 - 99 \times 10^9 \\ &= 99 \times 10^9 \text{ J} = 9.9 \times 10^{10} \text{ J} \end{aligned}$$

• CHECK POINT 2.4

1 (d) H_2 , O_2 , N_2 etc., are not polar dielectrics.

2 (c) Volume of 8 small drops = Volume of big drop

$$8 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3 \Rightarrow R = 2r$$

As, capacity is proportional to r , hence capacity become 2 times.

3 (c) Capacity of a spherical conductor is $C = 4\pi\epsilon_0 R$.

4 (c) For spherical conductor,

$$C = 4\pi\epsilon_0 R$$

$$\therefore V = \frac{4}{3} \pi R^3 \text{ and } A = 4\pi R^2$$

$$\therefore R = \frac{3V}{A}$$

$$\therefore \text{Capacitance, } C = 12\pi\epsilon_0 \frac{V}{A}$$

5 (d) Given, $2\pi R = 2 \Rightarrow R = \frac{1}{\pi}$

For sphere $C = 4\pi\epsilon_0 K R$

$$\Rightarrow C = \frac{1}{9 \times 10^9} \times \frac{1}{\pi} \times 80 \quad (\text{for water } K = 80)$$

Capacitance of sphere, $C = 2828.28 \text{ pF} \approx 2800 \text{ pF}$

6 (d) We have, $C = \frac{\epsilon_0 A}{d} \Rightarrow C \propto \frac{1}{d}$

Therefore, the capacity of parallel plate condenser depends on the separation between the plates.

7 (c) Since, aluminium is a metal and very thin, therefore field inside this will be zero. Hence, it would not affect the field in between the two plates, so capacity $= \frac{q}{V} = \frac{q}{Ed}$, remains unchanged.

8 (a) The potential difference across the parallel plate capacitor,
 $V = 10 - (-10) = 20 \text{ volt}$

$$\therefore \text{Capacitance} = \frac{Q}{V} = \frac{40}{20} = 2 \text{ F}$$

9 (d) $C = \frac{\epsilon_0 A}{d}$. As $A \rightarrow \frac{1}{2}$ times and $d \rightarrow 2$ times

$$\text{So, } C \rightarrow \frac{1}{4} \text{ times, i.e. } C' = \frac{1}{4} C = \frac{12}{4} = 3 \mu\text{F}$$

10 (d) Capacitance, $C_{\text{medium}} = K C_{\text{air}}$

$$\Rightarrow K = \frac{C_{\text{medium}}}{C_{\text{air}}} = \frac{110}{50} = 2.20$$

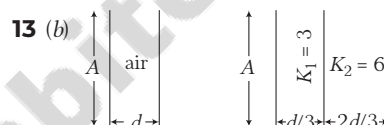
11 (d) Charge on capacitor, when its potential becomes 10 V,

$$Q = CV = 500 \times 10^{-6} \times 10 = 5000 \times 10^{-6} \mu\text{C}$$

$$\therefore \text{Required time} = \frac{5000 \times 10^{-6}}{100 \times 10^{-6}} = 50 \text{ s}$$

12 (b) $C = \frac{\epsilon_0 A}{d} = 1 \text{ pF}$ and $C = \frac{K \epsilon_0 A}{2d} = 2 \text{ pF}$

$$\therefore K = 4.$$



$$\text{Capacitance in air, } C_{\text{air}} = \frac{\epsilon_0 A}{d} = 9$$

Capacitance in medium,

$$\begin{aligned} \frac{1}{C_{\text{med}}} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{K_1 \epsilon_0 A} + \frac{d}{K_2 \epsilon_0 A} \\ \Rightarrow C_{\text{med}} &= \frac{K_1 K_2 \epsilon_0 A}{K_1 d_2 + K_2 d_1} \\ &= \frac{3 \times 6 \times \epsilon_0 A}{3 \times 2d/3 + 6 \times d/3} = \frac{18}{4} \times 9 = 40.5 \text{ pF} \end{aligned}$$

14 (a) Parallel plate capacitor, $C = K \epsilon_0 A / d$

As, given in figure, for series combination,

$$\frac{1}{C'} = \frac{1}{\frac{\epsilon_0 A}{d}} + \frac{1}{\frac{2\epsilon_0 A}{d}} \Rightarrow C = \frac{4}{3} \frac{\epsilon_0 A}{d}$$

• CHECK POINT 2.5

1 (c) Capacitor are in series, $\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C}$

$$\therefore \text{Capacitance, } C' = \frac{C}{3}$$

Total voltage of the series combination,

$$V' = V_1 + V_2 + V_3 = V + V + V = 3V$$

2 (c) In series, $\frac{1}{C} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$$\Rightarrow \text{Capacitance, } C = \frac{2}{3} \text{ F}$$

3 (d) Equivalent capacitance = $\frac{2 \times 3}{2 + 3} = \frac{6}{5} \mu\text{F}$

Total charge by $Q = CV = \frac{6}{5} \times 1000 = 1200 \mu\text{C}$

Potential (V) across $2 \mu\text{F}$ is $V = \frac{Q}{C} = \frac{1200}{2} = 600 \text{ V}$

$\therefore$ Potential on inner plates = $1000 - 600 = 400 \text{ V}$

4 (c) Capacitors are in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{8}$

$\Rightarrow C_{\text{eq}} = \frac{8}{13} \mu\text{F}$

Total charge, $Q = C_{\text{eq}}V = \frac{8}{13} \times 13 = 8 \mu\text{C}$

Potential difference across $2 \mu\text{F}$ capacitor = $\frac{8}{2} = 4 \text{ V}$

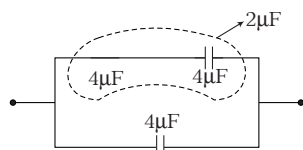
5 (b) Capacitance, $C_1 = \frac{C}{4}$ (series)

Capacitance, $C_2 = 4C$ (parallel)

The ratio of capacitance,

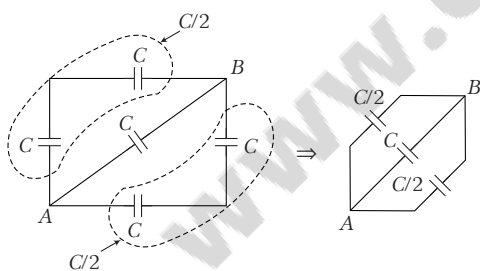
$\therefore \frac{C_1}{C_2} = \frac{C/4}{4C} = \frac{1}{16}$

6 (b) The given circuit can be drawn as follows



Effective capacitance, $C_{AB} = 2 + 4 = 6 \mu\text{F}$

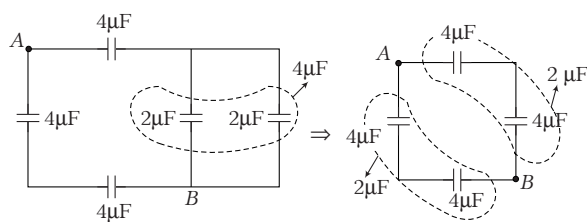
7 (a) The given circuit can be simplified as follows



Equivalent capacitance between A and B is

$C_{AB} = \frac{C}{2} + \frac{C}{2} + C = 2C$

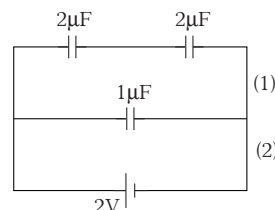
8 (c) The given circuit can be simplified as follows



Equivalent capacitance between A and B is $C_{AB} = 2 + 2 = 4 \mu\text{F}$

9 (d) Potential difference across both the lines is same, i.e. 2V. Hence, charge flowing in line (2).

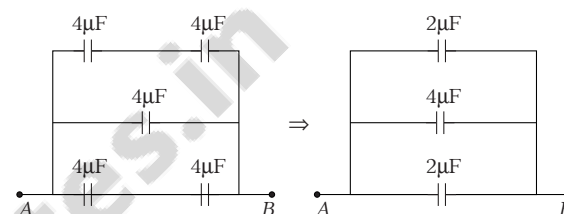
$Q = \left(\frac{2 \times 2}{2 + 2} \right) = 2 \mu\text{C}$



So, charge on each capacitor in line (2) is $2 \mu\text{C}$

and charge in line (1) is $Q = 2 \times 1 = 2 \mu\text{C}$.

10 (a)



Therefore, capacitor $2 \mu\text{F}$, $4 \mu\text{F}$ and $2 \mu\text{F}$ are in parallel.

So, equivalent capacitance between A and B

$C_{AB} = 2 + 4 + 2 = 8 \mu\text{F}$

11 (c) The energy stored in a capacitor is

$U = \frac{1}{2} CV^2$

So, potential difference, $V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2 \times 50}{100 \times 10^{-6}}} = 1000 \text{ V}$

12 (a) In series capacitance, $C' = \left(\frac{C_1}{n_1} \right)$ and $V' = 4V$

Energy, $U' = \frac{1}{2} C' V'^2 = \frac{1}{2} \left(\frac{C_1}{n_1} \right) (4V)^2$

In parallel capacitance, $C'' = n_2 C_2$ and $V'' = V$

$U'' = \frac{1}{2} C'' V''^2 = \frac{1}{2} (n_2 C_2) V^2$

Given, $\frac{1}{2} \left(\frac{C_1}{n_1} \right) (4V)^2 = \frac{1}{2} (n_2 C_2) V^2$

$\Rightarrow C_2 = \frac{16C_1}{n_1 n_2}$

13 (b) Energy of capacitor, $U = \frac{1}{2} \frac{Q^2}{C}$

$\therefore 1.21 U = \frac{1}{2} \frac{(Q + 2)^2}{C}$

$\therefore \frac{1.21}{1} = \frac{(Q + 2)^2}{Q^2}$

$\Rightarrow \sqrt{\frac{1.21}{1}} = \frac{Q + 2}{Q}$

$\Rightarrow 1.1Q = Q + 2$

Charge on the capacitor, $Q = 20 \text{ C}$

$$\begin{aligned}
 \text{14 (a) Initial energy} &= \frac{1}{2} \times 1 \times 10^{-6} \times (30)^2 = 450 \times 10^{-6} \text{ J} \\
 \text{Final energy} &= \frac{1}{2} (C_1 + C_2) V^2 \quad \left(\because V = \frac{V_1 C_1 + V_2 C_2}{C_1 + C_2} \right) \\
 &= \frac{1}{2} \times 3 \times 10^{-6} \times (10)^2 = 150 \times 10^{-6} \text{ J} \\
 \text{Loss of energy} &= (450 - 150) \times 10^{-6} \text{ J} = 300 \times 10^{-6} = 300 \mu\text{J}
 \end{aligned}$$

$$\text{15 (d) Initial energy stored in the capacitor,} \\
 U_i = \frac{1}{2} C V^2 = \frac{1}{2} C \times (50)^2 = \frac{1}{2} C (50)^2 \quad \dots (i)$$

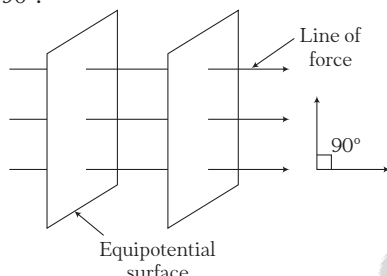
After 2s, when the potential drops by 10 V, the final potential is 40 V.

$$\text{Final energy stored in the capacitor, } U_f = \frac{1}{2} C (40)^2 \quad \dots (ii)$$

$$\text{Fraction of energy stored} = \frac{U_f}{U_i} = \frac{\frac{1}{2} C (40)^2}{\frac{1}{2} C (50)^2} = \left(\frac{40}{50} \right)^2 = 0.64$$

(A) Taking it together

- 1 (c) Angle between equipotential surface and line of force is 90° .



$$\text{2 (a) Potential, } V = E \cdot d$$

$$\therefore \text{Distance from the point charge, } d = \frac{V}{E} = \frac{3000}{500} = 6 \text{ m}$$

$$\text{3 (a) } \because \text{Potential difference} = \frac{\text{Work done}}{\text{Charge}}$$

$$\begin{aligned}
 \therefore W &= qV \\
 10 &= 5 \times V \\
 \therefore V &= 2 \text{ V}
 \end{aligned}$$

- 4 (a) A negative charge when moves from higher potential to lower potential, its velocity increases.

$$\text{7 (a) Work done, } W_{A \rightarrow B} = U_B - U_A = q (V_B - V_A)$$

$$\begin{aligned}
 \text{Potential difference, } V_B - V_A &= \frac{W_{A \rightarrow B}}{q} \\
 &= \frac{10 \times 10^{-3}}{5 \times 10^{-6}} = 2000 \text{ V} = 2 \text{ kV}
 \end{aligned}$$

- 10 (c) The electric dipole potential varies as $1/r$ at large distance, is not true, because $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2}$, i.e. $V \propto \frac{1}{r^2}$.

$$\text{11 (b) Energy, } U = \frac{1}{2} \frac{q^2}{V}$$

$\therefore$ It is in the form of potential energy.

$$\text{12 (a) } U = \frac{Q^2}{2C}$$

When a slab of dielectric constant K is inserted, then $C' = CK$

$$\begin{aligned}
 U' &= \frac{Q^2}{2C'} = \frac{Q^2}{2CK} \\
 \Rightarrow U' &= \frac{U_0}{K}
 \end{aligned}$$

- 13 (b) Potential at a point in a field is defined as the amount of work done in bringing a unit positive test charge (q) from infinity to that point along any arbitrary path.

$$\text{Potential, } V = \frac{W}{q}$$

$$\therefore \text{Work done, } W = qV$$

- 14 (a) Force between the plates of a parallel plate capacitor is given by

$$|F| = \frac{\sigma^2 A}{2\epsilon_0} = \frac{Q^2}{2\epsilon_0 A} = \frac{CV^2}{2d}$$

- 15 (c) The positively charged particle experiences electrostatic force along the direction of electric field, i.e. from high electrostatic potential to low electrostatic potential. As, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

- 16 (a) In this problem, the collection of charges, whose total sum is not zero, but with regard to great distance can be considered as a point charge. The equipotentials due to point charges are spherical in shape, as electric potential due to point charge q is given by

$$V = k_e \frac{q}{r}$$

This suggest that, electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, form spherical surface.

- 17 (a) Electron is moving in opposite direction of field, so field will produce an accelerating effect on electron.

- 18 (a) Capacitance of a metallic sphere,

$$C = 4\pi\epsilon_0 r = 1 \times 10^{-6}$$

$$\begin{aligned}
 \Rightarrow r &= 10^{-6} \times 9 \times 10^9 \\
 &= 9 \text{ km}
 \end{aligned}$$

- 19 (b) Unit of E in SI system is $E = \frac{F}{q_0}$ = newton/coulomb

$$\text{As, } E = - \frac{dV}{dr}$$

So, unit of E is also volt/metre.

$$\text{Also, } q = CV$$

$$\therefore q = CE d$$

$$E = \frac{q}{Cd} = \frac{qV}{qd} = \frac{W}{qd}$$

$$= \frac{\text{joule}}{\text{coulomb-metre}}$$

while J/C is the unit of electric potential.

$$\begin{aligned}
 (\because V = Ed) \\
 (\because C = \frac{q}{V})
 \end{aligned}$$

20 (a) Inside a sphere, potential remains constant.

21 (b) Radius of spherical conductor, $R = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

According to given situation, $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{5 \times 10^{-2}}$$

$$\Rightarrow \frac{q}{4\pi\epsilon_0} = 5 \times 10^{-2} V \quad \dots(i)$$

Again, electric potential at distance $r = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$ from the centre,

$$V' = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{30 \times 10^{-2}}$$

$$= \left(\frac{q}{4\pi\epsilon_0} \right) \times \frac{1}{30 \times 10^{-2}}$$

$$= \frac{5 \times 10^{-2}}{30 \times 10^{-2}} V$$

$$= \frac{V}{6} \quad [\text{from Eq. (i)}]$$

22 (a) Electric field is given by

$$E = - \frac{\Delta V}{\Delta r} = \frac{30 - (-10)}{2 \times 10^{-2}} = 2000 \text{ V/m}$$

23 (d) Potential at a point due to electric dipole

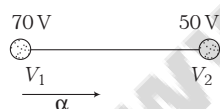
$$V = \frac{p \cos \theta}{r^2}$$

If $\theta = 0^\circ$, then V will be maximum

and if $\theta = 180^\circ$, then V will be minimum.

24 (c) $U = -pE \cos \theta$, U is minimum at $\theta = 0^\circ$.

25 (a) Kinetic energy gained by α -particle,



$$\text{KE} = q \cdot \Delta V = q(V_1 - V_2) = 2e(V_1 - V_2)$$

$$= 2 \times 1.6 \times 10^{-19} (70 - 50) = 40 \text{ eV}$$

26 (c) $\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = \frac{1}{2}m\left(\frac{qE}{m}\right)^2 t^2 \quad \left(\because a = \frac{qE}{m}\right)$

$$= \frac{E^2 q^2 t^2}{2m}$$

27 (a) Work done, $W = U_f - U_i = \frac{q_1 q_2}{4\pi\epsilon_0} \left[\frac{1}{r_f} - \frac{1}{r_i} \right]$

$$= (9 \times 10^9) (10^{-12}) (12) (8) \left[\frac{1}{0.06} - \frac{1}{0.1} \right] = 5.8 \text{ J}$$

28 (c) Capacitance, $C = 4\pi\epsilon_0 R$

$$= \frac{1}{9 \times 10^9} \times 6408 \times 10^3$$

$$= 712 \mu\text{F}$$

29 (d) If metallic slab fills the complete space between the plates or both plates are joined through a metallic wire, then capacitance became infinite.

30 (c) Energy stored, $W = \frac{Q^2}{2C}$

$$\therefore W \propto Q^2 \quad (\text{if } C \text{ is constant})$$

or $\frac{W}{W'} = \frac{Q^2}{(2Q)^2} = \frac{Q^2}{4Q^2}$

$$\therefore W' = 4W$$

31 (c) Heat produced in capacitor = Energy of charged capacitor

$$= \frac{1}{2} CV^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (100)^2 = 0.01 \text{ J}$$

32 (b) The energy stored, $U = \frac{1}{2} (nC) V^2$

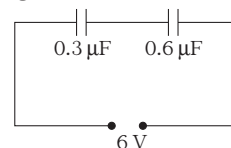
33 (d) Change in energy of condenser,

$$\Delta U = U_2 - U_1 = \frac{1}{2} C_2 V^2 - \frac{1}{2} C_1 V^2$$

$$= \frac{V^2}{2} (C_2 - C_1) = \frac{(100)^2}{2} (10 \times 10^{-6} - 2 \times 10^{-6})$$

$$= 4 \times 10^{-2} \text{ J}$$

34 (b) In series, charge remains same on both capacitors



$$\therefore U = \frac{Q^2}{2C} \text{ or } U \propto \frac{1}{C} \quad (\because Q_1 = Q_2)$$

$$\therefore \frac{U_1}{U_2} = \frac{C_2}{C_1} = \frac{0.6}{0.3}$$

$$\therefore U_1 : U_2 = 2 : 1$$

35 (b) In parallel combination, $C_{\text{net}} = C_1 + C_2$

$$V = \frac{q_{\text{net}}}{C_{\text{net}}} = \frac{C_1 V_0}{C_1 + C_2}$$

36 (c) Let Q_0 , C_0 , V_0 and U_0 be the charge, capacitance, potential difference and stored energy, respectively before the dielectric slab of dielectric constant K is inserted.

After inserting dielectric slab, charge will remain same, i.e.

$$Q' = Q_0.$$

Potential difference will decrease as $V' = \frac{V_0}{K}$

Stored energy will decrease as, $U = \frac{V_0}{K}$.

37 (b) Potential on parallel plate capacitor, $V = \frac{Q}{C}$.

Also, capacity of parallel plate capacitor is given by

$$C = \frac{\epsilon_0 K A}{d}$$

$$\therefore V = \frac{Qd}{\epsilon_0 K A}$$

$$\Rightarrow V \propto d$$

So, on increasing the distance between plates of capacitor, the potential difference between plates also increases.

$$38 \text{ (b) Potential difference} = V_i - V_o = \frac{q_i}{4\pi\epsilon_0} \left(\frac{1}{r_i} - \frac{1}{r_o} \right)$$

If q_i is positive, $V_i - V_o$ = positive or $V_i > V_o$.

39 (a) Dielectric constant,

$$K = \frac{\text{Permittivity of medium}}{\text{Permittivity of free space}}$$

$$K = \frac{\epsilon}{\epsilon_0}$$

$\therefore$ Permittivity of water, $\epsilon = K\epsilon_0$

$$= 81 \times 8.85 \times 10^{-12}$$

$$= 7.16 \times 10^{-10} \text{ MKS units}$$

40 (c) Both the conductors carry equal and opposite charges. So, after connecting by a wire, there will be no charge in any conductor. Hence, all the stored energy will be neutralised.

$$\text{Loss of energy} = 2 \left(\frac{1}{2} CV^2 \right) = CV^2$$

41 (d) Net charge distributes in direct ratio of capacity (or radius in case of spherical conductor)

$$\therefore q'_A = q_{\text{net}} \left[\frac{4}{4+6} \right] = 120 \left[\frac{4}{4+6} \right]$$

$$= 48 \mu\text{C}$$

$$\text{So, } \Delta q = 80 - 48 = 32 \mu\text{C}$$

$$42 \text{ (a) Force, } F_e = qE = q \left(\frac{V}{d} \right) = (4 \times 10^{-6}) \left(\frac{2000}{2 \times 10^{-3}} \right) = 4\text{N}$$

$$43 \text{ (a) } \left(\frac{q}{R} \right)_1 = \left(\frac{q}{R} \right)_2$$

$$\therefore V_1 = V_2$$

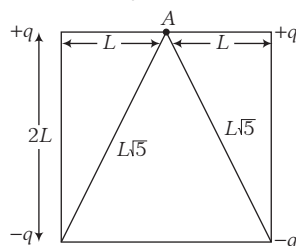
So, the charge will not flow at all.

44 (d) In redistribution of charges, there is always loss of energy, unless their potentials are same or $Q_1R_2 = Q_2R_1$.

45 (a) As per the given condition in the question, electricity will flow from the smaller sphere to the bigger one, when the smaller one is placed inside the bigger one and connected by wire.

$$46 \text{ (b) Work done, } W = \Delta U = U_f - U_i = (3U) - U = 2U$$

$$47 \text{ (b) Potential, } V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$



$$\text{Here, } V = 2V_{+ve} + 2V_{-ve}$$

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{2q}{L} - \frac{2q}{L\sqrt{5}} \right]$$

The electric potential at mid-point A,

$$V = \frac{2q}{4\pi\epsilon_0 L} \left(1 - \frac{1}{\sqrt{5}} \right)$$

48 (a) The potential at the centre of the sphere is 80 V because it remains same at each point inside the metallic hollow sphere.

49 (c) The energy stored in the capacitor,

$$U = \frac{1}{2} CV^2$$

$$U = \frac{1}{2} \left(\frac{A\epsilon_0}{d} \right) (Ed)^2 \quad \left(\because C = \frac{A\epsilon_0}{d} \text{ and } V = Ed \right)$$

$$U = \frac{1}{2} \epsilon_0 E^2 Ad$$

50 (d) Given, $q_1 = 5 \mu\text{C}$, $q_2 = 10 \mu\text{C}$ and $r = 0.5 \text{ m}$

$$\text{Work done, } W = K \frac{q_1 q_2}{r}$$

$$= 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 10 \times 10^{-6}}{0.5}$$

$$= 9 \times 10^{-1} \text{ J}$$

51 (c) Here, $q = 4 \times 10^{-3} \text{ C}$, $E = 5 \text{ V/m}$,

$$t = 10 \text{ s, } m = 2 \times 10^{-3} \text{ kg}$$

$$\text{The KE of charged particle} = \frac{q^2 E^2 t^2}{2m}$$

$$= \frac{(4 \times 10^{-3})^2 \times (5)^2 \times (10)^2}{2 \times 2 \times 10^{-3}}$$

$$\therefore \text{KE} = 10 \text{ J}$$

52 (d) The potential which is required to ionise the electron from outermost shell of mercury is called ionisation potential. The electric field strength is given by $E = \frac{V}{d}$.

where, d is distance between plates creating electric field.

$$\text{Given, } V = 10.39 \text{ V, } E = 1.5 \times 10^6 \text{ V/m}$$

Distance travelled by electron to gain ionisation energy,

$$\therefore d = \frac{V}{E} = \frac{10.39}{1.5 \times 10^6} \text{ m}$$

53 (a) By using charge conservation,

$$0.2 \times 600 = (0.2 + 1) V$$

$$V = \frac{0.2 \times 600}{1.2} = 100 \text{ V}$$

54 (d) Here, $q = 8 \times 10^{-18} \text{ C}$, $C = 100 \mu\text{F} = 10^{-4} \text{ F}$

$$\text{Potential, } V = \frac{q}{C} = \frac{8 \times 10^{-18}}{10^{-4}} = 8 \times 10^{-14} \text{ V}$$

$$\text{Work done} = \frac{1}{2} qV = \frac{1}{2} \times 8 \times 10^{-18} \times 8 \times 10^{-14}$$

$$= 32 \times 10^{-32} \text{ J}$$

55 (c) We have, $K = \frac{t}{t-d'}$

$$2 = \frac{1}{1-d'} \Rightarrow d' = \frac{1}{2} \text{ mm}$$

So, now distance $= 3 + \frac{1}{2} = 3.5 \text{ mm}$

56 (d) Electric field, $E = -\frac{dV}{dr} = -\frac{d}{dx}(3x^2) = -6x$

At point (2, 0, 1)

Electric field, $E = -12 \text{ Vm}^{-1}$

57 (c) Kinetic energy, $K = \frac{1}{2}mv^2 = eV$

$$\Rightarrow \text{Speed of the electron, } v = \sqrt{\frac{2eV}{m}}$$

58 (b) Kinetic energy,

$$= e(V_A - V_B) = 1.6 \times 10^{-19} (70 - 50) = 3.2 \times 10^{-18} \text{ J}$$

59 (c) By using, $W = Q \cdot \Delta V$

Potential difference,

$$\Delta V = \frac{W}{Q} = \frac{2}{20} = 0.1 \text{ V}$$

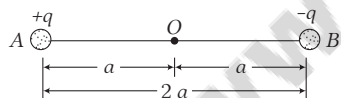
60 (b) By using, $KE = QV \Rightarrow 4 \times 10^{20} \times 1.6 \times 10^{-19} = 0.25 \times V$

$$\text{Potential difference, } V = \frac{4 \times 10^{20} \times 1.6 \times 10^{-19}}{0.25} = 256 \text{ V}$$

61 (a) By using $KE = QV \Rightarrow KE = 1.6 \times 10^{-19} \times 100$
 $= 1.6 \times 10^{-17} \text{ J}$

62 (d) Conducting surface behaves as equipotential surface.

63 (a) Potential at O due to charge at A,



$$\therefore V_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a}$$

Potential at O due to charge at B,

$$V_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{(-q)}{a}$$

$\therefore$ Potential at mid-point O,

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a} + \frac{1}{4\pi\epsilon_0} \cdot \frac{(-q)}{a} = 0$$

64 (b) As electric potential of spheres are same,

$$V_A = V_B$$

$$\Rightarrow E_A \cdot a = E_B \cdot b$$

$$\therefore \frac{\sigma_A a}{\epsilon_0} = \frac{\sigma_B b}{\epsilon_0} \text{ or } \frac{\sigma_A}{\sigma_B} = \frac{b}{a}$$

65 (a) Given, $C_1 = 2 \mu\text{F}$, $V_1 = 50 \text{ V}$

$$C_2 = 1 \mu\text{F}, V_2 = 20 \text{ V}$$

Common potential,

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{(2 \times 50 + 1 \times 20) \times 10^{-6}}{(2 + 1) \times 10^{-6}} = 40 \text{ V}$$

Loss of energy,

$$\begin{aligned} \Delta u &= \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2 \\ &= \frac{1}{2} \frac{2 \times 1}{2 + 1} (50 - 20)^2 \times 10^{-6} \\ &= 300 \times 10^{-6} = 300 \mu\text{J} \end{aligned}$$

66 (c) A, B, C, D, E lies on equipotential surface. On sphere, the surface potential is same. So,

$$W_{AB} = W_{AC} = W_{AD} = W_{AE} = q(V_f - V_i) = \text{zero}$$

67 (b) We have, $E_0 = -\frac{dV}{dx}$ or $dV = -E_0 dx$

On integrating both sides, we get

$$\int dV = - \int E_0 dx$$

$$\Rightarrow V_x = -x E_0$$

68 (a) Force, $F = qE = q \left(\frac{V}{d} \right)$

$\therefore$ Potential difference between two points,

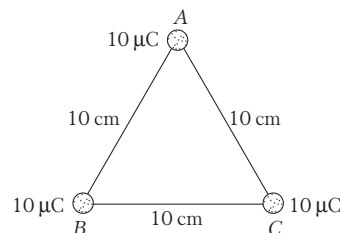
$$V = \frac{F \cdot d}{q} = \frac{(3000)(10^{-2})}{3} = 10 \text{ V}$$

69 (b) When charge particle enters in a potential field, then

$$\frac{1}{2} mv^2 = qV$$

$$\therefore v = \sqrt{\left(\frac{2qV}{m} \right)} \therefore \frac{v_A}{v_B} = \sqrt{\frac{q}{4q}} = \frac{1}{2}$$

70 (c)



$$U_{AB} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$U_{BC} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$U_{AC} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(10)(10) \times 10^{-12}}{10 \times 10^{-2}}$$

$$\begin{aligned} \therefore U_{\text{total}} &= U_{AB} + U_{BC} + U_{CA} \\ &= \frac{3}{4\pi\epsilon_0} \left[\frac{100 \times 10^{-12}}{10 \times 10^{-2}} \right] = 27 \text{ J} \end{aligned}$$

71 (b) When charge particle enters in uniform electric field, then force on charged particle, $F = qE$

Also, $F = ma$

$$\therefore ma = qE$$

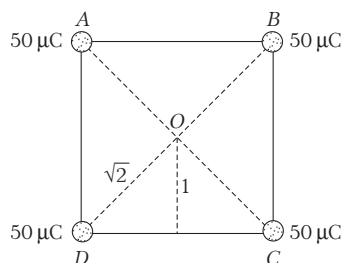
or acceleration of the particle,

$$a = \frac{qE}{m} = \frac{3 \times 10^{-3} \times 80}{20 \times 10^{-3}} = 12 \text{ m/s}^2$$

So, from equations of motion,

$$v = u + at = 20 + 12 \times 3 = 56 \text{ m/s}$$

72 (a)



$$\text{Potential, } V_A = V_B = V_C = V_D = V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{(50 \times 10^{-6})}{(\sqrt{2}/2)}$$

∴ Potential at the centre of square,

$$\begin{aligned} V_0 &= 4V \\ V_0 &= 4 \times \left[\frac{9 \times 10^9 \times 50 \times 10^{-6}}{\sqrt{2}} \right] \\ &= 90\sqrt{2} \times 10^4 \text{ V} \end{aligned}$$

Work done in bringing a charge ($q = 50 \mu\text{C}$) from ∞ to centre of the square is

$$\therefore W = qV_0 = 50 \times 10^{-6} \times 90\sqrt{2} \times 10^4 = 63.64 \text{ J} \approx 64 \text{ J}$$

73 (c)

$$\begin{aligned} U_{AB} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{(q)(-2q)}{a} \\ U_{BC} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{(-2q)(q)}{a} \\ U_{CA} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{(q)(q)}{2a} \end{aligned}$$

Potential energy of the system,

$$\begin{aligned} \therefore U_{\text{system}} &= \frac{1}{4\pi\epsilon_0} \cdot \left[\frac{-2q^2}{a} - \frac{2q^2}{a} + \frac{q^2}{2a} \right] \\ U_{\text{system}} &= \frac{-7q^2}{8\pi\epsilon_0 a} \end{aligned}$$

74 (b) When α -particle is accelerated through a potential difference V , then kinetic energy of α -particle,

$$\begin{aligned} K &= qV = (2e)V = 2 \times 1.6 \times 10^{-19} \times 10^6 \text{ J} \\ &= \frac{2 \times 1.6 \times 10^{-19} \times 10^6}{1.6 \times 10^{-19}} \text{ eV} = 2 \text{ MeV} \end{aligned}$$

75 (d) Since, we know that, $K = \frac{p^2}{2m}$

Also,

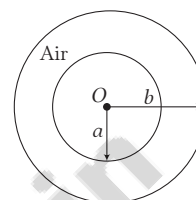
$$\begin{aligned} K &= qV \\ \therefore p &= \sqrt{2mqV} \end{aligned}$$

$$\therefore \text{Ratio of momenta, } \frac{p_e}{p_\alpha} = \sqrt{\frac{m_e q_e}{m_\alpha q_\alpha}} = \sqrt{\frac{m_e}{2m_\alpha}} \quad (\because q_\alpha = 2q_e)$$

76 (d) Velocity, $v = at = \left(\frac{qE}{m}\right)t \Rightarrow v \propto \frac{q}{m}$ or $\text{KE} \propto \frac{q^2}{m^2}$

$$\therefore \text{The ratio of kinetic energy, } \frac{K_1}{K_2} = \left(\frac{2}{1/2}\right)^2 = 16 : 1$$

77 (c) Capacity of spherical condenser, when outer sphere is earthed,



$$C_1 = 4\pi\epsilon_0 \cdot \frac{ab}{b-a}$$

Capacity of spherical condenser when inner sphere is earthed

$$C_2 = 4\pi\epsilon_0 b + \frac{4\pi\epsilon_0 ab}{b-a}$$

∴ Difference in their capacity $= C_2 - C_1 = 4\pi\epsilon_0 b$

78 (d) Work done, $W = U_f - U_i$

Here, $U_f = 0$

$$\begin{aligned} \therefore W &= -U_i = -\frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r} + \frac{q_2 q_3}{r} + \frac{q_3 q_1}{r} \right] \\ &= -\frac{(9 \times 10^9)}{0.1} [(1)(-2) + (1)(4) + (-2)(4)] \times 10^{-12} \\ &= 0.54 \text{ J} \end{aligned}$$

79 (d) Here, $\frac{1}{2}mv^2 = \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{r_i} - \frac{1}{r_f} \right)$

$$\frac{1}{2} \times 2 \times 10^{-3} \times v^2 = (10^{-9})(9 \times 10^9)(0.9)$$

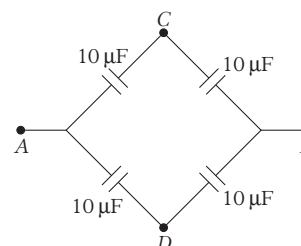
Speed of particle, $v = \sqrt{8.1 \times 10^3} \text{ m/s}$

$$= 90 \text{ m/s}$$

80 (b) Work done, $W = U_i - U_f = 6 \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{q \cdot q}{r} \right] - 0 = \frac{6q^2}{4\pi\epsilon_0 r}$

81 (d) Given circuit is balanced Wheatstone bridge circuit.

$$\text{For branch ACB, } C' = \frac{10 \times 10}{10 + 10} = 5 \mu\text{F}$$



For branch ADB , $C'' = \frac{10 \times 10}{10 + 10} = 5 \mu\text{F}$

There is no flow of charge in branch CD .

So, equivalent capacitance between AB ,

$$C_{AB} = C' + C'' = 5 + 5 = 10 \mu\text{F}$$

82 (a) When capacitors are connected in series, then

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{9} + \frac{1}{18}$$

$$\frac{1}{C_S} = \frac{1}{2}$$

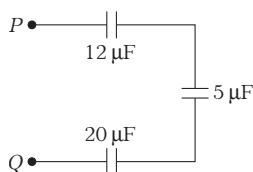
$$\Rightarrow C_S = 2 \mu\text{F}$$

When capacitors are joined in parallel, then

$$C_P = 3 + 9 + 18 = 30 \mu\text{F}$$

$$\therefore \frac{C_S}{C_P} = \frac{2}{30} = \frac{1}{15}$$

83 (b) In circuit, capacitors of capacitance $2 \mu\text{F}$ and $3 \mu\text{F}$ are in parallel. Their resultant capacitance is $5 \mu\text{F}$.

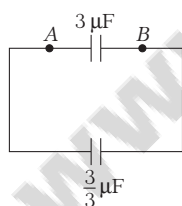


Now, capacitor $12 \mu\text{F}$, $5 \mu\text{F}$ and $20 \mu\text{F}$ are in series. So, their resultant capacity,

$$\frac{1}{C} = \frac{1}{5} + \frac{1}{20} + \frac{1}{12} = \frac{1}{3}$$

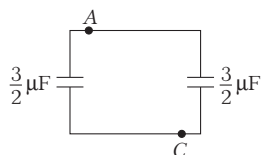
$$\therefore C = 3 \mu\text{F}$$

84 (a) Capacitance between A and B ,



$$C_{AB} = 3 + 1 = 4 \mu\text{F}$$

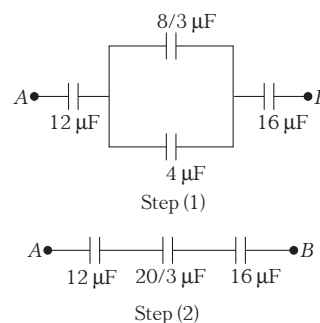
Capacitance between A and C ,



$$C_{AC} = \frac{3}{2} + \frac{3}{2} = 3 \mu\text{F}$$

$$\therefore \frac{C_{AB}}{C_{AC}} = \frac{4}{3}$$

85 (d) Given circuit can be reduced as following

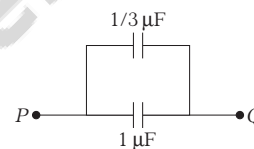


Hence, equivalent capacitance between A and B ,

$$\frac{1}{C_{AB}} = \frac{1}{12} + \frac{1}{20/3} + \frac{1}{16}$$

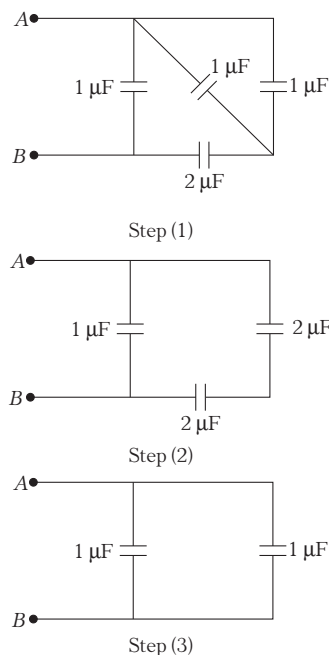
$$\therefore C_{AB} = \frac{240}{71} \mu\text{F}$$

86. (d) Given circuit can be simplified as shown,



$$\therefore C_{PQ} = \frac{1}{3} + 1 = \frac{4}{3} \mu\text{F}$$

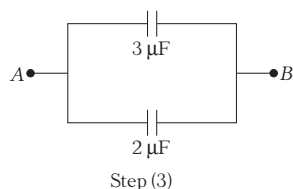
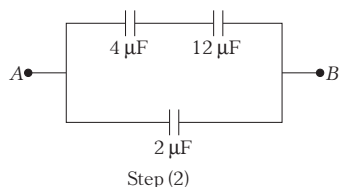
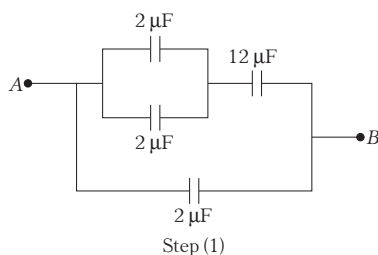
87 (b) Given circuit can be simplified as follows



So, equivalent capacitance between A and B ,

$$C_{AB} = 1 + 1 = 2 \mu\text{F}$$

88 (c) Given circuit can be simplified as,



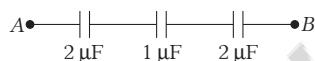
So, net capacitance between AB,

$$C_{AB} = 3 + 2 = 5 \mu\text{F}$$

89 (b) Capacitors C_1 and C_2 are in parallel and, they are in series with C_3 , then equivalent capacity between A and B,

$$C = \frac{C_P \times C_3}{C_P + C_3} = \frac{15 \times 4}{15 + 4} = \frac{60}{19} = 3.2 \mu\text{F}$$

90 (d) The situation can be simplified as follows,

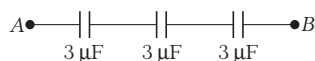


$\therefore$ Equivalent capacity between A and B,

$$\frac{1}{C_{AB}} = \frac{1}{2} + \frac{1}{1} + \frac{1}{2} = \frac{4}{2} = 2$$

$$\Rightarrow C_{AB} = \frac{1}{2} = 0.5 \mu\text{F}$$

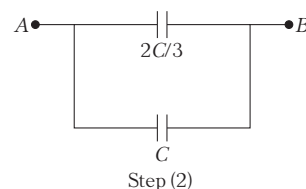
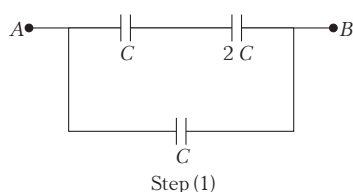
91 (a) The circuit can be redrawn as,



So, the capacitance between the point A and B,

$$C_{AB} = \frac{3}{3} = 1 \mu\text{F}$$

92 (c) Given circuit can be redrawn as follows,



Equivalent capacitance between A and B,

$$C_{AB} = \frac{2C}{3} + C = \frac{5}{3}C$$

93 (d) Let, here, C_0 = capacity of one capacitor.

$$C_S = \frac{C_0}{7} \Rightarrow C_0 = 7C_S$$

and

$$C_P = 7C_0$$

$\therefore$

$$C_P = 49C_S = 49C$$

94 (b) Potential, $V = \frac{q}{C}$ or $V \propto \frac{1}{C}$

V has reduced to $\frac{1}{8}$ th its original value. Therefore, C has increased 8 times, i.e. $k = 8$.

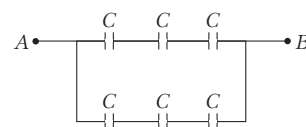
95 (c) Capacitance, $C' = \frac{K\epsilon_0 A}{d}$ or $C' \propto \frac{K}{d'}$

$$\therefore \frac{C'}{C} = \frac{Kd}{d'}$$

Here, $K = 4.0$, $d' = 2d$

$$\therefore C' = (10^{-12}) \left(\frac{4.0}{2} \right) = 2 \times 10^{-12} \text{ F}$$

96 (b)



The effective capacity of the combination,

$$\frac{2C}{3} = 4 \mu\text{F}$$

$\therefore$ Capacitance, $C = 6 \mu\text{F}$

97 (d) By the given arrangement, two capacitors are formed such a way that they are in parallel.

$$\therefore C_{\text{net}} = 2C = 2 \left(\frac{\epsilon_0 A}{d} \right)$$

98 (a) According to the question, $8 \left(\frac{4}{3} \pi r^3 \right) = \frac{4}{3} \pi R^3$

$\therefore$ Radius, $R = 2r$

Capacitance, $C = 4\pi\epsilon_0 r$

$$C' = 4\pi\epsilon_0 2r \Rightarrow C' = 2C$$

Therefore, the capacitance of the bigger drop as compared to each smaller drop is 2 times.

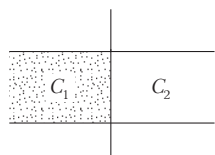
99 (a) Given, $C = 10 \mu\text{F} = 10^{-5} \text{ F}$

$$\Rightarrow \frac{\epsilon_0 A}{D} = 10^{-5} \text{ F}$$

...(i)

Now, both capacitors are in parallel.

$$\begin{aligned} \therefore C' &= C_1 + C_2 = \frac{\epsilon_0(A/2)}{d} + \frac{K\epsilon_0(A/2)}{D} \\ \Rightarrow C' &= \frac{\epsilon_0 A}{d} \left(\frac{1}{2} + 2 \right) = \frac{5\epsilon_0 A}{2d} = \frac{5}{2} \times 10^{-5} \text{ [from Eq. (i)]} \\ &= 25\mu\text{F} \end{aligned}$$



100 (d) Capacity of capacitor, $C = \frac{\epsilon_0 A}{d}$

$$\Rightarrow C \propto \frac{1}{d}$$

$\therefore$ Capacitance, $C' = 2C$

$\therefore$ Extra charge flow, $q = (2CV - CV) = CV$

$\therefore$ Work done, $W = qV = (CV)V = CV^2$

101 (b) $\frac{C'}{C} = \frac{\epsilon_0 A / (d-t)}{\epsilon_0 A / d} = \frac{d}{d-t}$

$$\Rightarrow \frac{d}{(5/7)d} = \frac{7}{5}$$

$$\therefore \text{Capacitance, } C' = \frac{7}{5} \times 25\mu\text{F} = 35\mu\text{F}$$

102 (b) On introducing dielectric K in a parallel plate capacitor, its capacity becomes,

$$C' = KC_0$$

$$\therefore C' = 5C_0$$

$$\text{Also, energy stored, } W_0 = \frac{q^2}{2C_0}$$

$$\therefore W' = \frac{q^2}{2C'} = \frac{q^2}{2 \times 5C_0}$$

$$\therefore \frac{W_0}{W'} = \frac{5}{1}$$

$$\therefore W' = \frac{W_0}{5}$$

103 (b) $\frac{C'}{C} = \frac{\epsilon_0 A / (d-t)}{\epsilon_0 A / d} = \frac{d}{d-t} = \frac{d}{d/2} = 2:1$

$$\left[\because \text{in question, thickness } (t) \text{ is } b = \frac{d}{2} \right]$$

104 (d) When charged capacitor is filled with a dielectric material of dielectric constant K , then charge remains constant, V becomes $\frac{1}{K}$ times and E becomes $\frac{1}{K}$ times.

So, new value of charge is Q_0 .

$$\text{New value of } V = \frac{V_0 \times 3}{9} = \frac{V_0}{3}$$

$$\text{New value of } E = \frac{E_0 \times 3}{9} = \frac{E_0}{3}$$

105 (d) Here capacitors of capacity $4\mu\text{F}$ each are in parallel, their equivalent capacity is $8\mu\text{F}$. Now, there is a combination of three capacitors in series of capacity $20\mu\text{F}$, $8\mu\text{F}$ and $12\mu\text{F}$ so, their resultant capacity,

$$\frac{1}{C} = \frac{1}{20} + \frac{1}{8} + \frac{1}{12}$$

$$\therefore C = \frac{120}{31} \mu\text{F}$$

$$\text{Total charge, } Q = CV = \frac{120}{31} \times 300 = 1161\mu\text{C}$$

$$\therefore \text{Charge through } 4\mu\text{F capacitor} = \frac{1161}{2} = 580\mu\text{C}$$

and potential difference across $4\mu\text{F}$ condenser,

$$V = \frac{q}{C} = \frac{580}{4} = 145\text{ V}$$

106 (c) As capacitors C_1 and C_2 are in series, then there should be equal charge on them, *i.e.*

charge on C_1 = charge on C_2

$$\therefore C_1(V_A - V_D) = C_2(V_D - V_B)$$

$$\text{or } C_1(V_1 - V_D) = C_2(V_D - V_2)$$

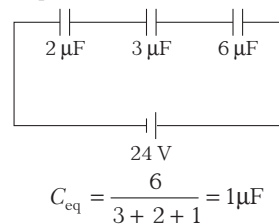
$$\text{or } C_1V_1 - C_1V_D = C_2V_D - C_2V_2$$

$$\text{or } V_D(C_1 + C_2) = C_1V_1 + C_2V_2$$

$\therefore$ The potential difference of point D ,

$$V_D = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

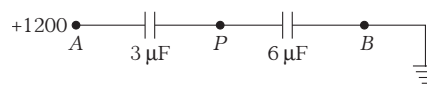
107 (a) Equivalent capacitance of circuit,



Total charge, $Q = 1 \times 24 = 24\mu\text{C}$

$$\text{Now, potential difference across } 6\mu\text{F capacitor} = \frac{24}{6} = 4\text{ V}$$

108 (c) Given, circuit can be reduced as,



Let potential at P be V_P and potential at B be V_B .

As capacitors $3\mu\text{F}$ and $6\mu\text{F}$ are in series, they have same charge.

$\therefore$ Charge on $3\mu\text{F}$ = Charge on $6\mu\text{F}$

$$\therefore C_1V_1 = C_2V_2$$

$$\text{or } 3(1200 - V_P) = 6(V_P - V_B)$$

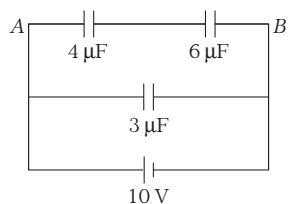
As B point is attached to earth.

$$\text{So, } V_B = 0$$

$$\therefore 1200 - V_P = 2V_P$$

$$V_P = 400\text{ V}$$

109 (b) The circuit can be redrawn as,



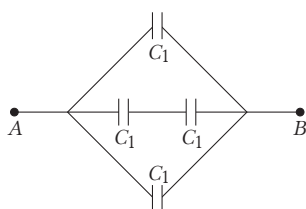
Here, $4 \mu\text{F}$ and $6 \mu\text{F}$ are in series. So, charge is same on both. Now, equivalent capacity between A and B,

$$C_{AB} = \frac{6 \times 4}{6 + 4} = 2.4 \mu\text{F}$$

So, charge on $4 \mu\text{F}$ capacitor,

$$Q = C_{AB} \times 10 = 2.4 \times 10 = 24 \mu\text{C}$$

110 (d) Equivalent capacitance between A and B,

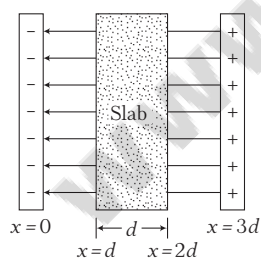


$$C_{AB} = C_1 + \frac{C_1}{2} + C_1 = \frac{5}{2} C_1$$

As charge, $Q = CV$

$$\begin{aligned} \text{So, } 1.5 \times 10^{-6} &= \frac{5}{2} C_1 \times 6 \Rightarrow C_1 = \frac{15}{5} \times 10^{-6} \\ &= 0.1 \times 10^{-6} \text{ F} = 0.1 \mu\text{F} \end{aligned}$$

111 (b) After the introduction of dielectric slab, direction of electric field remains perpendicular to plate and is directed from positive to negative plate.



$$\text{Electric field in air} = \frac{\sigma}{\epsilon_0}$$

$$\text{and electric field in dielectric} = \frac{\sigma}{K\epsilon_0}$$

Positive plate is at higher potential and negative plate is at lower potential. So, electric potential increases continuously as we move from $x = 0$ to $x = 3d$.

112 (c) Potential, $V = \frac{q_{\text{net}}}{C_{\text{net}}}$

$$\Rightarrow 40 = \frac{(2)(200)}{2 + C} \Rightarrow 80 + 40C = 400$$

$\therefore$ Capacity of second condenser, $C = 8 \mu\text{F}$

113 (d) We have,

$$q_i = q_f$$

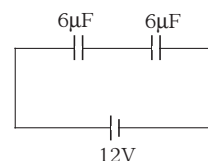
$$C_1 V_1 + C_2 V_2 = (C_1 + C_2) V$$

$$\therefore (2)(4) + (C)5 = (2 + C)4.6$$

$$0.4C = 1.2$$

$$C = 3 \text{ units}$$

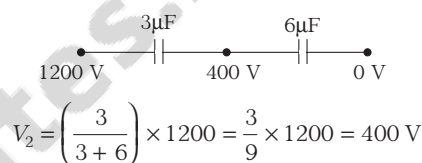
114 (b) The simplified circuit will be as under.



The potential drop 12 V will be equally distributed.

Potential difference across capacitor of $6 \mu\text{F}$ is 6 V. So, the voltage across a $2 \mu\text{F}$ capacitor is 6 V.

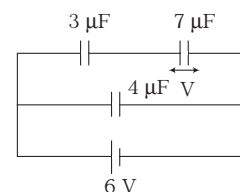
115 (b) PD across $6 \mu\text{F}$ capacitor,



$$V_2 = \left(\frac{3}{3 + 6} \right) \times 1200 = \frac{3}{9} \times 1200 = 400 \text{ V}$$

$$\therefore \text{Charge, } q = 4 \times 400 = 1600 \mu\text{C}$$

116 (b) In parallel, potential difference is same and in series, it distributes in inverse ratio of capacity.



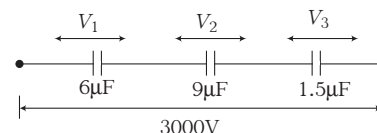
Potential difference,

$$V = 6 \left(\frac{3}{3 + 7} \right) = 1.8 \text{ V}$$

$\therefore$ Charge stored in $5 \mu\text{F}$ capacitor will be

$$q = CV = (5)(1.8) = 9 \mu\text{C}$$

117 (d) In series, potential difference distributes in inverse ratio of capacities. Hence,



$$V_1 : V_2 : V_3 = \frac{1}{6} : \frac{1}{9} : \frac{1}{1.5} = 1.5 : 1 : 6$$

$$V_2 = (3000) \left(\frac{1}{1 + 1.5 + 6} \right) = 352.9 \text{ V}$$

$$\approx 350 \text{ V}$$

118 (a) In the given figure capacitors $3 \mu\text{F}$, $3 \mu\text{F}$ and $3 \mu\text{F}$ (between R and S) are in series. If Q is the charge on each of these capacitors, then

$$30 = \frac{Q}{3} + \frac{Q}{3} + \frac{Q}{3}$$

$$Q = 30 \mu\text{C}$$

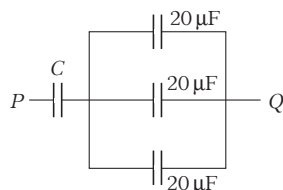
Potential difference between R and S

$$V = \frac{Q}{C} = \frac{30}{3}$$

$\Rightarrow$

$$V = 10 \text{ V}$$

119 (a)



$$C' = C_1 + C_2 + C_3 \\ = 20 + 20 + 20 = 60 \mu\text{F}$$

$$\frac{1}{C''} = \frac{1}{C} + \frac{1}{C'}$$

$$\frac{1}{30} = \frac{1}{C} + \frac{1}{60}$$

$$\frac{1}{C} = \frac{1}{30} - \frac{1}{60} \Rightarrow \frac{1}{C} = \frac{2-1}{60}$$

$$\frac{1}{C} = \frac{1}{60} \Rightarrow C = 60 \mu\text{F}$$

120 (d) Potential difference across C is 10 V.

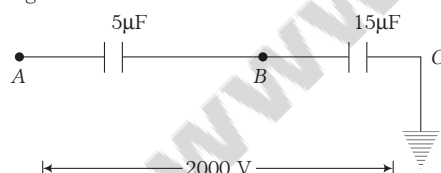
$$\therefore q = CV = 6 \times 10 = 60 \mu\text{C}$$

121 (a) Potential difference across $4 \mu\text{F}$ capacitor is 9 V and potential difference across $6 \mu\text{F}$ capacitor will be $(12 - 9) \text{ V} = 3 \text{ V}$.

$\therefore$ Charge stored in capacitor of $6 \mu\text{F}$,

$$q = CV = 6 \times 3 = 18 \mu\text{C}$$

122 (c) The given circuit can be redrawn as follow



$$(V_A - V_B) = \left(\frac{15}{5+15} \right) \times 2000$$

$$(V_A - V_B) = 1500 \text{ V}$$

$$2000 - V_B = 1500 \text{ V}$$

$$V_B = 500 \text{ V}$$

$$123 \text{ (c) Work done} = \frac{1}{2} \left(\frac{4C}{3} \right) V^2 = \frac{2CV^2}{3}$$

$$124 \text{ (c) Common potential, } V = \frac{6 \times 20 + 3 \times 0}{(6+3)} = \frac{120}{9} \text{ V}$$

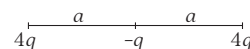
So, charge on $3 \mu\text{F}$ capacitor (by closing S_2)

$$Q_2 = 3 \times 10^{-6} \times \frac{120}{9} = 40 \mu\text{C}$$

125 (b) The energy stored when the plates are fully charged,

$$U = \frac{1}{2} (2C) V^2 = CV^2 = \left(\frac{\epsilon_0 A}{d} \right) (V^2)$$

126 (c) At the give positions all the charges are in equilibrium. But when they displaced slightly from their given position, they do not return back. So, they are in unstable equilibrium position.



$$127 \text{ (d) We have, } V \propto \frac{q}{R} \quad \left(\because V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R} \right)$$

$$\therefore \sigma = \frac{q}{4\pi R^2}$$

$$\therefore q \propto \sigma R^2 \text{ or } V \propto \sigma R$$

Potential is same, i.e. $V_1 = V_2$

$$\therefore \sigma_1 R_1 = \sigma_2 R_2$$

$$\Rightarrow \text{Ratio of } \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} = \frac{5}{4}$$

$$128 \text{ (c) } V = \frac{q}{4\pi\epsilon_0 r} - \frac{q}{4\pi\epsilon_0 3r} = \frac{q}{6\pi\epsilon_0 r}$$

Electric field intensity at a distance $3r$ is given by

$$E = \frac{q}{4\pi\epsilon_0 (3r)^2} = \frac{q}{4\pi\epsilon_0 \cdot 9r^2}$$

$$\text{Thus, } \frac{E}{V} = \frac{q(6\pi\epsilon_0 r)}{4\pi\epsilon_0 9r^2(q)} = \frac{1}{6r} \Rightarrow E = \frac{V}{6r}$$

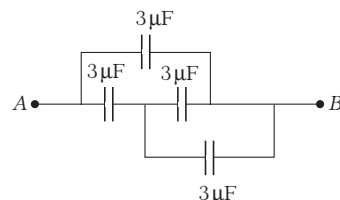
$$129 \text{ (d) Area} = \frac{1}{2} QV = \text{energy stored in the capacitor.}$$

130 (c) Charge, $q = nCV$

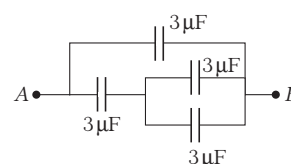
$$1 = n \times 1 \times 10^{-6} \times 110$$

$$\text{Number of capacitors, } n = \frac{1}{110 \times 10^{-6}} = \frac{100000}{11} \approx 9090$$

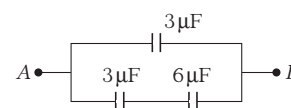
131 (d)

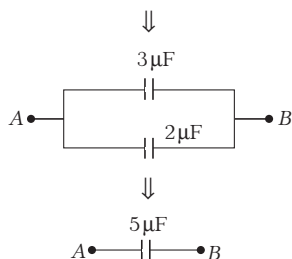


$\Downarrow$



$\Downarrow$





The effective capacitance between point A and B is $5 \mu\text{F}$.

- 132** (c) Charge, $Q = CV = 500 \times 10^{-6} \times 10 = 5 \times 10^{-3} \text{ C}$

$$\text{Now, } Q = qt \text{ or } t = \frac{Q}{q}$$

$$\text{or } t = \frac{5 \times 10^{-3}}{100 \times 10^{-6}} = \frac{1}{20} \times 1000 = 50 \text{ s}$$

- 133** (a) By using, $\frac{1}{2}m(v_1^2 - v_2^2) = QV$

$$\frac{1}{2} \times 10^{-3} [v_1^2 - (0.2)^2] = 10^{-8} (600 - 0)$$

Velocity of the ball at the point A, $v_1 = 22.8 \text{ cm s}^{-1}$

- 134** (c) In series combination,

$$\frac{1}{C_1} = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} = \frac{4+2+1}{4} = \frac{7}{4} \Rightarrow C_1 = \frac{4}{7} \mu\text{F}$$

In parallel combination, $C_1 = 1 + 2 + 4 = 7 \mu\text{F}$

$$\therefore \frac{C_1}{C_2} = \frac{4/7}{7} = \frac{4}{49}$$

- 135** (c) Electric force, $qE = ma$

$$\therefore a = \frac{qE}{m}$$

$$\Rightarrow a = \frac{1.6 \times 10^{-19} \times 1 \times 10^3}{9 \times 10^{-31}} = \frac{1.6 \times 10^{15}}{9} \text{ ms}^{-2}$$

Initially speed, $u = 5 \times 10^6 \text{ ms}^{-1}$ and final speed, $v = 0$

$$\therefore \text{From } v^2 = u^2 - 2as$$

$$\Rightarrow \text{Distance travelled by electron, } s = \frac{u^2}{2a}$$

$$s = \frac{(5 \times 10^6)^2 \times 9}{2 \times 1.6 \times 10^{15}} = 7 \text{ cm}$$

- 136** (a) Potential gradient is related with electric field according to the following relation, $E = \frac{-dV}{dr}$.

$$\mathbf{E} = -\frac{\partial V}{\partial r} \hat{\mathbf{r}} = \left[-\frac{\partial V}{\partial x} \hat{\mathbf{i}} - \frac{\partial V}{\partial y} \hat{\mathbf{j}} - \frac{\partial V}{\partial z} \hat{\mathbf{k}} \right]$$

$$= [\hat{\mathbf{i}}(2xy + z^3) + \hat{\mathbf{j}}x^2 + \hat{\mathbf{k}}3xz^2]$$

- 137** (c) Potential energy of the system,

$$\frac{-kqQ}{x} - \frac{kQq}{x} + \frac{kq^2}{2x} = 0$$

$$\Rightarrow \frac{-4kqQ + kq^2}{2x} = 0$$

$$\Rightarrow kq^2 = 4kQq$$

$$\text{Ratio, } \frac{q}{Q} = 4$$

- 138** (c) Electrostatic potential energy, $U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$

$$\text{Here, } q_1 = q_2 = 1.6 \times 10^{-19} \text{ C}$$

$$\text{and } r = 9 \times 10^{-15} \text{ m}$$

$$\therefore U = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{9 \times 10^{-15}}$$

$$= 2.56 \times 10^{-14} \text{ J}$$

- 139** (b) When capacitors are connected in parallel, then

$$C_p = C + C + C = 3C$$

According to question,

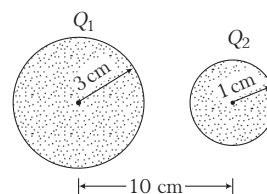


$$\therefore \text{In series, } C_s = \frac{3C \times C}{3C + C} \Rightarrow 3.75 = \frac{3C \times C}{4C}$$

$$\Rightarrow C = 5 \mu\text{F}$$

- 140** (c) The work done by a electrostatic force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

- 141** (a) Let Q_1 and Q_2 be the charge on the two conducting spheres of radii $R_1 (= 3 \text{ cm})$ and $R_2 (= 1 \text{ cm})$, respectively.



$$\therefore \text{Potential, } V_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R_1} \Rightarrow 10 = \frac{9 \times 10^9 \times Q_1}{3 \times 10^{-2}}$$

$$\Rightarrow \text{Charge, } Q_1 = \frac{10 \times 3 \times 10^{-2}}{9 \times 10^9} \quad \dots(i)$$

$$\text{and } V_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{R_2} \Rightarrow 10 = \frac{9 \times 10^9 \times Q_2}{1 \times 10^{-2}}$$

$$\Rightarrow \text{Charge, } Q_2 = \frac{10 \times 1 \times 10^{-2}}{9 \times 10^9} \quad \dots(ii)$$

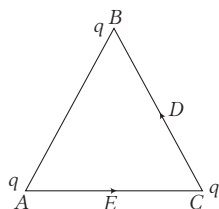
According to Coulomb's law, force of repulsion between them is

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} = \frac{9 \times 10^9 \times 10 \times 3 \times 10^{-2} \times 10 \times 1 \times 10^{-2}}{9 \times 10^9 \times 9 \times 10^9 \times (10 \times 10^{-2})^2}$$

[using Eqs. (i) and (ii)]

$$= \left(\frac{1}{3}\right) \times 10^{-9} \text{ N}$$

- 142 (a) We have, $AC = BC$



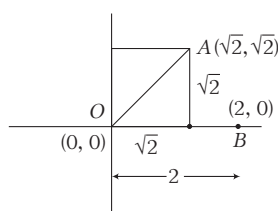
$$\therefore V_D = V_E$$

Workdone in taking a charge Q from D to E

$$W = Q(V_E - V_D)$$

$$W = 0$$

- 143 (b)



$$AO = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$$

$$AO = \sqrt{4} = 2 \quad \text{and} \quad BO = 2$$

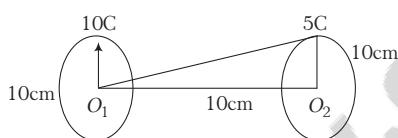
Potential difference between the points A and B

$$= V_A - V_B$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{AO} - \frac{1}{4\pi\epsilon_0} \frac{Q}{BO}$$

$$= 0 \quad (\because AO = BO)$$

- 144 (b)



Potential at the centre of the ring 1 is

$$V_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{10}{10} + \frac{5}{\sqrt{(10)^2 + (10)^2}} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{10}{10} + \frac{5}{10\sqrt{2}} \right]$$

Potential at the centre of the ring 2 is

$$V_2 = \frac{1}{4\pi\epsilon_0} \left[\frac{5}{10} + \frac{10}{\sqrt{(10)^2 + (10)^2}} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{5}{10} + \frac{10}{10\sqrt{2}} \right]$$

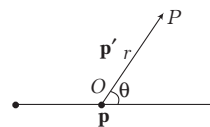
Work done, $W = q(V_1 - V_2)$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{10}{10} + \frac{5}{10\sqrt{2}} - \frac{5}{10} - \frac{10}{10\sqrt{2}} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{5}{10} - \frac{5}{10\sqrt{2}} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{2} - \frac{1}{2\sqrt{2}} \right] = \frac{q}{8\pi\epsilon_0} \left[\frac{\sqrt{2}-1}{\sqrt{2}} \right]$$

- 145 (d) As shown in figure, component of dipole moment along the line OP will be



$$p' = p \cos \theta$$

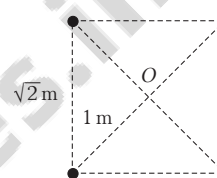
Hence, electric potential at point P will be

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p \cos \theta}{r^2}$$

- 146 (b) Total charge is independent of r .

Hence, option (b) is incorrect.

- 147 (c) Length of each side of square is $\sqrt{2}$ m, so distance of its centre from each corner is 1 m.



Potential at the centre,

$$V = 9 \times 10^9 \left[\frac{10 \times 10^{-6}}{1} + \frac{5 \times 10^{-6}}{1} - \frac{3 \times 10^{-6}}{1} + \frac{8 \times 10^{-6}}{1} \right]$$

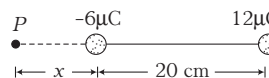
$$= 1.8 \times 10^5 \text{ V}$$

- 148 (a) By using, $W = Q(\mathbf{E} \cdot \Delta \mathbf{r})$

$$\Rightarrow W = Q[(e_1 \hat{i} + e_2 \hat{j} + e_3 \hat{k}) \cdot (a \hat{i} + b \hat{j})]$$

$$= Q(e_1 a + e_2 b)$$

- 149 (c) Point P will lie near the charge which is smaller in magnitude, i.e. $-6\mu\text{C}$. Hence, potential at P ,



$$V = \frac{1}{4\pi\epsilon_0} \frac{(-6 \times 10^{-6})}{x} + \frac{1}{4\pi\epsilon_0} \frac{(12 \times 10^{-6})}{(0.2 + x)} = 0$$

$$\Rightarrow x = 0.20 \text{ m}$$

- 150 (d) Work done, $W = 6 \times 10^{-6} (V_A - V_B)$, where

$$V_A = 10^{10} \left[\frac{(-5 \times 10^6)}{15 \times 10^{-2}} + \frac{2 \times 10^6}{5 \times 10^{-2}} \right] = \frac{1}{15} \times 10^6 \text{ V}$$

$$\text{and} \quad V_B = 10^{10} \left[\frac{(2 \times 10^6)}{15 \times 10^{-2}} - \frac{5 \times 10^6}{5 \times 10^{-2}} \right] = -\frac{13}{15} \times 10^6 \text{ V}$$

$$\therefore W = 6 \times 10^{-6} \left[\frac{1}{15} \times 10^6 - \left(-\frac{13}{15} \times 10^6 \right) \right] = 5.6 \text{ J}$$

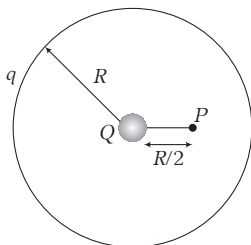
- 151 (d) $E_x = -\frac{dV}{dx} = -(-5) = 5$, $E_y = -\frac{dV}{dy} = -3$

$$\text{and} \quad E_z = -\frac{dV}{dz} = -\sqrt{15}$$

$$\therefore E_{\text{net}} = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(5)^2 + (-3)^2 + (-\sqrt{15})^2} = 7$$

- 152 (d) Electric potential at
- P
- ,

$$V = \frac{kQ}{R/2} + \frac{kq}{R} = \frac{2Q}{4\pi\epsilon_0 R} + \frac{q}{4\pi\epsilon_0 R}$$



- 153 (c) Charge
- $80\mu\text{C}$
- divided in both capacitors of capacitances
- $2\mu\text{F}$
- and
- $3\mu\text{F}$
- .

Charge on the $3\mu\text{F}$,

$$q_3 = \left(\frac{C_3}{C_2 + C_3} \right) \times Q$$

$$\Rightarrow q_3 = \left(\frac{3}{3+2} \right) \times 80 = \frac{3}{5} \times 80 = 48\mu\text{C}$$

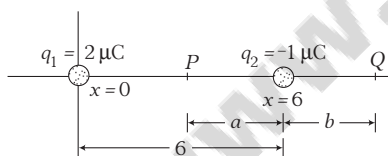
- 154 (d)
- $C_{\text{air}} = \frac{\epsilon_0 A}{d}$
- , with dielectric slab
- $C' = \frac{\epsilon_0 A}{\left(d - t + \frac{t}{K} \right)}$

$$\text{Given, } C' = \frac{4}{3}C \Rightarrow \frac{\epsilon_0 A}{\left(d - t + \frac{t}{K} \right)} = \frac{4}{3} \times \frac{\epsilon_0 A}{d}$$

The dielectric constant of the slab,

$$K = \frac{4t}{4t-d} = \frac{4(d/2)}{4(d/2)-d} = 2$$

- 155 (c) Let, potential will be zero at two points
- P
- and
- Q
- , then

At internal point P

$$\frac{1}{4\pi\epsilon_0} \times \left[\frac{2 \times 10^{-6}}{(6-a)} + \frac{(-1 \times 10^{-6})}{a} \right] = 0$$

$$\therefore a = 2$$

So, distance of P from origin, $x = 6 - 2 = 4$ At external point Q ,

$$\frac{1}{4\pi\epsilon_0} \times \left[\frac{2 \times 10^{-6}}{(6+b)} + \frac{(-1 \times 10^{-6})}{b} \right] = 0$$

$$\therefore b = 6$$

So, distance of Q from origin, $x = 6 + 6 = 12$

- 156 (b) Let radius of big drop be
- R
- and radius of small drops be
- r
- ,

 $\therefore$ Volume of big drop $= 8 \times$ volume of small drops

$$\text{or } \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3 \quad \text{or } R = 2r$$

Potential of big drop,

$$V_{\text{big}} = \frac{Q}{C} = \frac{8q}{(8)^{1/3}C_0} \quad (q \text{ is charge on small drop})$$

$$\therefore V_{\text{big}} = (8)^{2/3} V_{\text{small}} \quad \left(\because V_{\text{small}} = \frac{q}{C_0} \right)$$

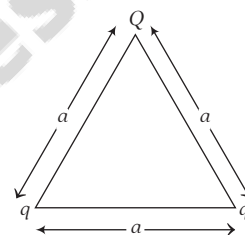
$$\therefore \frac{V_{\text{big}}}{V_{\text{small}}} = (8)^{2/3} = \frac{4}{1}$$

- 157 (c) We have,
- $8 \left(\frac{4}{3}\pi r^3 \right) = \frac{4}{3}\pi R^3, \Rightarrow R = 2r, Q = 8q$

$$\text{Further, } V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} = 50 \text{ V}$$

$$\begin{aligned} \text{Potential of the large drop, } V' &= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R} = \frac{1}{4\pi\epsilon_0} \cdot \frac{8q}{2r} \\ &= 4 \times 50 = 200 \text{ V} \end{aligned}$$

$$158 (a) U_{\text{net}} = \frac{1}{4\pi\epsilon_0} \left[\frac{Qq}{a} + \frac{qq}{a} + \frac{Qq}{a} \right]$$



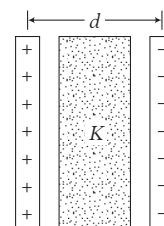
$$\therefore U_{\text{net}} = 0$$

$$2Qq + q^2 = 0 \Rightarrow Q = -\frac{q}{2}$$

- 159 (b) Potential of the system,

$$V = \frac{\text{Net charge}}{\text{Net capacity}} = \frac{q_1 + q_2}{4\pi\epsilon_0 (r_1 + r_2)}$$

- 160 (c) Potential difference between plates
- A
- and
- B
- ,

 V = Potential difference in air

+ Potential difference in medium

$$V = \frac{\sigma}{\epsilon_0} (d-t) + \frac{\sigma}{K\epsilon_0} t$$

$$\therefore V = \frac{\sigma}{\epsilon_0} \left[d - t + \frac{t}{K} \right] = \frac{Q}{A\epsilon_0} \left(d - t + \frac{t}{K} \right) \quad \left(\because \sigma = \frac{Q}{A} \right)$$

$$\therefore \text{Capacitance, } C = \frac{Q}{V} = \frac{Q}{\frac{Q}{A\epsilon_0} \left(d - t + \frac{t}{K} \right)}$$

$$= \frac{\epsilon_0 A}{d - t + \frac{t}{K}} = \frac{\epsilon_0 A}{d - t \left(1 - \frac{1}{K} \right)}$$

161 (b) According to question,

capacity of spherical condenser = capacity of parallel plate capacitor

$$\therefore 4\pi\epsilon_0 r = \frac{\epsilon_0 A}{d}$$

$$\begin{aligned}\therefore d &= \frac{A}{4\pi r} = \frac{\pi R^2}{4\pi r} \\ &= \frac{\pi(20 \times 10^{-3})^2}{4\pi \times 1} \\ &= 0.1 \text{ mm}\end{aligned}$$

162 (a) Three capacitors are in series therefore, their resultant capacity is given by

$$\frac{1}{C_S} = \frac{1}{\left(\frac{\epsilon_0 K_1 A}{d_1}\right)} + \frac{1}{\left(\frac{\epsilon_0 K_2 A}{d_2}\right)} + \frac{1}{\left(\frac{\epsilon_0 K_3 A}{d_3}\right)}$$

or
$$\frac{1}{C_S} = \frac{d_1}{\epsilon_0 K_1 A} + \frac{d_2}{\epsilon_0 K_2 A} + \frac{d_3}{\epsilon_0 K_3 A}$$

$$\frac{1}{C_S} = \frac{1}{\epsilon_0 A} \left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3} \right)$$

$$\therefore C_S = \frac{\epsilon_0 A}{\left(\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3} \right)}$$

163 (b)
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{4C} + \dots \infty$$

$$= \frac{1}{C} \left[1 + \frac{1}{2} + \frac{1}{2^2} + \dots \infty \right] = \frac{1}{C} \left[\frac{1}{1 - \frac{1}{2}} \right] = \frac{2}{C}$$

The equivalent capacitance between A and B,

$$C_{eq} = \frac{C}{2}$$

164 (a) The given figure is equivalent to two identical capacitors in parallel combination,

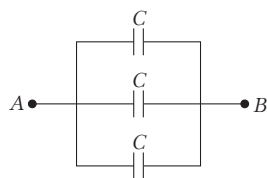
$$C = \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d} = \frac{2\epsilon_0 A}{d}$$

$\therefore$ Both the middle plates have same potential V_B .

165 (b) Given plates are equivalent to 3 identical capacitors in parallel combination. Hence, equivalent capacitance

$$\begin{aligned}C_P &= C + C + C = 3C \\ &= 3 \frac{\epsilon_0 A}{d}\end{aligned}$$

166 (b) Here, three capacitors are connected in parallel. So, their equivalent capacity



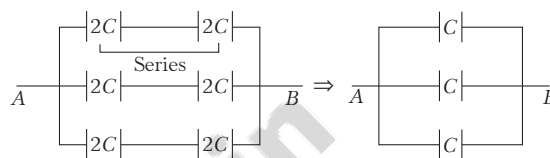
$$C_P = C + C + C = 3C$$

167 (b) We have,
$$\frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{(d + \Delta d) - t + \frac{t}{K}} \text{ or } K = \frac{t}{t - \Delta d}$$

Dielectric constant,
$$K = \frac{4 \times 10^{-3}}{4 \times 10^{-3} - 3.5 \times 10^{-3}}$$

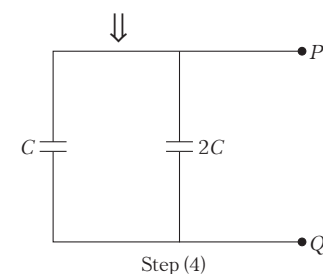
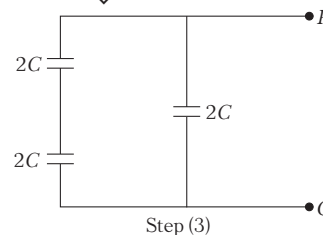
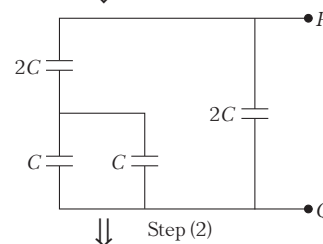
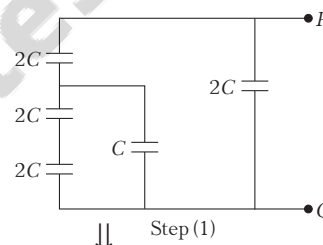
$$K = \frac{4 \times 10^{-3}}{0.5 \times 10^{-3}} = \frac{40}{5} = 8 \Rightarrow K = 8$$

168 (c) This diagram is symmetrical about line AB, hence point on perpendicular of AB, i.e. C, D and E are at same potential. So, remove capacitor between C and D, D and E.



The equivalent capacitance, $C_{eq} = C + C + C = 3C$

169 (a) The given circuit can be simplified in following way.



So, capacity between P and Q,

$$C_{PQ} = 2C + C = 3C$$

- 170** (d) Capacitors C_1 and C_2 are in series with C_3 in parallel with them.

Now,

$$C_1 = \frac{K_1 \epsilon_0 (A/2)}{(d/2)} = \frac{K_1 \epsilon_0 A}{d}$$

$$C_2 = \frac{K_2 \epsilon_0 (A/2)}{(d/2)} = \frac{K_2 \epsilon_0 A}{d} \text{ and } C_3 = \frac{K_3 \epsilon_0 A}{2d}$$

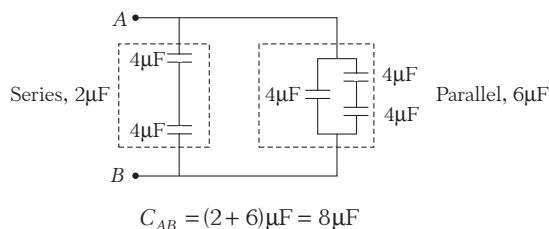
$$C_{\text{equivalent}} = C_3 + \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{K_3 \epsilon_0 A}{2d} + \frac{\left(\frac{K_1 \epsilon_0 A}{d}\right) \left(\frac{K_2 \epsilon_0 A}{d}\right)}{\frac{K_1 \epsilon_0 A}{d} + \frac{K_2 \epsilon_0 A}{d}}$$

$$= \frac{\epsilon_0 A}{d} \left(\frac{K_3}{2} + \frac{K_1 K_2}{K_1 + K_2} \right)$$

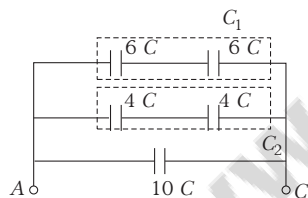
So, option (d) is correct.

- 171** (d) The simple circuit is as shown below,



- 172** (b) The given combination is a balanced Wheatstone bridge in parallel with 10 C .

Capacitance, $C_1 = \frac{6 \times 6}{6 + 6} = 3\text{ C}$



Capacitance, $C_2 = \frac{4 \times 4}{4 + 4} = 2\text{ C}$

The equivalent capacitance between the points A and C.

$$C_{AC} = (3 + 2 + 10)\text{ C} \Rightarrow C_{AC} = 15\text{ C}$$

- 173** (b) Potential difference between two points,

$$V_A - V_B = \int_A^B (2\hat{i} - 4\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= [2x - 4y]_{(0,0)}^{(3m, 4m)} = -10\text{ V}$$

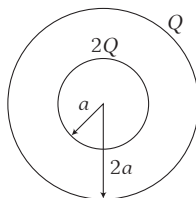
174 (c) $\frac{K(2Q)}{2a} + \frac{KQ}{2a} = 5$

or $\frac{K(3Q)}{2a} = 5$

$\therefore \frac{KQ}{a} = \frac{10}{3}$

Now potential of inner sphere,

$$V_m = \frac{K2Q}{a} + \frac{KQ}{2a} = \frac{5}{2} \frac{KQ}{a} = \frac{5}{2} \left(\frac{10}{3} \right) = \frac{25}{3}\text{ V}$$



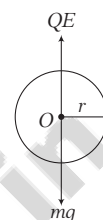
175 (d) We have, $V = \frac{KQ}{r} \Rightarrow 120 = \frac{KQ}{2}$

$\therefore Kq = 240\text{ units}$

When it is made to touch the bigger sphere, whole charge will transfer to bigger sphere.

$\therefore$ Potential of bigger sphere, $V = \frac{Kq}{6} = \frac{240}{6} = 40\text{ V}$

- 176** (b) Let radius of drop is r . According to Millikan, for balance of drop, $QE = mg$ or $Q \frac{V}{d} = \left[\left(\frac{4}{3} \pi r^3 \right) \rho \right] g$



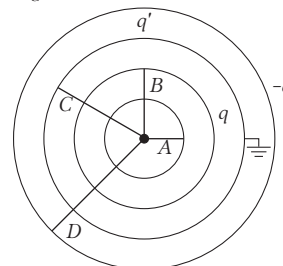
where, V is potential difference and ρ is density of drop.

$\therefore \frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2} \right)^3 \times \frac{V_2}{V_1}$

$\Rightarrow \frac{Q}{Q_2} = \left(\frac{r}{r/2} \right)^3 \times \frac{600}{2400} = 2 \Rightarrow Q_2 = Q/2$

- 177** (d) Shell C is now earthed.

$\therefore V_C = 0$



$\therefore \frac{K(q + q')}{3a} - \frac{Kq}{4a} = 0$

$\therefore q' = -\frac{q}{4}$

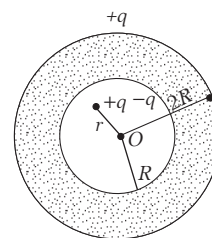
Now, potential difference,

$$V_A - V_C = \frac{Kq}{2a} - \frac{Kq/4}{3a} - \frac{Kq}{4a} - 0 = \frac{Kq}{6a}$$

- 178** (a) Potential difference only depends upon the inner charge.

So, if the shell is given a charge of $-3Q$, the new potential difference between the same two surface is V .

- 179** (a) The induced charges will be as under.



Potential at point O (the centre),

$$V_0 = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r} - \frac{q}{R} + \frac{q}{2R} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r} - \frac{1}{2R} \right]$$

180 (c) Potential at point P , $V = \frac{KQ}{R} - \frac{Kq}{x}$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} - \frac{q}{x} \right)$$

181 (b) Here, circuit is equivalent to two capacitors in parallel,

$$\therefore C_{eq} = C_1 + C_2$$

$$= \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{d} = \frac{2\epsilon_0 A}{d}$$

$$\therefore \text{Energy stored} = \frac{1}{2} C_{eq} V^2 = \frac{1}{2} \left(\frac{2\epsilon_0 A}{d} \right) V^2$$

$$= \frac{8.85 \times 10^{-12} \times 50 \times 10^{-4} \times 12 \times 12}{3 \times 10^{-3}}$$

$$= 2.1 \times 10^{-9} \text{ J}$$

182 (c) In figure, there is combination of two capacitors in parallel,

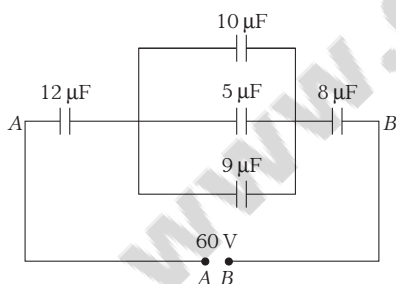
$$\therefore C_P = C_1 + C_2$$

$$= \frac{K_1 \epsilon_0 (A/2)}{d} + \frac{K_2 \epsilon_0 (A/2)}{d}$$

$$= \frac{2\epsilon_0 (A/2)}{d} + \frac{4\epsilon_0 (A/2)}{d}$$

$$= 2 \times \frac{10}{2} + 4 \times \frac{10}{2} = 30 \mu\text{F}$$

183 (d) Given circuit can be redrawn as follows



Equivalent capacitance of the circuit,

$$C_{AB} = \frac{24}{2+1+3} = 4 \mu\text{F}$$

Total charge given by battery,

$$q = C_{AB} \cdot V = 4 \times 60 = 240 \mu\text{C}$$

Charge on $5 \mu\text{F}$ capacitor, $q_2 = \left(\frac{5}{10+5+9} \right) \times 240 = 50 \mu\text{C}$

184 (a) We have, $K_i + U_i = K_f + U_f$

$$0 + 0 = \frac{1}{2} mv^2 + (-e) \frac{KQ}{R}$$

$\therefore$ Velocity, $v = \sqrt{\frac{2KQe}{mR}}$

185 (c) Charge on first capacitor $= C(-V) = -CV$

Charge on second capacitor $= (2C)(2V) = 4CV$

$\therefore$ Total charge on both capacitors $= 4CV - CV = 3CV$

$\therefore$ Common potential on them $= \frac{3CV}{3C} = V$

$\therefore$ Energy $= \frac{1}{2} (3C)V^2 = \frac{3}{2} CV^2$

186 (b) Charge, $q_1 = C_1 V = (15) \times 100 = 1500 \mu\text{C}$

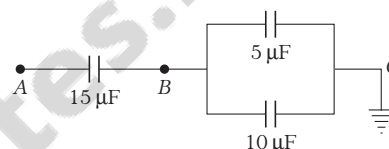
Charge, $q_2 = C_2 V = (1) \times 100 = 100 \mu\text{C}$

$\therefore$ Net charge, $q_{net} = q_1 + q_2 = 1600 \mu\text{C}$

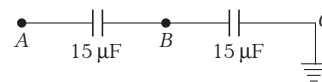
When dielectric is removed, $C_1 = \frac{C_1}{K} = \frac{15}{15} = 1 \mu\text{F}$

Now, common potential, $V = \frac{q_{net}}{C_1 + C_2} = \frac{1600}{1+1} = 800 \text{ V}$

187 (b) Given, circuit can be redrawn as



Step (1)



Step (2)

Potential difference between A and B ,

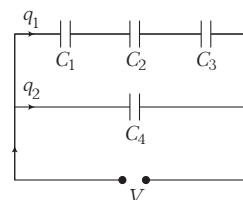
$$V_A - V_B = \left(\frac{15}{15+15} \right) \times 2000$$

$$\therefore V_A - V_B = 1000 \text{ V}$$

$$\therefore 2000 - V_B = 1000 \text{ V}$$

$$\therefore V_B = 1000 \text{ V}$$

188 (b) Given, circuit can be reduced to



For series combination of C_1, C_2, C_3 resultant capacity,

$$C_S = \frac{6C}{6C + 3C + 2C}$$

or $C_S = \frac{6C}{11}$

Now, the ratio of the charges on C_2 and C_4 ,

$$\frac{q_1}{q_2} = \frac{C_S}{C_4} = \frac{6C/11}{4C} = \frac{3}{22}$$

- 189** (c) Potential difference between two equipotential surfaces A and B .

$$V_A - V_B = kq \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

$$= kq \left(\frac{r_B - r_A}{r_A r_B} \right) = \frac{kqt_1}{r_A r_B}$$

or $t_1 = \frac{(V_A - V_B) r_A r_B}{kq}$ or $t_1 \propto r_A r_B$

Similarly, $t_2 \propto r_B r_C$

Since, $r_A < r_B < r_C$, therefore $r_A r_B < r_B r_C$.

$$\therefore t_1 < t_2$$

- 190** (c) We know that, $qE = mg$

$$\frac{qQ}{\epsilon_0 A} = mg$$

or the charge of the oil drop,

$$q = \frac{\epsilon_0 A mg}{Q} = \frac{8.85 \times 10^{-12} \times 2 \times 10^{-2} \times 2.5 \times 10^{-7} \times 10}{5 \times 10^{-7}}$$

$$= 8.85 \times 10^{-13} \text{ C}$$

- 191** (c) The capacitance of parallel plate capacitor filled with dielectric block has thickness d_1 and dielectric constant K_2 is given by

$$C_1 = \frac{K_1 \epsilon_0 A}{d_1}$$

Similarly, capacitance of parallel plate capacitor filled with dielectric block has thickness d_2 and dielectric constant K_2 is given by

$$C_2 = \frac{K_2 \epsilon_0 A}{d_2}$$

Since, the two capacitors are in series combination, the equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or $C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{K_1 \epsilon_0 A}{d_1} \frac{K_2 \epsilon_0 A}{d_2}}{\frac{K_1 \epsilon_0 A}{d_1} + \frac{K_2 \epsilon_0 A}{d_2}} = \frac{K_1 K_2 \epsilon_0 A}{K_1 d_2 + K_2 d_1} \dots (i)$

But, the equivalent capacitance is given by

$$C = \frac{K \epsilon_0 A}{d_1 + d_2}$$

On comparing, we have

$$K = \frac{K_1 K_2 (d_1 + d_2)}{K_1 d_2 + K_2 d_1}$$

- 192** (c) Minimum number of condensers in each row = $\frac{3000}{500} = 6$

If C_S is capacity of 6 condensers in a row,

$$\frac{1}{C_S} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 6$$

$$\Rightarrow C_S = \frac{1}{6} \mu\text{F}$$

Let there be m such rows in parallel.

$$\text{Total capacity} = m \times C_S$$

$$2 = m \times \frac{1}{6}$$

$$\therefore m = 12$$

$$\text{Total number of capacitors} = 6 \times 12 = 72$$

- 193** (d) **Case I** When the capacitors are joined in series,

$$U_{\text{series}} = \frac{1}{2} \frac{C_1}{n_1} (4V)^2$$

Case II When the capacitors are joined in parallel,

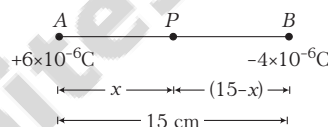
$$U_{\text{parallel}} = \frac{1}{2} (n_2 C_2) V^2$$

$$\text{Given, } U_{\text{series}} = U_{\text{parallel}}$$

$$\text{or } \frac{1}{2} \frac{C_1}{n_1} (4V)^2 = \frac{1}{2} (n_2 C_2) V^2$$

$$\Rightarrow C_2 = \frac{16 C_1}{n_2 n_1}$$

- 194** (b)



Let the potential be zero at point P at a distance x , from the charge $+6 \times 10^{-6} \text{ C}$ at A as shown in the figure. Potential at P ,

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{6 \times 10^{-6}}{x} + \frac{(-4 \times 10^{-6})}{(15-x)} \right]$$

$$0 = \frac{1}{4\pi\epsilon_0} \left[\frac{6 \times 10^{-6}}{x} - \frac{4 \times 10^{-6}}{15-x} \right]$$

$$0 = \frac{6 \times 10^{-6}}{x} - \frac{4 \times 10^{-6}}{15-x}$$

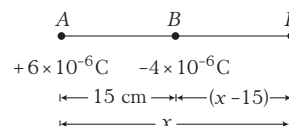
$$\frac{6 \times 10^{-6}}{x} = \frac{4 \times 10^{-6}}{15-x}$$

$$\Rightarrow 6(15-x) = 4x$$

$$90 - 6x = 4x \text{ or } 10x = 90$$

$$\Rightarrow x = \frac{90}{10} = 9 \text{ cm}$$

The other possibility is that point of zero potential P may lie on AB produced at a distance x from the charge $+6 \times 10^{-6} \text{ C}$ at A as shown in the figure.



Potential at P ,

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{6 \times 10^{-6}}{x} + \frac{(-4 \times 10^{-6})}{(x-15)} \right]$$

$$0 = \frac{1}{4\pi\epsilon_0} \left[\frac{6 \times 10^{-6}}{x} - \frac{4 \times 10^{-6}}{x-15} \right]$$

$$0 = \frac{6 \times 10^{-6}}{x} - \frac{4 \times 10^{-6}}{x-15}$$

$$\Rightarrow \frac{6}{x} = \frac{4}{x-15}$$

$$\Rightarrow 6x - 90 = 4x$$

$$\Rightarrow 2x = 90$$

$$\text{or } x = \frac{90}{2} = 45 \text{ cm}$$

Thus, electrostatic potential is zero at 9 cm and 45 cm from the charge $+6 \times 10^{-6} \text{ C}$ at A. Also, potential is zero at infinity.

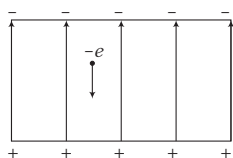
195 (c) We have, potential difference, $V_A - V_o = - \int_o^A E_x dx$

$$\text{Potential difference, } V_A - V_o = - \int_0^2 30x^2 dx$$

$$= -30 \left(\frac{2^3}{3} \right) = -80 \text{ V}$$

196 (d) As the field is upward, so the negatively charged electron experiences a downward force of the magnitude of eE , where E is the magnitude of the electric field.

The acceleration of the electron is $a_e = \frac{eE}{m_e}$.



where, m_e is the mass of the electron.

Starting from rest, the time taken by the electron to fall through a distance h is given by

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}} = \sqrt{\frac{2 \times 1.5 \times 10^{-2} \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 2 \times 10^4}}$$

$$= 2.9 \times 10^{-9} \text{ s}$$

197 (d) We have, $V(x) = \frac{20}{x^2 - 4}$

Electric field at $x = 4 \mu\text{m}$

$$E = \frac{-dV}{dx} = \frac{20}{(x^2 - 4)^2} (2x - 0) = \frac{160}{144} = \frac{10}{9} \text{ V}/\mu\text{m}$$

Direction of electric field E will be along positive x -direction.

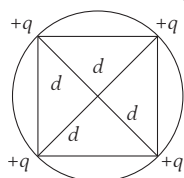
198 (b) Net electrostatic energy,

$$U = \frac{kQq}{a} + \frac{kq^2}{a} + \frac{kQq}{a\sqrt{2}} = 0$$

$$\Rightarrow \frac{kq}{a} \left(Q + q + \frac{Q}{\sqrt{2}} \right) = 0$$

$$\therefore \text{The value of } Q = -\frac{2q}{2 + \sqrt{2}}$$

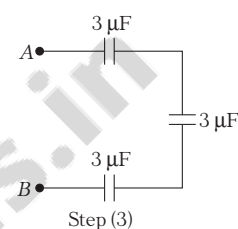
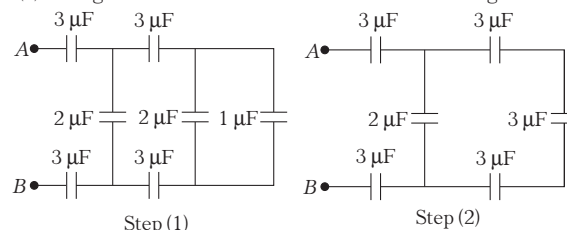
199 (b) Potential at centre due to all charges,



$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{d} + \frac{q}{d} + \frac{q}{d} + \frac{q}{d} \right]$$

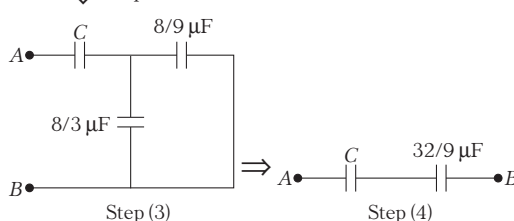
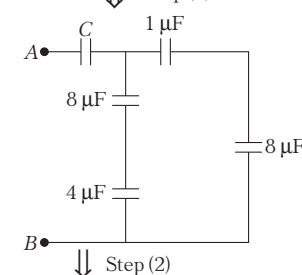
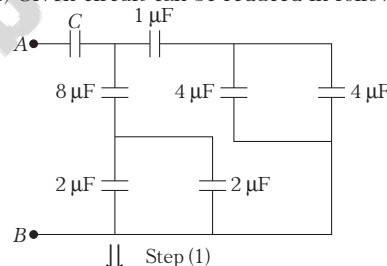
$$= \frac{1}{4\pi\epsilon_0} \frac{4q}{d} \text{ in SI unit} = \frac{4q}{d} \text{ in CGS unit}$$

200 (a) The given circuit can be reduced in following manner



$\therefore$ Resultant capacity between A and B,
 $C_{AB} = 1 \mu\text{F}$

201 (d) Given circuit can be reduced in following manner



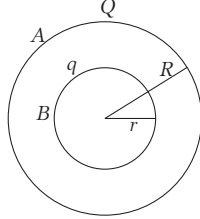
So, equivalent capacitance between A and B,

$$C_{eq} = 1 = \frac{\frac{32}{9} \times C}{\frac{32}{9} + C}$$

$\therefore$ The value of $C = \frac{32}{23} \mu\text{F}$

- 202 (c)** The potential of conducting sphere having charge Q at the surface, $A = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R}$.

The potential of conducting sphere having charge q at the surface, $A = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R}$



The potential at B is due to Q inside $= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R}$

The potential at B due to $q = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

$\therefore$ Potential at A , $V_A = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} + \frac{q}{R} \right)$

Potential at B , $V_B = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} + \frac{q}{r} \right)$

$\therefore V_B - V_A = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r} - \frac{q}{R} \right)$

- 203 (d)** Capacitance, $C = \frac{\epsilon_0 A}{d}$

When capacitor is half filled with a dielectric capacitance,

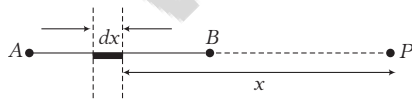
$$C' = \frac{\epsilon_0 A}{2d} + \frac{\epsilon_0 (5A)}{2d} = \frac{\epsilon_0 A}{2d} (1 + 5) = \frac{6\epsilon_0 A}{2d} = \frac{3\epsilon_0 A}{d}$$

$$\Rightarrow \Delta C = C' - C = \frac{3\epsilon_0 A}{d} - \frac{\epsilon_0 A}{d} = \frac{2\epsilon_0 A}{d}$$

Percentage change in capacitance,

$$\frac{\Delta C}{C} = \frac{\frac{2\epsilon_0 A}{d}}{\frac{\epsilon_0 A}{d}} \times 100\% = 200\%$$

- 204 (c)** We have, $dQ = \left(\frac{Q}{2l} \right) \cdot dx$



$$\text{Potential of small part, } dV = \frac{1}{4\pi\epsilon_0} \cdot \frac{dQ}{x} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{2l} \cdot \frac{dx}{x}$$

$$\therefore \text{Potential at } P, V = \int_{x=l}^{x=2l} dV = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{2l} \ln(3)$$

- 205 (b)** Work done, $W_{AB} = U_B - U_A = q_0(V_B - V_A)$

$$\begin{aligned} \text{Potential, } V_B - V_A &= - \int_A^B E dr \\ &= - \int_{3a}^{2a} \frac{\lambda}{2\pi\epsilon_0 r} dr = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{3}{2}\right) \end{aligned}$$

$$\therefore \text{Work done, } W_{A \rightarrow B} = \frac{q_0 \lambda}{2\pi\epsilon_0} \ln\left(\frac{3}{2}\right)$$

- 206 (a)** $\Delta U = U_f - U_i$

$$\begin{aligned} &= \left[\frac{Kq_1 q_2}{0.3} + \frac{Kq_1 q_3}{0.4} + \frac{Kq_2 q_3}{0.1} \right] - \left[\frac{Kq_1 q_2}{0.3} + \frac{Kq_1 q_3}{0.4} + \frac{Kq_2 q_3}{0.5} \right] \\ &= 8Kq_2 q_3 = 8q_2 \frac{q_3}{4\pi\epsilon_0} \end{aligned}$$

According to the question, the change in the potential energy is $\frac{q_3}{4\pi\epsilon_0} K$.

$$\therefore K = 8q_2$$

- 207 (c)** $q_A = \frac{Q}{2}$

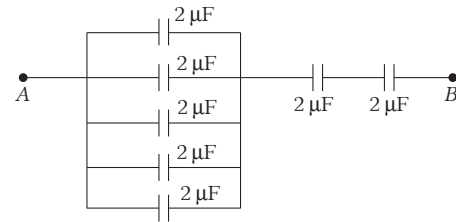
When B is earthed

$$\begin{aligned} V_B &= 0 \\ \therefore \frac{Kq_B}{a} + \frac{K(Q/2)}{d} &= 0 \\ \therefore q_B &= \frac{-Qa}{2d} \end{aligned}$$

When C is earthed

$$\begin{aligned} V_C &= 0 \\ \therefore \frac{Kq_C}{a} + \frac{K(Q/2)}{d} - \frac{K(Qa/2d)}{d} &= 0 \\ \therefore q_C &= \frac{Qa^2}{2d^2} - \frac{Qa}{2d} = \frac{Qa}{2d} \left(\frac{a-d}{d} \right) \end{aligned}$$

- 208 (a)** From concept of series and parallel combination, we can easily find that in option (a) the resultant capacity is $\frac{10}{11} \mu\text{F}$.

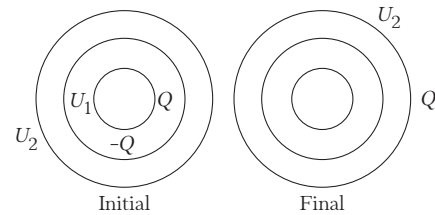


The circuit can be redrawn as



$$\therefore \text{Equivalent capacity, } C_{eq} = \frac{10 \times 1}{10 + 1} = \frac{10}{11} \mu\text{F}$$

- 209 (c)** Heat produced $= U_i - U_f = (U_1 + U_2) - U_2 = U_1$



$$\text{Heat produced} = \frac{q^2}{2C} = \frac{q^2}{2 \left[4\pi\epsilon_0 \left\{ \frac{a \times 2a}{2a - a} \right\} \right]} = \frac{Kq^2}{4a}$$

210 (c) $F = \frac{K \cdot q^2}{l^2}$ or $\frac{Kq^2}{l} = F \cdot l = W$

Now, $W = U_f - U_i = \frac{3K \cdot q^2}{2l} - \frac{3Kq^2}{l}$
 $= -\frac{3Kq^2}{2l} = -\frac{3}{2} Fl$

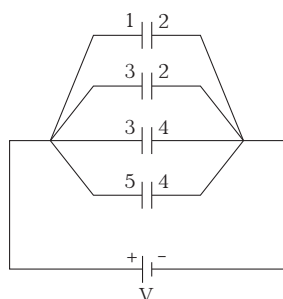
- 211 (b) Applying charge distribution law on C_1 and C_2 at steady state. Charge on C_1 ,

$$Q_1 = \left(\frac{C_1}{C_1 + 2C_1} \right) \times Q = \frac{Q}{3}$$

Charge on C_2 ,

$$Q_2 = \left(\frac{2C_1}{C_1 + 2C_1} \right) \times Q = \frac{2}{3} Q$$

- 212 (c) Given circuit can be redrawn as shown



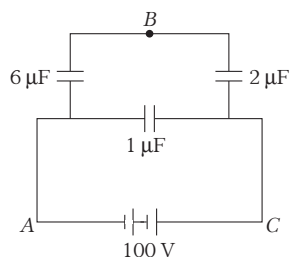
Capacity of each capacitor, $C = \frac{\epsilon_0 A}{d}$

So, magnitude of charge on each capacitor = Magnitude of charge on each plate = $\frac{\epsilon_0 A}{d} V$.

As plate 1 is connected with +ve terminal of battery, so charge on plate 1 = $+\frac{\epsilon_0 A}{d} V$.

Plate 4 comes twice and it is connected with negative terminal of battery. So, charge on plate 4 = $-\frac{2\epsilon_0 A V}{d}$

- 213 (c) The given circuit can be reduced as follows. (Resistance does not matter in considering equivalent capacitance)



$$C_{eq} = \frac{6 \times 2}{6 + 2} + 1 = \frac{5}{2} \mu F$$

Total charge, $Q = CV = \frac{5}{2} \times 100 = 250 \mu C$

So, charge in $6 \mu F$ branch = $CV = \left(\frac{6 \times 2}{6 + 2} \right) \times 100 = 150 \mu C$

$$\therefore V_{AB} = \frac{150}{6} = 25 V$$

$$\text{and } V_{BC} = 100 - V_{AB} = 100 - 25 = 75 V$$

(B) Medical entrance special format questions

• Assertion and reason

- 1 (d) Electric potential of a charged conductor depends not only on the amount of charge and volume but also on the shape of the conductor. Hence, if their shapes are different, they may have different electric potential.

- 2 (b) Potential energy of a system of two charges, $U = K \frac{q_1 q_2}{r}$.

$\therefore$ When two positive point charges move away from each other, then their potential energy decreases and work done by electrostatic force can always be expressed in terms of a potential energy, when the particle moves from a point.

- 3 (d) Battery is disconnected from the capacitor.

So, $Q = \text{constant}$

$$\text{Energy} = \frac{Q^2}{2C} = \frac{Q^2 d}{2\epsilon_0 A}$$

$$\Rightarrow \text{Energy} \propto d$$

- 4 (c) Energy supplied by battery = $qV = (CV)V = CV^2$

$$\text{Energy stored} = \frac{1}{2} CV^2$$

$$\therefore \text{Energy lost} = CV^2 - \frac{1}{2} CV^2 = \frac{1}{2} CV^2$$

Therefore, half energy is lost.

- 5 (b) $C = \frac{\epsilon_0 k A}{d} \Rightarrow C \propto \frac{k}{d}$

$$\therefore \frac{C_1}{C_2} = \frac{k_1}{d_1} \times \frac{d_2}{k_2} = \frac{k}{d} \times \frac{d/2}{3k} = \frac{1}{6}$$

$$C_2 = 6C_1$$

Again for capacity of a capacitor, $C = \frac{k\epsilon_0 A}{d}$

Therefore, capacity of a capacitor depends upon the medium between two plates of capacitor.

• Statement based questions

- 1 (b) The torque acting on the dipole is given by $\tau = \mathbf{p} \times \mathbf{E}$, the potential energy of the dipole is given by $U = -\mathbf{p} \cdot \mathbf{E}$ and the resultant force on the dipole is zero.

- 2 (b) Electrostatic field at the surface of a conductor is normal to the surface.

- 3 (c) In this problem, the electric field intensity E and electric potential V are related as $E = -\frac{dV}{dr}$.

Electric field intensity, $E = 0$ suggest that, $\frac{dV}{dr} = 0$

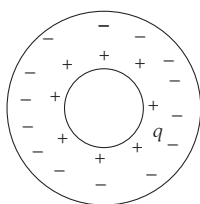
This imply that $V = \text{constant}$.

Thus, $E = 0$ inside the charged conducting sphere causes, the same electrostatic potential 100 V at any point inside the sphere.

Note V equals zero does not necessary imply that $E = 0$ e.g., the electric potential at any point on the perpendicular bisector due to electric dipole is zero but E not.

$E = 0$ does not necessary imply that $V = 0$ e.g., the electric field intensity at any point inside the charged spherical shell is zero but there may exist non-zero electric potential.

- 4 (a) When a charge density is given to inner cylinder an electric field will be produced between the inner and outer cylinder. Hence, a potential difference will appear between the two cylinders.



- 5 (c) Since, potential around Q_1 is positive and that around Q_2 is negative, so Q_1 is positive and Q_2 is negative in nature. Also, potential at A is zero, i.e.

$$\frac{|kQ_1|}{r_1} + \frac{kQ_2}{r_2} = 0 \Rightarrow r_1 > r_2$$

$$\therefore |Q_1| > |Q_2|$$

Also, at point C, potential is maximum, so it is a point of unstable equilibrium.

• Match the columns

1 (a) $U \propto \frac{1}{r}$ and $F \propto \frac{1}{r^2}$

2 (c) $q = CV, U = \frac{1}{2} CV^2, C = \frac{K\epsilon_0 A}{d}$ or $\frac{\epsilon_0 A}{d-t}$

- 3 (b) A dielectric slab is when inserted to fill the space between the plates and battery is removed, then

Quantity	Battery is removed
Capacity	$C' = KC$
Charge	$Q' = Q$
Potential	$V' = V/K$
Electric field	$E' = E/K$
Energy	$U' = U/K$

4 (c) (A) Capacitance, $C = \frac{\epsilon_0 A}{d}$

$$= \frac{8.85 \times 10^{-12} \times 0.20}{0.01} = 1.77 \times 10^{-10} \text{ (SI unit)}$$

(B) Charge on each plate, $q = CV = 3000 \times 1.77 \times 10^{-10}$

$$= 5.31 \times 10^{-7} \text{ (SI unit)}$$

(C) $\therefore C = \frac{\epsilon_0 A}{d-t + \frac{t}{K}}$

$$\Rightarrow 5.31 \times 10^{-10} = \frac{8.85 \times 10^{-12} \times 0.20}{0.01 - 0.01 + \frac{0.01}{K}}$$

$$\Rightarrow K = \frac{5.31 \times 10^{-10} \times 0.01}{8.85 \times 10^{-12} \times 0.20}$$

Dielectric constant, $K = 3$

(D) Capacitance, $C = \frac{q}{V}$

$$\Rightarrow C = \frac{5.31 \times 10^{-7}}{1000}$$

$$\Rightarrow C = 5.31 \times 10^{-10} \text{ (SI unit)}$$

(C) Medical entrances' gallery

- 1 (d) Given, volume, $V = 0.2 \text{ m}^3$

Electric potential = 5 V = constant

Electric field = ?

We know that for constant electric potential, the value of electric field is zero.

i.e. $E = \frac{-dV}{dr} = \frac{-d(5)}{dr} = 0$

- 2 (b) Given, $C_0 = 6 \mu\text{F}$

and $C_m = 30 \mu\text{F}$

$\therefore$ As, dielectric constant, $K = \epsilon_r = \frac{C_m}{C_0} = \frac{30}{6} = 5$

Permittivity of the medium,

$$\epsilon_m = K \times \epsilon_0 = 5 \times \epsilon_0$$

$$= 5 \times 8.85 \times 10^{-12}$$

$$= 0.44 \times 10^{-10} \text{ C}^2 \text{N}^{-1} \text{m}^{-2}$$

- 3 (a) Given, electric dipole moment, $p = 16 \times 10^{-9} \text{ C-m}$

Distance, $r = 0.6 \text{ m}$

Angle, $\theta = 60^\circ \Rightarrow \cos 60^\circ = \frac{1}{2}$

Electric potential at a point which is at a distance r at some angle θ from electric dipole is

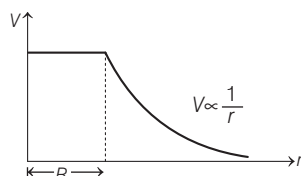
$$V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2} = \frac{9 \times 10^9 \times 16 \times 10^{-9} \times \frac{1}{2}}{(0.6)^2}$$

$$= 2 \times 10^2 = 200 \text{ V}$$

- 4 (b) Since, electric potential remains constant inside the metallic spherical shell and same at the surface of spherical shell.

Outside the spherical shell, $V \propto \frac{1}{r}$

Hence, variation of potential (V) with distance r is given as



- 5 (b) Capacitance of parallel plate capacitor when medium is air

$$C_0 = \frac{\epsilon_0 A}{d} \quad \dots (i)$$

According to second condition,

$$A' = A, t = d/2, K = 4$$

$$\therefore \text{Capacitance, } C = \frac{\epsilon_0 A}{(d-t) + \frac{t}{K}} = \frac{\epsilon_0 A}{\left(d - \frac{d}{2}\right) + \frac{d/2}{4}}$$

$$= \frac{\epsilon_0 A}{\frac{d}{2} + \frac{d}{8}} = \frac{8}{5} \cdot \frac{\epsilon_0 A}{d}$$

$$\therefore \frac{C}{C_0} = \frac{\frac{8}{5} \cdot \frac{\epsilon_0 A}{d}}{\frac{\epsilon_0 A}{d}}$$

$$\Rightarrow \frac{C}{C_0} = \frac{8}{5}$$

$$\Rightarrow C : C_0 = 8 : 5$$

- 6 (a) Given, capacitance of a capacitor,
 $C = 15 \text{ nF} = 15 \times 10^{-9} \text{ F}$,
 $\epsilon_r = 2.5$

Electric field, $E = 30 \times 10^6 \text{ V/m}$

Potential difference, $V = 30 \text{ V}$

Area of plate = ?

If d be the distance between the plates, then

$$d = \frac{V}{E} = \frac{30}{30 \times 10^6} = 10^{-6} \text{ m}$$

Capacitance of capacitor,

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$15 \times 10^{-9} = \frac{8.85 \times 10^{-12} \times 2.5 \times A}{10^{-6}}$$

$$\Rightarrow A = 6.7 \times 10^{-4} \text{ m}^2$$

- 7 (c) Given, potential difference,

$$V(x) = -x^2 y \text{ volt}$$

$$\therefore \mathbf{E} = -\Delta V = -\left[\hat{i} \frac{\partial}{\partial x} (-x^2 y) + \hat{j} \frac{\partial}{\partial y} (-x^2 y)\right]$$

$$= -[-2xy \hat{i} - x^2 \hat{j}]$$

$$= 2xy \hat{i} + x^2 \hat{j}$$

$$\mathbf{E} \text{ at } (1, 2) = 2 \times 1 \times 2 \hat{i} + 1^2 \hat{j} = 4\hat{i} + \hat{j} \text{ Vm}^{-1}$$

- 8 (c) In a parallel plate capacitor, the capacity of capacitor,

$$C = \frac{K \epsilon_0 A}{d}$$

$$\therefore C \propto A$$

So, the capacity of capacitor increases, if area of the plate is increased.

- 9 (b) The electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed \text{ or } V \propto d$$

Hence, if the plates are pulled apart, the potential difference increases.

- 10 (c) As we know, the total work in transferring a charge to a parallel plate capacitor is given as

$$W = \frac{Q^2}{2C} \quad \dots (i)$$

We can also write a relation for work done as, $W = F \cdot d \dots (ii)$

From Eqs. (i) and (ii), we get

$$W = \frac{Q^2}{2C} = Fd$$

$$\Rightarrow F = \frac{Q^2}{2Cd} \quad \dots (iii)$$

As, the capacitance of a parallel plate capacitor is given as

$$C = \frac{\epsilon_0 A}{d}$$

Substituting the value of C in Eq. (iii), we get

$$F = \frac{Q^2 d}{2\epsilon_0 A d} = \frac{Q^2}{2\epsilon_0 A}$$

Thus, it means electrostatic force is independent of the distance between the plates.

- 11 (c) This combination forms a GP,

$$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

$$\text{Sum of infinite GP, } S = \frac{a}{1-r} \Rightarrow S = \frac{1}{1-1/2} = \frac{1}{1/2} = 2$$

Hence, capacitance of combination, $C_\infty = 2 \times 1 \mu\text{F} = 2 \mu\text{F}$

- 12 (a) This combination is same as the two capacitors are connected in series and distance between plate of each capacitor is $d/2$. So,

$$C_1 = \frac{K\epsilon_0 A}{d/2} \text{ and } C_2 = \frac{\epsilon_0 A}{d/2}$$

$$\text{Hence, } C_{\text{net}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$= \frac{\left(\frac{2K\epsilon_0 A}{d}\right) \left(\frac{2\epsilon_0 A}{d}\right)}{\left(\frac{2K\epsilon_0 A}{d} + \frac{2\epsilon_0 A}{d}\right)}$$

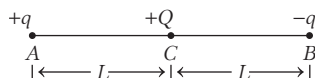
$$C_{\text{net}} = \frac{2KA\epsilon_0}{(K+1)d}$$

13 (b) ∴ Capacitance of a parallel plate capacitor, $C = \frac{\epsilon_0 A}{d}$

$$\Rightarrow A = \frac{Cd}{\epsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.854 \times 10^{-12}}$$

$$\Rightarrow A = 1130 \text{ km}^2$$

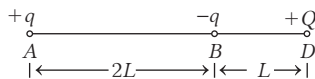
14 (d) In first case, when charge $+Q$ is situated at C.



Electric potential energy of system,

$$U_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q(-q)}{2L} + \frac{1}{4\pi\epsilon_0} \cdot \frac{(-q)Q}{L} + \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{L}$$

In second case, when charge $+Q$ is moved from C to D.



Electric potential energy of system in this case,

$$U_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q(-q)}{2L} + \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{3L} + \frac{1}{4\pi\epsilon_0} \cdot \frac{(-q)(Q)}{L}$$

Work done $(\Delta U) = U_2 - U_1$

$$\begin{aligned} &= \left(-\frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{2L} + \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{3L} - \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{L} \right) \\ &\quad - \left(-\frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{2L} - \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{L} + \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{L} \right) \\ &= \frac{qQ}{4\pi\epsilon_0} \left[\frac{1}{3L} - \frac{1}{L} \right] = \frac{qQ}{4\pi\epsilon_0} \cdot \frac{(1-3)}{3L} \\ &= -\frac{2qQ}{12\pi\epsilon_0 L} = -\frac{qQ}{6\pi\epsilon_0 L} \end{aligned}$$

15 (c) The two capacitors are connected in parallel.

The equivalent capacitance,

$$C' = C + \frac{C}{2} = \frac{3C}{2}$$

Work done in charging the equivalent capacitor is stored in the form of potential energy,

$$W = U = (1/2)C'V^2$$

$$W = \frac{1}{2} \times \frac{3C}{2} \times V^2$$

$$W = \frac{3}{4} CV^2$$

16 (d) At closest distance r , the whole kinetic energy of charge q is converted into potential energy.

$$\therefore \frac{1}{2} mv^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q \cdot q}{r}$$

$$\Rightarrow r = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Q \cdot q}{mv^2}$$

$$\text{In next case, } r' = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Qq}{m(2v)^2}$$

$$\Rightarrow r' = \frac{1}{4} \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{2Qq}{mv^2} \right)$$

$$\Rightarrow r' = \frac{r}{4}$$

17 (d) The arrangement of n metal plates separated by dielectric acts as parallel combination of $(n-1)$ capacitors.

$$\text{Therefore, } C = \frac{(n-1)K\epsilon_0 A}{d}$$

$$\text{Here, } C = 100 \text{ pF} = 100 \times 10^{-12} \text{ F,}$$

$$K = 4, \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2},$$

$$A = \pi r^2 = 3.14 \times (1 \times 10^{-2})^2$$

$$\text{and } d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

$$\therefore 100 \times 10^{-12} = \frac{(n-1) \times 4 \times (8.85 \times 10^{-12}) \times 3.14 \times (1 \times 10^{-2})^2}{1 \times 10^{-3}}$$

$$\Rightarrow n = 9.99 \approx 10$$

18 (c) Consider the given figure,

When the switch S is connected to point 1, then initial energy stored in the capacitor can be given as

$$E_1 = \frac{1}{2} \times 2 \times 10^{-6} \times V^2.$$

When the switch S is connected to point 2, energy dissipated on connection across $8 \mu\text{F}$ will be

$$\begin{aligned} E_2 &= \frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2} \right) \cdot V^2 \\ &= \frac{1}{2} \times \frac{2 \times 10^{-6} \times 8 \times 10^{-6}}{10^{-5}} \times V^2 \\ &= \frac{1}{2} \times (1.6 \times 10^{-6}) \times V^2 \end{aligned}$$

Therefore, % loss of energy

$$= \frac{E_2}{E_1} \times 100 = \frac{\frac{1}{2} \times 16 \times 10^{-6} \times V^2}{\frac{1}{2} \times 2 \times 10^{-6} \times V^2} = 80\%$$

19 (c) Given capacitor is equivalent to capacitors K_1, K_2 and K_3 in parallel and part of K_4 in series with them.

$$\begin{aligned} \frac{1}{C_{\text{eq}}} &= \frac{1}{(C_1 + C_2 + C_3)} + \frac{1}{C_4} \\ \Rightarrow \frac{1}{\frac{K\epsilon_0 A}{d}} &= \frac{1}{\frac{2}{3} [K_1 + K_2 + K_3] \frac{\epsilon_0 A}{d}} + \frac{1}{2(K_4) \frac{\epsilon_0 A}{d}} \\ \Rightarrow \frac{2}{K} &= \frac{3}{[K_1 + K_2 + K_3]} + \frac{1}{K_4} \end{aligned}$$

20 (d) When a parallel plate air capacitor connected to a cell of emf V , then charge stored will be

$$\begin{aligned} q &= CV \\ \Rightarrow V &= \frac{q}{C} \end{aligned}$$

Also energy stored is $U = \frac{1}{2} CV^2 = \frac{q^2}{2C}$

As the battery is disconnected from the capacitor, the charge will not be destroyed, i.e. $q' = q$ with the introduction of dielectric in the gap of capacitor, so the new capacitance will be

$$C' = CK$$

$$\Rightarrow V' = \frac{q}{C'} = \frac{q}{CK}$$

The new energy stored will be

$$\begin{aligned} U' &= \frac{q^2}{2CK} \\ \Delta U &= U' - U \\ &= \frac{q^2}{2C} \left(\frac{1}{K} - 1 \right) = \frac{1}{2} CV^2 \left(\frac{1}{K} - 1 \right) \end{aligned}$$

Hence, option (d) is incorrect.

21 (b) Displacement, $s = ut + \frac{1}{2} at^2 = \frac{1}{2} at^2$ or $t = \sqrt{\frac{2s}{a}}$

As, s is same.

$$\therefore t \propto \frac{1}{\sqrt{a}} \Rightarrow \frac{t_2}{t_1} = \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{F_e/M_e}{F_p/M_p}} = \sqrt{\frac{M_p}{M_e}}$$

22 (b) As, potential of spherical charge (Q) having radius r is given by $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$

Also, capacitance, $C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}}$

$$\therefore C = 4\pi\epsilon_0 r$$

If $C = 50 \times 10^{-12}$ F, then radius of the sphere,

$$\begin{aligned} r &= \frac{50 \times 10^{-12}}{4\pi\epsilon_0} \\ &= 50 \times 10^{-12} \times 9 \times 10^9 \\ &= 50 \times 9 \times 10^{-3} \\ &= 450 \times 10^{-3} \text{ m} \\ &= 0.450 \text{ m} = 45 \text{ cm} \end{aligned}$$

23 (d) There are 91 plates forming 90 capacitors.

All the capacitors are connected in parallel, then the equivalent capacitance is

$$C_{eq} = (90) C = (90) (3 \text{ pF}) = 270 \text{ pF}$$

24 (b) We have, $mg = qE$

$$\begin{aligned} q &= \frac{mg}{E} = \frac{mg}{V/d} = \frac{1.96 \times 10^{-15} \times 10}{(400/0.02)} \\ &= \frac{1.96 \times 10^{-14}}{2 \times 10^4} = 0.98 \times 10^{-18} \\ &= 9.8 \times 10^{-19} \approx 6e \end{aligned}$$

25 (b) At distance of closest approach, total energy of particle is converted into potential energy.

Let charge on α -particle is q_1 and charge on nucleus is q_2 , then in first case

$$K = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_0} \quad \dots(i)$$

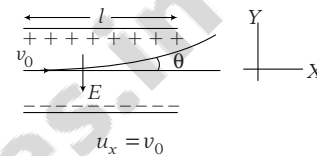
In second case, (let distance of closest approach is r_0')

$$2K = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_0'} \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{2K}{K} = \frac{\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_0'}}{\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_0}} = \frac{1}{r_0'} \times \frac{r_0}{1} = \frac{r_0}{r_0'} \Rightarrow r_0' = \frac{r_0}{2}$$

26 (b) Initial velocity of the electron along x-direction is given by



As, applied electric field is vertical,

$$v_x = u_x = v_0 \Rightarrow u_y = 0 \text{ and } v_y = u_y + a_y t$$

or $v_y = 0 + \frac{eE}{m} \times \frac{l}{v_0} \quad (\because l = v_0 t)$

or $v_y = \frac{eEl}{mv_0}$

We can write, $\tan \theta = \frac{v_y}{v_x}$

$$\Rightarrow \tan \theta = \frac{eEl}{mv_0} \times \frac{1}{v_0} = \frac{eEl}{mv_0^2}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{eEl}{mv_0^2} \right)$$

27 (a) Energy stored in a parallel plate capacitor of capacitance C and separated by a distance d is

$$V_0 = \frac{1}{2} CV^2 = \frac{1}{2} \left(\frac{\epsilon_0 A}{d} \right) V^2$$

If a slab of dielectric constant K and thickness d is introduced between plates of capacitor, then

$$C' = \frac{\epsilon_0 A}{d - d \left(1 - \frac{1}{K} \right)} = \frac{K\epsilon_0 A}{d}$$

$\therefore$ The new energy of the system is given by

$$U' = \frac{1}{2} C' V^2 = \frac{1}{2} \left(\frac{K\epsilon_0 A}{d} \right) V^2 = K \left(\frac{1}{2} \frac{\epsilon_0 A}{d} V^2 \right) = KV_0$$

28 (a) The time constant,

$$\tau = CR = 100 \times 2 = 200 \mu\text{s}$$

The charge on capacitor after time t ,

$$q = \epsilon C (1 - e^{-t/RC})$$

According to question, $q = 0.99 \epsilon C$

$$\Rightarrow 0.99 = 1 - e^{-t/200 \mu\text{s}}$$

$$\Rightarrow \frac{t}{200 \mu s} = \ln(0.01)$$

This gives, $t = 920 \mu s = 0.92 \text{ ms}$

- 29** (c) After isolation of capacitor, charge is constant.

$$\text{Capacity, } C = \frac{\epsilon_0 A}{d}$$

$\therefore$ Capacitance decreases with increase in distance.

$$V = \frac{Q}{C}$$

$\therefore$ Potential increases with decrease in capacitance (C).

- 30** (a) Inside the spherical shell potential is same.

$$\therefore V_1 = V_2$$

$$\text{Also, } V \propto \frac{1}{d}, \text{ i.e. } V_1 = V_2 > V_3$$

- 31** (a) The equivalent capacitance between the points P and N is given by

$$\frac{1}{C_{eq}} = \frac{1}{(10 + 20)} + \frac{1}{30} = \frac{2}{30} = \frac{1}{15}$$

$$\therefore C_{eq} = 15 \mu F$$

Charge on equivalent capacitor having capacitance $15 \mu F$ is

$$Q = C_{eq} V = 15 \times 30 = 450 \mu C$$

- 32** (c) When length x of the dielectric slab is placed between the plates of the capacitor, then

$$C_1 = \frac{K \epsilon_0 t x}{d} \quad \left(\because C = K \frac{\epsilon_0 A}{d} \right)$$

$$\text{and similarly, } C_2 = \frac{\epsilon_0 t(L - x)}{d}$$

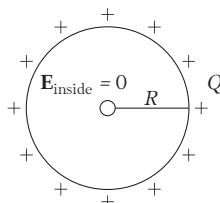
$$\therefore \text{Net capacitance, } C_{net} = C_1 + C_2 = \frac{\epsilon_0 t}{d} [L + x(K - 1)]$$

Thus, energy stored in the capacitor,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} C_{net} V^2 = \frac{\epsilon_0 t V^2}{2d} [L + x(K - 1)]$$

- 33** (b) As, we know, $E_{inside} = 0$

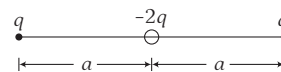
$$\text{and } V_{inside} = V_{surface} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$



- 34** (c) Graph (c) will be the right graph. The electric field inside the dielectrics will be less than the electric field outside the dielectrics. The electric field inside the dielectric could not be zero.

As, $K_2 > K_1$, the drop in electric field for dielectric slab K_2 must be more than dielectric slab K_1 .

- 35** (d) Potential energy of the system,



$$\begin{aligned} U &= \frac{1}{4\pi\epsilon_0} \left[\frac{q(-2q)}{r_{12}} + \frac{q(-2q)}{r_{13}} + \frac{q(-2q)}{r_{23}} \right] \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{q(-2q)}{a} + \frac{q(-2q)}{a} + \frac{q(-2q)}{2a} \right] \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{-2q^2}{a} - \frac{2q^2}{a} + \frac{q^2}{2a} \right] \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{-4q^2}{a} + \frac{q^2}{2a} \right] = \frac{-7q^2}{8\pi\epsilon_0 a} \end{aligned}$$

- 36** (c) Let the charge on the inner shell be q' .

The total charge will be zero.

$$\text{So, } \frac{Kq'}{r_1} + \frac{Kq}{r_2} = 0 \quad (\because r_2 > r_1)$$

$$\therefore q' = \frac{-Kq/r_2}{K/r_1} \Rightarrow q' = -\left(\frac{r_1}{r_2}\right)q$$

- 37** (c) Given, $q = 3 \text{ nC} = 3 \times 10^{-9} \text{ C}$

$$\text{and } r = 9 \text{ cm} = 9 \times 10^{-2} \text{ m}$$

We know that, potential due to point charge is given by

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} = \frac{9 \times 10^9 \times 3 \times 10^{-9}}{9 \times 10^{-2}} = 3 \times 10^2 \text{ V}$$

$\therefore$ Electric potential, $V = 300 \text{ V}$

- 38** (d) For the combination of three capacitors in series,

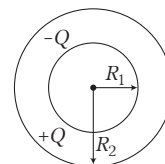
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow \frac{1}{C} = \frac{1}{3} + \frac{1}{6} + \frac{1}{6} \Rightarrow C = \frac{6}{4} = 1.5 \mu F$$

The charge stored in this circuit, $q = CV = 1.5 \times 120 = 180 \mu C$

The potential difference across the $3 \mu F$

$$q = CV, \quad V = \frac{q}{C} = \frac{180}{3} = 60 \text{ V}$$

- 39** (c) We know that,



For concentric spherical shells ($R_2 > R_1$),

$$V_{R_1} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R_1} \quad \dots(i)$$

$$V_{R_2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R_2} \quad \dots(ii)$$

The common potential of the capacitors,

$$V = V_{R_1} - V_{R_2} = \frac{1}{4\pi\epsilon_0} \cdot Q \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

If the capacitance of two concentric spherical shell be C , then

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\epsilon_0} \left(\frac{R_2 - R_1}{R_2 R_1} \right)} = 4\pi\epsilon_0 \frac{R_1 R_2}{(R_2 - R_1)}$$

40 (a) Given, $C_1 = 10 \text{ pF} = 10 \times 10^{-12} \text{ F}$

$$C_2 = 20 \text{ pF} = 20 \times 10^{-12} \text{ F}$$

$$V_1 = 200 \text{ V and } V_2 = 100 \text{ V}$$

where, C_1 = capacitance of 1st capacitor,

C_2 = capacitance of 2nd capacitor,

V_1 = voltage across 1st capacitor

and V_2 = voltage across 2nd capacitor.

$$\text{We know that, } V_1 = \frac{q_1}{C_1} \text{ and } V_2 = \frac{q_2}{C_2} \Rightarrow q_1 = V_1 C_1 \quad \dots(i)$$

$$q_2 = V_2 C_2 \quad \dots(ii)$$

So, common potential of capacitors,

$$\begin{aligned} V &= \frac{q_1 + q_2}{C_1 + C_2} = \frac{V_1 C_1 + V_2 C_2}{C_1 + C_2} \\ &= \frac{200 \times 10 \times 10^{-12} + 100 \times 20 \times 10^{-12}}{10 \times 10^{-12} + 20 \times 10^{-12}} \\ &= \frac{200 \times 10 + 100 \times 20}{10 + 20} \\ &= \frac{2000 + 2000}{30} = \frac{4000}{30} = 133.3 \text{ V} \end{aligned}$$

41 (b) Clearly, the capacitors form Wheatstone bridge arrangement.

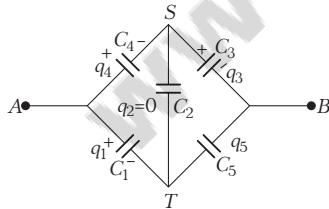
$$C_1 = C_2 = C_3 = C_4$$

$$\Rightarrow \frac{C_4}{C_1} = \frac{C_3}{C_2}$$

By symmetry, $q_1 = q_4$, $q_3 = q_5$

$$\text{Also, } q_1 + q_4 = q_3 + q_5$$

$$\Rightarrow q_1 = q_4 = q_3 = q_5 \Rightarrow q_2 = 0$$



Hence, the bridge is balanced. The point S and T are at the same potential. No charge can accumulate on C_2 , which thus become ineffective.

Between points A and B , two series combination (C_4, C_3) and (C_1, C_5) are connected in parallel.

Then, effective capacitance,

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{4} \Rightarrow C' = \frac{4}{2} = 2 \mu\text{F}$$

Similarly, $C'' = 2 \mu\text{F}$

Hence, equivalent capacitance between A and B is

$$C = C' + C'' = 2 + 2 = 4 \mu\text{F}$$

42 (a) Potential due to a uniformly charged sphere is

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \quad \dots(i)$$

Also,

$$\sigma = \frac{Q}{4\pi R^2} = \frac{Q \epsilon_0}{4\pi \epsilon_0 R^2}$$

$$\frac{\sigma R^2}{\epsilon_0} = \frac{1 \times Q}{4\pi \epsilon_0} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $V = \frac{\sigma R}{\epsilon_0}$

Potential due to outer sphere, $V_1 = \frac{\sigma R}{\epsilon_0}$

Potential due to inner sphere, $V_2 = \frac{\sigma r}{\epsilon_0}$

Net potential at centre, $V = V_1 + V_2 = \frac{\sigma}{\epsilon_0} (R + r)$

43 (b) The electric field is maximum at B , because electric field is directed along decreasing potential, i.e. $V_B > V_C > V_A$.

$$\text{44 (b) Potential, } V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} = \frac{(9 \times 10^9) \times (5 \times 10^{-8})}{0.1} = 4500 \text{ V}$$

45 (d) As, the capacitors are identical,

$$\therefore C_1 = C_2 = C$$

$$C_S = \frac{C_1 C_2}{C_1 + C_2} = \frac{C \cdot C}{2C} = \frac{C}{2}$$

$$C_P = C + C = 2C$$

$$\therefore \frac{C_S}{C_P} = 1:4$$

46 (d) Given, $V_1 = 120 \text{ V}$ and $V_2 = 200 \text{ V}$

Here given that potential is zero for each capacitor, then their charge must be same, i.e. $C_1 V_1 = C_2 V_2$

$$120 C_1 = 200 C_2$$

$$12 C_1 = 20 C_2$$

$$\Rightarrow 3 C_1 = 5 C_2$$

47 (b) As, $F = mg = qE$

$$\therefore E = \frac{mg}{q} \text{ or } q = \frac{mg}{E}$$

$$\text{Also, } E = \frac{V}{d}$$

$$\therefore q = \frac{mgd}{V} = \frac{(10^{-6} \times 10 \times 1 \times 10^{-3})}{500} = 2 \times 10^{-11} \text{ C}$$

48 (c) When two metal spheres are joined by a wire, charge flows from one at higher potential to the other at lower potential, till they acquire the same potential.

$$\therefore V_1 = V_2$$

$$\frac{q_1}{4\pi\epsilon_0 r_1} = \frac{q_2}{4\pi\epsilon_0 r_2} \Rightarrow \frac{r_1}{r_2} = \frac{q_1}{q_2} = \frac{15 \text{ mC}}{45 \text{ mC}} = \frac{1}{3}$$

Final charge on first sphere,

$$q_1 = \frac{1}{3} (q_1 + q_2) = \frac{1}{3} \times 60 = 20 \text{ mC}$$

- 49 (a) Electric potential at their common centre of concentric spheres is given by

$$V = \frac{\sigma}{\epsilon_0} (R + r)$$

where, σ is surface charge density.

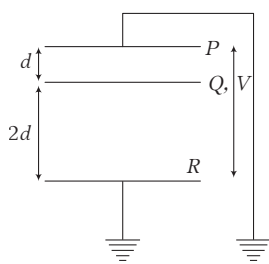
- 50 (c) In the given arrangement, plate Q is common for two capacitors which are connected in parallel.

$$\therefore C_{\text{eff}} = C_P = C_1 + C_2$$

$$\Rightarrow C_P = \frac{\epsilon_0 A}{d} + \frac{\epsilon_0 A}{2d} = \frac{3\epsilon_0 A}{2d}$$

Let V be the potential difference across the capacitor, which is equal to the potential of the plate Q .

So, the arrangement can be shown as,



$$\text{Potential, } V = \frac{Q}{C_P} = \frac{8.85 \times 10^{-8}}{\frac{3 \epsilon_0 A}{2d}} = \frac{2d \times 8.85 \times 10^{-8}}{3 \times 8.85 \times 2}$$

$$= \frac{2 \times 2 \times 10^{-3} \times 8.85 \times 10^{-8}}{3 \times 8.85 \times 10^{-12} \times 2} = 6.67 \text{ V}$$

- 51 (d) We know that, potential, $V = \frac{q}{4\pi\epsilon_0 r}$

Here, $r_1 = r$ and $r_2 = 2r$

$$\text{So, } V_1 = \frac{q}{4\pi\epsilon_0 r} \text{ and } V_2 = \frac{q}{4\pi\epsilon_0 (2r)}$$

$$\text{Hence, } \frac{V_1}{V_2} = \frac{q}{4\pi\epsilon_0 r} \times \frac{4\pi\epsilon_0 (2r)}{q}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{2}{1}; V_2 = \frac{V_1}{2}$$

In question, $V_1 = 12 \text{ V}$ (given)

$$\text{So, } V_2 = \frac{12}{2} = 6 \text{ V}$$

- 52 (c) Let, the charge on the inner sphere be Q , then the charge induced on the inner surface of the outer sphere is $-Q$.
 $\therefore$ Electric potential V of the inner sphere is given by

$$V = \frac{Q}{4} - \frac{Q}{6}$$

But $V = 3 \text{ esu}$

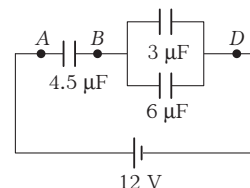
$$\therefore 3 = \frac{Q}{4} - \frac{Q}{6}$$

$$\Rightarrow Q \left[\frac{1}{4} - \frac{1}{6} \right] = 3 \Rightarrow \frac{Q}{12} = 3$$

$\therefore$ Charge on the inner sphere, $Q = 36 \text{ esu}$

- 53 (c) The given circuit capacitance between B and D is

$$C_1 = (3 + 6) \mu\text{F} = 9 \mu\text{F}$$



Capacitance between A and D ,

$$\frac{1}{C_2} = \frac{1}{4.5} + \frac{1}{9} = \frac{2+1}{9} = \frac{3}{9} = \frac{1}{3}$$

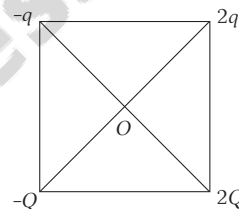
$$\Rightarrow C_2 = 3 \mu\text{F}$$

Charge on $C_2 = (3 \mu\text{F})(12 \text{ V}) = 36 \mu\text{C}$

So, the potential difference between A and B

$$= \frac{\text{Charge}}{\text{Capacitance}} = \frac{36 \mu\text{C}}{4.5 \mu\text{F}} = 8 \text{ V}$$

- 54 (a)



If potential at centre is zero, then

$$V_1 + V_2 + V_3 + V_4 = 0$$

$$-\frac{KQ}{r} - \frac{Kq}{r} + \frac{K2Q}{r} + \frac{K2q}{r} = 0$$

$$-Q - q + 2q + 2Q = 0$$

$$\Rightarrow Q = -q$$

- 55 (a) Let R and r be the radii of bigger and each smaller drop, respectively.

$$\therefore \frac{4}{3} \pi R^3 = 8 \times \frac{4}{3} \pi r^3$$

$$\Rightarrow R = 2r \quad \dots(i)$$

The capacitance of a smaller spherical drop is

$$C = 4\pi\epsilon_0 r \quad \dots(ii)$$

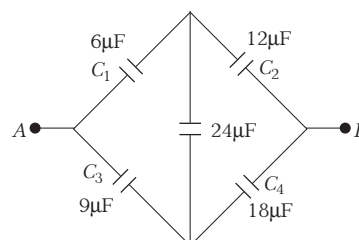
The capacitance of bigger drop is

$$C' = 4\pi\epsilon_0 R = 2 \times 4\pi\epsilon_0 r \quad (\because R = 2r)$$

$$= 2C \quad [\text{from Eq. (ii)}]$$

$$\therefore C = \frac{C'}{2} = \frac{1}{2} \mu\text{F} \quad (\because C' = 1 \mu\text{F})$$

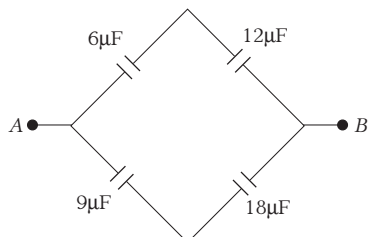
- 56 (a) The given circuit can be redrawn as,



It is a balance Wheatstone bridge type network,

$$\text{i.e.} \quad \frac{C_1}{C_2} = \frac{C_3}{C_4} = \frac{1}{2}$$

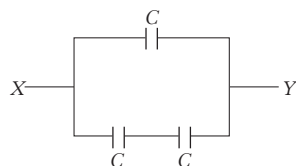
$\therefore$ $24 \mu\text{F}$ capacitor can be neglected.



Hence, equivalent capacitance between A and B

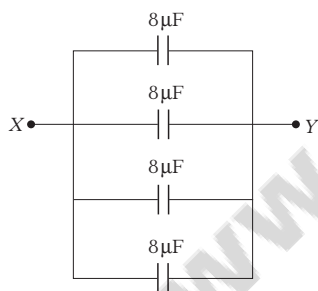
$$= 4 + 6 = 10 \mu\text{F}$$

57 (d) The given circuit can be redrawn as



$$C_{\text{eq}} = \frac{3C}{2} = \frac{3\epsilon_0 A}{2d}$$

58 (d) The given circuit can be redrawn as



$$\therefore C_{xy} = 4 \times 8 = 32 \mu\text{F}$$

59 (d) It will act like three capacitors which are connected in parallel, so equivalent capacitance,

$$C = C_1 + C_2 + C_3 = 3 + 3 + 3 = 9 \mu\text{F}$$

60 (d) Here, $C_{13} = C_1 + C_3 = 9 + 9 = 18 \mu\text{F}$

$$C_{2-13} = \frac{C_2 \cdot C_{13}}{C_2 + C_{13}} = \frac{9 \times 18}{9 + 18} = 6 \mu\text{F}$$

Equivalent capacitance between the point A and B.

$$C_{\text{eq}} = C_{2-13} + C_4 = 6 + 9 = 15 \mu\text{F}$$

61 (b) Heat produced in a wire is equal to energy stored in capacitor.

$$\begin{aligned} H &= \frac{1}{2} CV^2 = \frac{1}{2} \times (2 \times 10^{-6}) \times (200)^2 \\ &= 10^{-6} \times 200 \times 200 \\ &= 4 \times 10^{-2} \text{ J} \end{aligned}$$

62 (a) Volume of 8 liquid drops = Volume of large liquid drop

$$\therefore \left(\frac{4}{3} \pi r^3 \right) \times 8 = \frac{4}{3} \pi R^3$$

$$\Rightarrow 2r = R \quad \dots(i)$$

According to charge conservation,

$$8q = Q \quad \dots(ii)$$

$$\text{We have,} \quad V \propto \frac{1}{r}$$

$$\text{or} \quad \frac{V_1}{V_2} = \frac{r_2}{r_1}$$

$$\Rightarrow \frac{20}{V_2} = \frac{2r}{r}$$

$$\Rightarrow \text{Potential on each drop, } V_2 = 10 \text{ V}$$

63 (c) The electric field, $\mathbf{E} = 5\hat{i} - 3\hat{j}$ kV/m

$$\therefore \text{Magnitude, } E = \sqrt{25 + 9} = \sqrt{34} \text{ kV/m}$$

$$\begin{aligned} \text{Distance, } d &= \sqrt{(10-4)^2 + (3-0)^2} \\ &= \sqrt{(6)^2 + (3)^2} \\ &= \sqrt{36 + 9} = \sqrt{45} \text{ m} \end{aligned}$$

Potential difference between points A and B,

$$V = E \cdot d = \sqrt{34} \times \sqrt{45} = 39 \text{ kV}$$

64 (d) Force between the plates of a parallel plate capacitor,

$$|F| = \frac{V^2 A}{2 \epsilon_0} = \frac{Q^2}{2 \epsilon_0 A}$$

65 (a) Store energy in capacitor of $3 \mu\text{F}$,

$$\begin{aligned} U_1 &= \frac{1}{2} \times C_1 V^2 \\ &= \frac{1}{2} \times 3 \times (6)^2 \times 10^{-6} \\ &= 54 \times 10^{-6} \text{ J} \end{aligned}$$

Store energy in capacitor of $4 \mu\text{F}$,

$$\begin{aligned} U_2 &= \frac{1}{2} C_2 V^2 \\ &= \frac{1}{2} \times 4 \times (6)^2 \times 10^{-6} \\ &= 72 \times 10^{-6} \text{ J} \end{aligned}$$

When both capacitors are connected in series,

$$\begin{aligned} C_{\text{eq}} &= \frac{C_1 C_2}{C_1 + C_2} \\ &= \frac{3 \times 4}{3 + 4} = \frac{12}{7} \mu\text{F} \end{aligned}$$

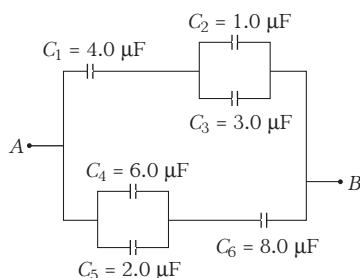
$$\text{Energy lost, } U = \frac{1}{2} C_{\text{eq}} (V_1 - V_2)^2$$

$$= \frac{1}{2} \times \frac{12}{7} \times (0)^2 \times 10^{-6} = 0$$

$$\text{Total energy} = U_1 + U_2$$

$$= 1.26 \times 10^{-4} \text{ J}$$

- 66 (a) In given figure, C_2 and C_3 are in parallel.



$$\therefore C' = C_2 + C_3 = 4 \mu\text{F}$$

As C' and C_1 are in series, $\frac{1}{C_{S1}} = \frac{1}{C'} + \frac{1}{C_1} = \frac{1}{4} + \frac{1}{4}$

$$\Rightarrow C_{S1} = 2 \mu\text{F}$$

Similarly, C_4 and C_5 are in parallel, $C'' = 6 + 2 = 8 \mu\text{F}$

C'' and C_6 are in series, $\frac{1}{C_{S2}} = \frac{1}{C''} + \frac{1}{C_6} = \frac{1}{8} + \frac{1}{8}$

$$\Rightarrow C_{S2} = 4 \mu\text{F}$$

Now, C_{S1} and C_{S2} are in parallel, $C = 4 + 2 = 6 \mu\text{F}$

- 67 (d) A positively charged body can have positive, negative or zero potential.

When we ground the charged body, potential difference between body and ground is zero but not the charge and same for negatively charged body.

- 68 (a) The work done against the force of repulsion in moving the two charges closer, increases the potential energy of the system.

- 69 (b) Earth is a conducting sphere of large capacitance.

$$V = q/C$$

As, C is very large, so $V \rightarrow 0$ for all finite charges.

Hence, earth is a good conductor.

- 70 (a) A current flows in the circuit during the time, the capacitor is charged. After the capacitor gets fully charged, the current stops flowing. It means when a capacitor is connected to a battery, a current flows in the circuit for sometime, then reduces to zero.

- 71 (c) For a point charge, $V \propto \frac{1}{r}$

$$\text{For a dipole, } V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^2} \quad [\text{on axial position}]$$

$$\text{i.e., } V \propto \frac{1}{r^2}$$

The electric dipole potential varies as $1/r$ at large distance is not true.

CHAPTER 03

Current Electricity

We have read that the directional flow of charge in a conductor under a potential difference maintained between the ends of the conductor, constitutes an **electric current** in the conductor. In this chapter, we will study about the basic properties of electric current, property of batteries and how they cause current and energy transfer in a circuit. In this chapter, we will use the concepts of current, potential difference, resistance and electromotive force.

ELECTRIC CURRENT

Electric current is defined as the rate of flow of charge through any cross-sectional area of the conductor. It is denoted by I .

If electric current is steady, then it can be expressed as

$$I = \frac{\text{Total charge flowing } (q)}{\text{Total time taken } (t)} \Rightarrow I = \frac{q}{t} = \frac{ne}{t}$$

where, n = number of carriers (electrons) of electricity and e = electronic charge.

But current is not always steady, so it can be defined in two ways

(i) **Average current** If a charge Δq flows through a conductor in the time interval t to $(t + \Delta t)$, then average current is defined as $I_{\text{av}} = \frac{\Delta q}{\Delta t}$.

(ii) **Instantaneous current** Current at any instant of time is called instantaneous current. If a charge dq flows through a conductor in small time dt , i.e. limit of Δt tending to zero, then $I_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$.

Important points about electric current

- (i) Current is a scalar quantity.
- (ii) Its SI unit is ampere (A) and CGS unit is emu and is also called biot (Bi) or ab ampere, i.e. $|A| = \left(\frac{1}{10}\right) \text{ Bi (ab amp)}$.
- (iii) One ampere is the current through a wire, if a charge of one coulomb flows through any cross-section of the wire in one second, i.e.

$$1 \text{ ampere (A)} = \frac{1 \text{ coulomb (C)}}{1 \text{ second (s)}} = 1 \text{ coulomb per second} = 1 \text{ Cs}^{-1}$$

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- (iv) As a matter of convention, the direction of flow of positive charge gives the direction of current. This is called **conventional current**. The direction of flow of electrons gives the direction of electronic current. Therefore, the direction of electronic current is opposite to that of conventional current.

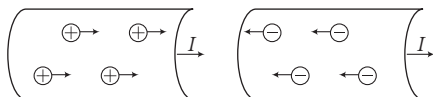


Fig. 3.1 Flow of electronic current is equivalent to the flow of conventional current in opposite direction

- (v) As charge is conserved and current is the rate of flow of charge, i.e. charge entering per second at one end of conductor is equal to the charge leaving per second the other end.

Note

- (i) In case, a charge q is revolving in a circle of radius r with uniform velocity v , current is given by

$$i = \frac{q}{t} = qf = \frac{q}{2\pi/\omega} = \frac{qv}{2\pi r}$$

- (ii) Total charge in time interval t_1 to t_2 can be given as

$$Q = \int_{t_1}^{t_2} I \cdot dt$$

Area under the graph I versus t in the interval t_1 to t_2 as shown in the figure

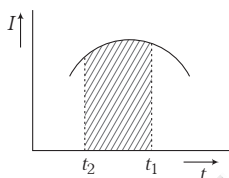


Fig. 3.2 Area under I versus t graph

So, average current in this interval,

$$I_{av} = \frac{Q}{t_2 - t_1} = \frac{\int_{t_1}^{t_2} I \cdot dt}{t_2 - t_1} = \frac{\text{Area under the } I \text{ versus } t \text{ graph}}{\text{Time interval}}$$

Example 3.1 How many electrons pass through a bulb in 1 min, if the current is 400 mA? (Take, $e = 1.6 \times 10^{-19}$ C)

Sol. Given, current, $I = 400 \text{ mA} = 400 \times 10^{-3} \text{ A}$

Time, $t = 1 \text{ min} = 60 \text{ s}$

Charge, $e = 1.6 \times 10^{-19} \text{ C}$

Thus, charge passing through a bulb in 1 min is

$$q = I \times t = 400 \times 10^{-3} \times 60 = 24 \text{ C}$$

Also, $q = ne$

$\therefore$ Number of electrons,

$$n = \frac{q}{e} = \frac{24}{1.6 \times 10^{-19}} = 1.5 \times 10^{20} \text{ electrons}$$

Example 3.2 10^{20} electrons, each having a charge of $1.6 \times 10^{-19} \text{ C}$, pass from a point A towards another point B in 0.1 s. What is the current in ampere? What is its direction?

Sol. Given, $n = 10^{20}$, $e = 1.6 \times 10^{-19} \text{ C}$ and $t = 0.1 \text{ s}$

As we know,

$$\text{Current, } I = \frac{q}{t} = \frac{ne}{t} = \frac{(10^{20} \times 1.6 \times 10^{-19}) \text{ C}}{0.1 \text{ s}} = 160 \text{ A}$$

Direction of current is from point B to A (in the direction opposite to flow of electrons).

Example 3.3 A wire carries a current of 2 A. What is the charge that has flowed through its cross-section in 1.0 s? How many electrons does this correspond to?

Sol. Given, $i = 2 \text{ A}$ and $t = 1.0 \text{ s}$

As, current, $i = \frac{q}{t}$

$$\therefore q = it = 2 \times 1 = 2 \text{ C}$$

Also, $q = ne$

$$\therefore \text{Number of electrons, } n = \frac{q}{e} = \frac{2}{1.6 \times 10^{-19}} = 1.25 \times 10^{19}$$

Example 3.4 If an electron revolves in a circle of radius $\pi/2 \text{ cm}$ with uniform speed $6 \times 10^5 \text{ m/s}$. Find the electric current. (Take, $\pi^2 = 10$)

Sol. Time period is given by

$$t = \frac{2\pi r}{v}$$

$$\text{Here, } r = \left(\frac{\pi}{2}\right) \text{ cm} = \frac{\pi}{2} \times 10^{-2} \text{ m and } v = 6 \times 10^5 \text{ m/s}$$

$$\begin{aligned} \text{So, electric current, } i &= \frac{ev}{2\pi r} = \frac{1.6 \times 10^{-19} \times 6 \times 10^5}{2\pi \times \frac{\pi}{2} \times 10^{-2}} \\ &= 9.6 \times 10^{-13} \text{ A} \end{aligned}$$

Example 3.5 If the amount of charge flowed in time t through a cross-section of wire is $q = \beta t - \gamma t^2$, where β and γ are constants.

(i) Find the current in terms of t .

(ii) Sketch i versus t graph.

Sol. Given, $q = \beta t - \gamma t^2$

(i) As we know, current in a wire is given by

$$i = \frac{dq}{dt} = \frac{d}{dt} (\beta t - \gamma t^2) = \beta - 2\gamma t$$

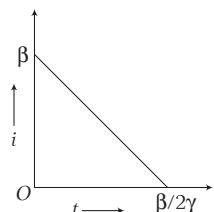
(ii) To plot a graph i versus t , we will compare the given current with the equation of a straight line, i.e.

$$y = mx + c$$

So, at $t = 0$, $i = \beta$

and when $i = 0$, then $t = \frac{\beta}{2\gamma}$.

which means the graph for given current equation is a straight line with negative slope, i.e. -2γ and positive intercept, i.e. β .



So, the plot will be as shown alongside.

Example 3.6 The current in a wire varies with time according to the relation,

$$i = 3(A) + 2(A/s)t$$

- How many coulombs of charge pass through a cross-section of the wire in the time interval between $t = 0$ to $t = 4$ s?
- What constant current would transport the same charge in the same time interval?

Sol. Given, $i = 3(A) + 2(A/s)t$

- Current through a cross-sectional area is given by

$$i = \frac{dq}{dt}$$

$$\therefore dq = i dt$$

On integrating both sides between the given time interval, i.e. 0 to 4 s, we get

$$\int_0^q dq = \int_0^4 i dt$$

$$\begin{aligned} \therefore \text{Charge, } q &= \int_0^4 (3 + 2t) dt = \left[3t + \frac{2t^2}{2} \right]_0^4 \\ &= [3t + t^2]_0^4 = [12 + 16] = 28 \text{ C} \end{aligned}$$

- Therefore, the current flowing through wire would be

$$i = \frac{q}{t} = \frac{28}{4} = 7 \text{ A}$$

Current density

Current density at any point inside a conductor is defined as “the amount of charge flowing per second through a unit area held normal to the direction of the flow of charge at that point”.

Current density is a vector quantity and its direction is along the motion of the positive charge as shown in Fig. 3.3 (a).

$$\text{Current density, } \mathbf{J} = \frac{q/t}{A} = \frac{I}{A}$$

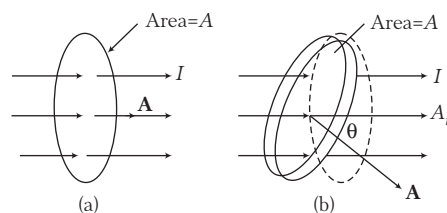


Fig. 3.3 Direction of current density

If the cross-sectional area A is not perpendicular to the current and normal to the area makes an angle θ with the direction of the current as shown in Fig. 3.3 (b), then

$$A_n = A \cos \theta$$

$$\therefore \text{Current density, } \mathbf{J} = \frac{I}{A_n} = \frac{I}{A \cos \theta} \text{ or } I = \mathbf{J} \cdot \mathbf{A}$$

If area is finite,

$$I = \int \mathbf{J} \cdot d\mathbf{A}$$

where, $d\mathbf{A}$ is a small element of the given surface.

The SI unit of current density is ampere per square metre, i.e. Am^{-2} and its dimensional formula is $[\text{AL}^{-2}]$.

Example 3.7 An electron beam has an aperture of 2 mm^2 . A total of 7×10^{16} electrons flow through any perpendicular cross-section per second. Calculate the current density in the electron beam.

Sol. As we know, current density in the electron beam,

$$J = \frac{I}{A} = \frac{ne}{t} / A = \frac{ne}{tA}$$

Substituting the values, we get

$$\begin{aligned} J &= \frac{7 \times 10^{16} \times 1.6 \times 10^{-19}}{1 \times 2 \times 10^{-6}} \\ &= 5.6 \times 10^3 \text{ Am}^{-2} \end{aligned}$$

Electric current in conductors

The electric current in a conductor can be explained by the electron theory. In an atom of a substance, the electrons in the orbits close to the nucleus are bound to it under the strong attraction of the nuclear positive charge, but the electrons far from the nucleus experience a very feeble force.

Hence, the outer electrons can be removed easily from the atom (by rubbing or by heating the substance). In fact, a few outer electrons, leave their atoms and move freely within the substance (in the vacant spaces between the atoms). These electrons, called **free electrons** or **conduction electrons**, carry the charge in the substance from one place to the other.

Therefore, the electrical conductivity of a solid substance depends upon the number of free electrons in it. In metals, this number is quite large ($\approx 10^{29}/\text{m}^3$). Hence, metals are good conductors of electricity. Silver is the best conductor of electricity than are copper, gold and aluminium, respectively.

There are some other materials in which the electrons will be bound and they will not be accelerated, even if the electric field is applied, *i.e.* **no current flow on applying electric field**. Such materials are called **insulators**, *e.g.* wood, plastic, rubber, etc.

Drift velocity

It is defined as “the average velocity with which the free electrons in a conductor get drifted towards the positive end of the conductor under the influence of an electric field applied across the conductor”.

It is given by
$$v_d = \frac{eE}{m}\tau$$

where, e = charge on electron, E = electric field,
 m = mass of the electron and τ = relaxation time.
 The electric current relates with drift velocity as

$$i = neAv_d$$

$$\Rightarrow v_d = \frac{i}{neA} = \frac{J}{ne}$$

Hence, current density is also given by

$$J = \frac{i}{A} = nev_d$$

The direction of drift velocity for electrons in a metal is opposite to that of applied electric field. The drift velocity of electron is very small of the order of 10^{-4} ms^{-1} as compared to thermal speed ($\approx 10^5 \text{ m/s}$) of electron at room temperature.

Relaxation time (τ)

As free electrons move in a conductor, they continuously collide with positive ions. The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time.

$$\tau = \frac{\text{mean free path}}{\text{rms velocity of electrons}} = \frac{\lambda}{v_{\text{rms}}}$$

With rise in temperature v_{rms} increases, consequently τ decreases.

Mobility (μ)

Drift velocity per unit electric field is called **mobility of electron**.

$$\mu = \frac{v_d}{E} = \frac{e\tau}{m}$$

Its unit is $\text{m}^2/\text{V}\cdot\text{s}$.

Mobility of free electrons is independent of electric field.

Example 3.8 Find the current flow through a copper wire of length 0.2 m, area of cross-section 1 mm^2 , when connected to a battery of 4 V. (Take, electron mobility is $4.5 \times 10^{-6} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$, charge on an electron is $1.6 \times 10^{-19} \text{ C}$ and density of electrons in copper is $8.5 \times 10^{28} \text{ m}^{-3}$)

Sol. Given, length of copper wire, $l = 0.2 \text{ m}$

Cross-sectional area, $A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$

Potential difference, $V = 4 \text{ V}$

Electron mobility, $\mu = 4.5 \times 10^{-6} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

Charge on an electron, $e = 1.6 \times 10^{-19} \text{ C}$

Density of electrons in copper, $n = 8.5 \times 10^{28} \text{ m}^{-3}$

We know that, electric field set up across the conductor,

$$E = \frac{V}{l} = \frac{4}{0.2} = 20 \text{ V m}^{-1}$$

Mobility of electrons, $\mu = \frac{v_d}{E}$

$$\Rightarrow v_d = 4.5 \times 10^{-6} \times 20 = 9 \times 10^{-5} \text{ m/s}$$

So, the current flow through the copper wire is

$$I = nev_d A = 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 9 \times 10^{-5} \times 10^{-6} = 1.224 \text{ A}$$

Example 3.9 An aluminium wire of diameter 0.24 cm is connected in series to a copper wire of diameter 0.16 cm. The wire carry an electric current of 10 A. Find (i) current density in the aluminium wire (ii) and drift velocity of electrons in the copper wire. (Take, number of electrons per cubic metre volume of copper, $n = 8.4 \times 10^{28}$)

Sol. (i) Radius of Al wire, $r = \frac{0.24}{2} = 0.12 \text{ cm} = 0.12 \times 10^{-2} \text{ m}$

Area of cross-section of Al wire, $A = \pi r^2$

$$= 3.14 \times (0.12 \times 10^{-2})^2$$

$$= 4.5 \times 10^{-6} \text{ m}^2$$

$$\therefore \text{Current density, } J = \frac{I}{A} = \frac{10}{4.5 \times 10^{-6}} = 2.2 \times 10^6 \text{ Am}^{-2}$$

(ii) Radius of copper wire, $r = \frac{0.16}{2} \text{ cm} = 0.08 \times 10^{-2} \text{ m}$

Area of cross-section of copper wire,

$$A = \pi r^2 = 3.14 \times (0.08 \times 10^{-2})^2 = 2 \times 10^{-6} \text{ m}^2$$

Also, $n = 8.4 \times 10^{28} \text{ m}^{-3}$, $e = 1.6 \times 10^{-19} \text{ C}$, $I = 10 \text{ A}$

$\therefore$ Drift velocity of electrons in the copper wire,

$$v_d = \frac{I}{enA} = \frac{10}{1.6 \times 10^{-19} \times 8.4 \times 10^{28} \times 2 \times 10^{-6}} = 3.7 \times 10^{-4} \text{ ms}^{-1}$$

Example 3.10 What is the drift velocity of electrons in a silver wire of length 2 m, having cross-sectional area $6.14 \times 10^{-6} \text{ m}^2$ and carrying a current of 5 A? (Take, atomic weight of silver = 108, density of silver = $7.5 \times 10^3 \text{ kg/m}^3$, charge on electron = $1.6 \times 10^{-19} \text{ C}$ and Avogadro's number = 6.023×10^{26} per kg atom)

Sol. First, we will calculate, n = number of electrons per unit volume.

Imagine the volume of silver to be 1 m^3 .

Now, its mass = density $\times$ volume = $7.5 \times 10^3 \times 1$

$$= 7.5 \times 10^3 \text{ kg}$$

Now, number of moles of silver,

$$= \frac{7.5 \times 10^3}{108 \times 10^{-3}} = 0.069 \times 10^6 = 6.9 \times 10^4 \text{ mol}$$

So, number of silver atoms in this can be calculated by multiplying the number of moles by Avogadro's number.

$$\begin{aligned} \text{i.e., number of silver atoms} &= 6.9 \times 10^4 \times 6.023 \times 10^{23} \\ &\approx 4.16 \times 10^{28} \text{ atoms} \end{aligned}$$

Now, since the valency of silver is one, we can assume each atom of silver contributes one electron. So, finally

$$\Rightarrow n = 4.16 \times 10^{28} \text{ per m}^3$$

Given, $I = 5 \text{ A}$ and $A = 6.14 \times 10^{-6} \text{ m}^2$

We use the formula, drift velocity,

$$\begin{aligned} v_d &= \frac{I}{neA} = \frac{5}{4.16 \times 10^{28} \times 1.602 \times 10^{-19} \times 6.14 \times 10^{-6}} \\ &= 1.22 \times 10^{-4} \text{ m/s} \end{aligned}$$

CHECK POINT 3.1

- The current through a wire depends on time as $I = 3t^2 + 2t + 5$. The charge flowing through the cross-section of the wire in time interval between $t = 0$ to $t = 2 \text{ s}$ is
(a) 22 C (b) 20 C (c) 18 C (d) 5 C
- The charge on an electron is $1.6 \times 10^{-19} \text{ C}$. How many electrons strike the screen of a cathode ray tube each second when the beam current is 16 mA?
(a) 10^{17} (b) 10^{19}
(c) 10^{-19} (d) 10^{-17}
- A conductor carries a current of 0.2 A. In 30 s, how many electrons will flow through the cross-section of the conductor? (Take, $q = 1.6 \times 10^{-19} \text{ C}$)
(a) 0.375×10^{19} (b) 375×10^{19}
(c) 3.75×10^{19} (d) 37.5×10^{19}
- In a closed circuit, the current I (in ampere) at an instant of time t (in second) is given by $I = 4 - 0.08t$. The number of electrons flowing in 50 s through the cross-section of the conductor is
(a) 1.25×10^{19} (b) 6.25×10^{20}
(c) 5.25×10^{19} (d) 2.25×10^{20}
- Drift velocity v_d varies with the intensity of electric field as per the relation,
(a) $v_d \propto E$ (b) $v_d \propto \frac{1}{E}$
(c) $v_d = \text{constant}$ (d) $v_d \propto E^2$
- When current flows through a conductor, then the order of drift velocity of electrons will be
(a) 10^{10} cms^{-1} (b) 10^{-2} cms^{-1}
(c) 10^4 cms^{-1} (d) 10^{-1} cms^{-1}
- The number density of free electron in a copper conductor is $8.5 \times 10^{28} \text{ m}^{-3}$. How long does an electron take to drift from one end of a wire, 3.0 m long to its other end? The area of cross-section of the wire is $2.0 \times 10^{-6} \text{ m}^2$ and it is carrying a current of 3.0 A.
(a) $2.73 \times 10^4 \text{ s}$ (b) $4.73 \times 10^4 \text{ s}$
(c) $5 \times 10^4 \text{ s}$ (d) $6 \times 10^8 \text{ s}$

OHM'S LAW

It states that, "the current I flowing through a conductor is always directly proportional to the potential difference V across the ends of the conductor", provided that the physical conditions (temperature, mechanical strain, etc) are kept constant.

Mathematically, $I \propto V$

or $V \propto I$

or $V = IR$

where, R is resistance of the conductor.

Graph between V and I for a metallic conductor is a straight line as shown. At different temperature, V - I curves are different.

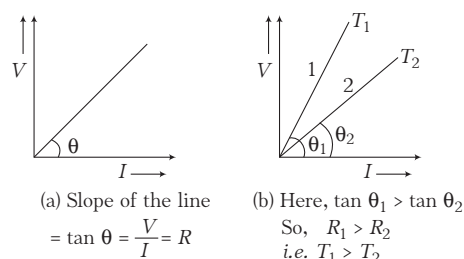


Fig. 3.4

Note The substances which obey Ohm's law, e.g. copper, silver, aluminium are known as **ohmic substances**. The devices or substances which don't obey Ohm's law, e.g. gases, crystal rectifiers, thermionic valve, transistors, etc., are known as **non-ohmic** or **non-linear conductors**.

Resistance and resistivity

The property of a substance by virtue of which it opposes the flow of current through it, is known as the **resistance**.

It is defined as ‘the ratio of the potential difference applied across the ends of the conductor to the current flowing through’.

$$\text{Mathematically, } R = \frac{V}{I}$$

The resistance of the conductor depends upon the following factors

(i) It is directly proportional to the length of the conductor,
i.e. $R \propto l$... (i)

(ii) It is inversely proportional to the area of the cross-section of the conductor,
i.e. $R \propto \frac{1}{A}$... (ii)

From Eqs. (i) and (ii), we get

$$R \propto \frac{l}{A} \Rightarrow R = \rho \frac{l}{A} \quad \dots \text{(iii)}$$

where, ρ is the constant of proportionality known as **resistivity or specific resistance** of the conductor.

Resistivity is the intrinsic property of the substance. It is independent of shape and size of the body and depends upon the nature of the material.

Example 3.11 Resistance of a conductor of length l and area of cross-section A is R . If its length is doubled and area of cross-section is halved, then find its new resistance.

Sol. Initial length = l , area = A

$$\text{So, initial resistance, } R = \rho \frac{l}{A}$$

$$\text{Final length, } l' = 2l, \text{ area, } A' = \frac{A}{2}$$

$$\text{New resistance, } R' = \rho \frac{l'}{A'} = \rho \frac{2l}{\left(\frac{A}{2}\right)} = 4\rho \frac{l}{A} = 4R$$

If its length is doubled and area of cross-section is halved, then new resistance becomes four times the initial value.

Unit and dimensional formula of resistance

The SI unit of resistance is ohm and is denoted by symbol Ω .

$$1 \text{ ohm } (\Omega) = \frac{1 \text{ volt (V)}}{1 \text{ ampere (A)}} = 1 \text{ volt/ampere (or V/A)}$$

The resistance of a conductor is said to be one ohm, if one ampere of current flows, when a potential difference of one volt is applied across the ends of the conductor.

Its dimensional formula is $[ML^2T^{-3}A^{-2}]$.

Note Any conducting material that offers some resistance is known as **resistor**.

Conductance and conductivity

Conductance

Reciprocal of resistance is known as conductance G , i.e.

$$G = \frac{1}{R}$$

Its unit is $1/\Omega$ or Ω^{-1} or Siemen.

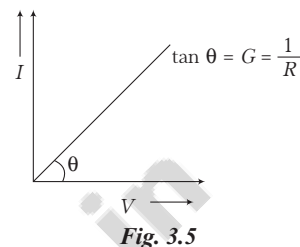


Fig. 3.5

Conductivity

Reciprocal of resistivity is called conductivity (σ), i.e.

$$\sigma = \frac{1}{\rho}$$

Its unit is mho/m.

The dimensional formula of conductivity is $[M^{-1}L^{-3}T^3A^2]$.

Vector form of Ohm's law

Electric field in a conductor of length l and having potential difference V at its ends is given by

$$V = El$$

$$\text{Also, } V = IR \Rightarrow El = \frac{\rho l}{A}$$

$$\Rightarrow E = \frac{\rho}{A} I$$

$$\therefore \mathbf{E} = \rho \mathbf{J}$$

$$\text{or } \boxed{\mathbf{J} = \sigma \mathbf{E}} \quad \dots \text{(iv)}$$

The above equation is the vector form of Ohm's law.

Also, current density,

$$J = nev_d = ne \left(\frac{eE\tau}{m} \right)$$

$$J = \left(\frac{ne^2\tau}{m} \right) E$$

Comparing with Eq. (iv), we get

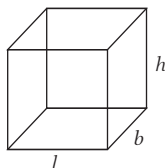
$$\text{Conductivity, } \sigma = \frac{ne^2\tau}{m}$$

$$\text{and resistivity, } \rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

Example 3.12 All the edges of a block with parallel faces are unequal and its longest edge is four times its shortest edge. Find the ratio of the maximum to the minimum resistance between different faces.

Sol. Let l = longest edge, b = shortest edge.

According to question, $l/b = 4$



Resistance between different faces of block

(i) Area, $A_1 = (l \times b)$

$$\therefore \text{Resistance, } R_1 = \rho \frac{l}{A} = \frac{\rho h}{lb}$$

(ii) Area, $A_2 = (b \times h)$

$$\therefore \text{Resistance, } R_2 = \frac{\rho l}{bh} = R_{\max}$$

(iii) Area, $A_3 = (l \times h)$

$$\therefore \text{Resistance, } R_3 = \frac{\rho b}{lh} = R_{\min}$$

$$\text{Hence, the ratio of } \frac{R_{\max}}{R_{\min}} = \frac{l^2}{b^2} = \frac{(4b)^2}{b^2} = 16$$

Example 3.13 Calculate the electric field in a copper wire of cross-sectional area 2.0 mm^2 carrying a current of 1 A . The conductivity of copper is $6.25 \times 10^7 \text{ Sm}^{-1}$.

Sol. Given, area, $A = 2 \text{ mm}^2 = 2 \times 10^{-6} \text{ m}^2$

Current, $I = 1 \text{ A}$

Conductivity, $\sigma = 6.25 \times 10^7 \text{ Sm}^{-1}$

$$\text{As, current density, } J = \frac{I}{A} = \sigma E$$

$$\therefore \text{Electric field, } E = \frac{I}{A\sigma} = \frac{1}{2 \times 10^{-6} \times 6.25 \times 10^7} = 8 \times 10^{-3} \text{ Vm}^{-1}$$

Example 3.14 A copper wire has a resistance of 10Ω and an area of cross-section 1 mm^2 . A potential difference of 10 V exists across the wire. Calculate the drift velocity of electrons, if the number of electrons per cubic metre in copper is 8×10^{28} electrons.

Sol. Given, $R = 10 \Omega$, $A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$

$$V = 10 \text{ V}, n = 8 \times 10^{28} \text{ electrons/m}^3$$

Now, electric current, $I = enAv_d$

Using Ohm's law,

$$\begin{aligned} \therefore \frac{V}{R} &= enAv_d \text{ or } v_d = \frac{V}{enAR} \\ &= \frac{10}{1.6 \times 10^{-19} \times 8 \times 10^{28} \times 10^{-6} \times 10} \\ &= 0.078 \times 10^{-3} \text{ ms}^{-1} = 0.078 \text{ mm/s} \end{aligned}$$

Example 3.15 A current of 1 A is flowing through a copper wire of length 0.1 m and cross-section $1 \times 10^{-6} \text{ m}^2$.

(i) If the specific resistance of copper is $1.7 \times 10^{-8} \Omega\text{-m}$, then calculate the potential difference across the ends of the wire.

(ii) Determine current density in the wire.

(Take, density of copper $= 8.9 \times 10^3 \text{ kg m}^{-3}$, atomic weight $= 63.5$ and $N = 6.02 \times 10^{26}$ per kg-atom)

Sol. Given, $I = 1 \text{ A}$, $l = 0.1 \text{ m}$, $A = 1 \times 10^{-6} \text{ m}^2$,

$$\rho = 1.7 \times 10^{-8} \Omega\text{-m}, d = 8.9 \times 10^3 \text{ kg m}^{-3}.$$

$$\begin{aligned} \text{(i) Resistance of wire is, } R &= \frac{\rho l}{A} = \frac{1.7 \times 10^{-8} \times 0.1}{1 \times 10^{-6}} \\ &= 1.7 \times 10^{-3} \Omega \end{aligned}$$

$$\therefore \text{Potential difference, } V = IR = 1 \times 1.7 \times 10^{-3} = 1.7 \times 10^{-3} \text{ V}$$

$$\text{(ii) Current density, } J = \frac{I}{A} = \frac{1}{1 \times 10^{-6}} = 1 \times 10^6 \text{ Am}^{-2}$$

Example 3.16 Find the time of relaxation between two collisions and free path of electrons in copper at room temperature. (Take, resistivity of copper $= 1.7 \times 10^{-8} \Omega\text{-m}$, density of electrons in copper $= 8.5 \times 10^{28} \text{ m}^{-3}$, charge on an electron $= 1.6 \times 10^{-19} \text{ C}$, mass of electron $= 9.1 \times 10^{-31} \text{ kg}$ and drift velocity of free electrons $= 1.6 \times 10^{-4} \text{ ms}^{-1}$)

Sol. Given, $\rho = 1.7 \times 10^{-8} \Omega\text{-m}$, $n = 8.5 \times 10^{28} \text{ m}^{-3}$,

$$e = 1.6 \times 10^{-19} \text{ C}, m_e = 9.1 \times 10^{-31} \text{ kg},$$

$$v_d = 1.6 \times 10^{-4} \text{ ms}^{-1}$$

$$\text{We know that, } \rho = \frac{m_e}{ne^2\tau}$$

$\therefore$ Relaxation time,

$$\begin{aligned} \tau &= \frac{m_e}{e^2 n \rho} = \frac{9.1 \times 10^{-31}}{(1.6 \times 10^{-19})^2 \times 8.5 \times 10^{28} \times 1.7 \times 10^{-8}} \\ &= 2.5 \times 10^{-14} \text{ s} \end{aligned}$$

Mean free path of electron (distance covered between two collisions) $= v_d \tau$

$$= 1.6 \times 10^{-4} \times 2.5 \times 10^{-14}$$

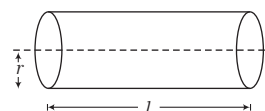
$$= 4 \times 10^{-18} \text{ m}$$

Example 3.17 A wire has a resistance R .

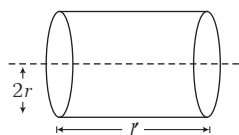
(i) If radius of cross-section of a cylindrical wire is doubled, then find the ratio of initial to final resistance.

(ii) If length of wire is increased by 10%, then find the percentage increase in its resistance.

(iii) If length of wire is increased by 20%, then find the percentage increase in its resistance.



Sol. (i) According to the question, we draw a figure given below



We have, $A_1 l_1 = A_2 l_2$

$$\pi r^2 l = \pi (2r)^2 l_2 \Rightarrow l_2 = l/4$$

$$\text{Resistance, } R_1 = \frac{\rho l}{\pi r^2}, R_2 = \frac{\rho l_2}{A_2} = \frac{\rho l/4}{4\pi r^2} = \frac{1}{16} \frac{\rho l}{\pi r^2}$$

$$\text{Ratio of initial to final resistance, } \frac{R_1}{R_2} = 16$$

(ii) Given, $l_1 = l, A_1 = A, l_2 = l + 10\% \text{ of } l$
 $= l + 0.1l = 1.1l, A_2 = A'$

We have, $A_1 l_1 = A_2 l_2 \Rightarrow Al = A' (1.1l)$

$$\text{Area, } A' = \frac{A}{1.1}$$

$$\text{Resistance, } R = \frac{\rho l}{A}, R' = \frac{\rho l_2}{A_2} = \frac{\rho \times l \times 1.1}{A/1.1} = \frac{\rho l}{A} (1.1)^2$$

$$\frac{R'}{R} = (1.1)^2 = 1.21$$

Percentage increase in resistance,

$$\left(\frac{R'}{R} - 1 \right) \times 100 = (1.21 - 1) \times 100 = 21\%$$

(iii) Given, $l_1 = l, A_1 = A$

$$l_2 = l \left(1 + \frac{20}{100} \right) = \frac{6l}{5}, A_2 = A'$$

We have, $A_1 l_1 = A_2 l_2$

$$Al = A' \frac{6l}{5} \Rightarrow A' = \frac{5A}{6}$$

$$\therefore \frac{R_2}{R_1} = \frac{\frac{\rho l_2}{A_2}}{\frac{\rho l_1}{A_1}} = \frac{l_2}{l_1} \cdot \frac{A_1}{A_2} = \frac{6l/5}{l} \cdot \frac{A}{5A/6} = \frac{36}{25}$$

Percentage increase in resistance,

$$\left(\frac{R_2}{R_1} - 1 \right) \times 100 = \left(\frac{36}{25} - 1 \right) \times 100 = 44\%$$

Temperature dependence of resistivity

Resistivity of a material depends on the temperature.

$$\text{As, } \rho \propto \frac{1}{\tau}$$

When temperature increases in a conductor, average speed of free electrons increases, hence relaxation time decreases. Thus, resistivity increases.

Resistivity of a metal conductor is given by

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

where, ρ = resistivity at temperature T , ρ_0 = resistivity at temperature T_0 and α = temperature coefficient of resistivity.

$$\text{i.e. } \alpha = \frac{\rho - \rho_0}{\rho_0(T - T_0)}$$

In terms of resistance,

$$R \propto \rho$$

$$\therefore R = R_0 [1 + \alpha(T - T_0)]$$

$$\Rightarrow \alpha = \frac{R - R_0}{R_0(T - T_0)}$$

where, R_0 = resistance of conductor at 0°C

and R_t = resistance of conductor at $t^\circ\text{C}$.

Important points related to resistivity

For metals, α is positive as their resistivity increases with rise in temperature. The graph of ρ plotted against T would be a straight line as shown in Fig. 3.6 (a).

For semiconductors, α is negative as their resistivity decreases with rise in temperature. Variation in resistivity with the temperature for semiconductor is shown in Fig. 3.6 (b).

The resistivity of **alloys** also increases with rise in temperature. The graph between resistivity and temperature is shown in Fig. 3.6 (c).

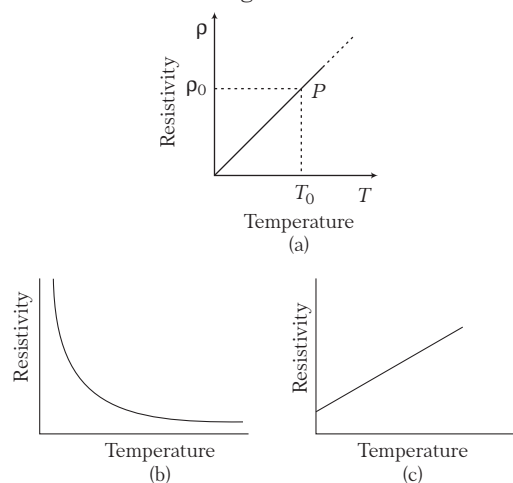


Fig. 3.6 Resistivity as the function of temperature for (a) metals, (b) semiconductors and (c) alloys

Example 3.18 The resistance of a thin silver wire is 1.0Ω at 20°C . The wire is placed in a liquid bath and its resistance rises to 1.2Ω . What is the temperature of the bath? (Take, α for silver = $3.8 \times 10^{-3} \text{ per } ^\circ\text{C}$)

Sol. We know that, $R(T) = R_0[1 + \alpha(T - T_0)]$

Here, $R(T) = 1.2 \Omega$, $R_0 = 1.0 \Omega$

$$\alpha = 3.8 \times 10^{-3} \text{ per } ^\circ\text{C}$$

and $T_0 = 20^\circ\text{C}$

Substituting the values, we get

$$1.2 = 1.0[1 + 3.8 \times 10^{-3}(T - 20^\circ)]$$

$$\text{or } 3.8 \times 10^{-3}(T - 20^\circ) = 0.2$$

Solving this, we get $T = 72.6^\circ \text{C}$

Example 3.19 Resistance of platinum wire in a platinum resistance thermometer at melting ice, boiling water and at a hot bath are 6Ω , 6.5Ω and 6.2Ω , respectively. Find the temperature of hot bath.

Sol. Given, $R_0 = 6 \Omega$, $R_{100} = 6.5 \Omega$ and $R_t = 6.2 \Omega$

The temperature of platinum resistance thermometer (t) is given by

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100^\circ \text{C} = \frac{6.2 - 6}{6.5 - 6} \times 100^\circ \text{C} \\ = \frac{0.2}{0.5} \times 100^\circ \text{C} = 40^\circ \text{C}$$

Example 3.20 The temperature coefficient of resistance of a wire is $0.00145^\circ \text{C}^{-1}$. At 100°C , its resistance is 2Ω . At what temperature, the resistance of the wire will be 3Ω ?

Sol. Using the relation, $R = R_0[1 + \alpha T]$

$$R_1 = R_0[1 + \alpha t_1]$$

$$\therefore 2 = R_0[1 + 0.00145 \times 100] \quad \dots(i)$$

$$\text{and } R_2 = R_0[1 + \alpha t_2]$$

$$\therefore 3 = R_0[1 + 0.00145 \times t_2] \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{3}{2} = \frac{1 + 0.00145 \times t_2}{1 + 0.00145 \times 100}$$

$$\text{or } t_2 = 494.8^\circ \text{C} = 767.8 \text{K}$$

Colour code for carbon resistor

In electrical and electronic circuits, there are two major types of resistors, which are being used, i.e. wire wound resistors and carbon resistor, offering resistances over a wide range. Wire wound resistors have resistances upto few hundred ohm, whereas for higher resistances, resistors are mostly made up of carbon.

To know the value of resistance of carbon resistors, colour code is used. These codes are printed in the form of set of rings or strips. By reading the values of colour bands, we can estimate the value of resistance.

The carbon resistor has normally four coloured rings or bands say A, B, C and D as shown in the Fig. 3.7.

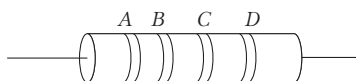


Fig. 3.7 Carbon resistor

Colour bands A and B indicates the first two significant figures of resistance in ohm.

Band C indicates the decimal multiplier, i.e. the number of zeros that follows the two significant figures A and B.

Band D indicates the tolerance in percent as per the indicated value. In other words, it represents the percentage accuracy of the indicated value.

The tolerance in the case of gold is $\pm 5\%$ and in silver is $\pm 10\%$. If only three bands are marked on carbon resistor, then it indicates a tolerance of 20% . (i.e. no colour)

$R = AB \times C \pm D\%$, where D is tolerance.

Sequence of colour code for carbon resistor

Letters as an aid to memory	Colour	Figure (A, B)	Multiplier (C)
B	Black	0	10^0
B	Brown	1	10^1
R	Red	2	10^2
O	Orange	3	10^3
Y	Yellow	4	10^4
G	Green	5	10^5
B	Blue	6	10^6
V	Violet	7	10^7
G	Grey	8	10^8
W	White	9	10^9

Note To remember the sequence of colour code following sentence should be kept in memory. B B ROY Great Britain Very Good Wife.

Example 3.21 How will you represent a resistance of $3700 \Omega \pm 10\%$ by colour code?

Sol. The value of resistance = $3700 \Omega \pm 10\%$

$$\Rightarrow R = 37 \times 10^2 \pm 10\%$$

The colour assigned to numbers 3, 7 and 2 are orange, violet and red. For $\pm 10\%$ accuracy, the colour is silver.

Hence, the bands of colour on carbon resistor in sequence are orange, violet, red and silver.

Combination of resistances

There are two types of combination of resistances, i.e. series combination and parallel combination.

1. Series combination

In this combination, resistors are connected end-to-end, i.e. second end of first resistor is connected to first end of the second resistor and so on.

In series combination, same current flows through each resistance but potential difference distributes in the ratio of their resistance, i.e. $V \propto R$. So, the total potential difference is equal to the sum of potential difference applied across the combination of resistors.

Equivalent resistance in series combination is given by

$$R_{eq} = R_1 + R_2 + R_3$$

i.e. equivalent resistance is greater than the maximum value of resistance in the combination.

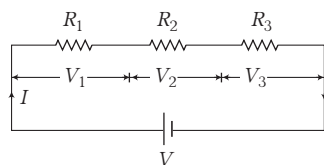


Fig. 3.8 Resistances in series

If n identical resistors of resistance R each, are connected in series, then equivalent resistance, $R_{eq} = nR$ and potential difference across each resistor is $V' = V/n$.

2. Parallel combination

In this combination, first end of all resistors are connected to one point and second end of all resistors are connected to other point.

Same potential difference appears across each resistor but current distributes in the reverse ratio of their resistance.

i.e. $I \propto \frac{V}{R}$. So, the total current is equal to the sum of currents through each resistance.

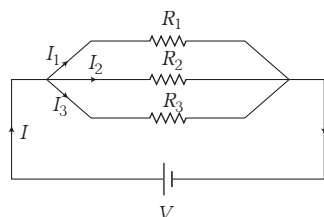


Fig. 3.9 Resistances in parallel

Equivalent resistance in parallel combination is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

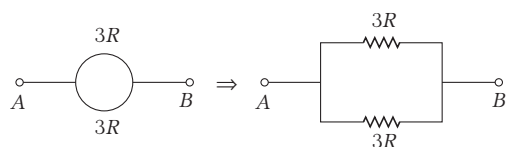
i.e. equivalent resistance is less than the minimum value of resistance in the combination.

If n identical resistors of resistance R each, are connected in parallel, then equivalent resistance, $R_{eq} = \frac{R}{n}$ and

current through each resistor is $I' = \frac{I}{n}$.

Example 3.22 A wire of resistance $6R$ is bent in the form of a circle. What is the effective resistance between the ends of the diameter?

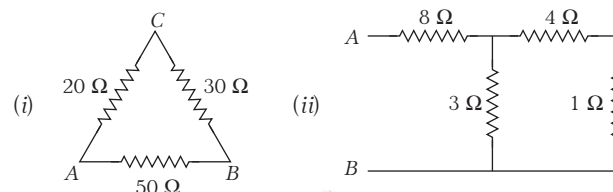
Sol.



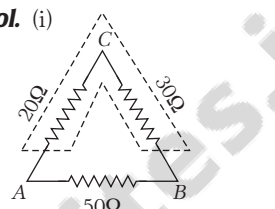
As shown in figure, the two resistances of value $3R$ each are in parallel with each other. So, the resistance between the ends A and B of a diameter is

$$R' = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{3R \times 3R}{3R + 3R} = \frac{9R^2}{6R} = \frac{3}{2}R$$

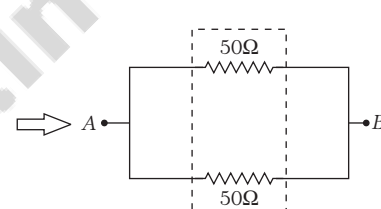
Example 3.23 Find the equivalent resistance between A and B in the following cases.



Sol. (i)

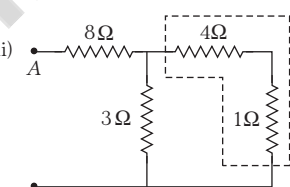


From the above dotted region, $20\ \Omega$ and $30\ \Omega$ are in series,
 $\Rightarrow R = 20 + 30 = 50\ \Omega$

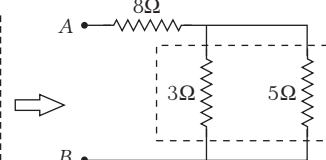


From the above dotted region, $50\ \Omega$ and $50\ \Omega$ are in parallel,
 $\Rightarrow R_{eq} = \frac{50 \times 50}{50 + 50} = \frac{2500}{100} = 25\ \Omega$

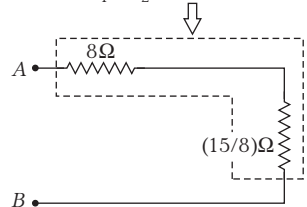
(ii)



From the above dotted region, $4\ \Omega$ and $1\ \Omega$ are in series,
 $\Rightarrow R = 4 + 1 = 5\ \Omega$



From the above dotted region, $3\ \Omega$ and $5\ \Omega$ are in parallel,
 $\Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{5} \Rightarrow R = \frac{15}{8}\ \Omega$



From the above dotted region, we have $8\ \Omega$ and $(15/8)\ \Omega$ are in series,
 $\Rightarrow R_{eq} = 15/8 + 8 = 79/8\ \Omega$

Example 3.24 A letter 'A' consists of a uniform wire of resistance $0.2\ \text{ohm per cm}$. The sides of letter are each $20\ \text{cm}$ long and the cross-piece in the middle is $10\ \text{cm}$ long while apex angle is 60° . Find the resistance of the letter between the two ends of the legs.

Sol. Clearly it is given that,

$$AB = BC = CD = DE = BD = 10\ \text{cm}$$

$$\text{and } R_1 = R_2 = R_3 = R_4 = R_5 = 2\ \Omega$$

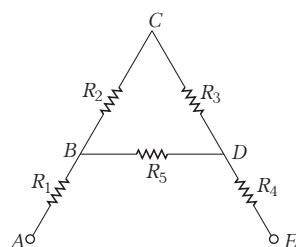
As R_2 and R_3 are in series, their combined resistance

$$= 2 + 2 = 4 \Omega$$

This combination is in parallel with $R_5 (= 2 \Omega)$.

Hence, resistance between points of B and D is given by

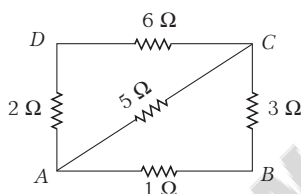
$$\begin{aligned} \frac{1}{R} &= \frac{1}{4} + \frac{1}{2} \\ \Rightarrow \frac{1}{R} &= \frac{6}{8} \\ \Rightarrow R &= \frac{4}{3} \Omega \end{aligned}$$



Now, resistance R_1, R, R_4 form a series combination. So, resistance between the ends A and E is

$$R' = 4 + \frac{4}{3} + 4 = 9.34 \Omega$$

Example 3.25 In the given network of resistors, find the equivalent resistance between the points A and B and between the points A and D .



Sol. Resistance between A and D ,

As we can see from the circuit, 1Ω and 3Ω are in series,

$$\Rightarrow R_1 = 1 + 3 = 4 \Omega$$

Now, R_1 and 5Ω are in parallel, $R_2 = \frac{4 \times 5}{4 + 5} = \frac{20}{9} \Omega$

Now, R_2 and 6Ω are in series, $R_3 = \frac{20}{9} + 6 = \frac{74}{9} \Omega$

Now, R_3 and 2Ω are in parallel, $R_4 = \frac{\frac{74}{9} \times 2}{\left(\frac{74}{9} + 2\right)} = \frac{37}{23} \Omega$

Resistance between A and B ,

The resistors $AD (= 2 \Omega)$ and $DC (= 6 \Omega)$ are in series to give a total resistance $R' = 8 \Omega$. The resistance $R' (= 8 \Omega)$ and the resistor $AC (= 5 \Omega)$ are in parallel. Their equivalent

resistance is, $R'' = \frac{5 \times 8}{5 + 8} = \frac{40}{13} \Omega$

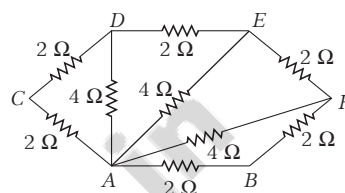
Now, R'' and 3Ω are in series,

$$R''' = R'' + 3 = \frac{40}{13} + 3 = \frac{79}{13} \Omega$$

Now, R''' and 1Ω are in parallel, so resistance between A and B ,

$$(R''''') = \frac{1 \times 79/13}{1 + 79/13} = \frac{79}{92} \Omega$$

Example 3.26 Find the effective resistance between A and B for the network shown in the figure below



Sol. At points A and D , a series combination of $2 \Omega, 2 \Omega$

resistance (along AC and CD) is in parallel with 4Ω resistance (along AD), therefore resistance between A and D

$$= \frac{1}{\frac{1}{2+2} + \frac{1}{4}} = 2 \Omega$$

Similarly, proceeding this way the resistance between A and F

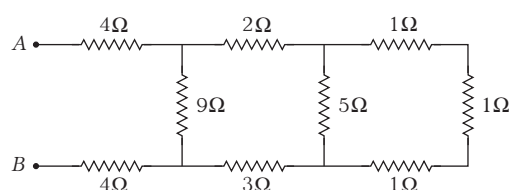
$$= \frac{1}{\frac{1}{2+2} + \frac{1}{4}} = 2 \Omega$$

Finally, resistance between A and B

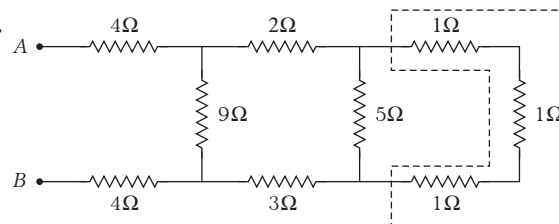
$$= \frac{1}{\frac{1}{2+2} + \frac{1}{2}} = \frac{4}{3} \Omega$$

Thus, the effective resistance between A and B is $\frac{4}{3} \Omega$.

Example 3.27 Find the equivalent resistance between A and B .

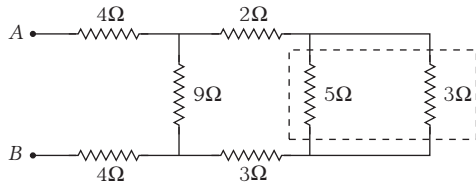


Sol.



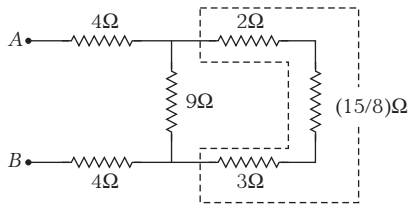
As per the dotted region, 1Ω , 1Ω and 1Ω are in series,

$$R = 1 + 1 + 1 = 3\Omega$$



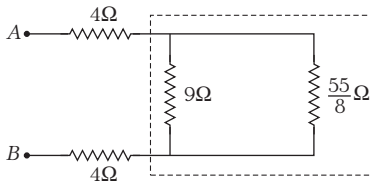
In dotted region, 3Ω and 5Ω are in parallel,

$$\Rightarrow R_1 = \frac{3 \times 5}{3 + 5} = \frac{15}{8}\Omega$$



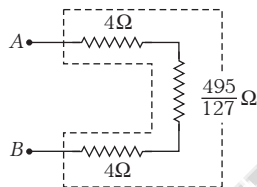
In dotted region, 2Ω , 3Ω and $(15/8)\Omega$ are in series,

$$\Rightarrow R_2 = 2 + 3 + \frac{15}{8} = \frac{55}{8}\Omega$$



In dotted region, $(55/8)\Omega$ and 9Ω are in parallel,

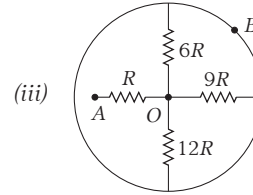
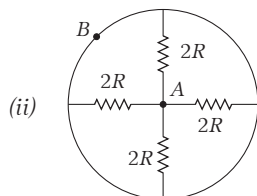
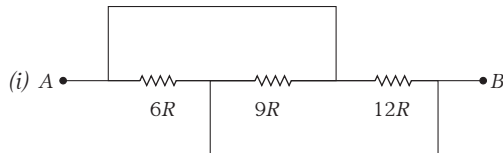
$$\Rightarrow R_3 = \frac{\frac{55}{8} \times 9}{\frac{55}{8} + 9} = \frac{495}{127}\Omega$$



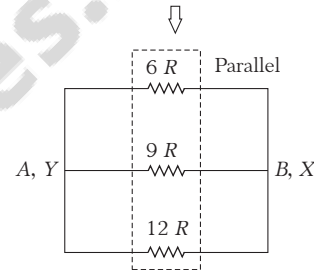
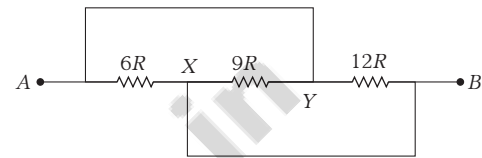
In dotted region, 4Ω , 4Ω and $(495/127)\Omega$ are in series,

$$\Rightarrow R_{eq} = 4 + 4 + \frac{495}{127} \Rightarrow R_{eq} = \frac{1511}{127}\Omega$$

Example 3.28 Find the equivalent resistance between A and B.



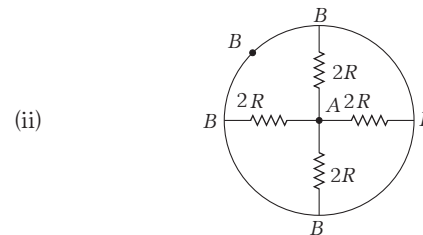
Sol. (i) The points connected by a conducting wire are at same potential. Then, redraw the diagram, by placing the points of same potential at one place and then solve for equivalent resistance.



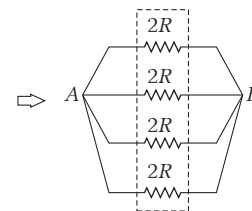
Hence, from the new figure A and Y are at same potential; B and X are at same potential.

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{6R} + \frac{1}{9R} + \frac{1}{12R} = \frac{13}{36R}$$

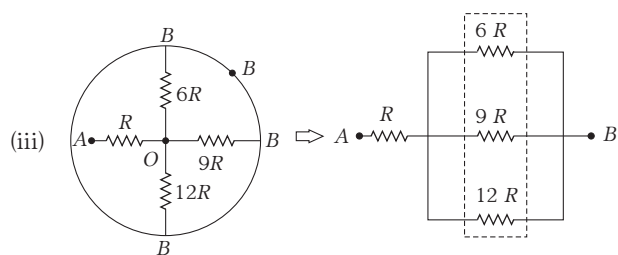
$$R_{eq} = \frac{36R}{13} = 2.77R$$



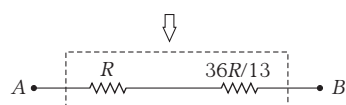
Similarly, placing the points of same potential at one place, then the equivalent resistance is



$$\text{In parallel, } R_{eq} = \frac{2R}{4} = 0.5R$$

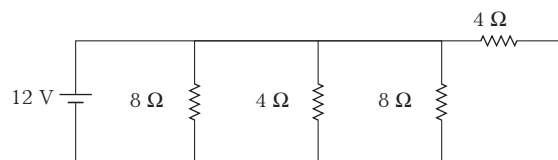


$$\text{In parallel, } \frac{1}{R'} = \frac{1}{6R} + \frac{1}{9R} + \frac{1}{12R} = \frac{13}{36R} \Rightarrow R' = \frac{36}{13}R$$

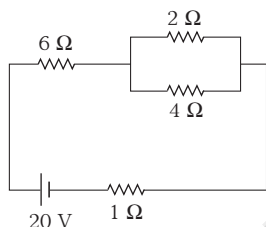


$$\text{In series, } R_{\text{eq}} = R + \frac{36R}{13} = \frac{49}{13}R = 3.77R$$

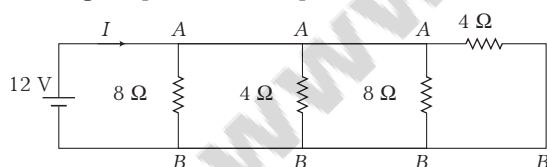
Example 3.29 (i) Determine the current supplied by the battery in the circuit as shown



(ii) Find currents in resistances 2Ω and 4Ω .



Sol. (i) Placing the points of same potential as shown

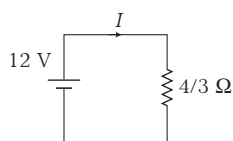


All resistances in the circuit are in parallel,

$$\Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{2+1+2+1}{8} = \frac{6}{8} = \frac{3}{4}$$

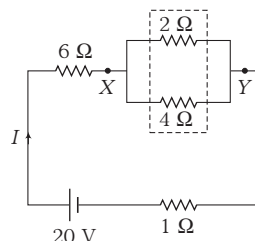
$$R_{\text{eq}} = (4/3)\Omega$$

Now, the circuit becomes



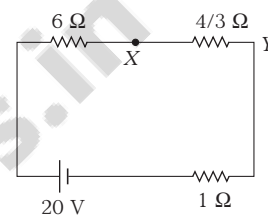
$$\text{So, current supplied by battery, } I = \frac{V}{R} = \frac{12 \times 3}{4} = 9\text{ A}$$

(ii)



In dotted region, resistances are in parallel,

$$\Rightarrow R_1 = \frac{4 \times 2}{4 + 2} = \frac{8}{6} = \frac{4}{3}\Omega$$

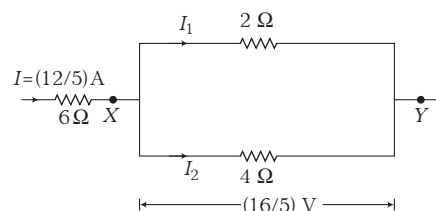


All resistance in the circuit are in series,

$$R_{\text{eq}} = 6 + 1 + \frac{4}{3} = \frac{25}{3}\Omega$$

$$\therefore \text{Current supplied by battery, } I = \frac{V}{R} = \frac{20 \times 3}{25} = \frac{12}{5}\text{ A}$$

In order to calculate the current in resistance 2Ω and 4Ω , we can redraw the circuit as



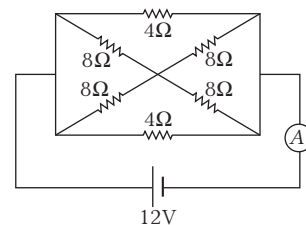
Potential difference across

$$XY, V_{XY} = IR_1 = \left(\frac{12}{5}\right)\left(\frac{4}{3}\right) = \frac{16}{5}\text{ V}$$

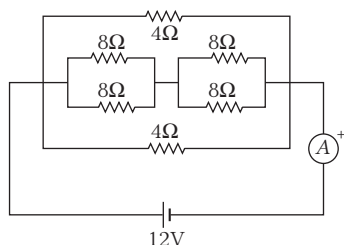
2Ω and 4Ω are parallel across XY. Hence, they have same potential.

$$\Rightarrow I_1 = \frac{V_{XY}}{2} = \frac{16}{5 \times 2} = \frac{8}{5}\text{ A and } I_2 = \frac{V_{XY}}{4} = \frac{16}{5 \times 4} = \frac{4}{5}\text{ A}$$

Example 3.30 Calculate the current shown by the ammeter A in the circuit shown in figure.



Sol. The given circuit can be redrawn as



From the above figure, the two $8\ \Omega$ resistances are connected in parallel, so equivalent resistance, $R_{eq} = \frac{8 \times 8}{8 + 8} = 4\ \Omega$.

These two combinations are connected in series, so equivalent resistance = $4 + 4 = 8\ \Omega$.

Now, we have resistances of $4\ \Omega$, $8\ \Omega$ and $4\ \Omega$ connected in parallel, so

$$\Rightarrow \frac{1}{R} = \frac{1}{4} + \frac{1}{8} + \frac{1}{4} = \frac{5}{8} \quad \text{or} \quad R = \frac{8}{5}\ \Omega$$

Also,

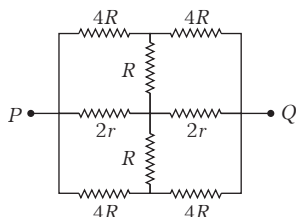
$$V = 12V$$

(given)

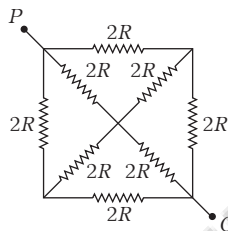
$$\therefore \text{Current, } I = \frac{V}{R} = \frac{12}{8/5} = \frac{15}{2} = 7.5\ \text{A}$$

Example 3.31 Find the equivalent resistance between P and Q .

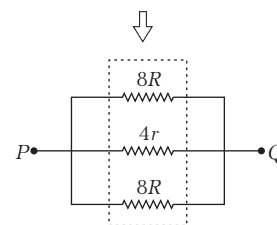
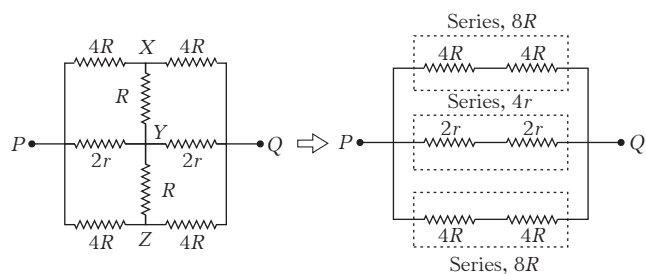
(i)



(ii)



Sol. (i) It can be seen that, this diagram is symmetrical about PQ , so points on the perpendicular bisector of PQ , i.e. X, Y and Z are at same potential. So, in this type of diagrams, to calculate the equivalent resistance, we can remove the resistances at the same potential, i.e. the resistances between X and Y, Y and Z , are redundant and can be removed.

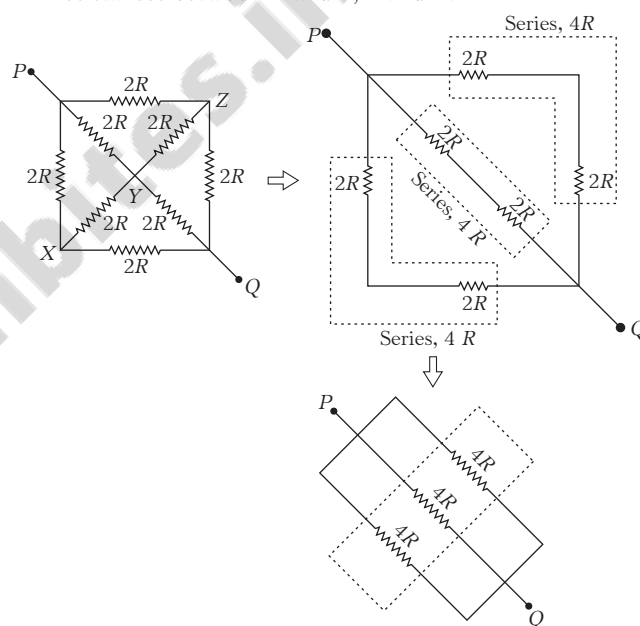


All resistances in the circuit are in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{8R} + \frac{1}{4r} + \frac{1}{8R} = \frac{R+r}{4Rr}$$

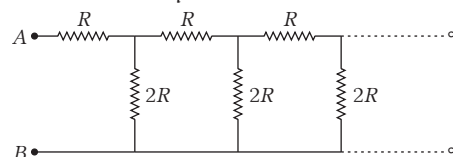
$$\Rightarrow R_{eq} = \frac{4Rr}{R+r}\ \Omega$$

(ii) Similarly as in (i), we see that there is symmetry about PQ and X, Y and Z are at same potential. So, remove resistances between X and $Y; Y$ and Z .

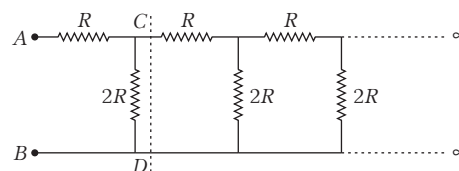


$$\text{In parallel, } R_{eq} = \frac{4R}{3}\ \Omega$$

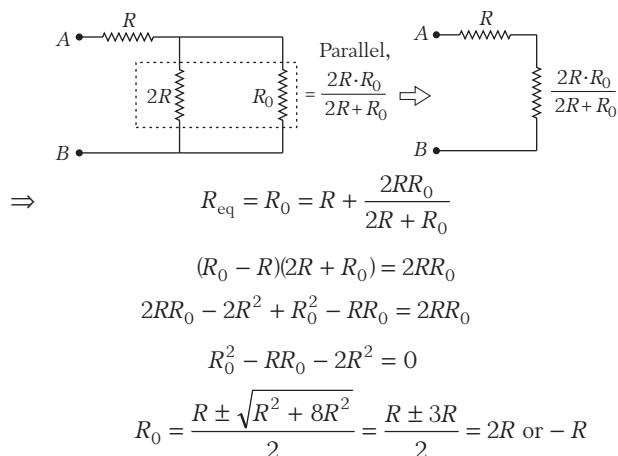
Example 3.32 Find the equivalent resistance between A and B .



Sol.



Here, we have infinite pairs of R and $2R$. Suppose, the equivalent resistance is R_0 between C and D , i.e. excluding one pair near AB (since, pairs are infinite, equivalent resistance will remain same, if we include pair near AB).



Equivalent resistance between A and B = $2R$ ($\because$ equivalent resistance cannot be negative).

Cells, emf and internal resistance

An electric cell is a source of energy that maintains a continuous flow of charge in a circuit. Electric cell changes chemical energy into electrical energy.

Electromotive force (emf) of a cell (ϵ)

Electric cell has to do some work in maintaining the current through a circuit. The work done by the cell in moving unit positive charge through the whole circuit (including the cell) is called the **electromotive force (emf) of the cell**.

If during the flow of q coulomb of charge in an electric circuit, the work done by the cell is W , then

$$\text{emf of the cell, } \epsilon = \frac{W}{q}$$

Its unit is joule/coulomb or volt.

If $W = 1$ joule and $q = 1$ coulomb, then $\epsilon = 1$ volt, *i.e.* if in the flow of 1 coulomb of charge, the work done by the cell is 1 joule, then the emf of the cell is 1 volt.

Internal resistance (r)

Internal resistance of a cell is defined as the resistance offered by the electrolyte of the cell to the flow of current through it. It is denoted by r and its unit is ohm.

Internal resistance of a cell depends on the following factors

- It is directly proportional to the separation between the two plates of the cell.
- It is inversely proportional to the plates area dipped into the electrolyte.
- It depends on the nature, concentration and temperature of the electrolyte and increases with increasing the concentration of electrolyte.

Terminal potential difference (V)

Terminal potential difference of a cell is defined as the potential difference between the two terminals of the cell in a closed circuit (*i.e.* when current is drawn from the cell). It is represented by V and its unit is volt.

Terminal potential difference of a cell is always less than the emf of the cell (*i.e.* $V < \epsilon$). In closed circuit, the current flows through the circuit including the cell, due to internal resistance of the cell, there is some fall of potential.

This is the amount of potential by which the terminal potential difference is less than the emf of the cell.

Relation between terminal potential difference, emf and internal resistance of a cell

- If no current is drawn from the cell, *i.e.* the cell is in open circuit, then emf of the cell will be equal to the terminal potential difference of the cell.

$$I = 0 \text{ or } V = \epsilon$$

- Consider a cell of emf ϵ and internal resistance r is connected across an external resistance R .

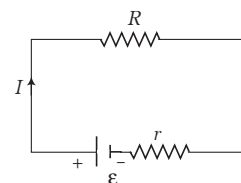


Fig. 3.10

$$\text{Current drawn from the cell, } I = \frac{\epsilon}{R + r} \quad \dots(i)$$

$$\text{Now from Ohm's law, } V = IR \Rightarrow I = \frac{V}{R} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{V}{R} = \frac{\epsilon}{R + r} \Rightarrow r = \left(\frac{\epsilon}{V} - 1 \right) R$$

Charging of cell

During charging of a cell, the positive terminal (electrode) of the cell is connected to positive terminal of battery charger and negative terminal (electrode) of the cell is connected to negative terminal of battery charger.

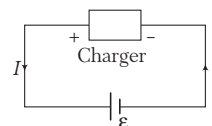


Fig. 3.11

In this process, current flows from positive electrode to negative electrode of the cell. From figure above, $V = \epsilon + Ir$

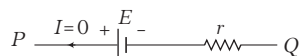
Thus, the terminal potential difference of a cell becomes greater than the emf of the cell.

The potential difference across internal resistance of the cell is called **lost voltage**, as it is not indicated by a voltmeter. Its value is equal to Ir .

Potential difference (V) across the terminals of a cell

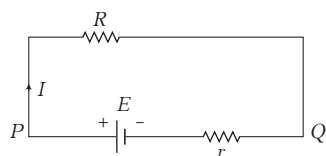
(i) When the cell is in open circuit,

$$\therefore V = V_P - V_Q = E, \text{ i.e. } V = E$$

**Fig. 3.12**

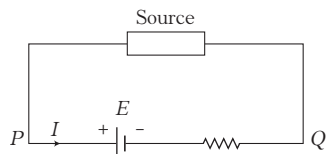
(ii) Discharging of cell when cell is supplying current,

$$\therefore V = V_P - V_Q = E - Ir, \text{ i.e. } V < E$$

**Fig. 3.13**

(iii) Charging of cell when cell is taking current,

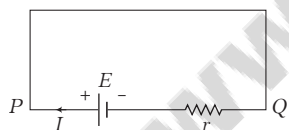
$$\therefore V = V_P - V_Q = E + Ir, \text{ i.e. } V > E$$

**Fig. 3.14**

(iv) When the cell is short circuited, i.e. external resistance is zero.

$$\Rightarrow I = E/r$$

$$\therefore V = V_P - V_Q = E - Ir = 0, \text{ i.e. } V = 0$$

**Fig. 3.15**

Example 3.33 The reading on a high resistance voltmeter when a cell is connected across it is 3 V. When the terminals of the cell are also connected to a resistance of 4 Ω , then the voltmeter reading drops to 1.2 V. Find the internal resistance of the cell.

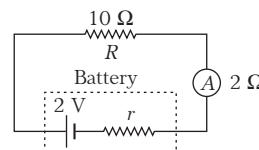
Sol. Given, $E = 3 \text{ V}$, $R = 4 \Omega$ and $V = 1.2 \text{ V}$

As, internal resistance,

$$r = R \left(\frac{E - V}{V} \right) = 4 \left(\frac{3 - 1.2}{1.2} \right) \Omega = 6 \Omega$$

Example 3.34 A battery of emf 2V and internal resistance r is connected in series with a resistor of 10 Ω through an ammeter of resistance 2 Ω . The ammeter reads 50 mA. Draw the circuit diagram and calculate the value of r .

Sol. Total resistance = $10 + 2 + r = (12 + r) \Omega$



Now, current = 50 mA = $50 \times 10^{-3} \text{ A}$ and emf = 2 V.

$$\text{So, resistance} = \frac{\text{emf}}{\text{current}}$$

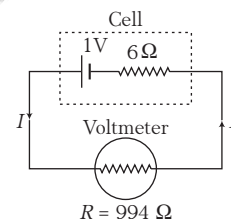
$$12 + r = \frac{2}{50 \times 10^{-3}} = 40$$

$$\Rightarrow r = 40 - 12 = 28 \Omega$$

Example 3.35 A voltmeter of resistance 994 Ω is connected across a cell of emf 1 V and internal resistance 6 Ω . Find the potential difference across the voltmeter, that across the terminals of the cell and percentage error in the reading of the voltmeter.

Sol. Given, $E = 1 \text{ V}$, $r = 6 \Omega$

and resistance of voltmeter, $R = 994 \Omega$



Current in the circuit is

$$I = \frac{E}{R + r} = \frac{1}{(994 + 6)} = 1 \times 10^{-3} \text{ A}$$

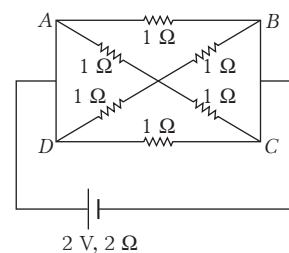
The potential difference across the voltmeter is

$$V = IR = 1 \times 10^{-3} \times 994 = 9.94 \times 10^{-1} \text{ V}$$

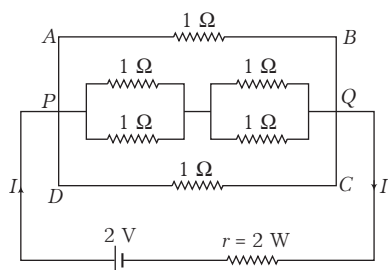
The same will be the potential difference across the terminals of the cell. The voltmeter used to measure the emf of the cell will read 0.994 V. Hence, the percentage error is

$$\frac{E - V}{E} \times 100 = \frac{1 - 0.994}{1} \times 100 = 0.6\%$$

Example 3.36 Find the current drawn from a cell of emf 2 V and internal resistance 2 Ω connected to the network given below.



Sol. The equivalent circuit is shown below



Resistance in arm $AB = 1\ \Omega$

Resistance in arm $PQ = \frac{1 \times 1}{1+1} + \frac{1 \times 1}{1+1} = \frac{1}{2} + \frac{1}{2} = 1\ \Omega$

Resistance in arm $DC = 1\ \Omega$

These three resistances are connected in parallel.

Their equivalent resistance R is given by

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = \frac{3}{1}$$

or

$$R = \frac{1}{3}\ \Omega$$

Current drawn from the cell,

$$I = \frac{E}{R+r} = \frac{2}{\left(\frac{1}{3} + 2\right)}$$

$$= \frac{2 \times 3}{7} = \frac{6}{7}\text{ A}$$

Grouping of cells

Series grouping

In series grouping, anode of one cell is connected to cathode of other cell and so on as shown below.

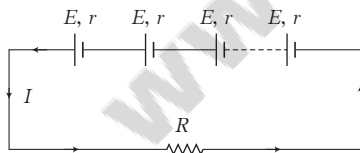


Fig. 3.16

If n identical cells of emf E and internal resistance r each, are connected in series, then

(i) equivalent emf of the combination, $E_{\text{eq}} = nE$.

(ii) equivalent internal resistance of the combination, $r_{\text{eq}} = nr$.

(iii) main current = Current from each cell $= I = \frac{nE}{R + nr}$.

(iv) potential difference across external resistance, $V = IR$.

(v) potential difference across each cell, $V' = \frac{V}{n}$.

If external resistance is much higher than the total internal resistance, then cells should be connected in series to get the maximum current.

Note If dissimilar plates of cells are connected together, then their emf's are added to each other while, if their similar plates are connected together their emf's are subtracted. While their internal resistances are always additive.

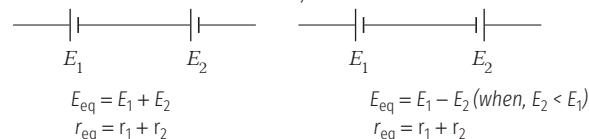


Fig. 3.17

Parallel grouping

In parallel grouping of cells, all anodes of cells are connected at one point and all cathodes of cells are connected together at other point as shown below.

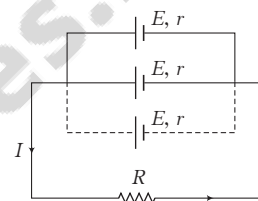


Fig. 3.18

If n identical cells of emf E and internal resistance r each are connected in parallel, then

(i) equivalent emf of the combination, $E_{\text{eq}} = E$.

(ii) equivalent internal resistance, $R_{\text{eq}} = r/n$.

(iii) main current, $I = \frac{E}{R + r/n}$.

(iv) current from each cell, $I' = \frac{I}{n}$.

(v) potential difference across external resistance = potential difference across each cell $= V = IR$.

If external resistance is much smaller than the total internal resistance, then cells should be connected in parallel to get the maximum current.

Mixed grouping

If n identical cells of emf E and internal resistance r each, are connected in a row and such m rows are connected in parallel as shown in Fig. 3.19, then

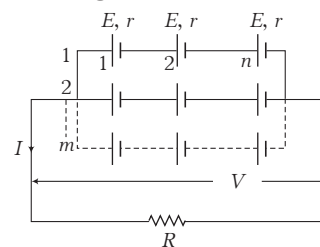


Fig. 3.19

- (i) Equivalent emf of the combination,

$$E_{\text{eq}} = nE$$

- (ii) Equivalent internal resistance,

$$r_{\text{eq}} = \frac{nr}{m}$$

- (iii) Main current flowing through the load,

$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr}$$

- (iv) Potential difference across load,

$$V = IR$$

- (v) Current from each cell,

$$I' = \frac{I}{n}$$

- (vi) Potential difference across each cell,

$$V' = \frac{V}{n}$$

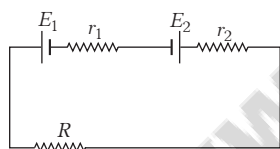
- (vii) Total number of cells = mn

In mixed grouping of cells, the current through the external resistance would be maximum, if the external resistance is equal to the total internal resistance of the cells, i.e. $R = \frac{nr}{m}$.

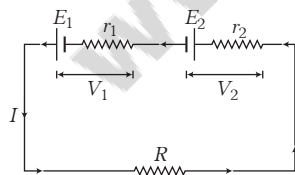
Example 3.37. In the circuit shown in figure,

$$E_1 = 10 \text{ V}, E_2 = 4 \text{ V}, r_1 = r_2 = 1 \Omega \text{ and } R = 2 \Omega.$$

Find the potential difference across battery 1 and battery 2.



Sol. Net emf of the circuit = $E_1 + E_2 = 14 \text{ V}$



Total resistance of the circuit,

$$= R + r_1 + r_2 = 4 \Omega$$

$\therefore$ Current in the circuit,

$$I = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{14}{4} = 3.5 \text{ A}$$

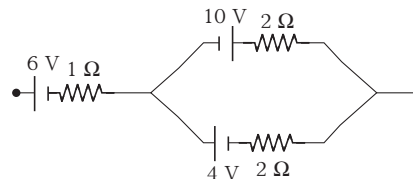
Now, potential difference across battery 1,

$$V_1 = E_1 + Ir_1 = 10 + (3.5)(1) = 13.5 \text{ V}$$

and potential difference across battery 2,

$$V_2 = E_2 + Ir_2 = 4 + (3.5)(1) = 7.5 \text{ V}$$

Example 3.38 Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



Sol. The given combination consists of two batteries in parallel and resultant of these two in series with the third one.

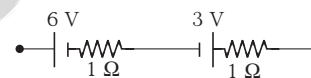
For parallel combination, we can apply,

$$E_{\text{eq}} = \frac{\frac{E_1}{r_1} - \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{10}{2} - \frac{4}{2}}{\frac{1}{2} + \frac{1}{2}} = 3 \text{ V}$$

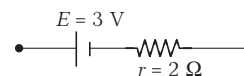
$$\text{Further, } \frac{1}{r_{\text{eq}}} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{2} + \frac{1}{2} = 1$$

$$\therefore r_{\text{eq}} = 1 \Omega$$

Now, this is in series with the third one, i.e.



The equivalent emf of these two is $(6 - 3) \text{ V}$ or 3 V and the internal resistance will be $(1 + 1) \Omega$ or 2Ω , i.e.

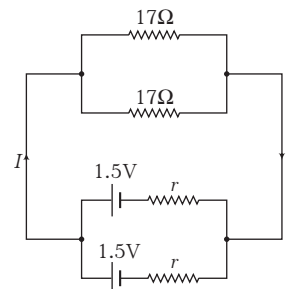


Example 3.39 Two identical cells of emf 1.5 V each joined in parallel provide supply to an external circuit consisting of two resistances of 17Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of cells to be 1.4 V . Calculate the internal resistance of each cell.

Sol. Given, $E = 1.5 \text{ V}$ and $V = 1.4 \text{ V}$

Resistance of external circuit = Total resistance of two resistances of 17Ω connected in parallel

$$\Rightarrow R = \frac{R_1 R_2}{R_1 + R_2} = \frac{17 \times 17}{17 + 17} = 8.5 \Omega$$



Let r' be the total internal resistance of the two cells.

Then,

$$r' = R \left(\frac{E - V}{V} \right) = 8.5 \left(\frac{1.5 - 1.4}{1.4} \right) \Omega = 0.6 \Omega$$

As the two cells of internal resistance r each have been connected in parallel, therefore

$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} \quad \text{or} \quad \frac{1}{0.6} = \frac{2}{r}$$

or $r = 0.6 \times 2 = 1.2 \Omega$

Example 3.40 Find the minimum number of cells required to produce an electric current of 1.5 A through a resistance of 30Ω . Given that the emf of each cell is 1.5 V and internal resistance is 1.0Ω .

Sol. As, $\frac{nr}{m} = R$

$$\therefore \frac{n \times 1}{m} = 30 \quad \text{or} \quad n = 30m \quad \dots(i)$$

$$\text{Current, } I = \frac{nE}{2R} \quad \text{or} \quad 1.5 = \frac{n \times 1.5}{2 \times 30} \quad \text{or} \quad n = 60 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$m = 60/30 = 2$$

and $mn = 120$

Example 3.41 36 cells, each of internal resistance 0.5Ω and emf of 1.5 V , are used to send maximum current through an external circuit of 2Ω resistance. Find the best mode of grouping them and the maximum current through the external circuit.

Sol. Given, $E = 1.5 \text{ V}$, $r = 0.5 \Omega$ and $R = 2 \Omega$

$$\text{Total number of cells, } mn = 36 \quad \dots(i)$$

For maximum current in the mixed grouping,

$$\frac{nr}{m} = R \quad \text{or} \quad \frac{n \times 0.5}{m} = 2 \quad \dots(ii)$$

Multiplying Eqs. (i) and (ii), we get

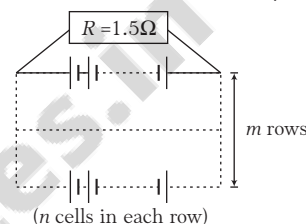
$$0.5n^2 = 72 \quad \text{or} \quad n^2 = 144$$

$$\therefore n = 12 \quad \text{and} \quad m = \frac{36}{12} = 3$$

Thus, for maximum current there should be three rows in parallel, each containing 12 cells in series.

$$\therefore \text{Maximum current} = \frac{mnE}{mR + nr} = \frac{36 \times 1.5}{3 \times 2 + 12 \times 0.5} = 4.5 \text{ A}$$

Example 3.42 12 cells, each of emf 1.5 V and internal resistance of 0.5Ω , are arranged in m rows each containing n cells connected in series, as shown. Calculate the values of n and m for which this combination would send maximum current through an external resistance of 1.5Ω .



Sol. For maximum current through the external resistance, external resistance = total internal resistance of cells

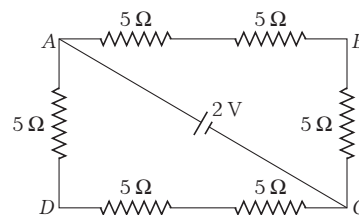
$$\begin{aligned} \text{or} \quad R &= \frac{nr}{m} \\ \therefore 1.5 &= \frac{n \times 0.5}{\frac{12}{n}} \quad (\because mn = 12) \end{aligned}$$

$$\text{or} \quad 36 = n^2 \quad \text{or} \quad n = 6 \quad \text{and} \quad m = 2$$

CHECK POINT 3.2

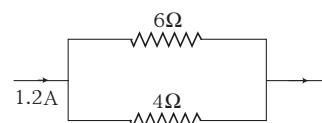
- Calculate the amount of charge flowed in 2 min through a wire of resistance 10Ω , when a potential difference of 20 V is applied across its ends.
(a) 120 C (b) 240 C (c) 20 C (d) 4 C
- If e , τ and m respectively, represent the electron density, relaxation time and mass of the electron, then the resistance R of a wire of length l and area of cross-section A will be
(a) $\frac{ml}{ne^2\tau A}$ (b) $\frac{m\tau^2 A}{ne^2 l}$ (c) $\frac{ne^2\tau A}{2ml}$ (d) $\frac{ne^2 A}{2m\tau l}$
- Four wires are made of the same material and are at the same temperature. Which one of them has highest electrical resistance?
(a) Length = 50 cm , diameter = 0.5 mm
(b) Length = 100 cm , diameter = 1 mm
(c) Length = 200 cm , diameter = 2 mm
(d) Length = 300 cm , diameter = 3 mm
- Carbon resistors, used in electronic circuits are marked for their value of resistance and tolerance by a colour code. A given carbon resistor has colour scheme brown, red, green and gold. Its value in ohm is
(a) $52 \times 10^6 \pm 10\%$ (b) $24 \times 10^5 \pm 5\%$
(c) $12 \times 10^4 \pm 10\%$ (d) $12 \times 10^5 \pm 5\%$

- The potential difference between points A and B of the following figure is



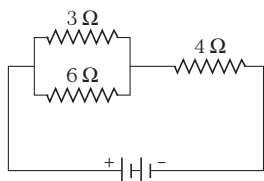
- (a) $\frac{2}{3} \text{ V}$ (b) $\frac{8}{9} \text{ V}$ (c) $\frac{4}{3} \text{ V}$ (d) 2 V

- In the figure given below, the current passing through 6Ω resistor is



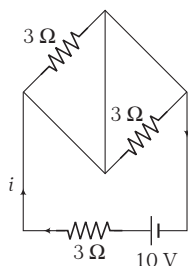
- (a) 0.40 A (b) 0.48 A
(c) 0.72 A (d) 0.80 A

7. In the figure given below, current passing through the $3\ \Omega$ resistor is 0.8 A , then potential drop through $4\ \Omega$ resistor is



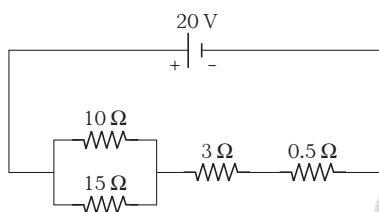
- (a) 9.6 V (b) 2.6 V (c) 4.8 V (d) 1.2 V

8. Current i as shown in the circuit will be



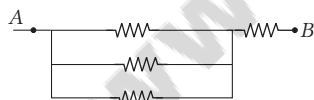
- (a) 10 A (b) $\frac{10}{3}\text{ A}$ (c) zero (d) infinite

9. In the figure given below, the current flowing through $10\ \Omega$ resistance is



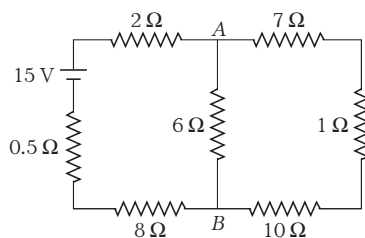
- (a) 12 A (b) 1.2 A (c) 0.8 A (d) 0.4 A

10. If all the resistors shown have the value $2\ \Omega$ each, the equivalent resistance over AB is



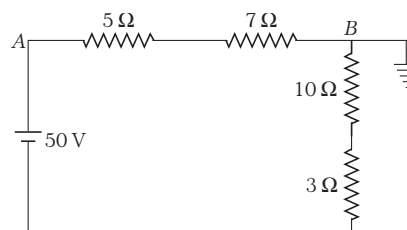
- (a) $2\ \Omega$ (b) $4\ \Omega$ (c) $1\frac{2}{3}\ \Omega$ (d) $2\frac{2}{3}\ \Omega$

11. The current drawn from the battery in circuit diagram shown is



- (a) 1 A (b) 2 A (c) 1.5 A (d) 3 A

12. In the circuit shown, the point B is earthed. The potential at the point A is

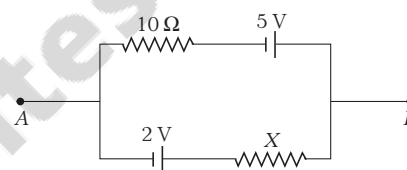


- (a) 14 V (b) 24 V
(c) 26 V (d) 50 V

13. A current of 2 A passes through a cell of emf 1.5 V having internal resistance of $0.15\ \Omega$. The potential difference measured in volt, across both the ends of the cell will be

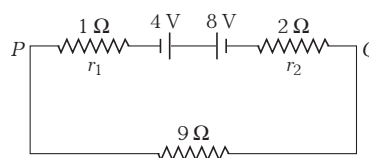
- (a) 1.35 (b) 1.50
(c) 1.00 (d) 1.20

14. If V_{AB} is 4 V in the given figure, then resistance X will be



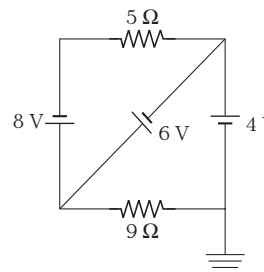
- (a) $5\ \Omega$ (b) $10\ \Omega$
(c) $15\ \Omega$ (d) $20\ \Omega$

15. Two batteries of emf 4 V and 8 V with internal resistances $1\ \Omega$ and $2\ \Omega$ are connected in a circuit with a resistance of $9\ \Omega$ as shown in figure. The current and potential difference between the points P and Q are



- (a) $\frac{1}{3}\text{ A}$ and 3 V (b) $\frac{1}{6}\text{ A}$ and 4 V
(c) $\frac{1}{9}\text{ A}$ and 9 V (d) $\frac{1}{2}\text{ A}$ and 12 V

16. The current flowing through $5\ \Omega$ resistance is



- (a) 10 A (b) 1 A
(c) 2.5 A (d) 0.4 A

KIRCHHOFF'S LAWS

Many electric circuits cannot be reduced to simple series-parallel combinations. Kirchhoff's laws (or rules) are used to solve these complicated electric circuits. These rules are basically the expressions of conservation of electric charge and of energy. *e.g.* two circuits that cannot be broken down are shown in Fig. 3.20.

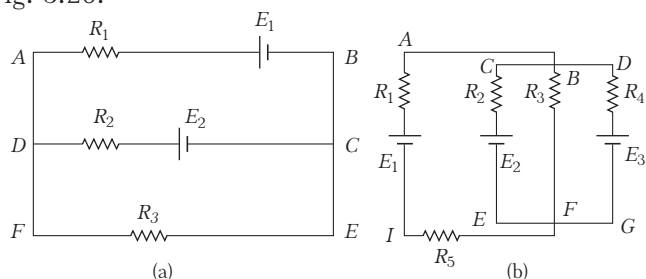


Fig. 3.20

However, it is always possible to analyse such circuits by applying two rules, derived by Kirchhoff in 1845 and 1846. The two terms related to Kirchhoff's laws are given below

Junction

A junction in a circuit is a point, where three or more conductors meet. Junctions are also called **nodes** or **branch points**. For example, in Fig. 3.20 (a) points D and C are junctions. Similarly, in Fig. 3.20 (b) points B and F are junctions.

Loop

A loop is any closed conducting path. For example, in Fig. 3.20 (a) ABCDA, DCEFD and ABEFA are loops. Similarly, in Fig. 3.20 (b), CBFEC, BDGFB are loops.

Kirchhoff's junction rule

The algebraic sum of the currents meeting at a point or at a junction in an electric circuit is always zero.

i.e.
$$\sum_{\text{junction}} i = 0$$

This law can also be written as, "the sum of all the currents directed towards a point (node) in a circuit is equal to the sum of all the currents directed away from that point (node)".

This law is also known as **Kirchhoff's Current Law (KCL)**.

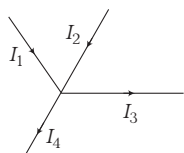


Fig. 3.21

Thus, in Fig. 3.21 according to KCL, $I_1 + I_2 = I_3 + I_4$.

The junction rule is based on **conservation of electric charge**.

Kirchhoff's loop rule

This law states that the algebraic sum of change in potential around any closed loop involving resistors and cells in the loop, is zero. It means that, in any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistances and currents flowing through them. It is also known as **loop rule**. *i.e.* $\sum \Delta V = 0$
 closed loop

This law is also known as **Kirchhoff's Voltage Law (KVL)**. The loop rule is based on **conservation of energy**.

Sign conventions for the application of Kirchhoff's laws

For the application of Kirchhoff's laws, the following sign conventions are to be considered.

- (i) The change in potential in traversing a resistance in the direction of current is $-IR$ while in the opposite direction is $+IR$.



Fig. 3.22

- (ii) The change in potential in traversing an emf source from negative to positive terminal is $+E$ while in the opposite direction is $-E$, irrespective of the direction of current in the circuit.

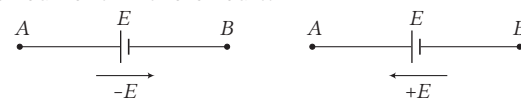


Fig. 3.23

To calculate potential difference between two points by using Kirchhoff's law

While using Kirchhoff's laws to calculate the potential difference, the following points should be considered

- (i) Start from a point on the loop and go along the loop, either anti-clockwise or clockwise, to reach the same point again, but balance currents at junction as per KCL.
- (ii) If moving along the direction of the current, there will be potential drop across a resistance and if moving in the opposite direction, there will be potential gain.
- (iii) The net sum of all these potential differences should be zero, using the KVL rule.

Now, let us consider a circuit as shown in Fig. 3.24.

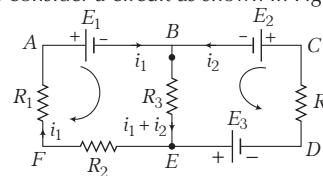


Fig. 3.24

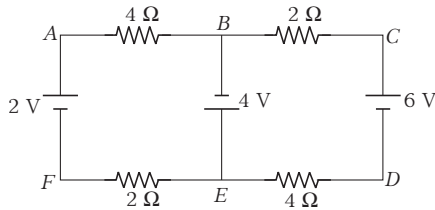
In closed loop ABEFA,

$$-E_1 - (i_1 + i_2)R_3 - i_1R_2 - i_1R_1 = 0$$

In closed loop BCDEB,

$$E_2 + i_2R_4 + E_3 - (i_1 + i_2)R_3 = 0$$

Example 3.43 Find currents in different branches of the electric circuit shown in figure.



Sol. Applying Kirchhoff's first law (junction law) at junction B,

$$i_1 = i_2 + i_3 \quad \dots(i)$$

Applying Kirchhoff's second law in loop 1 (ABEFA),

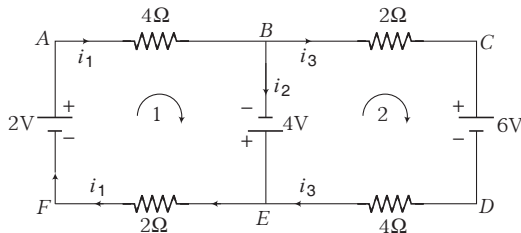
$$-4i_1 + 4 - 2i_1 + 2 = 0 \quad \dots(ii)$$

Applying Kirchhoff's second law in loop 2 (BCDEB),

$$-2i_3 - 6 - 4i_3 - 4 = 0 \quad \dots(iii)$$

Solving Eqs. (i), (ii) and (iii), we get

$$i_1 = 1 \text{ A}$$



$$i_3 = -\frac{5}{3} \text{ A} \Rightarrow i_2 = \frac{8}{3} \text{ A}$$

Here, negative sign of i_3 implies that current i_3 is in opposite direction of what we have assumed.

Example 3.44 In above example, find the potential difference between points F and C.

Sol. Let us reach from F to C via A and B,

$$V_F + 2 - 4i_1 - 2i_3 = V_C$$

$$\therefore V_F - V_C = 4i_1 + 2i_3 - 2$$

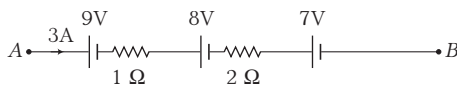
Substituting, $i_1 = 1 \text{ A}$

and $i_3 = -(5/3) \text{ A}$, we get

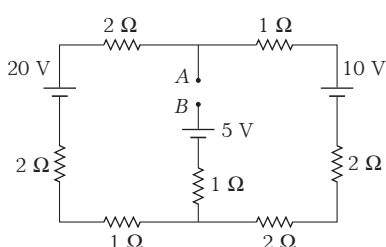
$$V_F - V_C = -(4/3) \text{ V}$$

Here, negative sign implies that $V_F < V_C$.

Example 3.45 (i) Find the potential difference between the points A and B.



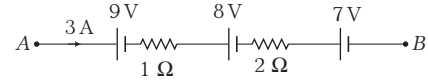
(ii)



(a) Find the potential difference between the points A and B.

(b) Find current through 20 V cell, if points A and B are connected.

Sol. (i)

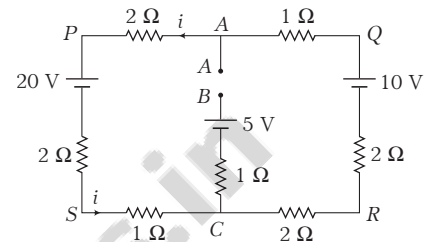


When we move from A to B, using KVL,

$$V_A - 9 - 3 \times 1 - 8 - 3 \times 2 - 7 = V_B$$

$$\Rightarrow V_A - V_B = 33 \text{ V}$$

(ii) (a)



No current flows in the branch CB as AB is not connected.

Let current in the circuit APSCRQA be i .

Using KVL,

$$V_A - i \times 2 - 20 - i \times 2 - i \times 1 - i \times 2 - i \times 2 + 10 - i \times 1 = V_A$$

$$\Rightarrow 10i = -10 \Rightarrow i = -1 \text{ A}$$

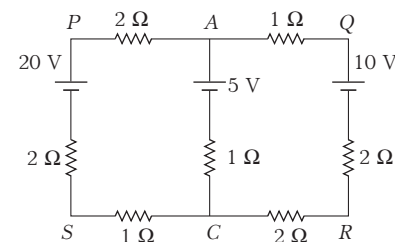
i.e., direction of i is opposite.

Now, A to B path will be APSCB,

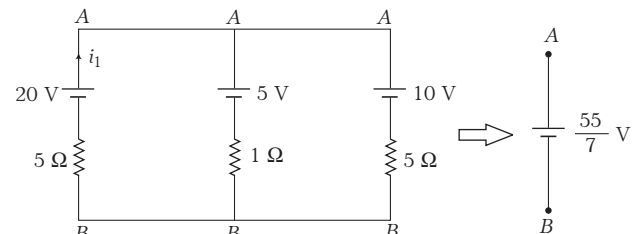
$$V_A - i \times 2 - 20 - i \times 2 - i \times 1 + 5 = V_B$$

$$V_A - V_B = 15 + 5i = 15 + 5 \times (-1) = 10 \text{ V}$$

(b) Now, if the points A and B are connected.



Resistance 2Ω , 2Ω and 1Ω are in series along APSC and resistance 1Ω , 2Ω and 2Ω are also in series along AQRC.



So, the equivalent voltage will be

$$E = \frac{\frac{20}{5} + \frac{5}{1} + \frac{10}{5}}{\frac{1}{5} + \frac{1}{1} + \frac{1}{5}} = \frac{4 + 5 + 2}{\frac{1+5+1}{5}} = \frac{11 \times 5}{7} = \frac{55}{7} \text{ V}$$

∴ Potential difference between points A and B,

$$V_A - V_B = \frac{55}{7} \text{ V}$$

For cell of emf 20 V,

$$V_A - V_B = 20 - 5i$$

$$\frac{55}{7} = 20 - 5i \Rightarrow 5i = 20 - \frac{55}{7} \approx 12 \text{ A}$$

∴ Current through cell of 20 V, $i = 2.4 \text{ A}$

Electrical energy and power

Electrical energy and power in electrical circuits or components are described below.

Electrical energy

It is defined as the total work done W by the source of emf V in maintaining the electric current I in the circuit for a specified time t .

According to Ohm's law, we have $V = IR$

Total charge that crosses the resistor is given by $q = It$

Energy gained is given by $E = W = Vq = VIt$

$$= (IR) It = I^2 R t \quad [\because V = IR]$$

$$= \left(\frac{V}{R}\right)^2 R t = \frac{V^2 t}{R} \quad \left(\because I = \frac{V}{R}\right)$$

The SI unit of electrical energy is joule (J), where

$$1 \text{ joule} = 1 \text{ volt} \times 1 \text{ ampere} \times 1 \text{ second} = 1 \text{ watt} \times 1 \text{ second}.$$

Electrical Power

It is defined as the rate of electrical energy supplied per unit time to maintain the flow of electric current through a conductor.

$$\text{Mathematically, } P = \frac{W}{t} = VI = I^2 R = \frac{V^2}{R}$$

The SI unit of power is watt (W), where

$$1 \text{ watt} = 1 \text{ volt} \times 1 \text{ ampere} = 1 \text{ ampere-volt}.$$

It can be defined as, the power of an electric circuit is called **one watt**, if one ampere current flows in it against a potential difference of one volt. The bigger units of electrical power are kilowatt (kW) and megawatt (MW)

where, $1 \text{ kW} = 1000 \text{ W}$ and $1 \text{ MW} = 10^6 \text{ W}$.

Commercial unit of electrical power is horse power (HP), where, $1 \text{ HP} = 746 \text{ W}$.

Heating effects of current

An electric current through a resistor increases its thermal energy. Also, there are other situations in which an electric current can produce or absorb thermal energy. This effect is called heating effect of electric current.

When some potential difference V is applied across a resistance R , charge q flows through the circuit in time t , then the heat absorbed or produced is given by

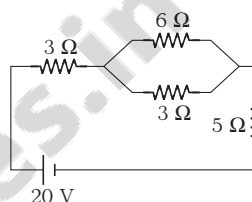
$$W = qV = Vit = i^2 R t = \frac{V^2 t}{R} \text{ joule}$$

$$\text{or } W = \frac{Vit}{J} = \frac{i^2 R t}{J} = \frac{V^2 t}{JR} \text{ cal}$$

where, J is the joule's mechanical equivalent of heat (4.21 J/cal).

These relations are also called **Joule's law of heating**.

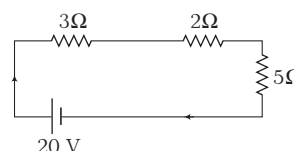
Example 3.46 In the given network of resistors, find the heat developed across each resistance in 2s.



Sol. The 6Ω and 3Ω resistances are in parallel. So, their combined resistance is

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \quad \text{or } R = 2 \Omega$$

The equivalent simple circuit can be drawn as shown.

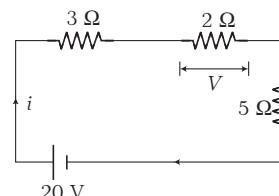


Current in the circuit,

$$i = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{20}{3 + 2 + 5} = 2 \text{ A}$$

$$V = iR = (2)(2) = 4 \text{ V}$$

i.e. potential difference across 6Ω and 3Ω resistances same as 4 V. Now,



$$H_{3\Omega} = i^2 R t = (2)^2 (3) (2) = 24 \text{ J}$$

$$H_{6\Omega} = \frac{V^2}{R} t = \frac{(4)^2}{6} (2) = \frac{16}{3} \text{ J}$$

$$H_{3\Omega} = \frac{V^2}{R} t = \frac{(4)^2 (2)}{3} = \frac{32}{3} \text{ J}$$

and

$$H_{5\Omega} = i^2 R t = (2)^2 (5) (2) = 40 \text{ J}$$

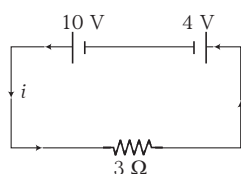
Electricity consumption

To measure the electrical energy consumed commercially, joule is not sufficient. So, a special unit **kilowatthour** is used in place of joule. It is also called **1 unit of electrical energy**. 1 kilowatt hour or 1 unit of electrical energy is the amount of energy dissipated in 1 hour in a circuit, when the electric power in the circuit is 1 kilowatt.

$$1 \text{ kilowatt hour (kWh)} = 3.6 \times 10^6 \text{ joule (J)}$$

Note Resistance of electrical appliance On electrical appliances (bulbs, geysers, heaters, etc.) wattage, voltage printed are called rated values. The resistance of any electrical appliance can be calculated by rated power and rated voltage by using $R = V_R^2 / P_R$.

Example 3.47 In the following figure, find



- the power supplied by 10 V battery,
- the power consumed by 4 V battery and
- the power dissipated in 3 Ω resistance.

Sol. Net emf of the circuit = $(10 - 4) = 6 \text{ V}$

Total resistance of the circuit = 3 Ω

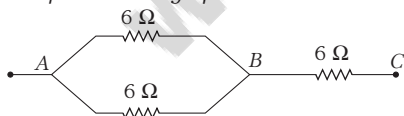
$$\therefore \text{Current in the circuit, } i = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{6}{3} = 2 \text{ A}$$

$$(i) \text{ Power supplied by 10 V battery} = Ei = (10)(2) = 20 \text{ W}$$

$$(ii) \text{ Power consumed by 4 V battery} = Ei = (4)(2) = 8 \text{ W}$$

$$(iii) \text{ Power dissipated in } 3 \Omega \text{ resistance} = i^2 R = (2)^2(3) = 12 \text{ W}$$

Example 3.48 In the following figure, each of the three resistances, has rating of 24 W and resistance of 6 Ω. Find the maximum power rating of the circuit.



Sol. To find maximum current i ,

$$\text{we use, } P = i^2 R \Rightarrow i^2 = \frac{P}{R} = \frac{24}{6} = 4 \Rightarrow i = 2 \text{ A}$$

$$\text{Resistance between A and B is } \frac{1}{R_1} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \Rightarrow R = 3 \Omega$$

and resistance between A and C, $R_{eq} = 3 + 6 = 9 \Omega$

If we make 2 A current flow through the given circuit, 1 A will flow through each of 6 Ω in parallel and 2 A through 6 Ω in series. This is the maximum current the circuit can hold.

$$\text{So, power of circuit} = i^2 R = 2 \times 2 \times 9 = 36 \text{ W}$$

Power consumption in a combination of bulbs

Series combination of bulbs

(i) Total power consumed is given by

$$\frac{1}{P_{eq}} = \frac{1}{P_1} + \frac{1}{P_2} + \dots$$

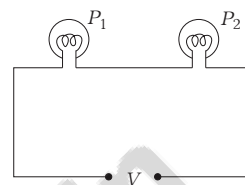


Fig. 3.25 Series combination of two bulbs

(ii) $P_{\text{consumed}} (\text{brightness}) \propto V \propto R \propto \frac{1}{P_{\text{rated}}}$, i.e. in series

combination bulb of lesser wattage will give more bright light and potential difference appearing across it will be more.

Parallel combination of bulbs

(i) Total power consumed is given by

$$P_{\text{total}} = P_1 + P_2 + \dots$$

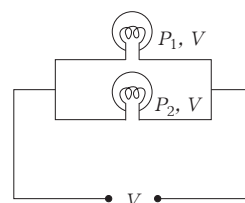


Fig. 3.26 Parallel combination of two bulbs

(ii) $P_{\text{consumed}} (\text{brightness}) \propto P_{\text{rated}} \propto I \propto \frac{1}{R}$, i.e. in parallel

combination, bulb of greater wattage will give more bright light and more current will pass through it.

Applications of heating effects of current

- Filament of electric bulb is made up of tungsten which has high resistivity and high melting point.
- Electric devices having heating elements like heater, geyser or press are made up of nichrome which has high resistivity and high melting point.
- Fuse wire is made up of tin-lead alloy, since it should have low melting point and high resistivity.

It is used in series as a safety device in an electric circuit and is designed, so as to melt and thereby open the circuit, if the current exceeds a predetermined value due to some fault.

Example 3.49 Two bulbs having rating of 60 W-220 V and 100 W-220 V are joined (i) in series and (ii) in parallel. Which of the two will glow brighter in each case?

Sol. Suppose both are used at 220 V supply in both the cases. In case of **parallel combination**, same voltage will be appear on each bulb, hence heat produced will be proportional to $1/R$. As, bulb of greater wattage will glow more. Hence, 100 W bulb will glow brighter. In case of **series combination**, same current flows through each bulb, hence heat produced will be proportional to R ($P = i^2 R$). As we know, higher the wattage, lower the resistance, then 60 W bulb will have higher resistance. That means more heat will be produced in 60 W bulb in this case, so this bulb will glow brighter.

Example 3.50 Two bulbs having rating of 40 W-220 V and 100 W-220 V are joined in series and alternately, (i) 300 V and (ii) 440 V is applied. Find out which bulb will fuse in each case.

Sol. We first have to find out the maximum current each bulb can bear. This can be calculated from rating of the bulb,

$$P = VI \Rightarrow 40 = 220 I \Rightarrow I = \frac{40}{220} = 0.18 \text{ A}$$

This is the maximum current 40 W bulb can bear.

Similarly, 100 W bulb can bear $\frac{100}{220} = 0.45 \text{ A}$.

Now, find out the resistance of each bulb, $R = \frac{V^2}{P}$

$$\Rightarrow \text{Resistance of 40 W bulb} = \frac{220}{40} \times 220 = 1210 \Omega$$

$$\text{Resistance of 100 W bulb} = \frac{220 \times 220}{100} = 484 \Omega$$

These are joined in series, so total resistance = 1694 Ω

(i) Current in each bulb when joined with 300 V

$$\text{i.e. } I = \frac{300}{1694} = 0.177 \text{ A}$$

This current will flow in each, so no bulb will fuse as it is less than their maximum permissible current.

(ii) When they are joined with 440 V in series then current will be = $\frac{440}{1694} = 0.26 \text{ A}$.

This current is less than maximum permissible current of 100 W bulb but more than that of 40W bulb. Hence, 40 W bulb will be fused and 100 W bulb will remain safe.

Example 3.51 In above example, if we join the bulbs in parallel and 300 V is applied, which of the two bulbs will fuse?

Sol. When they are joined in parallel and 300 V is applied on them, then both will get 300 V. Since, their rating is 220 V, naturally, current flowing through them will be more than maximum possible value. Hence, both will fuse out.

Example 3.52 Two coils of power 60 W and 100 W and both operating at 220 V takes time 2 min and 1.5 min separately to boil certain amount of water. If they are joined (i) in series and (ii) in parallel, then find the ratio of time taken by them to boil the same water in the two cases.

Sol. When they are joined in series, then total power,

$$P = \frac{P_1 P_2}{P_1 + P_2} = \frac{60 \times 100}{160} = 37.5 \text{ W}$$

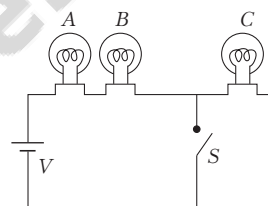
When they are joined in parallel, then total power = 60 + 100 = 160 W

Time taken to boil the water will be inversely proportional to power, so ratio of time taken in the two cases will be 160 : 37.5.

Example 3.53 Figure shows three identical bulbs A, B and C, which are connected to a battery of supply voltage V. When the switch S is closed, then discuss the change in

(i) the illumination of the three bulbs.

(ii) the power dissipated in the circuit.



Sol. When the switch S is open,

$$V_A = V_B = V_C = V/3$$

and

$$P_A = P_B = P_C = \frac{(V/3)^2}{R} = \frac{V^2}{9R} = P \text{ (say)}$$

(i) When the switch S is closed, then the bulb C is short circuited and hence there will be no current through C. So, $P_C = 0$

$$V_A = V_B = \frac{V}{2}$$

$$\text{So, } P_A = P_B = \frac{(V/2)^2}{R} = \frac{V^2}{4R} = \frac{9}{4} P$$

Therefore, the intensity of illumination of each of the bulb A and B become $9/4$ times of the initial value but the intensity of the bulb C becomes zero.

(ii) The power dissipated in the circuit before closing the switch is

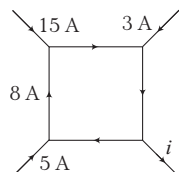
$$P_i = P_A + P_B + P_C = 3P$$

The power dissipated after closing the switch is

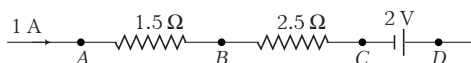
$$\begin{aligned} P_f &= P_A + P_B + P_C \\ &= \frac{9}{4} P + \frac{9}{4} P + 0 \\ &= \frac{9}{2} P \end{aligned}$$

CHECK POINT 3.3

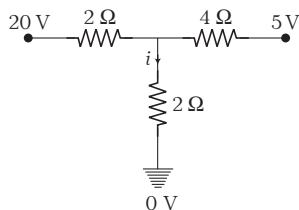
1. The figure shows a network of currents. The current i will be



- (a) 3 A (b) 13 A
(c) 23 A (d) -3 A
2. In the circuit element given here, if the potential difference at point B , $V_B = 0$, then the potential difference and D are



- (a) $V_A = -1.5$ V, $V_D = +2$ V (b) $V_A = -1.5$ V, $V_D = +0.5$ V
(c) $V_A = +1.5$ V, $V_D = +0.5$ V (d) $V_A = +1.5$ V, $V_D = -0.5$ V
3. Three resistances are connected to form a T-shape as shown in the figure. Then, the current i in the $2\ \Omega$ resistor is



- (a) 0.93 A (b) 4.5 A
(c) 2.5 A (d) 1.57 A
4. How much work is required to carry a $6\ \mu\text{C}$ charge from the negative terminal to the positive terminal of a 9 V battery?
(a) 54×10^{-3} J (b) 54×10^{-6} J
(c) 54×10^{-9} J (d) 54×10^{-12} J
5. Two resistors R and $2R$ are connected in series in an electric circuit. The thermal energy developed in R and $2R$ are in the ratio
(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1
6. The resistor of resistance R is connected to 25 V supply and heat produced in it is $25\ \text{Js}^{-1}$. The value of R is
(a) $225\ \Omega$ (b) $1\ \Omega$ (c) $25\ \Omega$ (d) $50\ \Omega$
7. Just as electricity is supplied at 220 V for domestic use in India, it is supplied at 110 V in USA. If the resistance of 60 W bulb for use in India is R , then that of 60 W bulb for use in USA will be
(a) $R/4$ (b) $R/2$
(c) R (d) $2R$
8. If R_1 and R_2 are respectively, the filament resistances of 200 W bulb and 100 W bulb designed to operate on the same voltage, then
(a) R_1 is two times R_2 (b) R_2 is two times R_1
(c) R_2 is four times R_1 (d) R_1 is four times R_2

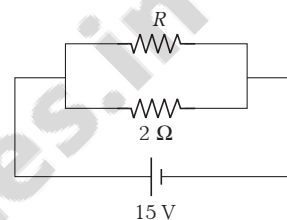
9. The electric bulb have tungsten filaments of same length. If one of them gives 60 W and other 100 W, then

- (a) 100 W bulb has thicker filament
(b) 60 W bulb has thicker filament
(c) Both filaments are of same thickness
(d) it is not possible to get different wattage unless the lengths are different

10. How many calories of heat will be produced approximately in 210 W electric bulb in 5 min?

- (a) 800000 cal (b) 63000 cal
(c) 1050 cal (d) 15000 cal

11. If in the circuit, power dissipation is 150 W, then R is



- (a) $2\ \Omega$ (b) $6\ \Omega$ (c) $5\ \Omega$ (d) $4\ \Omega$

12. A wire when connected to 220 V mains supply has power dissipation P_1 . Now, the wire is cut into two equal pieces, which are connected in parallel to the same supply. Power dissipation in this case is P_2 . Then, $P_2 : P_1$ is

- (a) 1 (b) 4 (c) 2 (d) 3

13. Two electric bulbs, one of 200 V-40 W and other 200 V-100 W are connected in series to a 200 V line, then the potential drop across

- (a) the two bulbs is zero (b) both the bulbs is 200 V
(c) 40 W bulb is more (d) 100 W bulb is more

14. Three identical bulbs are connected in series and these together dissipate a power P . Now, if the bulbs are connected in parallel, then the power dissipated will be

- (a) $\frac{P}{3}$ (b) $3P$ (c) $9P$ (d) $\frac{P}{9}$

15. A and B are two bulbs connected in parallel. If A is glowing brighter than B , then the relation between R_A and R_B is

- (a) $R_A = R_B$ (b) $R_B > R_A$
(c) $R_A > R_B$ (d) None of these

16. Some electric bulbs are connected in series across a 220 V supply in a room. If one bulb is fused, then remaining bulbs are connected again in series (after removing the fused bulb) across the same supply. The illumination in the room will

- (a) increase (b) decrease
(c) remain the same (d) not continuous

17. Electric bulbs of 50 W-100 V glowing at full power are to be used in parallel with battery 120 V, $10\ \Omega$. Maximum number of bulbs that can be connected, so that they glow in full power is

- (a) 2 (b) 8
(c) 4 (d) 6

MEASURING INSTRUMENTS FOR CURRENT AND VOLTAGE

There are various instruments like galvanometer, ammeter and voltmeter which can be used to detect current and voltage, depending on the range.

Galvanometer

It is an instrument used to detect small current passing through it by showing deflection. It can be converted into voltmeter (for measuring voltage) and ammeter (for measuring current).

Ammeter

It is an instrument used to measure current and is always connected in series with the circuit element through which current is to be measured. Smaller the resistance of an ammeter, more accurate will be its reading, as it will not change the circuit current. An ammeter is said to be ideal, if its resistance r is zero.

Conversion of galvanometer into ammeter

A galvanometer can be converted into an ammeter by **connecting a low resistance (called shunt S) in parallel to the galvanometer of resistance G .**

Hence, only a small amount of current pass through galvanometer and remaining will pass through the shunt.

G and S are parallel and hence have equal potential difference, i.e. $i_g G = (i - i_g) S$.

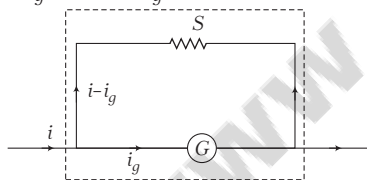


Fig. 3.27 Ammeter

$$\therefore \text{Required shunt resistance, } S = \frac{i_g}{(i - i_g)} G$$

Example 3.54 What shunt resistance is required to make the 1 mA, 20 Ω galvanometer into an ammeter with a range of 0 to 50 mA?

Sol. Given, $i_g = 1\text{mA} = 10^{-3}\text{ A}$, $G = 20\ \Omega$

$$i = 50 \times 10^{-3}\text{ A}$$

$$\begin{aligned} \text{Substituting in } S &= \left(\frac{i_g}{i - i_g} \right) G \\ &= \frac{(10^{-3})(20)}{(50 \times 10^{-3}) - (10^{-3})} \\ &= 0.408\ \Omega \end{aligned}$$

Voltmeter

It is an instrument used to measure potential difference and is always connected in parallel with the circuit element across which potential difference is to be measured.

Greater the resistance of voltmeter, more accurate will be its reading, as only small amount of current pass through it, by not changing the circuit current. A voltmeter is said to be ideal, if its resistance is infinite.

Conversion of galvanometer into voltmeter

A galvanometer can be converted into voltmeter by **connecting a large resistance R in series with the galvanometer.**

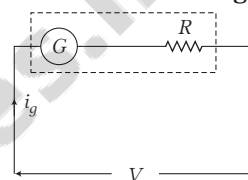


Fig. 3.28 Voltmeter

According to Ohm's law, $V = i_g (G + R)$

$$\text{or required resistance, } R = \frac{V}{i_g} - G$$

Example 3.55 How can we make a galvanometer with $G = 20\ \Omega$ and $i_g = 1\text{ mA}$ into a voltmeter with a maximum range of 10 V?

Sol. Using, $R = \frac{V}{i_g} - G$

$$\text{We have, } R = \frac{10}{10^{-3}} - 20 = 9980\ \Omega$$

Thus, a resistance of 9980 Ω is to be connected in series with the galvanometer to convert it into the voltmeter of desired range.

Wheatstone bridge

It is an arrangement of four resistances used to measure one of them, in terms of the other three as shown in Fig. 3.29.

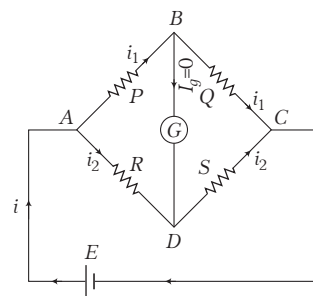


Fig. 3.29 A Wheatstone bridge

The bridge is said to be balanced, when deflection in galvanometer is zero ($I_g = 0$), i.e. no current flows through the galvanometer (branch BD). In the balanced condition,

$$\frac{P}{Q} = \frac{R}{S}$$

On mutually changing the position of cell and galvanometer, this condition will not change.

Note Different forms of Wheatstone bridge are shown below

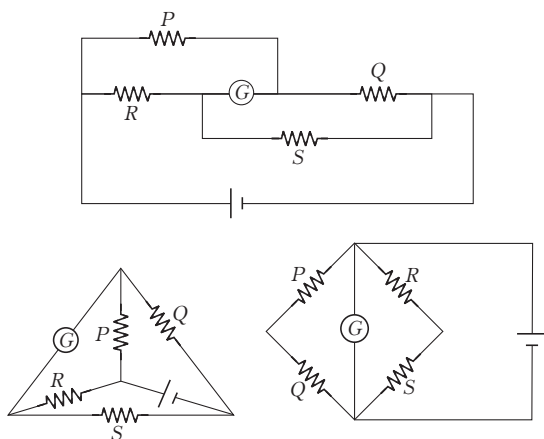
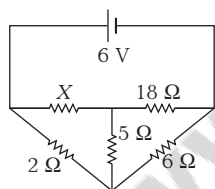


Fig. 3.30

Example 3.56 Find out the magnitude of resistance X in the circuit shown in figure, when no current flows through the $5\ \Omega$ resistor.

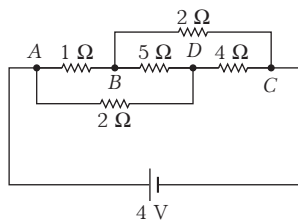


Sol. As no current flows through the middle $5\ \Omega$ resistor, the circuit represents a balanced Wheatstone bridge.

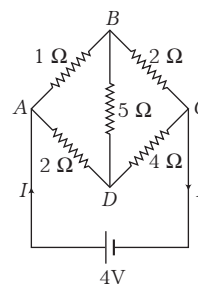
$$\therefore \frac{X}{18} = \frac{2}{6}$$

$$\text{or } X = \frac{2}{6} \times 18 = 6\ \Omega$$

Example 3.57 Calculate the current drawn from the battery by the network of resistors shown in figure.



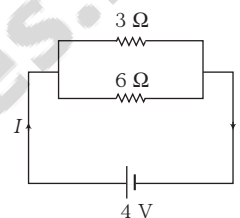
Sol. The given network is equivalent to the circuit shown in figure below



Now, the circuit represents a balanced Wheatstone bridge.

$$\therefore \frac{1}{2} = \frac{2}{4}$$

The resistance of $5\ \Omega$ in arm BD is ineffective. The equivalent circuit reduces to the circuit shown in figure.



$$\text{Equivalent resistance, } R_{eq} = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

$\therefore$ Current drawn from the battery,

$$I = \frac{V}{R_{eq}} = \frac{4}{2} = 2\text{ A}$$

Meter bridge

A meter bridge is **slide wire bridge** or **Carey Foster bridge**. It is an instrument that works on the principle of Wheatstone bridge.

It consists of a straight and uniform wire along a meter scale (AC) and by varying the taping point B as shown in Fig. 3.31, the bridge is balanced.

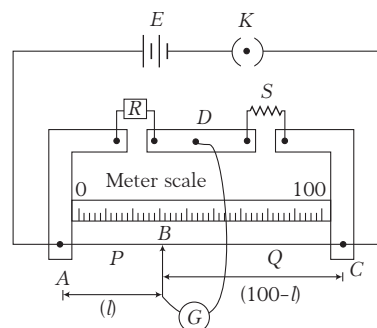


Fig. 3.31 Measuring unknown resistance using meter bridge

∴ At balancing situation of bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \frac{l}{100-l} = \frac{R}{S}$$

$$\Rightarrow S = \frac{100-l}{l} \times R$$

Applications of meter bridge

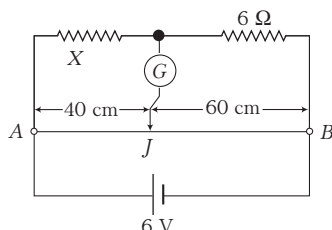
(i) It is used to measure an unknown resistance by

$$\text{using, } S = \frac{R(100-l)}{l}$$

(ii) To compare the two unknown resistances by using,

$$\frac{R}{S} = \frac{l}{100-l}$$

Example 3.58 In the following circuit, a meter bridge is shown in its balanced state. The meter bridge wire has a resistance of $1 \Omega\text{-cm}$. Calculate the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.



Sol. In balanced condition, no current flows through the galvanometer.

Here, P = resistance of wire AJ = 40Ω

Q = resistance of wire JB = 60Ω

$R = X, S = 6 \Omega$

In the balanced condition,

$$\frac{P}{Q} = \frac{R}{S}$$

$$\text{or } \frac{40}{60} = \frac{X}{6}$$

$$\text{or } X = 4 \Omega$$

Total resistance of wire AB = 100Ω

Total resistance of resistances X and 6Ω connected in series
 $= 4 + 6 = 10 \Omega$

This series combination is in parallel with wire AB .

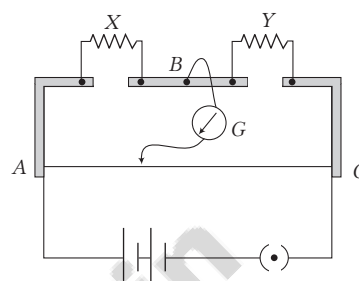
$$\therefore \text{Equivalent resistance} = \frac{10 \times 100}{10 + 100} = \frac{100}{11} \Omega$$

emf of the battery = 6 V

∴ Current drawn from the battery,

$$I = \frac{\text{emf}}{\text{resistance}} = \frac{6}{100/11} = 0.66 \text{ A}$$

Example 3.59 The given figure shows the experimental set up of a meter bridge. The null point is found to be 60 cm away from the end A with X and Y in position as shown. When a resistance of 15Ω is connected in series with Y , then the null point is found to shift by 10 cm towards the end A of the wire. Find the position of null point, if a resistance of 30Ω were connected in parallel with Y .



Sol. In first case,

$$\frac{X}{Y} = \frac{60}{40} \quad \text{or} \quad \frac{X}{Y} = \frac{3}{2} \quad \dots(i)$$

In second case,

$$\frac{X}{Y+15} = \frac{50}{50} = 1 \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{X}{Y} \times \frac{Y+15}{X} = \frac{3}{2} \times 1$$

or

$$1 + \frac{15}{Y} = \frac{3}{2}$$

or

$$Y = 30 \Omega$$

$$X = \frac{3}{2}Y = \frac{3}{2} \times 30 = 45 \Omega$$

When a resistance of 30Ω is connected in parallel with Y , then the resistance in the right gap becomes

$$Y' = \frac{30Y}{30+Y} = \frac{30 \times 30}{30+30} = 15 \Omega$$

Suppose the null point occurs at $l \text{ cm}$ from end A .

Then,

$$\frac{X}{15} = \frac{l}{100-l}$$

or

$$\frac{45}{15} = \frac{l}{100-l}$$

or

$$300 - 3l = l$$

or

$$4l = 300$$

or

$$l = 75 \text{ cm}$$

Potentiometer

The potentiometer is an instrument that can be used to measure the emf or the internal resistance of an unknown source. It is a device which does not draw any current through the circuit to measure the potential difference.

Hence, it is equivalent to an ideal voltmeter. It also has a number of other useful applications.

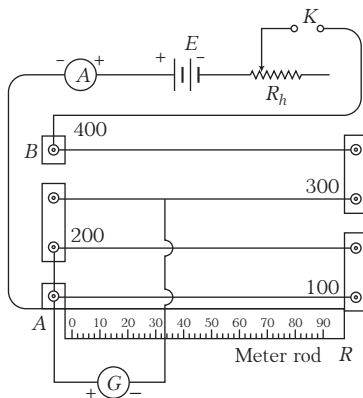


Fig. 3.32 Potentiometer

Principle of potentiometer

The potentiometer works on the principle that, when a constant amount of current flows through a wire of uniform cross-section and composition, then the potential drop across the wire is directly proportional to its length, *i.e.*

$$V \propto l \Rightarrow V = kl \quad \dots(i)$$

where, k is the constant of proportionality.

Also, by Ohm's law,

$$V = IR = \rho \frac{l}{A} I \quad \dots(ii)$$

If the current I is constant, then $\frac{\rho I}{A}$ would be constant.

So, comparing Eqs. (i) and (ii), we will have

$$\Rightarrow k = \rho \frac{I}{A}$$

k is also known as **potential gradient**, which is the potential drop per unit length of the potentiometer wire.

$$\text{i.e. } k = \frac{V}{l}$$

The SI unit of potential gradient is Vm^{-1} and CGS unit is Vcm^{-1} .

Applications of potentiometer

Some important applications of potentiometer are given below.

(i) To compare the emfs of two cells

Consider two cells of emfs E_1 and E_2 is to be compared. The positive terminals of both the cells are connected to terminal A of potentiometer and the negative terminals of both cells are connected to terminals 1 and 2 of a 2-way key, while its common terminal is connected to a jockey J through a galvanometer G . A battery of emf E , ammeter A ,

rheostat R_h and 1-way key K are connected between the terminals A and B of the potentiometer.

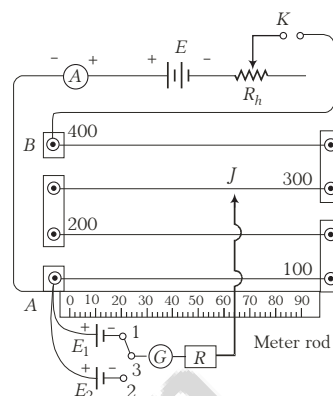


Fig. 3.33 Circuit for comparing emf's of two cells

Now, to compare emfs of two cells having a constant current passing through the wire between terminals A and B, the current is kept constant by using rheostat. If the plug is put in the gap between terminals 1 and 3 of 2-way key, then the emf E_1 of the cell is given by

$$E_1 = (xl_1) I \quad \dots(i)$$

where, x = resistance per unit length of potentiometer wire ($\because l_1$ = balancing length)

Now, when the key is put in the gap between terminals 2 and 3 after removing it from the gap between 1 and 3, then the emf E_2 is given by

$$E_2 = (xl_2) I \quad (\because l_2 = \text{balancing length}) \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

(ii) To measure internal resistance of a cell

Now, to find the internal resistance r of a cell of emf E , let E' be emf of the battery. A constant current I is maintained through the potentiometer wire with the help of rheostat.

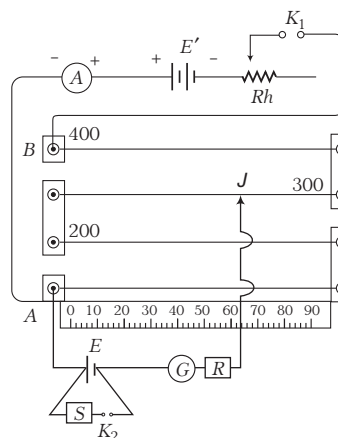


Fig. 3.34 Circuit for determining internal resistance of cell

Now, the plug in key K_2 is kept out and the jockey J is moved on the potentiometer wire to balance the emf E of the cell, whose internal resistance r is to be determined. Suppose l_1 be the balancing length of the potentiometer wire between point A and jockey J . If x is resistance per unit length of wire, then emf of cell is given by

$$E = xl_1 I \quad \dots(i)$$

Introduce some resistance, says S from the resistance box S and now put in the plug key K_2 . The potential difference V between the two terminals of the cell is given by

$$V = xl_2 I \quad \dots(ii)$$

[$\because l_2 = \text{balancing length}$]

On dividing Eq. (i) by Eq. (ii), we have

$$\frac{E}{V} = \frac{l_1}{l_2}$$

The internal resistance of the cell is given by $r = \left(\frac{E}{V} - 1 \right) S$

Now, substituting values of E/V in above equation, we get

$$\Rightarrow r = \left(\frac{l_1}{l_2} - 1 \right) S$$

where, S is the resistance of the resistance box.

Note

- (i) The sensitiveness of potentiometer means the smallest potential difference that can be measured with its help.
- (ii) A potentiometer can also be used to compare unknown resistances and to calibrate a voltmeter or an ammeter.
- (iii) A balance point is obtained on the potentiometer wire, if the fall of potential along the potentiometer wire due to driving cell is greater than the emf of the cell to be balanced.

Example 3.60 A potentiometer wire is 10 m long and has a resistance of 18Ω . It is connected to a battery of emf 5V and internal resistance 2Ω . Calculate the potential gradient along the wire.

Sol. Given, $l = 10 \text{ m}$, $R = 18 \Omega$, $E = 5 \text{ V}$ and $r = 2 \Omega$

Current through the potentiometer wire,

$$I = \frac{E}{R + r} = \frac{5}{18 + 2} = \frac{5}{20} = \frac{1}{4} \text{ A}$$

$$\therefore \text{Potential gradient} = \frac{IR}{l} = \frac{1}{4} \times \frac{18}{10} = 0.45 \text{ V m}^{-1}$$

Example 3.61 A cell can be balanced against 110 cm and 100 cm of potentiometer wire respectively, when in open circuit and when short circuited through a resistance of 10Ω . Find the internal resistance of the cell.

Sol. Given, $l_1 = 110 \text{ cm}$, $l_2 = 100 \text{ cm}$, $R = 10 \Omega$ and $r = ?$

$$\therefore r = \left(\frac{l_1 - l_2}{l_2} \right) R$$

$$\Rightarrow r = \frac{110 - 100}{100} \times 10 = 1 \Omega$$

Example 3.62 In a potentiometer arrangement, a cell of emf 2.25 V gives a balance point at 30.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 60.0 cm , then what is the emf of the second cell?

Sol. Given, $E_1 = 2.25 \text{ V}$, $l_1 = 30.0 \text{ cm}$,

$$l_2 = 60.0 \text{ cm and } E_2 = ?$$

As we know that in case of potentiometer, the potential gradient remains constant.

So,

$$E \propto l$$

$\therefore$

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

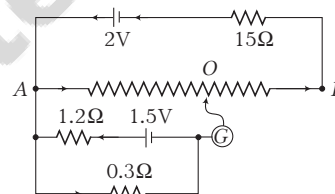
$\Rightarrow$

$$\frac{2.25}{E_2} = \frac{30.0}{60.0}$$

$\therefore$

$$E_2 = \frac{2.25 \times 60}{30} = 4.5 \text{ V}$$

Example 3.63 AB is 1 m long uniform wire of 10Ω resistance. Other data are as shown in figure. Calculate (i) potential gradient along AB and (ii) length AO , when galvanometer shows no deflection.



Sol. (i) Total resistance of the primary circuit

$$= 15 + 10 = 25 \Omega \text{ and emf} = 2 \text{ V}$$

$\therefore$ Current in the wire AB ,

$$I = \frac{2}{25} = 0.08 \text{ A}$$

Potential difference across the wire AB

$$= \text{Current} \times \text{Resistance of wire } AB$$

$$= 0.08 \times 10 = 0.8 \text{ V}$$

Potential gradient

$$= \frac{\text{Potential difference}}{\text{Length}} = \frac{0.8}{100} = 0.008 \text{ V cm}^{-1}$$

(ii) Resistance of secondary circuit $= 1.2 + 0.3 = 1.5 \Omega$

$$\text{emf} = 1.5 \text{ V}$$

$$\text{Current in the secondary circuit} = \frac{1.5}{1.5} = 1.0 \text{ A}$$

The same current is flowing in 0.3Ω resistor.

Potential difference between points A and O

$$= \text{Potential difference across } 0.3 \Omega \text{ resistor in the zero deflection condition}$$

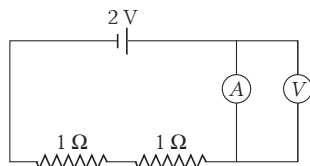
$$= \text{Current} \times \text{Resistance} = 1.0 \times 0.3 = 0.3 \text{ V}$$

$$\text{Length } AO = \frac{\text{Potential difference}}{\text{Potential gradient}}$$

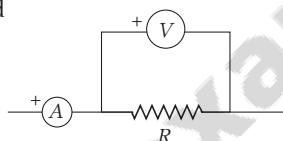
$$= \frac{0.3 \text{ V}}{0.008 \text{ V cm}^{-1}} = 37.5 \text{ cm}$$

CHECK POINT 3.4

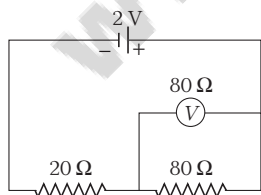
1. In the circuit shown, A and V are ideal ammeter and voltmeter, respectively. Reading of the voltmeter will be



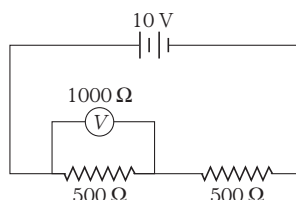
- (a) 2 V (b) 1 V (c) 0.5 V (d) zero
2. The net resistance of a voltmeter should be large to ensure that
- it does not get overheated
 - it does not draw emissive current
 - it can measure large potential difference
 - it does not appreciably change the potential difference to be measured
3. Two galvanometers A and B require 3 mA and 6 mA respectively, to produce the same deflection of 10 divisions. Then,
- A is more sensitive than B
 - B is more sensitive than A
 - both A and B are equally sensitive
 - sensitiveness of B is twice that of A
4. An ammeter A , a voltmeter V and a resistance R are connected as shown in the figure. If the voltmeter reading is 1.6 V and the ammeter reading is 0.4 A, then R is



- (a) equal to 4 Ω
 (b) greater than 4 Ω
 (c) less than 4 Ω
 (d) between 3 Ω and 4 Ω
5. In the following circuit, the emf of the cell is 2 V and the internal resistance is negligible. The resistance of the voltmeter is 80 Ω . The reading of the voltmeter will be

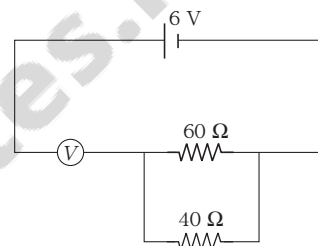


- (a) 0.80 V (b) 1.60 V (c) 1.33 V (d) 2.00 V
6. What is the reading of voltmeter in the figure?



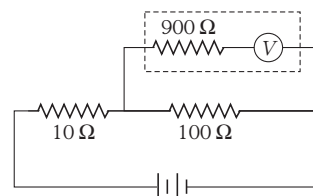
- (a) 3 V (b) 2 V
 (c) 5 V (d) 4 V

7. A galvanometer of 25 Ω and having full scale deflection for a current of 10 mA is changed into voltmeter of range 100 V by connecting a resistance R in series with the galvanometer. The resistance R (in ohm) is
- 10000
 - 975
 - 10025
 - 9975
8. An ammeter and a voltmeter are joined in series to a cell. Their readings are A and V , respectively. If a resistance is now joined in parallel with the voltmeter, then
- both A and V will decrease
 - both A and V will increase
 - A will decrease, V will increase
 - A will increase, V will decrease
9. In the circuit shown in the figure, the voltmeter reading is



- (a) 2.4 V (b) 3.4 V
 (c) 4.0 V (d) 6.0 V

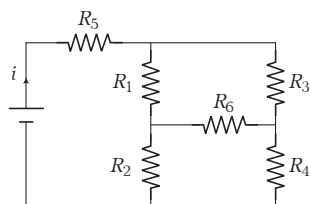
10. To send 10% of the main current through a moving coil galvanometer of resistance 99 Ω , the shunt required is
- 9.9 Ω
 - 10 Ω
 - 11 Ω
 - 9 Ω
11. The potential difference across the 100 Ω resistance in the circuit is measured by a voltmeter of 900 Ω resistance. The percentage error made in reading the potential difference is



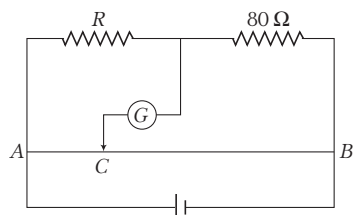
- (a) $\frac{10}{9}$ (b) 0.1 (c) 1.0 (d) 10.0

12. A microammeter has a resistance of 100 Ω and a full scale range of 50 μ A. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combinations.
- 50 V range with 10 k Ω resistance in series
 - 10 V range with 200 k Ω resistance in series
 - 10 mA range with 1 Ω resistance in parallel
 - None of the above
13. The percentage error in measuring resistance with a meter bridge can be minimised by adjusting the balancing point close to
- 20 cm
 - 50 cm
 - 80 cm
 - 100 cm

14. When an additional resistance of $1980\ \Omega$ is connected in series with a voltmeter, then the scale division reads 100 times larger value. Resistance of the voltmeter is
 (a) $10\ \Omega$ (b) $20\ \Omega$
 (c) $30\ \Omega$ (d) $40\ \Omega$
15. In the given circuit, it is observed that the current I is independent of the value of the resistance R_6 . Then, the resistance values must satisfy

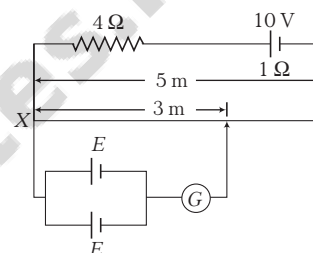


- (a) $R_1 R_2 R_3 = R_3 R_4 R_5$
 (b) $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$
 (c) $R_1 R_4 = R_2 R_3$
 (d) $R_1 R_3 = R_2 R_4 = R_5 R_6$
16. AB is a wire of uniform resistance. The galvanometer G shows no current when the length $AC = 20\text{ cm}$ and $CB = 80\text{ cm}$. The resistance R is equal to



- (a) $320\ \Omega$ (b) $8\ \Omega$ (c) $20\ \Omega$ (d) $40\ \Omega$

17. A potentiometer is used for the comparison of emf of two cells E_1 and E_2 . For cell E_1 , the deflection point is obtained at 20 cm and for E_2 , the deflection point is obtained at 30 cm . The ratio of their emfs will be
 (a) $2/3$ (b) $3/2$
 (c) 1 (d) 2
18. In a potentiometer experiment, the galvanometer shows no deflection, when a cell is connected across 60 cm of the potentiometer wire. If the cell is shunted by a resistance of $6\ \Omega$, the balance is obtained across 50 cm of the wire. The internal resistance of the cell is
 (a) $0.5\ \Omega$ (b) $0.6\ \Omega$
 (c) $1.2\ \Omega$ (d) $1.5\ \Omega$
19. A resistance of $4\ \Omega$ and a wire of length 5 m and resistance $5\ \Omega$ are joined in series and connected to a cell of emf 10 V and internal resistance $1\ \Omega$. A parallel combination of two identical cells is balanced across 3 m of the wire. The emf E of each cell is



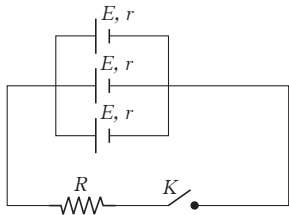
- (a) 1.5 V (b) 3.0 V
 (c) 0.67 V (d) 1.33 V
20. Potentiometer wire of length 1 m is connected in series with $490\ \Omega$ resistance and 2 V battery. If 0.2 mV/cm is the potential gradient, then the resistance of the potentiometer wire is
 (a) $4.9\ \Omega$ (b) $7.9\ \Omega$ (c) $5.9\ \Omega$ (d) $6.9\ \Omega$

Chapter Exercises

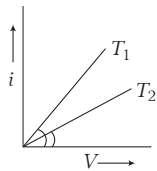
(A) Taking it together

Assorted questions of the chapter for advanced level practice

- The terminal potential difference of a cell is greater than its emf when it is
 - being discharged
 - open circuit
 - being charged
 - being either charged or discharged
- For measurement of potential difference, potentiometer is preferred in comparison to voltmeter because
 - potentiometer is more sensitive than voltmeter
 - the resistance of potentiometer is less than voltmeter
 - potentiometer is cheaper than voltmeter
 - potentiometer does not take current from the circuit
- What is immaterial for an electric fuse wire?
 - Its specific resistance
 - Its radius
 - Its length
 - Current flowing through it
- Conductivity increases in the order of
 - Al, Ag, Cu
 - Al, Cu, Ag
 - Cu, Al, Ag
 - Ag, Cu, Al
- By mistake, a voltmeter is connected in series and an ammeter in parallel. When the circuit is switched on
 - Only the ammeter will be damaged
 - Only the voltmeter will be damaged
 - Both ammeter and voltmeter will be damaged
 - Neither the ammeter nor the voltmeter will be damaged
- If E is the emf of a cell of internal resistance r and external resistance R , then potential difference across R is given as
 - $V = E/(R + r)$
 - $V = E$
 - $V = E/(1 + r/R)$
 - $V = E/(1 + R/r)$
- When n cells are joined in parallel combination as shown, the strength of the current i is given by



 - $\frac{nE}{R + nr}$
 - $\frac{E}{R + (r/n)}$
 - $\frac{E}{r + Rn}$
 - None of these
- A student has 10 resistors of resistance r each. The minimum resistance made by him from given resistors is
 - $10r$
 - $\frac{r}{10}$
 - $\frac{r}{100}$
 - $\frac{r}{5}$
- A wire has resistance $12\ \Omega$. It is bent in the form of a circle. The effective resistance between the two points on any diameter is equal to
 - $12\ \Omega$
 - $6\ \Omega$
 - $3\ \Omega$
 - $24\ \Omega$
- A steady current (i) is flowing through a conductor of uniform cross-section. Any segment of the conductor has
 - zero charge
 - only positive charge
 - only negative charge
 - charge proportional to current i
- There are n cells, each of emf E and internal resistance r , connected in series with an external resistance R . One of the cells is wrongly connected, so that it sends current in the opposite direction. The current flowing in the circuit is
 - $\frac{(n-1)E}{(n+1)r + 2}$
 - $\frac{(n-1)E}{nr + R}$
 - $\frac{(n-2)E}{nr + R}$
 - $\frac{(n-2)E}{(n-2)r + R}$
- The maximum power dissipated in an external resistance R , when connected to a cell of emf E and internal resistance r , will be
 - $\frac{E^2}{r}$
 - $\frac{E^2}{2r}$
 - $\frac{E^2}{3r}$
 - $\frac{E^2}{4r}$
- The current-voltage graph for a given metallic wire at two different temperatures T_1 and T_2 is shown in the figure. The temperatures T_1 and T_2 are related as



 - $T_1 > T_2$
 - $T_1 < T_2$
 - $T_1 = T_2$
 - $T_1 > 2T_2$

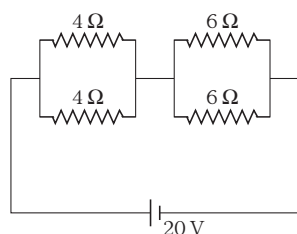
- 14** Which of the following characteristics of electrons determines the current in a conductor?
[NCERT Exemplar]

(a) Drift velocity only
(b) Thermal velocity only
(c) Both drift velocity and thermal velocity
(d) Neither drift nor thermal velocity

- 15** An ammeter and a voltmeter of resistance R are connected in series to an electric cell of negligible internal resistance. Their readings are A and V , respectively. If another resistance R is connected in parallel with the voltmeter, then potential across
- (a) Both A and V will increase
(b) Both A and V will decrease
(c) A will decrease and V will increase
(d) A will increase and V will decrease

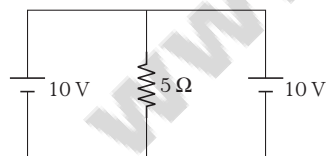
- 16** The resistance of a wire is $10\ \Omega$. Its length is increased by 10% by stretching. The new resistance will be
- (a) $12\ \Omega$ (b) $1.2\ \Omega$ (c) $13\ \Omega$ (d) $11\ \Omega$

- 17** Four resistances are connected in a circuit as shown in the given figure. The electric current flowing through $4\ \Omega$ and $6\ \Omega$ resistance is respectively



(a) 2 A and 4 A (b) 1 A and 2 A
(c) 1 A and 1 A (d) 2 A and 2 A

- 18** Current through the $5\ \Omega$ resistor is



(a) 2A (b) 4A
(c) zero (d) 1A

- 19** A cell which has an emf of 1.5 V is connected in series with an external resistance of $10\ \Omega$. If the potential difference across the cell is 1.25 V, then the internal resistance of the cell (in Ω) is
- (a) 2 (b) 0.25 (c) 1.5 (d) 0.3

- 20** A piece of wire of resistance $4\ \Omega$ is bent through 180° at its mid-point and the two halves are twisted together, then the resistance is
- (a) $8\ \Omega$ (b) $1\ \Omega$ (c) $2\ \Omega$ (d) $5\ \Omega$

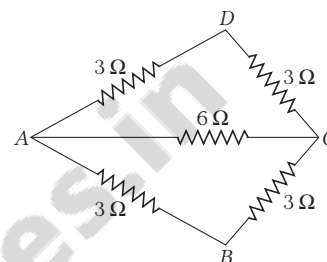
- 21** A wire of resistance R is divided into 10 equal parts. These parts are connected in parallel, the equivalent resistance of such connection will be

(a) $0.01 R$ (b) $0.1 R$
(c) $10 R$ (d) $100 R$

- 22** Three resistors each of $2\ \Omega$ are connected together in a triangular shape. The resistance between any two vertices will be

(a) $4/3\ \Omega$ (b) $3/4\ \Omega$ (c) $3\ \Omega$ (d) $6\ \Omega$

- 23** The effective resistance between the points A and B in the figure is



(a) $5\ \Omega$ (b) $2\ \Omega$ (c) $3\ \Omega$ (d) $4\ \Omega$

- 24** Two resistances are joined in parallel whose resultant is $\frac{6}{8}\ \Omega$. One of the resistance wire is broken

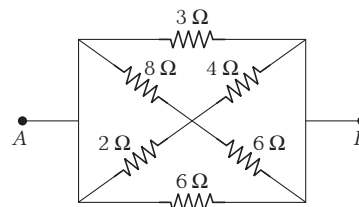
and the effective resistance becomes $2\ \Omega$. Then, the resistance in ohm of the wire that got broken was

(a) $3/5$ (b) 2 (c) $6/5$ (d) 3

- 25** A wire of resistance $9\ \Omega$ is broken in two parts. The length ratio being 1 : 2. The two pieces are connected in parallel. The net resistance will be

(a) $2\ \Omega$ (b) $3\ \Omega$ (c) $4\ \Omega$ (d) $6\ \Omega$

- 26** In the network shown, the equivalent resistance between A and B is



(a) $\frac{4}{3}\ \Omega$ (b) $\frac{3}{4}\ \Omega$ (c) $\frac{24}{17}\ \Omega$ (d) $\frac{17}{24}\ \Omega$

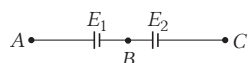
- 27** To send 10% of main current through a moving coil galvanometer of resistance $99\ \Omega$, shunt required is

(a) $9\ \Omega$ (b) $11\ \Omega$
(c) $10\ \Omega$ (d) $9.9\ \Omega$

- 28** A 1250 W heater operates at 115 V. What is the resistance of the heating coil?

(a) $1.6\ \Omega$ (b) $13.5\ \Omega$
(c) $1250\ \Omega$ (d) $10.6\ \Omega$

- 29** The resistance of a wire at 20°C is $20\ \Omega$ and at 500°C is $60\ \Omega$. At which temperature its resistance will be $25\ \Omega$?
 (a) 50°C (b) 60°C (c) 70°C (d) 80°C
- 30** A conducting wire of cross-sectional area $1\ \text{cm}^2$ has 3×10^{23} charge carriers per metre³. If wire carries a current $24\ \text{mA}$, then drift velocity of carriers is
 (a) $5 \times 10^{-2}\ \text{ms}^{-1}$ (b) $0.5\ \text{ms}^{-1}$
 (c) $5 \times 10^{-3}\ \text{ms}^{-1}$ (d) $5 \times 10^{-6}\ \text{ms}^{-1}$
- 31** At room temperature, copper has free electron density of $8.4 \times 10^{28}\ \text{m}^{-3}$. The electron's drift velocity in a copper conductor of cross-sectional area of $10^{-6}\ \text{m}^2$ and carrying a current of $5.4\ \text{A}$, will be
 (a) $4\ \text{ms}^{-1}$ (b) $0.4\ \text{ms}^{-1}$
 (c) $4\ \text{cms}^{-1}$ (d) $0.4\ \text{mms}^{-1}$
- 32** A metal rod of length $10\ \text{cm}$ and a rectangular cross-section of $1\ \text{cm} \times (1/2)\ \text{cm}$ is connected to a battery across opposite faces. The resistance will be
 [NCERT Exemplar]
 (a) maximum when the battery is connected across $1\ \text{cm} \times (1/2)\ \text{cm}$ faces
 (b) maximum when the battery is connected across $10\ \text{cm} \times (1)\ \text{cm}$ faces
 (c) maximum when the battery is connected across $10\ \text{cm} \times (1/2)\ \text{cm}$ faces
 (d) same irrespective of the three faces
- 33** Two cells of emfs approximately $5\ \text{V}$ and $10\ \text{V}$ are to be accurately compared using a potentiometer of length $400\ \text{cm}$.
 [NCERT Exemplar]
 (a) The battery that runs the potentiometer should have voltage of $8\ \text{V}$
 (b) The battery of potentiometer can have a voltage of $15\ \text{V}$ and R adjusted so that the potential drop across the wire slightly exceeds $10\ \text{V}$
 (c) The first portion of $50\ \text{cm}$ of wire itself should have a potential drop of $10\ \text{V}$
 (d) Potentiometer is usually used for comparing resistances and not voltages
- 34** The resistivity of a potentiometer wire is $40 \times 10^{-8}\ \Omega\text{-m}$ and its area of cross-section is $8 \times 10^{-6}\ \text{m}^2$. If $0.2\ \text{A}$ current is flowing through the wire, the potential gradient will be
 (a) $10^{-2}\ \text{V/m}$ (b) $10^{-1}\ \text{V/m}$
 (c) $3.2 \times 10^{-2}\ \text{V/m}$ (d) $1\ \text{V/m}$
- 35** Two cells of emfs E_1 and E_2 ($E_1 > E_2$) are connected as shown in figure.



When a potentiometer is connected between A and B , the balancing length of the potentiometer wire is

$300\ \text{cm}$. On connecting the same potentiometer between A and C , the balancing length is $100\ \text{cm}$.

The ratio $\frac{E_1}{E_2}$ is

- (a) $3 : 1$ (b) $1 : 3$
 (c) $2 : 3$ (d) $3 : 2$
- 36** A voltmeter of resistance $998\ \Omega$ is connected across a cell of emf $2\ \text{V}$ and internal resistance $2\ \Omega$. The potential difference across the voltmeter is
 (a) $1.99\ \text{V}$ (b) $3.5\ \text{V}$
 (c) $5\ \text{V}$ (d) $6\ \text{V}$
- 37** A wire $50\ \text{cm}$ long and $1\ \text{mm}^2$ in cross-section carries a current of $4\ \text{A}$ when connected to a $2\ \text{V}$ battery. The resistivity of the wire is
 (a) $2 \times 10^{-7}\ \Omega\text{-m}$ (b) $5 \times 10^{-7}\ \Omega\text{-m}$
 (c) $4 \times 10^{-6}\ \Omega\text{-m}$ (d) $1 \times 10^{-6}\ \Omega\text{-m}$
- 38** Three resistances P, Q, R each of $2\ \Omega$ and an unknown resistance S form the four arms of a Wheatstone's bridge circuit. When a resistance of $6\ \Omega$ is connected in parallel to S , the bridge gets balanced. What is the value of S ?
 (a) $2\ \Omega$ (b) $3\ \Omega$
 (c) $6\ \Omega$ (d) $1\ \Omega$
- 39** A $2\ \text{V}$ battery, a $990\ \Omega$ resistor and a potentiometer of $2\ \text{m}$ length, all are connected in series. If the resistance of potentiometer wire is $10\ \Omega$, then the potential gradient of the potentiometer wire is
 (a) $0.05\ \text{Vm}^{-1}$ (b) $0.5\ \text{Vm}^{-1}$
 (c) $0.01\ \text{Vm}^{-1}$ (d) $0.1\ \text{Vm}^{-1}$
- 40** The electron with charge $q = (1.6 \times 10^{-19}\ \text{C})$ moves in an orbit of radius $5 \times 10^{-11}\ \text{m}$ with a speed of $2.2 \times 10^6\ \text{ms}^{-1}$, around an atom. The equivalent current is
 (a) $1.12 \times 10^{-6}\ \text{A}$ (b) $1.12 \times 10^{-3}\ \text{A}$
 (c) $1.12 \times 10^{-9}\ \text{A}$ (d) $1.12\ \text{A}$
- 41** A potentiometer having the potential gradient of $2\ \text{mV/cm}$ is used to measure the difference of potential across a resistance of $10\ \Omega$ in same circuit. If a length of $50\ \text{cm}$ of the potentiometer wire is required to get the null point, then the current passing through the $10\ \Omega$ resistor is (in mA)
 (a) 1 (b) 2 (c) 5 (d) 10
- 42** The n rows each containing m cells in series are joined in parallel. Maximum current is taken from this combination across an external resistance of $3\ \Omega$. If the total number of cells used are 24 and internal resistance of each cell is $0.5\ \Omega$, then
 (a) $m = 8, n = 3$ (b) $m = 6, n = 4$
 (c) $m = 12, n = 2$ (d) $m = 2, n = 12$

- 43** A 100 V voltmeter of internal resistance $20\text{ k}\Omega$ in series with a high resistance R is connected to a 110 V line. The voltmeter reads 5 V, the value of R is

(a) $210\text{ k}\Omega$ (b) $315\text{ k}\Omega$ (c) $420\text{ k}\Omega$ (d) $440\text{ k}\Omega$

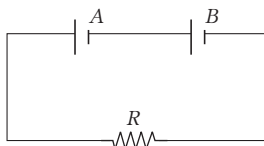
- 44** A cell supplies a current i_1 through a resistance R_1 and a current i_2 through a resistance R_2 . The internal resistance of this cell is

(a) $R_2 - R_1$ (b) $\frac{i_1 R_2 - i_2 R_1}{i_1 - i_2}$
 (c) $\frac{i_2 R_2 - i_1 R_1}{i_1 - i_2}$ (d) $\left(\frac{i_1 + i_2}{i_1 - i_2}\right) \sqrt{R_1 R_2}$

- 45** Out of five resistances of $R\ \Omega$ each, 3 are connected in parallel and are joined to the rest 2 in series. Find the resultant resistance.

(a) $(3/7)R\ \Omega$ (b) $(7/3)R\ \Omega$
 (c) $(7/8)R\ \Omega$ (d) $(8/7)R\ \Omega$

- 46** Two batteries A and B each of emf 2 V are connected in series to an external resistance $R = 1\ \Omega$. If the internal resistance of battery A is $1.9\ \Omega$ and that of B is $0.9\ \Omega$. What is the potential difference between the terminals of battery A ?

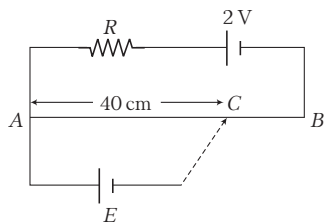


(a) 2 V (b) 3.8 V (c) zero (d) 4.8 V

- 47** For a cell of emf 2 V , a balance is obtained for 50 cm of the potentiometer wire. If the cell is shunted by a $2\ \Omega$ resistor, the balance is obtained across 40 cm of the wire. Find the internal resistance of the cell.

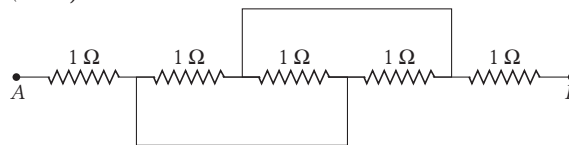
(a) $0.25\ \Omega$ (b) $0.50\ \Omega$
 (c) $0.80\ \Omega$ (d) $1.00\ \Omega$

- 48** AB is a potentiometer wire of length 100 cm and its resistance is $10\ \Omega$. It is connected in series with a resistance $R = 40\ \Omega$ and a battery of emf 2 V and negligible internal resistance. If a source of unknown emf E is balanced by 40 cm length of the potentiometer wire, the value of E is



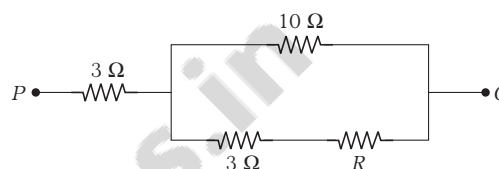
(a) 0.8 V (b) 1.6 V
 (c) 0.08 V (d) 0.16 V

- 49** Equivalent resistance between the points A and B is (in Ω)



(a) $\frac{1}{5}$ (b) $\frac{5}{4}$ (c) $\frac{7}{3}$ (d) $\frac{7}{2}$

- 50** In the circuit shown here, what is the value of the unknown resistance R , so that the total resistance of the circuit between points P and Q is also equal to R ?

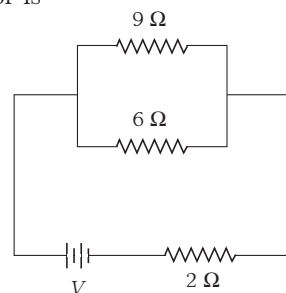


(a) $3\ \Omega$ (b) $\sqrt{39}\ \Omega$ (c) $\sqrt{69}\ \Omega$ (d) $10\ \Omega$

- 51** Two wires of same metal have the same length but their cross-sections are in the ratio $3 : 1$. They are joined in series. The resistance of the thicker wire is $10\ \Omega$. The total resistance of the combination will be

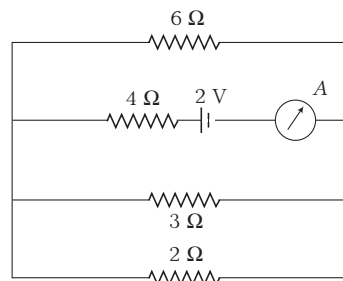
(a) $40\ \Omega$ (b) $\frac{40}{3}\ \Omega$ (c) $\frac{5}{2}\ \Omega$ (d) $100\ \Omega$

- 52** If power dissipated in the $9\ \Omega$ resistor in the circuit shown is 36 W , the potential difference across the $2\ \Omega$ resistor is



(a) 8 V (b) 10 V (c) 2 V (d) 4 V

- 53** The reading of the ammeter in the following figure will be



(a) 0.8 A (b) 0.6 A (c) 0.4 A (d) 0.2 A

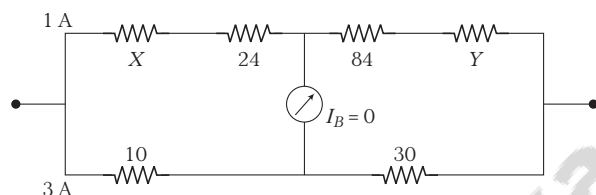
- 54** A wire of length 100 cm is connected to a cell of emf 2 V and negligible internal resistance. The resistance of the wire is $3\ \Omega$. The additional resistance required to produce a potential drop of 1 milli volt per cm is

(a) $60\ \Omega$ (b) $47\ \Omega$
(c) $57\ \Omega$ (d) $35\ \Omega$

- 55** Two uniform wires *A* and *B* are of same metal and have equal masses. The radius of wire *A* is twice that of wire *B*. The total resistance of *A* and *B* when connected in parallel is

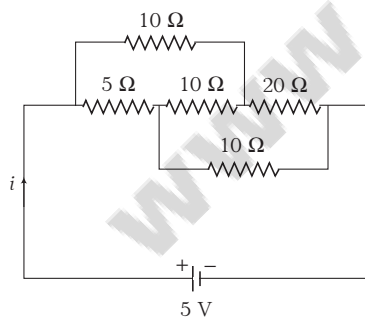
(a) $4\ \Omega$ when the resistance of wire *A* is $4.25\ \Omega$
(b) $5\ \Omega$ when the resistance of wire *A* is $4\ \Omega$
(c) $4\ \Omega$ when the resistance of wire *B* is $4.25\ \Omega$
(d) $5\ \Omega$ when the resistance of wire *B* is $4\ \Omega$

- 56** In the given circuit, the resistances are given in ohm. The current through the $10\ \Omega$ resistance is 3 A while that through the resistance *X* is 1 A. No current passes through the galvanometer. The values of the unknown resistances *X* and *Y* are respectively (in ohm)



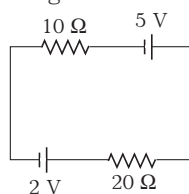
(a) 14 and 54 (b) 12 and 6
(c) 6 and 12 (d) 6 and 6

- 57** The current *i* drawn from the 5 V source will be



(a) 0.33 A (b) 0.5 A
(c) 0.67 A (d) 0.17 A

- 58** The current in the given circuit is



(a) 0.3 A (b) 0.4 A (c) 0.1 A (d) 0.2 A

- 59** Two resistors $400\ \Omega$ and $800\ \Omega$ are connected in series with a 6 V battery. The potential difference measured by voltmeter of $10\ \text{k}\Omega$ across $400\ \Omega$ resistor is

(a) 2 V (b) 1.95 V (c) 3.8 V (d) 4 V

- 60** A battery of emf *E* has an internal resistance *r*. A variable resistance *R* is connected to the terminals of the battery. A current *i* is drawn from the battery. *V* is the terminal potential difference. If *R* alone is gradually reduced to zero, which of the following best describes *i* and *V*?

(a) *i* approaches zero, *V* approaches *E*
(b) *i* approaches $\frac{E}{r}$, *V* approaches zero
(c) *i* approaches $\frac{E}{r}$, *V* approaches *E*
(d) *i* approaches infinity, *V* approaches *E*

- 61** Two resistances are connected in two gaps of a meter bridge. The balance point is 20 cm from the zero end. A resistance of $15\ \Omega$ is connected in series with the smaller of the two. The null point shifts to 40 cm. The value of the smaller resistance in ohm is

(a) 3 (b) 6
(c) 9 (d) 12

- 62** A battery of four cells in series, each having an emf of 1.4 V and an internal resistance of $2\ \Omega$ is to be used to charge a small 2 V accumulator of negligible internal resistance. What is the charging current?

(a) 0.1 A (b) 0.2 A (c) 0.3 A (d) 0.45 A

- 63** The length of a wire of a potentiometer is 100 cm, and the emf of its cell is *E* volt. It is employed to measure the emf of a battery whose internal resistance is $0.5\ \Omega$. If the balance point is obtained at *l* = 30 cm from the positive end, the emf of the battery is

(a) $\frac{30E}{100.5}$
(b) $\frac{30E}{100 - 0.5}$
(c) $\frac{30(E - 0.5i)}{100}$, where *i* is the current in the potentiometer wire
(d) $\frac{30E}{100}$

- 64** When a resistance of $100\ \Omega$ is connected in series with a galvanometer of resistance *R*, then its range is *V*. To double its range, a resistance of $1000\ \Omega$ is connected in series. Find the value of *R*.

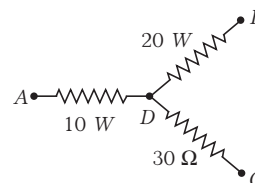
(a) $700\ \Omega$ (b) $800\ \Omega$
(c) $900\ \Omega$ (d) $100\ \Omega$

- 65** Two wires of the same material but of different diameters carry the same current i . If the ratio of their diameters is 2 : 1, then the corresponding ratio of their mean drift velocities will be
 (a) 4 : 1 (b) 1 : 1
 (c) 1 : 2 (d) 1 : 4
- 66** Two bulbs consume same power when operated at 200 V and 300 V, respectively. When these bulbs are connected in series across a DC source of 500 V, then
 (a) ratio of potential differences across them is 3/2
 (b) ratio of potential differences across them is 9/4
 (c) ratio of powers consumed across them is 4/9
 (d) ratio of powers consumed across them is 2/3
- 67** A factory is served by a 220 V supply line. In a circuit protected by a fuse marked 10 A, the maximum number of 100 W lamps in parallel that can be turned on, is
 (a) 11 (b) 22
 (c) 33 (d) 66
- 68** A tap supplies water at 22°C, a man takes 1 L of water per min at 37°C from the geyser. The power of geyser is
 (a) 525 W (b) 1050 W (c) 1775 W (d) 2100 W
- 69** The mean free path of electrons in a metal is 4×10^{-8} m. The electric field which can give an average 2 eV energy to an electron in the metal will be in unit of Vm^{-1} ?
 (a) 8×10^7 (b) 5×10^{-11} (c) 8×10^{-11} (d) 5×10^7
- 70** You are given two resistances R_1 and R_2 . By using them singly, in series and in parallel, you can obtain four resistances of 1.5 Ω , 2 Ω , 6 Ω and 8 Ω . The values of R_1 and R_2 are
 (a) 1 Ω , 7 Ω (b) 1.5 Ω , 6.5 Ω
 (c) 3 Ω , 5 Ω (d) 2 Ω , 6 Ω
- 71** A potentiometer having the potential gradient of 2 mVcm^{-1} is used to measure the difference of potential across a resistance of 10 Ω . If a length of 50 cm of the potentiometer wire is required to get the null points, then the current passing through 10 Ω resistor is (in mA)
 (a) 1 (b) 2 (c) 5 (d) 10
- 72** A galvanometer of resistance 50 Ω is connected to a battery of 3 V along with a resistance of 2950 Ω in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 20 divisions, the resistance in series should be
 (a) 4450 Ω (b) 5050 Ω
 (c) 5550 Ω (d) 6050 Ω

- 73** When a galvanometer is shunted by resistance S , then its current capacity increases n times. If the same galvanometer is shunted by another resistance S' , then its current capacity will increase by n' , which is given by

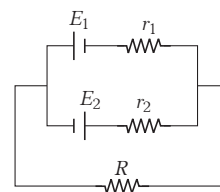
$$(a) \frac{(n+1)S}{S'} \quad (b) \frac{S(n-1)+S'}{S'} \quad (c) \frac{n+S}{S'} \quad (d) \frac{S(n-1)-S'}{S'}$$

- 74** The tungsten filaments of two electric bulbs are of the same length. If one of them gives 25 W power and the other 60 W power, then
 (a) Both the filaments are of same thickness
 (b) 25 W bulb has thicker filament
 (c) 60 W bulb has thicker filament
 (d) Both the filaments have same cross-sectional area
- 75** Three unequal resistors in parallel are equivalent to a resistance 1 Ω . If two of them are in the ratio 1 : 2 and if no resistance value is fractional, then the largest of the three resistances (in ohms) is
 (a) 4 (b) 6 (c) 8 (d) 12
- 76** In the circuit given here, the points A, B and C are at 70 V, zero, 10 V, respectively. Then,



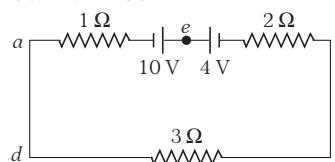
- (a) the point D will be at a potential of 60 V
 (b) the point D will be at a potential of 20 V
 (c) currents in the paths AD, DB and DC are in the ratio of 1 : 2 : 3
 (d) currents in the paths AD, DB and DC are in the ratio of 3 : 2 : 1

- 77** The current in the resistance R will be zero if



- (a) $E_1 r_1 = E_2 r_2$ (b) $\frac{E_1}{r_1} = \frac{E_2}{r_2}$
 (c) $(E_1 + E_2) r_1 = E_1 r_2$ (d) $(E_1 - E_2) r_1 = E_2 r_1$

- 78** The magnitude and direction of the current in the circuit shown will be



- (a) $\frac{7}{3}$ A from a to e (b) $\frac{7}{3}$ A from b to e
(c) 1 A from b to e (d) 1 A from a to e

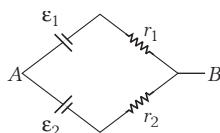
79 Consider a current carrying wire (current I) in the shape of a circle. Note that as the current progresses along the wire, the direction of J (current density) changes in an exact manner, while the current I remains unaffected. The agent that is essentially responsible for

[NCERT Exemplar]

- (a) source of emf
(b) electric field produced by charges accumulated on the surface of wire
(c) the charges just behind a given segment of wire which push them just the right way by repulsion
(d) the charges ahead

80. Two batteries of emf ϵ_1 and ϵ_2 ($\epsilon_2 > \epsilon_1$) and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.

[NCERT Exemplar]



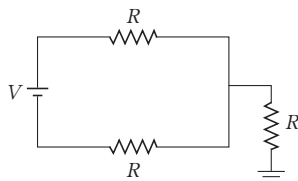
- (a) The equivalent emf ϵ_{eq} of the two cells is between ϵ_1 and ϵ_2 , i.e. $\epsilon_1 < \epsilon_{eq} < \epsilon_2$
(b) The equivalent emf ϵ_{eq} is smaller than ϵ_1
(c) The ϵ_{eq} is given by $\epsilon_{eq} = \epsilon_1 + \epsilon_2$ always
(d) ϵ_{eq} is independent of internal resistances r_1 and r_2

81 A resistance R is to be measured using a meter bridge, student chooses the standard resistance S to be 100Ω . He finds the null point at $l_1 = 2.9$ cm. He is told to attempt to improve the accuracy. Which of the following is a useful way?

[NCERT Exemplar]

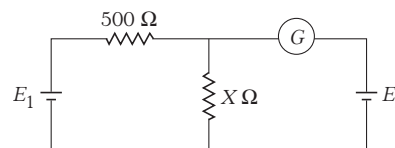
- (a) He should measure l_1 more accurately
(b) He should change S to 1000Ω and repeat the experiment
(c) He should change S to 3Ω and repeat the experiment
(d) He should given up hope of a more accurate measurement with a meter bridge

82 The current drawn from the battery shown in the figure is



- (a) $\frac{V}{R}$ (b) $\frac{V}{2R}$ (c) $\frac{2V}{R}$ (d) $\frac{3V}{2R}$

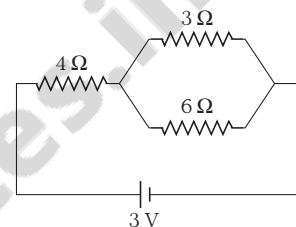
83 In the below circuit, the battery E_1 has an emf of 12 V and zero internal resistance. While the battery E_2 has an emf of 2 V.



If the galvanometer G reads zero, then the value of the resistance X in ohms is

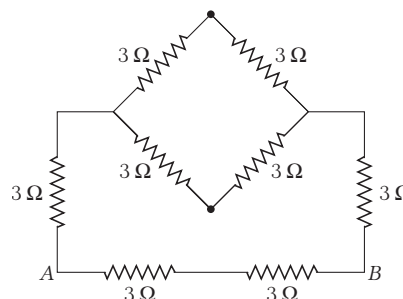
- (a) 250 (b) 100
(c) 50 (d) 200

84 The potential drop across the 3Ω resistor is



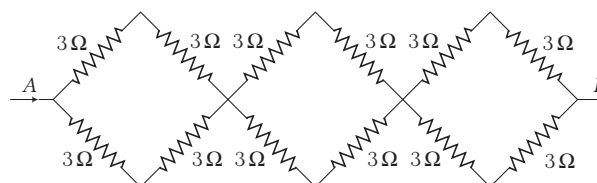
- (a) 1 V (b) 1.5 V
(c) 2 V (d) 3 V

85 Equivalent resistance between A and B will be



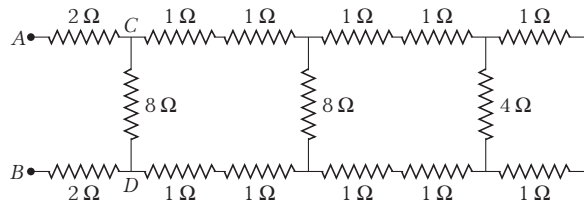
- (a) 2Ω (b) 18Ω
(c) 6Ω (d) 3.6Ω

86 In the network of resistors shown in the figure, the equivalent resistance between A and B is



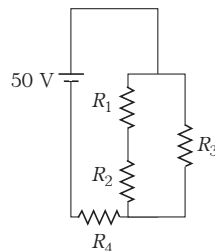
- (a) 54Ω (b) 18Ω
(c) 36Ω (d) 9Ω

- 87 In the figure shown, the total resistance between A and B is



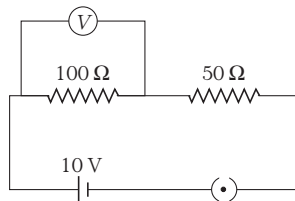
- (a) $12\ \Omega$ (b) $4\ \Omega$ (c) $6\ \Omega$ (d) $8\ \Omega$

- 88 The potential difference in volt across the resistance R_3 in the circuit shown in figure, is ($R_1 = 15\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 30\ \Omega$, $R_4 = 35\ \Omega$)



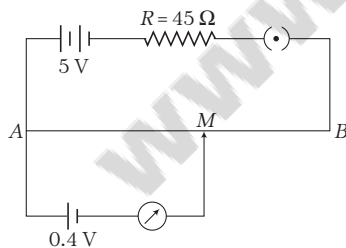
- (a) 5
(b) 7.5
(c) 15
(d) 12.5

- 89 In the given circuit, the voltmeter records 5 V. The resistance of the voltmeter in Ω is



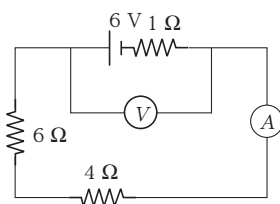
- (a) 200 (b) 100 (c) 10 (d) 50

- 90 In given figure, the potentiometer wire AB has a resistance of $5\ \Omega$ and length 10 m. The balancing length AM for the emf of 0.4 V is



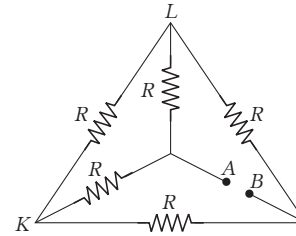
- (a) 0.4 m (b) 4 m (c) 0.8 m (d) 8 m

91. In the circuit shown below, the readings of the ammeter and voltmeter are



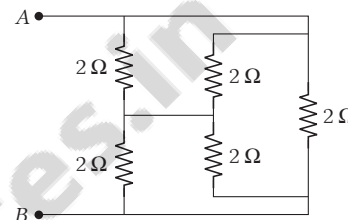
- (a) 6 A, 60 V (b) 0.6 A, 6 V
(c) 6 A, 6 V (d) $(6/11)\text{ A}$, $(6/11)\text{ V}$

- 92 Each of the resistance in the network shown in the figure is equal to R . The resistance between the terminals A and B is



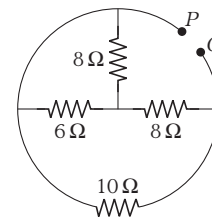
- (a) R (b) $5R$ (c) $3R$ (d) $6R$

- 93 Find the equivalent resistance across AB.



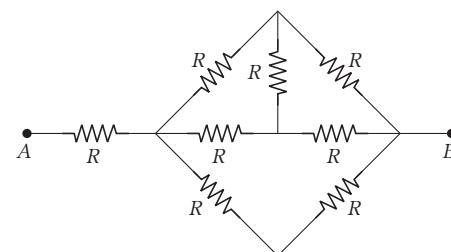
- (a) $1\ \Omega$ (b) $2\ \Omega$ (c) $3\ \Omega$ (d) $4\ \Omega$

- 94 The equivalent resistance between P and Q in the figure is approximately



- (a) $6\ \Omega$ (b) $5\ \Omega$ (c) $7.5\ \Omega$ (d) $20\ \Omega$

- 95 In the given network of resistances, the effective resistance between A and B is

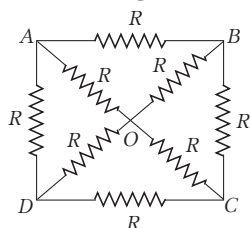


- (a) $\frac{5}{3}R$ (b) $\frac{8}{3}R$ (c) $5R$ (d) $8R$

- 96 A source of emf $E = 15\text{ V}$ and having negligible internal resistance is connected to a variable resistance, so that the current in the circuit increases with time as $i = 1.2t + 3$. Then, the total charge that will flow in first five second will be

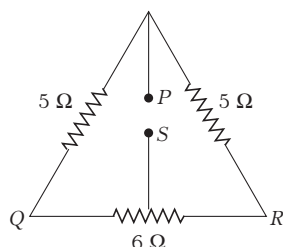
- (a) 10 C (b) 20 C
(c) 30 C (d) 40 C

- 97 The effective resistance between points A and C for the network shown in figure is



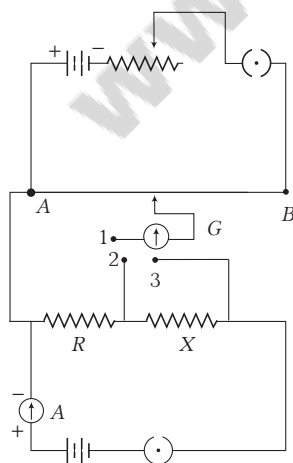
- (a) $\frac{2}{3}R$ (b) $\frac{3}{2}R$ (c) $2R$ (d) $\frac{1}{2}R$

- 98 Three resistances 5Ω , 5Ω and 6Ω are connected as shown in figure. If the point S divides the resistance 6Ω into two equal halves, the resistance between points P and S is



- (a) 11Ω (b) 8Ω (c) 6Ω (d) 4Ω

- 99 A potentiometer circuit is set up as shown. The potential gradient across the potentiometer wire, is k volt/cm and the ammeter, present in the circuit, reads 1.0 A when two way key is switched off. The balance points, when the key between the terminals (i) 1 and 2 (ii) 1 and 3, is plugged in, are found to be at lengths l_1 cm and l_2 cm, respectively. The magnitudes of the resistors R and X in ohm, are then, respectively equal to



- (a) $k(l_2 - l_1)$ and kl_2 (b) kl_1 and $k(l_2 - l_1)$
(c) $k(l_2 - l_1)$ and kl_2 (d) kl_1 and kl_2

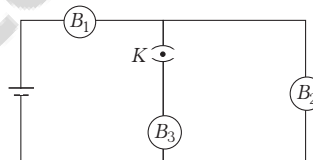
- 100 An electric immersion heater of 1.08 kW is immersed in water. After the water has reached a temperature of 100°C , how much time will be required to produce 100 g of steam?

- (a) 50 s (b) 420 s
(c) 105 s (d) 210 s

- 101 A moving coil galvanometer is converted into an ammeter reading upto 0.03 A by connecting a shunt of resistance $4r$ across it and into an ammeter reading upto 0.06 A when a shunt of resistance r is connected across it. What is the maximum current which can be sent through this galvanometer, if no shunt is used?

- (a) 0.01 A (b) 0.02 A
(c) 0.03 A (d) 0.04 A

- 102 B_1 , B_2 and B_3 are the three identical bulbs connected to a battery of steady emf with key K closed. What happens to the brightness of the bulbs B_1 and B_2 when the key is opened?

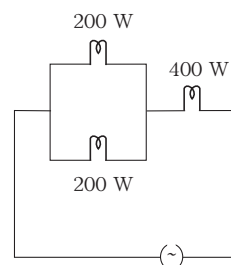


- (a) Brightness of the bulb B_1 increases and that of B_2 decreases
(b) Brightness of the bulbs B_1 and B_2 increases
(c) Brightness of the bulb B_1 decreases and B_2 increases
(d) Brightness of the bulbs B_1 and B_2 decreases

- 103 The scale of a galvanometer of resistance 100Ω contains 25 divisions. It gives a deflection of one division on passing a current of 4×10^{-4} A. The resistance (in ohm) to be added to it, so that it may become a voltmeter of range 2.5 V is

- (a) 150 (b) 170
(c) 110 (d) 220

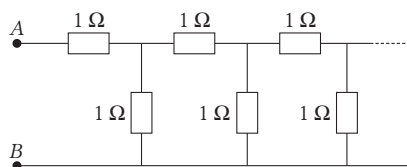
- 104 Three electric bulbs of 200 W, 200 W and 400 W are connected as shown in figure. The resultant power of the combination is



- (a) 800 W (b) 400 W
(c) 200 W (d) 600 W

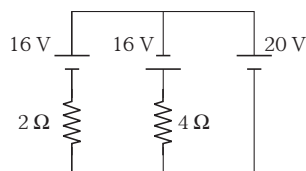
- 105** Two electric bulbs rated 50 W and 100 V are glowing at full power, when used in parallel with a battery of emf 120 V and internal resistance $10\ \Omega$. The maximum number of bulbs that can be connected in the circuit when glowing at full power, is
 (a) 6 (b) 4 (c) 2 (d) 8

- 106** The equivalent resistance between points A and B of an infinite network of resistances, each of $1\ \Omega$, connected as shown, is



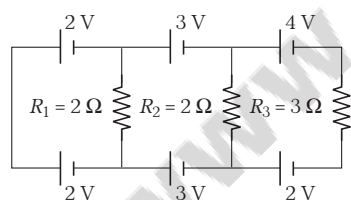
- (a) infinite (b) $2\ \Omega$
 (c) $\frac{1+\sqrt{5}}{2}\ \Omega$ (d) zero

- 107** In the given figure, the current through the 20 V battery is



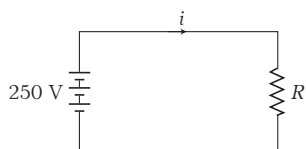
- (a) 11 A (b) 12 A (c) 7 A (d) 14 A

- 108** The current in resistance R_3 in the given circuit is



- (a) 1 A (b) $\frac{2}{3}$ A (c) 0.25 A (d) 0.50 A

- 109** In the circuit shown in figure, the resistance R has a value that depends on the current. Specifically R is $20\ \Omega$ when i is zero and the amount of increase in resistance is numerically equal to one-half of the current.

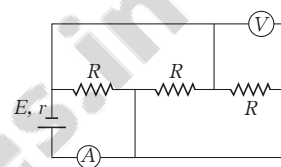


What is the value of current i in circuit?

- (a) 15 A (b) 10 A
 (c) 20 A (d) 5 A

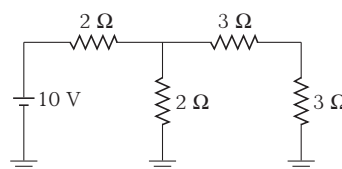
- 110** The charge flowing in a conductor varies with time as $Q = at - bt^2$, then the current
 (a) reaches a maximum and then decreases
 (b) falls to zero after $t = \frac{a}{2b}$
 (c) changes at a rate of $(-2b)$
 (d) Both (b) and (c)

- 111** In the circuit shown in figure, ammeter and voltmeter are ideal. If $E = 4\text{ V}$, $R = 9\ \Omega$ and $r = 1\ \Omega$, then readings of ammeter and voltmeter are



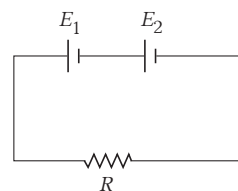
- (a) 1 A, 3 V (b) 2 A, 3 V
 (c) 3 A, 4 V (d) 4 A, 4 V

- 112** In the circuit shown, the current in $3\ \Omega$ resistance is



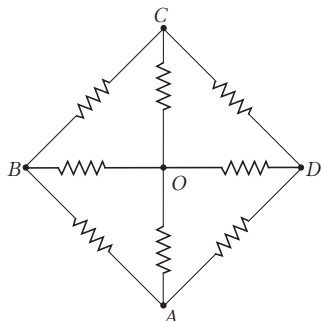
- (a) 1 A (b) $\frac{1}{7}$ A
 (c) $\frac{5}{7}$ A (d) $\frac{15}{7}$ A

- 113** Under what conditions, current passing through the resistance R can be increased by short circuiting the battery of emf E_2 ? The internal resistances of the two batteries are r_1 and r_2 , respectively.



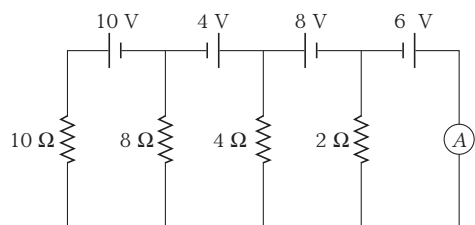
- (a) $E_2 r_1 > E_1 (R + r_2)$
 (b) $E_1 r_2 < E_2 (R + r_1)$
 (c) $E_2 r_2 < E_1 (R + r_2)$
 (d) $E_1 r_2 > E_2 (R + r_1)$

- 114** In the arrangement shown, the magnitude of each resistance is $1\ \Omega$. The equivalent resistance between O and A is given by



- (a) $\frac{14}{13}\ \Omega$ (b) $\frac{3}{4}\ \Omega$ (c) $\frac{2}{3}\ \Omega$ (d) $\frac{5}{6}\ \Omega$

- 115** Find the reading of the ideal ammeter connected in the given circuit. Assume that the cells have negligible internal resistance.

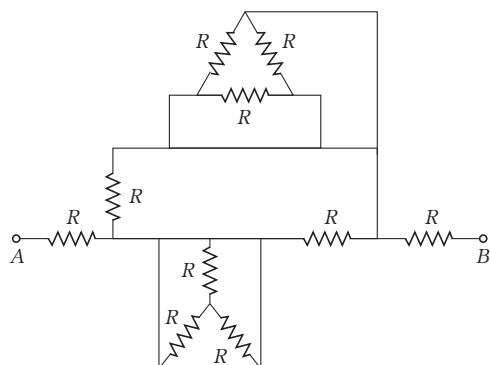


- (a) 0.8 A (b) 0.25 A (c) 1.95 A (d) 1.0 A

- 116** A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 V, the resistance (in ohms) needed to be connected in series with the coil will be
(a) 99995 (b) 9995 (c) 10^3 (d) 10^5

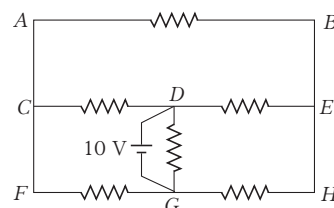
- 117** It takes 16 min to boil some water in an electric kettle. Due to some defect it becomes necessary to remove 10% turns of heating coil of the kettle. After repairs, how much time will it take to boil the same mass of water?
(a) 17.7 min (b) 14.4 min (c) 20.9 min (d) 13.9 min

- 118** Equivalent resistance between points A and B is



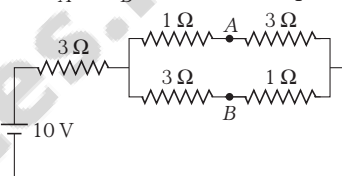
- (a) $3R$ (b) $4R$
(c) $\frac{5R}{2}$ (d) $\frac{7R}{2}$

- 119** All resistances shown in circuit are $2\ \Omega$ each. The current in the resistance between D and E is



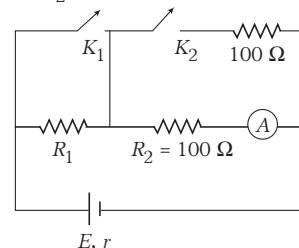
- (a) 5 A (b) 2.5 A (c) 1 A (d) 7.5 A

- 120** A battery of emf 10 V is connected to a group of resistances as shown in figure. The potential difference $V_A - V_B$ between the points A and B is



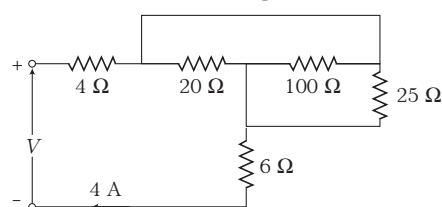
- (a) -2 V (b) 2 V
(c) 5 V (d) $\frac{20}{11}$ V

- 121** In the circuit shown, when key K_1 is closed, then the ammeter reads I_0 whether K_2 is open or closed. But when K_1 is open the ammeter reads $I_0/2$, when K_2 is closed.



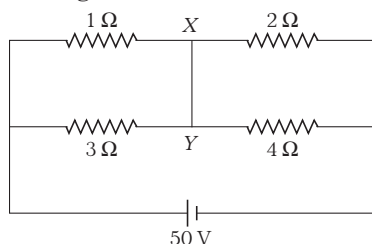
- Assuming that ammeter resistance is much less than R_2 , the values of r and R_1 in ohms are
(a) 100, 50 (b) 50, 100
(c) 0, 100 (d) 0, 50

- 122** In the circuit shown in figure, V must be



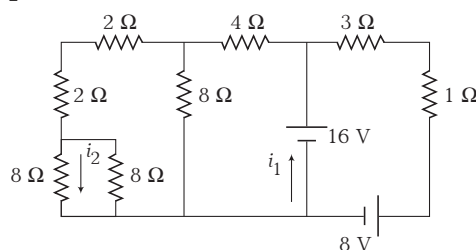
- (a) 50 V (b) 80 V
(c) 100 V (d) 1290 V

123 Current through wire XY of circuit shown is



- (a) 1 A (b) 4 A
(c) 2 A (d) 3 A

124 In the circuit shown in figure, the ratio of currents i_1/i_2 is



- (a) 2 (b) 8 (c) 0.5 (d) 4

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1 Assertion If a current flows through a wire of non-uniform cross-section, potential difference per unit length of wire is same throughout the length of wire.

Reason Current through the wire is same at all cross-sections.

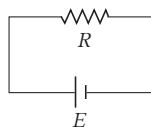
2 Assertion In our houses when we start switching on different light buttons, main current goes on increasing.

Reason Different connections in houses are in parallel. When we start switching on different light buttons, then net resistance of the circuit decreases. Therefore, main current increases.

3 Assertion Resistance of an ammeter is less than the resistance of a milliammeter.

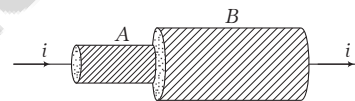
Reason Value of shunt required in case of ammeter is more than a milliammeter.

4 Assertion In the circuit shown in figure, battery is ideal. If a resistance R_0 is connected in parallel with R , then power across R will increase.



Reason Current drawn from the battery will increase.

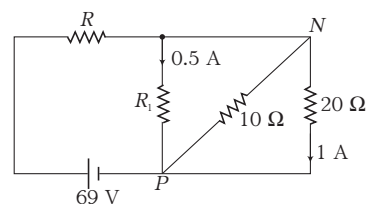
5 Assertion Two resistance wires shown in figure are of same material. They have equal length. More heat is generated in wire A.



Reason In series $H \propto R$ and resistance of wire A is more.

Statement based questions

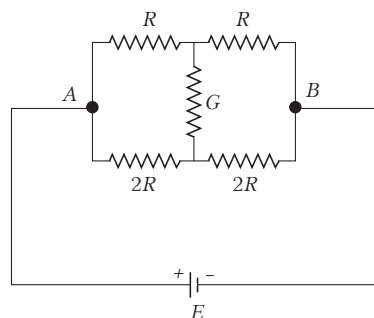
1 For the circuit shown in the figure, which of the following statement is incorrect?



- (a) The current through NP is 0.5 A.
(b) The value of $R_1 = 40 \Omega$.
(c) The value of $R = 14 \Omega$.
(d) The potential difference across $R = 49$ V.

2 Two non-ideal batteries are connected in series. Consider the following statements.
I. The equivalent emf is larger than either of the two emfs.
II. The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Each of I and II is correct
(b) I is correct but II is wrong
(c) II is correct but I is wrong
(d) Each of I and II is wrong

- 3 Consider the following statements regarding the network shown in the figure.



- I. The equivalent resistance of the network between points A and B is independent of value of G .
 - II. The equivalent resistance of the network between points A and B is $4/3 R$.
 - III. The current through G is zero.
- Which of the above statement(s) is/are true?
- (a) Only I
 - (b) Only II
 - (c) Both II and III
 - (d) I, II and III

- 4 Two non-ideal unidentical batteries are connected in parallel with positive terminals. Consider the following statements.

- I. The equivalent emf is smaller than either of the two emfs.
 - II. The equivalent internal resistance is smaller than either of the two internal resistances.
- (a) Both I and II are correct
 - (b) I is correct but II is wrong
 - (c) II is correct but I is wrong
 - (d) Both I and II are wrong

- 5 If $V_A - V_B = V_0$ and the value of each resistance is R , then



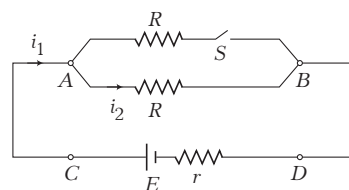
- I. net resistance between AB is $\frac{R}{2}$.
- II. net resistance between AB is $\frac{3R}{5}$.
- III. current through CD is $\frac{V_0}{R}$.
- IV. current through EF is $\frac{2V_0}{3R}$.

Which of the statement(s) is/are correct?

- (a) I and II
- (b) I and III
- (c) II and III
- (d) All of these

Match the columns

- 1 In the circuit diagram shown in figure, match the following two columns when switch S is closed. Choose the correct option from codes given below.

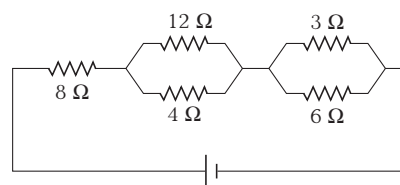


Column I	Column II
A. Current (i_1)	(p) will increase
B. Current (i_2)	(q) will decrease
C. Potential difference across A and B	(r) will remain same
D. Potential difference across C and D	(s) may increase or decrease. It will depend on value of r

Codes

A	B	C	D	A	B	C	D
(a) p	q	q	q	(b) q	p	r	r
(c) p	s	r	q	(d) s	s	r	p

- 2 In the circuit diagram shown in figure, potential difference across 3Ω resistance is 20 V . Then, match the following two columns and choose the correct option from codes given below.



Column I	Column II
A. Potential difference across 6Ω resistance	(p) 30 V
B. Potential difference across 4Ω resistance	(q) 40 V
C. Potential difference across 12Ω resistance	(r) 20 V
D. Potential difference across 8Ω resistance	(s) 80 V

Codes

A	B	C	D
(a) p	r	s	q
(b) p	r	r	s
(c) r	p	p	s
(d) r	q	s	p

(C) Medical entrances' gallery

Collection of questions asked in NEET and various medical entrance exams

- 1 A charged particle having drift velocity of $7.5 \times 10^{-4} \text{ ms}^{-1}$ in an electric field of $3 \times 10^{-10} \text{ Vm}^{-1}$, has a mobility (in $\text{m}^2 \text{V}^{-1} \text{s}^{-1}$) of [NEET 2020]

(a) 2.5×10^6 (b) 2.5×10^{-6}
(c) 2.25×10^{-15} (d) 2.25×10^{15}

- 2 The color code of a resistance is given below



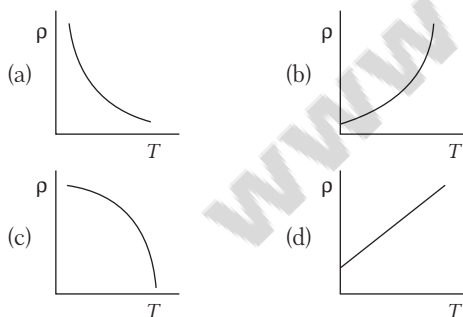
The values of resistance and tolerance respectively, are [NEET 2020]

(a) $47 \text{ k}\Omega$, 10% (b) $4.7 \text{ k}\Omega$, 5%
(c) 470Ω , 5% (d) $470 \text{ k}\Omega$, 5%

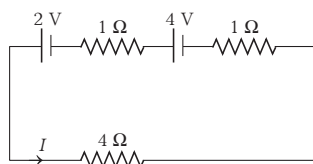
- 3 A resistance wire connected in the left gap of a metre bridge balances a 10Ω resistance in the right gap at a point which divides the bridge wire in the ratio 3 : 2. If the length of the resistance wire is 1.5 m, then the length of 1Ω of the resistance wire is [NEET 2020]

(a) $1.0 \times 10^{-1} \text{ m}$ (b) $1.5 \times 10^{-1} \text{ m}$
(c) $1.5 \times 10^{-2} \text{ m}$ (d) $1.0 \times 10^{-2} \text{ m}$

- 4 Which of the following graph represents the variation of resistivity (ρ) with temperature (T) for copper? [NEET 2020]



- 5 For the circuit shown in the figure, the current I will be [NEET 2020]

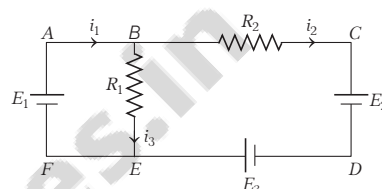


(a) 0.75 A (b) 1 A
(c) 1.5 A (d) 0.5 A

- 6 Two solid conductors are made up of same material, have same length and same resistance. One of them has a circular cross-section of area A_1 and the other one has a square cross-section of area A_2 . The ratio A_1 / A_2 is [NEET 2020]

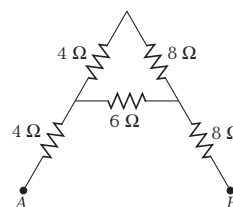
(a) 1.5 (b) 1 (c) 0.8 (d) 2

- 7 For the circuit given below, the Kirchhoff's loop rule for the loop BCDEB is given by the equation [NEET 2020]



(a) $-i_2 R_2 + E_2 - E_3 + i_3 R_1 = 0$
(b) $i_2 R_2 + E_2 - E_3 - i_3 R_1 = 0$
(c) $i_2 R_2 + E_2 + E_3 + i_3 R_1 = 0$
(d) $-i_2 R_2 + E_2 + E_3 + i_3 R_1 = 0$

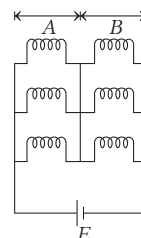
- 8 The equivalent resistance between A and B for the mesh shown in the figure is [NEET 2020]



(a) 7.2Ω (b) 16Ω (c) 30Ω (d) 4.8Ω

- 9 Six similar bulbs are connected as shown in the figure with a DC source of emf E and zero internal resistance.

The ratio of power consumption by the bulbs when (i) all are glowing and (ii) in the situation when two from section A and one from section B are glowing, will be [NEET 2019]

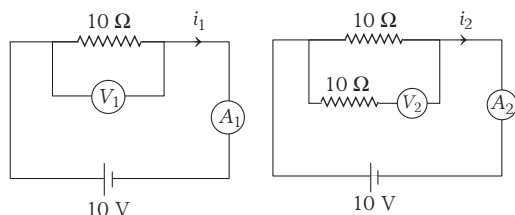


(a) 9 : 4 (b) 1 : 2 (c) 2 : 1 (d) 4 : 9

- 10** Which of the following acts as a circuit protection device? [NEET 2019]

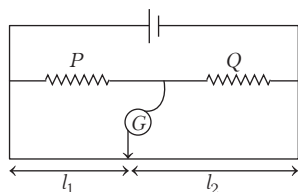
(a) Inductor (b) Switch (c) Fuse (d) Conductor

- 11** In the circuits shown below, the readings of voltmeters and the ammeters will be [NEET 2019]



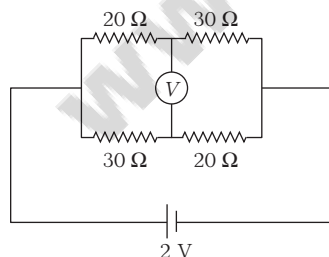
(a) $V_1 = V_2$ and $i_1 > i_2$ (b) $V_1 = V_2$ and $i_1 = i_2$
 (c) $V_2 > V_1$ and $i_1 > i_2$ (d) $V_2 > V_1$ and $i_1 = i_2$

- 12** The meter bridge shown in the balance position with $\frac{P}{Q} = \frac{l_1}{l_2}$. If we now interchange the positions of galvanometer and cell, will the bridge work? If yes, that will be balanced condition? [NEET (Odisha) 2019]



(a) Yes, $\frac{P}{Q} = \frac{l_2 - l_1}{l_2 + l_1}$ (b) No, no null point
 (c) Yes, $\frac{P}{Q} = \frac{l_2}{l_1}$ (d) Yes, $\frac{P}{Q} = \frac{l_1}{l_2}$

- 13** The reading of an ideal voltmeter in the circuit shown is [NEET (Odisha) 2019]



(a) 0.6 V (b) 0 V (c) 0.5 V (d) 0.4 V

- 14** For a wire $\frac{R}{l} = \frac{1}{2}$ and length of wire is $l = 5$ cm.

If potential difference of 1 V is applied across it, then current through wire will be [AIIMS 2019]

(a) 40 A (b) 4 A (c) 25 A (d) 2.5 A

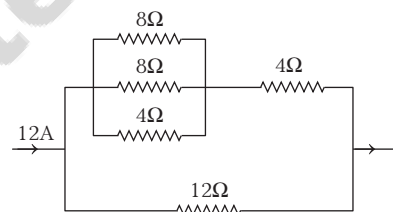
- 15** A current of 10 A is passing through a metallic wire of cross-sectional area $4 \times 10^{-6} \text{ m}^2$. If the density of the aluminium conductor is 2.7 g/cc, considering aluminium gives 1 electron per atom for conduction, then find the drift velocity of the electrons if molecular weight of aluminium is 27 g. [AIIMS 2019]

(a) $1.6 \times 10^{-4} \text{ m/s}$ (b) $3.6 \times 10^{-4} \text{ m/s}$
 (c) $2.6 \times 10^{-4} \text{ m/s}$ (d) $1.5 \times 10^{-4} \text{ m/s}$

- 16** A circuit contain two resistances R_1 and R_2 are in series. Find the ratio of input voltage to voltage of R_2 . [JIPMER 2019]

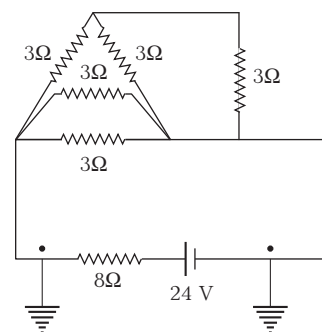
(a) $\frac{R_2}{R_1 + R_2}$ (b) $\frac{R_1 + R_2}{R_2}$
 (c) $\frac{R_1 + R_2}{R_1}$ (d) $\frac{R_1}{R_1 + R_2}$

- 17** In the given circuit, find voltage across 12Ω resistance. [JIPMER 2019]



(a) 12 V (b) 36 V (c) 72 V (d) 48 V

- 18** Find the current in the 8Ω resistance in the given circuit. [JIPMER 2019]

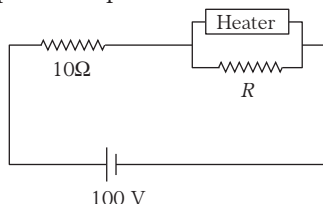


(a) 2 A (b) 3 A
 (c) 4 A (d) 5 A

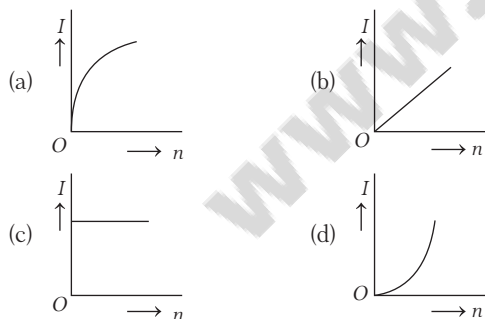
- 19** If resistivity of copper is $172 \times 10^{-8} \Omega\text{-m}$ and number of free electrons in copper is $8.5 \times 10^{28} / \text{m}^3$. Find the mobility. [JIPMER 2019]

(a) $4.25 \times 10^{-3} \text{ m}^2 / \text{C}\Omega$ (b) $6.8 \times 10^{-3} \text{ m}^2 / \text{C}\Omega$
 (c) $8.5 \times 10^{-3} \text{ m}^2 / \text{C}\Omega$ (d) $3.4 \times 10^{-3} \text{ m}^2 / \text{C}\Omega$

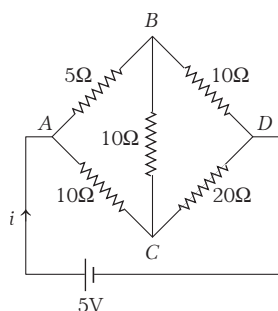
- 20** In the given circuit, if power rating of heater is 1000 W at 100 V, find the resistance R in figure so that heater produces power of 62.5 W. [JIPMER 2019]



- (a) 5 Ω (b) 7 Ω (c) 10 Ω (d) 8 Ω
- 21** A carbon resistor of (47 ± 4.7) k Ω is to be marked with rings of different colours for its identification. The colour code sequence will be [NEET 2018]
- (a) Yellow - Green - Violet - Gold
 (b) Yellow - Violet - Orange - Silver
 (c) Violet - Yellow - Orange - Silver
 (d) Green - Orange - Violet - Gold
- 22** A set of n equal resistors, of value R each, are connected in series to a battery of emf E and internal resistance R . The current drawn is I . Now, the n resistors are connected in parallel to the same battery. Then, the current drawn from battery becomes $10I$. The value of n is [NEET 2018]
- (a) 20 (b) 11 (c) 10 (d) 9
- 23** A battery consists of a variable number n of identical cells (having internal resistance r each) which are connected in series. The terminals of the battery are short-circuited and the current I is measured. Which of the graphs shows the correct relationship between I and n ? [NEET 2018]



- 24** Find current (i) in circuit shown in figure. [NEET 2018]



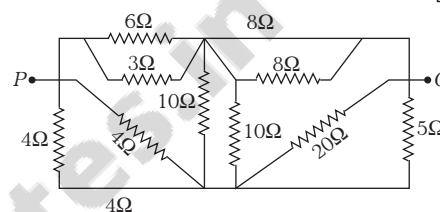
- (a) 0.5 A (b) 0.2 A
 (c) 1 A (d) 2 A

- 25 Assertion** Terminal voltage of a cell is greater than emf of cell during charging of the cell. [AIIMS 2018]

Reason The emf of a cell is always greater than its terminal voltage.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

- 26** The effective resistance between P and Q in given figure is [AIIMS 2018]



- (a) 2 Ω (b) 3 Ω (c) 5 Ω (d) 6 Ω

- 27 Assertion** Bulb generally get fused when they are switched ON or OFF.

Reason When we switch ON or OFF a circuit, current changes in it rapidly. [AIIMS 2018]

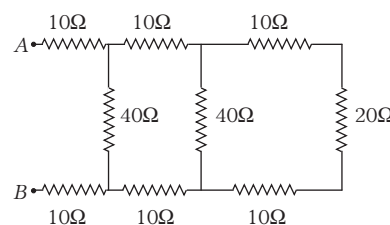
- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct and Reason is incorrect.
 (d) Assertion is incorrect and Reason is correct.
- 28** A current i is flowing through the wire of diameter d having drift velocity of electrons v_d in it. What will be new drift velocity when diameter of wire is made $d/4$? [JIPMER 2018]

- (a) $4v_d$ (b) $\frac{v_d}{4}$ (c) $16v_d$ (d) $\frac{v_d}{16}$

- 29** The resistance of a wire is R ohm. If it is melted and stretched to n times its original length, its new resistance will be [NEET 2017]

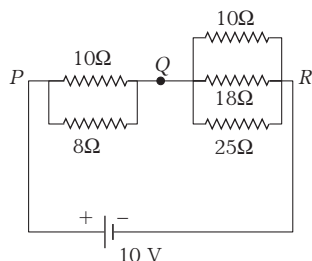
- (a) nR (b) $\frac{R}{n}$ (c) n^2R (d) $\frac{R}{n^2}$

- 30** Find the value of R_{net} between A and B . [NEET 2017]

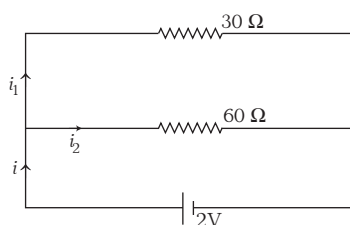


- (a) 60 Ω (b) 40 Ω (c) 70 Ω (d) 20 Ω

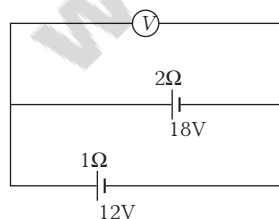
- 31 Find $V_P - V_Q$ in the circuit shown in below figure. [NEET 2017]



- (a) 6.68 V (b) 8 V (c) 4.65 V (d) 7 V
- 32 Find the value of i in shown below figure. [NEET 2017]



- (a) 0.2A (b) 0.1A (c) 0.3A (d) 0.4A
- 33 N lamps each of resistance r are fed by a machine of resistance R . If light emitted by any lamp is proportional to the square of the heat produced, prove that the most efficient way of arranging them is to place them in parallel arcs, each containing n lamps, where n is the integer nearest to [NEET 2017]
- (a) $\left(\frac{r}{NR}\right)^{3/2}$ (b) $\left(\frac{NR}{r}\right)^{1/2}$
 (c) $(NRr)^{3/2}$ (d) $(NRr)^{1/2}$
- 34 Two batteries, one of emf 18V and internal resistance 2Ω and the other of emf 12V and internal resistance 1Ω are connected as shown in figure. The voltmeter V will record a reading of [AIIMS 2017]



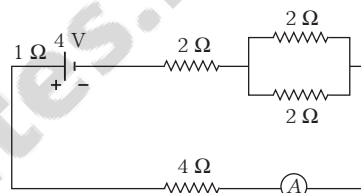
- (a) 14 V (b) 15 V (c) 18 V (d) 30 V
- 35 **Assertion** A potentiometer is preferred over that of a voltmeter for measurement of emf of a cell.
Reason Potentiometer does not draw any current from the cell. [AIIMS 2017]
- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

- (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct Reason is incorrect.
 (d) Assertion is incorrect but Reason incorrect.

- 36 You are given resistance wire of length 50 cm and a battery of negligible resistance. In which of the following cases is largest amount of heat generated? [JIPMER 2017]

- (a) When the wire is connected to the battery directly.
 (b) When the wire is divided into two parts and both the parts are connected to the battery in parallel.
 (c) When the wire is divided into four parts and all the four parts are connected to the battery in parallel.
 (d) When only half of the wire is connected to the battery.

- 37 The current passing through the ideal ammeter in the circuit given below is [KCET 2017]

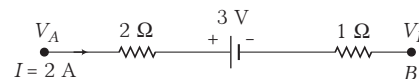


- (a) 1.25A (b) 1A (c) 0.75A (d) 0.5 A
- 38 A potentiometer wire is 100 cm long and a constant potential difference is maintained across it. Two cells are connected in series first to support one another and then in opposite direction. The balance points are obtained at 50 cm and 10 cm from the positive end of the wire in the two cases. The ratio of emfs is [NEET 2016]
- (a) 5 : 4 (b) 3 : 4 (c) 3 : 2 (d) 5 : 1

- 39 The charge flowing through a resistance R varies with time t as $Q = at - bt^2$, where a and b are positive constants. The total heat produced in R is [NEET 2016]

- (a) $\frac{a^3 R}{3b}$ (b) $\frac{a^3 R}{2b}$ (c) $\frac{a^3 R}{b}$ (d) $\frac{a^3 R}{6b}$

- 40 The potential difference ($V_A - V_B$) between the points A and B in the given figure is [NEET 2016]

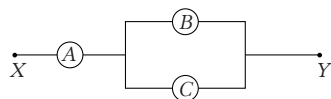


- (a) -3 V (b) +3 V (c) +6 V (d) +9 V
- 41 A filament bulb (500 W, 100 V) is to be used in a 230 V main supply. When a resistance R is connected in series, it works perfectly and the bulb consumes 500 W. The value of R is [NEET 2016]
- (a) 230 Ω (b) 46 Ω
 (c) 26 Ω (d) 13 Ω

- 42** A potentiometer wire has length 4 m and resistance $8\ \Omega$. The resistance that must be connected in series with the wire and an accumulator of emf 2 V, so as to get a potential gradient of 1 mV per cm on the wire is [CBSE AIPMT 2015]

(a) $32\ \Omega$ (b) $40\ \Omega$ (c) $44\ \Omega$ (d) $48\ \Omega$

- 43** A, B and C are voltmeters of resistance R , $1.5R$ and $3R$ respectively, as shown in the figure. When some potential difference is applied between X and Y, then the voltmeter readings are V_A , V_B and V_C , respectively. Then, [CBSE AIPMT 2015]

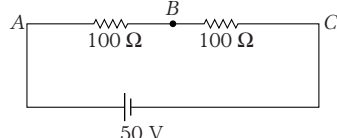


(a) $V_A = V_B = V_C$ (b) $V_A \neq V_B = V_C$
(c) $V_A = V_B \neq V_C$ (d) $V_A \neq V_B \neq V_C$

- 44** Across a metallic conductor of non-uniform cross-section, a constant potential difference is applied. The quantity which remain(s) constant along the conductor is [CBSE AIPMT 2015]

(a) current density (b) current
(c) drift velocity (d) electric field

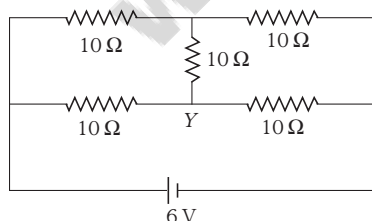
- 45** Consider the diagram shown below [AIIMS 2015]



A voltmeter of resistance $150\ \Omega$ is connected across A and B. The potential drop across B and C measured by voltmeter is

(a) 29 (b) 27 V (c) 31 V (d) 30 V

- 46** Each resistor shown in the figure has a resistance of $10\ \Omega$ and the battery has the emf 6 V. What will be the current supplied by the battery? [UK PMT 2015]



(a) 0.6 A (b) 1.2 A (c) 1.8 A (d) 0.3 A

- 47** A $1\ \Omega$ resistance in series with an ammeter is balanced by 75 cm of potentiometer wire. A standard cell of emf 1.02 V is balanced by 50 cm. The ammeter shows a reading of 1.5 A. Then, the error in ammeter reading is [EAMCET 2015]

(a) 0.03 A (b) 3A (c) 1.3 A (d) 0.3 A

- 48** The range of voltmeter is 10 V and its internal resistance is $50\ \Omega$. To convert it to a voltmeter of range 15 V, how much resistance is to be added?

(a) Add $25\ \Omega$ resistor in parallel [EAMCET 2015]
(b) Add $25\ \Omega$ resistor in series
(c) Add $125\ \Omega$ resistor in parallel
(d) Add $125\ \Omega$ resistor in series

- 49** Identify the wrong statement. [Kerala CEE 2015]

(a) Charge is a vector quantity
(b) Current is a scalar quantity
(c) Charge can be quantised
(d) Charge is additive in nature
(e) Charge is conserved

- 50** When the rate of flow of charge through a metallic conductor of non-uniform cross-section is uniform, then the quantity that remains constant along the conductor is [Kerala CEE 2015]

(a) current density (b) electric field
(c) electric potential (d) drift velocity
(e) current

- 51** The resistance of a carbon resistor of colour code Red-Red-Green-Silver is (in $k\Omega$) [Kerala CEE 2015]

(a) $2200 \pm 5\%$ (b) $2200 \pm 10\%$
(c) $220 \pm 10\%$ (d) $220 \pm 5\%$
(e) $2200 \pm 1\%$

- 52** The slope of the graph showing the variation of potential difference V on X-axis and current on Y-axis gives conductor [Kerala CEE 2015]

(a) resistance (b) resistivity
(c) reciprocal of resistance (d) conductivity
(e) impedance

- 53** Two wires of equal length and equal diameter and having resistivities ρ_1 and ρ_2 are connected in series. The equivalent resistivity of the combination is [Guj. CET 2015]

(a) $\frac{\rho_1 + \rho_2}{2}$ (b) $\rho_1 + \rho_2$
(c) $\frac{\rho_1 \rho_2}{\rho_1 + \rho_2}$ (d) $\sqrt{\rho_1 \rho_2}$

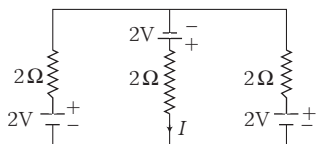
- 54** A galvanometer of resistance $50\ \Omega$ is connected to a battery of 8 V along with a resistance of $3950\ \Omega$ in series. A full scale deflection of 30 divisions is obtained in the galvanometer. In order to reduce this deflection to 15 divisions, the resistance in series should be ... Ω . [Guj. CET 2015]

(a) 1950 (b) 7900
(c) 2000 (d) 7950

- 55** Choose the correct statement. [CG PMT 2015]

(a) Kirchhoff's first law of electricity is based on conservation of charge while the second law is based on conservation of energy.

- (b) Kirchhoff's first law of electricity is based on conservation of energy while the second law is based on conservation of charge.
 (c) Kirchhoff's both laws are based on conservation of charge.
 (d) Kirchhoff's both laws are based on conservation of energy.
- 56** A metal plate weighing 750 g is to be electroplated with 0.05% of its weight of silver. If a current of 0.8 A is used, find the time (approx.) needed for depositing the required weight of silver (ECE of silver is $11.8 \times 10^{-7} \text{ kgC}^{-1}$) [CG PMT 2015]
 (a) 5 min 32 s (b) 6 min 37 s
 (c) 4 min 16 s (d) 6 min 10 s
- 57** A DC ammeter has resistance 0.1Ω and its current ranges 0-100 A. If the range is to be extended to 0-500, then the following shunt resistance will be required [CG PMT 2015]
 (a) 0.010Ω (b) 0.011Ω
 (c) 0.025Ω (d) 0.25Ω
- 58** The current I shown in the circuit is [WB JEE 2015]



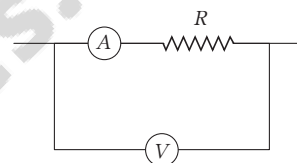
- (a) 1.33 A (b) zero (c) 2 A (d) 1 A
- 59** A metal wire of circular cross-section has a resistance R_1 . The wire is now stretched without breaking, so that its length is doubled and the density is assumed to remain the same. If the resistance of the wire now becomes R_2 , then $R_2 : R_1$ is [WB JEE 2015]
 (a) 1 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 4
- 60** Consider the combination of resistor,



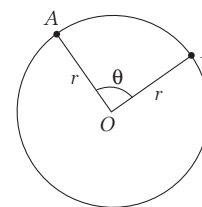
The equivalent resistance between a and b is [UP CPMT 2015]

- (a) $\frac{R}{6}$ (b) $\frac{2R}{3}$ (c) $\frac{R}{3}$ (d) $3R$
- 61** A potentiometer wire of length 100 cm has a resistance of 10Ω . It is connected in series with a resistance and a cell of emf 2V having negligible internal resistance. A source of emf 10 mV is balanced against a length of 40 cm of the potentiometer wire. The value of external resistance is [UP CPMT 2015]
 (a) 760Ω (b) 640Ω (c) 790Ω (d) 840Ω

- 62** Three resistances 2Ω , 3Ω and 4Ω are connected in parallel. The ratio of currents passing through them when a potential difference is applied across its ends will be [KCET 2015]
 (a) 5 : 4 : 3 (b) 6 : 3 : 2
 (c) 4 : 3 : 2 (d) 6 : 4 : 3
- 63** Four identical cells of emf $\mathcal{E}$ and internal resistance r are to be connected in series. Suppose, if one of the cell is connected wrongly, then the equivalent emf and effective internal resistance of the combination is [KCET 2015]
 (a) $2\mathcal{E}$ and $4r$ (b) $4\mathcal{E}$ and $4r$
 (c) $2\mathcal{E}$ and $2r$ (d) $4\mathcal{E}$ and $2r$
- 64** In the circuit shown alongside, the ammeter and the voltmeter readings are 3A and 6V, respectively. Then, the value of the resistance R is [KCET 2015]



- (a) $< 2 \Omega$ (b) 2Ω
 (c) $\geq 2 \Omega$ (d) $> 2 \Omega$
- 65** The resistance of a bulb filament is 100Ω at a temperature of 100°C . If its temperature coefficient of resistance be 0.005 per $^\circ \text{C}$, then its resistance will become 200Ω at a temperature [KCET 2015]
 (a) 500°C (b) 300°C (c) 200°C (d) 400°C
- 66** A and B are the two points on a uniform ring of radius r . The resistance of the ring is R and $\angle AOB = \theta$ as shown in the figure. The equivalent resistance between points A and B is [Guj. CET 2015]



- (a) $\frac{R(2\pi - \theta)}{4\pi}$ (b) $\frac{R\theta}{2\pi}$
 (c) $R \left(1 - \frac{\theta}{2\pi} \right)$ (d) $\frac{R}{4\pi^2} (2\pi - \theta)^2$
- 67** The resistance in the two arms of the meter bridge are 5Ω and $R \Omega$, respectively. When the resistance R is shunted with an equal resistance, then the new balance point is at $1.6 l_1$. The resistance R is [CBSE AIPMT 2014]
 (a) 10Ω (b) 15Ω (c) 20Ω (d) 25Ω

- 67** A potentiometer circuit has been set up for finding the internal resistance of a given cell. The main battery, used across the potentiometer wire, has an emf of 2.0 V and a negligible internal resistance. The potentiometer wire itself is 4 m long. When the resistance R connected across the given cell, then has values of

(i) infinity (ii) 9.5Ω

The balancing lengths of the potentiometer wire are found to be 3 m and 2.85 m, respectively.

The value of internal resistance of the cell is

[CBSE AIPMT 2014]

- (a) 0.25Ω (b) 0.95Ω
(c) 0.5Ω (d) 0.75Ω

- 69** In an ammeter, 0.2% of main current passes through the galvanometer. If resistance of galvanometer is G , then the resistance of ammeter will be

[CBSE AIPMT 2014]

- (a) $\frac{1}{499} G$ (b) $\frac{499}{500} G$ (c) $\frac{1}{500} G$ (d) $\frac{500}{499} G$

- 70** A carbon film resistor has colour code green, black, violet, gold. The value of the resistor is [KCET 2014]

- (a) $50 \text{ M}\Omega$ (b) $500 \text{ M}\Omega$
(c) $500 \pm 5\% \text{ M}\Omega$ (d) $500 \pm 10\% \text{ M}\Omega$

- 71** A uniform wire of resistance 9Ω is joined end-to-end to form a circle. Then, the resistance of the circular wire between any two diametrically points is

[Kerala CEE 2014]

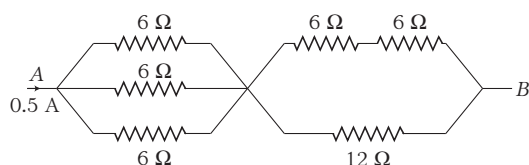
- (a) 6Ω (b) 3Ω (c) $\frac{9}{4} \Omega$ (d) $\frac{3}{2} \Omega$
(e) 1Ω

- 72** The equivalent resistance of two resistors connected in series is 6Ω and their parallel equivalent resistance is $\frac{4}{3} \Omega$. What are the values of

resistances? [KCET 2014]

- (a) $4 \Omega, 6 \Omega$ (b) $8 \Omega, 1 \Omega$
(c) $4 \Omega, 2 \Omega$ (d) $6 \Omega, 2 \Omega$

- 73** Six resistances are connected as shown in figure. If total current flowing is 0.5 A, then the potential difference $V_A - V_B$ is [EAMCET 2014]



- (a) 8 V (b) 6 V (c) 2 V (d) 4 V

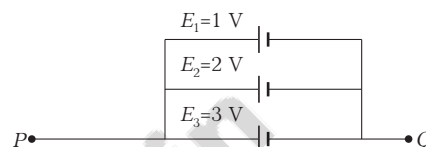
- 74** Four cells, each of emf E and internal resistance r , are connected in series across an external resistance

R . By mistake one of the cells is connected in reverse. Then, the current in the external circuit is

[WB JEE 2014]

- (a) $\frac{2E}{4r + R}$ (b) $\frac{3E}{4r + R}$ (c) $\frac{3E}{3r + R}$ (d) $\frac{2E}{3r + R}$

- 75** A circuit consists of three batteries of emf $E_1 = 1 \text{ V}$, $E_2 = 2 \text{ V}$ and $E_3 = 3 \text{ V}$ and internal resistance 1Ω , 2Ω and 1Ω respectively which are connected in parallel as shown in figure. The potential difference between points P and Q is [WB JEE 2014]



- (a) 1.0 V (b) 2.0 V
(c) 2.2 V (d) 3.0 V

- 76** Two resistors of resistances 2Ω and 6Ω are connected in parallel. This combination is then connected to a battery of emf 2 V and internal resistance 0.5Ω . What is the current flowing through the battery? [KCET 2014]

- (a) 4 A (b) $\frac{4}{3} \text{ A}$
(c) $\frac{4}{17} \text{ A}$ (d) 1 A

- 77** The dimensions of mobility of charge carriers are [Kerala CEE 2014]

- (a) $[M^{-2}T^2A]$ (b) $[M^{-1}T^2A]$
(c) $[M^{-2}T^3A]$ (d) $[M^{-1}T^3A]$
(e) $[M^{-1}T^3A^{-1}]$

- 78** The temperature coefficient of resistance of an alloy used for making resistor is [Kerala CEE 2014]

- (a) small and positive (b) small and negative
(c) large and positive (d) large and negative
(e) zero

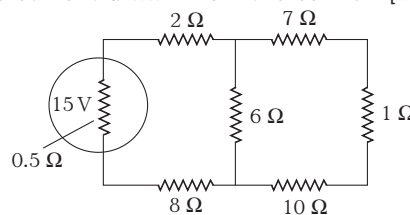
- 79** A wire of resistance 4Ω is stretched to twice its original length. In the process of stretching, its area of cross-section gets halved. Now, the resistance of the wire is [EAMCET 2014]

- (a) 8Ω (b) 16Ω (c) 1Ω (d) 4Ω

- 80** In Wheatstone bridge, three resistors P , Q and R are connected in three arms in order and 4th arm of resistance s , is formed by two resistors s_1 and s_2 connected in parallel. The condition for bridge to be balanced is, $\frac{P}{Q}$ [MHT CET 2014]

- (a) $\frac{R(s_1 + s_2)}{s_1 s_2}$ (b) $\frac{s_1 s_2}{R(s_1 + s_2)}$ (c) $\frac{R s_1 s_2}{(s_1 + s_2)}$ (d) $\frac{(s_1 + s_2)}{R s_1 s_2}$

- 81** An electron in potentiometer experiences a force 2.4×10^{-19} N. The length of potentiometer wire is 6m. The emf of the battery connected across the wire is (electronic charge = 1.6×10^{-19} C) [MHT CET 2014]
 (a) 6 V (b) 9 V
 (c) 12 V (d) 15 V
- 82** A galvanometer having internal resistance 10Ω requires 0.01 A for a full scale deflection. To convert this galvanometer to a voltmeter of full scale deflection at 120 V, we need to connect a resistance of [UK PMT 2014]
 (a) 11990Ω in series (b) 11990Ω in parallel
 (c) 12010Ω in series (d) 12010Ω in parallel
- 83** In potentiometer experiment, a cell of emf 1.25 V gives balancing length of 30 cm. If the cell is replaced by another cell, then balancing length is found to be 40 cm. What is the emf of second cell? [KCET 2014]
 (a) ≈ 1.5 V (b) ≈ 1.67 V
 (c) ≈ 1.47 V (d) ≈ 1.37 V
- 84** Potentiometer measures the potential difference more accurately than a voltmeter because [UK PMT 2014]
 (a) it does not draw current from external circuit
 (b) it draws a heavy current from external circuit
 (c) it has a wire of high resistance
 (d) it has a wire of low resistance
- 85** In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2Ω , the balancing becomes 120 cm. The internal resistance of the cell is [UK PMT 2014]
 (a) 1Ω (b) 0.5Ω
 (c) 4Ω (d) 2Ω
- 86** A galvanometer has a coil of resistance 100Ω and gives full scale deflection for 30 mA current. If it is to work as a voltmeter of 30 V, the resistance required to be added is [UK PMT 2014]
 (a) 500Ω (b) 900Ω
 (c) 1000Ω (d) 1800Ω
- 87** When 4 A current flows for 2 min in an electroplating experiment, then m gram of silver is deposited. Then, the amount (in gram) of silver deposited by 6 A current flowing for 40 s is [NEET 2013]
 (a) $4m$ (b) $\frac{m}{2}$ (c) $2m$ (d) $\frac{m}{4}$
- 88** A wire of resistance 4Ω is stretched to twice its original length. The resistance of stretched wire would be [NEET 2013]
 (a) 2Ω (b) 4Ω
 (c) 8Ω (d) 16Ω
- 89** The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10Ω is [NEET 2013]
 (a) 0.2Ω (b) 0.5Ω (c) 0.8Ω (d) 1.0Ω
- 90** The resistances of the four arms P, Q, R and S in a Wheatstone's bridge are 10Ω , 30Ω , 30Ω and 90Ω , respectively. The emf and internal resistance of the cell are 7V and 5Ω , respectively. If the galvanometer resistance is 50Ω , then the current drawn from the cell will be [NEET 2013]
 (a) 1.0 A (b) 0.2 A (c) 0.1 A (d) 2.0 A
- 91** An electron revolves in a circle at the rate of 10^{19} rounds per second. The equivalent current is ($e = 1.6 \times 10^{-19}$ C) [J & K CET 2013]
 (a) 1.0 A (b) 1.6 A (c) 2.0 A (d) 2.6 A
- 92.** A silver wire of radius 0.1 cm carries a current of 2A. If the charge density in silver is $5.86 \times 10^{28} \text{ m}^{-3}$, then the drift velocity is [J & K CET 2013]
 (a) $0.2 \times 10^{-3} \text{ ms}^{-1}$ (b) $0.4 \times 10^{-4} \text{ ms}^{-1}$
 (c) $0.68 \times 10^{-4} \text{ ms}^{-1}$ (d) $7 \times 10^{-4} \text{ ms}^{-1}$
- 93** A 1 m long wire of diameter 0.31 mm has a resistance of 4.2Ω . If it is replaced by another wire of same material of length 1.5 m and diameter 0.155 mm, then the resistance of wire is [J & K CET 2013]
 (a) 25.2Ω (b) 0.6Ω (c) 26.7Ω (d) 0.8Ω
- 94** 24 cells of emf 1.5 V each having internal resistance of 1Ω are connected to an external resistance of 1.5Ω . To get maximum current, [J & K CET 2013]
 (a) all cells are connected in series combination
 (b) all cells are connected in parallel combination
 (c) 4 cells in each row are connected in series and 6 such rows are connected in parallel
 (d) 6 cells in each row are connected in series and 4 such rows are connected in parallel
- 95** The temperature coefficient of the resistance of a wire is $0.00125 \text{ per } ^\circ\text{C}$. At 300 K its resistance is 1Ω . The resistance of wire will be 2Ω at [J&K CET 2013]
 (a) 1154 K (b) 1100 K (c) 1400 K (d) 1127 K
- 96** The emf of a cell E is 15 V as shown in the figure with an internal resistance of 0.5Ω . Then, the value of the current drawn from the cell is [EAMCET 2013]



- (a) 3 A (b) 2 A (c) 5 A (d) 1 A

- 97** Copper and carbon wires are connected in series and the combined resistor is kept at 0°C . Assuming the combined resistance does not vary with temperature, the ratio of the resistances of carbon and copper wires at 0°C is (temperature coefficients of resistivity of copper and carbon respectively are $4 \times 10^{-3}/^\circ\text{C}$ and $-0.5 \times 10^{-3}/^\circ\text{C}$) [EAMCET 2013]
- (a) 4 (b) 8 (c) 6 (d) 2

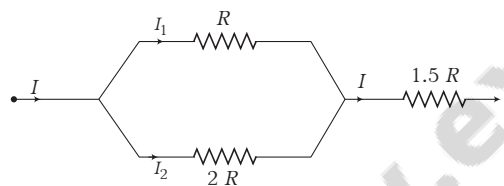
- 98** Three conductors draw currents of 1 A, 2 A and 3 A respectively, when connected in turn across a battery. If they are connected in series and the combination is connected across the same battery, then the current drawn will be [Karnataka CET 2013]

- (a) $\frac{2}{7}$ A (b) $\frac{3}{7}$ A
(c) $\frac{4}{7}$ A (d) $\frac{5}{7}$ A

- 99** Masses of three wires of copper are in the ratio of 1 : 3 : 5 and their lengths are in the ratio of 5 : 3 : 1. The ratio of their electrical resistances is [Karnataka CET 2013]

- (a) 1 : 3 : 5 (b) 5 : 3 : 1
(c) 1 : 15 : 125 (d) 125 : 15 : 1

- 100** In the circuit diagram, heat produces in R , $2R$ and $1.5R$ are in the ratio of [Karnataka CET 2013]



- (a) 4 : 2 : 3 (b) 8 : 4 : 27
(c) 2 : 4 : 3 (d) 27 : 8 : 4

- 101** Which one of the following electrical meter has the smallest resistance? [Kerala CET 2013]

- (a) Ammeter (b) Milliammeter
(c) Galvanometer (d) Voltmeter
(e) Millivoltmeter

- 102** Two wires of the same material having equal area of cross-section have length L and $2L$. Their respective resistances are in the ratio [Kerala CET 2013]

- (a) 2 : 1 (b) 1 : 1
(c) 1 : 2 (d) 1 : 3
(e) 4 : 1

- 103** Two bulbs 60 W and 100 W designed for voltage 220 V are connected in series across 220 V source. The net power dissipated is [Kerala CET 2013]

- (a) 80 W (b) 160 W (c) 37.5 W (d) 60 W
(e) 120 W

- 104** The drift speed of electrons in copper wire of diameter d and length l is v . If the potential difference across the wire is doubled, then the new drift speed becomes [Kerala CET 2013]

- (a) v (b) $2v$
(c) $3v$ (d) $v/2$
(e) $v/4$

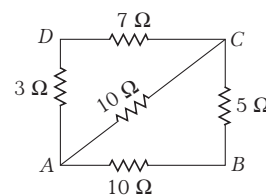
- 105** A potentiometer wire of length 10 m and resistance $10\ \Omega$ per metre is connected in series with a resistance box and a 2 volt battery. If a potential difference of 100 mV is balanced across the whole length of potentiometer wire, then the resistance introduced in the resistance box will be [MP PMT (2013)]

- (a) $1900\ \Omega$ (b) $900\ \Omega$
(c) $190\ \Omega$ (d) $90\ \Omega$

- 106** If a wire is stretched to four times its length, then the specific resistance of the wire will [MP PMT 2013]

- (a) become 4 times (b) become $1/4$ times
(c) become 16 times (d) remain the same

- 107** For the circuit shown in figure given below, the equivalent resistance between points A and B is [MP PMT 2013]

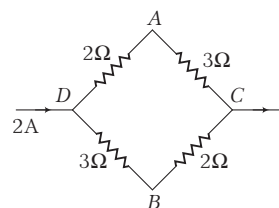


- (a) $10\ \Omega$ (b) $5\ \Omega$
(c) $\frac{10}{3}\ \Omega$ (d) $2\ \Omega$

- 108** Two resistors of $6\ \Omega$ and $9\ \Omega$ are connected in series to a 120 V source. The power consumed by $6\ \Omega$ resistor is [MP PMT 2013]

- (a) 384 W (b) 616 W
(c) 1500 W (d) 1800 W

- 109** A current of 2A flows in the arrangement of conductors as shown in below figure. The potential difference between points A and B ($V_A - V_B$) will be [UP CPMT 2013]

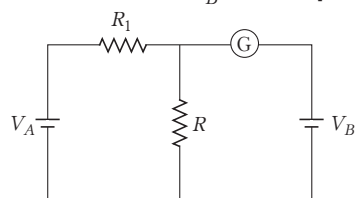


- (a) +1 V (b) -1 V
(c) +2 V (d) -2 V

- 110** A cell of emf E and internal resistance r supplies current for the same time t through external resistance R_1 and R_2 separately. If the heat developed in both the cases is the same, then the internal resistance r will be [UP CPMT 2013]

(a) $r = \sqrt{R_1 + R_2}$ (b) $r = \sqrt{R_1 \times R_2}$
 (c) $r = \frac{R_1 + R_2}{2}$ (d) $r = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

- 111** In the circuit shown, the cells A and B have negligible resistances. For $V_A = 12\text{ V}$, $R_1 = 500\ \Omega$ and $R = 100\ \Omega$ the galvanometer (G) shows no deflection. The value of V_B is [CBSE AIPMT 2012]



- (a) 4 V (b) 2 V (c) 12 V (d) 6 V
- 112** A millivoltmeter of 25 mV range is to be converted into an ammeter of 25 A range. The value (in ohm) of necessary shunt will be [CBSE AIPMT 2012]
- (a) 0.001 (b) 0.01 (c) 1 (d) 0.05
- 113** If voltage across a bulb rated 220 V-100 W drops by 2.5% of its rated value, then the percentage of the rated value by which the power would decrease is [CBSE AIPMT 2012]
- (a) 20% (b) 2.5% (c) 5% (d) 10%

- 114** 6 Ω and 12 Ω resistors are connected in parallel. This combination is connected in series with a 10 V battery and 6 Ω resistor. What is the potential difference between the terminals of the 12 Ω resistor? [AIIMS 2012]
- (a) 4 V (b) 16 V (c) 2 (d) 8 V

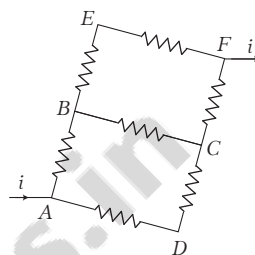
- 115** Charge passing through a conductor of cross-section area $A = 0.3\text{ m}^2$ is given by $q = 3t^2 + 5t + 2$ in coulomb, where t is in second. What is the value of drift velocity at $t = 2\text{ s}$? (Given, $n = 2 \times 10^{25}/\text{m}^3$) [AIIMS 2012]
- (a) $0.77 \times 10^{-5}\text{ m/s}$ (b) $1.77 \times 10^{-5}\text{ m/s}$
 (c) $2.08 \times 10^{-5}\text{ m/s}$ (d) $0.57 \times 10^{-5}\text{ m/s}$

- 116** A galvanometer having resistance of 50 Ω requires a current of 100 μA to give full scale deflection. How much resistance is required to convert it into an ammeter of range of 10 A? [BCECE (Mains) 2012]
- (a) $5 \times 10^{-3}\ \Omega$ in series (b) $5 \times 10^{-4}\ \Omega$ in parallel
 (c) $10^5\ \Omega$ in series (d) $10^5\ \Omega$ in parallel

- 117** Two cells when connected in series are balanced on 8 m on a potentiometer. If the cells are connected with polarities of one of the cells is reversed, then they balance on 2 m. The ratio of emfs of two cells is [BCECE (Mains) 2012]

(a) 3 : 4 (b) 4 : 3
 (c) 3 : 5 (d) 5 : 3

- 118** In the given circuit diagram if each resistance is of 10 Ω , then the current in arm AD will be [BCECE (Mains) 2012]



(a) $\frac{i}{5}$ (b) $\frac{2i}{5}$
 (c) $\frac{3i}{5}$ (d) $\frac{4i}{5}$

- 119** When current i is flowing through a conductor, the drift velocity is v . If the value of current through the conductor and its area of cross-section is doubled, then new drift velocity will be [BCECE Mains 2012]

(a) $4v$ (b) $\frac{v}{2}$
 (c) $\frac{v}{4}$ (d) v

- 120** A wire having resistance 12 Ω is bent in the form of an equilateral triangle. The effective resistance between any two corners of the triangle will be [BCECE Mains 2012]

(a) 6 Ω (b) $\frac{8}{3}\ \Omega$
 (c) 9 Ω (d) 12 Ω

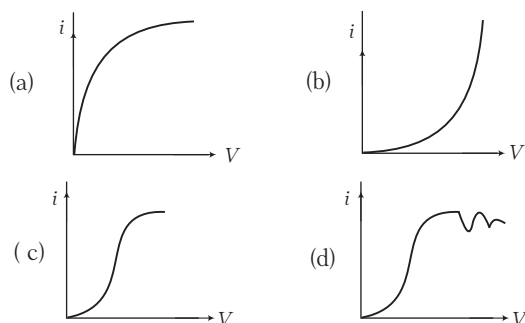
- 121** When a current of $(2.5 \pm 0.5)\text{ A}$ flows through a wire, it develops a potential difference of $(20 \pm 1)\text{ V}$, then the resistance of wire is [UP CPMT 2012]

(a) $(8 \pm 2)\ \Omega$ (b) $(8 \pm 1.6)\ \Omega$
 (c) $(8 \pm 1.5)\ \Omega$ (d) $(8 \pm 3)\ \Omega$

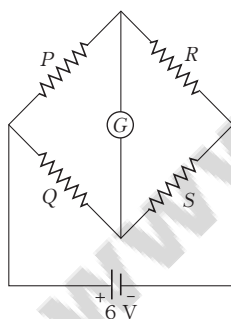
- 122** To draw the maximum current from a combination of cells, how should the cells be grouped? [UP CPMT 2012]

- (a) Parallel
 (b) Series
 (c) Depends upon the relative values of internal and external resistance
 (d) Mixed grouping

- 123** The variation between V - i has shown by graph for heating filament [UP CPMT 2012]

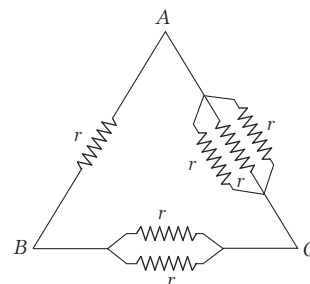


- 124** Two bulbs when connected in parallel to a source take 60 W each. The power consumed, when they are connected in series with the same source is [UP CPMT 2012]
- (a) 15 W (b) 30 W
(c) 60 W (d) 120 W
- 125** A voltmeter of range 2 V and resistance $300\ \Omega$ cannot be converted into ammeter of range [Manipal 2012]
- (a) 1 A (b) 1 mA
(c) 100 mA (d) 10 mA
- 126** In the Wheatstone network given, $P = 10\ \Omega$, $Q = 20\ \Omega$, $R = 15\ \Omega$, $S = 30\ \Omega$, the current passing through the battery (of negligible internal resistance) is [Manipal 2012]



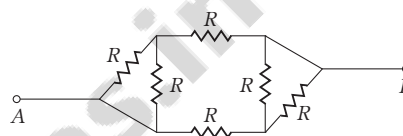
- (a) 0.36 A (b) zero
(c) 0.18 A (d) 0.72 A
- 127** A current of 5 A is passing through a metallic wire of cross-sectional area $4 \times 10^{-6}\ \text{m}^2$. If the density of charge carriers of the wire is $5 \times 10^{26}\ \text{m}^{-3}$, the drift velocity of the electrons will be [Manipal 2012]
- (a) $1 \times 10^2\ \text{ms}^{-1}$ (b) $1.56 \times 10^{-2}\ \text{ms}^{-1}$
(c) $1.56 \times 10^{-3}\ \text{ms}^{-1}$ (d) $1 \times 10^{-2}\ \text{ms}^{-1}$
- 128.** Six resistances each of value $r = 5\ \Omega$ are connected between points A, B and C as shown in the figure. If R_1 , R_2 and R_3 are the net resistance between A and B, between B and C and between A and C

C respectively, then $R_1 : R_2 : R_3$ will be equal to [AMU 2012]



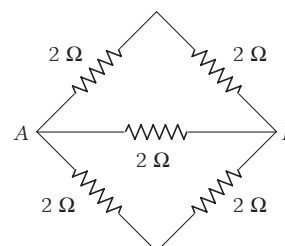
- (a) 6:3:2 (b) 1:2:3 (c) 5:4:3 (d) 4:3:2

- 129** The equivalent resistance between A and B of network shown in figure is [UP CPMT 2012]



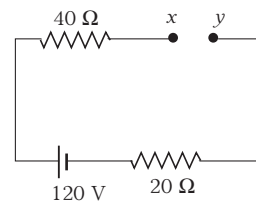
- (a) $\frac{3R}{4}$ (b) $\frac{4R}{3}$ (c) $6R$ (d) $2R$

- 130** Each resistance shown in figure is $2\ \Omega$. The equivalent resistance between A and B is [AFMC 2012]



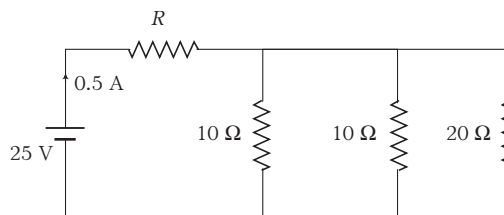
- (a) $2\ \Omega$ (b) $1\ \Omega$ (c) $4\ \Omega$ (d) $5\ \Omega$

- 131** In the circuit shown, the potential difference between x and y will be [JCECE 2012]



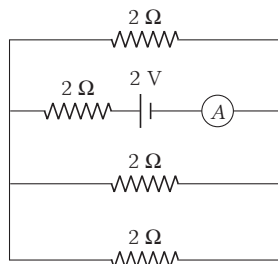
- (a) zero (b) 120 V (c) 60 V (d) 20 V

- 132** For the circuit shown in figure, [JCECE 2012]



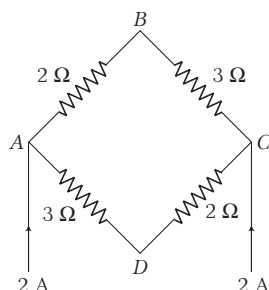
- (a) resistance $R = 46 \Omega$
 (b) current through 20Ω resistance is 0.1 A
 (c) potential difference across the middle resistance is 2 V
 (d) All of the above are true

133 An ammeter connected in the circuit as shown in figure shows a reading of [BHU Screening 2012]



- (a) $\frac{1}{2} \text{ A}$ (b) $\frac{3}{4} \text{ A}$ (c) $\frac{1}{8} \text{ A}$ (d) 2 A

134 If a current of 2 A flows through resistances connected as shown in figure, the potential difference $V_A - V_B$ is [BHU Screening 2012]



- (a) -1 V (b) $+1 \text{ V}$
 (c) -2 V (d) $+2 \text{ V}$

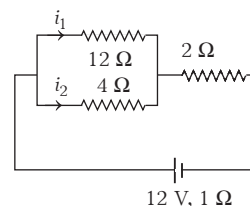
135 Two batteries of emfs 2 V and 1 V of internal resistances 1Ω and 2Ω respectively are connected in parallel. The effective emf of the combination is [Kerala CEE 2011]

- (a) $\frac{3}{2} \text{ V}$ (b) $\frac{5}{3} \text{ V}$ (c) $\frac{3}{5} \text{ V}$ (d) 2 V
 (e) 5 V

136 Two cells with the same emf E and different internal resistances r_1 and r_2 are connected in series to an external resistance R . What is the value of R , if the potential difference across the first cell is zero? [WB JEE 2011]

- (a) $\sqrt{r_1 r_2}$ (b) $r_1 + r_2$
 (c) $r_1 - r_2$ (d) $\frac{r_1 + r_2}{2}$

137 In the circuit shown below, the currents i_1 and i_2 are [KCET 2011]



- (a) $1.5 \text{ A}, 0.5 \text{ A}$ (b) $0.5 \text{ A}, 1.5 \text{ A}$
 (c) $1 \text{ A}, 3 \text{ A}$ (d) $3 \text{ A}, 1 \text{ A}$

ANSWERS

CHECK POINT 3.1

1. (a) 2. (a) 3. (c) 4. (b) 5. (a) 6. (b) 7. (a)

CHECK POINT 3.2

1. (b) 2. (a) 3. (a) 4. (d) 5. (c) 6. (b) 7. (c) 8. (b) 9. (b) 10. (d)
 11. (a) 12. (b) 13. (d) 14. (d) 15. (a) 16. (d)

CHECK POINT 3.3

1. (c) 2. (d) 3. (b) 4. (b) 5. (a) 6. (c) 7. (a) 8. (b) 9. (a) 10. (d)
 11. (b) 12. (b) 13. (c) 14. (c) 15. (b) 16. (a) 17. (c)

CHECK POINT 3.4

1. (d) 2. (d) 3. (a) 4. (b) 5. (c) 6. (d) 7. (d) 8. (d) 9. (d) 10. (c)
 11. (c) 12. (b) 13. (b) 14. (b) 15. (c) 16. (c) 17. (a) 18. (c) 19. (b) 20. (a)

(A) Taking it together

1. (c)	2. (d)	3. (c)	4. (b)	5. (a)	6. (c)	7. (b)	8. (b)	9. (b)	10. (a)
11. (c)	12. (d)	13. (b)	14. (a)	15. (c)	16. (a)	17. (d)	18. (a)	19. (a)	20. (b)
21. (a)	22. (a)	23. (b)	24. (c)	25. (a)	26. (a)	27. (b)	28. (d)	29. (d)	30. (c)
31. (d)	32. (a)	33. (b)	34. (a)	35. (d)	36. (a)	37. (d)	38. (b)	39. (c)	40. (b)
41. (d)	42. (c)	43. (c)	44. (c)	45. (b)	46. (c)	47. (b)	48. (d)	49. (c)	50. (c)
51. (a)	52. (b)	53. (c)	54. (c)	55. (a)	56. (d)	57. (b)	58. (c)	59. (b)	60. (b)
61. (c)	62. (d)	63. (d)	64. (c)	65. (d)	66. (c)	67. (b)	68. (b)	69. (d)	70. (d)
71. (d)	72. (a)	73. (b)	74. (c)	75. (b)	76. (d)	77. (b)	78. (d)	79. (b)	80. (a)
81. (c)	82. (b)	83. (b)	84. (a)	85. (d)	86. (d)	87. (d)	88. (c)	89. (b)	90. (d)
91. (d)	92. (a)	93. (a)	94. (b)	95. (a)	96. (c)	97. (a)	98. (d)	99. (b)	100. (d)
101. (b)	102. (c)	103. (a)	104. (a)	105. (b)	106. (c)	107. (a)	108. (b)	109. (b)	110. (d)
111. (a)	112. (c)	113. (d)	114. (c)	115. (c)	116. (b)	117. (b)	118. (c)	119. (b)	120. (b)
121. (d)	122. (b)	123. (c)	124. (b)						

(B) Medical entrance special format questions**• Assertion and reason**

1. (d) 2. (a) 3. (b) 4. (d) 5. (a)

• Statement based questions

1. (a) 2. (d) 3. (d) 4. (c) 5. (b)

• Match the columns

1. (a) 2. (c)

(C) Medical entrances' gallery

1. (a)	2. (c)	3. (a)	4. (b)	5. (b)	6. (b)	7. (b)	8. (b)	9. (a)	10. (c)
11. (b)	12. (d)	13. (d)	14. (a)	15. (c)	16. (b)	17. (d)	18. (b)	19. (a)	20. (a)
21. (b)	22. (c)	23. (c)	24. (a)	25. (c)	26. (b)	27. (a)	28. (c)	29. (c)	30. (b)
31. (c)	32. (b)	33. (b)	34. (a)	35. (a)	36. (c)	37. (d)	38. (c)	39. (d)	40. (d)
41. (c)	42. (a)	43. (a)	44. (b)	45. (c)	46. (a)	47. (a)	48. (b)	49. (a)	50. (e)
51. (b)	52. (c)	53. (a)	54. (a)	55. (a)	56. (b)	57. (c)	58. (a)	59. (c)	60. (c)
61. (c)	62. (d)	63. (a)	64. (a)	65. (b)	66. (d)	67. (b)	68. (c)	69. (a)	70. (c)
71. (c)	72. (c)	73. (d)	74. (a)	75. (b)	76. (d)	77. (b)	78. (a)	79. (b)	80. (a)
81. (b)	82. (b)	83. (b)	84. (a)	85. (d)	86. (b)	87. (b)	88. (d)	89. (b)	90. (b)
91. (b)	92. (c)	93. (a)	94. (d)	95. (d)	96. (d)	97. (b)	98. (*)	99. (d)	100. (b)
101. (a)	102. (c)	103. (c)	104. (b)	105. (c)	106. (d)	107. (b)	108. (a)	109. (a)	110. (b)
111. (b)	112. (a)	113. (c)	114. (a)	115. (b)	116. (b)	117. (d)	118. (b)	119. (d)	120. (b)
121. (a)	122. (c)	123. (a)	124. (b)	125. (b)	126. (a)	127. (b)	128. (c)	129. (a)	130. (a)
131. (b)	132. (d)	133. (b)	134. (d)	135. (b)	136. (c)	137. (b)			

Hints & Explanations

• CHECK POINT 3.1

1 (a) Current, $I = \frac{dq}{dt} = 3t^2 + 2t + 5$

$$q = \int_{t=0}^{t=2} (3t^2 + 2t + 5) dt$$

Charge, $q = [t^3 + t^2 + 5t]_0^2 = [8 + 4 + 10] = 22 \text{ C}$

2 (a) Current, $i = \frac{q}{t}$ or $i = \frac{ne}{t} \Rightarrow 16 \times 10^{-3} = \frac{n \times 1.6 \times 10^{-19}}{1}$

$\therefore$ Number of electrons, $n = 10^{17}$

3 (c) We have, current, $i = \frac{ne}{t}$

$\therefore$ Number of electrons, $n = \frac{it}{e} = \frac{0.2 \times 30}{1.6 \times 10^{-19}} = 3.75 \times 10^{19}$

4 (b) Given, $I = 4 - 0.08t$

$\Rightarrow \frac{dq}{dt} = 4 - 0.08t$

$\Rightarrow q = \int_0^{50} (4 - 0.08t) dt$

$\Rightarrow ne = \left[4t - \frac{0.08t^2}{2} \right]_0^{50} = 100$

$\Rightarrow n = \frac{100}{e} = \frac{100}{1.6 \times 10^{-19}} = 6.25 \times 10^{20} \text{ electrons}$

5 (a) Drift velocity, $v_d = \frac{eE\tau}{m}$

$\therefore v_d \propto E$

6 (b) The order of drift velocity of electrons is $10^{-4} \text{ ms}^{-1} = 10^{-2} \text{ cms}^{-1}$

7 (a) Drift velocity,

$$v_d = \frac{i}{neA} = \frac{3}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-6}} = 0.11 \times 10^{-3} \text{ ms}^{-1}$$

$\therefore$ Time, $t = \frac{\text{length of the wire}}{v_d} = \frac{3}{0.11 \times 10^{-3}} = 2.73 \times 10^4 \text{ s}$

• CHECK POINT 3.2

1 (b) We have, $i = \frac{V}{R} = \frac{q}{t}$

$\therefore$ Charge, $q = \frac{Vt}{R} = \frac{20 \times 2 \times 60}{10} = 240 \text{ C}$

2 (a) Resistance, $R = \rho \frac{l}{A}$ and resistivity, $\rho = \frac{m}{ne^2\tau}$

$\therefore R = \frac{ml}{ne^2\tau A}$

3 (a) $R = \rho \frac{l}{A}$

Since, wires are made of same material, so resistivity ρ of all wires will be same.

Therefore, $R \propto \frac{l}{A}$

$\Rightarrow R \propto \frac{l}{\pi r^2}$

$R \propto \frac{l}{\left(\frac{d}{2}\right)^2}$

For length = 50 cm and diameter = 0.5 mm

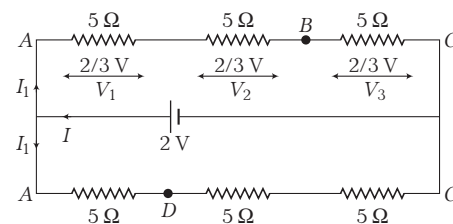
Resistance will be maximum.

4 (d) Colour code for carbon resistor,

A	B	C	D
Brown	Red	Green	Gold
↓	↓	↓	↓
1	2	10^5	5%

$R = AB \times C \pm D$
 $= 12 \times 10^5 \pm 5\%$

5 (c) Given, circuit can be redrawn as follows



Here, $I_1 = \frac{I}{2} = \frac{V}{2R_{eq}} = \frac{2}{2 \times \left(\frac{15}{2}\right)} = \frac{2}{15} \text{ A}$

So, potential across each resistance,

$V' = I_1 R = \frac{2}{15} \times 5 = \frac{2}{3} \text{ V}$

$\therefore$ Potential difference across,

$AB = \frac{2}{3} + \frac{2}{3} = \frac{4}{3} \text{ V}$

6 (b) Potential difference across the circuit

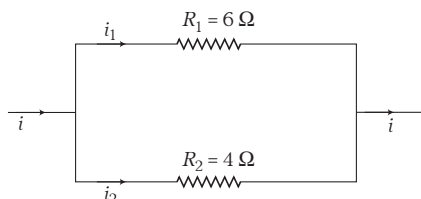
$= i \left(\frac{R_1 R_2}{R_1 + R_2} \right) = 1.2 \times \left(\frac{6 \times 4}{6 + 4} \right) = 2.88 \text{ V}$

So, current through 6Ω resistance

$= \frac{2.88}{6} = 0.48 \text{ A}$

Alternative solution

From current division rule, $i_1 = i \left(\frac{R_2}{R_1 + R_2} \right)$



$$= 1.2 \left(\frac{4}{6 + 4} \right) = 1.2 \left(\frac{4}{10} \right) = 0.48 \text{ A}$$

- 7 (c)** Current divides according to resistance, so current in 6Ω resistance is $\frac{0.8}{2} = 0.4 \text{ A}$

So, total current in circuit is $0.8 + 0.4 = 1.2 \text{ A}$

$\therefore$ Potential drop across $4 \Omega = 1.2 \times 4 = 4.8 \text{ V}$

- 8 (b)** Two resistances are short circuited.

So, only third resistance will be considered and hence,

$$i = V/R = 10/3 \text{ A}$$

- 9 (b)** Net resistance, $R_{\text{net}} = \frac{10 \times 15}{10 + 15} + 3 + 0.5 = 9.5 \Omega$

$$\therefore I = \frac{V}{R_{\text{net}}} = \frac{20}{9.5} = 2.1 \text{ A}$$

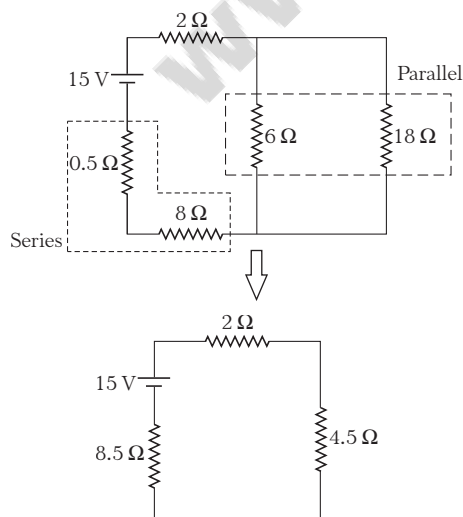
According to current division rule, current through 10Ω resistance is

$$I_1 = \left(\frac{15}{10 + 15} \right) \times 2.1 = 1.2 \text{ A}$$

- 10 (d)** Resistance, $R_{AB} = \frac{R}{3} + R = \frac{2}{3} + 2$

$$R_{AB} = \frac{8}{3} = 2 \frac{2}{3} \Omega$$

- 11 (a)** The circuit can be reduced as follows



So, equivalent resistance across battery,

$$R_{\text{eq}} = 8.5 + 2 + 4.5 = 15 \Omega$$

Hence, current from the battery,

$$i = \frac{15}{15} = 1 \text{ A}$$

- 12 (b)** As B is connected to the earth, so potential at B is $V_B = 0$.

Now, current in the given circuit,

$$i = \frac{V}{R_{\text{net}}} = \frac{50}{5 + 7 + 10 + 3} = 2 \text{ A}$$

Potential difference between A and B is

$$V_A - V_B = 2 \times 12$$

$$\text{or } V_A - 0 = 24$$

$$V_A = 24 \text{ V}$$

- 13 (d)** We have, $V = E - ir$

$$= 1.5 - 2 \times 0.15$$

$$V = 1.20 \text{ V}$$

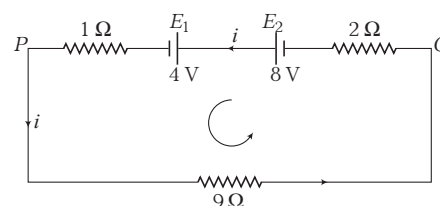
- 14 (d)** Potential difference between A and B is given by

$$V_A - V_B = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$\therefore 4 = \frac{5X + 2 \times 10}{X + 10}$$

$$\Rightarrow X = 20 \Omega$$

- 15 (a)**



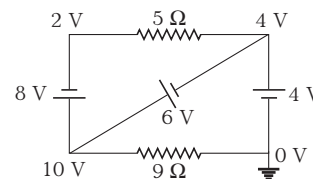
$$E_1 = 4 \text{ V and } E_2 = 8 \text{ V}$$

As, $E_2 > E_1$, so current flows from Q to P .

$$\therefore i = \frac{8 - 4}{12} = \frac{1}{3} \text{ A}$$

$$\therefore \text{Potential difference across } PQ = \frac{1}{3} \times 9 = 3 \text{ V}$$

- 16 (d)** The potentials of different points are as shown in below figure



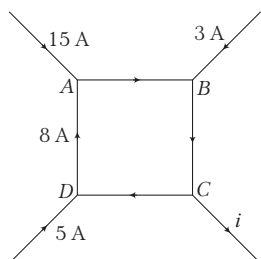
Current through 5Ω resistance

$$= \frac{\text{Potential difference}}{\text{Resistance}}$$

$$= \frac{(4 - 2)}{5} = 0.4 \text{ A}$$

• CHECK POINT 3.3

1 (c)



Applying Kirchhoff's first law at junction A, B, C, D

At A, $i_{AB} = 15 + 8 = 23 \text{ A}$

At B, $i_{BC} = 23 + 3 = 26 \text{ A}$

At D, $i_{CD} = 8 - 5 = 3 \text{ A}$

At C, $i_{CD} + i = i_{BC}$

or $3 + i = 26$

$\therefore i = 23 \text{ A}$



Potential difference between A and B,

$$V_A - V_B = 1 \times 1.5 \text{ or } V_A - 0 = 1.5 \quad (\because V_B = 0, \text{ given})$$

$\therefore V_A = 1.5 \text{ V}$

Now, potential difference between B and C,

$$V_B - V_C = 1 \times 2.5 = 2.5 \text{ V}$$

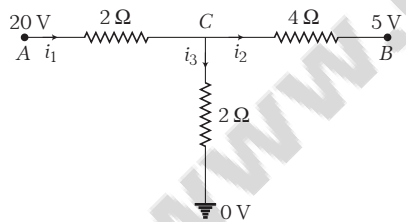
$\therefore 0 - V_C = 2.5 \text{ V}$

$\Rightarrow V_C = -2.5 \text{ V}$

Hence, potential difference between C and D,

$$V_C - V_D = -2 \text{ V or } -2.5 - V_D = -2 \text{ or } V_D = -0.5 \text{ V}$$

3 (b) Let at junction C, potential is V .



$$\therefore \frac{V_A - V_C}{2} + \frac{V_B - V_C}{4} = \frac{V_C - 0}{2}$$

$$\frac{20 - V}{2} + \frac{5 - V}{4} = \frac{V - 0}{2}$$

$$V = 9 \text{ V}$$

$$\text{Current, } i_3 = i = \frac{9}{2} = 4.5 \text{ A}$$

4 (b) Work done, $W = qV = 6 \times 10^{-6} \times 9 = 54 \times 10^{-6} \text{ J}$

5 (a) In series, i is same, so $H = i^2 R t$. Therefore, $H \propto R$.

$$\therefore \frac{H_1}{H_2} = \frac{R}{2R} = \frac{1}{2}$$

$$\Rightarrow H_1 : H_2 = 1 : 2$$

$$6 \text{ (c) Resistance, } R = \frac{V^2}{P} = \frac{25 \times 25}{25} = 25 \Omega$$

$$7 \text{ (a) Resistance, } R = \frac{V^2}{P} \text{ or } R \propto V^2$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{V_1^2}{V_2^2} \Rightarrow \frac{R}{R_2} = \left(\frac{220}{110} \right)^2 = 4 \Rightarrow R_2 = \frac{R}{4}$$

$$8 \text{ (b) Power, } P = \frac{V^2}{R}$$

$$\therefore P \propto \frac{1}{R}$$

$$\text{So, } \frac{P_1}{P_2} = \frac{R_2}{R_1}$$

$$\frac{200}{100} = \frac{R_2}{R_1}$$

$$R_2 = 2R_1$$

$$9 \text{ (a) Power, } P = \frac{V^2}{R}$$

$$\therefore P \propto \frac{1}{R}$$

$$\text{Also, resistance of wire } \propto \frac{1}{(\text{radius of filament})^2}$$

$$\therefore P \propto (\text{radius of filament})^2$$

So, 100 W bulb has thicker filament.

$$10 \text{ (d) } H = \frac{V^2 t}{R \times J} = \frac{Pt}{J} = \frac{210 \times 5 \times 60}{4.2} = 15000 \text{ cal}$$

$$11 \text{ (b) } P = \frac{V^2}{R} \Rightarrow 150 = \frac{(15)^2}{\left(\frac{2R}{2+R} \right)}$$

$$\therefore \frac{2+R}{2R} = \frac{2}{3}$$

$$\therefore \text{Resistance, } R = 6 \Omega$$

$$12 \text{ (b) } R' = \frac{R}{4} \text{ and } P = \frac{V^2}{R}$$

$$\therefore P_2 = 4P_1 \Rightarrow \frac{P_2}{P_1} = 4$$

13 (c) $R_{40} > R_{100}$. In series, potential difference distributes in direct ratio of resistance.

14 (c) When bulbs are in series, $P = \frac{V^2}{3R}$... (i)

When bulbs are connected in parallel,

$$P' = \frac{V^2}{(R/3)} = \frac{3V^2}{R} = 3 \times 3P \quad [\text{from Eq. (i)}]$$

$$= 9P$$

15 (b) It is known that in parallel combination,

$$P_{\text{consumed}} \propto \text{Brightness} \propto \frac{1}{R}$$

According to question, $P_A > P_B$ (given), therefore $R_B > R_A$

16 (a) $P = \frac{V^2}{R}$. As, R_{net} will decrease, so P will increase.

17 (c) When each bulb is glowing at full power.

$$\text{Current from each bulb, } i = \frac{50}{100} = \frac{1}{2} \text{ A}$$

$$\text{So, main current, } i = \frac{n}{2} \text{ A} \quad (\text{for parallel circuit})$$

$$\text{Also, } E = V + ir$$

$$120 = 100 + \left(\frac{n}{2}\right) \times 10$$

$$n = 4$$

• CHECK POINT 3.4

1 (d) Ammeter is parallel with voltmeter, therefore its reading will be zero.

4 (b) If voltmeter is ideal, then R should be $\frac{1.6}{0.4} = 4 \Omega$. If it is non-ideal, R should be greater than 4Ω .

5 (c) Total resistance of given circuit

$$= \frac{80}{1+1} + 20 = 40 + 20 = 60 \Omega$$

$$\therefore \text{Main current, } i = \frac{2}{60} = \frac{1}{30} \text{ A}$$

Now, in parallel, there are two resistances of 80Ω each (one of voltmeter and other 80Ω resistance). So, current is equally distributed in 80Ω resistance and voltmeter, i.e. $\frac{1}{60} \text{ A}$ of current flows through each.

$\therefore$ Potential difference across 80Ω resistance or voltmeter reading $= \frac{1}{60} \times 80 = 1.33 \text{ V}$

6 (d) Equivalent resistance of circuit,

$$R_{\text{eq}} = 500 + \frac{1000}{3} = \frac{2500}{3} \Omega$$

$\therefore$ Current drawn from the cell,

$$i = \frac{10}{2500/3} = \frac{3}{250} \text{ A}$$

$$\text{Reading of voltmeter} = \frac{3}{250} \times \frac{1000}{3} = 4 \text{ V}$$

7 (d) $V = IR$

$$100 = (10 \times 10^{-3})(25 + R)$$

$$\therefore \text{Resistance, } R = 9975 \Omega$$

8 (d) When some resistance is connected in parallel with voltmeter the effective resistance get decreased. So, A will increase and V will decrease.

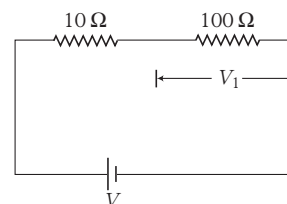
10 (c) We have, $i_g G = (i - i_g)S$

$$10 \times 99 = (90)S$$

$$S = \frac{10 \times 99}{90}$$

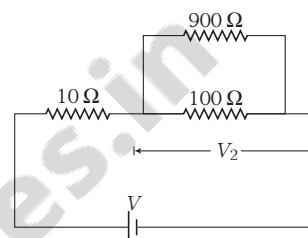
$$\therefore \text{Shunt, } S = 11 \Omega$$

11 (c) Before connecting the voltmeter, let the potential difference across 100Ω is V_1 .



$$\therefore V_1 = \left(\frac{100}{100 + 10}\right) \times V = \frac{10}{11} V$$

After connecting the voltmeter across 100Ω ,



$$\text{Equivalent resistance} = \frac{100 \times 900}{100 + 900} = 90 \Omega$$

Let this time potential difference is V_2 .

$$\therefore V_2 = \left(\frac{90}{90 + 10}\right) V = \frac{9}{10} V$$

$$\text{Magnitude of \% error} = \frac{\frac{10}{11} V - \frac{9}{10} V}{\frac{10}{11} V} \times 100 = 1.0$$

12 (b) (i) $50 \neq (50 \times 10^{-6})(100 + 10^4)$. Therefore, (a) is wrong.

(ii) $10 \approx (50 \times 10^{-6})(100 + 200 \times 10^3)$. Therefore, (b) is correct.

(iii) $\frac{50 \times 10^{-6}}{10 \times 10^{-3} - 50 \times 10^{-6}} \neq \frac{1}{100}$. Therefore, (c) is wrong.

14 (b) $(i_g R_V)(100) = i_g (R_V + 1980)$

$$\therefore \text{Resistance, } R_V = 20 \Omega$$

15 (c) $\frac{R_3}{R_4} = \frac{R_1}{R_2}$ (balanced Wheatstone bridge)

$$\text{or } R_1 R_4 = R_2 R_3$$

16 (c) The galvanometer shows no current, it means this is a balanced Wheatstone bridge, so

$$\frac{R}{80} = \frac{AC}{BC} = \frac{20}{80}$$

$$\therefore \text{Resistance, } R = 20 \Omega$$

17 (a) In potentiometer, the ratio of emfs is equal to the ratio of no deflection lengths.

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} = \frac{2}{3}$$

$$18 \text{ (c) } r = \frac{(l_1 - l_2)}{l_2} \times R = \left(\frac{60 - 50}{50} \right) \times 6 = 1.2 \Omega$$

19 (b) For potentiometer, emf

$$E \propto l \text{ or } E = kl$$

where, k is a constant.

$$\text{Also, } E = \frac{V}{l} = \frac{iR}{L} \times l$$

$$\therefore E = \frac{E'}{(R_1 + R_2 + r)} \times \frac{R}{L} \times l$$

$$\therefore E = \frac{10}{5 + 4 + 1} \times \frac{5}{5} \times 3 = 3 \text{ V}$$

$$20 \text{ (a) Potential gradient, } x = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L}$$

$$\Rightarrow \frac{0.2 \times 10^{-3}}{10^{-2}} = \frac{2}{(R + 490 + 0)} \times \frac{R}{1}$$

$$\Rightarrow R = 4.9 \Omega$$

(A) Taking it together

1 (c) During the charging of battery terminal, potential difference is always greater than emf of circuit.

$$V = E + ir$$

2 (d) Potentiometer works on null deflection method. In balance condition, no current flow in secondary circuit.

3 (c) We cannot increase the rating of fuse wire of lower value just by increasing its length. For it, we shall have to make fuse wire thick.

4 (b) Specific resistance of silver, copper and aluminium are $1.6 \times 10^{-8} \Omega \cdot \text{m}$, $1.7 \times 10^{-8} \Omega \cdot \text{m}$ and $2.7 \times 10^{-8} \Omega \cdot \text{m}$, respectively.

$$\text{Since, conductivity } (\sigma) = \frac{1}{\text{resistivity } (\rho)}$$

$$\text{Hence, } \sigma_{\text{Al}} < \sigma_{\text{Cu}} < \sigma_{\text{Ag}}$$

Thus, correct sequence is Al, Cu, Ag.

5 (a) Voltmeter has high resistance. So, most of the main current will flow through ammeter which is in parallel. So, it will burn out. No damage will occur to voltmeter.

$$6 \text{ (c) } V = E - ir = E - \left(\frac{E}{R + r} \right) r = E \left(\frac{R}{R + r} \right) = \frac{E}{1 + r/R}$$

8 (b) The minimum resistance can be achieved when we connect all resistances in parallel.

$$\text{So, equivalent resistance of combination} = \frac{r}{10}.$$

9 (b) When wire is bent in the form of a circle, then between two points in any diameter, it is equivalent to two resistances in parallel, $R_{\text{eq}} = \frac{12}{2} = 6 \Omega$

10 (a) As steady current is flowing through the conductor, hence the number of electrons entering from one end and outgoing from the other end of any segment is equal. Hence, charge will be zero.

11 (c) Out of n cells, two cells will cancel out each other's emf. So, net emf $= (n - 2) E$.

$$\text{Total resistance} = R + nr$$

$$\text{Current, } i = \frac{(n - 2)E}{nr + R}$$

$$12 \text{ (d) Current, } i = \frac{E}{r + R}$$

$$\text{Power, } P = i^2 R \Rightarrow P = \frac{E^2 R}{(r + R)^2}$$

Power will be maximum, when $r = R$.

$$P_{\text{max}} = \frac{E^2}{4r}$$

13 (b) Slope of the V - i curve at any point is equal to reciprocal of resistance at that point.

From the curve, slope for $T_1 >$ slope for T_2

$$R_{T_1} < R_{T_2}$$

$\Rightarrow$ Also at higher temperature resistance will be higher, so

$$T_2 > T_1$$

14 (a) The relationship between current and drift speed is given by $i = neAv_d$

Here, I is the current and v_d is the drift velocity.

$$\text{So, } I \propto v_d$$

Thus, only drift velocity determines the current in a conductor.

15 (c) Current in the circuit will increase because another resistance is connected in parallel to the circuit and hence potential drop across the ammeter will decrease. So, the potential difference over voltmeter will increase because total potential difference over ammeter and voltmeter is equal to emf (constant).

16 (a) Resistance of wire, $R = \rho L/A$

On stretching the wire, the volume of the wire remains same.

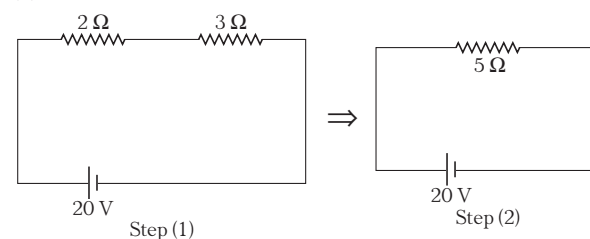
$$\Rightarrow \frac{A'}{A} = \frac{l}{l'}$$

$$\text{So, new resistance, } R' = \rho l'/A'$$

$$\Rightarrow \frac{R'}{R} = \left(\frac{l'}{l} \right) \left(\frac{A}{A'} \right) = \frac{l'}{l} \cdot \frac{l}{l'} = \left(\frac{l'}{l} \right)^2 = \left(\frac{1.1l}{l} \right)^2 = 1.21$$

$$\therefore R' = 1.21R = 1.21 \times 10 = 12.1 \Omega \approx 12 \Omega$$

17 (d) Circuit can be reduced as follows



So, equivalent resistance of circuit, $R_{\text{eq}} = 5 \Omega$

$$\therefore \text{Current in the circuit} = \frac{20}{5} = 4 \text{ A}$$

As in parallel, current is divided according to resistance, so current flowing through each resistance $= 2 \text{ A}$.

- 18 (a) Potential difference = 10 V

$$\text{So, } i = \frac{10}{5} = 2 \text{ A}$$

- 19 (a) Internal resistance, $r = R \left(\frac{E}{V} - 1 \right) = (10) \left(\frac{1.5}{1.25} - 1 \right) = 2 \Omega$

- 20 (b) For twisted wire, there are two halves each of resistance 2Ω in parallel.

$$\text{So, } R_{\text{eq}} = \frac{2 \times 2}{2 + 2} = 1 \Omega$$

- 21 (a) When wire is divided in 10 equal parts, then each part will have a resistance = $R/10 = r$.

Let equivalent resistance be r_R , then

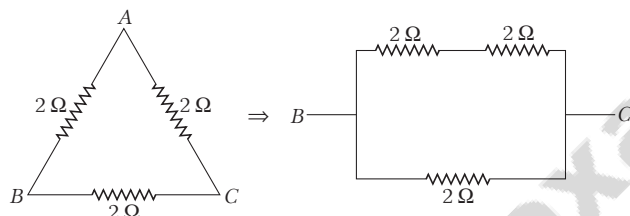
$$\frac{1}{r_R} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \dots 10 \text{ times}$$

$$\therefore \frac{1}{r_R} = \frac{10}{r} = \frac{10}{(R/10)} = \frac{100}{R}$$

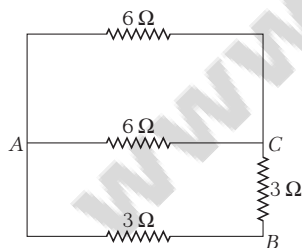
$$\therefore r_R = \frac{R}{100} = 0.01R$$

- 22 (a) Equivalent resistance between B and C.

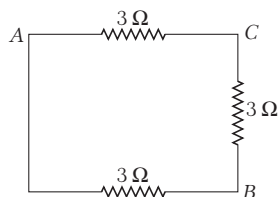
$$R_{BC} = \frac{(2 + 2) \times 2}{2 + 2 + 2} = \frac{8}{6} = \frac{4}{3} \Omega$$



- 23 (b) Given circuit can be reduced to



So, equivalent resistance between points A and B is equal to



$$R_{\text{eq}} = \frac{6 \times 3}{6 + 3} = 2 \Omega$$

- 24 (c) Given, $\frac{R_1 R_2}{R_1 + R_2} = \frac{6}{8}$... (i)

When one resistance say R_2 is broken, then

$$R_1 = 2 \Omega \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$R_2 = \frac{6}{5} \Omega$$

- 25 (a) We have, $R = \rho \frac{l}{A}$

$$R \propto l$$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{1}{2}$$

$$\Rightarrow R_2 = 2R_1 \quad \dots (i)$$

$$\therefore R_1 + R_2 = 9 \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

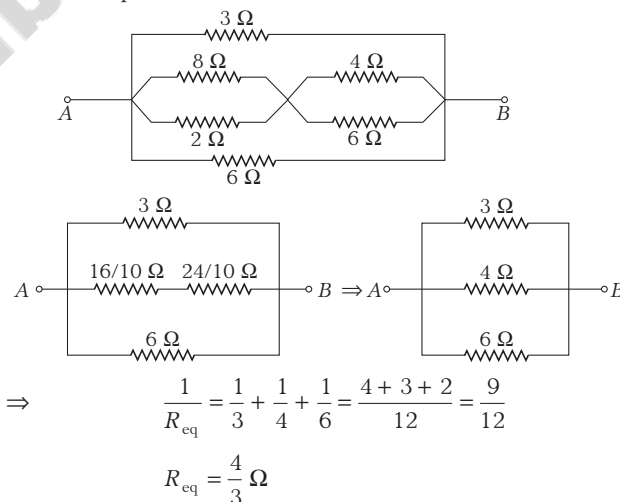
$$R_1 + 2R_1 = 9$$

$$\Rightarrow R_1 = 3 \Omega$$

$$\therefore R_2 = 2R_1 = 2 \times 3 = 6 \Omega$$

$$\text{Net resistance, } R_{\text{net}} = \frac{R_1 R_2}{R_1 + R_2} = \frac{3 \times 6}{3 + 6} = 2 \Omega$$

- 26 (a) The simplified circuit is as shown below



$$\Rightarrow \frac{1}{R_{\text{eq}}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{4 + 3 + 2}{12} = \frac{9}{12}$$

$$R_{\text{eq}} = \frac{4}{3} \Omega$$

- 27 (b) Let resistance, $R = 100 \Omega$

$$\therefore R' = 100 + 100 \times \frac{10}{100} = 110 \Omega$$

$$\therefore \text{Required shunt, } \Delta R = R' - R = 110 - 100 = 10 \Omega$$

- 28 (d) Given, $V = 115 \text{ V}$

$$\text{and } P = 1250 \text{ W}$$

$$\text{We know, } R = \frac{V^2}{P}$$

$$\text{So, } R = \frac{(115)^2}{1250} = 10.58 \Omega$$

$$\text{Resistance, } R = 10.6 \Omega$$

- 29** (d) Resistance, $R_t = R_0(1 + \alpha t)$

$$20 = R_0(1 + 20\alpha) \text{ and } 60 = R_0(1 + 500\alpha)$$

From these equations, we can find

$$R_0 = 18.33 \, \Omega$$

$$\alpha = 4.54 \times 10^{-3} \, ^\circ\text{C}^{-1}$$

$$R_t = 25 \, \Omega$$

Again,

$$R_t = R_0(1 + \alpha t)$$

$$25 = 18.33(1 + 4.54 \times 10^{-3} t)$$

We find,

$$t = 80^\circ\text{C}$$

- 30** (c) The current i crossing area of cross-section A , can be expressed in terms of drift velocity v_d and the moving charges as

$$i = nev_d A$$

where, n is number of charge carriers per unit volume and e the charge on the carrier.

$$\therefore v_d = \frac{i}{neA} = \frac{24 \times 10^{-3}}{(3 \times 10^{23})(1.6 \times 10^{-19})(10^{-4})} = 5 \times 10^{-3} \text{ ms}^{-1}$$

- 31** (d) Drift velocity in a copper conductor,

$$v_d = \frac{i}{neA} = \frac{5.4}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6}} = 0.4 \times 10^{-3} \text{ ms}^{-1} = 0.4 \text{ mms}^{-1}$$

- 32** (a) The resistance of wire is given by, $R = \rho \frac{l}{A}$

For greater value of R , l must be higher and A should be lower and it is possible only when the battery is connected across

$$1 \text{ cm} \times \left(\frac{1}{2}\right) \text{ cm (area of cross-section } A).$$

- 33** (b) In potentiometer experiment, the emf of a cell can be measured, if the potential drop along the potentiometer wire is more than the emf of the cell to be determined. Here value of emfs of two cells are given as 5V and 10V, therefore the potential drop along the potentiometer wire must be more than 10 V.

- 34** (a) Potential gradient $= \frac{V}{L} = \frac{iR}{L} = \frac{i \rho L}{LA} = \frac{i \rho}{A}$
- $$= \frac{0.2 \times 40 \times 10^{-8}}{8 \times 10^{-6}} = 10^{-2} \text{ V/m}$$

- 35** (d) When potentiometer is connected between A and B , then it measures only E_1 and when connected between A and C , then it measures $E_1 - E_2$.

$$\therefore \frac{E_1}{E_1 - E_2} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{E_1 - E_2}{E_1} = \frac{l_2}{l_1}$$

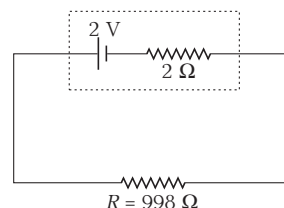
$$\Rightarrow 1 - \frac{E_2}{E_1} = \frac{100}{300}$$

$$\Rightarrow \frac{E_2}{E_1} = 1 - \frac{1}{3}$$

$$\Rightarrow \frac{E_2}{E_1} = \frac{2}{3}$$

$$\Rightarrow \frac{E_1}{E_2} = \frac{3}{2}$$

- 36** (a) From Ohm's law,



$$i = \frac{E}{R + r} = \frac{2}{998 + 2} = 2 \times 10^{-3} \text{ A}$$

Potential difference across the voltmeter is

$$V = iR = (2 \times 10^{-3}) \times 998 = 1.996 \text{ V} \approx 1.99 \text{ V}$$

- 37** (d) Resistance, $R = \frac{V}{i} = \frac{2}{4} = 0.5 \, \Omega$

$$\text{Resistance, } R = \frac{\rho l}{A} \Rightarrow 0.5 = \frac{\rho \times 0.5}{1 \times 10^{-6}}$$

Resistivity of wire,

$$\rho = \frac{0.5 \times 10^{-6}}{0.5} = 1 \times 10^{-6} \, \Omega \cdot \text{m}$$

- 38** (b) In order to balance the bridge, the effective value of S must be equal to $2 \, \Omega$.

$$\text{So, } 2 = \frac{S \times 6}{S + 6}$$

$$2S + 12 = 6S \Rightarrow 4S = 12 \Rightarrow S = 3 \, \Omega$$

- 39** (c) Current in the potentiometer wire $= \frac{2}{990 + 10} = 2 \times 10^{-3} \text{ A}$

Potential drop over wire $= 2 \times 10^{-3} \times 10 = 2 \times 10^{-2} \text{ V} (\because V = IR)$

$$\text{Potential gradient} = \frac{2 \times 10^{-2}}{2} = 0.01 \text{ V/m} \quad \left(\because k = \frac{V}{l} \right)$$

- 40** (b) Current, $i = qf = q \left(\frac{v}{2\pi r} \right) = \frac{1.6 \times 10^{-19} (2.2 \times 10^6)}{(2\pi) (5 \times 10^{-11})} = 1.12 \times 10^{-3} \text{ A}$

- 41** (d) We have, $k \times l = i \times R$

$$(2 \times 10^{-3}) (50) = 10 \times i$$

The current passing through the resistor,

$$i = 10 \times 10^{-3} \text{ A} = 10 \text{ mA}$$

- 42** (c) Total cells $= m \times n = 24 \Rightarrow mn = 24$... (i)

For maximum current in the circuit,

$$R = \frac{mr}{n} \Rightarrow 3 = \frac{m}{n} \times (0.5) \Rightarrow m = 6n$$
 ... (ii)

On solving Eqs. (i) and (ii), we get $m = 12$ and $n = 2$

43 (c) Current, $I_V = \frac{V_V}{R_V} = \frac{5}{20 \times 10^3} = 0.25 \times 10^{-3} \text{ A}$

$\therefore V = I_V(R + r)$
 $110 = 0.25 \times 10^{-3}(R + 20 \times 10^3)$
 $R = 420 \text{ k } \Omega$

44 (c) We have, $i_1 = \frac{E}{r + R_1}$ and $i_2 = \frac{E}{r + R_2}$

From these two equations, we get

$$r = \frac{i_2 R_2 - i_1 R_1}{i_1 - i_2}$$

45 (b) $\frac{1}{R_p} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$

$$R_p = \frac{R}{3}$$

and $R_s = R + R = 2R \Omega$

$$R_{\text{net}} = R_p + R_s$$

$$R_{\text{net}} = 2R + \frac{R}{3} = \frac{7R}{3} \Omega$$

46 (c) $i = \frac{2 + 2}{1 + 1.9 + 0.9} = \frac{4}{3.8} \text{ A}$

For cell A, $E = V + ir$

$$V = E - ir = 2 - \frac{4}{3.8} \times 1.9$$

$$V = 0 \text{ (zero)}$$

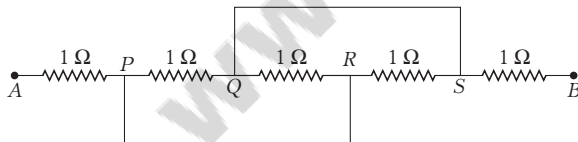
47 (b) Internal resistance,

$$r = \left(\frac{l_1}{l_2} - 1 \right) R = \left(\frac{50}{40} - 1 \right) \times 2 = 0.50 \Omega$$

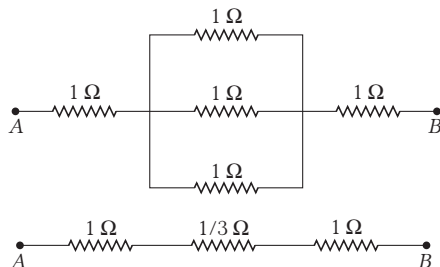
48 (d) $E = \frac{e}{(R + R_h + r)} \cdot \frac{R}{L} \times l$

$$= \frac{2}{(10 + 40 + 0)} \times \frac{10}{1} \times 0.4 = 0.16 \text{ V}$$

49 (c)



In circuit, resistance between PQ, QR and RS are in parallel. Now circuit reduces to



$\therefore R_{AB} = 1 + 1 + \frac{1}{3} = \frac{7}{3} \Omega$

50 (c) Resistance, $R = 3 + 10 || (3 + R) = 3 + \frac{10(3 + R)}{13 + R}$

Solving this equation, we get

$$R = \sqrt{69} \Omega$$

51 (a) Resistance, $R = \rho \cdot \frac{l}{A}$

For same material and same length, $\frac{R_2}{R_1} = \frac{A_1}{A_2} = \frac{3}{1}$

$\therefore R_2 = 3R_1$

Resistance of thick wire, $R_1 = 10 \Omega$

(given)

$\therefore$ Resistance of thin wire, $R_2 = 3 \times 10 = 30 \Omega$

Total resistance of series combination $= 10 + 30 = 40 \Omega$

52 (b) Electric power, $P = i^2 R$

$\therefore$ Current, $i = \sqrt{\frac{P}{R}}$

For resistance of 9Ω ,

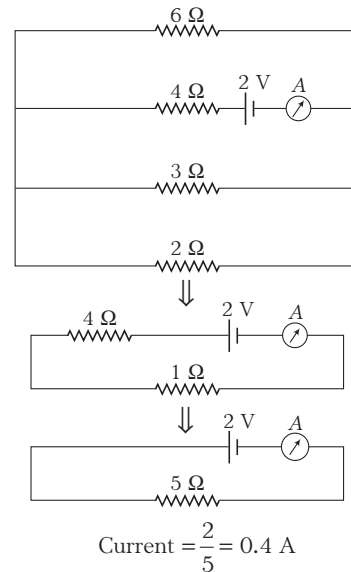
$$i_1 = \sqrt{\frac{36}{9}} = \sqrt{4} = 2 \text{ A}$$

$$i_2 = \frac{i_1 \times R}{6} = \frac{2 \times 9}{6} = 3 \text{ A}$$

$$i = i_1 + i_2 = 2 + 3 = 5 \text{ A}$$

$\Rightarrow V_2 = iR_2 = 5 \times 2 = 10 \text{ V}$

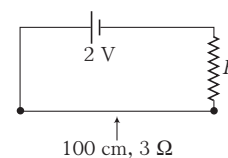
53 (c) The equivalent circuit can be drawn as



54 (c) Total potential drop across the given wire

$$= (1 \times 10^{-3})(10^2) = 0.1 \text{ V}$$

Therefore, potential difference across R should be 1.9 V.



Now,

$$\frac{1.9}{0.1} = \frac{R}{3} \text{ or } R = 57 \Omega$$

55 (a) We have, $r_A = 2r_B$
 $\therefore A_A = 4 A_B$
 or $l_A = \frac{l_B}{4}$
 $\therefore R_B = 16R_A$
 $R_{\text{net}} = \left(\frac{16}{17}\right) R_A = \frac{R_B}{17}$

If $R_{\text{net}} = 4$, then $R_A = 4.25 \Omega$ or $R_B = 68 \Omega$.

56 (d) Balanced condition for Wheatstone's bridge,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

We have, $\frac{24 + X}{84 + Y} = \frac{10}{30} = \frac{1}{3}$... (i)

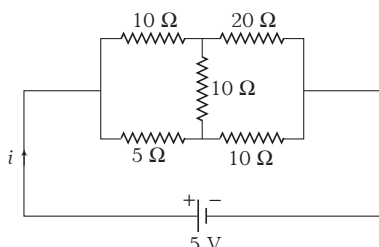
and $\frac{3}{1} = \frac{24 + 84 + X + Y}{10 + 30}$ [$\because V = IR$]

$$= \frac{108 + X + Y}{40}$$
 ... (ii)

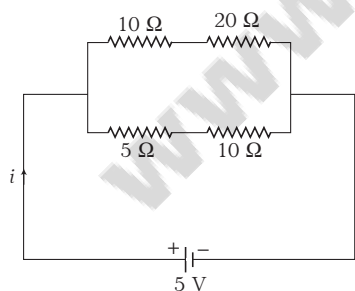
Solving Eqs. (i) and (ii), we get

$$X = Y = 6 \Omega$$

57 (b) The given circuit can be redrawn as



It is a balanced Wheatstone bridge and hence, no current flows in the middle resistor, so equivalent circuit would be as shown in figure.



10 Ω and 20 Ω resistances are in series,

$$\therefore R' = 10 \Omega + 20 \Omega = 30 \Omega$$

Similarly, 5 Ω and 10 Ω are in series, $R' = 15 \Omega$

$$\text{Now, } R' \text{ and } R' \text{ are in parallel, } R = \frac{15 \times 30}{15 + 30} = 10 \Omega$$

$$\text{So, } i = \frac{V}{R} = \frac{5}{10} = 0.5 \text{ A}$$

58 (c) We know that, when current flow is same then resistors are connected in series, hence resultant resistance is

$$R' = R_1 + R_2 = 10 \Omega + 20 \Omega = 30 \Omega$$

Also since, cells are connected in opposite polarities, the resultant emf is

$$E = E_1 - E_2 = 5 \text{ V} - 2 \text{ V} = 3 \text{ V}$$

$$\text{From Ohm's law, } E = iR \Rightarrow i = \frac{E}{R} = \frac{3}{30} = 0.1 \text{ A}$$

59 (b) Here, the resistances of 400 Ω and 10000 Ω are in parallel, so their effective resistance R_p will be

$$R_p = \frac{400 \times 10000}{400 + 10000} = \frac{5000}{13} \Omega$$

Total resistance of the circuit,

$$R = \frac{5000}{13} + 800 = \frac{15400}{13} \Omega$$

$$\text{Current in the circuit, } i = \frac{6}{15400/13} = \frac{39}{7700} \text{ A}$$

Potential difference across voltmeter,

$$V = iR_p = \frac{39}{7700} \times \frac{5000}{13} = 1.95 \text{ V}$$

60 (b) Current, $i = \frac{E}{R + r}$. When R decreases to 0, $i = \frac{E}{r}$ and $V = IR = 0$.

$$61 \text{ (c) Initially, } \frac{P}{Q} = \frac{l}{100 - l} = \frac{20}{100 - 20} = \frac{20}{80} = \frac{1}{4}$$

When a resistance of 15 Ω is connected in series with the smaller of the two, i.e. with P , then

$$\frac{P + 15}{Q} = \frac{40}{60} \Rightarrow \frac{P}{Q} + \frac{15}{Q} = \frac{2}{3}$$

$$\frac{1}{4} + \frac{15}{Q} = \frac{2}{3} \Rightarrow \frac{15}{Q} = \frac{5}{12}$$

$$\Rightarrow Q = 36 \Omega$$

$$\text{Resistance, } P = \frac{1}{4} \times 36 = 9 \Omega$$

62 (d) Effective voltage on charging

$$= 4 \times 1.4 - 2 = 3.6 \text{ V}$$

Total resistance = 8 Ω

$$\Rightarrow i = V/R = 3.6/8 = 0.45 \text{ A}$$

63 (d) Since, on balancing no current flows through the battery so its internal resistance will not be affected.

$$\text{Potential gradient} = \frac{E}{100}$$

$$\text{So, balancing emf} = \frac{E}{100} \times 30 = \frac{30E}{100}$$

64 (c) When a resistance of 100 Ω is connected in series, then the current flowing will be

$$i = \frac{V}{100 + R} \quad \dots(i)$$

When a resistance of 1000 Ω is connected in series, the range of galvanometer gets doubled.

$$\text{Current, } i = \frac{2V}{1100 + R} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{V}{100 + R} = \frac{2V}{1100 + R} \Rightarrow R = 900 \Omega$$

65 (d) Drift velocity, $v_d = \frac{i}{nAe} = \frac{i \times 4}{n\pi D^2 e}$

i.e. $v_d \propto \frac{1}{D^2}$

$$\therefore \frac{v_{d1}}{v_{d2}} = \frac{D_2^2}{D_1^2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

66 (c) Power, $P = \frac{V^2}{R}$

$$\therefore R = \frac{V^2}{P} \text{ or } R \propto V^2$$

i.e. $\frac{R_1}{R_2} = \left[\frac{200}{300}\right]^2 = \frac{4}{9}$

When connected in series, then potential drop and power consumed are in the ratio of their resistances. So,

$$\frac{P_1}{P_2} = \frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{4}{9}$$

67 (b) Current required by each bulb,

$$i = \frac{P}{V} = \frac{100}{220} \text{ A}$$

If n bulbs are joined in parallel, then $ni = i_{\text{fuse}}$

or $n \times \frac{100}{220} = 10 \text{ or } n = 22$

68 (b) Rise in temperature = 15°C

Amount of water = 1L = 1000 cc = 1000 g

Heat supplied in cal = $1000 \times 15 \times 1$

= $1000 \times 15 \times 4.2$ (in form of joule)

$$\therefore \text{Power} = \frac{1000 \times 15 \times 4.2}{60} = 1050 \text{ W or (J/s)}$$

69 (d) Average energy = 2 eV

$$\Rightarrow eV_0 = 2eV_0$$

$$\Rightarrow V_0 = 2 \text{ volt}$$

$$\therefore \text{Electric field, } E = \frac{V_0}{\text{Mean free path}} = \frac{2}{4 \times 10^{-8}} = 5 \times 10^7 \text{ V/m}$$

70 (d) By joining 2Ω and 6Ω in parallel, we get 1.5Ω and joining them in series, we get 8Ω .

So, values of R_1 and R_2 will be 2Ω and 6Ω .

71 (d) $10 \times i = 2 \times 50$

$$i = \frac{100}{10} = 10 \text{ mA}$$

72 (a) Initially, $R_1 = 50 + 2950 = 3000 \Omega$

$$E = 3V \Rightarrow I_1 = \frac{3}{3000} = 1 \times 10^{-3} \text{ A}$$

To reduce deflection to 20,

$$I_2 = \frac{2}{3} \times 1 \times 10^{-3} \text{ A}$$

$$\therefore R \times \frac{2}{3} \times 10^{-3} = 3000 \times 1 \times 10^{-3}$$

$$\Rightarrow R = \frac{3000 \times 3}{2} = 4500 \Omega$$

So, resistance to be added = $4500 - 50 = 4450 \Omega$

73 (b) We have, shunt, $S = \frac{G}{(n-1)}$... (i)

where, G is resistance of galvanometer.

Again, $S' = \frac{G}{(n'-1)}$... (ii)

$$\therefore \frac{S}{S'} = \frac{(n'-1)}{(n-1)}$$

$$\Rightarrow \frac{S(n-1)}{S'} = (n'-1)$$

$$\Rightarrow n' = \frac{S(n-1) + S'}{S'}$$

74 (c) Higher the power, lower is the resistance and lower the resistance, thicker will be the element.

75 (b) Given, $\frac{R_1}{R_2} = \frac{1}{2}$

Let, third resistance is R .

$$\text{So, } \frac{1}{a} + \frac{1}{2a} + \frac{1}{R} = 1$$

$$\therefore a = \frac{3R}{2(R-1)}$$

As resistance is not fractional.

(given)

$$\therefore \frac{R}{R-1} = 2$$

$$\text{So, } R = 2 \Omega$$

$$\therefore R_1 = a = 3 \Omega \text{ and } R_2 = 2a = 6 \Omega$$

76 (d) Applying Kirchhoff's law at point D , we get

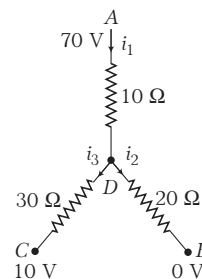
$$i_1 = i_2 + i_3$$

$$\frac{V_A - V_D}{10} = \frac{V_D - 0}{20} + \frac{V_D - V_C}{30}$$

$$\text{or } 70 - V_D = \frac{V_D}{2} + \frac{V_D - 10}{3}$$

$$\Rightarrow V_D = 40 \text{ V} \Rightarrow i_1 = \frac{70 - 40}{10} = 3 \text{ A}$$

$$i_2 = \frac{40 - 0}{20} = 2 \text{ A}$$



and

$$i_3 = \frac{40 - 10}{30} = 1 \text{ A}$$

$$77 \text{ (b)} E_{\text{net}} = \frac{E_1/r_1 - E_2/r_2}{1/r_1 + 1/r_2}$$

Current through R will be zero, if $E_{\text{net}} = 0$

$$\therefore \frac{E_1}{r_1} = \frac{E_2}{r_2}$$

$$78 \text{ (d)} i = \frac{\text{Net emf}}{\text{Net resistance}} = \frac{10 - 4}{1 + 2 + 3} = 1 \text{ A from } a \text{ to } e.$$

79 (b) Electric field produced by charges accumulated on the surface of wire, provides force to the electrons to change the direction of current density.

80 (a) We know that in the given circuit,

$$\epsilon_{\text{eq}} = \frac{\epsilon_1 r_2 + \epsilon_2 r_1}{r_1 + r_2}$$

Here, ϵ_{eq} is weighted average of ϵ_1 and ϵ_2 .

Naturally, its value will lie between the value of ϵ_1 and ϵ_2 ,

i.e. ϵ_{eq} will be less than ϵ_2 and more than ϵ_1 .

81 (c) We know that percentage of error can be minimised by adjusting the balance point near the middle of the meter scale, i.e. around 50 cm. This can be achieved by adjusting the value of S . For balance point to be at 2.9 cm, we have the equation,

$$\frac{R}{S} = \frac{2.9}{97.1}$$

$$\Rightarrow R = \frac{2.9}{97.1} \times 100 = 3 \Omega$$

For balance point to be in the middle, the equation becomes

$$\frac{R}{S} = \frac{1}{1}$$

$$R = S$$

Here, $R = 3 \Omega$

$$\Rightarrow S = 3 \Omega$$

So, the value of S should be changed from 100Ω to 3Ω to obtain reading in the mean position.

82 (b) No current will flow through the grounded wire.

$$\therefore I = \frac{V}{2R}$$

83 (b) We have, $I_G = 0$

$$\therefore V_X = E_2 = 2 \text{ V}$$

$$\frac{V_{500 \Omega}}{V_X} = \frac{(12 - 2)}{2} = \frac{500}{X}$$

$$\therefore X = 100 \Omega$$

$$84 \text{ (a)} \text{ Equivalent resistance of circuit, } R = 4 + \frac{3 \times 6}{3 + 6} = 6 \Omega$$

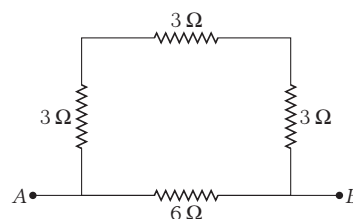
$$\text{and current through battery, } i = \frac{3}{6} = 0.5 \text{ A}$$

$$\text{Potential difference over } 4 \Omega = 0.5 \times 4 = 2 \text{ V}$$

$$\text{Potential difference across the resistor of } 3 \Omega$$

$$\text{over } 6 \Omega = 3 - 2 = 1 \text{ V}$$

85 (d) The given circuit can be reduced to



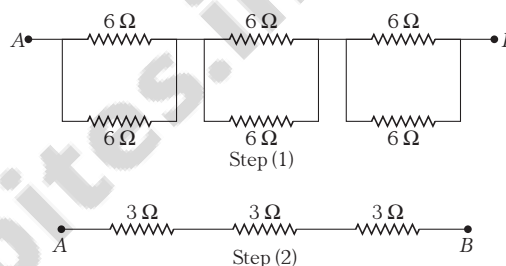
So, equivalent resistance between A and B,

$$R_{AB} = (3 + 3 + 3) \parallel 6 = 9 \parallel 6$$

$$= \frac{9 \times 6}{9 + 6} = \frac{9 \times 6}{15} = \frac{18}{5} \Omega$$

$$\Rightarrow R_{AB} = 3.6 \Omega$$

86 (d) The network can be redrawn as follows

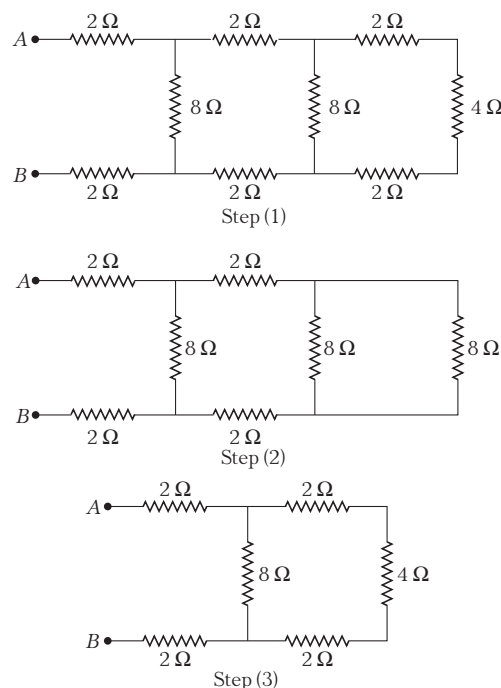


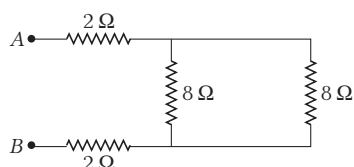
Now, there are three resistances in series.

So, equivalent resistance,

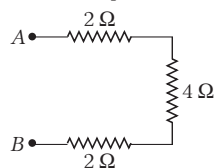
$$R_{\text{eq}} = 3 + 3 + 3 = 9 \Omega$$

87 (d) The circuit can be reduced as,





Step (4)



Step (5)

Now, equivalent resistance between A and B is

$$R_{eq} = 2 + 4 + 2 = 8 \Omega$$

88 (c) Total resistance of the circuit,

$$R = R_4 + \frac{(R_1 + R_2) \times R_3}{(R_1 + R_2) + R_3} = 35 + \frac{(15 + 15) \times 30}{(15 + 15) + 30}$$

$$= 35 + \frac{30 \times 30}{30 + 30} = 50 \Omega$$

Current in circuit, $i = \frac{50}{50} = 1 \text{ A}$

Current through R_3 , $i' = i_2 = \frac{1}{2} \text{ A}$

Potential difference across R_3 ,

$$V_3 = i' \times R_3 = \frac{1}{2} \times 30 = 15 \text{ V}$$

89 (b) Potential difference across 100Ω resistance is 5 V . As voltmeter and 100Ω resistance are in parallel. It means equivalent resistance of voltmeter and 100Ω should be 50Ω .

So, resistance of voltmeter must be 100Ω .

90 (d) Emf of cell, $E = xil = \frac{iR}{L} \times l$

$$\therefore E = \frac{e}{(R_1 + R_2 + r)} \times \frac{R}{L} \times l$$

$$0.4 = \frac{5}{(5 + 45 + 0)} \times \frac{5}{10} \times l$$

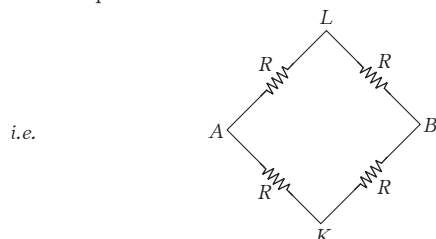
$$\therefore l = 8 \text{ m}$$

91 (d) Net resistance of circuit, $R = 1 + 6 + 4 = 11 \Omega$

$$I = \frac{6}{11} \text{ A}$$

$$\therefore V = \frac{6}{11} \times 1 = \frac{6}{11} \text{ V}$$

92 (a) The given circuit forms a balanced Wheatstone bridge between points A and B.

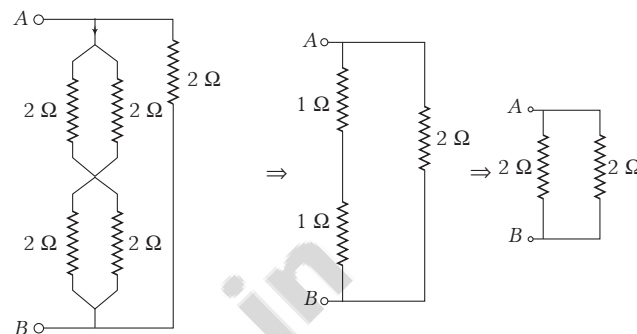


$$R_{AB} = (R + R) \parallel (R + R)$$

$$= 2R \parallel 2R$$

$$= \frac{2R \times 2R}{2R + 2R} = R$$

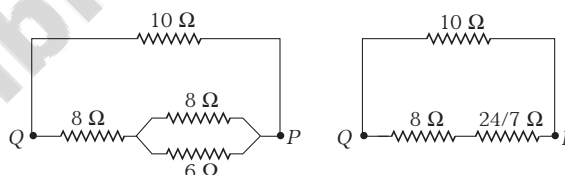
93 (a) The simple circuit is as shown below,



$\therefore$ The equivalent resistance across AB,

$$R_{eq} = \frac{2 \times 2}{2 + 2} = 1 \Omega$$

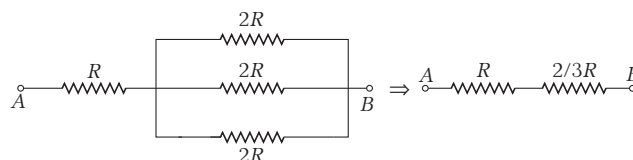
94 (b) The simple circuit is as shown below,



$$R_{net} = \frac{10 \times (8 + 24/7)}{10 + (8 + 24/7)}$$

$$= 5.3 \Omega \approx 5 \Omega$$

95 (a) The given circuit consists of a balanced Wheatstone bridge.



$$\therefore R_{net} = R + \frac{2}{3}R = \frac{5}{3}R$$

96 (c) $q = \int_0^5 i dt = \int_0^5 (1.2t + 3) dt = 30 \text{ C}$

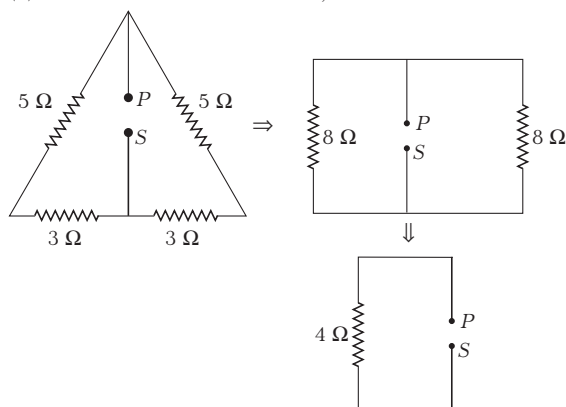
97 (a) As $R_{AB} = R_{AO}$ and $R_{BC} = R_{OC}$, so points O and B will be at same potential and hence, resistance R_{OB} becomes ineffective. Similarly, as $R_{AO} = R_{AD}$ and $R_{OC} = R_{DC}$, resistance R_{OD} becomes ineffective.

So, excluding R_{BO} and R_{OD} , equivalent resistance R_{eq} of the given network between points A and C will be given by

$$\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R}$$

$$\Rightarrow R_{eq} = \frac{2}{3}R$$

- 98 (d) The circuit can be drawn as,



Resistance between points P and $S = 4\ \Omega$

- 99 (b) In the first case, potential difference balances against resistance $R, V_1 = k l_1 \Rightarrow R = k l_1$ ($\because I = 1\text{A}$)
In the second case, potential difference balances against $R + X$,

$$V_2 = k l_2 \Rightarrow R + X = k l_2$$

$\therefore$ Potential difference balance over $X = k(l_2 - l_1)\ \Omega$

- 100 (d) Power = 1.08 kW = 1080 W

$$P \times t = E \Rightarrow 1080 \times t = m \times L$$

$$1080 \times t = 100 \times 540 \times 4.2 \Rightarrow t = 210\text{ s}$$

- 101 (b) For an ammeter, $\frac{i_g}{i} = \frac{S}{G + S} \Rightarrow i_g G = (i - i_g)S$

$$\therefore i_g G = (0.03 - i_g) 4r \quad \dots(i)$$

$$\text{and } i_g G = (0.06 - i_g)r \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$0.12 - 4i_g = 0.06 - i_g$$

Maximum current through the galvanometer, $i_g = 0.02\text{ A}$

- 102 (c) When key K is opened, bulb B_3 will not draw any current from the source. So, that terminal voltage of source increases. Hence, power consumed by bulb increases. So, light of the bulb B_2 becomes more. The brightness of bulb B_1 decreases.

- 103 (a) Maximum current possible in galvanometer,

$$I_{\max} = 25 \times 4 \times 10^{-4}\text{ A} = 10^{-2}\text{ A}$$

$$\text{So, } 10^{-2} = \frac{2.5}{100 + R}$$

$$100 + R = 250$$

$$\text{Resistance, } R = 150\ \Omega$$

- 104 (a) Let, the bulb 400 W is having resistance value of R . For 200 W, necessary value of resistance will be $2R$.

Total value of resistance in the circuit will be $R + R = 2R$

If I is the maximum current in the circuit, then $I^2 R = 400\text{ W}$

$$\text{Power of circuit as a whole} = I^2 \times 2R = 2 \times I^2 R = 2 \times 400 = 800\text{ W}$$

- 105 (b) Maximum current possible in bulb = $\frac{50}{100} = 0.5\text{ A}$

$$\text{Resistance of each bulb} = \frac{V^2}{P} = \frac{100 \times 100}{50} = 200\ \Omega$$

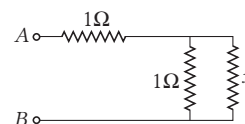
If n be the number of bulbs possible, then total resistance of

$$\text{circuit} = \frac{200}{n} + 10$$

Maximum current in the circuit = $0.5 \times n$

$$\text{So, } \frac{120}{\frac{200}{n} + 10} = 0.5n \Rightarrow n = 4$$

- 106 (c) Let $R_{AB} = x$. Then,



$$R_{AB} = 1 + \frac{x}{1+x} \quad \text{or} \quad x = 1 + \frac{x}{1+x}$$

$$\therefore x + x^2 = 1 + x + x \quad \text{or} \quad x^2 - x - 1 = 0$$

$$x = \frac{1 + \sqrt{1+4}}{2} = \frac{1 + \sqrt{5}}{2}\ \Omega$$

- 107 (a) Potential difference across $4\ \Omega$ resistance = $20 + 16 = 36\text{ V}$

$$\text{Current through } 4\ \Omega \text{ resistance} = \frac{36}{4} = 9\text{ A [from top to bottom]}$$

$$\text{Similarly, current through } 2\ \Omega \text{ resistance} = \frac{20 - 16}{2} = 2\text{ A}$$

Therefore, total current through 20 V battery will be 11 A.

- 108 (b) Current through R_1 and R_2 comes out to be zero (potential difference = 0).

$$\therefore \text{Current through } R_3 = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{(4 + 3 + 2) - (2 + 3 + 2)}{3} = \frac{2}{3}\text{ A}$$

- 109 (b) According to the question, $R = \left(20 + \frac{i}{2}\right)\ \Omega$

$$\text{Now, current, } i = \frac{250}{R} = \frac{250}{20 + (i/2)}$$

$$\Rightarrow i^2 + 40i - 500 = 0$$

Solving, we get

$$i = 10\text{ A}$$

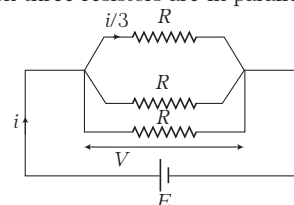
- 110 (d) According to question,

$$Q = at - bt^2 \Rightarrow i = \frac{dQ}{dt} = a - 2bt$$

$$i = 0 \text{ at } t = \frac{a}{2b}$$

$$\frac{di}{dt} = -2b$$

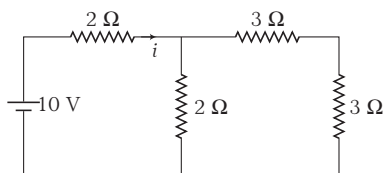
- 111 (a) The given three resistors are in parallel,



$$\therefore \text{Current, } i = \frac{E}{r + R/3} = \frac{4}{1 + 3} = 1 \text{ A}$$

$$V = \frac{i}{3} R = E - ir = 4 - 1(1) = 3 \text{ V}$$

112 (c) Resistance, $R_{\text{net}} = 2 + \frac{6 \times 2}{6 + 2} = \frac{7}{2} \Omega$



$$\therefore i = \frac{10}{(7/2)} = \frac{20}{7} \text{ A}$$

According to current division rule,

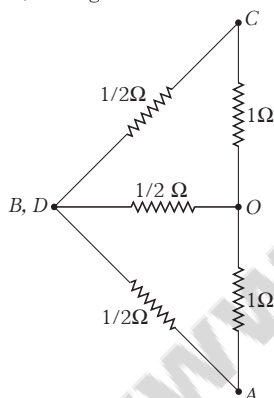
$$i_{3\Omega} = \left(\frac{2}{2+6} \right) i = \frac{1}{4} \cdot i = \frac{5}{7} \text{ A}$$

113 (d) Current, $i_1 = \frac{E_1 + E_2}{r_1 + r_2 + R}$ and $i_2 = \frac{E_1}{R + r_1}$

$$\therefore i_2 > i_1$$

$$\therefore \frac{E_1}{R + r_1} > \frac{E_1 + E_2}{r_1 + r_2 + R} \text{ or } E_1 r_2 > E_2 (R + r_1)$$

114 (c) Both B and D are symmetrically located with respect to points O. Hence, the figure can be folded as shown in figure.

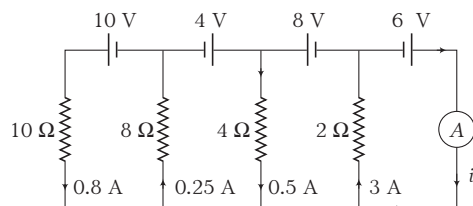


The given circuit forms Wheatstone bridge with DO or BO ineffective.

$$\therefore R_s = 1 + \frac{1}{2} + \frac{1}{2} = 2 \Omega$$

$$\text{Resistance across AO} = \frac{2 \times 1}{2 + 1} = \frac{2}{3} \Omega$$

115 (c) By finding potential difference across any resistance, we can find current through each resistance directly.



$$\therefore i = 3 + 0.25 - 0.5 - 0.8 = 1.95 \text{ A}$$

116 (b) Voltage sensitivity = $\frac{\text{Current sensitivity}}{\text{Resistance of galvanometer } G}$

$$G = \frac{\text{Current sensitivity}}{\text{Voltage sensitivity}} = \frac{10}{2} = 5 \Omega$$

$$\text{Full scale deflection current, } i_g = \frac{150}{10} = 15 \text{ mA}$$

$$\text{Voltage to be measured, } V = 150 \times 1 = 150 \text{ V}$$

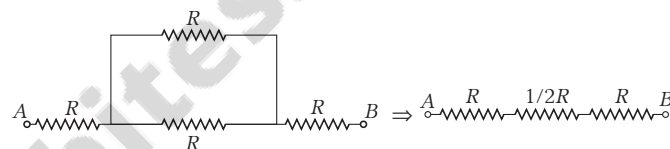
$$\text{Hence, } R = \frac{V}{i_g} - G = \frac{150}{15 \times 10^{-3}} - 5 = 9995 \Omega$$

117 (b) We have, $\frac{V^2}{R} \times 16 = H$ ($\because H = P \times t$)

$$\frac{V^2}{0.9R} \times t = H \Rightarrow \frac{V^2}{R} \times 16 = \frac{V^2}{0.9R} \times t$$

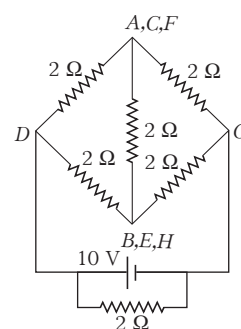
$$\therefore \text{Time, } t = 16 \times 0.9 = 14.4 \text{ min}$$

118 (c) Topmost and bottommost figures are short-circuited. Simplified circuit is shown below



$$R_{\text{eq}} = R + R + \frac{R}{2} = \frac{5R}{2}$$

119 (b) Due to balanced Wheatstone bridge, resistance between A and B can be removed.

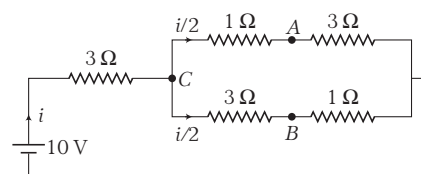


The current between D and E,

$$I_{DE} = \frac{10}{R_{DE} + R_{HG}} = \frac{10}{2 + 2} = 2.5 \text{ A}$$

120 (b) Resistance between upper branch and lower branch in parallel part is same, so equal amount of current flows through them.

Let main current is i .



$$\therefore i = \frac{V}{R_{\text{eq}}}$$

Equivalent resistance of circuit,

$$R_{eq} = 3 + 2 = 5 \Omega \Rightarrow i = \frac{10}{5} = 2 \text{ A}$$

So, current in each branch = 1 A

$$\text{Now, } V_C - V_A = 1 \times 1 = 1 \text{ V} \quad \dots(i)$$

$$\text{and } V_C - V_B = 1 \times 3 = 3 \text{ V} \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we have

$$V_A - V_B = 3 - 1 = 2 \text{ V}$$

121 (d) When K_1 is closed, R_1 is short-circuited.

$$\text{When } K_2 \text{ is open, } I_0 = \frac{E}{r + R_2} = \frac{E}{r + 100} \quad \dots(i)$$

$$\text{When } K_2 \text{ is closed, } I_0 = \frac{1}{2} \left[\frac{E}{r + 50} \right] \quad \dots(ii)$$

From these two equations, we get $r = 0$

$$\text{When } K_1 \text{ is open and } K_2 \text{ is closed, } \frac{I_0}{2} = \frac{E}{2(R_1 + 50)} \quad \dots(iii)$$

From Eqs. (i) and (iii), we have

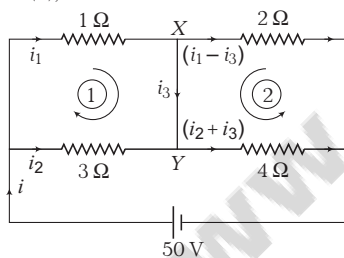
$$R_1 = 50 \Omega$$

122 (b) In the circuit, we can see that 20Ω , 100Ω and 25Ω are in parallel.

$$\text{Net resistance of circuit} = 4 + 6 + \frac{1}{\frac{1}{20} + \frac{1}{100} + \frac{1}{25}} = 20 \Omega$$

$$\therefore V = iR = 4 \times 20 = 80 \text{ V}$$

123 (c) Let current through XY is i_3 . Applying Kirchhoff's law to loops (1) and (2),



$$i_1 + 0 \times i_3 - 3i_2 = 0$$

$$\therefore i_1 = 3i_2 \quad \dots(i)$$

$$\text{and } -2(i_1 - i_3) + 4(i_2 + i_3) = 0$$

$$\text{So, } 2i_1 - 4i_2 = 6i_3 \quad \dots(ii)$$

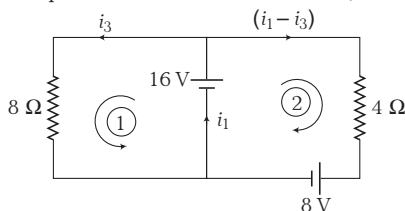
$$\text{Also, } 50 = i_1 + 2(i_1 - i_3)$$

$$\therefore 3i_1 - 2i_3 = 50 \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$i_3 = 2 \text{ A}$$

124 (b) The simplified circuit can be drawn as,



Applying KVL in loop (1) and (2), we get

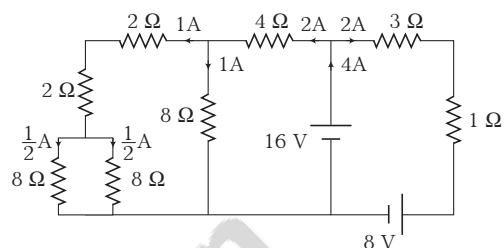
$$8i_3 = 16$$

$$\Rightarrow i_3 = 2 \text{ A} \quad \dots(i)$$

$$\text{and } 4(i_1 - i_3) = 16 - 8 = 8$$

$$\Rightarrow i_1 = 2 + i_3 = 4 \text{ A} \quad [\text{using Eq. (i)}]$$

The current in a circuit is distributed depending on the value of resistance as shown below.



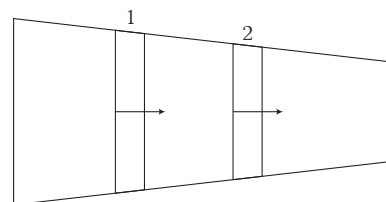
$$\therefore i_2 = \frac{1}{2} \text{ A}$$

$$\Rightarrow \frac{i_1}{i_2} = \frac{4}{(1/2)} = 8$$

(B) Medical entrance special format questions

• Assertion and reason

1 (d) R_1 and R_2 are resistances per unit length.



and

$$V_1 = i_1 R_1$$

$$V_2 = i_2 R_2$$

$$R_2 > R_1$$

$$\left(\text{as } R \propto \frac{1}{A} \right)$$

As,

$$i_1 = i_2$$

Hence,

$$V_2 > V_1$$

3 (b) The resistance of milliammeter becomes high because of increased number of turns of coil, so the torque produced in the coil is not decreased due to low value of current. Milliammeter generally do not have shunt because the main current is already very low. Even if there is shunt, its value is kept high so that current diversion through it is least in case of milliammeter.

4 (d) In both cases potential difference across R is E .

$$\therefore P = \frac{E^2}{R}$$

i.e., power remains same.

In second case, net resistance will decrease. Therefore, main current will increase.

5 (a) $R = \rho \frac{l}{A}$ or $R \propto \frac{1}{A}$

Area of cross-section of wire A is less. Hence, its resistance is more.

Also, $H \propto R$, so more heat is generated in wire A.

• Statement based questions

- 1 (a) In parallel circuit, voltage remains same, so

$$V_{NP} = 20 \times 1 = 20 \text{ V}$$

$$\therefore I_{NP} = \frac{20}{10} = 2 \text{ A}$$

Current through R,

$$I_R = 0.5 + 2 + 1 = 3.5 \text{ A}$$

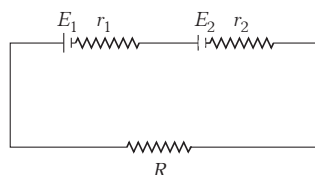
$$V_R = 69 - 20 = 49 \text{ V}$$

$$\therefore R = \frac{49}{3.5} = 14 \Omega$$

$$\text{Also, } V_{R_1} = 20 = 0.5 \times R_1$$

$$\Rightarrow R_1 = 40 \Omega$$

- 2 (d) When two non-ideal batteries of emfs E_1 and E_2 are connected as given below.

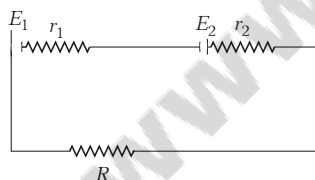


Then, equivalent emf, $E = E_1 + E_2$

$\therefore$ Hence, $|E| > |E_1|$ and $|E| > |E_2|$

Equivalent internal resistance, $r_{eq} = r_1 + r_2$

Again, when two non-ideal batteries of emfs E_1 and E_2 are connected as given below.



Then, equivalent emf, $E' = E_1 - E_2$ ($\because E_1 > E_2$)

Hence, $|E'|$ may be greater than or equal to or less than the individual value of E_1 or E_2 .

Equivalent internal resistance, $r = r_1 + r_2$

Hence, each of statement I and II is wrong.

$$\begin{aligned} 3 (d) R_{\text{eff}} &= \frac{(P+Q)(R+S)}{(P+Q+R+S)} \\ &= \frac{(R+R)(2R+2R)}{(R+R+2R+2R)} \end{aligned}$$

The equivalent resistance of the network between points A and B,

$$R_{\text{eff}} = \frac{4}{3} R$$

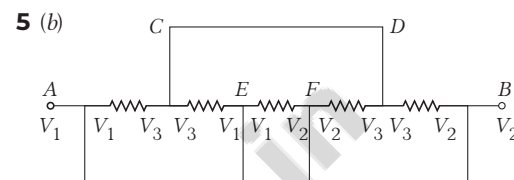
In this case, equivalent resistance of the network between points A and B is independent of the value of G, as no current flows through it.

$$4 (c) E_{\text{eq}} = \frac{E_1/r_1 + E_2/r_2}{1/r_1 + 1/r_2}$$

$$= \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = E_1 \left[\frac{r_2 + r_1(E_2/E_1)}{r_1 + r_2} \right]$$

Therefore, E_{eq} can be greater than, less than or equal to E_1 depending upon the condition whether

$$E_2 > E_1, E_2 = E_1 \text{ or } E_2 < E_1$$



$$\therefore R_{AB} = \frac{R}{2}$$

$$\Rightarrow I_{CD} = \frac{V_0}{R}, I_{EF} = \frac{V_0}{R}$$

• Match the columns

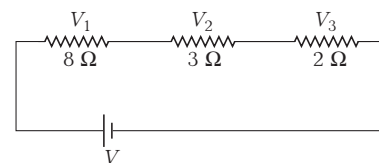
- 1 (a) In parallel connections, net resistance will decrease. Therefore, main current i_1 will increase.

$$V_{AB} = V_{CD} = E - i_1 r$$

With increase in i_1 , V_{AB} or V_{CD} will decrease. Further, $i_2 = \frac{V_{AB}}{R}$.

V_{AB} is decreasing. Therefore, i_2 will also decrease.

- 2 (c) The simple circuit is shown in below figure



As, 3Ω and 6Ω are in parallel connection.

Hence, potential difference across 6Ω is also 20 V.

$$\begin{aligned} \text{And, for } 8 \Omega, \frac{V_1}{8} &= \frac{20}{3} + \frac{20}{6} = \frac{60}{6} = 10 \\ V &= 80 \text{ V} \end{aligned}$$

Similarly, for 12Ω and 4Ω ,

$$\begin{aligned} \frac{V_1}{8} &= \frac{V_2}{3} \\ \Rightarrow \frac{80}{8} &= \frac{V_2}{3} \\ \Rightarrow V_2 &= 30 \text{ V} \end{aligned}$$

(C) Medical entrances' gallery

- 1 (a) Given, drift velocity, $v_d = 7.5 \times 10^{-4}$ m/s

Electric field, $E = 3 \times 10^{-10}$ Vm⁻¹

Mobility, $\mu = ?$

$$\text{As, } \mu = \frac{v_d}{E} = \frac{7.5 \times 10^{-4}}{3 \times 10^{-10}} \\ = 2.5 \times 10^6 \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$$

- 2 (c) According to the carbon colour code for resistors,

Code of yellow = 4

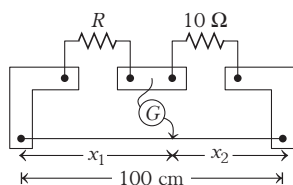
Code of violet = 7

Code of brown, i.e. multiplier = 10^1

Code of gold, i.e. tolerance = $\pm 5\%$

Hence, resistance of resistor = $47 \times 10^1 \Omega$, 5%
= 470Ω , 5%

- 3 (a) According to the question, the metre bridge is shown below,



Given, $\frac{x_1}{x_2} = \frac{3}{2}$

At balance condition in metre bridge,

$$\frac{R}{10} = \frac{x_1}{x_2}$$

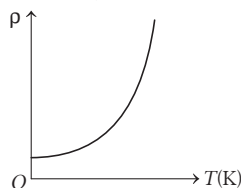
$$\Rightarrow R = \frac{x_1}{x_2} \times 10 = \frac{3}{2} \times 10 = 15 \Omega$$

Now, length of given wire of resistance 15Ω is 1.5 m.

Therefore, length of 1Ω resistance wire is

$$= \frac{1.5}{15} = \frac{1}{10} = 0.1 \text{ or } 1 \times 10^{-1} \text{ m}$$

- 4 (b) Resistivity of copper (a metal) as a function of temperature increases with the increase in temperature as shown below,



For copper at 0K, value of resistivity is $1.7 \times 10^{-8} \Omega\text{-m}$.

Hence, correct option is (b).

- 5 (b) Applying KVL in the loop, we get

$$4I + I \cdot 1 - 4 + I \cdot 1 - 2 = 0$$

$$\Rightarrow 6I = 6 \Rightarrow I = 1 \text{ A}$$

- 6 (b) Given, $R_1 = R_2$, $l_1 = l_2$

$$\text{Since, resistance, } R = \rho \cdot \frac{l}{A} \Rightarrow \frac{R_1}{R_2} = \frac{l_1}{l_2} \cdot \frac{A_2}{A_1}$$

$$\Rightarrow \frac{R_1}{R_1} = \frac{l_1}{l_1} \cdot \frac{A_2}{A_1}$$

$$\Rightarrow 1 = \frac{A_2}{A_1}$$

$$\text{or } \frac{A_1}{A_2} = 1$$

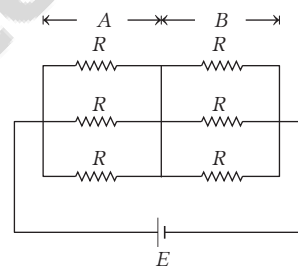
- 7 (b) Applying KVL rule in loop BCDEB,

$$R_2 i_2 + E_2 - E_3 - i_3 R_1 = 0$$

- 8 (b) Equivalent resistance between points A and B is

$$R_{AB} = [(4 + 8) \parallel 6] + 4 + 8 = [12 \parallel 6] + 12 \\ = \left[\frac{12 \times 6}{12 + 6} \right] + 12 = [4] + 12 = 16 \Omega$$

- 9 (a) **Case I** When all bulbs are glowing, then the circuit can be realised as shown in the figure below.



$\therefore$ The equivalent resistance of this circuit is

$$R_{eq} = R_A + R_B$$

As, section A has three parallel resistance, so equivalent resistance,

$$R_A = \frac{R}{3}$$

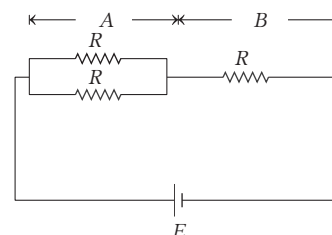
Similarly, for section B, equivalent resistance, $R_B = \frac{R}{3}$

$$\therefore R_{eq} = \frac{R}{3} + \frac{R}{3} = \frac{2R}{3} \quad \dots (i)$$

Thus, power consumed in this circuit,

$$P_1 = \frac{V^2}{R} = \frac{E^2}{R_{eq}} = \frac{3E^2}{2R} \quad [\text{using Eq. (i)}] \dots (ii)$$

Case II When two from section A and one from section B glow, the circuit can be realised as shown in the figure below.



∴ Equivalent resistance of section A,

$$R_A = \frac{R}{2}$$

and of section B,

$$R_B = R$$

Thus, equivalent resistance of the entire circuit becomes

$$R_{eq} = R_A + R_B = \frac{R}{2} + R = \frac{3R}{2} \quad \dots (iii)$$

∴ Power consumed by this circuit,

$$P_2 = \frac{V^2}{R} = \frac{E^2}{R_{eq}} = \frac{2E^2}{3R} \quad [\text{using Eq. (iii)}] \dots (iv)$$

So, ratio of power in two cases is obtained from Eqs. (ii)

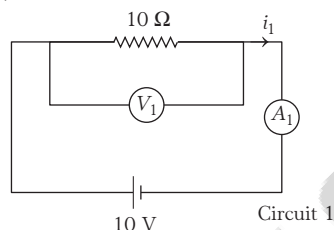
$$\text{and (iv), } \frac{P_1}{P_2} = \frac{3E^2}{2R} \times \frac{3R}{2E^2} = \frac{9}{4} \text{ or } 9:4$$

- 10 (c)** Among given devices fuse is used in electric circuit as a protection device.

It helps in preventing excessive amount of current to flow in the circuit or from short circuiting.

It has low melting point and low resistivity, so when excess amount of current flows in the circuit, then due to excessive amount of heat, it melts and breaks the circuit.

- 11 (b)** For an ideal voltmeter, the resistance is infinite and for an ideal ammeter, the resistance is zero.

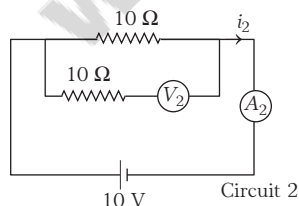


So, the current in circuit 1 is

$$R \times i = V \text{ or } 10 i_1 = 10 \Rightarrow i_1 = \frac{10}{10} = 1 \text{ A}$$

$$\therefore V_1 = i_1 \times R = 1 \times 10 = 10 \text{ V}$$

Similarly, for circuit 2, the addition of 10Ω to voltmeter does not affect the current and hence



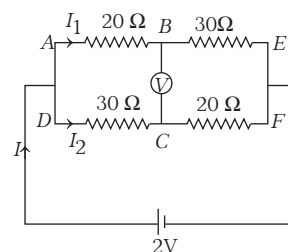
$$10 i_2 = 10 \Rightarrow i_2 = \frac{10}{10} = 1 \text{ A}$$

$$V_2 = i_2 R = 1 \times 10 = 10 \text{ V}$$

$$\therefore V_1 = V_2 \text{ and } i_1 = i_2$$

- 12 (d)** The balance condition still remains the same, if the jockey points the same point P given in the initial condition, for which there is no deflection in the galvanometer or no current will be drawn from the cell. Thus, the bridge will work as usual and balance condition is same, $P/Q = l_1/l_2$.

- 13 (d)** The given circuit diagram can be drawn as shown below



The equivalent resistance of circuit is given by

$$\begin{aligned} \frac{1}{R_{eq}} &= \frac{1}{R_{AE}} + \frac{1}{R_{DF}} \\ &= \frac{1}{(20 + 30)} + \frac{1}{(30 + 20)} \\ &= \frac{1}{50} + \frac{1}{50} = \frac{2}{50} \end{aligned}$$

$$\Rightarrow R_{eq} = 25 \Omega$$

$$\text{The current in circuit, } I = \frac{V}{R} = \frac{2}{25} \text{ A}$$

As the resistance of two branches is same, i.e. 50Ω .

So, the current $I_1 = I_2$

$$\Rightarrow I = I_1 + I_2$$

$$\Rightarrow \frac{2}{25} = 2I_1$$

$$\Rightarrow I_2 = I_1 = \frac{1}{25} \text{ A}$$

∴ The voltage across AB,

$$V_1 = I_1 R_1 = \frac{1}{25} \times 20 \text{ V}$$

and voltage across CD,

$$V_2 = I_2 R_2 = \frac{1}{25} \times 30 \text{ V}$$

∴ Voltmeter reading

$$= V_2 - V_1 = \frac{30}{25} - \frac{20}{25} = \frac{10}{25} = 0.4 \text{ V}$$

- 14 (a)** Given, for a wire, $\frac{R}{l} = \frac{1}{2}$

Length of wire, $l = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

$$\therefore R = \frac{l}{2} = 2.5 \times 10^{-2} \Omega$$

Potential difference, $V = 1 \text{ V}$ or $IR = 1$

$$I = \frac{1}{R} = \frac{1}{2.5 \times 10^{-2}} = \frac{100}{2.5} = 40 \text{ A}$$

- 15 (c)** Given, current, $I = 10 \text{ A}$

Area of cross-section, $A = 4 \times 10^{-6} \text{ m}^2$

Density of conductors, $\rho = 2.7 \text{ g/cc} = 2.7 \times 10^3 \text{ kg/m}^3$

Molecular weight of aluminium, $M_w = 27 \text{ g} = 27 \times 10^{-3} \text{ kg}$

If n be the total number of electrons in the conductor per unit volume, then

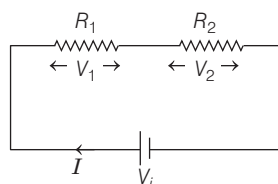
$$\begin{aligned}
 n &= \frac{\text{Total number of electrons}}{\text{Volume of conductor (V)}} \\
 &= \frac{\text{Number of atoms per mole} \times \text{Number of moles}}{V} \\
 &= \frac{\text{Avogadro number}}{V} \times \left(\frac{M}{M_w} \right) \\
 &= 6 \times 10^{23} \times \frac{\rho}{M_w} = 6 \times 10^{23} \times \frac{2.7 \times 10^3}{27 \times 10^{-3}}
 \end{aligned}$$

$$\therefore n = 6 \times 10^{28}$$

We know that, drift velocity,

$$\begin{aligned}
 v_d &= \frac{I}{neA} = \frac{10}{6 \times 10^{28} \times 1.6 \times 10^{-19} \times 4 \times 10^{-6}} \\
 &= 2.6 \times 10^{-4} \text{ m/s}
 \end{aligned}$$

- 16 (b)** The situation is shown in the circuit diagram.



Current flowing through the circuit, $I = \frac{V_i}{R_1 + R_2}$

Voltage across R_2 , $V_2 = IR_2$

$$V_2 = \frac{V_i R_2}{R_1 + R_2} \Rightarrow \frac{V_i}{V_2} = \frac{R_1 + R_2}{R_2}$$

- 17 (d)** If R_1 be the equivalent resistance of parallel resistors 8Ω , 8Ω and 4Ω , then

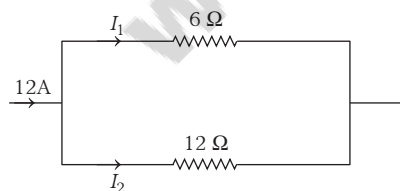
$$\frac{1}{R_1} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{4}{8}$$

$$\Rightarrow R_1 = 2\Omega$$

$\therefore$ Total resistance of upper branch,

$$R_2 = R_1 + 4 = 2 + 4 = 6\Omega$$

Now, circuit can be redrawn as



By current division rule, current through 12Ω resistor,

$$I_2 = \frac{12 \times 6}{12 + 6} = 12 \times \frac{1}{3} = 4\text{A}$$

$\therefore$ Voltage across 12Ω resistor,

$$V = I_2 R = 4 \times 12 = 48\text{V}$$

- 18 (b)** As the other end of 8Ω resistor is grounded, which is at zero potential, the potential difference across 8Ω resistor = $24 - 0 = 24\text{V}$

So, current in 8Ω resistor,

$$I = \frac{V}{R} = \frac{24 - 0}{8} = 3\text{A}$$

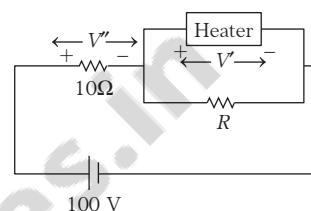
- 19 (a)** Given, resistivity of copper, $\rho = 1.72 \times 10^{-8} \Omega\text{-m}$

Electrons density, $n = 8.5 \times 10^{28} / \text{m}^3$

$$\begin{aligned}
 \therefore \text{Mobility } (\mu) &= \frac{1}{\rho ne} = \frac{1}{1.72 \times 10^{-8} \times 8.5 \times 10^{28} \times 1.6 \times 10^{-19}} \\
 &\approx 4.25 \times 10^{-3} \text{ m}^2/\text{C } \Omega
 \end{aligned}$$

- 20 (a)** Power rating of heater, $P = 1000\text{W}$

Voltage rating of heater, $V = 100\text{V}$



$\therefore$ Resistance of heater,

$$R_1 = \frac{V^2}{P} = \frac{(100)^2}{1000} = 10\Omega$$

According to question, power dissipated in heater,

$$P' = 62.5\text{W}$$

$\therefore$ Voltage (V') across heater can be calculated as

$$P' = \frac{(V')^2}{R_1}$$

$$\Rightarrow (V')^2 = P' \times R_1 = 62.5 \times 10$$

$$\Rightarrow V' = 25\text{V (across heater)}$$

$\therefore$ Voltage across 10Ω resistor,

$$V'' = 100 - 25 = 75\text{V}$$

Current in 10Ω resistor

$$= \frac{V''}{10} = \frac{75}{10} = 7.5\text{A}$$

Current in heater resistor

$$= \frac{V'}{10} = \frac{25}{10} = 2.5\text{A}$$

So, current in R , $I = 7.5 - 2.5 = 5\text{A}$

Now, $V = IR$

$$\Rightarrow R = V / I = \frac{25}{5} = 5\Omega$$

- 21 (b)** Given, $R = (47 \pm 4.7)\text{k}\Omega$

$$= 47 \times 10^3 \pm 10\% \Omega$$

As per the colour code for carbon resistors, the colour assigned to numbers, 4-Yellow, 7-Violet, 3-Orange.

For $\pm 10\%$ accuracy, the colour is silver. Hence, the bands of colours on carbon resistor are in sequence Yellow, Violet, Orange and Silver.

- 22** (c) When n equal resistors of resistances R are connected in series, then the current drawn is given as

$$I = \frac{E}{nR + r}$$

where, nR = equivalent resistance of n resistors in series and r = internal resistance of battery.

Given, $r = R$

$$\Rightarrow I = \frac{E}{nR + R} = \frac{E}{R(n + 1)} \quad \dots(i)$$

Similarly, when n equal resistors are connected in parallel, then the current drawn is given as

$$I' = \frac{E}{\frac{R}{n} + R}$$

where, $\frac{R}{n}$ = equivalent resistance of n resistors in parallel.

Given, $I' = 10I$

$$\Rightarrow 10I = \frac{E}{\frac{R}{n} + R} = \frac{nE}{(n + 1)R} \quad \dots(ii)$$

Substituting the value of I from Eq. (i) in Eq. (ii), we get

$$10 \left(\frac{E}{R(n + 1)} \right) = \frac{nE}{R(n + 1)}$$

$$\Rightarrow n = 10$$

- 23** (c) If n identical cells are connected in series, then equivalent emf of the combination,

$$E_{eq} = nE$$

Equivalent internal resistance,

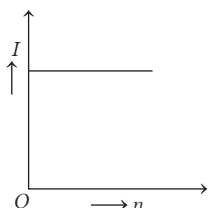
$$r_{eq} = nr$$

$$\therefore \text{Current, } I = \frac{E_{eq}}{r_{eq}} = \frac{nE}{nr}$$

$$\text{or } I = \frac{E}{r} = \text{constant}$$

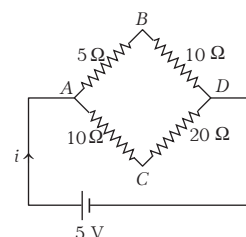
Thus, current (I) is independent of the number of cells (n) present in the circuit.

Therefore, the graph showing the relationship between I and n would be as shown below



- 24** (a) Given circuit satisfies the Wheatstone bridge condition, so no current flows in the branch BC and it behaves like an open circuit.

So, circuit becomes as shown below



Now, in the above circuit, $R_{ABD} = 5 + 10 = 15 \Omega$

$$R_{ACD} = 10 + 20 = 30 \Omega$$

Resistance R_{ABD} and R_{ACD} are in parallel, so

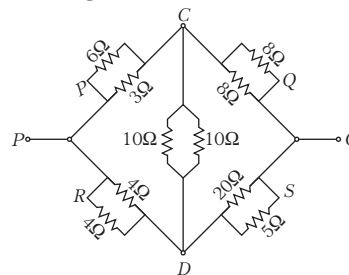
$$R_{net} = \frac{15 \times 30}{30 + 15} = \frac{450}{45} = 10 \Omega$$

$$\therefore i = \frac{E}{R_{net}} = \frac{5}{10} = 0.5 \text{ A}$$

- 25** (c) During charging, $E = V - ir$ (due to reversed current). In case of charging, emf of a cell is less than its terminal voltage while in case of discharging emf is greater than terminal voltage.

Therefore, Assertion is correct but Reason is incorrect.

- 26** (b) Figure is equivalent to the one shown below. It is a Wheatstone's bridge in which



$$P = \frac{6 \times 3}{6 + 3} = 2 \Omega$$

$$Q = \frac{8 \times 8}{8 + 8} = \frac{64}{16} = 4 \Omega$$

$$R = \frac{4 \times 4}{4 + 4} = \frac{16}{8} = 2 \Omega$$

$$\text{and } S = \frac{20 \times 5}{20 + 5} = 4 \Omega$$

$$\text{We find that, } \frac{P}{Q} = \frac{2}{4} = \frac{R}{S}$$

i.e., the bridge is balanced and resistance of arm CD is ineffective. Effective resistance between P and Q ,

$$R_{PQ} = \frac{(P + Q)(R + S)}{P + Q + R + S} = \frac{(2 + 4)(2 + 4)}{2 + 4 + 2 + 4} = \frac{6 \times 6}{12} = 3 \Omega$$

- 27** (a) Switching results in high decay/growth rate of current which results in a high current when bulb is turned ON or OFF (due to back emf). So, a bulb is most likely to get fused when it is just turned ON or OFF.

- 28 (c) $\therefore$ Current, $I = nAev_d$

or $v_d \propto \frac{1}{A}$

If diameter of wire is $d/4$, then area will be $A/16$, so new drift velocity $= 16v_d$.

- 29 (c) Volume of material remains same in stretching.

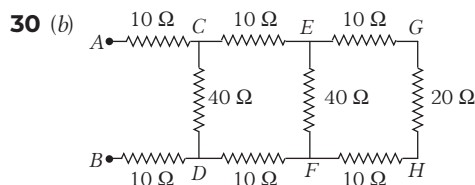
As volume remains same, $A_1 l_1 = A_2 l_2$

Given, $l_2 = nl_1$

$\therefore$ New area, $A_2 = A_1 l_1 / l_2 = A_1 / n$

Resistance of wire after stretching,

$$R_2 = \rho \frac{l_2}{A_2} = \rho \frac{nl_1}{A_1/n} = \left(\rho \frac{l_1}{A_1} \right) n^2 = n^2 R \quad \left[\because R = \left(\rho \frac{l_1}{A_1} \right) \right]$$



In the circuit, the branch $EGHF$ have three resistances of 10Ω , 20Ω and 10Ω respectively, which are connected in series combination.

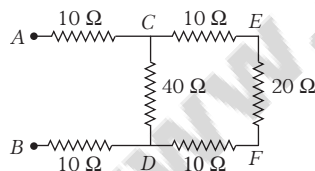
So, their equivalent resistance is given by

$$R_1 = 10 + 20 + 10 = 40 \Omega$$

This R_1 resistance is in parallel with 40Ω resistance which is connected in the branch EF . So, their equivalent resistance,

$$R_2 = \frac{40 \times 40}{40 + 40} = 20 \Omega$$

Now, circuit becomes

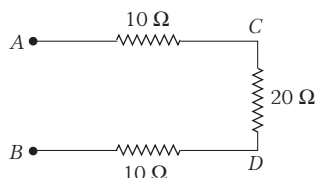


Now, in the branch $CEFD$ 10Ω , 20Ω and 10Ω resistance are connected in series combination and their equivalent resistance is given by

$$R_3 = 10 + 20 + 10 = 40 \Omega$$

This R_3 is in parallel with 40Ω resistance which is in branch CD . Their equivalent resistance, $R_4 = \frac{40 \times 40}{40 + 40} = 20 \Omega$

Now, circuit becomes



The net resistance between A and B,

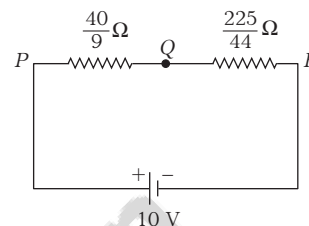
$$R_{\text{net}} = 10 + 20 + 10 = 40 \Omega$$

- 31 (c) In the given circuit, net resistance between P and Q points is R_{PQ} and net resistance between Q and R points is R_{QR} , then

$$R_{PQ} = \frac{10 \times 8}{10 + 8} = \frac{80}{18} = \frac{40}{9} \Omega$$

$$R_{QR} = \frac{10 \times 18 \times 25}{10 \times 18 + 18 \times 25 + 25 \times 10} = \frac{4500}{880} = \frac{225}{44} \Omega$$

So, circuit will be



In the series circuit, voltages will be divide in the ratio of their resistance.

$$\therefore \frac{R_{PQ}}{R_{QR}} = \frac{V_{PQ}}{V_{QR}}$$

$$\therefore \frac{\left(\frac{40}{9} \right)}{\left(\frac{225}{44} \right)} = \frac{V_{PQ}}{V_{QR}} \quad \text{or} \quad \frac{1760}{2025} = \frac{V_{PQ}}{V_{QR}}$$

$$\Rightarrow \frac{1760}{1760 + 2025} = \frac{V_{PQ}}{V_{PQ} + V_{QR}}$$

$$\Rightarrow \frac{1760}{3785} = \frac{V_{PQ}}{10}$$

$$\Rightarrow V_{PQ} = 4.65 \text{ V}$$

- 32 (b) $\therefore$ 60Ω and 30Ω resistors are connected in parallel.

So, their net resistance

$$R_{\text{net}} = \frac{30 \times 60}{30 + 60} = \frac{1800}{90} = 20 \Omega$$

$$\therefore i = \frac{2}{R_{\text{net}}} = \frac{2}{20} = 0.1 \text{ A}$$

- 33 (b) As each arc containing n lamps, hence resistance of each arc $= nr$, number of arcs $= N/n$

Equivalent resistance S is given by

$$\frac{1}{S} = \sum \frac{1}{nr} = \frac{N}{n} \left(\frac{1}{nr} \right)$$

$$S = \frac{n^2 r}{N}$$

$$\therefore \text{Total resistance} = R + S = R + \frac{n^2 r}{N}$$

If E is the emf of the machine, current entering the arcs is $E / (R + S)$ and in each arc is $nE / (R + S)N$.

Hence, current passing through each lamp,

$$I = \frac{nE}{N(R + n^2 r / N)} = \frac{E}{N} \left[\frac{R}{n} + \frac{nr}{N} \right]^{-1}$$

Now, heat produced per second in the lamps is $H = NrI^2$.

Since, light emitted is proportional to H^2 , therefore light produced is maximum when H^2 and hence H is maximum or $\left[\frac{R}{n} + \frac{nr}{N}\right]$ is minimum.

Hence, we can write, $\frac{R}{n} + \frac{nr}{N} = \left[\left(\frac{R}{n}\right)^{1/2} - \left(\frac{nr}{N}\right)^{1/2}\right]^2 + 2\left(\frac{Rr}{N}\right)^{1/2}$

This is minimum, when $\sqrt{\frac{R}{n}} - \sqrt{\frac{nr}{N}} = 0$

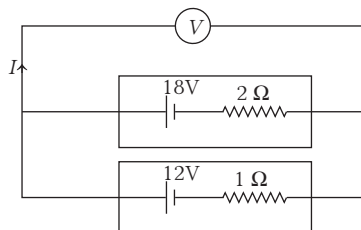
or very small or n is closely equal to $(NR/r)^{1/2}$.

- 34** (a) It is clear that the two cells oppose each other, hence the effective emf in closed circuit is $18 - 12 = 6\text{V}$ and net resistance is $1 + 2 = 3\Omega$ (because in the closed circuit the internal resistance of two cells are in series).

The current in circuit will be in direction of arrow shown in figure

$$I = \frac{\text{effective emf}}{\text{total resistance}} = 6/3 = 2\text{A}$$

The potential difference across V will be same as the terminal voltage of either cell.



Since, current is drawn from the cell of 18V, hence

$$V_1 = E_1 - Ir_1 = 18 - (2 \times 2) = 18 - 4 = 14\text{V}$$

- 35** (a) When a voltmeter is connected across the two terminals of a cell, it draws a small current from the cell, so it measures terminal potential difference between the two terminals of the cell, which is always less than the emf of the cell. On the other hand, when a potentiometer is used for the measurement of emf of cell, it does not draw any current from the cell. Hence, it accurately measures the emf of cell. Thus, a potentiometer is preferred over a voltmeter.

- 36** (c) Let R be the resistance of the wire, then

(i) The heat generated is $H_1 = \frac{V^2 t}{R}$.

- (ii) Resistance of each part will be $R/2$. When they are connected in parallel, the resistance will be $R/4$.

Hence, $H_2 = 4V^2 t/R$.

- (iii) In case of four wires connected in parallel, the resistance will be $R/8$.

$\therefore H_3 = \frac{8V^2 t}{R}$

(iv) $H_4 = \frac{V^2 t}{R/2} = \frac{2V^2 \cdot t}{R}$

Hence, largest amount of heat will be generated in case of four parts connected in parallel.

- 37** (d) Here, 2Ω and 2Ω are in parallel.

$$\begin{aligned} \therefore \frac{1}{R} &= \frac{1}{2} + \frac{1}{2} \\ \Rightarrow R &= \frac{2 \times 2}{2 + 2} = 1\Omega \end{aligned}$$

Now, resistance 1Ω , 2Ω , 4Ω and 1Ω are in series.

$\therefore R_{\text{net}} = 1 + 2 + 4 + 1 = 8\Omega$

Hence, current, $I = \frac{V}{R} = \frac{4}{8} = 0.5\text{A}$

- 38** (c) According to question, emf of the cell is directly proportional to the balancing length, i.e., $E \propto l$... (i)

Now, in the first case, cells are connected in series to support one another, i.e. net emf $= E_1 + E_2$

From Eq. (i), $E_1 + E_2 = 50\text{ cm}$ (given) ... (ii)

Again cells are connected in series but in opposite direction, i.e. net emf $= E_1 - E_2$

From Eq. (i), $E_1 - E_2 = 10\text{ cm}$ (given) ... (iii)

From Eqs. (ii) and (iii),

$$\begin{aligned} \frac{E_1 + E_2}{E_1 - E_2} &= \frac{50}{10} \\ \Rightarrow \frac{E_1}{E_2} &= \frac{5 + 1}{5 - 1} = \frac{6}{4} = \frac{3}{2} \end{aligned}$$

- 39** (d) Given, charge, $Q = at - bt^2$

$\therefore$ We know that current, $I = \frac{dQ}{dt}$... (i)

So, Eq. (i) can be written as

$$\begin{aligned} I &= \frac{d}{dt}(at - bt^2) \\ \Rightarrow I &= a - 2bt \end{aligned} \quad \dots (ii)$$

For maximum value of t ,

$$\begin{aligned} \frac{dQ}{dt} &= 0 \\ \text{or } \frac{d}{dt}(at - bt^2) &= 0 \end{aligned}$$

$\Rightarrow a - 2bt = 0$

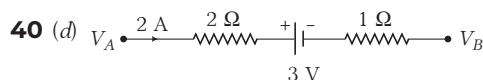
$\therefore t = \frac{a}{2b}$... (iii)

$\therefore$ The total heat produced (H) can be given as

$$\begin{aligned} H &= \int_0^t I^2 R dt = \int_0^{a/2b} (a - 2bt)^2 R dt \\ &= \int_0^{a/2b} (a^2 + 4b^2 t^2 - 4abt) R dt \\ H &= \left[a^2 t + 4b^2 \frac{t^3}{3} - \frac{4abt^2}{2} \right]_0^{a/2b} R \end{aligned}$$

Solving above equation, we get

$\Rightarrow H = \frac{a^3 R}{6b}$



Applying KVL,

$$V_A + \Sigma V = V_B + 2 \times 2 + 2 \times 1$$

$$V_A - V_B - 3 = 4 + 2; V_A - V_B = 9 \text{ V}$$

41 (c) If a rated voltage and power are given, then

$$P_{\text{rated}} = \frac{V_{\text{rated}}^2}{R}$$

∴ Resistance of bulb,

$$R_b = \frac{100 \times 100}{500} = 20 \Omega$$

And current in the bulb, $i = \frac{P}{V}$

$$\Rightarrow i = \frac{500}{100} = 5 \text{ A}$$

∴ Resistance R is connected in series,

$$\therefore \text{Current, } i = \frac{E}{R_{\text{net}}} = \frac{230}{R + R_b}$$

$$\Rightarrow R + 20 = \frac{230}{5} = 46$$

$$\therefore R = 26 \Omega$$

42 (a) Given, $l = 4 \text{ m}$

R = potentiometer wire resistance = 8Ω

$$\text{Potential gradient} = \frac{dV}{dl} = 1 \text{ mV cm}^{-1}$$

So, for 400 cm, $\Delta V = 400 \times 1 \times 10^{-3} = 0.4 \text{ V}$

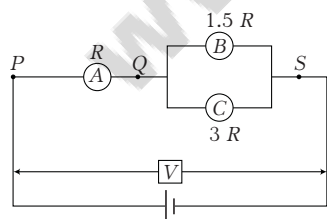
Let a resistor R_S is connected in series, so as

$$\Delta V = \frac{V}{R + R_S} \times R$$

$$\Rightarrow 0.4 = \frac{2}{8 + R_S} \times 8$$

$$\Rightarrow 8 + R_S = \frac{16}{0.4} = 40 \Rightarrow R_S = 32 \Omega$$

43 (a) The equivalent resistance between Q and S is given by



$$\frac{1}{R'} = \frac{1}{1.5R} + \frac{1}{3R} = \frac{2+1}{3R} \Rightarrow R' = R$$

Now, $V_{PQ} = V_A = IR$

Also, $V_{QS} = V_B = V_C = IR$

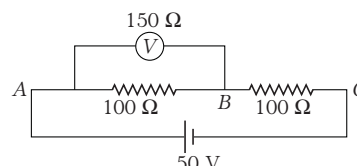
Hence, $V_A = V_B = V_C$

44 (b) As the cross-sectional area of the conductor is non-uniform, so current density will be different.

$$\text{As, } I = JA \quad \dots(i)$$

It is clear from Eq. (i), when area increases the current density decreases so the number of flow of electrons will be same and thus the current will be constant.

45 (c) When voltmeter is connected across A and B , the equivalent resistance of the circuit is



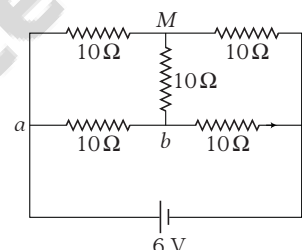
$$R_{\text{eq}} = 100 + \frac{150 \times 100}{100 + 150} = 100 + \frac{15000}{250} = 100 + 60 = 160 \Omega$$

$$\therefore \text{Current, } i = \frac{50}{160} = \frac{5}{16} \text{ A}$$

Therefore, potential drop across B and C is

$$V_{BC} = i R_{BC} = \frac{5}{16} \times 100 = \frac{500}{16} = 31.25 \text{ V} \approx 31 \text{ V}$$

46 (a) Figure can be redrawn as



$$\text{As, } \frac{10}{10} = \frac{10}{10} \quad (\text{from figure})$$

So, circuit is said to be Wheatstone bridge in balanced condition and hence current through arm Mb is zero, i.e. no current flows.

Now, the equivalent resistance of the circuit is given by

$$\frac{1}{R_{\text{eq}}} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} \Rightarrow R_{\text{eq}} = 10 \Omega$$

∴ Current supplied by the battery,

$$i = \frac{V}{R} = \frac{6}{10} = 0.6 \text{ A}$$

47 (a) Suppose, actual current through the ammeter is I .

Now, we can write

$$Ir = kl_1 \Rightarrow I \times (1) = k(75) \quad \dots(i)$$

where, k is a constant.

$$\text{Similarly, } 1.02 = k(50) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{I}{1.02} = \frac{75}{50} = \frac{3}{2}$$

$$\Rightarrow I = \frac{3}{2} (1.02) = 3 (0.51) = 1.53 \text{ A}$$

Thus, error in ammeter reading is

$$\Delta I = (1.53 - 1.5) \text{ A} = 0.03 \text{ A}$$

- 48 (b)** The current in case of voltmeter of range 10 V and resistance 50 Ω is

$$I = \frac{10}{50} \text{ A} \quad \dots(i)$$

Let X be the resistance connected in order to make a voltmeter with range 15 V having current,

$$I = \frac{15}{X} \text{ A} \quad \dots(ii)$$

Equating Eqs. (i) and (ii), we get

$$\frac{15}{X} = \frac{10}{50} = \frac{(15) \times (50)}{(10)} = 75 \Omega$$

Then, $R = 75 - 50 = 25 \Omega$ must be connected in series, because $V \propto R$ when current is constant.

- 50 (e)** Current remains constant throughout the metallic conductor. Current density $J = \frac{I}{A}$ is not constant because cross-sectional area is a variable parameter. Drift velocity $v_d = \frac{I}{neA}$ is not constant. Since, $v_d \propto \frac{1}{A}$.

- 51 (b)** Given,
Colour coding of resistance = Red, Red, Green, Silver
First band value = 2
Second band value = 2
Third band value = 10^5
Fourth band value = $\pm 10\%$
Now, $R = 22 \times 10^5 \pm 10\% \Omega$

$$R = 2200 \pm 10\% \text{ k}\Omega$$

- 52 (c)** Slope = $\frac{I}{V}$

$$\therefore V = IR$$

$$\therefore \frac{I}{V} = \frac{1}{R}$$

Hence, slope is reciprocal of resistance.

- 53 (a)** The resistance of two wires are

$$R_1 = \rho_1 \frac{l}{A} \text{ and } R_2 = \rho_2 \frac{l}{A}$$

Now, equivalent resistance of series connection of wire

$$R_{eq} = R_1 + R_2 = \rho_1 \frac{l}{A} + \rho_2 \frac{l}{A} = \frac{l}{A} (\rho_1 + \rho_2)$$

The equivalent resistance R_{eq} can be given by $R_{eq} = \rho_{eq} \frac{2l}{A}$

$$\Rightarrow \rho_{eq} \frac{2l}{A} = \frac{l}{A} (\rho_1 + \rho_2)$$

$$\text{Hence, } \rho_{eq} = \frac{\rho_1 + \rho_2}{2}$$

- 54 (a)** Total resistance = $50 + 3950 = 4000 \Omega$

For this circuit, deflection of resistance R be

$$\Rightarrow \frac{4000}{R} = \frac{30}{15} \Rightarrow R = \frac{4000}{2} = 2000 \Omega$$

Then, resistance in series should be = $2000 - 50 = 1950 \Omega$

- 55 (a)** Kirchhoff's first law states that, algebraic sum of currents meeting at a point in a circuit is zero. It is based on conservation of charge.

Kirchhoff's second law states that, the algebraic sum of potential differences around a closed loop is zero. It is based on the law of conservation of energy.

- 56 (b)** Using the formula, $m = ZI$, where m = mass of silver

$$\text{deposited} = 0.05\% \text{ of } 750 \text{ g} = 0.375 \text{ g} = 3.75 \times 10^{-4} \text{ kg}$$

Current passing through it, $I = 0.8 \text{ A}$

$$Z = \text{ECE of silver} = 11.8 \times 10^{-7} \text{ kgC}^{-1}$$

$\therefore$ The time needed for depositing silver is given by

$$t = \frac{m}{ZI} = \frac{3.75 \times 10^{-4}}{11.8 \times 10^{-7} \times 0.8} = 397 \text{ s} = 6 \text{ min } 37 \text{ s}$$

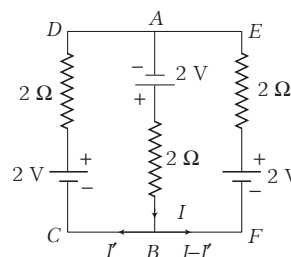
- 57 (c)** The shunt resistance required is given by

$$S = \left(\frac{I_{DC}}{I - I_{DC}} \right) R_{DC} = \left(\frac{100}{500 - 100} \right) \times 0.1 = \frac{100}{400} \times 0.1$$

$$\therefore S = 0.025 \Omega$$

If this value of shunt resistance is connected in parallel with the DC ammeter, then the range will be extended to 0-500 A.

- 58 (a)** The circuit diagram can be redrawn as the potential between A and B is



For the loop ABCDA, $+2 - 2I' + 2 - 2I = 0$

$$2 = I + I'$$

...(i)

For the loop ABFEA,

$$2 - 2I + 2 - 2(I - I') = 0$$

$$4 - 4I + 2I' = 0$$

$$2 = 2I - I'$$

...(ii)

From Eqs. (i) and (ii), we get

$$4 = 3I \Rightarrow I = 4/3 = 1.33 \text{ A}$$

- 59 (c)** As we know that, $R_1 = \rho \frac{l}{a} = \rho \frac{l^2}{V}$

where, l = length of wire,

a = area of cross-section of the wire

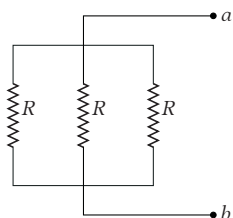
and V = volume of the wire.

As, $R_1 \propto l^2$

$$\Rightarrow \frac{R_1}{R_2} = \left(\frac{l_1}{l_2} \right)^2 = \left(\frac{1}{2} \right)^2$$

$$\Rightarrow R_2 : R_1 = 4 : 1$$

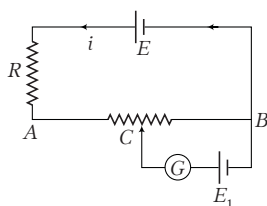
- 60 (c) The combination of resistors can be redrawn as



Therefore, equivalent resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \Rightarrow R_{eq} = \frac{R}{3}$$

- 61 (c) From the circuit (potentiometer),



$V_{CB} = E$ (if no current is drawn from the battery)

$$\Rightarrow \left[\frac{E_1}{R + R_{AB}} \right] R_{CB} = E \quad \dots(i)$$

Given, $E_1 = 2 \text{ V}$ and $R_{AB} = 10 \Omega$

$$\therefore R_{CB} = \frac{40}{100} \times 10 = 4 \Omega \text{ and } E = 10 \times 10^{-3} \text{ V}$$

On solving Eq. (i), we get $R = 790 \Omega$

- 62 (d) We know that, potential drop across a resistance,

$$V = IR \Rightarrow V = \text{constant}$$

$$i.e. \quad I \propto \frac{1}{R}$$

$$I_1 : I_2 : I_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$$

- 63 (a) Total internal resistance does not change

$$\therefore R' = 4r$$

$$\text{Net emf, } E' = E(n - 2m)$$

Here, $n = \text{total number of cells} = 4$

$m = \text{wrong connection} = 1$

$$E' = E[4 - 2], i.e. E' = 2E$$

- 64 (a) Given, $I = 3 \text{ A}$ and $V = 6 \text{ V}$

We know that, $V = IR$

$$R = \frac{V}{I} \Rightarrow R = \frac{6}{3} = 2 \Omega$$

If the ammeter and voltmeter have resistance, then $R < 2 \Omega$.

- 65 (b) Given, $R_1 = 100 \Omega$

$$R_2 = 200 \Omega \Rightarrow T_1 = 100^\circ \text{ C}$$

$$\alpha = 0.005 \text{ per } ^\circ \text{ C}$$

We know that, new resistance of the bulb filament,

$$R_2 = R_1 [1 + \alpha(T_2 - T_1)]$$

$$200 = 100 [1 + 0.005(T_2 - 100)]$$

$$2 = 1 + [0.005(T_2 - 100)]$$

$$0.005(T_2 - 100) = 1$$

$$T_2 - 100 = \frac{1}{0.005}$$

$$\Rightarrow T_2 - 100 = \frac{1000}{5}$$

$$T_2 - 100 = 200$$

$$\Rightarrow T_2 = 200 + 100 = 300^\circ \text{ C}$$

- 66 (d) Consider the ring as two parts. As two resistances are joined in parallel between two points A and B, then two resistances would be

$$R_1 = \frac{R}{2\pi r} \cdot r\theta = \frac{R}{2\pi} \theta$$

$$\text{and } R_2 = \frac{R}{2\pi r} r(2\pi - \theta) = \frac{R}{2\pi} (2\pi - \theta)$$

Now, equivalent or effective resistance between A and B,

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2}$$

$$\Rightarrow R_{eq} = \frac{\frac{R}{2\pi} \theta \times \frac{R}{2\pi} (2\pi - \theta)}{\frac{R}{2\pi} [\theta + 2\pi - \theta]} = \left[\frac{R^2 \theta (2\pi - \theta)}{4\pi^2} \right]$$

$$= \frac{R^2 \theta (2\pi - \theta)}{4\pi^2} \times \frac{2\pi}{2\pi R} = \frac{R\theta(2\pi - \theta)}{4\pi^2}$$

- 67 (b) Case I For balanced point of meter bridge,

$$\frac{5}{R} = \frac{l_1}{100 - l_1} \quad \dots(i)$$

Case II When R is shunted with equal resistance, i.e. R

$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{R} \Rightarrow R' = R/2$$

$$\therefore \frac{5}{R/2} = \frac{1.6 l_1}{100 - 1.6 l_1} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$2 \left(\frac{l_1}{100 - l_1} \right) = \frac{1.6 l_1}{100 - 1.6 l_1} \Rightarrow l_1 = 25 \text{ cm}$$

$$\text{From Eq. (i), } \frac{5}{R} = \frac{25}{75} \Rightarrow R = 15 \Omega$$

- 68 (c) As, $r = \text{internal resistance,}$

$$\text{and } r = R_1 \left(\frac{l - l_1}{l_1} \right) \Rightarrow \frac{r l_1}{R_1} = l - l_1 \quad \dots(i)$$

$$\text{Also, } r = R_2 \left(\frac{l - l_2}{l_2} \right) \Rightarrow \frac{r l_2}{R_2} = l - l_2 \quad \dots(ii)$$

Subtracting Eq. (i) from Eq. (ii), we get

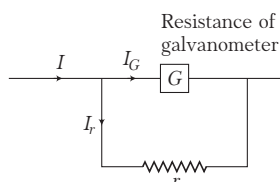
$$l_1 - l_2 = r \left[\frac{l_2}{R_2} - \frac{l_1}{R_1} \right]$$

$$\text{At, } R_1 = \infty, l_1 = 3 \text{ m and at, } R_2 = 9.5 \Omega, l_2 = 2.85 \text{ m}$$

$$\Rightarrow (3 - 2.85) = r \left[\frac{2.85}{9.5} - 0 \right]$$

$$\Rightarrow r = 0.5 \Omega$$

- 69 (a) As, current $I_G = \frac{2}{1000} I$



Also, $I_r = \left(\frac{G}{G+r} \right) I$

We know that, potential across G and shunt r are same.

$$\therefore V_G = V_r \Rightarrow I_G(G) = I_r r$$

$$\Rightarrow \frac{2IG}{1000} = \frac{GI}{G+r} (r) \Rightarrow 2(G+r) = 1000 r$$

$$\Rightarrow G+r = 500 r \Rightarrow \frac{G}{r} + 1 = 500$$

$$\Rightarrow G/r = 499 \Rightarrow r = \frac{1}{499} G$$

- 70 (c) Corresponding to the colours of the first and second bands, i.e. green and black, the figures are 5 and 0.

Corresponding to the colour of third band, i.e. violet, the multiplier is 10^7 . Therefore, the value of the resistance is $50 \times 10^7 \Omega$. The gold colour of the fourth band indicates the tolerance of $\pm 5\%$.

So, the value of the resistor is written as

$$50 \times 10^7 \Omega \pm 5\% = 500 \times 10^6 \Omega \pm 5\% = 500 \pm 5\% \text{ M}\Omega$$

- 71 (c) The required resistance = $\frac{\frac{R}{2} \times \frac{R}{2}}{\frac{R}{2} + \frac{R}{2}} = \frac{R}{4} = \frac{9}{4} \Omega$

- 72 (c) Let the values of resistances be R_1 and R_2 , respectively.

When R_1 and R_2 resistances are in series, then

$$R_1 + R_2 = 6 \text{ (according to question)} \quad \dots(i)$$

When R_1 and R_2 resistances are parallel, then

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{4}{3} \quad \dots(ii)$$

From Eq. (i), we get

$$\frac{R_1 R_2}{6} = \frac{4}{3} \Rightarrow R_1 R_2 = 4 \times 2 \Rightarrow R_1 R_2 = 8 \quad \dots(iii)$$

We know that,

$$R_1 - R_2 = \sqrt{(R_1 + R_2)^2 - 4R_1 R_2} = \sqrt{36 - 4 \times 8}$$

$$\Rightarrow R_1 - R_2 = \sqrt{4}$$

$$\Rightarrow R_1 - R_2 = 2 \quad \dots(iv)$$

From Eqs. (i) and (iv), we get

$$R_1 = 4 \Omega, R_2 = 2 \Omega$$

- 73 (d) Given, 6Ω , 6Ω and 6Ω are in parallel.

$$\text{So, } \frac{1}{R} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

$$\therefore R = \frac{6}{3} = 2 \Omega$$

Now, 6Ω and 6Ω are in series.

$$\text{So, } R' = 6 + 6 = 12 \Omega$$

$$\therefore \frac{1}{R''} = \frac{1}{R'} + \frac{1}{12} = \frac{1}{12} + \frac{1}{12} = \frac{2}{12}$$

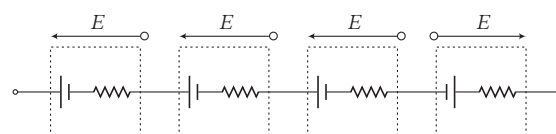
$$\Rightarrow R'' = 6 \Omega$$

The required resistance of the circuit = $2 + 6 = 8 \Omega$.

So, the potential difference,

$$V_A - V_B = iR = 0.5 \times 8 = 4 \text{ V}$$

- 74 (a) Total emf of the cell = $3E - E = 2E$



Total internal resistance = $4r$

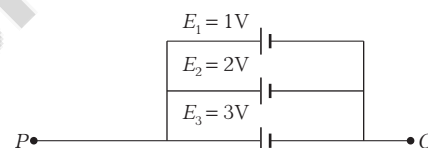
$\therefore$ Total resistance of the circuit = $4r + R$

So, the current in the external circuit

$$i = \frac{2E}{4r + R} \quad \left(\because i = \frac{V}{R} \right)$$

- 75 (b) As, resistances 1Ω , 2Ω and 1Ω are in parallel.

So, the required internal resistance,



$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1} = \frac{2+1+2}{2} \Rightarrow r = \frac{2}{5} \Omega$$

The potential difference between points P and Q ,

$$E = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{1/r} = \frac{\frac{1}{1} + \frac{2}{2} + \frac{3}{1}}{5/2}$$

$$= \frac{\frac{2+2+6}{2}}{5/2} = \frac{10/2}{5/2} = \frac{10}{5} \times \frac{2}{2} = 2 \text{ V}$$

- 76 (d) Given, $R_1 = 2 \Omega$, $R_2 = 6 \Omega$, $E = 2 \text{ V}$, $r = 0.5 \Omega$, $I = ?$

$\therefore R_1, R_2$ are in parallel combination.

$$\text{So, } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} \Rightarrow R = \frac{6}{4} = 1.5 \Omega$$

Then, the current in the circuit,

$$I = \frac{E}{r + R} = \frac{2}{0.5 + 1.5} = \frac{2}{2.0} = 1 \text{ A}$$

(as internal resistance, r is in series with other resistance)

- 77 (b) As, mobility = $\frac{\text{drift velocity}}{\text{electric field}}$

$$\Rightarrow \mu = \frac{v_d}{E} = \frac{\text{metre}^2}{\text{volt-second}} = \frac{[\text{L}]^2}{(\text{joule/coulomb})\text{-second}}$$

$$= \frac{[\text{L}]^2}{[\text{kg-metre}^2 \text{-second}^{-2}]\text{-second}} = \frac{[\text{L}^2]}{[\text{ML}^2\text{T}^{-2}\text{A}^{-1}]}\text{-second} = [\text{M}^{-1}\text{T}^2\text{A}]$$

79 (b) Resistance of wire, $R = \rho \frac{l}{A}$

Given, $R_1 = 4 \Omega$, $l_1 = l$, $A_1 = A$

$l_2 = 2l$, $A_2 = A/2$, $\rho = \text{constant}$

$$\therefore \frac{R_1}{R_2} = \frac{l_1/A_1}{l_2/A_2} = \frac{l_1 \times A_2}{l_2 \times A_1}$$

$$\Rightarrow \frac{4}{R_2} = \frac{l \times A/2}{2l \times A} \Rightarrow \frac{4}{R_2} = \frac{1}{4} \Rightarrow R_2 = 16 \Omega$$

80 (a) Balanced condition for Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{s} \quad \dots(i)$$

$$\text{where, } \frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2} \Rightarrow s = \frac{s_1 s_2}{s_1 + s_2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{P}{Q} = \frac{R(s_1 + s_2)}{s_1 s_2}$$

81 (b) Force experienced by electron, $F = qE$

where, E = electric field intensity.

$$\therefore 2.4 \times 10^{-19} = 1.6 \times 10^{-19} E$$

$$\Rightarrow E = 1.5 \text{ NC}^{-1}$$

Moreover, E = potential gradient

$$E = \frac{dV}{dl}$$

$$\Rightarrow E \times l = V$$

$$\Rightarrow V = 1.5 \times 6 = 9 \text{ V}$$

82 (b) Resistance, $R = \frac{V}{i_g} - G = \frac{120}{0.01} - 10 = 12000 - 10$

$$R = 11990 \Omega$$

To convert a galvanometer into a voltmeter, high resistance of value 11990Ω should be connected in parallel.

83 (b) Given, first balancing length, $l_1 = 30 \text{ cm}$

Second balancing length,

$$l_2 = 40 \text{ cm} \Rightarrow E_1 = 1.25 \text{ V}$$

$$E_2 = ?$$

So, according to the principle of potentiometer,

$$E_1 = K l_1 \quad \dots(i)$$

$$\text{and } E_2 = K l_2 \quad \dots(ii)$$

$$\therefore \frac{E_1}{E_2} = \frac{K l_1}{K l_2} \Rightarrow \frac{1.25}{E_2} = \frac{30}{40}$$

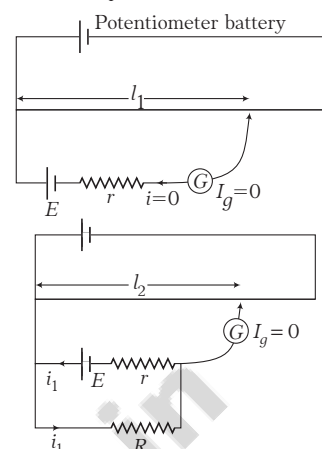
$$\Rightarrow E_2 = \frac{1.25 \times 40}{30}$$

$$\Rightarrow E_2 = \frac{5}{3} = 1.666 \text{ V} \approx 1.67 \text{ V}$$

84 (a) Potentiometer measures the potential difference more accurately than a voltmeter because while measuring emf (electromotive force), it does not draw any current from the source of known emf. Potentiometer has its own battery which maintains constant potential drop across the potentiometer wire.

85 (d) As, $k l_1 = E - r i = E - r(0) = E$

$$\Rightarrow k l_1 = E \quad \dots(i)$$



and

$$k l_2 = E - r i_1 = R i_1$$

$$\Rightarrow i_1 = \frac{E}{R + r} \Rightarrow k l_2 = \frac{R E}{R + r} \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{l_1}{l_2} = \frac{R + r}{R} = 1 + \frac{r}{R} \Rightarrow r = R \left(\frac{l_1}{l_2} - 1 \right)$$

We know that for internal resistance,

$$r = \left(\frac{l_1}{l_2} - 1 \right)$$

where, l_1 = balancing length of potentiometer wire = 240 cm,

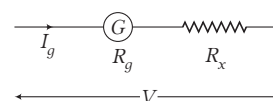
l_2 = balancing length after shunting = 120 cm

and R = shunting resistance = 2Ω .

Putting all the values, we get

$$\text{Internal resistance, } r = \left(\frac{240}{120} - 1 \right) \times 2 = (2 - 1) \times 2 = 2 \Omega$$

86 (b) Let, R_x resistance be connected in series to convert galvanometer to voltmeter.



According to the question, $R_g = 100 \Omega$, $I_g = 30 \text{ mA}$

(Current corresponding to full scale deflection)

$$\text{Now, we can write, } V = I_g \times R_g + I_g R_x = I_g (R_g + R_x) \quad \dots(i)$$

Given, $V = 30 \text{ V}$

From Eq. (i), we get

$$30 = (30 \times 10^{-3}) (100 + R_x)$$

$$\Rightarrow \frac{30}{30 \times 10^{-3}} = 100 + R_x$$

$$\Rightarrow 10^3 = 100 + R_x \Rightarrow R_x = 1000 - 100 = 900 \Omega$$

87 (b) By Faraday's law of electrolysis, $m = Z i t$

Given, $m_1 = m$ gram, $i_1 = 4 \text{ A}$, $i_2 = 6 \text{ A}$, $t_1 = 120 \text{ s}$

$$t_2 = 40 \text{ s}, m_2 = ?$$

$$\therefore \frac{m_1}{m_2} = \frac{i_1}{i_2} \Rightarrow \frac{m}{m_2} = \frac{4 \times 120}{6 \times 40}$$

$$\Rightarrow m_2 = \frac{m \times 6 \times 40}{4 \times 120} = \frac{m}{2}$$

88 (d) As $R \propto l^2$. When wire is stretched,
 $\therefore R' = n^2 R = 2^2 \times 4 = 4 \times 4 = 16 \Omega$

89 (b) As $I = \frac{E}{R+r}$ or $E = I(R+r)$

$$\Rightarrow 21 = 0.2(10+r)$$

$$10+r = \frac{21}{0.2} \times 10$$

$$\therefore r = 10.5 - 10 = 0.5 \Omega$$

90 (b) Effective resistance,

$$R_{\text{eff}} = \frac{40 \times 120}{120 + 40} = \frac{4800}{160} = 30 \Omega$$

$$\therefore \text{Current, } I = \frac{7}{(30+5)} = \frac{7}{35} = 0.2 \text{ A}$$

$$\left(\because I = \frac{E}{R+r} \right)$$

91 (b) Current, $I = qf$

$$\text{Given, } q = 1.6 \times 10^{-19} \text{ C and } f = 10^{19}$$

$$\therefore I = 1.6 \text{ A}$$

92 (c) $v_d = \frac{I}{nAe}$

$$\text{Given, } I = 2 \text{ A, } n = 5.86 \times 10^{28} \text{ m}^{-3}$$

$$A = \pi r^2 = \pi (0.1 \times 10^{-2})^2, e = 1.6 \times 10^{-19} \text{ C}$$

$$\therefore \text{Drift velocity, } v_d = 0.68 \times 10^{-4} \text{ ms}^{-1}$$

93 (a) As, $R \propto l/d^2 \Rightarrow R_2 = R_1 \left(\frac{l_2 d_1^2}{l_1 d_2^2} \right) = \frac{1.5(0.31)^2}{(0.155)^2} \times 4.2 = 25.2 \Omega$

94 (d) Suppose, m rows are connected in parallel and each row contains n identical cells (each cell having $E = 1.5 \text{ V}$ and $r = 1 \Omega$)

For maximum current in the external resistance R , the necessary condition

$$R = \frac{nr}{m}$$

$$1.5 = \frac{n \times 1}{m}$$

$$1.5m = n \quad \dots(i)$$

$$\text{Total cells} = 24 = n \times m \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$m = 4$$

$$n = 6$$

Therefore, 6 cells in each row are connected in series and 4 such rows are connected in parallel.

95 (d) $R_1 = R_0 (1 + \alpha t)$

$$1 = R_0 (1 + 0.00125 \times 27) \quad \dots(i)$$

$$2 = R_0 (1 + 0.00125 \times t) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$t = 854^\circ \text{C} = 1127 \text{ K}$$

96 (d) We know that,

$$i = \frac{E}{r + R_{\text{eff}}} = \frac{15}{0.5 + 14.5} = \frac{15}{15} = 1 \text{ A}$$

97 (b) Given, temperature coefficient of carbon,

$$\alpha_1 = 4 \times 10^{-3}/^\circ \text{C}$$

Temperature coefficient of copper,

$$\alpha_2 = -0.5 \times 10^{-3}/^\circ \text{C}$$

Hence, $R\alpha_1 = -R_2\alpha_2$

$$\frac{R_2}{R_1} = \frac{-4 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$$\Rightarrow \frac{R_2}{R_1} = \frac{8}{1}$$

98 (*) Let the three conductors having resistances R_1, R_2 and R_3 respectively, and the current drawn by them are 1A, 2A and 3A respectively, when connected in turn across a battery.

$$\therefore V = R_1 \quad V = 2R_2 \text{ and } V = 3R_3$$

$$\text{So, } R_1 = \frac{V}{1}, R_2 = \frac{V}{2} \text{ and } R_3 = \frac{V}{3}$$

When the conductors are connected in series, across the same battery, then

$$V = I[R_1 + R_2 + R_3]$$

$$\Rightarrow V = I \left[V + \frac{V}{2} + \frac{V}{3} \right]$$

$$\Rightarrow 1 = I \left[\frac{6+3+2}{6} \right] \text{ or } I = \frac{6}{11} \text{ A}$$

$$\textbf{99} \text{ (d) } R \propto l^2/m \Rightarrow R_1 : R_2 : R_3 = \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3}$$

$$= \frac{25}{1} : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$$

100 (b) We have, $I_1 = \frac{I \times 2R}{3R} = \frac{2I}{3}$

$$\therefore H_1 = I_1^2 R = \frac{4I^2}{9} \times R \quad \dots(i)$$

$$\text{Also, } I_2 = \frac{I \times R}{3R} = \frac{I}{3}$$

$$\therefore H_2 = I_2^2 (2R) = \frac{I^2}{9} \times 2R \quad \dots(ii)$$

$$\text{and } H_3 = I^2 (1.5R) \quad \dots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$H_1 : H_2 : H_3 = \frac{4I^2}{9} \times R : \frac{I^2}{9} \times 2R : I^2 \times 1.5R$$

$$= \frac{4}{9} : \frac{2}{9} : 1.5 = 4 : 2 : 13.5$$

$$= 8 : 4 : 27$$

101 (a) A milliammeter will have greater resistance than ammeter because (1) number of turns in the coil in milliammeter is increased to compensate low value of current to create large deflection. (2) milliammeter does not have usually a shunt in parallel because main current is very small. Ammeter usually have a shunt in parallel which makes overall resistance low.

- 102** (c) We know that, the resistance of any wire,

$$R = \frac{\rho L}{A}$$

In given case, $L_1 = L$, $L_2 = 2L$, ρ and A are constants.

$$\text{Hence, } \frac{R_1}{R_2} = \frac{L_1}{L_2} \Rightarrow \frac{R_1}{R_2} = \frac{L}{2L} \Rightarrow \frac{R_1}{R_2} = \frac{1}{2}$$

- 103** (c) Power when connected in series,

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

Given, $P_1 = 60 \text{ W}$ and $P_2 = 100 \text{ W}$

$$\text{Hence, } P = \frac{60 \times 100}{60 + 100}$$

$$\Rightarrow P = \frac{6000}{160} \Rightarrow P = 37.5 \text{ W}$$

- 104** (b) We know that, drift velocity, $v_d = \frac{e E \tau}{m} = \frac{e \tau V}{m l}$

Here, $v_d \propto v$

Given, condition $V_1 = V$ and $V_2 = 2V$

$$\text{Hence, } \frac{v_d}{v_d'} = \frac{V}{2V} \Rightarrow v_d' = 2v_d = 2v$$

- 105** (c) Using, $V = IR$

$$100 \times 10^{-3} = I \times 10$$

$$\Rightarrow I = 0.01 \text{ A}$$

$$\text{Also, } I = \frac{E}{R + R'}$$

$$\Rightarrow \frac{2}{10 + R'} = 0.01 \Rightarrow R' = 190 \Omega$$

- 106** (d) Specific resistance of a material is the characteristic property, it does not depends upon the dimensions of material, hence it does not depend upon length.

- 107** (b) 7Ω and 3Ω are in series, so total 10Ω and 10Ω are in parallel, so total resistance across $AC = 5 \Omega$. Now, 5Ω and 5Ω are in series, so they make 10Ω . Now, 10Ω and 10Ω are in parallel, so $R_{\text{net}} = 5 \Omega$.

- 108** (a) Equivalent resistance in series $= 6 + 9 = 15 \Omega$

$$\text{Current flow in circuit, } i = \frac{V}{R} = \frac{120}{15} = 8 \text{ A}$$

Voltage in 6Ω resistor, $V = iR = 8 \times 6 = 48 \text{ V}$

Power consumed by 6Ω resistor,

$$P = \frac{V^2}{R} = \frac{48 \times 48}{6} = 8 \times 48 = 384 \text{ W}$$

- 109** (a) Current through each arm DAC and $DBC = 1 \text{ A}$

$$V_D - V_A = 2 \text{ and } V_D - V_B = 3$$

$$\therefore V_A - V_B = 1 \text{ V}$$

- 110** (b) Heat $= i^2 R t$

$$\therefore \left(\frac{E}{R_1 + r} \right)^2 R_1 = \left(\frac{E}{R_2 + r} \right)^2 R_2 \quad [\because t \text{ is the same}]$$

On solving, we get

$$(R_1 - R_2)r^2 = (R_1 - R_2)R_1R_2$$

$$\Rightarrow r = \sqrt{R_1 R_2}$$

- 111** (b) Applying Kirchhoff's law, $500I + 100I = 12$

$$\text{So, } I = \frac{12 \times 10^{-2}}{6} = 2 \times 10^{-2} \text{ A}$$

$$\text{Hence, } V_B = 100 (2 \times 10^{-2}) = 2 \text{ V}$$

- 112** (a) The full scale deflection current,

$$i_g = \frac{25 \text{ mV}}{G} \text{ ampere}$$

where, G is the resistance of the meter.

The value of shunt required for converting it into ammeter of

$$\text{range } 25 \text{ A is } S = \frac{i_g G}{i - i_g}$$

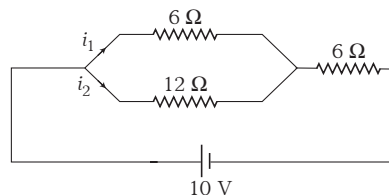
$$\Rightarrow S = \frac{25 \text{ mV}}{25} = 0.001 \Omega$$

- 113** (c) Power, $P = \frac{V^2}{R}$

$$\text{For small variation, } \frac{\Delta P}{P} \times 100\% = \frac{2 \times \Delta V}{V} \times 100\% = 2 \times 2.5 = 5\%$$

Therefore, power would decrease by 5%.

- 114** (a)



$$\text{Resistance, } R = \frac{6 \times 12}{6 + 12} = \frac{6 \times 12}{18} = 4 \Omega$$

Total resistance, $R_{\text{eq}} = 6 + 4 = 10 \Omega$

$$\text{Current, } i = \frac{V}{R} = \frac{10}{10} = 1 \text{ A}$$

The current in 12Ω resistor is

$$i_2 = i \left(\frac{R_1}{R_1 + R_2} \right) = 1 \times \left(\frac{6}{6 + 12} \right) \Rightarrow i_2 = \frac{1}{3} \text{ A}$$

The potential difference in 12Ω resistor,

$$V = i \times 12 = \frac{1}{3} \times 12 = 4 \text{ V}$$

- 115** (b) Given, $A = 0.3 \text{ m}^2$, $n = 2 \times 10^{25} / \text{m}^3$

$$q = 3t^2 + 5t + 2$$

$$i = \frac{dq}{dt} = 6t + 5 = 17 \text{ A (at } t = 2 \text{ s)}$$

We have current, $i = ne Av_d$

$$\begin{aligned} \text{Drift velocity, } v_d &= \frac{i}{ne A} = \frac{17}{2 \times 10^{25} \times 1.6 \times 10^{-19} \times 0.3} \\ &= \frac{17}{0.96 \times 10^{-6}} = 1.77 \times 10^{-5} \text{ ms}^{-1} \end{aligned}$$

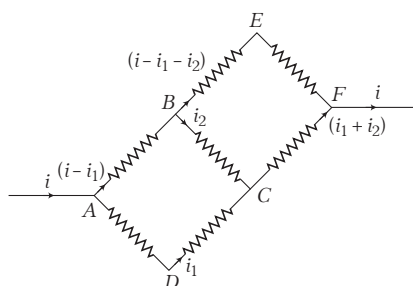
- 116 (b)** To convert a galvanometer into an ammeter, a resistance (shunt) is required to be connected in parallel to the galvanometer.

$$\text{By using, } S = \frac{Gi_g}{i - i_g} = \frac{50 \times 100 \times 10^{-6}}{(10 - 100 \times 10^{-6})} \approx 5 \times 10^{-4} \Omega$$

- 117 (d)** By using the relation,

$$\frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2} = \frac{(8 + 2)}{(8 - 2)} = \frac{5}{3} \quad \left(\begin{array}{l} \text{where, } E' + E'' = E_1 \\ \text{and } E' - E'' = E_2 \end{array} \right)$$

- 118 (b)** The current distribution in the circuit can be shown as



Applying Kirchhoff's law in mesh ABCDA,

$$-10(i - i_1) - 10i_2 + 20i_1 = 0$$

$$3i_1 - i_2 = i \quad \dots(i)$$

and in mesh BEFCB,

$$-20(i - i_1 - i_2) + 10(i_1 + i_2) + 10i_2 = 0$$

$$\Rightarrow 3i_1 + 4i_2 = 2i \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$i_1 = \frac{2i}{5}, \quad i_2 = \frac{i}{5} \Rightarrow i_{AD} = i_1 = \frac{2i}{5}$$

- 119 (d)** The drift velocity, $v_d = \frac{J}{ne}$

(where, J is current density $= \frac{i}{A}$)

So, $v_d \propto J$

$$\therefore J_1 = \frac{i}{A} \quad \text{and} \quad J_2 = \frac{2i}{2A} = \frac{i}{A} = J_1$$

So, $(v_d)_1 = (v_d)_2 = v$

- 120 (b)** As resistance, $R \propto \text{length}$

$$\therefore \text{Resistance of each arm} = \frac{12}{3} = 4 \Omega$$

So, effective resistance between any two corners,

$$R_{\text{eff}} = \frac{4 \times 8}{4 + 8} = \frac{32}{12} = \frac{8}{3} \Omega$$

- 121 (a)** We have, $R = \frac{V}{I} = \frac{20}{2.5} = 8 \Omega$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{1}{20} + \frac{0.50}{2.5} = \frac{1}{4}$$

$$\Delta R = \frac{R}{4} = \frac{8}{4} = 2 \Omega$$

$$\Rightarrow R = (8 \pm 2) \Omega$$

- 122 (c)** We have $R = \frac{nr}{m}$, so grouping of cells depends upon the relative values of internal and external resistance.

- 123 (a)** As the current in heating filament increases, it gets more heated. Hence, its temperature increases and resistance also increases, due to which the current decreases.

- 124 (b)** In series, $P = \frac{P_1 P_2}{P_1 + P_2}$

Given, $P_1 = P_2 = 60$

$$\therefore \text{Power, } P = \frac{60 \times 60}{60 + 60} = 30 \text{ W}$$

- 125 (b)** $I_g = \frac{2}{300} \text{ A} = \frac{2}{300} \times 1000 \text{ mA} = \frac{20}{3} \text{ mA}$

$$I_g = 6.67 \text{ mA}$$

As range of ammeter cannot be decreased but can be increased only, therefore the instrument cannot be converted to measure the range 1 mA.

- 126 (a)** The balanced condition for Wheatstone bridge is $\frac{P}{Q} = \frac{R}{S}$.

As is obvious from the given values, no current flows through galvanometer.

Now, P and R are in series, so

$$\text{Resistance, } R_1 = P + R = 10 + 15 = 25 \Omega$$

Similarly, Q and S are in series, so

$$\text{Resistance, } R_2 = Q + S = 20 + 30 = 50 \Omega$$

Net resistance of the network, as R_1 and R_2 are in parallel,

$$R_{\text{net}} = \frac{25 \times 50}{75} = \frac{50}{3} \Omega$$

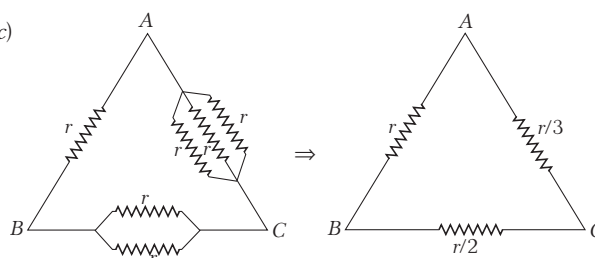
$$i = \frac{V}{R_{\text{net}}} = \frac{6 \times 3}{50} = 0.36 \text{ A}$$

- 127 (b)** Drift velocity, $v_d = \frac{I}{neA}$

$$\Rightarrow v_d = \frac{5}{(5 \times 10^{26}) \times (1.6 \times 10^{-19}) \times (4 \times 10^{-6})}$$

$$= 1.56 \times 10^{-2} \text{ ms}^{-1}$$

- 128 (c)**



Resistance between points A and B,

$$R_{AB} = r \parallel \left(\frac{r}{3} + \frac{r}{2} \right) = \frac{\left(r \times \frac{5}{6} r \right)}{\left(r + \frac{5}{6} r \right)} = \frac{5}{11} r$$

Resistance between points B and C,

$$R_{BC} = \frac{r}{2} \parallel \left(r + \frac{r}{3} \right) = \frac{\frac{r}{2} \times \frac{4}{3} r}{\frac{r}{2} + \frac{4}{3} r} = \frac{4}{11} r$$

Resistance between points C and A,

$$R_{CA} = \frac{r}{3} \parallel \left(\frac{r}{2} + r \right) = \frac{\left(\frac{r}{3} \times \frac{3r}{2} \right)}{\frac{r}{3} + \frac{3r}{2}} = \frac{3}{11} r$$

$$R_{AB} : R_{BC} : R_{CA} = 5 : 4 : 3$$

- 130** (a) Given circuit is a balanced Wheatstone bridge. So, diagonal resistance of 2Ω will be ineffective.

Equivalent resistance of upper arm $= 2 + 2 = 4 \Omega$

Equivalent resistance of lower arm $= 2 + 2 = 4 \Omega$

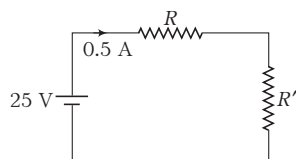
$$\therefore R_{AB} = \frac{4 \times 4}{4 + 4} = 2 \Omega$$

- 131** (b) In open circuit, current through the circuit $i = 0$

Potential difference between x and y will be

$$V = E = 120 \text{ V}$$

- 132** (d) $\frac{1}{R'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{20}$



$$\Rightarrow R' = \frac{20}{5} = 4 \Omega$$

Now, using Ohm's law, $i = \frac{25}{R + R'}$

$$\Rightarrow 0.5 = \frac{25}{R + 4}$$

$$\Rightarrow R + 4 = \frac{25}{0.5}$$

$$\Rightarrow R = 50 - 4 = 46 \Omega$$

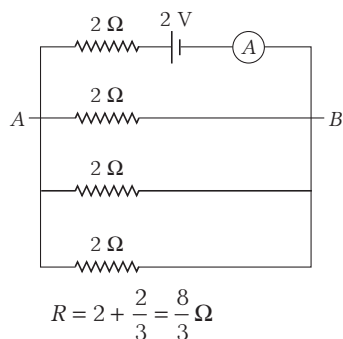
$$\text{Current through } 20 \Omega \text{ resistor} = \frac{0.5 \times 5}{20 + 5} = \frac{2.5}{25} = 0.1 \text{ A}$$

Potential difference across middle resistor

$$= \text{Potential difference across } 20 \Omega = 20 \times 0.1 = 2 \text{ V}$$

- 133** (b) The given circuit can be shown as the resistance between A and B $= \frac{2}{3} \Omega$.

Total resistance of the circuit



$$R = 2 + \frac{2}{3} = \frac{8}{3} \Omega$$

So, the reading of ammeter,

$$i = \frac{V}{R} = \frac{2}{8/3} = \frac{6}{8} = \frac{3}{4} \text{ A}$$

- 134** (d) The resistances of branches ABC and ADC are in parallel. The current entering at node A will equally divided in each parallel branch.

$$\therefore V_A - V_B = IR = 1 \times 2 = 2 \text{ V}$$

Point A is at greater potential with respect to point B, so

$$V_A - V_B = + 2 \text{ V}$$

- 135** (b) Equivalent potential in the given parallel circuit is given as

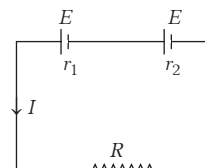
$$E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Here, $E_1 = 2 \text{ V}, E_2 = 1 \text{ V}, r_1 = 1 \Omega$

and $r_2 = 2 \Omega$

$$\Rightarrow E_{eq} = \frac{2 \times 2 + 1 \times 1}{2 + 1} = \frac{4 + 1}{3} = \frac{5}{3} \text{ V}$$

- 136** (c) Here are two batteries with emf E each and the internal resistances r_1 and r_2 , respectively.



Hence, we have $I(R + r_1 + r_2) = 2E$

$$\text{Thus, } I = \frac{2E}{R + r_1 + r_2} \quad \dots (i)$$

Now, the potential difference across the first cell would be equal to $V = E - Ir_1$. From the question, $V = 0$, hence

$$E = Ir_1 = \frac{2Er_1}{R + r_1 + r_2} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow R + r_1 + r_2 = 2r_1$$

$$\text{Hence, } R = r_1 - r_2$$

- 137** (b) The net resistance,

$$R = \frac{12 \times 4}{12 + 4} + 2 = \frac{48}{16} + 2 = 5 \Omega$$

Electric current,

$$i = \frac{E}{R + r} = \frac{12}{5 + 1} = 2 \text{ A}$$

From current division rule,

$$i_2 = \left(\frac{12}{12 + 4} \right) \times 2 = \frac{3}{4} \times 2 = 1.5 \text{ A}$$

$$\text{and } i_1 = 2 - 1.5 = 0.5 \text{ A}$$

CHAPTER 04

Magnetic Effect of Current and Moving Charges

In previous unit, we have studied about the electric current, its measurement and the thermal effects of current. In this unit, we shall study about magnetic effects of current. Earlier, it was thought that there is no connection between electricity and magnetism. However in the year 1820, Oersted realised that electricity and magnetism were related to each other. He showed experimentally that the electric current through a straight wire causes noticeable deflection of the magnetic compass needle held near the wire.

He also found that the iron fillings sprinkled around the wire arrange themselves in concentric circles with wire as the centre in the plane perpendicular to the wire. This shows that the magnetic field is associated with a moving charge or a current carrying conductor. The branch of physics which deals with the magnetism due to electric current is called **electromagnetism**.

MAGNETIC FIELD

The space in the surroundings of a magnet, in which magnetic influence of moving charges or a current carrying conductor can be experienced is called **magnetic field**. Magnetic field is a vector quantity and is denoted by **B**. The SI unit of magnetic field is weber/m^2 or tesla (T) and its CGS unit is gauss or maxwell/cm^2 .

$$1 \text{ tesla} = 10^4 \text{ gauss}$$

Magnetic field due to moving charges (Oersted's experiment)

Oersted found experimentally that a magnetic field is established around a current carrying conductor just as it occurs around a magnet. In this experiment, Oersted placed a magnetic compass near a current carrying conductor and observed that the compass needle shows deflection. Also, when the current is reversed, the needle shows deflection in opposite direction.

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1 Magnetic field

- Biot-Savart's law
(Magnetic field due to a current carrying conductor)
- Applications of Biot-Savart's law

2 Ampere's circuital law

- Applications of Ampere's circuital law

3 Force on a moving charge in a uniform magnetic field

- Motion of a charged particle in combined electric and magnetic fields : Lorentz force
- Cyclotron

4 Force on a current carrying conductor in a magnetic field

- Force between two parallel current carrying conductors
- Magnetic force between two moving charges
- Magnetic dipole moment
- The moving coil galvanometer (MCG)

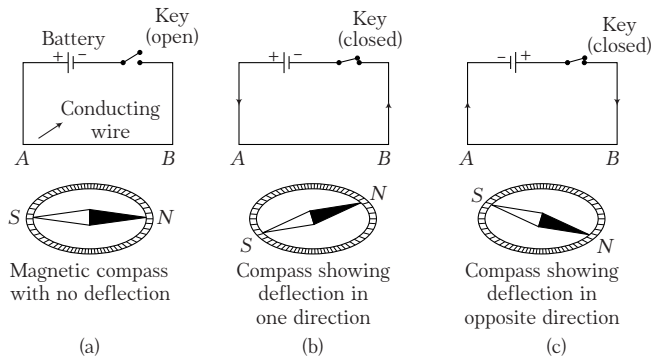


Fig. 4.1 Oersted's experiment

Thus, from the above figures, he concluded that a magnetic field is established around a current carrying wire due to which such deflections are visible.

Oersted's experiment shows the connectivity between the magnetic field and electric current and as electric current means moving charges, he also concluded that moving charges also produce magnetic field in their surroundings.

Biot-Savart's law (Magnetic field due to a current carrying conductor)

Biot-Savart's law is an experimental law predicted by Biot and Savart. This law deals with the magnetic field produced at a point due to a small current element (a part of any conductor carrying current).

Consider a wire XY carrying current I . Let dl be the infinitesimal element of the conductor, dB be the magnetic field at point P at a distance r from the element.

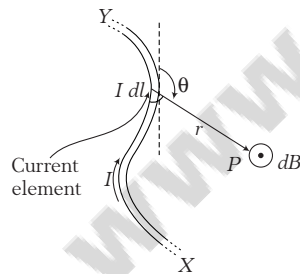


Fig. 4.2 A long current carrying conductor

Biot-Savart's law states that, the magnitude of the magnetic field induction dB at the point P due to the current element is

- directly proportional to the current flowing through the conductor, i.e. $dB \propto I$... (i)
- directly proportional to the length of the element, i.e. $dB \propto dl$... (ii)
- directly proportional to the sine of angle between the length of element and line joining the element to the point, i.e. $dB \propto \sin \theta$... (iii)

- inversely proportional to the square of distance between the element and the point, i.e. $dB \propto 1/r^2$... (iv)

Combining all the above four relations, we get

$$dB \propto \frac{Idl \sin \theta}{r^2}$$

This relation is called **Biot-Savart's law**.

If conductor is placed in air or vacuum, then magnitude of magnetic field is given by

$$|dB| = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

where, $\frac{\mu_0}{4\pi}$ is the **proportionality constant**.

μ_0 is called the **permeability of free space (or vacuum)**.

In SI unit, $\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}$ or Wb/A-m and its dimensions are $[\text{MLT}^{-2}\text{A}^{-2}]$. The value of $\frac{\mu_0}{4\pi}$ in CGS system is unity.

In vector form, Biot-Savart's law can be written as

$$dB \propto \frac{Idl \times \mathbf{r}}{|\mathbf{r}|^3} = \frac{\mu_0}{4\pi} \cdot \frac{Idl \times \mathbf{r}}{r^3}$$

The direction of dB is represented by the right hand screw rule or right hand thumb rule.

Similarly, magnetic field induction at point P due to current through entire wire is

$$\mathbf{B} = \int \frac{\mu_0}{4\pi} \cdot \frac{Idl \times \mathbf{r}}{r^3}$$

or

$$\mathbf{B} = \int \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin \theta}{r^2}$$

Biot-Savart's law in a medium

If the conductor is placed in a medium, then vector form of Biot-Savart's law is given as

$$dB = \frac{\mu}{4\pi} \cdot \frac{Idl \times \mathbf{r}}{|\mathbf{r}|^3} = \frac{\mu_0 \mu_r}{4\pi} \cdot \frac{Idl \times \mathbf{r}}{r^3}$$

where, $\mu_r = \text{relative permeability} = \frac{\mu}{\mu_0} = 1$

(for air or vacuum)

and $\mu = \text{absolute permeability of the medium}$.

Biot-Savart's law in terms of current density $\mathbf{J}$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{(\mathbf{J} \times \mathbf{r})}{r^3} dV \quad \left(\because \mathbf{J} = \frac{I}{A} = \frac{Idl}{Adl} = \frac{Idl}{dV} \right)$$

Biot-Savart's law in terms of charge (q) and its velocity (v)

A moving charge constitutes current and the magnetic field associated with the charge is given as

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{q(\mathbf{v} \times \mathbf{r})}{r^3} \quad \left(\because Idl = \frac{q}{dt} dl = q \frac{dl}{dt} = qv \right)$$

Special conditions of Biot-Savart's law

Special conditions of Biot-Savart's law are given below

- (i) The magnetic field due to the current element can

$$\text{also be written as } d\mathbf{B} = \frac{1}{4\pi\epsilon_0 c^2} \frac{Id\mathbf{l} \times \mathbf{r}}{r^3}$$

where, $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ = speed of light in vacuum.

- (ii) When $\theta = 0^\circ$ or 180° , then magnetic field produced due to the current will be zero.
- (iii) When $\theta = 90^\circ$, i.e. r is perpendicular to dl , then magnetic field will be $d\mathbf{B} = \frac{\mu_0 idl}{4\pi r^2}$, which is maximum.

Rules to find direction of magnetic field

- (i) **Right hand palm rule** If we spread our right hand in such a way that thumb is towards the direction of current and fingers are towards that point where we have to find the direction of magnetic field, then direction of magnetic field will be perpendicular to the palm.

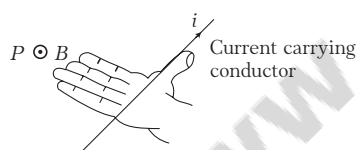


Fig. 4.3 Right hand palm rule

- (ii) **Maxwell's right handed screw rule** If a right handed cork screw is rotated such that its tip moves in the direction of flow of current through the conductor, then the rotation of the head of the screw gives the direction of magnetic field.

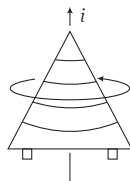


Fig. 4.4 Right handed screw rule

- (iii) **Right hand thumb rule** If a straight current carrying conductor is held in the right hand such

that the thumb represents the direction of flow of current, then the direction of the folding fingers will represent the direction of magnetic field.

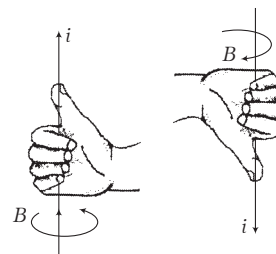


Fig. 4.5 Right hand thumb rule

Note

- (i) If magnetic field is directed perpendicular and into the plane of the paper, it is represented by $\otimes$ (cross) whereas if magnetic field is directed perpendicular and out of the plane of the paper, it is represented by $\odot$ (dot).

(ii)

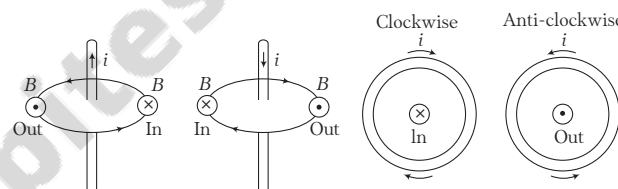


Fig. 4.6 Direction of magnetic field

In Magnetic field is away from the observer or perpendicular inwards.

Out Magnetic field is towards the observer or perpendicular outwards.

Applications of Biot-Savart's law

Applications of Biot-Savart's law are given below

Magnetic field surrounding a thin, straight current carrying conductor

Magnetic field due to a current carrying wire at a point P which lies at a perpendicular distance d from the wire (as shown in figure) is given as

$$B = \frac{\mu_0}{4\pi} \frac{i}{d} (\sin\alpha + \sin\beta)$$

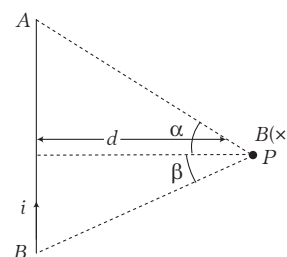


Fig. 4.7 A current carrying wire subtend angles α and β at a point P

Note down the following points regarding the above equation

- (i) For an infinitely long straight wire, $\alpha = \beta = 90^\circ$

$$\therefore B = \frac{\mu_0 i}{4\pi d} (\sin 90^\circ + \sin 90^\circ) = \frac{\mu_0 i}{4\pi d} (1 + 1) = \frac{\mu_0 i}{2\pi d}$$

- (ii) When wire is semi-infinite (at the foot of long wire),

$$\alpha = \frac{\pi}{2} \text{ and } \beta = 0^\circ$$

$$\therefore B = \frac{\mu_0 i}{4\pi d} \left(\sin 0^\circ + \sin \frac{\pi}{2} \right) \Rightarrow B = \frac{\mu_0 i}{4\pi d}$$

- (iii) For axial position of wire, i.e. when point P lies on axial position of current carrying conductor, then magnetic field at P ,

$$B = 0 \quad [\text{since, } \theta = 0^\circ]$$

- (iv) $B \propto 1/d$, i.e. B - d graph for an infinitely long straight wire is a rectangular hyperbola as shown in figure.

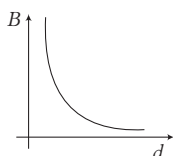


Fig. 4.8 B - d graph for an infinitely long straight wire

Magnetic field for different configurations of the straight conductors

Magnitude and direction of magnetic fields associated with different configurations of the straight conductors are given below

- (i) $B_P = \frac{\mu_0 i}{4\pi a} [\sin \alpha] \otimes$
- (ii) $B_P = \frac{\mu_0 i}{4\pi a} [\sin \alpha - \sin \beta] \otimes$
- (iii) $B_P = \frac{\mu_0 i}{2\pi a} \otimes$
- (iv) $B_P = \frac{\mu_0 i}{4\pi a} \otimes$
- (v) $B_P = \frac{\mu_0 i}{4\pi a} [\sin \alpha + 1] \otimes$
- (vi) $B_O = \frac{8\mu_0 i}{4\pi} \frac{\sqrt{a^2 + b^2}}{ab} \otimes$
When $a = b$, $B_O = \frac{8\sqrt{2} \mu_0 i}{4\pi a}$

Example 4.1 A current of 10 A is flowing east to west in a long wire kept in the east-west direction. Find magnetic field in a horizontal plane at a distance of (i) 10 cm north, (ii) 20 cm south from the wire and in a vertical plane at a distance of (iii) 40 cm downwards (iv) 50 cm upwards.

Sol. The magnitude of the magnetic field at a distance r from a long wire carrying a current i is given by

$$B = \frac{\mu_0 i}{2\pi r}$$

$$\text{where, } \frac{\mu_0}{2\pi} = 2 \times 10^{-7} \text{ T mA}^{-1}.$$

- (i) The magnetic field in a horizontal plane at a distance of 10 cm ($= 0.10$ m) north from the wire,

$$B_N = (2 \times 10^{-7}) \frac{10}{0.10} = 2 \times 10^{-5} \text{ T}$$

The current in the wire is from east to west. So, according to the right hand palm rule, the direction of the field at the point towards north will be downwards in a vertical plane.

- (ii) The magnetic field at a distance of 20 cm ($= 0.20$ m) south from the wire, $B_S = (2 \times 10^{-7}) \frac{10}{0.20} = 1 \times 10^{-5} \text{ T}$

The direction of the field will be upward in the vertical plane.

- (iii) The magnetic field at a distance of 40 cm ($= 0.40$ m) from the wire downwards in the vertical plane,

$$B_D = (2 \times 10^{-7}) \frac{10}{0.40} = 5 \times 10^{-6} \text{ T}$$

The field will be in a horizontal plane pointing south.

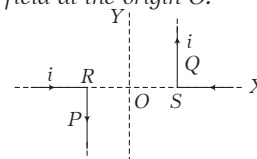
- (iv) The magnetic field at a distance of 50 cm ($= 0.50$ m) above the wire in the vertical plane,

$$B_U = (2 \times 10^{-7}) \frac{10}{0.50} = 4 \times 10^{-6} \text{ T}$$

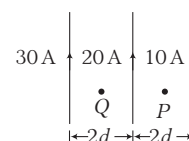
The field will be in a horizontal plane pointing north.

Example 4.2

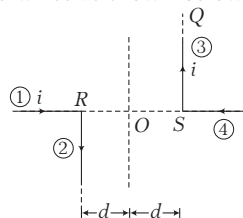
- (i) A pair of stationary and infinitely long bent wires are placed in the XY -plane as shown in figure. The wires carry currents 10 A each. The segments P and Q are parallel to the Y -axis such as $OS = OR = 0.02$ m. Find the magnitude and direction of the magnetic field at the origin O .



- (ii) Three long wires carrying currents 10 A, 20 A and 30 A are placed parallel to each other as shown below. Points P and Q are in the midway of wires. Find ratio of magnetic fields at points P and Q .



Sol. (i) Consider the wires as shown below



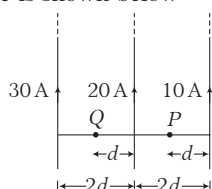
Magnetic field, $B_1 = B_4 = 0$ $[\because \theta = 0^\circ]$

Magnetic field, $B_2 = B_3 = \frac{\mu_0 i}{4\pi d} \odot$

Magnetic field at the centre O,

$$\begin{aligned} B_O &= B_1 + B_2 + B_3 + B_4 = 0 + \frac{\mu_0 i}{4\pi d} + \frac{\mu_0 i}{4\pi d} = \frac{\mu_0 i}{2\pi d} \odot \\ &= \frac{\mu_0}{2\pi} \times \frac{10}{0.02} = \frac{\mu_0}{4\pi} \times 1000 \\ &= 10^{-7} \times 10^3 = 10^{-4} \text{ T (downward)} \quad (\because \mu_0 = 4\pi \times 10^{-7}) \end{aligned}$$

(ii) The given figure is shown below



Magnetic field at the point Q,

$$B_Q = \frac{\mu_0}{2\pi} \left[\frac{30}{d} - \frac{20}{d} - \frac{10}{3d} \right] = \frac{\mu_0}{2\pi} \cdot \frac{20}{3d} \otimes$$

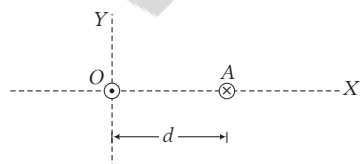
Magnetic field at the point P,

$$B_P = \frac{\mu_0}{2\pi} \left[\frac{30}{3d} + \frac{20}{d} - \frac{10}{d} \right] = \frac{\mu_0}{2\pi} \cdot \frac{20}{d} \otimes$$

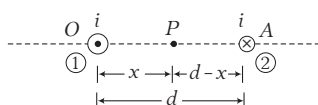
The ratio of magnetic fields at points P and Q,

$$\therefore \frac{B_P}{B_Q} = 3$$

Example 4.3 Two long wires carrying same currents in opposite directions are placed at separation d as shown in figure. Predict variation of magnetic field as one moves from one wire to another along X-axis excluding the points O and A.



Sol. The given situation can be shown as



Between the wires magnetic field at point P,

Due to current carrying wire (1), $B_1 = \frac{\mu_0 i}{2\pi x}$, upward

Due to current carrying wire (2), $B_2 = \frac{\mu_0 i}{2\pi(d-x)}$, upward

Now, net magnetic field at point P,

$$B_P = B_1 + B_2 = \frac{\mu_0 i}{2\pi} \left(\frac{1}{x} + \frac{1}{d-x} \right) = \frac{\mu_0 i}{2\pi} \cdot \frac{d}{x(d-x)}, \text{ upward}$$

B_P is minimum, if $x(d-x)$ is maximum.

$x(d-x)$ is maximum, if $x = d-x$

$$\Rightarrow x = d/2$$

(the product of two parts is maximum, if parts are equal)

As, $x = \frac{d}{2}$, B_P is minimum.

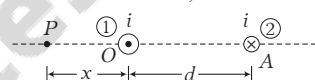
$$(B_P)_{\min} = \frac{2\mu_0 i}{\pi d}, \text{ upward}$$

As, $x \rightarrow 0, B \rightarrow \infty$
 $x \rightarrow d, B \rightarrow \infty$

From $x = 0$ to $x = d/2$, magnetic field decreases and reaches to minimum value at $x = d/2$.

From $x = d/2$ to $x = d$, magnetic field increases and tends to infinite at $x = d$.

Left of O At x distance from O,



Magnetic field at point P due to current carrying wire 1,

$$B_1 = \frac{\mu_0 i}{2\pi x}, \text{ downward}$$

Similarly, magnetic field at point P due to current carrying wire 2,

$$B_2 = \frac{\mu_0 i}{2\pi(x+d)}, \text{ upward}$$

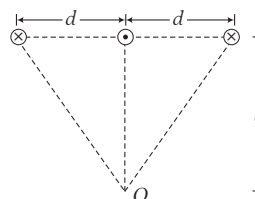
Magnetic field at the point P,

$$B_P = \frac{\mu_0 i}{2\pi} \left(\frac{1}{x} - \frac{1}{x+d} \right), \text{ downward}$$

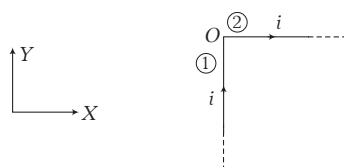
Example 4.4

(i) A very long wire carrying a current i is bent at right angles. Find magnetic field at a point lying on a perpendicular to the wire, drawn through the point of bending, at a distance d from it.

(ii) Three long wires carrying same current are placed as shown in figure. Find magnetic field at point O.



Sol. (i) A very long wire is bent at right angles as shown below

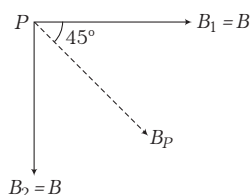


Magnetic field at point P at height d above point O ,

Due to wire (1), $B_1 = \frac{\mu_0 i}{4\pi d}$, towards $+X$ -axis

Due to wire (2), $B_2 = \frac{\mu_0 i}{4\pi d}$, towards $-Y$ -axis

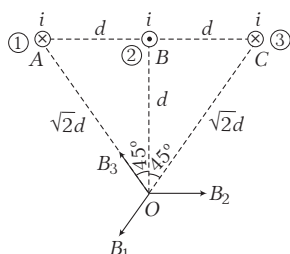
B_1 is perpendicular to B_2 .



Net magnetic field at the point P ,

$$B_P = \sqrt{2}B = \frac{\sqrt{2}\mu_0 i}{4\pi d} = \frac{\mu_0 i}{2\sqrt{2}\pi d}$$

(ii) The given figure can be drawn as

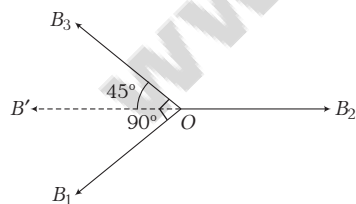


Magnetic field at the point O ,

Due to wire (1), $B_1 = \frac{\mu_0 i}{2\pi\sqrt{2}d}$

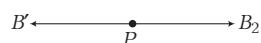
Due to wire (2), $B_2 = \frac{\mu_0 i}{2\pi d}$

Due to wire (3), $B_3 = \frac{\mu_0 i}{2\pi\sqrt{2}d}$



Resultant of B_1 and B_3 ,

$$B' = 2B_1 \cos 45^\circ = \frac{\mu_0 i}{2\pi d} \quad (\because B_1 = B_3)$$



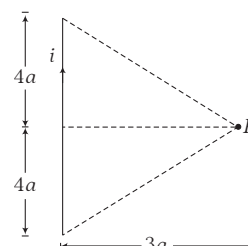
$$B' = B_2$$

$\therefore$ Magnetic field at the centre O ,

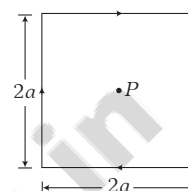
$$B_O = B' - B_2 = 0$$

Example 4.5 Evaluate magnitude and direction of magnetic field at point P in the following cases.

(i)

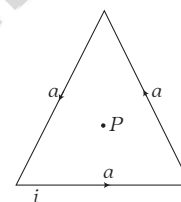


(ii)



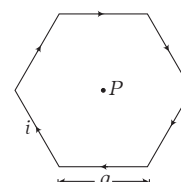
P is the centre of square.

(iii)



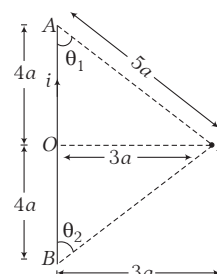
P is the centre of equilateral triangle.

(iv)



P is the centre of regular hexagon.

Sol. (i) The given figure can be drawn as



Magnetic field at centre P ,

$$\begin{aligned} B_P &= \frac{\mu_0 i}{4\pi(OP)} [\cos \theta_1 + \cos \theta_2] \\ &= \frac{\mu_0 i}{4\pi(3a)} [\cos \theta + \cos \theta] = \frac{\mu_0 i}{6\pi a} \cos \theta \end{aligned}$$

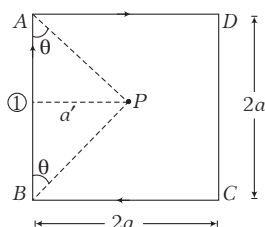
Use Pythagoras theorem, we find the value

$$PA = \sqrt{(3a)^2 + (4a)^2} \\ = \sqrt{25a^2} = 5a$$

From the figure, $\cos \theta = \frac{PA}{OA} = \frac{4}{5}$

$$\therefore B_P = \frac{\mu_0 i}{6\pi a} \cdot \frac{4}{5} = \frac{2\mu_0 i}{15\pi a} \otimes$$

(ii) The given figure can be drawn as



Magnetic field at centre P due to one side of square, say AB ,

$$B_1 = \frac{\mu_0 i}{4\pi a'} (\cos \theta + \cos \theta) \quad (\because a' = a, \theta = 45^\circ)$$

$$B_1 = \frac{\mu_0 i}{4\pi a} \cdot 2 \cdot \frac{1}{\sqrt{2}} = \frac{\mu_0 i}{2\sqrt{2}\pi a}$$

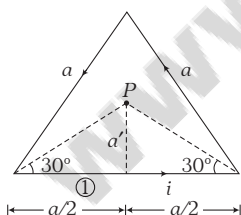
So, magnetic field at centre P due to square loop,

$$B_P = 4B_1 = \frac{\sqrt{2}\mu_0 i}{\pi a}$$

Since, current is clockwise, magnetic field at centre P will be inside the plane of paper.

$$B_P = \frac{\sqrt{2}\mu_0 i}{\pi a} \otimes$$

(iii) The given figure can be drawn as



From the figure,

$$\tan 30^\circ = \frac{a'}{a/2} \Rightarrow a' = \frac{a}{2\sqrt{3}} \quad \text{and} \quad \theta_1 = \theta_2 = 30^\circ$$

$$\text{Magnetic field, } B_1 = \frac{\mu_0 i}{4\pi a'} [\cos 30^\circ + \cos 30^\circ] \\ = \frac{\mu_0 i}{4\pi \cdot \frac{a}{2\sqrt{3}}} \cdot 2 \cdot \frac{\sqrt{3}}{2} = \frac{3\mu_0 i}{2\pi a}$$

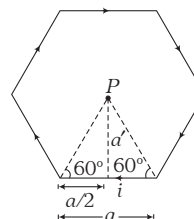
So, magnetic field at centre P due to triangular loop,

$$B_P = 3B_1 = \frac{9\mu_0 i}{2\pi a}$$

Since, current is anti-clockwise, magnetic field at centre P will be outside the plane of paper.

$$B_P = \frac{9\mu_0 i}{2\pi a} \odot$$

(iv)



Magnetic field,

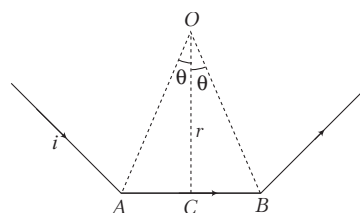
$$B_1 = \frac{\mu_0 i}{4\pi a'} (\cos 60^\circ + \cos 60^\circ) \\ = \frac{\mu_0 i}{4\pi \cdot \frac{a}{2} \cdot \tan 60^\circ} \cdot 2 \cos 60^\circ = \frac{\mu_0 i}{4\pi \cdot \frac{a}{2} \cdot \sqrt{3}} \cdot 2 \times \frac{1}{2} \\ = \frac{\mu_0 i}{2\pi a \sqrt{3}}$$

Magnetic field at $P = 6$ times the magnetic field due to one side

$$\therefore B_P = 6B_1 = \frac{\sqrt{3}\mu_0 i}{\pi a} \otimes$$

Example 4.6 A wire shaped to a regular hexagon of side 2 cm carries a current of 2 A. Find the magnetic field at the centre of the hexagon.

Sol.



From geometry of the figure, we have

$$\Rightarrow \frac{BC}{OC} = \tan \theta \quad [\because BC = 1 \text{ cm (given)}]$$

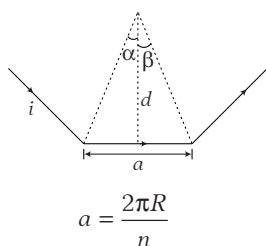
$$\therefore \frac{1}{r} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow r = \sqrt{3} \text{ cm}$$

Net magnetic field at $O = 6$ times the magnetic field due to one side

$$\therefore B_0 = 6 \left[\frac{\mu_0}{2\pi} \frac{i}{r} (\sin \theta + \sin \theta) \right] \\ = \frac{6 (2 \times 10^{-7}) (2) \left(\frac{1}{2} + \frac{1}{2} \right)}{\sqrt{3} \times 10^{-2}} \quad [\because \theta = 30^\circ] \\ = 1.38 \times 10^{-4} \text{ T}$$

Example 4.7 A rectangular polygon of n sides is formed by bending a wire of total length $2\pi R$ which carries a current i . Find the magnetic field at the centre of the polygon.

Sol. One side of the polygon is



$$a = \frac{2\pi R}{n}$$

Since, angle = $\frac{\text{arc}}{\text{radius}}$

$$\therefore \alpha + \beta = \frac{a}{R} = \frac{2\pi R/n}{R} = \frac{2\pi}{n}$$

$$\Rightarrow \alpha + \beta = \frac{2\pi}{n}$$

$$\Rightarrow \alpha + \alpha = \frac{2\pi}{n} \quad (\because \alpha = \beta)$$

$$\Rightarrow \alpha = \pi/n$$

$$\text{Hence, } \alpha = \beta = \frac{\pi}{n}$$

$$\text{Again, } \frac{d}{(a/2)} = \cot \alpha$$

$$\therefore d = \left(\frac{a}{2}\right) \cot \alpha = \left(\frac{\pi R}{n}\right) \cot \left(\frac{\pi}{n}\right)$$

All sides of the polygon produce the magnetic field at the centre in same direction (here). Hence, net magnetic field,

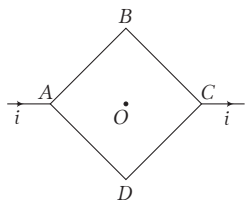
$$B = (n) (\text{magnetic field due to one side})$$

$$= n \left\{ \frac{\mu_0}{4\pi} \frac{i}{d} (\sin \alpha + \sin \beta) \right\}$$

$$\text{or } B = n \left(\frac{\mu_0}{4\pi} \right) \left(\frac{in}{\pi R} \right) \left(\tan \frac{\pi}{n} \right) \left(2 \sin \frac{\pi}{n} \right)$$

$$\text{or } B = \left(\frac{\mu_0 i}{2R} \right) \frac{\left\{ \sin \left(\frac{\pi}{n} \right) / \frac{\pi}{n} \right\}^2}{\cos (\pi / n)}$$

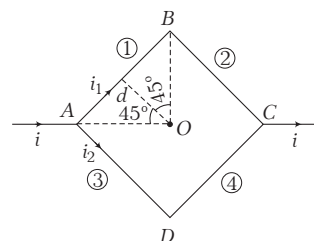
Example 4.8 Consider the following figure in which ABCD is a square of edge a . Resistance of the wire ABC is R_0 and that of ADC is $2R_0$. Find magnitude and direction of magnetic field at the centre O of the square.



Sol. Given, $R_{ABC} = R_1 = R_0$, $R_{ADC} = R_2 = 2R_0$

$$i_1 = \frac{2R_0}{R_0 + 2R_0} \cdot i = \frac{2i}{3}$$

$$i_2 = i - i_1 = \frac{i}{3}$$



$$\text{Magnetic field, } B_1 = \frac{\mu_0 i_1}{4\pi d} (\sin 45^\circ + \sin 45^\circ) = \frac{\mu_0 i_1}{2\sqrt{2} \pi d}$$

$$= \frac{\mu_0 (2i/3)}{2\sqrt{2} \pi \left(\frac{a}{2}\right)} = \frac{2\mu_0 i}{3\sqrt{2} \pi a} = B_2, \otimes$$

$$\text{Magnetic field, } B_3 = \frac{\mu_0 i_2}{4\pi d} (\sin 45^\circ + \sin 45^\circ)$$

$$= \frac{\mu_0 (i/3)}{2\sqrt{2} \pi d} = \frac{\mu_0 (i/3)}{2\sqrt{2} \pi \left(\frac{a}{2}\right)} = \frac{\mu_0 i}{3\sqrt{2} \pi a} = B_4, \odot$$

Magnetic field at the centre O ,

$$B_O = (B_1 + B_2) - (B_3 + B_4)$$

$$= \frac{4\mu_0 i}{3\sqrt{2} \pi a} - \frac{2\mu_0 i}{3\sqrt{2} \pi a}$$

$$= \frac{\sqrt{2} \mu_0 i}{3\pi a}, \otimes$$

Magnetic field at the centre of a circular current carrying coil

Consider a circular current carrying coil of radius a and carrying current i . Magnetic field at the centre O due to the current element dl is

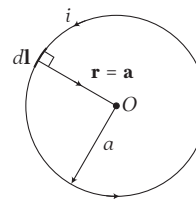


Fig. 4.9 Circular current carrying coil

$$dB = \frac{\mu_0 I}{4\pi} \cdot \frac{dl \times r}{r^3}$$

Here, $dl \perp r$, so $\theta = 90^\circ$

$$dB = \frac{\mu_0}{4\pi} \frac{i dl \sin 90^\circ}{a^2}$$

Therefore, magnetic field at the centre due to the whole

$$\text{circular loop, } B = \int dB = \frac{\mu_0 i}{4\pi a^2} \int dl = \frac{\mu_0 i}{4\pi a^2} (2\pi a)$$

$$B = \frac{\mu_0 i}{2a}$$

Direction of this field is outward perpendicular to the plane of the paper.

Note If the loop has N turns, then magnetic field, $B = \frac{\mu_0 NI}{2a}$.

Magnetic field due to an arc

Magnetic field due to an arc of a circular current carrying coil at the centre is given by

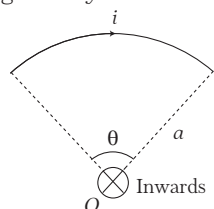


Fig. 4.10 Magnetic field due to an arc

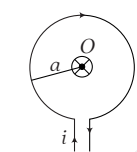
$$B = \left(\frac{\theta}{2\pi} \right) \frac{\mu_0 i}{2a} = \frac{\mu_0}{4\pi} \left(\frac{i}{a} \right) \theta \quad \text{or} \quad B = \left(\frac{\mu_0}{4\pi} \right) \left(\frac{i}{a} \right) \theta$$

Here, θ is to be substituted in radians.

Magnetic field for different configurations of the circular coil

Magnitude and direction of magnetic fields associated with different configurations of the circular coil are given below

(i)



$$B_O = \frac{\mu_0 i}{2a} \otimes$$

(ii)

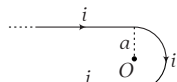


$$B_O = \frac{\mu_0 i}{2a} \odot$$

(iii)

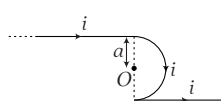
$$B_O = \frac{\mu_0 i}{4a} \otimes$$

(iv)



$$B_O = \frac{\mu_0 i}{2\pi a} + \frac{\mu_0 i}{4a} \otimes$$

(v)



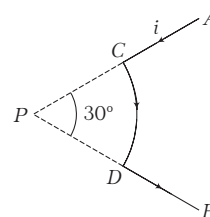
$$B_O = \frac{\mu_0 i}{4a} \otimes$$

(vi)



$$B_O = 0 \text{ (for any value of } \theta \text{)}$$

Example 4.9 A current path shaped as shown in figure produces a magnetic field at P , the centre of the arc. If the arc subtends an angle of 30° and the radius of the arc is 0.6 m. What is the magnitude of the field at P , if the current is 3.0 A?



Sol. Here, point P is along the length of straight wires CA and DE , hence

$$B_{CA} = B_{DE} = 0$$

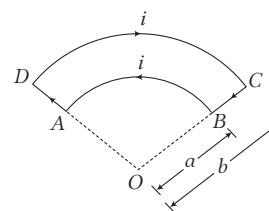
Magnetic field due to an arc of a circle at the centre,

$$B = \left(\frac{\theta}{2\pi} \right) \frac{\mu_0 i}{2R} = \left(\frac{\mu_0}{4\pi} \right) \left(\frac{i}{R} \right) \theta$$

Here, $\theta = 30^\circ = \frac{\pi}{6}$ rad, $i = 3$ A and $R = 0.6$ m

$$\begin{aligned} B &= \left(\frac{\mu_0}{4\pi} \right) \left(\frac{3}{0.6} \right) \left(\frac{\pi}{6} \right) \\ &= \frac{10^{-7} \times 3 \times \pi}{0.6 \times 6} \\ &= 2.6 \times 10^{-7} \text{ T} \end{aligned}$$

Example 4.10 Figure shows a current loop having two circular arcs joined by two radial lines. Find the magnetic field B at the centre O .



Sol. Magnetic field at O due to wires CB and AD will be zero.

Magnetic field due to arc BA ,

$$B_1 = \left(\frac{\theta}{2\pi} \right) \left(\frac{\mu_0 i}{2a} \right)$$

Direction of field B_1 is out of the plane of the figure.

Similarly, magnetic field at O due to arc DC ,

$$B_2 = \left(\frac{\theta}{2\pi} \right) \left(\frac{\mu_0 i}{2b} \right)$$

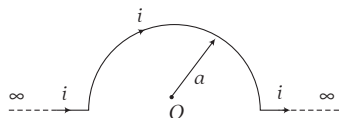
Direction of B_2 is into the plane of the figure.

The resultant field at O ,

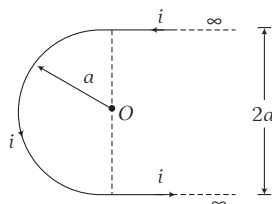
$$B = B_1 - B_2 = \frac{\mu_0 i \theta (b - a)}{4\pi ab}, \text{ out of the plane of figure.}$$

Example 4.11 What is the magnitude and direction of magnetic field at point O in the following cases?

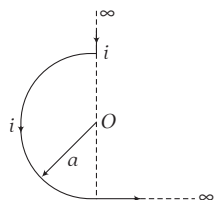
(i)



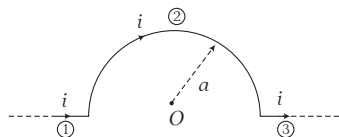
(ii)



(iii)



Sol. (i) The given figure can be drawn as

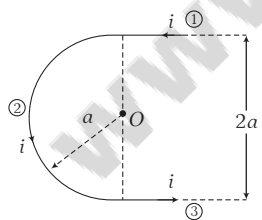


Since, point O is along the length of straight wires 1 and 3, hence $B_1 = B_3 = 0$.

Net magnetic field at the centre point O ,

$$B_O = B_2 = \frac{\mu_0 i}{4a} \otimes$$

(ii) The given figure can be drawn as



Since, point O is the foot of long wires 1 and 3,

$$\therefore B_1 = B_3 = \frac{\mu_0 i}{4\pi a} \odot$$

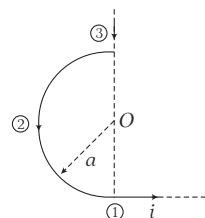
Due to semi-circle (2),

$$B_2 = \frac{\mu_0 i}{4a} \odot$$

Net magnetic field at the centre point O ,

$$B_O = B_1 + B_2 + B_3 = \frac{\mu_0 i}{4\pi a} + \frac{\mu_0 i}{4a} + \frac{\mu_0 i}{4\pi a} = \frac{\mu_0 i}{4a} \left(\frac{2}{\pi} + 1 \right) \odot$$

(iii) The given figure can be drawn as



$$\text{Magnetic field, } B_1 = \frac{\mu_0 i}{4\pi a} \odot$$

$$\text{Magnetic field, } B_2 = \frac{\mu_0 i}{4a} \odot$$

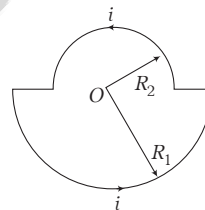
$$\text{Magnetic field, } B_3 = 0$$

Net magnetic field at the centre point O ,

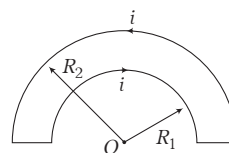
$$B_O = B_1 + B_2 + B_3 = \frac{\mu_0 i}{4\pi a} + \frac{\mu_0 i}{4a} + 0 = \frac{\mu_0 i}{4a} \left(\frac{1}{\pi} + 1 \right) \odot$$

Example 4.12 Find the magnitude and direction of magnetic field at point O in the following cases.

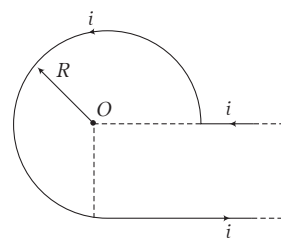
(i)



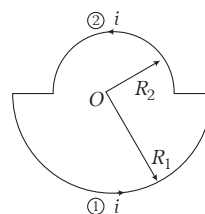
(ii)



(iii)



Sol. (i) The given figure can be drawn as



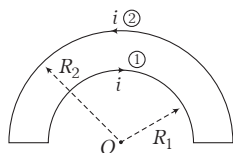
Magnetic field at O due to straight wires is zero.

$$\text{Now, } B_1 = \frac{\mu_0 i}{4R_1} \odot \text{ and } B_2 = \frac{\mu_0 i}{4R_2} \odot$$

Net magnetic field at the centre point O ,

$$B_O = B_1 + B_2 = \frac{\mu_0 i}{4} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \odot$$

(ii) The given figure can be drawn as



Magnetic field at O due to straight wires is zero.

Now, $B_1 = \frac{\mu_0 i}{4R_1} \otimes$

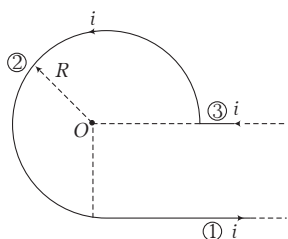
and $B_2 = \frac{\mu_0 i}{4R_2} \odot$

$\Rightarrow B_1 > B_2$ ($\because R_1 < R_2$)

Net magnetic field at the centre O ,

$$B_O = B_1 - B_2 = \frac{\mu_0 i}{4} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \otimes$$

(iii) The given figure can be drawn as



Magnetic field, $B_1 = \frac{\mu_0 i}{4\pi R} \odot$

Magnetic field, $B_2 = \frac{\mu_0 i}{2R} \cdot \frac{3}{4} \odot$

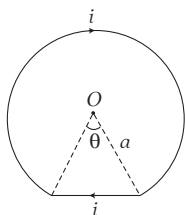
Magnetic field, $B_3 = 0$

Net magnetic field at the centre O ,

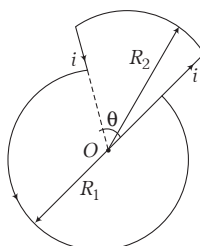
$$B_O = B_1 + B_2 + B_3 = \frac{\mu_0 i}{4R} \left(\frac{1}{\pi} + \frac{3}{2} \right) \odot$$

Example 4.13 Evaluate the magnitude and direction of magnetic field at point O in the following cases.

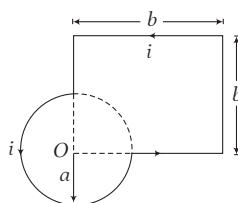
(i)



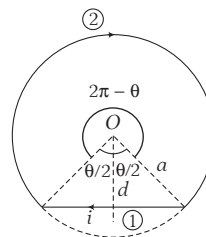
(ii)



(iii)



Sol. (i) The given figure can be drawn as



From the figure, $d = a \cos \frac{\theta}{2}$

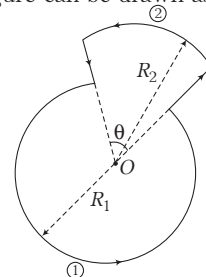
$$B_1 = \frac{\mu_0 i}{4\pi d} \left(\sin \frac{\theta}{2} + \sin \frac{\theta}{2} \right) = \frac{\mu_0 i}{4\pi \cdot a \cos \frac{\theta}{2}} \cdot 2 \sin \frac{\theta}{2} = \frac{\mu_0 i}{2\pi a} \tan \frac{\theta}{2} \otimes$$

and $B_2 = \frac{\mu_0 i (2\pi - \theta)}{2a} \otimes$

Net magnetic field at the centre O ,

$$B_O = B_1 + B_2 = \frac{\mu_0 i}{2\pi a} \left(\tan \frac{\theta}{2} + \pi - \frac{\theta}{2} \right) \otimes$$

(ii) The given figure can be drawn as



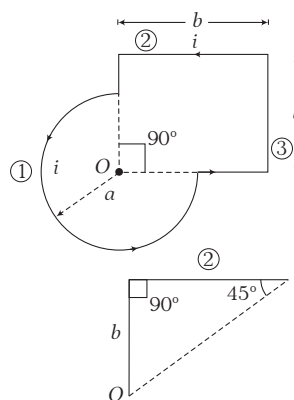
Magnetic field, $B_1 = \frac{\mu_0 i}{2R_1} \cdot \frac{(2\pi - \theta)}{2\pi} \odot$

Magnetic field, $B_2 = \frac{\mu_0 i}{2R_2} \cdot \frac{\theta}{2\pi} \odot$

Net magnetic field at the centre O ,

$$B_O = B_1 + B_2 = \left[\frac{\mu_0 i}{4\pi R_1} (2\pi - \theta) + \frac{\mu_0 i}{4\pi R_2} \theta \right] \odot$$

(iii) The given figure can be drawn as



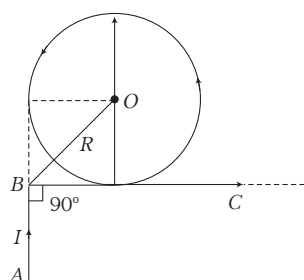
$$\text{Magnetic field, } B_1 = \frac{\mu_0 i}{2a} \cdot \frac{3}{4} = \frac{3\mu_0 i}{8a} \odot$$

$$\begin{aligned} \text{Magnetic field, } B_2 &= \frac{\mu_0 i}{4\pi b} (\cos 90^\circ + \cos 45^\circ) \\ &= \frac{\mu_0 i}{4\sqrt{2}\pi b} \odot = B_3 \end{aligned}$$

Net magnetic field at the centre O,

$$B_O = B_1 + (B_2 + B_3) = \left[\frac{3\mu_0 i}{8a} + \frac{\mu_0 i}{2\sqrt{2}\pi b} \right] \odot$$

Example 4.14 What is the magnetic field at the centre of the circular loop (as shown in figure), when a single wire is bent to form a circular loop and also extends to form straight section?



Sol. From figure, magnetic field due to AB,

$$B_1 = \frac{\mu_0 I}{4\pi R} \left[\sin \frac{\pi}{2} - \sin \frac{\pi}{4} \right] \otimes = \frac{\mu_0 I}{4\pi R} \left[1 - \frac{1}{\sqrt{2}} \right] \otimes$$

$$\text{Magnetic field due to circular loop, } B_2 = \frac{\mu_0 I}{2R} \odot$$

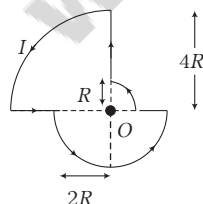
Magnetic field due to straight wire BC,

$$B_3 = \frac{\mu_0 I}{4\pi R} \left[\sin \frac{\pi}{2} + \sin \frac{\pi}{4} \right] \odot = \frac{\mu_0 I}{4\pi R} \left[1 + \frac{1}{\sqrt{2}} \right] \odot$$

$\therefore$ Net magnetic field, $B = B_2 + B_3 - B_1$

$$\frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R} \left(1 + \frac{1}{\sqrt{2}} \right) - \frac{\mu_0 I}{4\pi R} \left(1 - \frac{1}{\sqrt{2}} \right) = \frac{\mu_0 I}{2R} \left(1 + \frac{1}{\sqrt{2}\pi} \right) \odot$$

Example 4.15 Find the magnetic field at the centre O of the loop shown in the figure.



Sol. Due to straight parts, there is no magnetic field at O. There are three fields at O due to three arcs.

$$\text{As, field due to arc, } B = \frac{\mu_0 i}{4\pi r} \theta$$

$$B_1 = \frac{\mu_0 i}{4\pi(4R)} \left(\frac{\pi}{2} \right) \odot \quad (\text{due to arc of radius } 4R)$$

$$B_2 = \frac{\mu_0 i}{4\pi(2R)} (\pi) \odot \quad (\text{due to arc of radius } 2R)$$

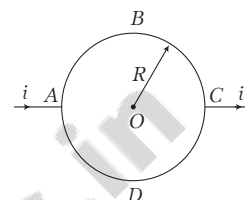
$$B_3 = \frac{\mu_0 i}{4\pi R} \left(\frac{\pi}{2} \right) \odot \quad (\text{due to arc of radius } R)$$

As, the current is anti-clockwise in all three arcs, therefore all the fields are perpendicular to plane and outwards.

$$\Rightarrow B_{\text{net}} = B_1 + B_2 + B_3$$

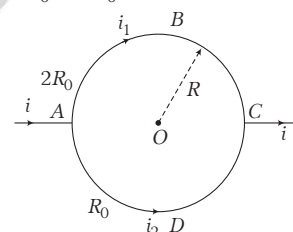
$$\therefore B_{\text{net}} = \frac{\mu_0 i}{R} \left[\frac{1}{32} + \frac{1}{8} + \frac{1}{8} \right] = \frac{9\mu_0 i}{32R}$$

Example 4.16 In the given figure, resistance of wire ABC is twice that of wire ADC. Find the magnitude and direction of magnetic field at the centre O.



Sol. Given, $R_{ABC} = R_1 = 2R_0$, $R_{ADC} = R_2 = R_0$

$$i_1 = \frac{R_0}{R_0 + 2R_0} \cdot i = \frac{i}{3} \Rightarrow i_2 = i - i_1 = \frac{2i}{3}$$



$$\text{Magnetic field, } B_1 = \frac{\mu_0 i_1}{2R} \cdot \frac{1}{2} = \frac{\mu_0 i}{12R} \otimes$$

$$\text{Magnetic field, } B_2 = \frac{\mu_0 i_2}{2R} \cdot \frac{1}{2} = \frac{\mu_0 i}{6R} \odot$$

Net magnetic field at the centre O,

$$B_O = B_2 - B_1 = \frac{\mu_0 i}{12R} \odot$$

Magnetic field on the axis of a circular current carrying coil

If a coil of radius a is carrying current i , then magnetic field on its axis at a distance r from its centre is given by

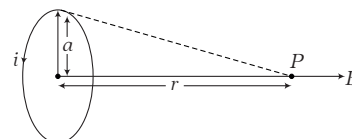


Fig. 4.11 Magnetic field on the axis of a circular current carrying coil

$$B_{\text{axis}} = \frac{\mu_0}{2} \frac{Nia^2}{(a^2 + r^2)^{3/2}}$$

where, N = number of turns in coil.

- (i) At centre, $r = 0$

$$\Rightarrow B_{\text{centre}} = \frac{\mu_0}{2} \cdot \frac{Ni}{a} = \frac{\mu_0 Ni}{2a} = B_{\text{max}}$$
- (ii) At far points,
 $r \gg a, B \approx \frac{\mu_0}{2} \left(\frac{Nia^2}{r^3} \right) = \frac{\mu_0}{2} \frac{Nia^2}{r^3} \Rightarrow B \propto \frac{1}{r^3}$
- (iii) B - r graph shows the variation of magnetic field at various position on the axis of circular current carrying coil. Magnetic field is maximum at the centre and decreases as we move away from the centre on the axis of the loop.

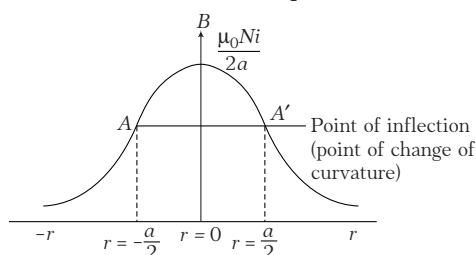


Fig. 4.12 B - r graph at the axis of coil (ring)

Direction of magnetic field on the axis of the circular coil

Direction of magnetic field on the axis of a circular loop can be obtained using the right hand thumb rule. This rule states that, if the fingers of right hand are curled along the current, then the stretched thumb will point towards the direction of magnetic field as shown in figures given below.

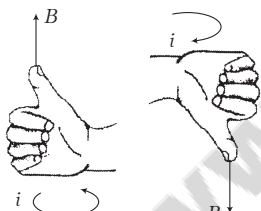


Fig. 4.13 Right hand thumb rule

Magnetic field due to circular current carrying coil apart from axis

The magnetic field at any point except the axis is mathematically difficult to calculate. The magnetic field lines due to a circular current carrying coil are shown below in the figure, which will give some idea of the field.

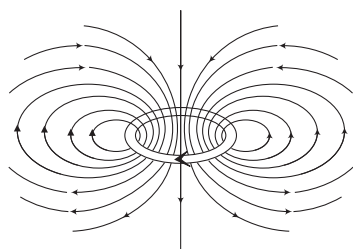


Fig. 4.14 Magnetic field lines of a current carrying loop

Magnetic field due to a set of two circular current carrying coils (case of Helmholtz)

Consider the set up of two co-axial coils of same radius such that distance between their centres is equal to their radius. This arrangement is called **Helmholtz**.

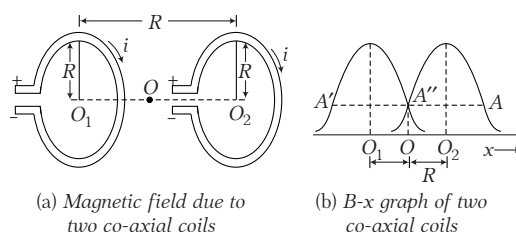


Fig. 4.15

At axial mid-point O , the magnetic field is given by

$$B = \frac{8\mu_0 Ni}{5\sqrt{5} R} = 0.716 \frac{\mu_0 Ni}{R} = 1.432 B_{\text{max}}$$

where, $B_{\text{max}} = \frac{\mu_0 Ni}{2R}$

Example 4.17 The magnetic field B due to a current carrying circular loop of radius 12 cm at its centre is 0.5×10^{-4} T. Find the magnetic field due to this loop at a point on the axis at a distance of 5.0 cm from the centre.

Sol. Magnetic field at the centre of a circular loop,

$$B_1 = \frac{\mu_0 i}{2R}$$

and that at an axial point,

$$B_2 = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$$

$$\text{Thus, } \frac{B_2}{B_1} = \frac{R^3}{(R^2 + x^2)^{3/2}}$$

$$\text{or } B_2 = B_1 \left[\frac{R^3}{(R^2 + x^2)^{3/2}} \right]$$

Substituting the given values in above equation, we get

$$\begin{aligned} B_2 &= (0.5 \times 10^{-4}) \left[\frac{(12)^3}{(144 + 25)^{3/2}} \right] \\ &= 3.9 \times 10^{-5} \text{ T} \end{aligned}$$

AMPERE'S CIRCUITAL LAW

It states that, the line integral of magnetic field $\mathbf{B}$ around any closed path (Amperian loop) or circuit is equal to μ_0 times the total current crossing the area bounded by the closed path, provided that the magnetic field inside the loop remains constant. Thus,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (i_{\text{net}})$$

In the above relation, we will use a sign convention given by right hand rule. According to this rule, fingers of the right hand are curled in the sense that the boundary is traversed in the loop integral $\oint \mathbf{B} \cdot d\mathbf{l}$, then the direction of the thumb gives the sense in which the current I is treated as positive.

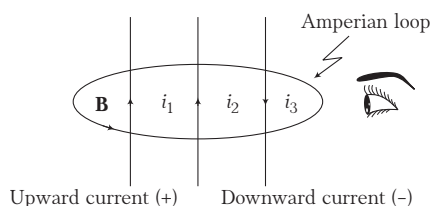


Fig. 4.16 Sign convention for the current

In this way, currents i_1 , i_2 and i_3 shown in the diagram are respectively taken as positive and negative.

$$\therefore i_{\text{net}} = i_1 + i_2 - i_3$$

$$\Rightarrow \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (i_1 + i_2 - i_3)$$

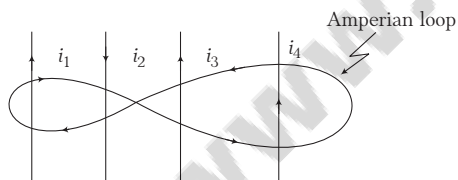
Its simplified form is $Bl = \mu_0 i_{\text{net}}$.

This simplified equation can be used under the following conditions

- At every point of the closed path, $\mathbf{B} \parallel d\mathbf{l}$.
- Magnetic field has the same magnitude B at all places on the closed path.
- Ampere's circuital law holds for steady currents which do not change with time.

Example 4.18 Write equation for Ampere's circuital law for the Amperian loop as shown below.

(Traverse in the direction shown by arrow mark put on it.)



Sol. If observed from the top, then left side portion of the loop is traversed clockwise while the other portion is traversed anti-clockwise. Therefore, by sign convention

$$i_1 \longrightarrow (+), i_2 \longrightarrow (-)$$

$$i_3 \longrightarrow (+), i_4 \longrightarrow (+)$$

$$\text{Thus, } \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (i_1 + i_3 + i_4 - i_2)$$

Applications of Ampere's circuital law

Ampere's circuital law can be applied to calculate magnetic field associated with symmetric distributed current carrying configurations which are discussed below

Magnetic field due to a cylindrical wire

- (i) **Outside the cylinder** ($r > R$) To find magnetic field outside at point P , we will assume Amperian loops as shown below

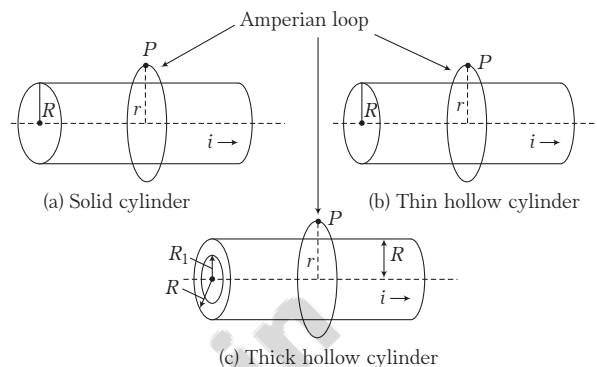


Fig. 4.17 Applications of Ampere's circuital law for a point P outside the cylinder

In all the above cases, magnetic field outside the wire at P ,

$$\oint \mathbf{B} \cdot d\mathbf{l} = B(2\pi r)$$

$$\Rightarrow B_{\text{out}} = \frac{\mu_0 i}{2\pi r}$$

$$\text{In all the above cases, } B_{\text{surface}} = \frac{\mu_0 i}{2\pi R}$$

- (ii) **Inside the hollow cylinder** ($r < R$) Magnetic field inside the hollow cylinder is zero because no current is enclosed by Amperian loops as shown below

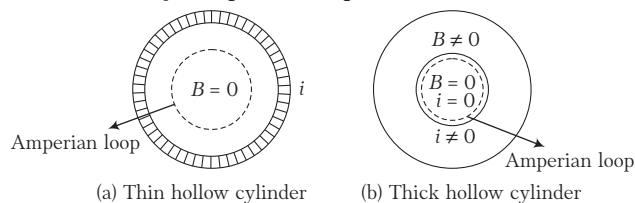


Fig. 4.18 Magnetic field inside the hollow cylinder

- (iii) **Inside the solid cylinder** ($r < R$) Current i' enclosed by Amperian loop is less than the total current (i).

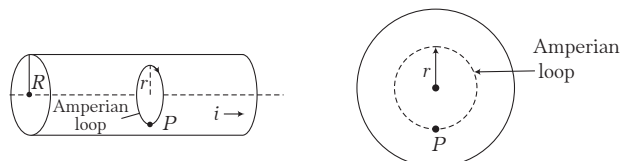


Fig. 4.19 Amperian loop inside a solid cylinder

Current density is uniform, i.e. $J = J'$

$$\Rightarrow i' = i \times \frac{A'}{A} = i \left(\frac{r^2}{R^2} \right)$$

Hence, at inside point, $\oint \mathbf{B}_{\text{in}} \cdot d\mathbf{l} = \mu_0 i'$

$$\Rightarrow \quad B_{\text{in}} = \frac{\mu_0}{2\pi} \cdot \frac{ir}{R^2}$$

- (iv) **Inside the thick portion of hollow cylinder** Current enclosed by the Amperian loop of radius r is

$$\text{given by } i' = i \times \frac{A'}{A} = i \times \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$$

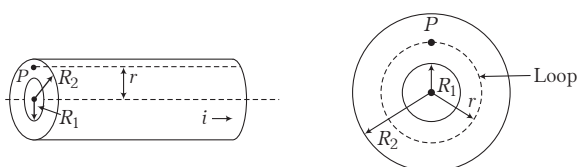


Fig. 4.20 Amperian loop inside the thick portion of the hollow cylinder

Hence, at point P, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i'$

$$\Rightarrow \quad B_{\text{in}} = \frac{\mu_0 i}{2\pi r} \cdot \frac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$$

Variation in magnetic field with radius

- (i) The variation in magnetic field due to infinite long solid cylindrical conductor along its radius is as shown in figure

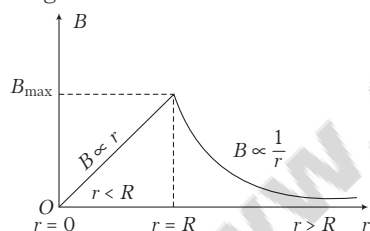


Fig. 4.21 B-r graph for solid cylinder

Note that inside the wire, $B = 0$ as $r = 0$.

Also, the magnetic field is continuous at the surface of the wire.

- (ii) The variation in magnetic field due to infinite long hollow cylindrical conductor along its radius is as shown in figure

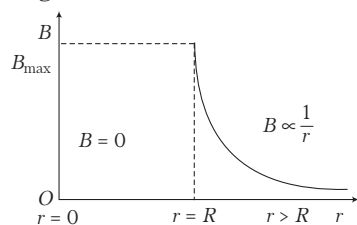


Fig. 4.22 B-r graph for hollow cylinder

Magnetic field outside the cylindrical conductor does not depend upon nature (thick/thin or solid/hollow) of the conductor as well as its radius of cross-section.

Magnetic field due to a solenoid

A cylindrical coil of many wound turns of insulated wire with diameter of the coil smaller than its length is called a **solenoid**.

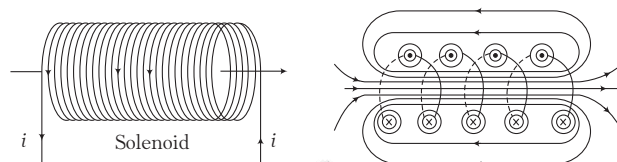


Fig. 4.23 Solenoid and its magnetic field

A magnetic field is produced around and within the solenoid as shown in above figure. The magnetic field within the tightly wound solenoid (ideal solenoid) is uniform and parallel to the axis of solenoid.

Outside the ideal solenoid, magnetic field is zero as shown in the figure below

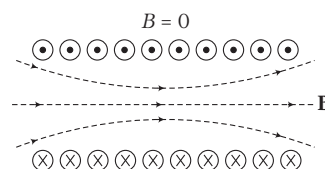


Fig. 4.24 Magnetic field (outside) due to ideal solenoid

- (i) **Finite length solenoid** If N = total number of turns, l = length of the solenoid and n = number of turns per unit length $= N/l$.

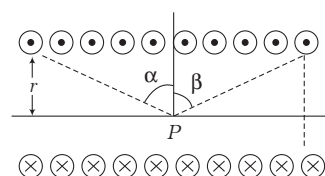


Fig. 4.25 Magnetic field due to finite length solenoid

Magnetic field inside the solenoid at point P is given by

$$B = \frac{\mu_0 n i}{2} [\sin \alpha + \sin \beta]$$

- (ii) **Infinite length solenoid** If the solenoid is of infinite length and the point is well inside it, i.e. $\alpha = \beta = (\pi/2)$

Then,

$$B_{\text{in}} = \mu_0 n i$$

If the solenoid is of infinite length and the point is near one end, i.e. $\alpha = 0$ and $\beta = (\pi/2)$, then

$$B_{\text{end}} = \frac{1}{2} (\mu_0 n i) \quad \left(B_{\text{end}} = \frac{1}{2} B_{\text{in}} \right)$$

(iii) Variation in magnetic field with distance d from its centre

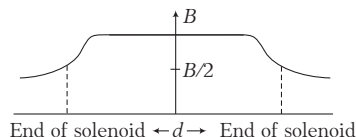


Fig. 4.26 B-d graph for solenoid

Note

- (i) Magnetic field produced by solenoid is directed along its axis.
- (ii) Magnetic field inside the solenoid is uniform.
- (iii) Magnetic field outside the volume of the ideal solenoid (tightly wound) approaches to zero.

Magnetic field due to a toroid

A toroid can be considered as a ring shaped closed solenoid. Hence, it is like an endless cylindrical solenoid.

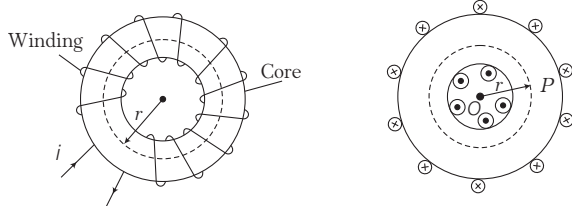


Fig. 4.27 Toroid and its magnetic field

Consider a toroid having n turns per unit length. Magnetic field at a point P in the figure is given as

$$B = \frac{\mu_0 N i}{2\pi r} = \mu_0 n i \quad \text{where, } n = \frac{N}{2\pi r}$$

Note

- (i) Magnetic field outside the volume of toroid is always zero.
- (ii) Magnetic field at the centre of toroid is always zero.

Magnetic field due to an infinitely large current carrying sheet

Consider an infinite sheet of current with linear current density J (A/m). Due to symmetry of field line pattern above and below the sheet is uniform.

Magnetic field at point P due to infinitely large current carrying sheet is given as

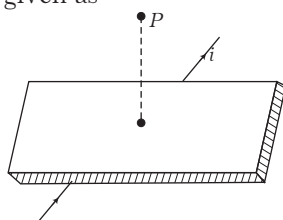


Fig. 4.28

$$B = \frac{\mu_0 J}{2}$$

Example 4.19 A 0.8 m long solenoid has 800 turns and has a field density of 2.52×10^{-3} T at its centre. Find the current in the solenoid.

Sol. Number of turns per unit length,

$$n = \frac{N}{l} = \frac{800}{0.8} = 1000 \text{ turns/m}$$

We know that, $B = \mu_0 n i$

$$\therefore \text{Current in the solenoid, } i = \frac{B}{\mu_0 n} = \frac{2.52 \times 10^{-3}}{4\pi \times 10^{-7} \times 1000} = 2.0 \text{ A}$$

Example 4.20 A copper wire having a resistance of $0.01 \Omega/\text{m}$ is used to wind a 400 turns solenoid of radius 1.0 cm and length 20 cm. Find the emf of a battery which when connected across the solenoid would produce a magnetic field of 10^{-2} T near the centre of the solenoid.

Sol. Length of wire used $= 2\pi r \times \text{Number of turns}$
 $= (2\pi \times 1.0 \times 10^{-2} \times 400) \text{ m}$

Resistance per unit length $= 0.01 \Omega/\text{m}$

$\therefore$ Total resistance of wire,

$$R = 2\pi \times 1.0 \times 10^{-2} \times 400 \times 0.01$$

$$= 8\pi \times 10^{-2} \Omega$$

Number of turns per unit length,

$$n = N/l = \frac{400}{20 \times 10^{-2}} = 2000 \text{ turns/m}$$

Magnetic field, $B = \mu_0 n i = \mu_0 n \frac{V}{R}$

$$\text{emf of the battery, } V = \frac{BR}{\mu_0 n} = \frac{10^{-2} \times 8\pi \times 10^{-2}}{4\pi \times 10^{-7} \times 2000} = 1 \text{ V}$$

Example 4.21 A coil wrapped around a toroid has inner radius of 20.0 cm and an outer radius of 25.0 cm. If the wire wrapping makes 800 turns and carries a current of 12.0 A. Find the maximum and minimum values of the magnetic field within the toroid.

Sol. Let a and b be the inner and outer radii of the toroid, respectively. Then, maximum value of magnetic field,

$$B_{\text{max}} = \mu_0 n I = \frac{\mu_0 N}{2\pi a} I = \frac{4\pi \times 10^{-7} \times 800 \times 12.0}{2\pi \times 20.0 \times 10^{-2}}$$

$$= 9.6 \times 10^{-3} \text{ T} = 9.6 \text{ mT}$$

Minimum value of magnetic field,

$$B_{\text{min}} = \mu_0 n I = \mu_0 \frac{NI}{2\pi b}$$

$$B_{\text{min}} = \frac{4\pi \times 10^{-7} \times 800 \times 12.0}{2\pi \times 25.0 \times 10^{-2}}$$

$$= 7.68 \times 10^{-3} \text{ T} = 7.68 \text{ mT}$$

Example 4.22 A solenoid 50 cm long has 4 layers of windings of 350 turns each. The radius of the lowest layer is 1.4 cm. If the current carried is 6.0 A, find the magnitude of magnetic field (i) near the centre of the solenoid on its axis and off its axis, (ii) near its ends on its axis (iii) and outside the solenoid near its centre.

- Sol.** (i) The magnitude of the magnetic field at (or near) the centre of the solenoid is given by

$$B = \mu_0 n I$$

where, n is the number of turns per unit length.

This expression for B can also be used, if the solenoid has more than one layer of windings because the radius of wire does not effect this equation. Therefore,

$$n = \frac{\text{Number of turns per layer} \times \text{Number of layers}}{\text{Length of the solenoid}}$$

$$= \frac{350 \times 4}{0.5} = 2800 \text{ turns/m}$$

$$\text{Now, } I = 6.0 \text{ A, } \mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$$

$$\text{and } n = 2800 \text{ turns/m}$$

$$\therefore B = 4\pi \times 10^{-7} \times 2800 \times 6 = 2.1 \times 10^{-2} \text{ T}$$

Since, for an infinitely long solenoid, the internal field near the centre is uniform over the entire cross-section, therefore this value of B is for both on and off its axis.

- (ii) Magnetic field at the ends of the solenoid,

$$B_{\text{end}} = \frac{\mu_0 n I}{2} = \frac{2.1 \times 10^{-2}}{2} = 1.05 \times 10^{-2} \text{ T}$$

- (iii) The outside field near the centre of a long solenoid is negligible as compared to the internal field.

CHECK POINT 4.1

1. Which of the following gives the value of magnetic field due to a small current element according to Biot-Savart's law?

- (a) $\frac{i \Delta l \sin \theta}{r^2}$ (b) $\frac{\mu_0}{4\pi} \frac{i \Delta l \sin \theta}{r}$
 (c) $\frac{\mu_0}{4\pi} \frac{i \Delta l \sin \theta}{r^2}$ (d) $\frac{\mu_0}{4\pi} \frac{i \Delta l \sin \theta}{r^3}$

2. Magnetic field at a distance r from an infinitely long straight conductor carrying steady current varies as

- (a) $1/r^2$ (b) $1/r$
 (c) $1/r^3$ (d) $1/\sqrt{r}$

3. The strength of the magnetic field at a distance r near a long straight current carrying wire is B . The magnetic field at a distance $r/2$ will be

- (a) $\frac{B}{2}$ (b) $\frac{B}{4}$
 (c) $2B$ (d) $4B$

4. The current is flowing in south direction along a power line. The direction of magnetic field above the power line (neglecting earth's field) is

- (a) south (b) east
 (c) north (d) west

5. Two infinitely long, thin, insulated, straight wires lie in the XY -plane along the X and Y -axes, respectively. Each wire carries a current I respectively in the positive x -direction and positive y -direction. The magnetic field will be zero at all points on the straight line with equation

- (a) $y = x$ (b) $y = -x$
 (c) $y = x - 1$ (d) $y = -x + 1$

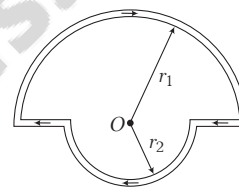
6. The magnetic field produced at the centre of a current carrying circular coil of radius r is

- (a) directly proportional to r (b) inversely proportional to r
 (c) directly proportional to r^2 (d) inversely proportional to r^2

7. A particle carrying a charge equal to 100 times the charge on an electron, is rotating one rotation per second in a circular path of radius 0.8 m. The value of the magnetic field produced at the centre will be (μ_0 = permeability for vacuum)

- (a) $\frac{10^{-7}}{\mu_0} \text{ T}$ (b) $10^{-17} \mu_0 \text{ T}$
 (c) $10^{-6} \mu_0 \text{ T}$ (d) $10^{-7} \mu_0 \text{ T}$

8. In the figure shown, there are two semicircles of radii r_1 and r_2 in which a current i is flowing. The magnetic induction at the centre O will be



- (a) $\frac{\mu_0 i}{4} (r_1 + r_2)$ (b) $\frac{\mu_0 i}{4} (r_1 - r_2)$
 (c) $\frac{\mu_0 i}{4} \left[\frac{r_1 + r_2}{r_1 r_2} \right]$ (d) $\frac{\mu_0 i}{4} \left[\frac{r_2 - r_1}{r_1 r_2} \right]$

9. A current of 0.1 A circulates around a coil of 100 turns and having a radius equal to 5 cm. The magnetic field set up at the centre of the coil is ($\mu_0 = 4\pi \times 10^{-7} \text{ Wb/A-m}$)

- (a) $5\pi \times 10^{-5} \text{ T}$ (b) $8\pi \times 10^{-5} \text{ T}$
 (c) $4\pi \times 10^{-5} \text{ T}$ (d) $2\pi \times 10^{-5} \text{ T}$

10. Two concentric coils each of 10 turns are situated in the same plane. Their radii are 2 cm and 4 cm and they carry 0.2 A and 0.3 A currents respectively in opposite directions. The magnetic field (in Wb/m^2) at the centre is

- (a) $(35/4) \mu_0$ (b) $(\mu_0/80)$ (c) $(7/80) \mu_0$ (d) $(25/2) \mu_0$

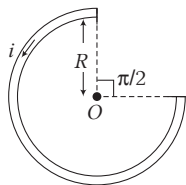
11. A given length of a wire carries a steady current. It is bent first to form a circular plane coil of one turn. If a loop of same length is now bent more sharply to give a double loop of smaller radius, then the magnetic field at the centre caused by the same current is

- (a) a quarter of its first value
 (b) unaltered
 (c) four times of its first value
 (d) two times of its first value

12. An arc of a circle of radius R subtends an angle $\pi/2$ at the centre. It carries a current i . The magnetic field at the centre will be

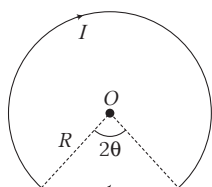
- (a) $\frac{\mu_0 i}{2R}$ (b) $\frac{\mu_0 i}{8R}$ (c) $\frac{\mu_0 i}{4R}$ (d) $\frac{2\mu_0 i}{5R}$

13. A current i ampere flows in a circular arc of radius R , which subtends an angle $3\pi/2$ radian at its centre. The magnetic induction B at the centre is



- (a) $\frac{\mu_0 i}{R}$ (b) $\frac{\mu_0 i}{2R}$
(c) $\frac{2\mu_0 i}{R}$ (d) $\frac{3\mu_0 i}{8R}$

14. A current I flows through a closed loop as shown in figure. The magnetic field at the centre O is



- (a) $\frac{\mu_0 I}{2\pi R} (\pi - \theta + \tan \theta)$ (b) $\frac{\mu_0 I}{2\pi R} (\pi - \theta + \sin \theta)$
(c) $\frac{\mu_0 I}{2\pi R} (\theta + \sin \theta)$ (d) None of these

15. Magnetic field due to a ring having n turns at a distance x on its axis is proportional to (if r = radius of ring)

- (a) $\frac{r}{(x^2 + r^2)}$ (b) $\frac{r}{(x^2 + r^2)^{3/2}}$
(c) $\frac{nr^2}{(x^2 + r^2)^{3/2}}$ (d) $\frac{n^2 r^2}{(x^2 + r^2)^{3/2}}$

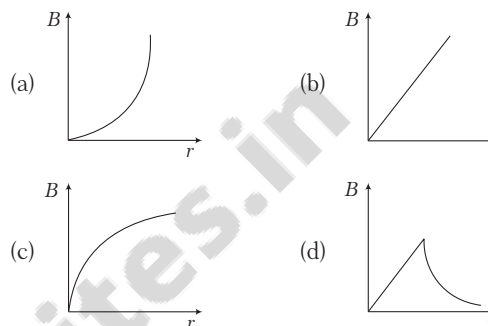
16. The ratio of the magnetic field at centre of a current carrying coil of the radius a and at a distance a from centre of the coil to the axis of coil is

- (a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$
(c) $\frac{1}{2\sqrt{2}}$ (d) $2\sqrt{2}$

17. A circular current carrying coil has a radius R . The distance from the centre of the coil on the axis of the coil, where the magnetic induction is $(1/8)$ th of its value at the centre of coil is

- (a) $\sqrt{3}R$ (b) $R/\sqrt{3}$ (c) $(2/\sqrt{3})R$ (d) $R/2\sqrt{3}$

18. Which of the following figures shows the magnetic flux density B at a distance r from a long straight rod carrying a steady current i ?



19. While using Ampere's law to determine the magnetic field inside a straight long solenoid, the loop that is taken is

- (a) a circular loop, co-axial with the solenoid
(b) a rectangular loop in a plane perpendicular to the axis of the solenoid
(c) a rectangular loop in a plane containing the axis of the solenoid, the loop being totally within the solenoid
(d) a rectangular loop in a plane containing the axis of the solenoid, the loop being partly inside the solenoid and partly outside it

20. A toroid has a core of inner radius 25 cm and outer radius 26 cm around which 3500 turns of a wire are wound. If the current in the wire is 11 A, the magnetic field inside the core of the toroid is

- (a) 1.5×10^{-3} T (b) 3.0×10^{-2} T
(c) 4.5×10^{-2} T (d) 5.0×10^{-2} T

FORCE ON A MOVING CHARGE IN A UNIFORM MAGNETIC FIELD

If a particle carrying a positive charge q and moving with velocity v enters in a magnetic field B , then it experiences a magnetic force F_m which is given by the expression

$$\mathbf{F}_m = q (\mathbf{v} \times \mathbf{B})$$

- (i) The magnitude of $\mathbf{F}_m$ is

$$F_m = Bqv \sin \theta$$

where, θ is the angle between $\mathbf{v}$ and $\mathbf{B}$.

- (ii) $\mathbf{F}_m$ is zero, when

- (a) $B = 0$, i.e. no magnetic field is present.
(b) $q = 0$, i.e. particle is neutral.

- (c) $v = 0$, i.e. charged particle is at rest.

- (d) $\theta = 0^\circ$ or 180° , i.e. particle is moving parallel or anti-parallel to the direction of magnetic field.

- (iii) $\mathbf{F}_m$ is maximum at $\theta = 90^\circ$ and this maximum value is Bqv .

Rules to find the direction of magnetic force ($\mathbf{F}_m$)

From the property of cross product, we can infer that $\mathbf{F}_m$ is perpendicular to both $\mathbf{v}$ and $\mathbf{B}$ or it is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$.

The exact direction of $\mathbf{F}_m$ can be determined by any of the following methods

- (i) Direction of $\mathbf{F}_m = (\text{sign of } q) (\text{direction of } \mathbf{v} \times \mathbf{B}) \dots (i)$

We can say that, if q is positive in Eq. (i), magnetic force is along $\mathbf{v} \times \mathbf{B}$ and if q is negative, magnetic force is in a opposite direction to $\mathbf{v} \times \mathbf{B}$.

- (ii) **Fleming's left hand rule** According to this rule, if the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other, then the central finger gives the direction of velocity of positive charge ($\mathbf{v}$), forefinger gives the direction of magnetic field ($\mathbf{B}$) and the thumb will give the direction of magnetic force ($\mathbf{F}_m$).

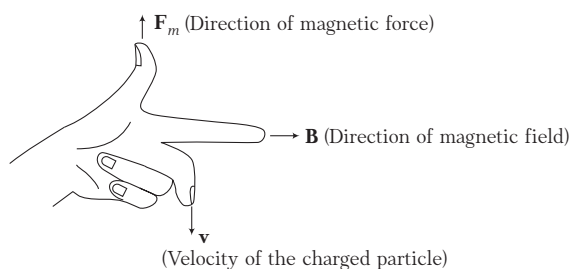


Fig. 4.29 Direction of $\mathbf{F}_m$ by Fleming's left hand rule

- (iii) **Right hand rule** Wrap the fingers of your right hand around the line perpendicular to the plane of $\mathbf{v}$ and $\mathbf{B}$ as shown in figure, so that they curl around with the sense of rotation from $\mathbf{v}$ to $\mathbf{B}$ through the smaller angle between them. Your thumb, then points in the direction of the force $\mathbf{F}_m$ on a positive charge.

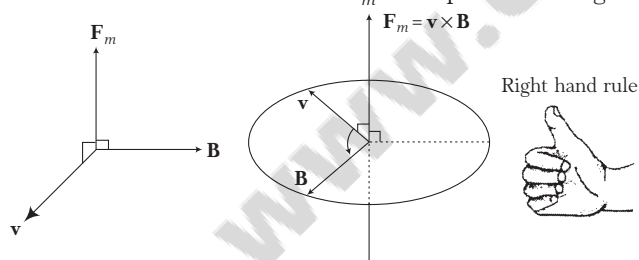


Fig. 4.30 Direction of $\mathbf{F}_m$ by right hand rule

Note If $\mathbf{F}_m \perp \mathbf{v} \Rightarrow \mathbf{F} \perp \frac{d\mathbf{s}}{dt}$. Therefore, $\mathbf{F}_m \perp d\mathbf{s}$ or the work done by the magnetic force in a static magnetic field is zero.
i.e. $W_{F_m} = 0$

Example 4.23 A charged particle of specific charge (i.e. charge per unit mass) 0.2 C/kg has velocity $(2\hat{i} - 3\hat{j}) \text{ ms}^{-1}$ at some instant in a uniform magnetic field $(5\hat{i} + 2\hat{j}) \text{ T}$. Find the acceleration of the particle at this instant.

Sol. The acceleration of the particle, $\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q}{m} (\mathbf{v} \times \mathbf{B})$

Given,

$$\frac{q}{m} = 0.2 \text{ C/kg}$$

$$\mathbf{v} = (2\hat{i} - 3\hat{j}) \text{ ms}^{-1} \text{ and } \mathbf{B} = (5\hat{i} + 2\hat{j}) \text{ T}$$

Therefore,

$$\begin{aligned} \mathbf{a} &= 0.2(2\hat{i} - 3\hat{j}) \times (5\hat{i} + 2\hat{j}) \\ &= 0.2[4\hat{k} + 15\hat{k}] = 3.8 \hat{k} \text{ ms}^{-2} \end{aligned}$$

Example 4.24 When a proton has a velocity $\mathbf{v} = (2\hat{i} + 3\hat{j}) \times 10^6 \text{ ms}^{-1}$, it experiences a force $\mathbf{F} = -(1.28 \times 10^{-13} \hat{k}) \text{ N}$. When its velocity is along the Z-axis, then it experiences a force along the X-axis. What is the magnetic field?

Sol. Magnetic force, $\mathbf{F}_m = q (\mathbf{v} \times \mathbf{B})$

For $\mathbf{F}_m$ to be along $(-\hat{k})$, magnetic field must be along $(-\hat{j})$.

Now, substituting all the values in above equation, we get

$$-(1.28 \times 10^{-13} \hat{k}) = (1.6 \times 10^{-19}) [(2\hat{i} + 3\hat{j}) \times (-B\hat{j})] \times 10^6$$

$$\therefore 1.28 = 1.6 \times 2 \times B$$

$$\text{or } B = \frac{1.28}{3.2} = 0.4$$

Therefore, the magnetic field, $\mathbf{B} = -B \hat{j} = (-0.4 \hat{j}) \text{ T}$

Example 4.25 A charged particle is projected in a magnetic field

$$\mathbf{B} = (3\hat{i} + 4\hat{j}) \times 10^{-2} \text{ T}$$

and the acceleration of the particle is found to be

$$\mathbf{a} = (x\hat{i} + 2\hat{j}) \text{ ms}^{-2}.$$

Find the value of x .

Sol. As,

$$\mathbf{F}_m \perp \mathbf{B}$$

i.e. the acceleration $\mathbf{a} \perp \mathbf{B} \Rightarrow \mathbf{a} \cdot \mathbf{B} = 0$

$$\text{or } (x\hat{i} + 2\hat{j}) \cdot (3\hat{i} + 4\hat{j}) \times 10^{-2} = 0$$

$$\text{or } (3x + 8) \times 10^{-2} = 0$$

$$\therefore \text{Value of } x = -\frac{8}{3}$$

Example 4.26 A magnetic field of $(4.0 \times 10^{-3} \hat{k}) \text{ T}$ exerts a force $(4.0 \hat{i} + 3.0 \hat{j}) \times 10^{-10} \text{ N}$ on a particle having a charge 10^{-9} C and moving in the XY-plane. Find the velocity of the particle.

Sol. Given, $\mathbf{B} = (4 \times 10^{-3} \hat{k}) \text{ T}$, $q = 10^{-9} \text{ C}$

and magnetic force, $\mathbf{F}_m = (4.0 \hat{i} + 3.0 \hat{j}) \times 10^{-10} \text{ N}$

Let velocity of the particle in XY-plane be

$$\mathbf{v} = v_x \hat{i} + v_y \hat{j}$$

Then, from the relation,

$$\mathbf{F}_m = q (\mathbf{v} \times \mathbf{B})$$

$$(4.0 \hat{i} + 3.0 \hat{j}) \times 10^{-10} = 10^{-9} [(v_x \hat{i} + v_y \hat{j}) \times (4 \times 10^{-3} \hat{k})]$$

$$= (4v_y \times 10^{-12} \hat{i} - 4v_x \times 10^{-12} \hat{j})$$

Comparing the coefficients of $\hat{i}$ and $\hat{j}$, we have

$$4 \times 10^{-10} = 4v_y \times 10^{-12}$$

$$\therefore v_y = 10^2 = 100 \text{ ms}^{-1}$$

$$\text{and } 3.0 \times 10^{-10} = -4v_x \times 10^{-12}$$

$$\therefore v_x = -75 \text{ ms}^{-1}$$

$$\begin{aligned} \therefore \mathbf{v} &= v_x \hat{i} + v_y \hat{j} \\ &= (-75\hat{i} + 100\hat{j}) \text{ m/s} \end{aligned}$$

Example 4.27 A charged particle carrying charge, $q = 1\mu\text{C}$ moves in uniform magnetic field with velocity, $v_1 = 10^6 \text{ ms}^{-1}$ at an angle 45° with X -axis in the XY -plane and experiences a force $F_1 = 5\sqrt{2} \text{ mN}$ along the negative Z -axis. When the same particle move with velocity, $v_2 = 10^6 \text{ ms}^{-1}$ along the Z -axis, then it experiences a force F_2 in y -direction. Find

(i) the magnitude and direction of the magnetic field.

(ii) the magnitude of the force F_2 .

Sol. F_2 is in y -direction when velocity is along Z -axis.

Therefore, magnetic field should be along X -axis. So, let

$$\mathbf{B} = B_0 \hat{i}$$

$$(i) \text{ Given, } \mathbf{v}_1 = v_1 \cos 45^\circ \hat{i} + v_1 \sin 45^\circ \hat{j}$$

$$= \frac{10^6}{\sqrt{2}} \hat{i} + \frac{10^6}{\sqrt{2}} \hat{j}$$

$$\text{and } \mathbf{F}_1 = -5\sqrt{2} \times 10^{-3} \hat{k}$$

$$\text{From the equation, } \mathbf{F}_1 = q(\mathbf{v}_1 \times \mathbf{B})$$

We have,

$$\begin{aligned} (-5\sqrt{2} \times 10^{-3}) \hat{k} &= (10^{-6}) \left[\left(\frac{10^6}{\sqrt{2}} \hat{i} + \frac{10^6}{\sqrt{2}} \hat{j} \right) \times (B_0 \hat{i}) \right] \\ &= -\frac{B_0}{\sqrt{2}} \hat{k} \end{aligned}$$

$$\therefore \frac{B_0}{\sqrt{2}} = 5\sqrt{2} \times 10^{-3}$$

$$\text{or } B_0 = 10^{-2} \text{ T}$$

Therefore, the magnetic field,

$$\mathbf{B} = (10^{-2} \hat{i}) \text{ T}$$

$$(ii) \text{ Here, } F_2 = B_0 q v_2 \sin 90^\circ$$

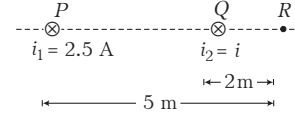
As, the angle between $\mathbf{B}$ and $\mathbf{v}$ in this case is 90° .

$$\therefore F_2 = (10^{-2})(10^{-6})(10^6) = 10^{-2} \text{ N}$$

Example 4.28 Two long parallel wires carrying current 2.5 A and i in the same direction (directed into plane of the paper) are held at P and Q respectively such that they are perpendicular to the plane of paper. The points P and Q are located at distance of 5 m and 2 m respectively from a collinear point R .

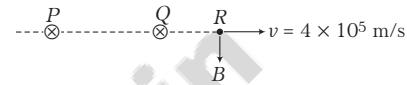
(i) An electron moving with a velocity of $4 \times 10^5 \text{ m/s}$ along positive x -direction experiences a force of magnitude $3.2 \times 10^{-20} \text{ N}$ at the point R . Find the value of i .

(ii) Find all positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed so that magnetic field at R is zero.



Sol. (i) Magnetic field at R due to wires,

$$\begin{aligned} B &= \frac{\mu_0}{2\pi} \left[\frac{i_1}{5} + \frac{i_2}{2} \right] = 2 \times 10^{-7} \left[\frac{2.5}{5} + \frac{i}{2} \right] \\ &= 10^{-7} (1 + i), \text{ along } Y\text{-axis} \end{aligned}$$



Angle between $\mathbf{v}$ and $\mathbf{B}$ is 90° .

Magnetic force, $F_m = Bqv$

$$3.2 \times 10^{-20} = 10^{-7} (1 + i) (1.6 \times 10^{-19}) (4 \times 10^5)$$

$$1 + i = 5$$

$\therefore$ Current, $i = 4 \text{ A}$

(ii) Third wire can be placed left of R or right of R at distance d . If it is placed left of R , current in it should be upward direction or if it is placed right of R , current in it should be in downward direction.

Now, $B_R = 0$

$$\Rightarrow \frac{\mu_0}{2\pi} \left[\frac{2.5}{5} + \frac{4}{d} \right] = \frac{\mu_0}{2\pi} \frac{2.5}{d}$$

$\therefore$ Position, $d = 1 \text{ m}$

Motion of a charged particle in uniform magnetic field

As we know that, the magnetic force on a charged particle is perpendicular to its velocity. Thus, this force does not do any work on the particle. Hence, the kinetic energy or the speed of the particle does not change due to the magnetic force.

Let a charged particle q be thrown in magnetic field $\mathbf{B}$ with a velocity $\mathbf{v}$ as shown in the figure. The magnetic force acting on the particle is given by $F = Bqv \sin \theta$, where θ is the angle between the velocity and the magnetic field.

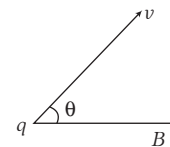


Fig. 4.31 Charged particle with velocity $\mathbf{v}$ at angle θ with the magnetic field

Depending on the initial conditions, the charged particle can follow different trajectories in a region of uniform magnetic field. Let us take them one-by-one.

Case I When θ is 0° or 180°

When $\theta = 0^\circ$ or 180° , the magnetic force $F_m = 0$. Hence, path of the charged particle is a straight line (undeviated) when it enters parallel or anti-parallel to magnetic field as shown below.

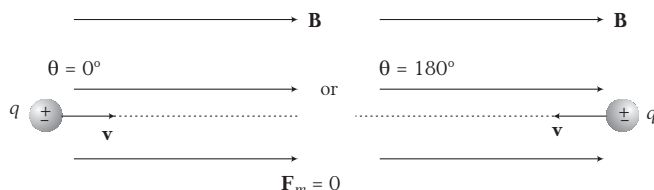


Fig. 4.32 A charged particle moving parallel or anti-parallel to magnetic field

Case II When θ is 90°

When $\theta = 90^\circ$, the magnetic force,

$$F_m = Bqv \sin 90^\circ = Bqv$$

This magnetic force is perpendicular to the velocity at every instant. Hence, path of the charged particle is a circle. The necessary centripetal force is provided by the magnetic force.

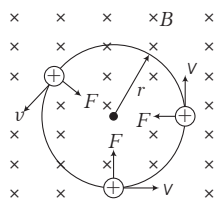


Fig. 4.33 A charged particle is moving in a circular path

Hence, if r be the radius of the circle, then

$$\frac{mv^2}{r} = Bqv \Rightarrow r = \frac{mv}{Bq}$$

This expression of r can be written in following different ways

$$r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq}$$

Here, p = momentum of particle and

$$K = \text{KE of particle} = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2Km}$$

Also, if the charged particle is accelerated by a potential difference of V volts, it acquires a KE given by

$$K = qV \Rightarrow r = \frac{\sqrt{2qVm}}{Bq}$$

(i) Further, time period of the circular path,

$$T = \frac{2\pi r}{v} = \frac{2\pi \left(\frac{mv}{Bq} \right)}{v} \Rightarrow T = \frac{2\pi m}{Bq}$$

(ii) Angular speed of the particle, $\omega = \frac{2\pi}{T} = \frac{Bq}{m}$

$$\therefore \omega = \frac{Bq}{m}$$

(iii) Frequency of rotation, $f = \frac{1}{T}$

$$\Rightarrow f = \frac{Bq}{2\pi m}$$

Note When a charged particle is projected perpendicular to a magnetic field, then

- (i) its path is circular in a plane perpendicular to the plane of magnetic field.
- (ii) the speed and kinetic energy of the particle remain constant.
- (iii) the velocity and momentum of the particle change only in direction.
- (iv) the time period of revolution, angular velocity and frequency of revolution are independent of velocity of the particle and radius of circular path.

Important points related to motion of a charged particle in uniform magnetic field

The speed of the particle in magnetic field does not change but the particle gets deviated. The deviation θ can be found in two ways,

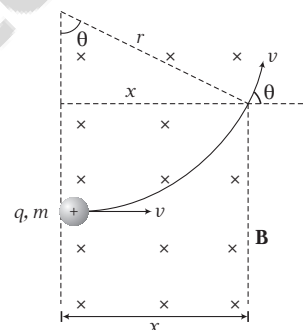


Fig. 4.34 Deviation θ of the charged particle in magnetic field

(i) After time t ,

$$\text{Deviation, } \theta = \omega t = \left(\frac{Bq}{m} \right) t \quad \left(\because \omega = \frac{Bq}{m} \right)$$

(ii) In terms of the length of the magnetic field (i.e. when the particle leaves the magnetic field), the

$$\text{deviation, } \theta = \sin^{-1} \left(\frac{x}{r} \right), \text{ if } x \leq r$$

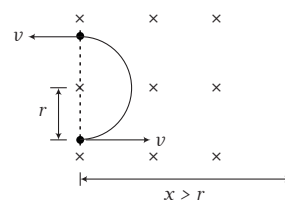


Fig. 4.35 Charged particle having 180° deviation

Since, $\sin \theta \geq 1$, this relation can be used only when $x \leq r$.

For $x > r$, the deviation will be 180° as shown in Fig. 4.34.

Example 4.29 What is the smallest value of B that can be set up at the equator to permit a proton having speed of 10^7 m s^{-1} to circulate around the earth? ($R = 6.4 \times 10^6 \text{ m}$, $m_p = 1.67 \times 10^{-27} \text{ kg}$)

Sol. From the relation, $R = \frac{mv}{Bq}$

$$\Rightarrow B = \frac{mv}{qR}$$

Substituting the given values in above equation, we get

$$B = \frac{(1.67 \times 10^{-27})(10^7)}{(1.6 \times 10^{-19})(6.4 \times 10^6)} = 1.6 \times 10^{-8} \text{ T}$$

Example 4.30 A proton and an α -particle, accelerated through same potential difference, enter in a region of uniform magnetic field with their velocities perpendicular to the field. Compare the radii of circular paths followed by them.

Sol. Let mass of proton be m and charge on proton be e .

Then, mass of α -particle = $4m$ and charge on α -particle = $2e$

When a charge q is accelerated by potential difference V , it acquires a kinetic energy, $E_k = qV$

$\therefore$ Momentum is given by $mv = \sqrt{2mE_k} = \sqrt{2mqV}$

$$\text{Radius, } r = \frac{mv}{qB} = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{2mV}{qB^2}}$$

$$\text{Thus, } \frac{r_p}{r_\alpha} = \sqrt{\frac{2mV}{eB^2}} \times \sqrt{\frac{2eB^2}{2(4m)V}} = \frac{1}{\sqrt{2}}$$

Example 4.31 (i) A charged particle having mass m and charge q is accelerated by a potential difference V , it flies through a uniform transverse magnetic field B . The field occupies a region of space d . Find the time interval for which it remains inside the magnetic field.

(ii) An α -particle is accelerated by a potential difference of 10^4 V . Find the change in its direction of motion if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 T .

(Take, $m_\alpha = 6.4 \times 10^{-27} \text{ kg}$)

(iii) A 10 g bullet having a charge of $4 \mu\text{C}$ is fired at a speed of 270 m s^{-1} in a horizontal direction. A vertical magnetic field of $500 \mu\text{T}$ exists in the space. Find the deflection of the bullet due to the magnetic field as it travels through 100 m . Make appropriate approximations.

Sol. (i) Kinetic energy of the charged particle,

$$K = \frac{1}{2} mv^2 = qV$$

$$\Rightarrow v = \sqrt{\frac{2qV}{m}}$$

$$\sin \theta = \frac{d}{R} = \frac{d}{mv / Bq} = \frac{Bqd}{mv}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{qBd}{mv} \right)$$

$$\text{Time interval, } t = \frac{\theta}{\omega} = \frac{m\theta}{Bq} = \frac{m}{qB} \sin^{-1} \left(\frac{qBd}{m} \right)$$

(ii) Given, $V = 10^4 \text{ V}$, $q = 2e = 2 \times 1.6 \times 10^{-19} \text{ C}$

$d = 0.1 \text{ m}$, $B = 0.1 \text{ T}$ and $m_\alpha = 6.4 \times 10^{-27} \text{ kg}$

$$\text{Kinetic energy, } K = \frac{1}{2} m_\alpha v^2 = qV = 2eV$$

$$\Rightarrow v = \sqrt{\frac{4eV}{m_\alpha}}$$

$$\therefore R = \frac{m_\alpha v}{Bq} = \frac{m_\alpha v}{2eB}$$

$$\text{Now, } \sin \theta = \frac{d}{R} = \frac{d}{m_\alpha v / B \cdot 2e} = \frac{2eBd}{m_\alpha v}$$

$$= \frac{2eBd}{m_\alpha \sqrt{\frac{4eV}{m_\alpha}}} = \sqrt{\frac{e}{m_\alpha V}} \cdot Bd$$

$$= \sqrt{\frac{1.6 \times 10^{-19}}{6.4 \times 10^{-27} \times 10^4}} \times 0.1 \times 0.1 = \frac{1}{2}$$

$\therefore$ Change in direction, $\theta = 30^\circ$

(iii) Given, $m = 10 \text{ g} = 10 \times 10^{-3} = 10^{-2} \text{ kg}$

$q = 4 \mu\text{C} = 4 \times 10^{-6} \text{ C}$, $v = 270 \text{ m/s}$

Magnetic field, $B = 500 \mu\text{T} = 500 \times 10^{-6} = 5 \times 10^{-4} \text{ T}$

$d = 100 \text{ m}$

$$\text{Radius, } R = \frac{mv}{Bq} = \frac{10^{-2} \times 270}{5 \times 10^{-4} \times 4 \times 10^{-6}} = 13.5 \times 10^8 \text{ m}$$

$$\sin \theta = \frac{d}{R} = \frac{100}{13.5 \times 10^8} = \frac{10^{-6}}{13.5}$$

Therefore, θ is very small.

$$\text{Now, deflection} = \frac{d^2}{2R} = \frac{(100)^2}{2 \times 13.5 \times 10^8} = 3.7 \times 10^{-6} \text{ m}$$

Case III When the charged particle is moving at an angle to the field other than 0° , 90° or 180°

In this case, velocity can be resolved in two components, one along $\mathbf{B}$ and another perpendicular to $\mathbf{B}$. Let the two components be $v_{\parallel}$ and $v_{\perp}$.

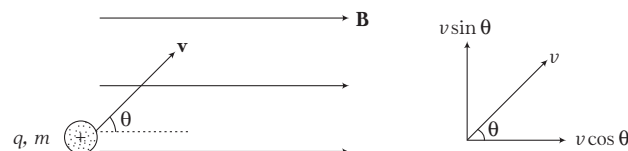


Fig. 4.36 Components of velocity of the charged particle

Then,

$$v_{\parallel} = v \cos \theta$$

and

$$v_{\perp} = v \sin \theta$$

The component perpendicular to field ($v_{\perp}$) gives a circular path and the component parallel to field ($v_{\parallel}$) gives a straight line path. The resultant path is a helix as shown in figure.

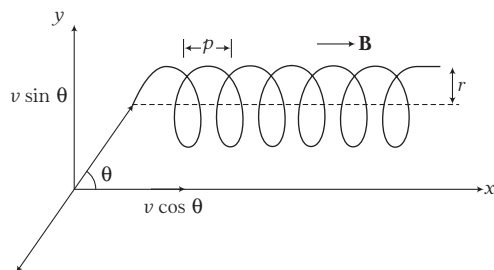


Fig. 4.37 Helical path of the charged particle

- (i) The radius of this helical path,

$$r = \frac{mv_{\perp}}{Bq} = \frac{mv \sin \theta}{Bq}$$

- (ii) Time period and frequency do not depend on velocity and so they are given as

$$T = \frac{2\pi m}{Bq} \quad \text{and} \quad f = \frac{Bq}{2\pi m}$$

- (iii) There is one more term associated with a helical path, that is pitch (p) of the helical path. Pitch is defined as the distance travelled along magnetic field in one complete cycle, i.e.

$$p = v_{\parallel} T = (v \cos \theta) \frac{2\pi m}{Bq}$$

$\therefore$

$$p = \frac{2\pi m v \cos \theta}{Bq}$$

Note After every pitch, the particle touches the X-axis (as shown in Fig. 4.36), i.e. $X = np$, where $n = 0, 1, 2, \dots$

Example 4.32 A beam of protons with a velocity of $4 \times 10^5 \text{ ms}^{-1}$ enters in a region of uniform magnetic field of 0.3 T. The velocity makes an angle of 60° with the magnetic field. Find the radius of the helical path taken by the proton beam and the pitch of the helix.

Sol. Velocity component along the field, $v_{\parallel} = 4 \times 10^5 \times \cos 60^\circ$
 $= 2 \times 10^5 \text{ ms}^{-1}$

Velocity component perpendicular to the field,

$$v_{\perp} = (4 \times 10^5) \sin 60^\circ = 2\sqrt{3} \times 10^5 \text{ ms}^{-1}$$

Proton will describe a circle in plane perpendicular to magnetic field with radius,

$$r = \frac{mv_{\perp}}{qB} = \frac{(1.67 \times 10^{-27}) \times (2\sqrt{3} \times 10^5)}{(1.6 \times 10^{-19}) \times (0.3)} = 1.2 \text{ cm}$$

Time taken to complete one revolution,

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2 \times 3.14 \times 0.012}{2\sqrt{3} \times 10^5} \text{ s}$$

Because of $v_{\parallel}$, protons will also move in the direction of magnetic field.

$\therefore$ Pitch of helix, $p = v_{\parallel} \times T$

$$= \frac{2 \times 10^5 \times 2 \times 3.14 \times 0.012}{2\sqrt{3} \times 10^5} \text{ m} = 0.044 \text{ m} = 4.4 \text{ cm}$$

Example 4.33 A beam of protons with a velocity $6 \times 10^5 \text{ ms}^{-1}$ enters a uniform magnetic field of 0.4 T at an angle of 37° to the magnetic field. Find the radius of the helical path taken by proton beam. Also, find the pitch of helix.

(Use, $\sin 37^\circ = 3/5$, $\cos 37^\circ = 4/5$ and $m_p \cong 1.6 \times 10^{-27} \text{ kg}$)

Sol. Radius of helical path, $r = \frac{mv_{\perp}}{Bq} = \frac{mv \sin \theta}{Bq}$
 $= \frac{(1.6 \times 10^{-27})(6 \times 10^5)(\sin 37^\circ)}{(0.4)(1.6 \times 10^{-19})}$
 $= 9 \times 10^{-3} \text{ m} = 0.9 \text{ cm}$

$$\text{Time period, } T = \frac{2\pi m}{Bq} = \frac{2\pi(1.6 \times 10^{-27})}{(0.4)(1.6 \times 10^{-19})} = 5\pi \times 10^{-8} \text{ s}$$

Pitch of helical path,

$$p = v_{\parallel} T = v \cos \theta T = (6 \times 10^5)(\cos 37^\circ)(5\pi \times 10^{-8})$$

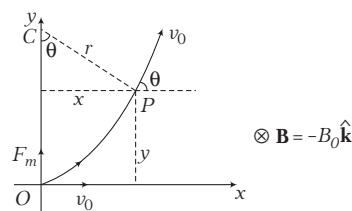
$$= 0.024\pi \text{ m}$$

Example 4.34 A particle of specific charge α enters a uniform magnetic field $\mathbf{B} = -B_0 \hat{\mathbf{k}}$ with velocity $\mathbf{v} = v_0 \hat{\mathbf{i}}$ from the origin. Find the time dependence of velocity and position of the particle.

Sol. In such type of problems, first of all see the angle between $\mathbf{v}$ and $\mathbf{B}$ because it only decides the path of the particle. Here, the angle is 90° . Therefore, the path is a circle. If it is a circle, see the plane of the circle (perpendicular to the magnetic field).

Here, the plane is X-Y.

Then, see the sense of the rotation.



$OC = CP = \text{radius of circle}$

Here, it will be anti-clockwise as shown in figure because at origin, the magnetic force is along positive y-direction (which can be seen from Fleming's left hand rule). Now, the deviation and radius of the particle are

$$\theta = \omega t = B_0 \alpha t \quad \text{and} \quad r = \frac{v_0}{B_0 \alpha} \quad \left(\because \alpha = \frac{q}{m} \right)$$

Now, according to the figure,

velocity of the particle at any time t ,

$$\mathbf{v}(t) = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}} = v_0 \cos \theta \hat{\mathbf{i}} + v_0 \sin \theta \hat{\mathbf{j}}$$

or $\mathbf{v}(t) = v_0 \cos(B_0 \alpha t) \hat{\mathbf{i}} + v_0 \sin(B_0 \alpha t) \hat{\mathbf{j}}$

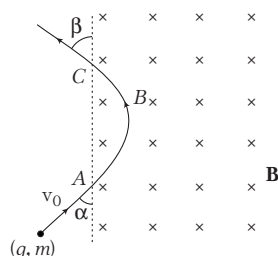
Position of particle at time t ,

$$\mathbf{r}(t) = x \hat{\mathbf{i}} + y \hat{\mathbf{j}} = r \sin \theta \hat{\mathbf{i}} + (r - r \cos \theta) \hat{\mathbf{j}}$$

Substituting the values of r and θ , we get

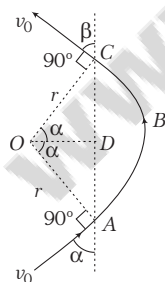
$$\mathbf{r}(t) = \frac{v_0}{B_0 \alpha} [\sin(B_0 \alpha t) \hat{\mathbf{i}} + \{1 - \cos(B_0 \alpha t)\} \hat{\mathbf{j}}]$$

Example 4.35 A charged particle (q, m) enters a uniform magnetic field $\mathbf{B}$ at angle α as shown in figure with speed v_0 . Find



- (i) the angle β at which it leaves the magnetic field,
- (ii) time spent by the particle in magnetic field and
- (iii) the distance AC .

Sol. (i) Here, velocity of the particle is in the plane of paper while the magnetic field is perpendicular to the paper inwards, i.e. angle between $\mathbf{v}$ and $\mathbf{B}$ is 90° . So, the path is a circle. The radius of the circle, $r = \frac{mv_0}{Bq}$



Now, O is the centre of the circle.

In $\triangle AOC$, $\angle OCD = \angle OAD$

or $90^\circ - \beta = 90^\circ - \alpha$

$\therefore \beta = \alpha$

(ii) $\angle COD = \angle DOA = \alpha$

(as $\angle OCD = \angle OAD = 90^\circ - \alpha$)

$\therefore \angle AOC = 2\alpha$

or $\text{length } ABC = r(2\alpha) = \frac{2mv_0}{Bq} \cdot \alpha$

$\therefore$ Time spent by the particle in magnetic field,

$$t_{ABC} = \frac{ABC}{v_0} = \frac{2m\alpha}{Bq}$$

Alternate method

$$t_{ABC} = \left(\frac{T}{2\pi} \right) (2\alpha) = \left(\frac{\alpha}{\pi} \right) \cdot T$$

$$= \left(\frac{\alpha}{\pi} \right) \left(\frac{2\pi m}{Bq} \right) = \frac{2m\alpha}{Bq}$$

(iii) Distance $AC = 2(AD) = 2(r \sin \alpha) = \frac{2mv_0}{Bq} \sin \alpha$

Motion of a charged particle in combined electric and magnetic fields : Lorentz force

Consider a point charge q moving with velocity $\mathbf{v}$ and is located at position vector $\mathbf{r}$ at a given time t . If electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are existing at that point, then force on the electric charge q is given by

$$\mathbf{F} = q [\mathbf{E} + \mathbf{v} \times \mathbf{B}] = \mathbf{F}_{\text{electric}} + \mathbf{F}_{\text{magnetic}}$$

This force was first given by H A Lorentz, hence it is called **Lorentz force**.

Here, magnetic force, $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = Bqv \sin \theta$ and electric force, $\mathbf{F}_e = q\mathbf{E}$.

The direction of magnetic force is same as that of $\mathbf{v} \times \mathbf{B}$, if charge is positive and opposite to that of $\mathbf{v} \times \mathbf{B}$, if charge q is negative.

Case I When $\mathbf{v}$, $\mathbf{E}$ and $\mathbf{B}$ are all collinear

In this situation, the magnetic force on it will be zero and only electric force will act. So, acceleration,

$$\mathbf{a} = \frac{\mathbf{F}_e}{m} = \frac{q\mathbf{E}}{m}$$

The particle will pass through the field following a straight line path (parallel field) with change in its speed. So, in this situation, speed, velocity, momentum and kinetic energy all will change without change in direction of motion as shown below.

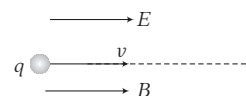


Fig. 4.38 Motion of a charged particle with $\mathbf{v} \parallel \mathbf{E} \parallel \mathbf{B}$

Case II When $\mathbf{v}$, $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular

In this situation, if $\mathbf{E}$ and $\mathbf{B}$ are such that $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = 0$, i.e. $\mathbf{a} = (\mathbf{F}/m) = 0$.

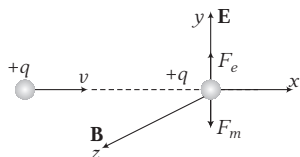


Fig. 4.39 Motion of a charged particle with $\mathbf{v} \perp \mathbf{E} \perp \mathbf{B}$

The particle will pass through the field with same velocity without any deviation in its path as shown in figure.

In this situation, as $F_e = F_m$, i.e. $qE = qvB$

$$v = E/B$$

This principle is used in velocity-selector to get a charged beam having a specific velocity.

Note If only magnetic field is present, we have to put $\mathbf{E} = 0 \Rightarrow \mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.

Similarly, if $\mathbf{B} = 0 \Rightarrow \mathbf{F} = q\mathbf{E}$

Example 4.36 A charge particle having charge 2 C is thrown with velocity $(2\hat{i} + 3\hat{j})$ m/s inside a region having $\mathbf{E} = 2\hat{j}$ V/m and magnetic field $5\hat{k}$ T. Find the initial Lorentz force acting on the particle.

Sol. Lorentz force is given by

$$\begin{aligned}\mathbf{F} &= q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 2[2\hat{j} + (2\hat{i} + 3\hat{j}) \times 5\hat{k}] \\ &= 2[2\hat{j} + 10(-\hat{j}) + 15\hat{i}] \\ &= (30\hat{i} - 16\hat{j}) \text{ N}\end{aligned}$$

Example 4.37 A proton beam passes without deviation through a region of space, where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 220$ kV/m and $B = 50$ mT. Then, the beam strikes a grounded target. Find the force imparted by the beam on the target if the beam current is equal to $i = 0.80$ mA.

Sol. Since, proton is moving in a straight line, hence net force is zero.

$$\begin{aligned}\therefore qE &= Bqv \\ \Rightarrow v &= \frac{E}{B}\end{aligned}$$

Also, current associated with the beam,

$$i = ne \Rightarrow n = i/e$$

where, n is number of protons/time.

Momentum of a proton $= mv$

Force, $F = nmv$

$$\begin{aligned}\Rightarrow F &= \frac{imE}{eB} = \frac{0.80 \times 10^{-3} \times 1.67 \times 10^{-27} \times 220 \times 10^3}{1.6 \times 10^{-19} \times 50 \times 10^{-3}} \\ &= 3.67 \times 10^{-5} \text{ N}\end{aligned}$$

Cyclotron

Cyclotron is a device used for accelerating positively charged particle (like α -particles, deuterons, etc.) with the help of uniform magnetic field upto energy of the order of MeV. It consists of two hollow metallic dees D_1 and D_2 . These are placed in a uniform magnetic field which is perpendicular to the plane of dees. An alternating voltage is applied between the dees.

As it uses the combination of both the fields (electric and magnetic), which are perpendicular to each other, hence are called **crossed fields**. The charged particle is accelerated is produced at centre point between the dees. The particle accelerates along circular path to acquire enough energy to carry out nuclear disintegration, etc.

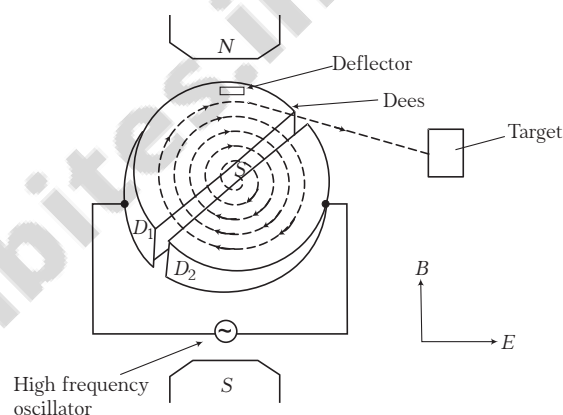


Fig. 4.40 Path described by a charged particle in a cyclotron

Cyclotron frequency Time taken by ion to describe a semicircular path is given by

$$t = \frac{\pi r}{v} = \frac{\pi m}{qB} \quad \left(\because r = \frac{mv}{qB} \right)$$

If T = time period of oscillating electric field, then

$$T = 2t = \frac{2\pi m}{qB}$$

$$\therefore \text{Cyclotron frequency, } \nu = \frac{1}{T} = \frac{qB}{2\pi m}$$

Note Cyclotron is based on the fact that the time period of one revolution is independent of its speed or radius of the orbit.

Maximum energy of particle Maximum energy gained

$$\text{by the charged particle, } E_{\max} = \left(\frac{q^2 B^2}{2m} \right) r_0^2$$

where, r_0 is maximum radius of the circular path followed by positive ion.

Limitations of Cyclotron

The cyclotron has following limitations

- (i) It is suitable only for accelerating heavy particles (like proton, deuteron, α -particle, etc). Electrons cannot be accelerated by the cyclotron because the mass of the electron is very small and a small increase in energy of the electron makes the electron to move with a very high speed. As a result of it, the electrons go quickly out of step with oscillating electric field.

- (ii) When a positive ion is accelerated by the cyclotron, then it moves with greater speed.

As the speed of ion becomes comparable with that of speed of light, the mass of the ion increases according to the relation,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where, m_0 = rest mass of the ion,

m = mass of the ion while moving with velocity v

and c = velocity of light.

Now, the time taken by the ion to describe semi-circular path,

$$t = \frac{\pi m}{qB} = \frac{\pi}{qB} \cdot \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

It shows that as v increases, t increases. Hence, the positive ion will take longer time to describe semi-circular path than the time for half-cycle of oscillating electric field.

As a result of it, the ion will not arrive in the gap between the two dees exactly at the instant when the polarity of the two dees is reversed and hence, it will not be accelerated further. Therefore, the ion cannot move with a speed beyond a certain limit in a cyclotron.

Example 4.38 The magnetic field applied on a cyclotron is 3.5 T. What will be the frequency of electric field that must be applied between the dees in order to accelerate protons?

Sol. The frequency of the field, $f = \frac{Bq}{2\pi m} = \frac{3.5 \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 1.6 \times 10^{-27}}$

$$= 5.57 \times 10^7 \text{ Hz}$$

Example 4.39 Magnetic field applied on a cyclotron is 0.7 T and radius of its dees is 1.8 m. What will be energy (in MeV) of the emergent protons?

Sol. Energy of the emergent protons,

$$E = \frac{q^2 B^2 r_0^2}{2m} = \frac{(1.6 \times 10^{-19})^2 \times (0.7)^2 \times (1.8)^2}{2 \times 1.67 \times 10^{-27}} \text{ J}$$

$$= \frac{(1.6 \times 10^{-19})^2 \times (0.7)^2 \times (1.8)^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-13}} \text{ MeV}$$

$$= 76.05 \text{ MeV}$$

Example 4.40 Deuterons in a cyclotron describe a circle of radius 32 cm just before emerging from the dees of the cyclotron. The frequency of the applied alternating voltage is 10 MHz. Find

(i) the magnetic flux density (i.e. the magnetic field).

(ii) the energy and speed of the deuterons upon emergence.

Sol. (i) Frequency of the applied emf = Cyclotron frequency

$$\text{or } f = \frac{Bq}{2\pi m}$$

$$\therefore B = \frac{2\pi mf}{q}$$

$$= \frac{(2)(3.14)(2 \times 1.67 \times 10^{-27})(10 \times 10^6)}{1.6 \times 10^{-19}}$$

$$= 1.31 \text{ T}$$

(ii) The speed of deuterons on the emergence from the

$$\text{cyclotron, } v = \frac{BqR}{m} = 2\pi f R$$

$$= (2)(3.14)(10 \times 10^6)(32 \times 10^{-2})$$

$$= 2.01 \times 10^7 \text{ ms}^{-1}$$

$$\therefore \text{Energy of deuterons} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times (2 \times 1.67 \times 10^{-27})(2.01 \times 10^7)^2 \text{ J}$$

$$= \frac{6.75 \times 10^{-13}}{1.6 \times 10^{-13}} \text{ MeV} = 4.22 \text{ MeV}$$

CHECK POINT 4.2

- A strong magnetic field is applied on a stationary electron, then the electron
 - moves in the direction of the field
 - moves in an opposite direction of the field
 - remains stationary
 - starts spinning
- A particle of mass m and charge Q moving with a velocity $\mathbf{v}$ enters a region of uniform magnetic field of induction $\mathbf{B}$. Then, its path in the region is
 - always circular
 - circular, if $\mathbf{v} \times \mathbf{B} = 0$
 - circular, if $\mathbf{v} \cdot \mathbf{B} = 0$
 - None of the above
- An electron is moving in a circular path of radius r with speed v in a transverse magnetic field B . The value of e/m for it will be

(a) $\frac{v}{Br}$	(b) $\frac{B}{rv}$
(c) Bvr	(d) $\frac{vr}{B}$
- An electron and a proton with equal momentum enter perpendicularly into a uniform magnetic field, then
 - the path of proton shall be more curved than that of electron
 - the path of proton shall be less curved than that of electron
 - Both are equally curved
 - path of both will be straight line
- A charged particle travels along a straight line with a speed v in a region, where both electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are present. It follows that
 - $|\mathbf{E}| = |\mathbf{B}|$ and the two fields are perpendicular
 - $|\mathbf{E}| = v|\mathbf{B}|$ and the two fields are perpendicular
 - $|\mathbf{B}| = v|\mathbf{E}|$ and the two fields are parallel
 - $|\mathbf{B}| = v|\mathbf{E}|$ and the two fields are perpendicular
- A proton, a deuteron and an α -particle having the same kinetic energy, are moving in circular trajectories in a uniform magnetic field. If r_p , r_d and r_α denote the radii of the trajectories of these particles respectively, then

(a) $r_\alpha = r_p < r_d$	(b) $r_\alpha > r_d > r_p$
(c) $r_\alpha = r_d > r_p$	(d) $r_p = r_d = r_\alpha$
- When a charged particle enters a uniform magnetic field, then its kinetic energy

(a) remains constant	(b) increases
(c) decreases	(d) becomes zero
- A proton of energy 8 eV is moving in a circular path in uniform magnetic field. The energy of an α -particle moving in the same magnetic field and along the same path will be

(a) 4 eV	(b) 2 eV	(c) 8 eV	(d) 6 eV
----------	----------	----------	----------
- A proton and an electron both moving with the same velocity v enter into a region of magnetic field directed perpendicular to the velocity of the particles. They will now move in circular orbits such that
 - their time periods will be same
 - the time period for proton will be higher
 - the time period for electron will be higher
 - their orbital radii will be same
- If a charged particle is describing a circle of radius r in a magnetic field with a time period T . Then,

(a) $T^2 \propto r^3$	(b) $T^2 \propto r$
(c) $T \propto r^2$	(d) $T \propto r^0$
- A particle is projected in a plane perpendicular to a uniform magnetic field. The area bounded by the path described by the particle is proportional to
 - the velocity
 - the momentum
 - the kinetic energy
 - None of these
- If electron velocity is $(2\hat{i} + 3\hat{j})$ and it is subjected to a magnetic field $4\hat{k}$, then
 - speed of electron will change
 - path of electron will change
 - Both (a) and (b)
 - None of the above
- A beam of protons with a velocity of $4 \times 10^5 \text{ ms}^{-1}$ enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The radius of helical path taken by proton beam is

(a) 0.036 m	(b) 0.012 m
(c) 0.024 m	(d) 0.048 m
- A proton and a deuteron, both having the same kinetic energy, enter perpendicularly into a uniform magnetic field B . For motion of proton and deuteron in circular path of radius R_p and R_d , respectively. The correct relation is

(a) $R_d = \sqrt{2}R_p$	(b) $R_d = R_p/\sqrt{2}$
(c) $R_d = R_p$	(d) $R_d = 2R_p$
- Two ions having masses in the ratio 1 : 1 and charges in the ratio 1 : 2 are projected perpendicular to the field of a cyclotron with speeds in the ratio 2 : 3. The ratio of the radii of circular paths along which the two particles move is

(a) 4 : 3	(b) 2 : 3
(c) 3 : 1	(d) 1 : 4
- Which of the particle will have minimum frequency of revolution when projected in a cyclotron with the same velocity perpendicular to a magnetic field?

(a) Li^+	(b) Electron
(c) Proton	(d) He^+

FORCE ON A CURRENT CARRYING CONDUCTOR IN A MAGNETIC FIELD

When a current carrying conductor is placed in a magnetic field, then it experiences a force in a direction perpendicular to both the direction of magnetic field and that of length of the conductor.

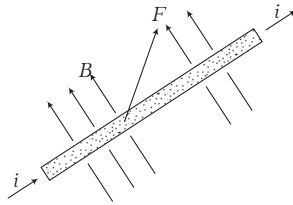


Fig. 4.41 Current carrying conductor in a uniform magnetic field

The magnetic force, $F = ilB \sin \theta$

In vector form, $\mathbf{F} = i (\mathbf{l} \times \mathbf{B})$... (i)

where, B = intensity of magnetic field,

i = current in the conductor,

l = length of the conductor

and θ = angle between the length of conductor and direction of magnetic field.

Cases

I. When $\theta = 90^\circ$ or $\sin \theta = 1$, then $F = ilB \times 1 = ilB$ (maximum).

Therefore, force will be maximum when the current carrying conductor is perpendicular to magnetic field.

II. When $\theta = 0^\circ$ or $\sin \theta = 0$, then $F = ilB \times 0 = 0$.

Thus, the force will be zero when the current carrying conductor is parallel to the magnetic field.

Direction of force on a current carrying conductor in a magnetic field

The direction of this force can be found out either by Fleming's left hand rule or by right hand palm rule as discussed below

Fleming's left hand rule

The direction of $\mathbf{F}_m$ or F can be given by Fleming's left hand rule. According to this rule, if the forefinger, the central finger and the thumb of the left hand are stretched in such a way that they are mutually perpendicular to each other, then the central finger gives the direction of current (or $\mathbf{l}$), forefinger gives the direction of magnetic field $\mathbf{B}$ and the thumb will give the direction of magnetic force $\mathbf{F}_m$.

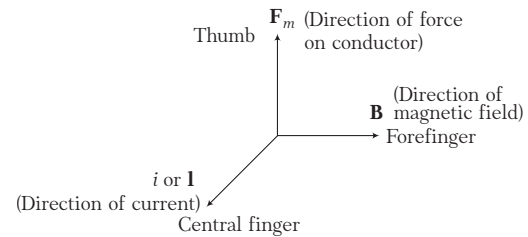


Fig. 4.42 Fleming's left hand rule

Right hand palm rule

Stretch the fingers and thumb of right hand at right angles to each other (as shown below). Now, if the fingers point in the direction of magnetic field $\mathbf{B}$ and thumb in the direction of current i , then normal to the palm will point in the direction of force.

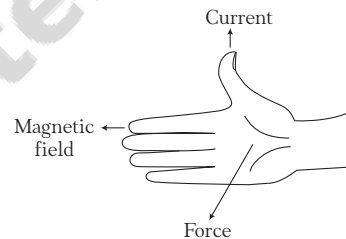


Fig. 4.43 Right hand palm rule for force on a current carrying conductor

Magnetic force on an arbitrarily shaped wire

For the magnetic force on an arbitrarily shaped wire segment, let us consider the magnetic force exerted on a small segment of vector length $d\mathbf{l}$.

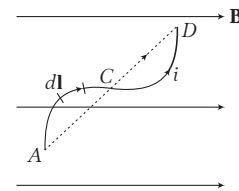


Fig. 4.44 Arbitrary shaped wire in a uniform magnetic field

$$d\mathbf{F}_m = i (d\mathbf{l} \times \mathbf{B}) \quad \dots (ii)$$

To calculate the total force $\mathbf{F}_m$ acting on the wire shown in figure, we will integrate Eq. (ii) over the length of the wire.

$$\therefore \mathbf{F}_m = i \left(\int_A^D d\mathbf{l} \right) \times \mathbf{B} \quad \dots (iii)$$

But the quantity $\int_A^D d\mathbf{l}$ represents the vector sum of all length elements from A to D. From the polygon law of

vector addition, the sum is equal to the vector $\mathbf{l}$ directed from A to D . Thus, $\mathbf{F}_m = i(\mathbf{l} \times \mathbf{B})$

or we can write, $\mathbf{F}_{ACD} = \mathbf{F}_{AD} = i(\mathbf{AD} \times \mathbf{B})$ in uniform magnetic field.

Here, AD = effective length of the wire (L_{eff}).

Magnetic force on a closed current carrying loop

When a closed current carrying loop is placed in uniform magnetic field, then the magnetic force on it is always zero. As the vector sum of $d\mathbf{l}$ is always zero, i.e. $\oint d\mathbf{l} = 0$.

$$\therefore \mathbf{F} = i \oint d\mathbf{l} \times \mathbf{B} = 0$$

Magnetic force on different configurations of the curve

Now, consider magnetic forces on current carrying conductors of different configurations as shown below. Here, magnetic force only depends upon effective length of the wires.

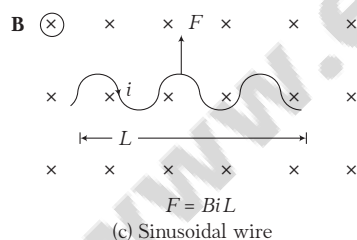
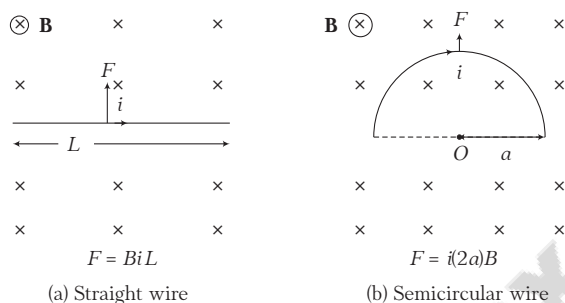


Fig. 4.45

Example 4.41 A wire of length a carries a current i along the Y -axis. A magnetic field exists which is given by

$$\mathbf{B} = B_0(3\hat{i} + 2\hat{j} + \hat{k}) \text{ T}$$

Calculate magnetic force in vector form and its magnitude.

Sol. Length of the wire, $\mathbf{l} = a\hat{j}$

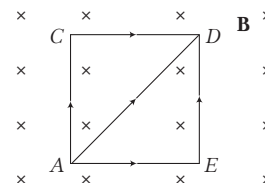
Magnetic force on the wire,

$$\begin{aligned} \mathbf{F} &= i(\mathbf{l} \times \mathbf{B}) = i[a\hat{j} \times B_0(3\hat{i} + 2\hat{j} + \hat{k})] \\ &= iaB_0[-3\hat{k} + 0 + \hat{i}] \\ &= B_0ia(\hat{i} - 3\hat{k}) \end{aligned}$$

$$\begin{aligned} \therefore |\mathbf{F}| &= F = B_0ia\sqrt{(1)^2 + (-3)^2} \\ &= \sqrt{10}B_0ia \end{aligned}$$

Example 4.42 A square of side 2.0 m is placed in a uniform magnetic field of 2.0 T in a direction perpendicular to the plane of the square inwards. Equal current, $i = 3.0 \text{ A}$ is flowing in the directions shown in figure.

Find the magnitude of magnetic force on the loop.



Sol. Force on wire ACD = Force on AD = Force on AED

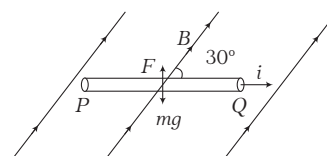
All these forces are acting along EC .

$\therefore$ Net force on the loop = $3(\mathbf{F}_{AD})$

$$\text{or } F_{\text{net}} = 3(i)(AD)(B) = (3)(3.0)(2\sqrt{2})(2.0) \text{ N} = 36\sqrt{2} \text{ N}$$

Example 4.43 A straight wire of length 30 cm and mass 60 mg lies in a direction 30° east of north. The earth's magnetic field at this site is horizontal and has a magnitude of 0.8 G . What current must be passed through the wire, so that it may float in air? (Take, $g = 10 \text{ ms}^{-2}$)

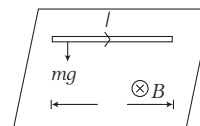
Sol. As shown in figure, if a current i is passed through the wire from end P towards Q , it will experience a force $BiL \sin \theta$ vertically up and hence will float, if



$$BiL \sin \theta = mg$$

$$\Rightarrow i = \frac{mg}{Bl \sin \theta} = \frac{60 \times 10^{-6} \times 10}{0.8 \times 10^{-4} \times 30 \times 10^{-2} \times 1/2} = 50 \text{ A}$$

Example 4.44 A straight wire of mass 200 g and length 1.5 m carries a current of 2 A . It is suspended in mid air by a uniform horizontal magnetic field B . What is the magnitude of the magnetic field?



Sol. Applying Fleming's left hand rule, we find that upward force F of magnitude IlB acts. For mid air suspension, this must be balanced by the force due to gravity.

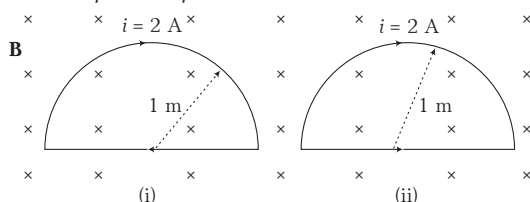
$$\therefore mg = IlB \Rightarrow B = \frac{mg}{Il}$$

Given, $m = 200 \text{ g} = 0.2 \text{ kg}$, $g = 9.8 \text{ ms}^{-2}$, $I = 2 \text{ A}$, $l = 1.5 \text{ m}$

Substituting these values in above equation, we get

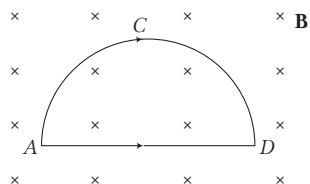
$$B = \frac{0.2 \times 9.8}{2 \times 1.5} = 0.65 \text{ T}$$

Example 4.45 In the figure shown, a semicircular wire loop is placed in a uniform magnetic field, $B = 1.0 \text{ T}$. The plane of the loop is perpendicular to the magnetic field. Current $i = 2 \text{ A}$ flows in the loop in the directions shown. Find the magnitude of the magnetic force in both the cases (i) and (ii). The radius of the loop is 1.0 m .



Sol. Refer Fig. (i) It forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero.

Refer Fig. (ii) In this case, although it forms a closed loop but current does not complete the loop. Hence, net force is not zero.

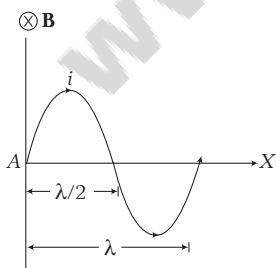


$$\begin{aligned} \mathbf{F}_{ACD} &= \mathbf{F}_{AD} \\ \therefore \mathbf{F}_{\text{loop}} &= \mathbf{F}_{ACD} + \mathbf{F}_{AD} = 2\mathbf{F}_{AD} \\ \therefore |\mathbf{F}_{\text{loop}}| &= 2 |\mathbf{F}_{AD}| = 2ilB \sin \theta \quad (\because l = 2r = 2.0 \text{ m}) \\ &= (2)(2)(2)(1) \sin 90^\circ = 8 \text{ N} \end{aligned}$$

Example 4.46 A wire carrying a current i is kept in the XY -plane along the curve $y = (2 \text{ cm}) A \sin \left(\frac{2\pi}{\lambda} x \right)$.

A magnetic field $\mathbf{B}$ exists in the z -direction. Find the magnitude of the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda/2$.

Sol. Consider the sinusoidal shaped wire as shown below



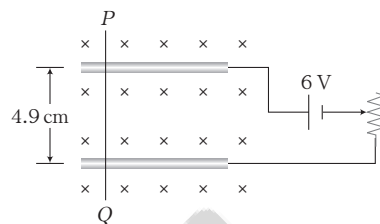
Effective length of the wire for $x = \lambda/2$ is

$$L_{\text{eff}} = \frac{\lambda}{2}$$

$\therefore$ Magnetic force on the wire,

$$F = i(L_{\text{eff}})B = i\left(\frac{\lambda}{2}\right)B = \frac{iB\lambda}{2}, \text{ upward}$$

Example 4.47 A wire PQ of mass 10 g is at rest on two parallel metal rails. The separation between the rails is 4.9 cm . A magnetic field of 0.80 T is applied perpendicular to the plane of the rails, directed downwards. The resistance of the circuit is slowly decreased. When the resistance decreases below to 20Ω , the wire PQ begins to slide on the rails. Calculate the coefficient of friction between the wire and the rails.



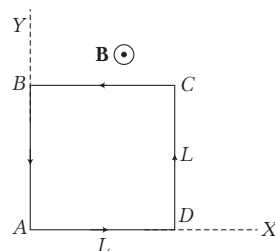
Sol. Wire PQ begins to slide when magnetic force is just equal to the force of friction, i.e. $\mu mg = ilB \sin \theta = ilB$ ($\because \theta = 90^\circ$)

$$\text{Here, } i = \frac{V}{R} = \frac{6}{20} = 0.3 \text{ A}$$

$$\therefore \mu = \frac{ilB}{mg} = \frac{(0.3)(4.9 \times 10^{-2})(0.8)}{(10 \times 10^{-3})(9.8)} = 0.12$$

Example 4.48 The magnetic field existing in a region is given by, $\mathbf{B} = B_0 \left(\frac{x}{L} + \frac{x^2}{L^2} \right) \hat{\mathbf{k}}$. A square loop of edge L carrying a current i is placed with its edges parallel to the X and Y -axes. Find the net magnetic force experienced by the loop.

Sol. Consider the situation shown below



For wire AB , $x = 0$, $\mathbf{B} = 0 \Rightarrow F_{AB} = 0$

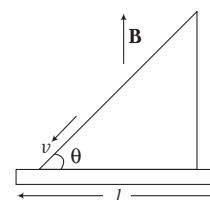
$$\text{For wire } CD, x = L, \mathbf{B} = B_0 \left(\frac{L}{L} + \frac{L^2}{L^2} \right) \hat{\mathbf{k}} = 2B_0 \hat{\mathbf{k}}$$

$$\Rightarrow F_{CD} = 2B_0 i L, \text{ towards right}$$

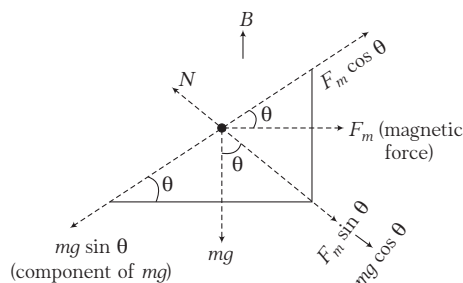
Force on BC is equal and opposite to force on DA .

$$\therefore (F_{\text{net}})_{\text{loop}} = F_{CD} = 2B_0 i L, \text{ towards right}$$

Example 4.49 A conducting rod of length l and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v . A current i is flowing in the conductor in a direction perpendicular to paper inwards. A vertically upwards magnetic field $\mathbf{B}$ exists in space. Then, find magnitude of magnetic field $\mathbf{B}$.



Sol. Consider the forces acting on the rod as given below



Hence, from figure,

$$F_m \cos \theta = mg \sin \theta \quad \dots(i)$$

Also,

$$F_m = B il \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$B = \frac{mg}{il} \tan \theta$$

Force between two parallel current carrying conductors

Consider two long wires 1 and 2 kept parallel to each other at a distance r and carrying currents i_1 and i_2 respectively in the same direction.

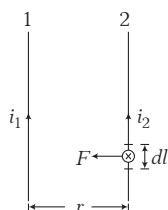


Fig. 4.46 Two long parallel wires carrying currents in the same direction

Magnetic field on wire 2 due to current in wire 1,

$$B = \frac{\mu_0}{2\pi} \cdot \frac{i_1}{r} \quad (\text{in } \otimes \text{ direction})$$

Magnetic force on a small element dl of wire 2 due to this magnetic field,

$$d\mathbf{F} = i_2 (d\mathbf{l} \times \mathbf{B})$$

Magnitude of this force,

$$dF = i_2 [(dl) (B) \sin 90^\circ] = i_2 (dl) \left(\frac{\mu_0}{2\pi} \frac{i_1}{r} \right) = \frac{\mu_0}{2\pi} \cdot \frac{i_1 i_2}{r} \cdot dl$$

Direction of this force is along $d\mathbf{l} \times \mathbf{B}$ or towards the wire 1.

The force per unit length of wire 2 due to wire 1,

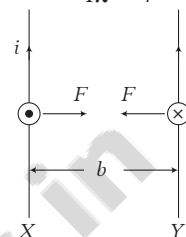
$$\frac{dF}{dl} = \frac{\mu_0}{2\pi} \frac{i_1 i_2}{r}$$

The same force acts on wire 1 due to wire 2. The wires attract each other if currents in the wires are flowing in the same direction and they repel each other if the currents are flowing in opposite directions.

Example 4.50 Currents of 10 A and 2 A are passed through two parallel wires A and B respectively in opposite directions. If the wire A is infinitely long and length of the wire B is 2 m, then find the force acting on the conductor B which is situated at 10 cm distance from A.

Sol. Force on a conductor of length l carrying current I_2 and placed at a distance r parallel to another infinitely long conductor carrying current I_1 ,

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} l \quad \dots(i)$$

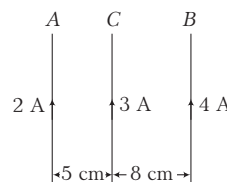


Here, $I_1 = 10 \text{ A}$, $I_2 = 2 \text{ A}$, $l = 2 \text{ m}$, $r = 10 \text{ cm} = 0.1 \text{ m}$

Substituting all these values in Eq. (i), we get

$$F = 10^{-7} \times \frac{2 \times 10 \times 2 \times 2}{0.1} = 8 \times 10^{-5} \text{ N}$$

Example 4.51 A and B are two infinitely long straight parallel conductors. C is another straight conductor of length 1 m kept parallel to A and B as shown in the figure. Then, find the magnitude and direction of the force experienced by C.



Sol. The force between the conductors is attractive, because the currents in them are in the same direction.

Mutual force between conductors A and C,

$$\begin{aligned} F_1 &= \frac{\mu_0}{2\pi} \frac{I_1 I_2 l}{r} \\ &= \frac{\mu_0}{2\pi} \frac{2 \times 3 \times 1}{0.05} = 2.40 \times 10^{-5} \text{ N} \quad (\text{towards A}) \end{aligned}$$

Mutual force between conductors B and C,

$$F_2 = \frac{\mu_0}{2\pi} \times \frac{4 \times 3 \times 1}{0.08} = 3 \times 10^{-5} \text{ N} \quad (\text{towards B})$$

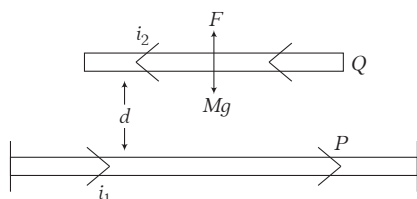
Hence, the resultant force experienced by C,

$$\begin{aligned} F_C &= F_2 - F_1 = (3 - 2.4) \times 10^{-5} \text{ N} \\ &= 0.6 \times 10^{-5} \text{ N} \quad (\text{towards B}) \end{aligned}$$

Example 4.52 A long horizontal wire P carries a current of 50 A. It is rigidly fixed. Another fine wire Q is placed directly above and parallel to P. The weight of wire Q is 0.075 Nm^{-1} and carries a current of 25 A. Find the perpendicular distance of wire Q from P so that wire Q remains suspended due to the magnetic repulsion. Also, indicate the direction of current in Q with respect to P.

Sol. The force per unit length between two parallel current carrying wires separated by a distance d is given by

$$\frac{dF}{dL} = \frac{\mu_0}{4\pi} \frac{2i_1 i_2}{d}$$



This force is repulsive if the current in the wires is in opposite direction (otherwise attractive).

So, in order that wire Q may remain suspended, the force F on it must be repulsive and equal to its weight, i.e. the current in the two wires must be in opposite directions and this force is given by

$$\Rightarrow F = Mg$$

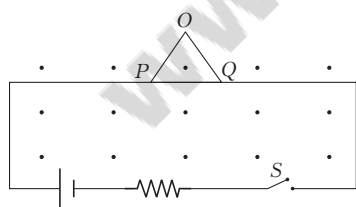
$$\text{i.e. } \frac{F}{L} = \frac{Mg}{L}$$

$$\Rightarrow \frac{\mu_0}{4\pi} \frac{2i_1 i_2}{d} = \frac{Mg}{L} \quad \left[\because \frac{dF}{dL} = \frac{\mu_0}{4\pi} \frac{2i_1 i_2}{d} \right]$$

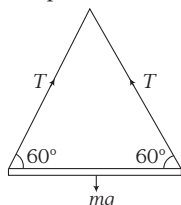
$$\Rightarrow d = 10^{-7} \times \frac{2 \times 50 \times 25}{0.075} = \frac{1}{3} \times 10^{-2} \text{ m}$$

$$\left(\because \frac{Mg}{L} = 0.075 \text{ Nm}^{-1} \right)$$

Example 4.53 The figure shows a rod PQ of length 20cm and mass 100g suspended through a fixed point O by two threads each of lengths 20 cm. A magnetic field of strength 0.5T exists in the vicinity of the wire PQ as shown in figure. The wire connecting PQ with the battery are loose and exert no force on PQ. (i) Find the tension in the threads when the switch S is open. (ii) A current of 2A is established when the switch S is closed. Find the tension in the threads now. (Take, $g = 10 \text{ ms}^{-2}$)



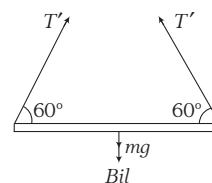
Sol. (i) When switch S is open,



$$2T \sin 60^\circ = mg$$

$$\therefore T = \frac{mg}{\sqrt{3}} = \frac{100 \times 10^{-3} \times 10}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ N}$$

(ii) When switch S is closed, current i flows in wire PQ. Magnetic force on PQ, $F_m = Bil$, downward



$$2T' \sin 60^\circ = mg + Bil$$

$$\therefore T' = \frac{mg + Bil}{\sqrt{3}} = \frac{100 \times 10^{-3} \times 10 + 0.5 \times 2 \times 0.2}{\sqrt{3}} = \frac{1 + 0.2}{\sqrt{3}} = \frac{1.2}{\sqrt{3}} \text{ N}$$

Magnetic force between two moving charges

Consider two charges q_1 and q_2 moving with velocities v_1 and v_2 respectively and at any instant, let the distance between them be r .

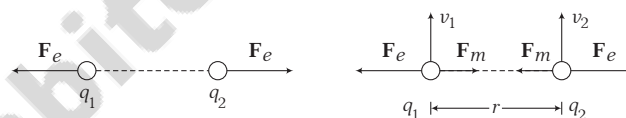


Fig. 4.47 Two moving charges

A magnetic force F_m will appear between them along with the electric force.

$$\text{Magnetic force between them, } F_m = \frac{\mu_0}{4\pi} \frac{q_1 q_2}{r^2} \frac{v_1 v_2}{r^2} \quad \dots(i)$$

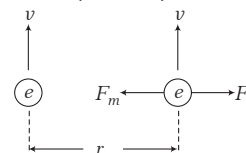
$$\text{Electric force between them, } F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{F_m}{F_e} = \left(\frac{v}{c} \right)^2 \quad \left(\because \mu_0 \epsilon_0 = \frac{1}{c^2} \right)$$

(for charges moving with same velocity.)

Example 4.54 Two electrons move parallel to each other with an equal velocity $v = 200 \times 10^3 \text{ m/s}$. Calculate the ratio of magnetic and electrical forces of the electrons.



Sol. Electrical force between electrons,

$$F_e = \frac{1}{4\pi\epsilon_0} \cdot \frac{e \cdot e}{r^2} \text{ (repulsive)}$$

Magnetic force between electrons,

$$F_m = \frac{\mu_0}{4\pi} \cdot \frac{e \cdot e \cdot v \cdot v}{r^2} \text{ (attractive)}$$

$$\therefore \frac{F_m}{F_e} = \mu_0 \epsilon_0 v^2 = \frac{v^2}{c^2} \quad \left(\because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \right)$$

$$= \left(\frac{200 \times 10^3}{3 \times 10^8} \right)^2 = 4.4 \times 10^{-7}$$

Magnetic dipole moment

Magnetic dipole moment associated with a current carrying loop is given as $\mathbf{M} = Ni\mathbf{A} \Rightarrow |\mathbf{M}| = NiA$

where, N = number of turns in the loop,

i = current in the loop

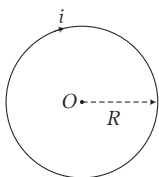
and A = area of cross-section of the loop.

It is a vector quantity directed along area vector $\mathbf{A}$ of the loop with anti-clockwise sense as positive. Its SI unit is A-m^2 .

Example 4.55 Find the magnetic moment of a thin round loop with current if the radius of the loop is equal to $R = 100 \text{ mm}$ and the magnetic induction at its centre is equal to $B = 8.0 \mu\text{T}$.

Sol. Magnetic field at centre O ,

$$B = \frac{\mu_0 i}{2R} \Rightarrow i = \frac{2RB}{\mu_0}$$



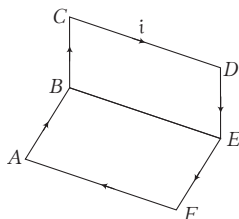
Magnetic moment of loop,

$$M = iA = i \cdot \pi R^2 = \frac{2RB}{\mu_0} \pi R^2$$

$$= \frac{2BR^3}{\mu_0 / \pi} = \frac{2 \times 8 \times 10^{-6} \times (0.1)^3}{4 \times 10^{-7}}$$

$$= 4 \times 10^{-2} \text{ Am}^{-2}$$

Example 4.56 Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is 10 cm long and current in the loop is $i = 2.0 \text{ A}$.



Sol. By assuming two equal and opposite currents in BE , two current carrying loops $ABEFA$ and $BCDEB$ are formed. Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

$$M_{\text{net}} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

$$\therefore M = iA = 2 \times (l \times b)$$

$$= 2 \times (0.1)(0.1) = 0.02 \text{ A-m}^2$$

$$M_{\text{net}} = (\sqrt{2})(0.02) = 0.028 \text{ A-m}^2$$

Example 4.57 In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius $5.1 \times 10^{-11} \text{ m}$ at a frequency of $6.8 \times 10^{15} \text{ Hz}$.

(i) What value of magnetic field is set-up at the centre of the orbit?

(ii) What is the equivalent magnetic moment?

Sol. (i) An electron moving around the nucleus is equivalent to a current, $i = qf$

$$\text{Magnetic field at the centre, } B = \frac{\mu_0 i}{2R} = \frac{\mu_0 qf}{2R}$$

Substituting the given values in above equation, we get

$$B = \frac{(4\pi \times 10^{-7})(1.6 \times 10^{-19})(6.8 \times 10^{15})}{2 \times 5.1 \times 10^{-11}} = 13.4 \text{ T}$$

(ii) The current carrying circular loop is equivalent to a magnetic dipole, with magnetic moment,

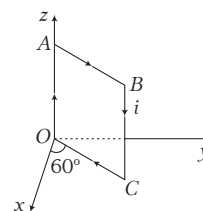
$$M = NiA = Nqf\pi R^2$$

Substituting all the values in above equation, we get

$$M = (1)(1.6 \times 10^{-19})(6.8 \times 10^{15})(3.14)(5.1 \times 10^{-11})^2$$

$$= 8.9 \times 10^{-24} \text{ A-m}^2$$

Example 4.58 A square loop $OABCO$ of side l carries a current i . It is placed as shown in figure. Find the magnetic moment of the loop.



Sol. Magnetic moment of the loop, $\mathbf{M} = i(\mathbf{BC} \times \mathbf{CO})$

Here,

$$\mathbf{BC} = -l\hat{\mathbf{k}}$$

$$\mathbf{CO} = -l \cos 60^\circ \hat{\mathbf{i}} - l \sin 60^\circ \hat{\mathbf{j}}$$

$$= -\frac{l}{2} \hat{\mathbf{i}} - \frac{\sqrt{3}l}{2} \hat{\mathbf{j}}$$

$$\therefore \mathbf{M} = i \left[(-l\hat{\mathbf{k}}) \times \left(-\frac{l}{2} \hat{\mathbf{i}} - \frac{\sqrt{3}l}{2} \hat{\mathbf{j}} \right) \right]$$

$$\text{or } \mathbf{M} = \frac{il^2}{2} (\hat{\mathbf{j}} - \sqrt{3} \hat{\mathbf{i}})$$

Torque on current carrying loop in a magnetic field

When a current carrying coil is placed in a magnetic field, the coil experiences a torque.

Consider a rectangular current carrying coil $PQRS$ having N turns and area A . When this coil is placed in a uniform

field $\mathbf{B}$ in such a way that the normal ($\hat{n}$) to the coil makes an angle θ with the direction of $\mathbf{B}$. The coil experiences a torque which is given by

$$\tau = NBiA \sin \theta$$

In vector form, $\tau = \mathbf{M} \times \mathbf{B}$

Special cases

- τ is zero when $\theta = 0^\circ$, i.e. the plane of the coil is perpendicular to the field.
- τ is maximum when $\theta = 90^\circ$, i.e. the plane of the coil is parallel to the field, $\tau_{\max} = NBiA$

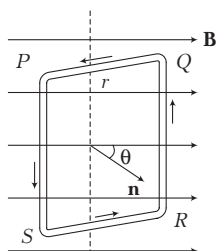
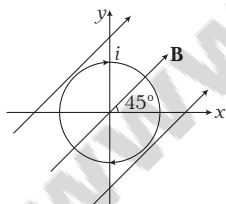


Fig. 4.48 Current carrying loop at an angle θ with magnetic field

Note

- Expression of torque is applicable for all shapes.
- Axis of rotation is parallel to τ , for loops rotating freely in magnetic field.

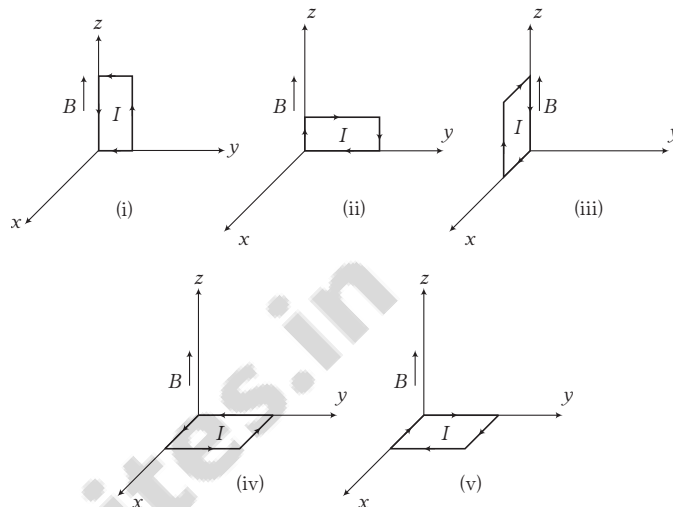
Example 4.59 A circular loop of radius $R = 20$ cm is placed in a uniform magnetic field $\mathbf{B} = 2$ T in XY-plane as shown in figure. The loop carries a current $i = 1.0$ A in the direction shown in figure. Find the magnitude of torque acting on the loop.



Sol. Here, $M = NiA = N\pi R^2 = (1)(1.0)(\pi)(0.2)^2 = 0.04\pi \text{ A-m}^2$
 $B = 2$ T and $\theta = \text{angle between } \mathbf{M} \text{ and } \mathbf{B} = 90^\circ$
 $\therefore$ Magnitude of torque, $|\tau| = (0.04\pi)(2) \sin 90^\circ = 0.25\pi \text{ N-m}$

Note Here, the angle 45° is given just to confuse the students. $\mathbf{M}$ is along negative z-direction (perpendicular to paper inwards) while $\mathbf{B}$ is in XY-plane. So, the angle between $\mathbf{M}$ and $\mathbf{B}$ is 90° not 45° . So, far as only magnitude of τ is concerned, there is no use of giving 45° . But when the direction of torque is desired, then this angle is used. Because in that case, you will write,
 $\mathbf{B} = 2 \cos 45^\circ \hat{i} + 2 \sin 45^\circ \hat{j} = \sqrt{2} (\hat{i} + \hat{j})$ T
 and $\mathbf{M} = -(0.04\pi) \hat{k} \text{ A-m}^2$
 $\therefore \tau = \mathbf{M} \times \mathbf{B} = (0.04\sqrt{2}\pi) (-\hat{j} + \hat{i}) = 0.18 (\hat{i} - \hat{j}) \text{ N-m}$

Example 4.60 A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in figure. What is the force in each case? Which case corresponds to stable equilibrium?



Sol. Here, $B = \text{uniform magnetic field} = 3000 \text{ G along Z-axis}$
 $= 3000 \times 10^{-4} \text{ T} = 0.3 \text{ T}$
 $\therefore$ Area of rectangular loop,
 $A = l \times b = 10 \times 5 = 50 \text{ cm}^2$
 $= 50 \times 10^{-4} \text{ m}^2$

Torque on the loop, $\tau = IAB \sin \theta$

where, θ is the angle between the normal to the plane of the loop and the direction of magnetic field.

(i) Here, $\theta = 90^\circ$

$$\therefore \tau = 12 \times 50 \times 10^{-4} \times 0.3 \times \sin 90^\circ$$

$$= 1.8 \times 10^{-2} \text{ N-m}$$

and acts along negative y-direction.

(ii) Here, $\theta = 90^\circ$

$$\therefore \tau = 12 \times 50 \times 10^{-4} \times 0.3 \times \sin 90^\circ$$

$$= 1.8 \times 10^{-2} \text{ N-m}$$

and acts along y-direction.

(iii) Here, $\theta = 90^\circ$

$$\therefore \tau = 12 \times 50 \times 10^{-4} \times 0.3 \times \sin 90^\circ$$

$$= 1.8 \times 10^{-2} \text{ N-m}$$

and acts along negative x-direction.

(iv) Here, $\theta = 0^\circ$

$$\therefore \tau = BIA \sin 0^\circ = 0$$

(v) Here, $\theta = 180^\circ$

$$\therefore \tau = 0$$

Net force on a planar loop in a uniform magnetic field is always zero, so force is zero in each case.

Case (iv) corresponds to stable equilibrium as $\mathbf{M}$ is aligned with $\mathbf{B}$.

The moving coil galvanometer (MCG)

It is a device which is based on the principle that when a current carrying coil is placed in a uniform magnetic field, then it experiences a torque.

The MCG consists of a multi-turn coil free to rotate about a vertical axis, in a uniform radial magnetic field. There is a cylindrical soft iron core to increase the sensitivity of the MCG.

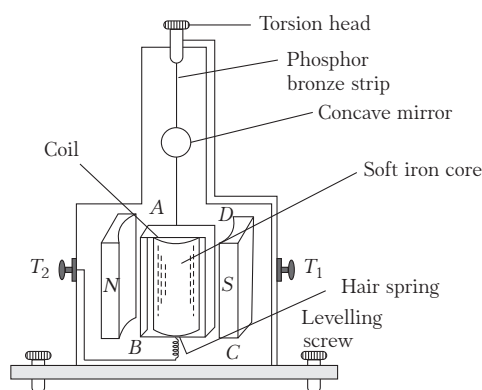


Fig. 4.49 Moving coil galvanometer

The plane of the coil always remains parallel to the direction of magnetic field, therefore $\theta = 90^\circ$. The deflecting torque has maximum value,

$$\tau = NIAB \quad \dots(i)$$

where, the symbols have their usual meaning. Since, the magnetic field is radial by design, we have taken $\sin\theta \approx 1$ in the above expression for the torque.

As the coil deflects, a restoring torque is set-up in the suspension fibre. If α is the angle of twist, the restoring torque,

$$\tau_{\text{rest}} = k\alpha \quad \dots(ii)$$

where, k is the torsional constant of the fibre. When the coil is in equilibrium,

$$NBIA = k\alpha$$

$$\Rightarrow I = G\alpha \Rightarrow \boxed{\alpha \propto I}$$

where, $G = \frac{k}{NBA}$ is the galvanometer constant.

This linear relationship between I and α makes the moving coil galvanometer useful for current measurement and detection.

Current sensitivity is the deflection per unit current flowing through it. It is given by

$$I_s = \frac{\alpha}{I} = \left(\frac{NAB}{k} \right)$$

Its SI unit is rad/A or div/A.

Voltage sensitivity is the deflection per unit voltage.

It is given by

$$V_s = \frac{\alpha}{V} = \left(\frac{NAB}{k} \right) \frac{I}{V}$$

$$\Rightarrow V_s = \frac{NAB}{k} \times \frac{I}{IR} = \frac{NAB}{kR} \quad (\because V = IR)$$

$$\text{Voltage sensitivity} = \frac{\text{Current sensitivity}}{R}$$

Its SI unit is rad/V or div/V.

Example 4.61 The coil of a moving coil galvanometer has an effective area of $5 \times 10^{-2} \text{ m}^2$. It is suspended in a magnetic field of $2 \times 10^{-2} \text{ Wb m}^{-2}$. If the torsional constant of the suspension fibre is $4 \times 10^{-9} \text{ Nm deg}^{-1}$, then find its current (in $\text{deg } \mu\text{A}^{-1}$) sensitivity.

Sol. Here, $N = 1$, $A = 5 \times 10^{-2} \text{ m}^2$,

$$B = 2 \times 10^{-2} \text{ Wb m}^{-2}, k = 4 \times 10^{-9} \text{ Nm deg}^{-1}$$

$$\begin{aligned} \text{Current sensitivity} &= \frac{NBA}{k} = \frac{1 \times 2 \times 10^{-2} \times 5 \times 10^{-2}}{4 \times 10^{-9}} \\ &= 0.25 \times 10^6 \text{ deg A}^{-1} \\ &= 0.25 \text{ deg } \mu\text{A}^{-1} \end{aligned}$$

Example 4.62 A moving coil galvanometer has 100 turns and each turn has an area of 2 cm^2 . The magnetic field produced by the magnet is 0.01 T . The deflection in the galvanometer coil is 0.05 rad when a current of 10 mA is passed through it. Find the torsional constant of the spiral spring.

Sol. Current, $i = \frac{k}{NAB} \theta$

$$\Rightarrow k = \frac{NABi}{\theta}$$

$$\text{or } k = \frac{100 \times 2 \times 10^{-4} \times 0.01 \times 10 \times 10^{-3}}{0.05}$$

$$\therefore k = 4.0 \times 10^{-5} \text{ Nm rad}^{-1}$$

Example 4.63 A current of 0.5 A is passed through the coil of a galvanometer having 500 turns and each turn has an average area of $3 \times 10^{-4} \text{ m}^2$. If a torque of 1.5 N-m is required for this coil carrying same current to set it parallel to a magnetic field, then calculate the strength of the magnetic field.

Sol. The magnetic moment of current loop,

$$M = NiA = 500 \times 0.5 \times 3 \times 10^{-4} = 0.075 \text{ A-m}^2$$

$$\text{Also, } \tau = \mathbf{M} \times \mathbf{B} \Rightarrow |\tau| = MB \sin\theta$$

where, θ = angle between B and A .

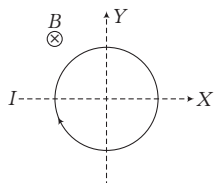
Here, $\theta = 90^\circ$

$$\therefore \tau = MB \sin 90^\circ$$

$$\Rightarrow B = \frac{\tau}{M} = \frac{1.5}{0.075} = 20 \text{ T}$$

CHECK POINT 4.3

1. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to



- (a) contract
(b) expand
(c) move towards +ve X-axis
(d) move towards -ve X-axis
2. Two parallel wires carrying currents in the same direction attract each other because of
(a) potential difference between them
(b) mutual inductance between them
(c) electric force between them
(d) magnetic force between them
3. Two parallel conductor A and B of equal length carry currents I and $10I$, respectively, in the same direction, then
(a) A and B will repel each other with same force
(b) A and B will attract each other with same force
(c) A will attract B but B will repel A
(d) A and B will attract each other with different forces
4. Two thin long parallel wires, separated by a distance d carry a current i ampere in the same direction. They will
(a) attract each other with a force of $\mu_0 i / 2\pi d^2$
(b) repel each other with a force of $\mu_0 i^2 / 2\pi d^2$
(c) attract each other with a force of $\mu_0 i^2 / 2\pi d$
(d) repel each other with a force of $\mu_0 i / 2\pi d$
5. Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is
(a) $-\frac{F}{3}$ (b) $\frac{F}{3}$ (c) $\frac{2F}{3}$ (d) $-\frac{2F}{3}$
6. The force per unit length between two long parallel wires A and B carrying current is 0.004 Nm^{-1} . The conductors are 0.01 m apart. If the current in conductor A is twice that of conductor B , then the current in the conductor B would be
(a) 5 A (b) 50 A
(c) 10 A (d) 100 A
7. Current i is carried in a wire of length L . If the wire is turned into a circular coil, the maximum magnitude of torque in a given magnetic field B will be
(a) $\frac{L^2 B^2}{2}$ (b) $\frac{L^2 B}{2}$ (c) $\frac{L^2 i B}{4\pi}$ (d) $\frac{L^2 B}{4\pi}$
8. A circular coil of 20 turns and radius 10 cm is placed in uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in coil is 5 A , then the torque acting on the coil will be
(a) 31.4 Nm (b) 3.14 Nm
(c) 0.314 Nm (d) zero
9. The pole pieces of the magnet used in a pivoted coil galvanometer are
(a) plane surfaces of a bar magnet
(b) plane surfaces of a horse-shoe magnet
(c) cylindrical surfaces of a bar magnet
(d) cylindrical surfaces of a horse-shoe magnet.
10. In a moving coil galvanometer, the deflection of the coil θ is related to the electrical current i by the relation
(a) $i \propto \tan\theta$ (b) $i \propto \theta$
(c) $i \propto \theta^2$ (d) $i \propto \sqrt{\theta}$
11. In order to increase the sensitivity of a moving coil galvanometer one should decrease
(a) the strength of its magnet
(b) the torsional constant of its suspension
(c) the number of turns in its coil
(d) the area of its coil
12. Two galvanometers A and B require current of 3 mA and 5 mA respectively, to produce the same deflection of I_0 division. Then,
(a) A is more sensitive than B
(b) B is more sensitive than A
(c) A and B are equally sensitive
(d) sensitiveness of B is $5/3$ times of that of A

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1** Biot-Savart's law indicates that the moving electrons (velocity, $\mathbf{v}$) produce a magnetic field $\mathbf{B}$ such that [NCERT Exemplar]

- (a) $\mathbf{B}$ is perpendicular to $\mathbf{v}$
- (b) $\mathbf{B}$ is parallel to $\mathbf{v}$
- (c) it obeys inverse cube law
- (d) it is along the line joining the electron and point of observation

- 2** In a cyclotron, a charged particle [NCERT Exemplar]

- (a) undergoes acceleration all the time
- (b) speeds up between the dees because of the magnetic field
- (c) speeds up in a dee
- (d) slows down within a dee and speeds up between dees

- 3** A proton is moving along the negative direction of X-axis in a magnetic field directed along the positive direction of Y-axis. The proton will be deflected along the negative direction of

- (a) X-axis
- (b) Y-axis
- (c) Z-axis
- (d) None of these

- 4** In a co-axial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero

- (a) outside the cable
- (b) inside the inner conductor
- (c) inside the outer conductor
- (d) in between the two conductors

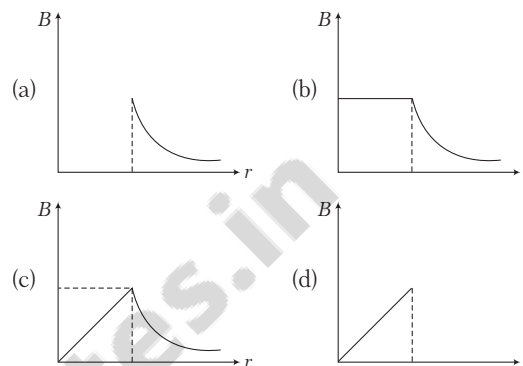
- 5** A current carrying circular loop of radius R is placed in the XY-plane with centre at the origin. Half of the loop with $x > 0$ is now bent, so that it now lies in the YZ-plane. Then, [NCERT Exemplar]

- (a) the magnitude of magnetic moment now diminishes
- (b) the magnetic moment does not change
- (c) the magnitude of $\mathbf{B}$ at $(0,0,z)$, $z > R$ increases
- (d) the magnitude of $\mathbf{B}$ at $(0,0,z)$, $z \gg R$ is unchanged

- 6** The maximum energy of a deuteron coming out of a cyclotron is 20 MeV. The maximum energy of proton that can be obtained from this accelerator is

- (a) 10 MeV
- (b) 20 MeV
- (c) 30 MeV
- (d) 40 MeV

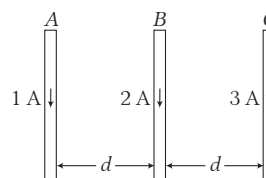
- 7** A long thin hollow metallic cylinder of radius R has a current i ampere. The magnetic induction B away from the axis at a distance r from the axis varies as shown in



- 8** The magnetic field at the centre of a circular coil of radius r carrying current I is B_1 . The field at the centre of another coil of radius $2r$ carrying same current I is B_2 . The ratio B_1/B_2 is

- (a) $1/2$
- (b) 1
- (c) 2
- (d) 4

- 9** Three long straight wires A, B and C are carrying currents as shown in figure. Then, the resultant force on B is directed



- (a) towards A
- (b) towards C
- (c) perpendicular to the plane of paper and outward
- (d) perpendicular to the plane of paper and inward

- 10** A wire of length 2 m carrying a current of 1 A is bent to form a circle. The magnetic moment of the coil is (in A-m²)

- (a) 2π
- (b) $\pi/2$
- (c) $\pi/4$
- (d) $1/\pi$

- 11** In hydrogen atom, an electron is revolving in the orbit of radius $0.53 \text{ \AA}$ with $6.6 \times 10^{15} \text{ rot/s}$. Magnetic field produced at the centre of the orbit is

- (a) 0.125 Wb/m^2
- (b) 1.25 Wb/m^2
- (c) 12.5 Wb/m^2
- (d) 125 Wb/m^2

- 12** A circular flexible loop of wire of radius r carrying a current I is placed in a uniform magnetic field B perpendicular to the plane of the circle, so that wire

comes under tension. If B is doubled, then tension in the loop

- (a) remains unchanged (b) is doubled
(c) is halved (d) becomes 4 times

- 13.** A current carrying conductor of length l is bent into two loops one by one. First loop has one turn of wire and the second loop has two turns of wire.

Compare the magnetic fields at the centre of the loops.

- (a) $B' = 4B$ (b) $4B' = B$ (c) $2B' = B$ (d) $B' = 2B$

- 14** Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\mathbf{B} = B_0 \hat{\mathbf{k}}$. Then,

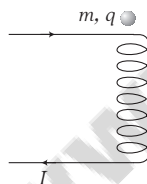
[NCERT Exemplar]

- (a) they have equal z -components of moment
(b) they must have equal charges
(c) they necessarily represent a particle, anti-particle pair
(d) the charge to mass ratio satisfy

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

- 15** When a certain length of wire is turned into one circular loop, then the magnetic field produced at the centre of coil due to some current flowing, is B_1 . If the same wire is turned into three loops to make a circular coil, then the magnetic induction at the centre of this coil for the same current will be
- (a) B_1 (b) $9B_1$ (c) $3B_1$ (d) $27B_1$

- 16** A long solenoid carrying a current I is placed with its axis vertical as shown in the figure. A particle of mass m and charge q is released from the top of the solenoid. Its acceleration is (g being acceleration due to gravity)



- (a) greater than g (b) less than g
(c) equal to g (d) None of these

- 17** A proton moves at a speed $v = 2 \times 10^6$ m/s in a region of constant magnetic field of magnitude $B = 0.05$ T. The direction of the proton when it enters this field is $\theta = 30^\circ$ to the field. When you look along the direction of the magnetic field, then the path is a circle, projected on a plane perpendicular to the magnetic field. How far will the proton move along the direction of B when two projected circles have been completed?

- (a) 4.35 m (b) 0.209 m (c) 2.82 m (d) 2.41 m

- 18** An electric current i enters and leaves a uniform circular wire of radius a through diametrically opposite points. A charged particle q moving along

the axis of the circular wire passes through its centre at speed v . The magnetic force acting on the particle when it passes through the centre has a magnitude

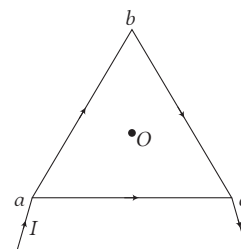
- (a) $qv \frac{\mu_0 i}{2a}$ (b) $qv \frac{\mu_0 i}{2\pi a}$ (c) $qv \frac{\mu_0 i}{a}$ (d) zero

- 19** A particle of mass m and charge q moves with a constant velocity v along the positive x -direction. It enters a region containing a uniform magnetic field B directed along the negative z -direction, extending from $x = a$ to $x = b$. The minimum value of v required, so that the particle can just enter the region $x > b$ is

- (a) qbB/m (b) $q(b-a)B/m$
(c) qaB/m (d) $q(b+a)B/2m$

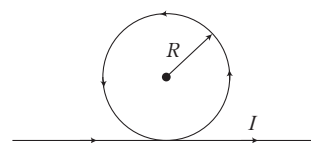
- 20.** A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19} C is projected in XY -plane with a speed of 2×10^6 m/s at an angle of 60° to the X -axis. If a uniform magnetic field of 0.14 T is applied along the Y -axis, then the path of the proton is
- (a) a circle of radius 0.2 m and time period $\pi \times 10^{-7}$ s
(b) a circle of radius 0.1 m and time period $2\pi \times 10^{-7}$ s
(c) a helix of radius 0.07 m and time period 0.5×10^{-6} s
(d) a helix of radius 0.14 m and time period 1.0×10^{-7} s

- 21** An equilateral triangle of side length l is formed from a piece of wire of uniform resistance. The current I is fed as shown in the figure. Then, the magnitude of the magnetic field at its centre O is



- (a) $\frac{\sqrt{3}\mu_0 I}{2\pi l}$ (b) $\frac{3\sqrt{3}\mu_0 I}{2\pi l}$ (c) $\frac{\mu_0 I}{2\pi l}$ (d) zero

- 22** An infinitely long conductor is bent into a circle as shown in figure. It carries a current I ampere and the radius of loop is R metre. The magnetic induction at the centre of loop is



- (a) $\frac{\mu_0 2I}{4\pi R} (\pi + 1)$ (b) $\frac{\mu_0 2I}{4\pi R} (\pi - 1)$
(c) $\frac{\mu_0 I}{8\pi R} (\pi + 1)$ (d) zero

23. Two identical coils carrying equal currents have a common centre and their planes are at right angles to each other. Find the ratio of the magnitudes of the resultant magnetic field at the centre and the field due to one coil alone.

(a) 2 : 1 (b) 1 : 1 (c) 1 : $\sqrt{2}$ (d) $\sqrt{2}$: 1

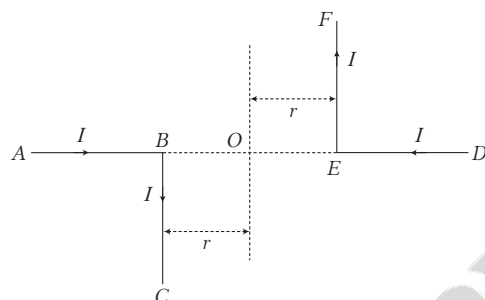
24. An electron moves in a circular orbit with a uniform speed v . It produces a magnetic field B at the centre of the circle. The radius of the circle is proportional to

(a) $\frac{B}{v}$ (b) $\frac{v}{B}$ (c) $\sqrt{\frac{v}{B}}$ (d) $\sqrt{\frac{B}{v}}$

25. Two wires of same length are shaped into a square and a circle. If they carry same current, then ratio of the magnetic moment is

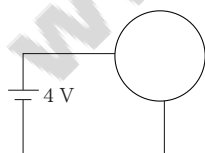
(a) 2 : π (b) π : 2 (c) π : 4 (d) 4 : π

26. Two long thin wires ABC and DEF are arranged as shown in the figure. The magnitude of the magnetic field at O is



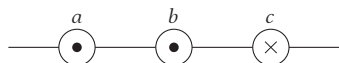
(a) $\frac{\mu_0 I}{4\pi r}$ (b) $\frac{\mu_0 I}{2\pi r}$ (c) $\frac{\mu_0 I}{2\sqrt{2}\pi r}$ (d) zero

27. A circular conductor of uniform resistance per unit length, is connected to a battery of 4 V. The total resistance of the conductor is 4 Ω . The net magnetic field at the centre of the conductor is



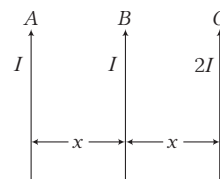
(a) $\frac{\mu_0}{2}$ (b) $\frac{8\mu_0}{3}$ (c) $2\mu_0$ (d) zero

28. Figure shows, three long straight wires parallel and equally spaced with identical currents as shown below. Then, the force acting on each wire F_a , F_b and F_c due to the other are related as



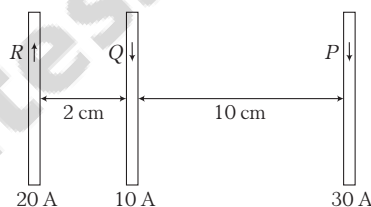
(a) $F_a > F_b > F_c$ (b) $F_b > F_c > F_a$
(c) $F_c > F_a > F_b$ (d) $F_b > F_a > F_c$

29. A , B and C are parallel conductors of equal length carrying currents I , I and $2I$, respectively. Distance between A and B is x . Distance between B and C is also x . F_1 is the force exerted by B on A and F_2 is the force exerted by C on A . Choose the correct answer.



(a) $F_1 = 2F_2$ (b) $F_2 = 2F_1$
(c) $F_1 = F_2$ (d) $F_1 = -F_2$

30. Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 cm length of wire Q is



(a) 1.4×10^{-4} N towards the right
(b) 1.4×10^{-4} N towards the left
(c) 2.6×10^{-4} N towards the right
(d) 2.6×10^{-4} N towards the left

31. A current of 10 A is flowing in a wire of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform magnetic field of 2 T. The angle between the magnetic field and the direction of the current is

(a) 30° (b) 45°
(c) 60° (d) 90°

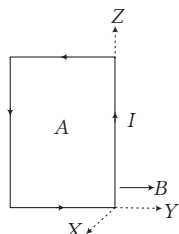
32. An ionised gas contains both positive and negative ions initially at rest. If it is subjected simultaneously to an electric field along the $+x$ -direction and a magnetic field along the $+z$ -direction, then

(a) positive ions deflect towards $+y$ -direction and negative ions $-y$ -direction
(b) all ions deflect towards $+y$ -direction
(c) all ions deflect towards $-y$ -direction
(d) positive ions deflect towards $-y$ -direction and negative ions towards $+y$ -direction

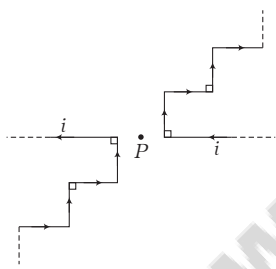
33. A charged particle P leaves the origin with speed $v = v_0$, at some inclination with the X -axis. There is a uniform magnetic field B along the X -axis. P strikes a fixed target T on the X -axis for a minimum value of $B = B_0$. P will also strike T , if

(a) $B = 2B_0$, $v = 2v_0$ (b) $B = 2B_0$, $v = v_0$
(c) Both are correct (d) Both are wrong

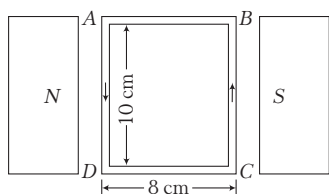
- 34** The rectangular coil of area A is in a field B . Find the torque about the Z -axis when the coil lies in the position shown and carries a current I .



- (a) IAB in negative Z -axis (b) IAB in positive Z -axis
(c) $2IAB$ in positive Z -axis (d) $2IAB$ in negative Z -axis
- 35** In hydrogen atom, the electron is making 6.6×10^{15} rev/s around the nucleus in an orbit of radius $0.528 \text{ \AA}$. The magnetic moment ($\text{A}\cdot\text{m}^2$) will be
- (a) 1×10^{-15} (b) 1×10^{-10} (c) 1×10^{-23} (d) 1×10^{-27}
- 36** Two infinitely long conductors carrying equal currents are shaped as shown in figure. All the short sections are of equal lengths. The point P is located symmetrically with respect to the two conductors. The magnetic field at P due to any one conductor is B . The total field at P is

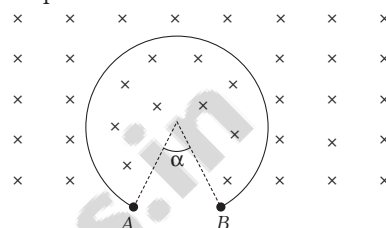


- (a) zero (b) B (c) $\sqrt{2} B$ (d) $2B$
- 37** A particle of mass m and having a positive charge q is projected from origin with speed v_0 along the positive X -axis in a magnetic field $\mathbf{B} = -B_0 \hat{\mathbf{k}}$, where B_0 is a positive constant. If the particle passes through $(0, y, 0)$, then y is equal to
- (a) $-\frac{2mv_0}{qB_0}$ (b) $\frac{mv_0}{qB}$ (c) $-\frac{mv_0}{qB}$ (d) $\frac{2mv_0}{qB_0}$
- 38** A 100 turns coil shown in figure carries a current of 2 A in a magnetic field $B = 0.2 \text{ Wb/m}^2$. The torque acting on the coil is



- (a) 0.32 N-m tending to rotate the side AD out of the page
(b) 0.32 N-m tending to rotate the side AD into the page
(c) 0.0032 N-m tending to rotate the side AD out of the page
(d) 0.0032 N-m tending to rotate the side AD into the page

- 39** A circular loop which is in the form of a major arc of a circle is kept in the horizontal plane and a constant magnetic field B is applied in the vertical direction such that the magnetic lines of forces go into the plane. If R is the radius of circle and it carries a current I in the clockwise direction, then the force on the loop will be



- (a) $BIR \tan \alpha$ (b) $2BIR \cos(\alpha/2)$
(c) $2BIR \sin(\alpha/2)$ (d) None of these
- 40.** Two protons are projected simultaneously from a fixed point with the same velocity v into a region, where there exists a uniform magnetic field. The magnetic field strength is B and it is perpendicular to the initial direction of v . One proton starts at time $t = 0$ and another proton at $t = \frac{\pi m}{2qB}$. The separation between them at time $t = \frac{\pi m}{qB}$ (where, m and q are the mass and charge of proton), will be approximately
- (a) $2 \frac{mv}{qB}$ (b) $\frac{\sqrt{2} mv}{qB}$ (c) $\frac{mv}{qB}$ (d) $\frac{mv}{2qB}$
- 41** A disc of radius R rotates with constant angular velocity ω about its own axis. Surface charge density of this disc varies as $\sigma = \alpha r^2$, where r is the distance from the centre of disc. Determine the magnetic field intensity at the centre of disc.
- (a) $\mu_0 \alpha \omega R^3$ (b) $\frac{\mu_0 \alpha \omega R^3}{6}$ (c) $\frac{\mu_0 \alpha \omega R^3}{8}$ (d) $\frac{\mu_0 \alpha \omega R^3}{3}$
- 42** A rigid circular loop of radius r and mass m lies in the XY -plane on a flat table and has a current i flowing in it. At this particular place, the earth's magnetic field is $\mathbf{B} = B_x \hat{\mathbf{i}} + B_z \hat{\mathbf{k}}$. The value of i , so that the loop starts tilting is

- (a) $\frac{mg}{\pi r \sqrt{B_x^2 + B_z^2}}$ (b) $\frac{mg}{\pi r B_x}$
(c) $\frac{mg}{\pi r B_z}$ (d) $\frac{mg}{\pi r \sqrt{B_x B_z}}$

- 43** Two circular coils 1 and 2 are made from the same wire but the radius of the 1st coil is twice that of the 2nd coil. What is the ratio of potential difference applied across them, so that the magnetic field at their centres is the same?

(a) 3 : 1 (b) 4 : 1 (c) 6 : 1 (d) 2 : 1

- 44** A charged particle with specific charge (charge per unit mass = $\frac{q}{m} = S$) moves undeflected through a region of space containing mutually perpendicular uniform electric and magnetic fields E and B . When the E field is switched off, then the particle will move in a circular path of radius

(a) $\frac{E}{BS}$ (b) $\frac{ES}{B}$ (c) $\frac{ES}{B^2}$ (d) $\frac{E}{B^2S}$

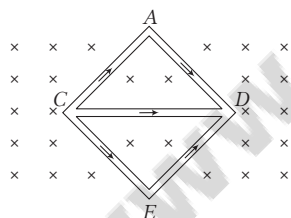
- 45** A large metal sheet carries an electric current along its surface. Current per unit length is λ .



Magnetic field induction near the metal sheet is

(a) $\lambda\mu_0$ (b) $\frac{\lambda\mu_0}{2}$ (c) $\frac{\lambda\mu_0}{2\pi}$ (d) $\frac{\mu_0}{2\pi\lambda}$

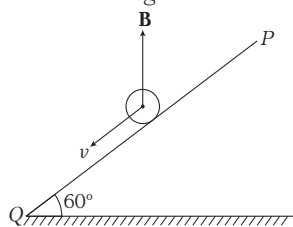
- 46** A current $i = 2\text{ A}$ is flowing in a wire frame as shown in figure. The frame is a combination of two equilateral triangles ACD and CDE of side 1 m. It is placed in uniform magnetic field $B = 4\text{ T}$ acting perpendicular to the plane of frame. The magnitude of magnetic force acting on the frame is



(a) 24 N (b) zero (c) 16 N (d) 8 N

- 47** A conducting stick of length $2L$ and mass m is moving down a smooth inclined plane of inclination 60° with constant speed 5 m/s. A current 2 A is flowing in the conductor perpendicular to the paper inwards. A vertically upward magnetic field B exists in space there. The magnitude of magnetic field B is

(a) $\frac{mg}{4L}$ (b) $\frac{mg}{L}$ (c) $\frac{\sqrt{3}mg}{4L}$ (d) $\frac{\sqrt{3}mg}{2L}$



- 48** A charge q is moving with a velocity $\mathbf{v}_1 = \hat{i}\text{ m/s}$ at a point in a magnetic field and experiences a force $\mathbf{F} = q[-\hat{j} + \hat{k}]\text{ N}$. If the charge is moving with a velocity $\mathbf{v}_2 = \hat{j}\text{ m/s}$ at the same point, then it experiences a force $\mathbf{F}_2 = q(\hat{i} - \hat{k})\text{ N}$. The magnetic induction $\mathbf{B}$ at that point is

(a) $(\hat{i} + \hat{j} + \hat{k})\text{ Wb/m}^2$ (b) $(\hat{i} - \hat{j} + \hat{k})\text{ Wb/m}^2$
(c) $(-\hat{i} + \hat{j} - \hat{k})\text{ Wb/m}^2$ (d) $(\hat{i} + \hat{j} - \hat{k})\text{ Wb/m}^2$

- 49** A square frame of side 1 m carries a current I , produces a magnetic field B at its centre. The same current is passed through a circular coil having the same perimeter as the square. The magnetic field at the centre of the circular coil is B' . The ratio B/B' is

(a) $\frac{8}{\pi^2}$ (b) $\frac{8\sqrt{2}}{\pi^2}$ (c) $\frac{16}{\pi^2}$ (d) $\frac{16}{\sqrt{2}\pi^2}$

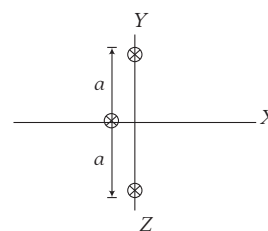
- 50** The magnetic field existing in a region is given by

$$\mathbf{B} = B_0 \left[1 + \frac{x}{l} \right] \hat{k}$$

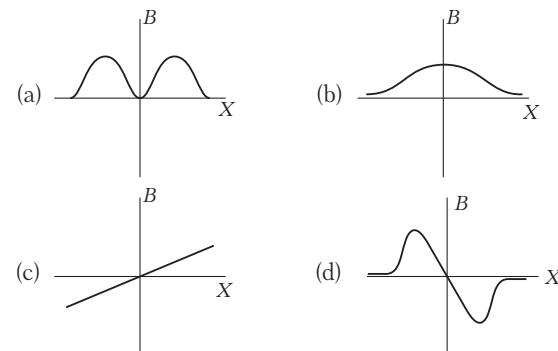
A square loop of edge l and carrying current I is placed with its edges parallel to the X and Y -axes. The magnitude of the net magnetic force experienced by the loop is

(a) $2B_0Il$ (b) zero
(c) B_0Il (d) $4B_0Il$

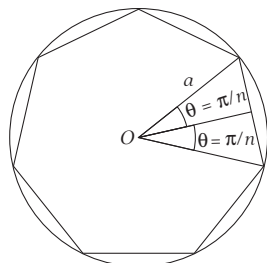
- 51** Two straight infinitely long current carrying wires are kept along Z -axis at the coordinates $(0, a, 0)$ and $(0, -a, 0)$ respectively, as shown in the figure. The current in each of the wire is equal and along negative Z -axis (into the plane of the paper).



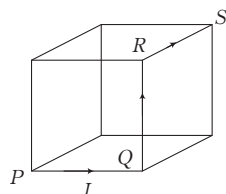
The variation of magnetic field on the X -axis will be approximately



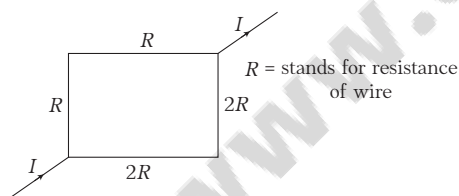
- 52** A wire is bent in the form of a regular polygon of n sides, is inscribed in a circle of radius a . If i ampere is the current flowing in the wire, then the magnetic field at the centre of the circle is



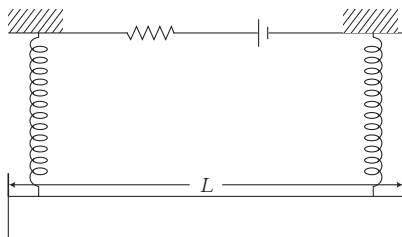
- (a) $\frac{\mu_0 i}{2\pi a} \tan \frac{\pi}{n}$ (b) $\frac{\mu_0 n i}{2\pi a} \tan \frac{\pi}{n}$
 (c) $\frac{2}{\pi} \frac{n i}{a} \mu_0 \tan \frac{\pi}{n}$ (d) $\frac{n i}{2a} \mu_0 \tan \frac{\pi}{n}$
- 53.** A wire $PQRS$ carrying a current I runs along three edges of a cube of side l as shown in figure. There exists a uniform magnetic field of magnitude B along one of the sides of the cube. The magnitude of the force acting on the wire is



- (a) zero (b) $\sqrt{3} IB$ (c) $\sqrt{2} IB$ (d) $2IB$
- 54** The magnetic field at the centre of square of side a is



- (a) $\frac{\sqrt{2} \mu_0}{\pi a}$ (b) $\frac{\sqrt{2} \mu_0 I}{3\pi a}$ (c) $\frac{2}{3} \frac{\mu_0 I}{a}$ (d) zero
- 55** A straight rod of mass m and length L is suspended from the identical springs as shown in figure. The spring is stretched a distance x_0 due to the weight of the wire.



The circuit has total resistance R . When the magnetic field perpendicular to the plane of paper is switched on, then springs are observed to extend further by the same distance. The magnetic field strength is

- (a) $\frac{2mgR}{LE}$ (b) $\frac{mgR}{LE}$ (c) $\frac{mgR}{2LE}$ (d) $\frac{mgR}{E}$

- 56** A particle of specific charge $q/m = \pi$ C/kg is projected from the origin towards positive X -axis with a velocity of 10 m/s in a uniform magnetic field $\mathbf{B} = -2\hat{k}$ T. The velocity $\mathbf{v}$ of the particle after time $t = \frac{1}{6}$ s will be

- (a) $(5\hat{i} + 5\sqrt{3}\hat{j})$ m/s (b) $10\hat{j}$ m/s
 (c) $(5\sqrt{3}\hat{i} + 5\hat{j})$ m/s (d) $-10\hat{j}$ m/s

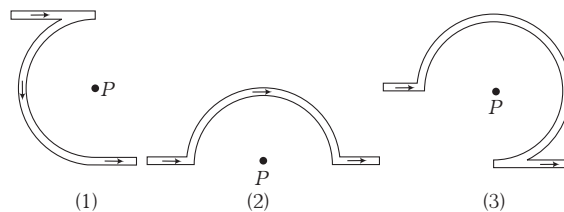
- 57** A charged particle enters into a uniform magnetic field with velocity v_0 perpendicular to it, the length of magnetic field is $x = (\sqrt{3}/2) R$, where R is the radius of the circular path of the particle in the field. The magnitude of change in velocity of the particle when it comes out of the field is

- (a) $2v_0$ (b) $\frac{v_0}{2}$ (c) $\frac{\sqrt{3}v_0}{2}$ (d) v_0

- 58** A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\mathbf{E}$ and $\mathbf{B}$ represent the electric and magnetic fields respectively, then this region of space may not have

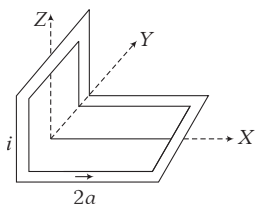
- (a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$
 (c) $E \neq 0, B = 0$ (d) $E \neq 0, B \neq 0$

- 59** Figure here shows three cases, in all cases the circular path has radius r and straight ones are infinitely long. For same current, the magnetic field at the centre P in cases (1), (2) and (3) have the ratio



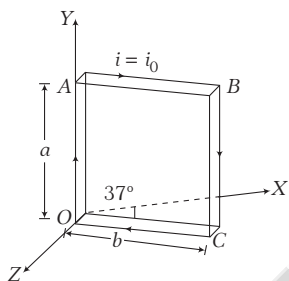
- (a) $\left(-\frac{\pi}{2}\right) : \left(\frac{\pi}{2}\right) : \left(\frac{3\pi}{4} - \frac{1}{2}\right)$
 (b) $\left(-\frac{\pi}{2} + 1\right) : \left(\frac{\pi}{2} + 1\right) : \left(\frac{3\pi}{4} + \frac{1}{2}\right)$
 (c) $-\frac{\pi}{2} : \frac{\pi}{2} : 3\frac{\pi}{4}$
 (d) $\left(-\frac{\pi}{2} - 1\right) : \left(\frac{\pi}{2} - \frac{1}{4}\right) : \left(\frac{3\pi}{4} + \frac{1}{2}\right)$

- 60** A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point $P(a, 0, a)$ is in the direction



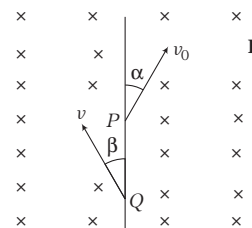
- (a) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$ (b) $\frac{1}{\sqrt{3}}(-\hat{j} + \hat{k} + \hat{i})$
 (c) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

- 61** A rectangular loop consists of N closed wrapped turns and has dimensions $a \times b$. The loop is hinged along the Y -axis. What is the magnitude of the torque exerted on the loop by a uniform magnetic field $B = B_0$ directed along the X -axis when current $i = i_0$ is in the direction shown. The torque acting on the loop is



- (a) $-\frac{4Ni_0abB_0}{5}\hat{j}$ (b) $\frac{4Ni_0abB_0}{5}\hat{i}$
 (c) $-\frac{4Ni_0abB_0}{3}\hat{j}$ (d) $\frac{-2Ni_0abB_0}{5}\hat{j}$

- 62** A particle of charge $-q$ and mass m enters a uniform magnetic field $\mathbf{B}$ (perpendicular to paper inwards) at P with a speed v_0 at an angle α and leaves the field at Q with speed v at angle β as shown in the figure. Then,



- (a) $\alpha \neq \beta$
 (b) $v \neq v_0$
 (c) $PQ = \frac{mv_0 \sin \alpha}{Bq}$
 (d) particle remains in the field for time $t = \frac{2m(\pi - \alpha)}{Bq}$

- 63** A square coil of edge L having n turns carries a current i . It is kept on a smooth horizontal plate. A uniform magnetic field B exists in a direction parallel to an edge. The total mass of the coil is M . What should be the minimum value of B for which the coil will start tipping over?

- (a) $\frac{Mg}{niL}$ (b) $\frac{Mg}{2niL}$ (c) $\frac{Mg}{4niL}$ (d) $\frac{2Mg}{niL}$

- 64** A long straight wire along the Z -axis carries a current I in the negative z -direction. The magnetic field vector $\mathbf{B}$ at a point having coordinates (x, y) in the $z = 0$ plane is

- (a) $\frac{\mu_0 I(y\hat{i} - x\hat{j})}{2\pi(x^2 + y^2)}$ (b) $\frac{\mu_0 I(x\hat{i} + y\hat{j})}{2\pi(x^2 + y^2)}$
 (c) $\frac{\mu_0 I(x\hat{j} - y\hat{i})}{2\pi(x^2 + y^2)}$ (d) $\frac{\mu_0 I(x\hat{i} - y\hat{j})}{2\pi(x^2 + y^2)}$

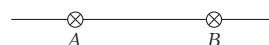
(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-6) These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
 (b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
 (c) If Assertion is true but Reason is false.
 (d) If Assertion is false but Reason is true.

- 1 Assertion** Two infinitely long wires A and B carry unequal currents both in inward direction.



Then, there is only one point (excluding the points at infinity), where net magnetic field is zero.

Reason That point lies between points A and B .

- 2 Assertion** When a charged particle moves perpendicular to a uniform magnetic field, then its momentum remains constant.

Reason Magnetic force acts perpendicular to the velocity of the particle.

3 Assertion An α -particle and a deuteron having same kinetic energy enter in a uniform magnetic field perpendicular to the field. Then, radius of circular path of α -particle will be more.

Reason $\frac{q}{m}$ ratio of an α -particle is equal to the $\frac{q}{m}$ ratio of a deuteron.

4 Assertion In a uniform magnetic field $\mathbf{B} = B_0 \hat{\mathbf{k}}$, if velocity of a charged particle is $v_0 \hat{\mathbf{i}}$ at $t = 0$, then it can have the velocity $v_0 \hat{\mathbf{j}}$ at some other instant.

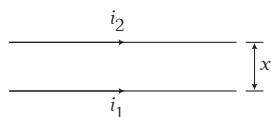
Reason In uniform magnetic field, acceleration of a charged particle is always zero.

5 Assertion If velocity of charged particle in a uniform magnetic field at some instant is $(a_1 \hat{\mathbf{i}} - a_2 \hat{\mathbf{j}})$ and at some other instant is $(b_1 \hat{\mathbf{i}} + b_2 \hat{\mathbf{j}})$, then

$$a_1^2 + a_2^2 = b_1^2 + b_2^2$$

Reason Magnetic force cannot change velocity of a charged particle.

6 Assertion Upper wire shown in figure is fixed. At a certain distance x , lower wire can remain in equilibrium.



Reason The above equilibrium of lower wire is stable equilibrium.

Statement based questions

1 An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following statement(s) is/are correct?

[NCERT Exemplar]

- (a) The electron will be accelerated along the axis.
- (b) The electron path will be circular about the axis.
- (c) The electron will experience a force at 45° to the axis and hence execute a helical path.
- (d) The electron will continue to move with uniform velocity along the axis of the solenoid.

2 H^+ , He^+ and O^{2+} ions having same kinetic energy pass through a region of space filled with uniform magnetic field B directed perpendicular to the velocity of ions. The masses of the ions H^+ , He^+ and O^{2+} are respectively, in the ratio 1 : 4 : 16. Which of the following statement(s) is/are correct?

- (a) H^+ ions will be deflected most
- (b) O^{2+} ions will be deflected least
- (c) He^+ and O^{2+} ions will suffer same deflection
- (d) All ions will suffer the same deflection

3 The coil of a moving coil galvanometer has an effective area of $4 \times 10^{-2} \text{ m}^2$. It is suspended in a magnetic field of $5 \times 10^{-2} \text{ Wbm}^{-2}$. If deflection in the galvanometer coil is 0.2 rad when a current of 5 mA is passed through it, then which of the following statement(s) is/are correct?

- (a) Torsional constant is $5 \times 10^{-6} \text{ N-m rad}^{-1}$.
- (b) Current sensitivity is 40 rad A^{-1} .
- (c) Torsional constant is $3 \times 10^{-3} \text{ N-m rad}^{-1}$.
- (d) Current sensitivity is 40 deg A^{-1} .

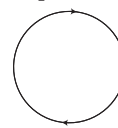
4 Which of the following statement(s) is/are correct?

- I. A flexible wire loop of irregular shape carrying current when placed in a uniform external magnetic field acquires a circular shape.
- II. For a given perimeter circular shape is having the greatest area.

- (a) Only I
- (b) Only II
- (c) Both I and II
- (d) None of these

5 Which of the following statement(s) is/are correct?

- I. A positive charged particle is rotating in a circle. Then, magnetic field ($\mathbf{B}$) at centre of circle and magnetic moment ($\mathbf{M}$) produced by motion of charged particle are parallel to each other.

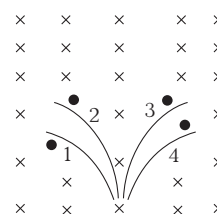


II. $\mathbf{M}$ and $\mathbf{B}$ are always parallel to each other.

- (a) Only I
- (b) Only II
- (c) Both I and II
- (d) None of these

Match the columns

1 Four particles; α -particle, deuteron, electron and a Cl^- ion enter in a transverse magnetic field perpendicular to it with same kinetic energy. Their paths are as shown in figure. Now, match the Column I with Column II and mark the correct option from the codes given below.



Column I		Column II	
(A)	Deuteron	(p)	Path-1
(B)	α -particle	(q)	Path-2
(C)	Electron	(r)	Path-3
(D)	Cl^-	(s)	Path-4

Codes

A	B	C	D	A	B	C	D
(a) p	q	r	s	(b) q	p	s	r
(c) s	r	p	q	(d) q	s	p	r

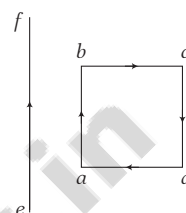
- 2** A charged particle is rotating in uniform circular motion in a uniform magnetic field. Let r = radius of circle, v = speed of particle, K = kinetic energy, a = magnitude of acceleration, p = magnitude of linear momentum, $\frac{q}{m} = \alpha$ = specific charge and ω = angular speed. Then, match the Column I with Column II and mark the correct option from the codes given below.

Column I		Column II	
(A)	If v is doubled	(p)	r will become two times
(B)	If B is doubled	(q)	ω will become two times
(C)	If p is doubled	(r)	a will become two times
(D)	If α is doubled	(s)	None

Codes

A	B	C	D
(a) p	q	p	q
(b) p	r	s	q
(c) s	r	s	q
(d) p	r	s	s

- 3** A square current carrying loop $abcd$ is placed near an infinitely long another current carrying wire ef . Now, match the Column I with Column II and mark the correct option from the codes given below.



Column I		Column II	
(A)	Net force on bc and da	(p)	zero
(B)	Net force on ab and cd	(q)	non-zero
(C)	Net force on complete loop $abcd$	(r)	rightwards
(D)	Net force on ab	(s)	leftwards

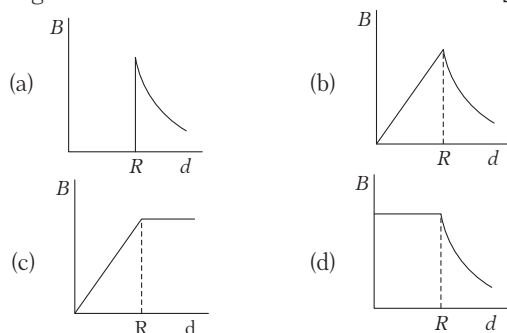
A	B	C	D	A	B	C	D
(a) p,s	r,q	p	s	(b) q,r	q,r	r	s
(c) p	q	q,s	q,s	(d) r	r	p	q

(C) Medical entrances' gallery

Collection of questions asked in NEET & various medical entrance exams

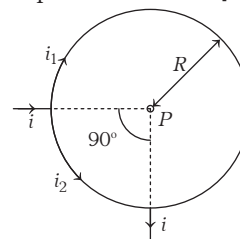
- 1** A long solenoid of 50 cm length having 100 turns carries a current of 2.5 A. The magnetic field at the centre of solenoid is [NEET 2020]
(Take, $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$)
(a) $3.14 \times 10^{-4} \text{ T}$ (b) $6.28 \times 10^{-5} \text{ T}$
(c) $3.14 \times 10^{-5} \text{ T}$ (d) $6.28 \times 10^{-4} \text{ T}$

- 2** A cylindrical conductor of radius R is carrying a constant current. The plot of the magnitude of the magnetic field B with the distance d from the centre of the conductor is correctly represented by the figure [NEET 2019]



- 3** Two toroids 1 and 2 have total number of turns 200 and 100 respectively with average radii 40 cm and 20 cm, respectively. If they carry same current i , the ratio of the magnetic fields along the two loops is [NEET (Odisha) 2019]
(a) 1 : 1 (b) 4 : 1 (c) 2 : 1 (d) 1 : 2

- 4** A straight conductor carrying current i splits into two parts as shown in the figure. The radius of the circular loop is R . The total magnetic field at the centre P of the loop is [NEET (Odisha) 2019]



- (a) zero
(b) $3 \mu_0 i / 32 R$, outward
(c) $3 \mu_0 i / 32 R$, inward
(d) $\frac{\mu_0 i}{2 R}$, inward

- 5 Ionised hydrogen atoms and α -particles with same momenta enters perpendicular to a constant magnetic field B . The ratio of their radii of their paths $r_H : r_\alpha$ will be [NEET 2019]

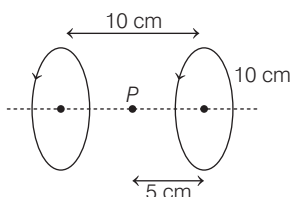
(a) 1 : 2 (b) 4 : 1 (c) 1 : 4 (d) 2 : 1

- 6 **Assertion** A charged particle is released from rest in magnetic field, then it will move in a circular path. [NEET 2019]

Reason Work done by magnetic field is non-zero.

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct, but Reason is not the correct explanation of Assertion.
(c) Assertion is correct, but Reason is incorrect.
(d) Both Assertion and Reason are incorrect.

- 7 Two circular loops having same radius ($R = 10$ cm) and same current $\frac{7}{2}$ A are placed along same axis as shown in the figure. If distance between their centres is 10 cm, find the value of net magnetic field at point P . [AIIMS 2019]



- (a) $\frac{50\mu_0}{\sqrt{5}}$ T (b) $\frac{28\mu_0}{\sqrt{5}}$ T
(c) $\frac{56\mu_0}{\sqrt{5}}$ T (d) $\frac{56\mu_0}{\sqrt{3}}$ T

- 8 A proton is projected with velocity $\mathbf{v} = 2\hat{\mathbf{i}}$ in a region where magnetic field, $\mathbf{B} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ μT and electric field, $\mathbf{E} = 10\hat{\mathbf{i}}$ V m^{-1} . Then, find out the net acceleration of proton. [AIIMS 2019]

- (a) 1400 ms^{-2} (b) 700 ms^{-2}
(c) 1000 ms^{-2} (d) 800 ms^{-2}

- 9 **Assertion** Electron moving perpendicular to $\mathbf{B}$ will perform circular motion.

Reason Force by magnetic field is perpendicular to velocity. [AIIMS 2019]

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Both Assertion and Reason are incorrect.

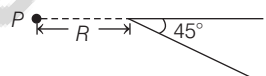
- 10 If two protons are moving with speed $v = 4.5 \times 10^5 \text{ ms}^{-1}$ parallel to each other, then find the value of ratio of electrostatic and magnetic force between them. [AIIMS 2019]

(a) 4.4×10^5 (b) 2.2×10^5 (c) 3.3×10^5 (d) 1.1×10^5

- 11 A metallic rod of mass per unit length 0.5 kg m^{-1} is lying horizontally on a smooth inclined plane which makes an angle of 30° with the horizontal. The rod is not allowed to slide down by flowing a current through it, when a magnetic field of induction 0.25 T is acting on it in the vertical direction. The current flowing in the rod to keep it stationary is [NEET 2018]

(a) 14.76 A (b) 5.98 A (c) 7.14 A (d) 11.32 A

- 12 A long straight wire carrying current I is bent at its mid-point to form an angle of 45° . Induction of magnetic field (in tesla) at point P , distant R from point of bending, is equal to [AIIMS 2018]



- (a) $\frac{(\sqrt{2}-1)\mu_0 I}{4\pi R}$ (b) $\frac{(\sqrt{2}+1)\mu_0 I}{4\pi R}$
(c) $\frac{(\sqrt{2}-1)\mu_0 I}{4\sqrt{2}\pi R}$ (d) $\frac{(\sqrt{2}+1)\mu_0 I}{4\sqrt{2}\pi R}$

- 13 An element $dl = dx \hat{\mathbf{i}}$ (where $dx = 1$ cm) is placed at the origin and carries a large current $i = 10 \text{ A}$. What is the magnetic field on the Y -axis at a distance of 0.5 m ? [AIIMS 2018]

- (a) $2 \times 10^{-8} \hat{\mathbf{k}} \text{ T}$ (b) $4 \times 10^{-8} \hat{\mathbf{k}} \text{ T}$
(c) $-2 \times 10^{-8} \hat{\mathbf{k}} \text{ T}$ (d) $-4 \times 10^{-8} \hat{\mathbf{k}} \text{ T}$

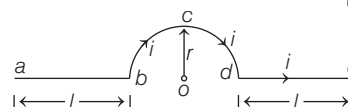
- 14 The magnetic moment of an electron orbiting in a circular orbit of radius r with a speed v is equal to [JIPMER 2018]

- (a) $evr/2$ (b) evr
(c) $er/2v$ (d) None of these

- 15 A current carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend upon [JIPMER 2018]

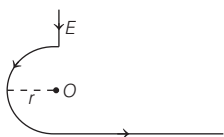
- (a) shape of loop (b) area of loop
(c) value of current (d) magnetic field

- 16 A long wire having a semicircular loop of radius r carries a current i as shown in figure. The magnetic induction at the centre O due to entire wire is [JIPMER 2017]



- (a) $\frac{\mu_0 i}{4r}$ (b) $\frac{\mu_0 i^2}{4r}$
 (c) $\frac{\mu_0 i}{4r^2}$ (d) None of these

- 17** In the given figure, what is the magnetic field induction at point O ? [JIPMER 2017]



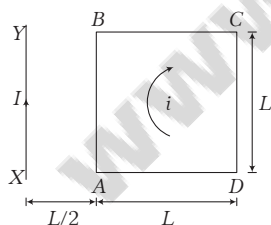
- (a) $\frac{\mu_0 I}{4\pi r}$ (b) $\frac{\mu_0 I}{4r} + \frac{\mu_0 I}{2\pi r}$
 (c) $\frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}$ (d) $\frac{\mu_0 I}{4r} - \frac{\mu_0 I}{4\pi r}$

- 18** A long wire carrying a steady current is bent into a circular loop of one turn. The magnetic field at the centre of the loop is B . It is then bent into a circular coil of n turns. The magnetic field at the centre of this coil of n turns will be [NEET 2016]
 (a) nB (b) n^2B (c) $2nB$ (d) $2n^2B$

- 19** An electron is moving in a circular path under the influence of a transverse magnetic field of 3.57×10^{-2} T. If the value of e/m is 1.76×10^{11} C/kg, the frequency of revolution of the electron is [NEET 2016]

- (a) 1 GHz (b) 100 MHz
 (c) 62.8 MHz (d) 6.28 MHz

- 20** A square loop $ABCD$ carrying a current i is placed near and coplanar with a long straight conductor XY carrying a current I , the net force on the loop will be [NEET 2016]

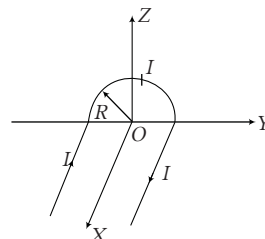


- (a) $\frac{\mu_0 i I}{2\pi}$ (b) $\frac{2\mu_0 i I L}{3\pi}$
 (c) $\frac{\mu_0 i I L}{2\pi}$ (d) $\frac{2\mu_0 i I}{3\pi}$

- 21** A long straight wire of radius a carries a steady current I . The current is uniformly distributed over its cross-section. The ratio of the magnetic fields B and B' at radial distances $\frac{a}{2}$ and $2a$ respectively, from the axis of the wire is [NEET 2016]

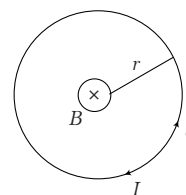
- (a) $\frac{1}{2}$ (b) 1 (c) 4 (d) $\frac{1}{4}$

- 22** A wire carrying current I has the shape as shown in adjoining figure. Linear parts of the wire are very long and parallel to X -axis while semicircular portion of radius R is lying in YZ -plane. Magnetic field at point O is [CBSE AIPMT 2015]



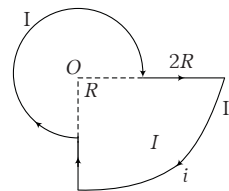
- (a) $B = \frac{\mu_0}{4\pi} \frac{I}{R} (\pi \hat{i} + 2\hat{k})$ (b) $B = -\frac{\mu_0}{4\pi} \frac{I}{R} (\pi \hat{i} - 2\hat{k})$
 (c) $B = -\frac{\mu_0}{4\pi} \frac{I}{R} (\pi \hat{i} + 2\hat{k})$ (d) $B = \frac{\mu_0}{4\pi} \frac{I}{R} (\pi \hat{i} - 2\hat{k})$

- 23** An electron moving in a circular orbit of radius r makes n rotations per second. The magnetic field produced at the centre has magnitude [CBSE AIPMT 2015]



- (a) $\frac{\mu_0 n e}{2\pi r}$ (b) zero
 (c) $\frac{\mu_0 n^2 e}{r}$ (d) $\frac{\mu_0 n e}{2r}$

- 24** Consider the circular loop having current i and with central point O . The magnetic field at the central point O is [AIIMS 2015]



- (a) $\frac{2\mu_0 i}{3\pi R}$ acting downward (b) $\frac{5\mu_0 i}{12R}$ acting downward
 (c) $\frac{6\mu_0 i}{11R}$ acting downward (d) $\frac{3\mu_0 i}{7R}$ acting upward

- 25** A proton is projected with a speed of 3×10^6 m/s horizontally from east to west. An uniform magnetic field of strength 2×10^{-3} T exists in the vertically upward direction. What would be the acceleration of proton? [UK PMT 2015]

- (a) 11.6×10^{11} m/s² (b) 17.4×10^{11} m/s²
 (c) 5.8×10^{11} m/s² (d) 2.9×10^{11} m/s²

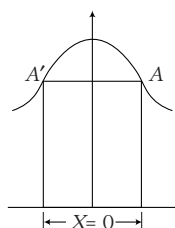
- 26** The magnetic field at the centre of a circular coil carrying current I ampere is B . If the coil is bent into smaller circular coil of n turns, then its magnetic field at the centre is B' . The ratio between B' and B is [Kerala CEE 2015]

(a) $1 : 1$ (b) $n : 1$
(c) $n^2 : 1$ (d) $2n : 1$
(e) $(n + 1) : 1$

- 27** Equal currents are passing through two very long and straight parallel wires in the same direction. They will [Guj. CET 2015]

(a) attract each other
(b) repel each other
(c) lean towards each other
(d) Neither attract nor repel each other

- 28** The variation of magnetic field B due to circular coil as the distance X varies is shown in the graph. Which of the following is false? [CG PMT 2015]



- (a) Points A and A' are known as points of zero curvature
(b) B varies linearly with X at points A and A'
(c) $\frac{dB}{dt} = 0$ at points A and A'
(d) $\frac{d^2B}{dt^2} = 0$ at points A and A'

- 29** Two particles A and B having equal charges, after being accelerated through the same potential difference enter into a region of uniform magnetic field and the particles describe circular paths of radii R_1 and R_2 , respectively. The ratio of the masses of A and B is [WB JEE 2015]

(a) $\sqrt{R_1/R_2}$ (b) R_1/R_2
(c) $(R_1/R_2)^2$ (d) $(R_2/R_1)^2$

- 30** There is a ring of radius r having linear charge density λ and rotating with a uniform angular velocity ω . The magnetic field produced by this ring at its own centre would be [UP CPMT 2015]

(a) $\frac{\lambda \omega^2}{2 - \mu_0}$ (b) $\frac{\mu_0 \lambda^2 \omega}{\sqrt{2}}$ (c) $\frac{\mu_0 \lambda \omega}{2}$ (d) $\frac{\mu_0 \lambda}{2 \omega^2}$

- 31** Two particles A and B having equal charges $+6\text{ C}$ after being accelerated through the same potential difference, enter in a region of uniform magnetic field and describe circular paths of radii 2 cm and

3 cm , respectively. The ratio of mass of A to that of B is [Manipal 2015]

(a) $1/3$ (b) $1/2$ (c) $4/9$ (d) $9/5$

- 32** A proton beam enters a magnetic field of 10^{-4} Wb/m^2 normally. If the specific charge of the proton is 10^{11} C/kg and its velocity is 10^9 m/s , then the radius of the circle described will be [KCET 2015]

(a) 100 m (b) 0.1 m (c) 1 m (d) 10 m

- 33** A cyclotron is used to accelerate [KCET 2015]

(a) Only negatively charged particles
(b) neutron
(c) Both positively and negatively charged particles
(d) Only positively charged particles

- 34** Two parallel beams of positron moving in the same direction will [Manipal 2015]

(a) not interact with each other
(b) repel each other
(c) attract each other
(d) be deflected normal to the plane containing two beams

- 35** Two concentric coils each of radius equal to $2\pi\text{ cm}$ are placed at right angles to each other. If 3 A and 4 A are the currents flowing through the two coils, respectively. The magnetic induction (in Wb/m^2) at the centre of the coils will be [KCET 2015]

(a) 5×10^{-5} (b) 12×10^{-5} (c) 7×10^{-5} (d) 10^{-5}

- 36** Two identical long conducting wires AOB and COD are placed at right angle to each other with one above other such that O is the common point for the two. The wires carry I_1 and I_2 currents, respectively. Point P is lying at distance d from O along a direction perpendicular to the plane containing the wires. The magnetic field at the point P will be [CBSE AIPMT 2014]

(a) $\frac{\mu_0}{2\pi d} \left(\frac{I_1}{I_2} \right)$ (b) $\frac{\mu_0}{2\pi d} (I_1 + I_2)$
(c) $\frac{\mu_0}{2\pi d} (I_1^2 - I_2^2)$ (d) $\frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{1/2}$

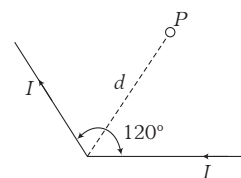
- 37** A solenoid has length 0.4 cm , radius 1 cm and 400 turns of wire. If a current of 5 A is passed through this solenoid, then what is the magnetic field inside the solenoid? [KCET 2014]

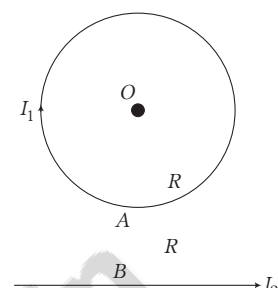
(a) $6.28 \times 10^{-4}\text{ T}$ (b) $6.28 \times 10^{-1}\text{ T}$
(c) $6.28 \times 10^{-7}\text{ T}$ (d) $6.28 \times 10^{-6}\text{ T}$

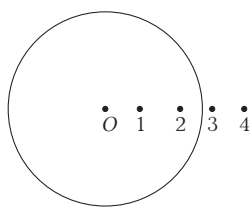
- 38** A toroid having 200 turns carries a current of 1 A . The average radius of the toroid is 10 cm . The magnetic field at any point in the open space inside the toroid is [Kerala CEE 2014]

(a) $4 \times 10^{-3}\text{ T}$ (b) zero
(c) $0.5 \times 10^{-3}\text{ T}$ (d) $3 \times 10^{-3}\text{ T}$
(e) $2 \times 10^{-3}\text{ T}$

- 39** The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is $54 \mu\text{T}$. What will be its value at the centre of the loop? [UK PMT 2014]
 (a) $200 \mu\text{T}$ (b) $250 \mu\text{T}$ (c) $125 \mu\text{T}$ (d) $75 \mu\text{T}$
- 40** A proton of mass m and charge q is moving in a plane with kinetic energy E . If there exists a uniform magnetic field B perpendicular to the plane motion, the proton will move in a circular path of radius [WB JEE 2014]
 (a) $\frac{2Em}{qB}$ (b) $\frac{\sqrt{2Em}}{qB}$ (c) $\frac{\sqrt{Em}}{2qB}$ (d) $\frac{\sqrt{2Eq}}{qB}$
- 41** If the velocity of charged particle has both perpendicular and parallel components while moving through a magnetic field, then what is the path followed by a charged particle? [KCET 2014]
 (a) Circular (b) Elliptical (c) Linear (d) Helical
- 42** A particle with charge q is moving along a circle of radius R with uniform speed v . The associated magnetic moment μ is given by [EAMCET 2014]
 (a) $\frac{1}{2} v^2 R$ (b) $\frac{1}{4} qvR$ (c) $\frac{1}{2} qvR$ (d) $\frac{1}{2} q^2 vR$
- 43** A charged particle of mass m and charge q moves along a circular path of radius r that is perpendicular to a magnetic field B . The time taken by the particle to complete one revolution is [UK PMT 2014]
 (a) $\frac{2\pi m q}{B}$ (b) $\frac{2\pi q^2 B}{m}$
 (c) $\frac{2\pi q B}{m}$ (d) $\frac{2\pi m}{qB}$
- 44** Magnetic induction produced at the centre of a circular loop carrying current is B . The magnetic moment of the loop of radius R is (where, μ_0 = permeability of free space) [MHT CET 2014]
 (a) $\frac{BR^2}{2\pi\mu_0}$ (b) $\frac{2\pi BR^3}{\mu_0}$ (c) $\frac{BR^2}{2\pi\mu_0}$ (d) $\frac{2\pi BR^2}{\mu_0}$
- 45** In cyclotron, for a given magnet, radius of the semicircle traced by positive ion is directly proportional to (where, v = velocity of positive ion) [MHT CET 2014]
 (a) v^{-2} (b) v^{-1}
 (c) v (d) v^2
- 46** When a magnetic field is applied on a stationary electron, then it [Kerala CEE 2014]
 (a) remains stationary
 (b) spins about its own axis
 (c) moves in the direction of the field
 (d) moves perpendicular to the direction of the field
 (e) moves opposite to the direction of the field
- 47** An electron in a circular orbit of radius 0.05 nm performs 10^{16} rev/s. The magnetic moment due to this rotation of electron is (in $\text{A}\cdot\text{m}^2$) [WB JEE 2014]
 (a) 2.16×10^{-23} (b) 3.21×10^{-22}
 (c) 3.21×10^{-24} (d) 1.26×10^{-23}
- 48** A circular coil of radius 10 cm and 100 turns carries a current 1 A. What is the magnetic moment of the coil? [KCET 2014]
 (a) $3.142 \times 10^4 \text{ A}\cdot\text{m}^2$ (b) $10^4 \text{ A}\cdot\text{m}^2$
 (c) $3.142 \text{ A}\cdot\text{m}^2$ (d) $3 \text{ A}\cdot\text{m}^2$
- 49** Two thin long conductors separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now, the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of force between them is [UK PMT 2014]
 (a) $-2F$ (b) $\frac{F}{3}$ (c) $-\frac{2F}{3}$ (d) $-\frac{F}{3}$
- 50** A charged particle experiences magnetic force in the presence of magnetic field. Which of the following statement is correct? [KCET 2014]
 (a) The particle is moving and magnetic field is perpendicular to the velocity
 (b) The particle is moving and magnetic field is parallel to the velocity
 (c) The particle is stationary and magnetic field is perpendicular to the velocity
 (d) The particle is stationary and magnetic field is parallel to the velocity
- 51** The ratio of magnetic dipole moment of an electron of charge e and mass m in Bohr's orbit in hydrogen atom to its angular momentum is [MHT CET 2014]
 (a) $\frac{e}{m}$ (b) $\frac{m}{e}$ (c) $\frac{2m}{e}$ (d) $\frac{e}{2m}$
- 52** A wire of length L metre carrying a current I ampere is bent in the form of a circle. The magnitude of the magnetic moment is [EAMCET 2014]
 (a) $\frac{L^2 I^2}{4\pi}$ (b) $\frac{LI}{4\pi}$ (c) $\frac{L^2 I}{4\pi}$ (d) $\frac{LI}{4\pi}$
- 53** A long conducting wire carrying a current I is bent at 120° (see figure). The magnetic field B at a point P on the right bisector of bending angle at a distance d from the bend is (μ_0 is the permeability of free space) [MP PMT 2014]



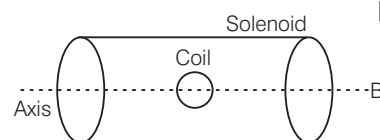
- (a) $\frac{2\mu_0 I}{2\pi d}$ (b) $\frac{\mu_0 I}{2\pi d}$
 (c) $\frac{\mu_0 I}{\sqrt{3}\pi d}$ (d) $\frac{\sqrt{3}\mu_0 I}{2\pi d}$
- 54** When a proton is released from rest in a room, it starts with an initial acceleration a_0 towards west. When it is projected towards north with a speed v_0 , it moves with an initial acceleration $3a_0$ towards west. The electric and magnetic fields in the room are [NEET 2013]
 (a) $\frac{ma_0}{e}$ west, $\frac{2ma_0}{ev_0}$ up (b) $\frac{ma_0}{e}$ west, $\frac{2ma_0}{ev_0}$ down
 (c) $\frac{ma_0}{e}$ east, $\frac{3ma_0}{ev_0}$ up (d) $\frac{ma_0}{e}$ east, $\frac{3ma_0}{ev_0}$ down
- 55** A current loop in a magnetic field [NEET 2013]
 (a) experiences a torque whether the field is uniform or non-uniform in all orientations
 (b) can be in equilibrium in one orientation
 (c) can be equilibrium in two orientations, both the equilibrium states are unstable
 (d) can be in equilibrium in two orientations, one stable while the other is unstable
- 56** A proton and helium nucleus are shot into a magnetic field at right angles to the field with same kinetic energy. Then, the ratio of their radii is [KCET 2013]
 (a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4
- 57** Two charged particles have charges and masses in the ratio 2 : 3 and 1 : 4, respectively. If they enter a uniform magnetic field and move with the same velocity, then the ratio of their respective time periods of revolution is [Kerala CEE 2013]
 (a) 3 : 8 (b) 1 : 4 (c) 3 : 5 (d) 1 : 6
 (e) 2 : 5
- 58** A current of 2 A is made to flow through a coil which has only one turn. The magnetic field produced at the centre is $4\pi \times 10^{-6}$ Wb/m². The radius of the coil is [MP PMT 2013]
 (a) 0.0001m (b) 0.01m (c) 0.1m (d) 0.001 m
- 59** A long straight wire is carrying a current of 12 A. The magnetic field at a distance of 8 cm is ($\mu_0 = 4\pi \times 10^{-7}$ N A²) [J & K CET 2013]
 (a) 2×10^{-4} Wb/m² (b) 3×10^{-5} Wb/m²
 (c) 4×10^{-4} Wb/m² (d) 4×10^{-5} Wb/m²
- 60** The magnetic field at a point on the axis of a long solenoid having 5 turns per cm length when a current of 0.8 A flows through it is [J & K CET 2013]
 (a) 5.024×10^{-4} Wb/m² (b) 6.024×10^{-4} Wb/m²
 (c) 7.024×10^{-4} Wb/m² (d) 8.024×10^{-4} Wb/m²
- 61** In the diagram, I_1 , I_2 are the strength of the currents in the loop and straight conductors respectively, $OA = AB = R$. The net magnetic field at the centre O is zero, then the ratio of the currents in the loop and the straight conductor is [KCET 2013]
- 
- (a) π (b) 2π (c) $\frac{1}{\pi}$ (d) $\frac{1}{2\pi}$
- 62** Two straight wires each 10 cm long are parallel to one another and separated by 2 cm. When the current flowing in them is 30 A and 40 A respectively, then the force experienced by either of the wires is [J & K CET 2013]
 (a) 1.2×10^{-3} N (b) 12×10^{-3} N
 (c) 11.2×10^{-3} N (d) 10.2×10^{-3} N
- 63** A charged particle with a velocity 2×10^3 ms⁻¹ passes undeflected through electric field and magnetic fields in mutually perpendicular directions. The magnetic field is 15 T. The magnitude of electric field will be [Karnataka CET 2013]
 (a) 1.5×10^3 NC⁻¹ (b) 2×10^3 NC⁻¹
 (c) 3×10^3 NC⁻¹ (d) 1.33×10^3 NC⁻¹
- 64** Two similar coils of radius R are lying concentrically with their planes at right angles to each other. The current flowing in them are I and $2I$, respectively. The resultant magnetic field induction at the centre will be [CBSE PMT 2012]
 (a) $\frac{\sqrt{5}\mu_0 I}{2R}$ (b) $\frac{3\mu_0 I}{2R}$
 (c) $\frac{\mu_0 I}{2R}$ (d) $\frac{\mu_0 I}{R}$
- 65** The adjacent figure shows the cross-section of a long rod with its length perpendicular to the plane of the paper. It carries constant current flowing along its length. B_1 , B_2 , B_3 and B_4 respectively, represent the magnetic fields due to the current in the rod at points 1, 2, 3 and 4 lying at different separations from the centre O , as shown in the figure. Which of the following shall hold true? [AMU 2012]



- (a) $B_1 > B_2 \neq 0$ (b) $B_2 > B_3 \neq 0$
 (c) $B_1 = B_2 = B_3 \neq 0$ (d) $B_3 > B_4 \neq 0$
- 66** A wire of one metre length carries a constant current. The wire is bent to form a circular loop. The magnetic field at the centre of this loop is B . The same is now bent to form a circular loop of smaller radius having four turns. The magnetic field at the centre of this new loop will be [BHU 2012]
 (a) $\frac{B}{2}$ (b) $4B$ (c) $\frac{B}{4}$ (d) $16B$
- 67** A proton is moving in a uniform magnetic field B in a circular path of radius a in a direction perpendicular to Z -axis along which field B exists. Calculate the angular momentum, if the radius is a and charge on proton is e . [Manipal 2012]
 (a) $\frac{Be}{a^2}$ (b) eB^2a (c) a^2eB (d) aeB
- 68** The magnetic field in a certain region of space is given by $\mathbf{B} = 8.35 \times 10^{-2} \hat{\mathbf{i}}$ T. A proton is shot into the field with velocity $\mathbf{v} = (2 \times 10^5 \hat{\mathbf{i}} + 4 \times 10^5 \hat{\mathbf{j}})$ m/s. The proton follows a helical path in the field. The distance moved by proton in the x -direction during the period of one revolution in the YZ -plane will be (Take, mass of proton = 1.67×10^{-27} kg) [AMU 2012]
 (a) 0.053 m (b) 0.136 m (c) 0.157 m (d) 0.236 m
- 69** A planar coil having 12 turns carries 15 A current. The coil is oriented with respect to the uniform magnetic field $\mathbf{B} = 0.2\hat{\mathbf{i}}$ T such that its directed area is $\mathbf{A} = 0.04\hat{\mathbf{i}}$ m². The potential energy of the coil in the given orientation is [AMU 2012]
 (a) 0 (b) + 0.72 J (c) +1.44 J (d) -1.44 J
- 70** A current i ampere flows along the inner conductor of a co-axial cable and returns along the outer conductor of the cable, then the magnetic induction at any point outside the conductor at a distance r metre from the axis is [JCECE 2012]
 (a) ∞ (b) zero (c) $\frac{\mu_0}{4\pi} \frac{2i}{r}$ (d) $\frac{\mu_0}{4\pi} \frac{2\pi i}{r}$
- 71** Two parallel long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, then the magnetic field midway between the wires is $10 \mu\text{T}$. When the direction of i_2 is reversed, then it becomes $40 \mu\text{T}$. Then, ratio of i_1/i_2 is [JCECE 2012]

- (a) 3 : 4 (b) 5 : 3
 (c) 7 : 11 (d) 11 : 7

- 72** When an electron beam passes through an electric field, they gain kinetic energy. If the same electron beam passes through a magnetic field, then their [BHU 2012]
 (a) energy and momentum both remain unchanged
 (b) potential energy increases
 (c) momentum increases
 (d) kinetic energy increases
- 73** If a steel wire of length l and magnetic moment M is bent into a semicircular arc, the new magnetic moment is [JCECE 2012]
 (a) $M \times l$ (b) $\frac{M}{l}$
 (c) $\frac{2M}{\pi}$ (d) M
- 74** A proton travelling at 23° w.r.t. the direction of a magnetic field of strength 2.6 mT experiences a magnetic force of 6.5×10^{-17} N. What is the speed of the proton? [DUMET 2011]
 (a) $2 \times 10^5 \text{ ms}^{-1}$ (b) $4 \times 10^5 \text{ ms}^{-1}$
 (c) $6 \times 10^5 \text{ ms}^{-1}$ (d) $6 \times 10^{-5} \text{ ms}^{-1}$
- 75** What uniform magnetic field applied perpendicular to a beam of electrons moving at $1.3 \times 10^6 \text{ ms}^{-1}$, is required to make the electrons travel in a circular arc of radius 0.35 m? [DUMET 2011]
 (a) 2.1×10^{-5} G (b) 6×10^{-5} T
 (c) 2.1×10^{-5} T (d) 6×10^{-5} G
- 76** Two very long straight parallel wires carry currents i and $2i$ in opposite directions. The distance between the wires is r . At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity v is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is [KCET 2011]
 (a) zero (b) $\frac{3\mu_0}{2\pi} \frac{iqv}{r}$ (c) $\frac{\mu_0}{\pi} \frac{iqv}{r}$ (d) $\frac{\mu_0}{2\pi} \frac{iqv}{r}$
- 77** The torque required to hold a small circular coil of 10 turns, area 1 mm^2 and carrying a current of $(21/44)\text{A}$ in the middle of a long solenoid of 10^3 turns / m carrying a current of 2.5 A with its axis perpendicular to the axis of the solenoid is [KCET 2011]



- (a) $1.5 \times 10^{-6} \text{ N-m}$ (b) $1.5 \times 10^{-8} \text{ N-m}$
 (c) $1.5 \times 10^{+6} \text{ N-m}$ (d) $1.5 \times 10^{+8} \text{ N-m}$

ANSWERS

CHECK POINT 4.1

1. (c)	2. (b)	3. (c)	4. (d)	5. (a)	6. (b)	7. (b)	8. (c)	9. (c)	10. (d)
11. (c)	12. (b)	13. (d)	14. (a)	15. (c)	16. (d)	17. (a)	18. (d)	19. (d)	20. (b)

CHECK POINT 4.2

1. (c)	2. (c)	3. (a)	4. (c)	5. (b)	6. (a)	7. (a)	8. (a)	9. (b)	10. (d)
11. (c)	12. (b)	13. (b)	14. (a)	15. (a)	16. (a)				

CHECK POINT 4.3

1. (b)	2. (d)	3. (b)	4. (c)	5. (d)	6. (c)	7. (c)	8. (d)	9. (d)	10. (b)
11. (b)	12. (a)								

(A) Taking it together

1. (a)	2. (a)	3. (c)	4. (a)	5. (a)	6. (d)	7. (a)	8. (c)	9. (a)	10. (d)
11. (c)	12. (b)	13. (a)	14. (d)	15. (b)	16. (c)	17. (a)	18. (d)	19. (b)	20. (c)
21. (d)	22. (a)	23. (d)	24. (c)	25. (c)	26. (b)	27. (d)	28. (b)	29. (c)	30. (c)
31. (a)	32. (d)	33. (c)	34. (b)	35. (c)	36. (a)	37. (d)	38. (a)	39. (c)	40. (b)
41. (b)	42. (b)	43. (b)	44. (d)	45. (b)	46. (a)	47. (c)	48. (a)	49. (b)	50. (c)
51. (d)	52. (b)	53. (c)	54. (b)	55. (b)	56. (a)	57. (d)	58. (c)	59. (a)	60. (d)
61. (a)	62. (d)	63. (b)	64. (a)						

(B) Medical entrance special format questions

• Assertion and reason

1. (b) 2. (d) 3. (d) 4. (c) 5. (c) 6. (c)

• Statement based questions

1. (d) 2. (c) 3. (b) 4. (b) 5. (a)

• Match the columns

1. (b) 2. (a) 3. (c)

(C) Medical entrances' gallery

1. (d)	2. (b)	3. (a)	4. (a)	5. (d)	6. (d)	7. (c)	8. (a)	9. (a)	10. (a)
11. (d)	12. (a)	13. (b)	14. (a)	15. (a)	16. (a)	17. (c)	18. (b)	19. (a)	20. (d)
21. (b)	22. (c)	23. (d)	24. (b)	25. (c)	26. (c)	27. (a)	28. (b)	29. (b)	30. (c)
31. (c)	32. (a)	33. (d)	34. (c)	35. (a)	36. (d)	37. (b)	38. (b)	39. (b)	40. (b)
41. (d)	42. (c)	43. (d)	44. (b)	45. (c)	46. (a)	47. (d)	48. (c)	49. (c)	50. (a)
51. (d)	52. (c)	53. (d)	54. (b)	55. (d)	56. (a)	57. (a)	58. (c)	59. (b)	60. (a)
61. (d)	62. (a)	63. (c)	64. (a)	65. (d)	66. (d)	67. (c)	68. (c)	69. (d)	70. (b)
71. (b)	72. (a)	73. (c)	74. (b)	75. (c)	76. (a)	77. (b)			

Hints & Explanations

• CHECK POINT 4.1

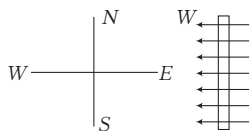
- 3 (c) Magnetic field at a distance r near a long straight current carrying wire is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2i}{r} \Rightarrow B \propto \frac{1}{r}$$

$$\therefore \frac{B_1}{B_2} = \frac{r_2}{r_1} \text{ or } \frac{B}{B_2} = \frac{r/2}{r}$$

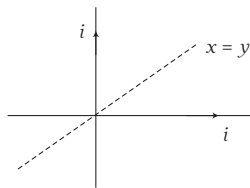
$$\Rightarrow B_2 = 2B$$

- 4 (d) Using Fleming's right-hand rule, the direction of magnetic field is towards west.



- 5 (a) According to right hand screw rule,

Along the line $x = y$, magnetic field due to two wires will be equal and opposite.



- 6 (b) Magnetic field at the centre of a current carrying circular coil of N turns is given by

$$B = \frac{\mu_0 Ni}{2r} \Rightarrow B \propto \frac{1}{r}$$

- 7 (b) Current produced in wire, $i = \frac{q}{t} = \frac{100 \times e}{1}$

$$\therefore i = 100e$$

Magnetic field at the centre of circular path,

$$= \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi \times 100e}{r} = \frac{\mu_0 \times 200e}{4r}$$

$$= \frac{\mu_0 \times 200 \times 1.6 \times 10^{-19}}{4 \times 0.8} = 10^{-17} \mu_0 \text{ T}$$

- 8 (c) The direction of magnetic field produced due to both semicircular parts will be perpendicular to the paper and inwards.

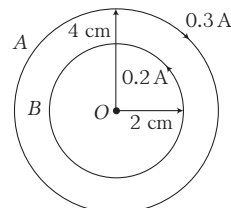
Magnetic induction at the centre O ,

$$B = B_1 + B_2 = \frac{\mu_0 i}{4r_1} + \frac{\mu_0 i}{4r_2} = \frac{\mu_0 i}{4} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

- 9 (c) Magnetic field at the centre of circular coil of N turns is given by

$$B = \frac{\mu_0 Ni}{2r} = \frac{4\pi \times 10^{-7} \times 100 \times 0.1}{2 \times 5 \times 10^{-2}} = 4\pi \times 10^{-5} \text{ T}$$

- 10 (d) The given situation is shown below



At centre O , magnetic field due to inner circular coil,

$$B_1 = \frac{\mu_0 Ni_1}{2r_1} = \frac{\mu_0 10 \times (0.2)}{2 \times (2 \times 10^{-2})} = 50\mu_0 \odot$$

Similarly, at centre O , magnetic field due to outer circular coil,

$$B_2 = \frac{\mu_0 Ni_2}{2r_2} = \frac{\mu_0 10 \times (0.3)}{2(4 \times 10^{-2})} = \frac{75\mu_0}{2} \otimes$$

$$\text{Net magnetic field, } B_O = B_1 - B_2 = 50\mu_0 - \frac{75\mu_0}{2}$$

$$= \frac{100\mu_0 - 75\mu_0}{2} = \frac{25}{2} \mu_0 \text{ Wb/m}^2 \odot$$

- 11 (c) The magnetic field, $B = \frac{\mu_0 Ni}{2R}$ or $B \propto \frac{N}{R}$

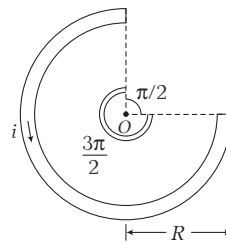
$$\frac{B_2}{B_1} = \frac{N_2}{N_1} \times \frac{R_1}{R_2} = \frac{2N_1}{N_1} \times \frac{R_1}{R_1/2} = 4 \Rightarrow B_2 = 4B_1 = 4B$$

$$B' = 4B$$

- 12 (b) Magnetic field at the centre of arc,

$$B = \frac{\mu_0}{4\pi} \frac{\theta i}{r} = \frac{\mu_0}{4\pi} \times \frac{\pi}{2} \times \frac{i}{R} = \frac{\mu_0 i}{8R}$$

- 13 (d) Magnetic field induction at centre O is given by



$$B = \frac{3}{4} \left(\text{Magnetic field due to whole circle} \right)$$

$$= \frac{3}{4} \left(\frac{\mu_0 i}{2R} \right) = \frac{3\mu_0 i}{8R}$$

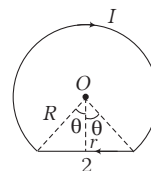
- 14 (a) The magnetic field at the centre O ,

$$B_O = B_1 + B_2$$

$$= \frac{\mu_0 I}{2R} \left[\frac{2\pi - 2\theta}{2\pi} \right] + \frac{\mu_0 I}{4\pi r} [\sin \theta + \sin \theta]$$

$$= \frac{\mu_0 I}{2R} \left[\frac{\pi - \theta}{\pi} \right] + \frac{\mu_0 I}{4\pi (R \cos \theta)} (2 \sin \theta)$$

$$= \frac{\mu_0 I}{2\pi R} [\pi - \theta + \tan \theta]$$



- 15 (c) Magnetic field due to a ring having n turns at a distance x on its axis is given by

$$B = \frac{\mu_0}{2} \cdot \frac{nir^2}{(x^2 + r^2)^{3/2}}$$

$$\therefore B \propto \frac{nr^2}{(x^2 + r^2)^{3/2}}$$

- 16 (d) Magnetic field at the centre of a current carrying coil having current I and radius a is given as

$$B_1 = \frac{\mu_0 I}{2a} \quad \dots(i)$$

Magnetic field on the axis of circular current carrying coil of radius ' a ' and current I at a distance x from centre is given as

$$B_2 = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

Here,

$$x = a$$

$$\therefore B_2 = \frac{\mu_0 I a^2}{2(a^2 + a^2)^{3/2}}$$

$$= \frac{\mu_0 I a^2}{2^{5/2} \cdot a^3}$$

$$= \frac{\mu_0 I}{2^{5/2} \cdot a} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \frac{B_1}{B_2} = \frac{\mu_0 I / 2a}{\mu_0 I / 2^{5/2} a} = \frac{2^{5/2}}{2}$$

$$= 2^{3/2} = 2 \cdot 2^{1/2} = 2\sqrt{2}$$

- 17 (a) According to the question,

$$\frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} = \frac{1}{8} \left(\frac{\mu_0 I}{2R} \right)$$

$$\therefore \frac{R^2}{(x^2 + R^2)^{3/2}} = \frac{1}{8R}$$

or $(x^2 + R^2)^{3/2} = 8R^3$

or $x^2 + R^2 = 4R^2$

$$\therefore x = \sqrt{3}R$$

- 18 (d) Inside the rod, $B \propto r$ and outside it, $B \propto \frac{1}{r}$.

Thus, graph (d) is correct option.

- 20 (b) Mean radius of toroid,

$$r = \frac{25 + 26}{2} = 25.5 \text{ cm} = 0.255 \text{ m}$$

Total number of turns, $N = 3500$

Current, $I = 1 \text{ A}$

$$\text{Number of turns per unit length, } n = \frac{N}{2\pi r} = \frac{3500}{2\pi \times 0.255}$$

Magnetic field inside the core of the toroid,

$$B = \mu_0 n I = 4\pi \times 10^{-7} \times \frac{3500}{2\pi \times 0.255} \times 11$$

$$= 3.0 \times 10^{-2} \text{ T}$$

• CHECK POINT 4.2

- 1 (c) For stationary electron, $v = 0$

$$\therefore F = Bqv \sin \theta = Bq \times 0 \times \sin \theta = 0, \text{ hence}$$

on a static charge, there is no effect of magnetic field, so electron remains stationary.

- 2 (c) Path is circular when $\mathbf{v} \perp \mathbf{B}$ or $\mathbf{v} \cdot \mathbf{B} = 0$.

- 3 (a) Radius of circular path, $r = \frac{mv}{qB}$

$$\text{For electron, } r = \frac{mv}{eB}$$

$$\therefore \frac{e}{m} = \frac{v}{Br}$$

- 4 (c) Radius of circular path of charged particle,

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

Since, electron and proton both have same momentum, therefore, the circular path of both will have the same radius.

- 5 (b) As, $\mathbf{F}_e = \mathbf{F}_m$, $q\mathbf{E} = q(\mathbf{v} \times \mathbf{B})$

$$\therefore \mathbf{E} = \mathbf{v} \times \mathbf{B} \Rightarrow |\mathbf{E}| = v |\mathbf{B}|$$

Therefore, the two fields ($\mathbf{E}$ and $\mathbf{B}$) are perpendicular.

- 6 (a) Radius, $r = \frac{\sqrt{2Km}}{Bq}$ or $r \propto \frac{\sqrt{m}}{q}$

$$\left(\frac{\sqrt{m}}{q} \right)_\alpha : \left(\frac{\sqrt{m}}{q} \right)_p : \left(\frac{\sqrt{m}}{q} \right)_d = \frac{\sqrt{4}}{2} : \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} = 1 : 1 : \sqrt{2}$$

Hence, $r_\alpha = r_p < r_d$.

- 7 (a) Work done by magnetic force is zero, hence according to work-energy theorem,

$$\text{Change in kinetic energy} = \text{Work done} = 0$$

So, kinetic energy remains constant.

- 8 (a) Radius of circular path of proton,

$$r = \frac{\sqrt{2Km}}{Bq} \text{ or } K \propto \frac{q^2}{m} \quad (\text{for same radius})$$

$$\Rightarrow K_\alpha = K \left(\frac{q'}{q} \right) \left(\frac{m}{m'} \right) = 8(2) \left(\frac{1}{4} \right) = 4 \text{ eV}$$

- 9 (b) When charged particle moving in circular path enters into a region of magnetic field, then time period of charged particle is given by

$$T = \frac{2\pi m}{qB}$$

$$\therefore T \propto m$$

$$\therefore m_p > m_e$$

$\therefore$ Time period of proton > Time period of electron.

- 10 (d) Time period of charged particle,

$$T = \frac{2\pi m}{Bq}$$

Hence, T is independent of r .

11 (c) Radius of particle, $r = \frac{\sqrt{2Km}}{Bq}$

$\therefore$ Area bounded by the path, $A = \pi r^2 = \frac{2\pi Km}{B^2 q^2}$ or $A \propto K$

- 12 (b) Velocity is in XY-plane and magnetic field along Z-axis. Therefore, path of the electron in magnetic field will be a circle. Magnetic force cannot change the speed of a particle but direction of its motion continuously changes. Hence, path of electron will change.

- 13 (b) Radius of helical path taken by proton beam,

$$r = \frac{mv \sin 60^\circ}{Bq} = \frac{(1.67 \times 10^{-27}) \times (4 \times 10^5) \sin 60^\circ}{0.3 \times 1.6 \times 10^{-19}}$$

$$= 0.012 \text{ m}$$

- 14 (a) For charged particle on circular path,

$$\frac{mv^2}{r} = qvB$$

$$\therefore r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB}$$

For proton, $R_p = \frac{\sqrt{2m_p E}}{qB} \quad \dots(i)$

For deuteron, $R_d = \frac{\sqrt{2m_d E}}{qB} \quad \dots(ii)$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{R_d}{R_p} = \sqrt{\frac{m_d}{m_p}} = \sqrt{2} \quad [\because m_d = 2m_p]$$

$$\therefore R_d = \sqrt{2} R_p$$

- 15 (a) Radius of the circular path, $r = \frac{mv}{qB}$

$$\Rightarrow \frac{r_1}{r_2} = \frac{m_1 v_1}{m_2 v_2} \times \frac{q_2}{q_1} = \frac{1 \times 2}{1 \times 3} \times \frac{2}{1} = \frac{4}{3}$$

- 16 (a) Frequency of revolution, $f = \frac{Bq}{2\pi m}$

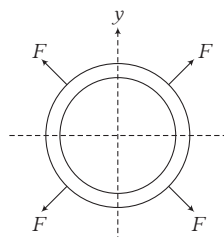
i.e. $f \propto \frac{q}{m}$

As q is same for all given particles, $f \propto \frac{1}{m}$. Mass of Li^+ is maximum, therefore its frequency will be minimum.

• CHECK POINT 4.3

- 1 (b) Applying Fleming's left-hand rule, if magnetic field is perpendicular to paper inwards and current in the loop is clockwise, the magnetic force F on each element of loop is radially outwards and loop has a tendency to expand outwards.

Also, when a current carrying loop is placed in uniform magnetic field, then net force on it is zero and loop cannot have translation motion.



- 2 (d) The parallel wires carrying currents in the same direction attract each other because magnetic forces on the two wires act towards each other.

- 3 (b) Two parallel conductor A and B of equal length carry currents I and $10I$ respectively in the same direction, then A and B attract each other with same force.

- 4 (c) Force per unit length,

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{d}$$

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i^2}{d}$$

$$(\because i = i_1 = i_2)$$

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \cdot \frac{i^2}{d}$$

$$(\text{attractive})$$

- 5 (d) Force between two conductors, $F \propto \frac{I_1 I_2}{d}$

When I_1 is changed to $2I_1$ and d is changed to $3d$.

$$\therefore F' \propto \frac{(2I_1)(I_2)}{3d} \propto \frac{2}{3} F$$

As, direction of current is reversed, so $F' = -2F/3$.

- 6 (c) Force per unit length, $\frac{F}{l} = \frac{\mu_0}{2\pi} \cdot \frac{i_A i_B}{r}$

Given, $i_A = 2i_B$

$$\therefore (0.004) = \frac{2 \times 10^{-7}}{0.01} (2i_B^2)$$

$$\therefore i_B = 10 \text{ A}$$

- 7 (c) Torque, $\tau_{\max} = NiAB = 1 \times i \times \pi r^2 \times B$

$$\left(\because 2\pi r = L \Rightarrow r = \frac{L}{2\pi} \right)$$

$$\therefore \tau_{\max} = \pi i \left(\frac{L}{2\pi} \right)^2 B = \frac{L^2 i B}{4\pi}$$

- 8 (d) Torque acting on the coil is given by

$$\tau = NiBA \sin \theta$$

As, magnetic field is normal to the plane of coil.

So, $\theta = 0^\circ$

$$\therefore \tau = 0$$

- 9 (d) The pole pieces of the magnet used in a pivoted coil galvanometer are cylindrical surfaces of a horse-shoe magnet.

- 10 (b) The electrical current in moving coil galvanometer

$$i = \frac{k\theta}{NAB}$$

$$\Rightarrow i \propto \theta$$

- 11 (b) We have sensitivity of a moving coil galvanometer

$$S = \frac{NAB}{k}$$

where, k is the torsional constant of its suspension.

In order to increase the sensitivity of a moving coil galvanometer, one should decrease the torsional constant of its suspension.

12 (a) Sensitivity, $S = \frac{\theta}{i}$

$$\frac{S_A}{S_B} = \frac{i_B}{i_A} \Rightarrow \frac{S_A}{S_B} = \frac{5}{3} \Rightarrow S_A > S_B$$

(A) Taking it together

1 (a) In Biot-Savart's law, magnetic field $\mathbf{B} \parallel i d\mathbf{l} \times \mathbf{r}$ and $i d\mathbf{l}$ due to flow of electron is in opposite direction of $\mathbf{v}$, so by direction of cross product of two vectors, $\mathbf{B} \perp \mathbf{v}$

2 (a) The charged particle undergoes acceleration as

(i) speeds up between the dees because of the oscillating electric field and

(ii) speed remain the same inside the dees because of the magnetic field but direction changes continuously.

3 (c) $\therefore$ Force, $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

$\therefore$ A proton is moving along the negative direction of X-axis in a magnetic field directed along the positive direction of Y-axis. Therefore, according to Fleming's left hand rule, force on proton in magnetic field, will act along negative direction of Z-axis.

So, the proton will be deflected along the negative direction of Z-axis

5 (a) The direction of magnetic moment of circular loop of radius R is placed in the XY-plane is along z-direction and its magnitude is given by $M = I(\pi R^2)$. When half of the loop with $x > 0$ is bent, such that it now lies in the YZ-plane, the magnitudes of magnetic moment of each semi-circular loop of radius R lie in the XY-plane and the YZ-plane having magnitude $M' = I(\pi R^2)/4$ each and the direction of magnetic moments are along z-direction and x-direction, respectively.

Their resultant, $M_{\text{net}} = \sqrt{M'^2 + M'^2} = \sqrt{2} M' = \sqrt{2} I(\pi R^2)/4$

So, $M_{\text{net}} < M$ or M diminishes.

6 (d) As, maximum energy of particle in cyclotron,

$$E_{\text{max}} = \frac{q^2 B^2}{2m} r_0^2 \Rightarrow E_{\text{max}} \propto \frac{q^2}{m} \left[\because \frac{B^2}{2} r_0^2 = \text{constant} \right]$$

Here, $q_d = q_p$, $m_d = 2m_p$ and $E_{\text{max}}(\text{deuteron}) = 20 \text{ MeV}$

$$(E_p)_{\text{max}} = (E_d)_{\text{max}} \left(\frac{q_p}{q_d} \right) \left(\frac{m_d}{m_p} \right) = 20 \times 2 = 40 \text{ MeV}$$

7 (a) For hollow metallic cylinder, magnetic field inside is zero while outside it the magnetic field is inversely proportional to distance from centre of cylinder.

So, variation is correctly shown by graph (a).

8 (c) We have, $B_1 = \frac{\mu_0 I}{2r}$, $B_2 = \frac{\mu_0 I}{4r}$

$$\therefore \frac{B_1}{B_2} = 2$$

9 (a) Magnetic force on wire B from both the wires is towards A.

10 (d) Given, $2\pi R = 2.0 \text{ m}$

$$\therefore R = \frac{1.0}{\pi} \text{ m}$$

Magnetic moment, $M = iA = (i)(\pi R^2)$

$$= (1)(\pi) \left(\frac{1.0}{\pi} \right)^2 = \frac{1}{\pi} \text{ A-m}^2$$

11 (c) Magnetic field produced at the centre of the orbit,

$$B = \frac{\mu_0}{4\pi} \frac{2\pi i}{r}$$

$$\text{Now, } i = \frac{q}{t} = qf$$

$$\begin{aligned} \therefore B &= \frac{\mu_0}{4\pi} \frac{2\pi (qf)}{r} \\ &= \frac{4\pi \times 10^{-7} \times 2 \times 3.14 \times 1.6 \times 10^{-19} \times 6.6 \times 10^{15}}{4\pi \times 0.53 \times 10^{-10}} \\ &= 12.5 \text{ Wb m}^{-2} \end{aligned}$$

12 (b) Force or tension in the loop, $F = Bil$

$$\Rightarrow F \propto B$$

If B is doubled, then tension in the loop is double.

13 (a) In 1st case

$$l = 2\pi r_1 \quad \dots (i)$$

$$\text{At centre, } B = \frac{\mu_0 i}{2r_1}$$

In 2nd case

$$l = 2 \times \pi r_2 \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$2 \times 2\pi r_2 = 2\pi r_1 \Rightarrow r_2 = r_1/2$$

$$\therefore B' = \frac{2 \times \mu_0 i}{2r_2} \quad (\because \text{there are 2 turns})$$

$$\Rightarrow \frac{B'}{B} = \frac{2r_1}{r_2} = 4 \Rightarrow B' = 4B$$

14 (d) For a given pitch (x) corresponding to charge particle, we

$$\text{have } \frac{q}{m} = \frac{2\pi v \cos \theta}{Bx} = \text{constant}$$

Since, charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\mathbf{B}$, so LHS of above equation for two particles should be same and of opposite sign. Therefore,

$$\left(\frac{e}{m} \right)_1 + \left(\frac{e}{m} \right)_2 = 0$$

15 (b) When wire is turned into n circular loops, then magnetic field produced is $B' = n^2 B = (3)^2 B_1 = 9 B_1$.

16 (c) As magnetic field and velocity of charge is in same direction, so magnetic force on it is zero. Hence, it falls with acceleration equal to g .

$$17 (a) \text{ Distance} = 2(\text{pitch}) = 2 \left(\frac{2\pi m}{Bq} \right) v \cos \theta$$

$$= \frac{4 \times \pi \times 1.6 \times 10^{-27} \times 2 \times 10^6 \times \cos 30^\circ}{0.05 \times 1.6 \times 10^{-19}} = 4.35 \text{ m}$$

- 18 (d) Velocity vector is parallel to magnetic field vector.

$$\therefore \theta = 0^\circ$$

Hence, force, $F = Bqv \sin 0 = 0$

- 19 (b) We have, $(b-a) = r = \frac{mv}{Bq}$

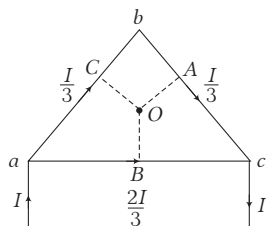
$$\therefore v_{\min} = \frac{(b-a)Bq}{m}$$

- 20 (c) Radius of circular path,

$$r = \frac{mv \cos 60^\circ}{Bq} = \frac{(1.67 \times 10^{-27})(2 \times 10^6) \cos 60^\circ}{0.14 \times 1.6 \times 10^{-19}} = 0.07 \text{ m}$$

$$\text{and } T = \frac{2\pi m}{Bq} = \frac{(2\pi)(1.67 \times 10^{-27})}{0.14 \times 1.6 \times 10^{-19}} = 0.5 \times 10^{-6} \text{ s}$$

- 21 (d) Distribution of current will be as shown below.



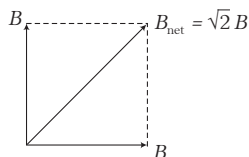
Net magnitude of magnetic field at the centre O is given by

$$\begin{aligned} B_{\text{net}} &= B_{ab} + B_{bc} - B_{ac} \\ &= \frac{\mu_0}{2\pi} \cdot \frac{I/3}{CO} + \frac{\mu_0}{2\pi} \cdot \frac{I/3}{AO} - \frac{\mu_0}{2\pi} \cdot \frac{2I/3}{BO} \\ &= \frac{\mu_0}{2\pi} \cdot \frac{I/3}{r} + \frac{\mu_0}{2\pi} \cdot \frac{I/3}{r} - \frac{\mu_0}{2\pi} \cdot \frac{2I/3}{r} \\ &= 0 \quad [\because AO = BO = CO = r] \end{aligned}$$

- 22 (a) B = (due to circular wire) + (due to straight wire)

$$= \frac{\mu_0 I}{2R} + \frac{\mu_0}{2\pi} \cdot \frac{I}{R} = \frac{\mu_0 I}{2R} \left(1 + \frac{1}{\pi} \right) = \frac{2\mu_0 I}{4R\pi} (1 + \pi)$$

- 23 (d) The magnitude of the resultant magnetic field at the common centre of coil is given as $B_{\text{net}} = \sqrt{2}B$



The ratio of the magnitude of the resultant magnetic field at the common centre and the magnetic field due to one coil alone at common centre.

$$\Rightarrow \frac{B_{\text{net}}}{B_{\text{at centre}}} = \frac{\sqrt{2}B}{B} = \frac{\sqrt{2}}{1}$$

- 24 (c) Magnetic field, $B = \frac{\mu_0 i}{2R} = \frac{\mu_0 (ef)}{2R} = \frac{(\mu_0)(e)\left(\frac{v}{2\pi R}\right)}{R}$

$$\Rightarrow R^2 = \frac{\mu_0 e v}{2\pi B}$$

$$\Rightarrow R^2 \propto \frac{v}{B}$$

$$\Rightarrow R \propto \sqrt{\frac{v}{B}}$$

- 25 (c) Let the length of each wire be L .

For square, length of each side $= L/4$

$$\text{Area of square} = \left(\frac{L}{4}\right)^2 = \frac{L^2}{16}$$

$$\text{For circle, } L = 2\pi r \Rightarrow r = \frac{L}{2\pi}$$

$$\text{Area of circle} = \pi r^2 = \pi \left(\frac{L}{2\pi}\right)^2 = \frac{L^2}{4\pi}$$

As, magnetic moment, $M = iA$

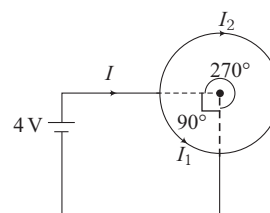
$$\Rightarrow M \propto A$$

$$\therefore \frac{M_{\text{square}}}{M_{\text{circle}}} = \frac{A_{\text{square}}}{A_{\text{circle}}} = \frac{L^2/16}{L^2/4\pi} = \frac{\pi}{4}$$

- 26 (b) At O , due to wire AB and DE , magnetic field will be zero. The combined effect of BC and DF is equivalent to that of an infinitely long wire, i.e. both fields are in same direction.

$$\Rightarrow B_O = \frac{2\mu_0 I}{4\pi r} = \frac{\mu_0 I}{2\pi r}$$

- 27 (d) The given figure is as shown



$$\frac{R_1}{R_2} = \frac{1}{3} \quad [\because \text{Resistance, } R \propto l]$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1} = 3 \quad [\because V = 4V]$$

$$\therefore B_1 = \frac{\mu_0 I_1}{2r} \times \frac{90^\circ}{360} \otimes \text{ and } B_2 = \frac{\mu_0 I_2}{2r} \times \frac{270^\circ}{360}$$

$$\therefore \frac{B_1}{B_2} = \frac{I_1}{I_2} \times \frac{90}{270} = 3 \times \frac{1}{3} = 1$$

$$\text{or } B_1 \otimes = B_2 \odot$$

$\therefore$ Net magnetic field at centre of conductor is zero.

- 28 (b) $F_a = F_{ab} - F_{ac} = \frac{\mu_0 I^2 l}{2\pi d} - \frac{\mu_0 I^2 l}{2\pi (2d)} = \frac{\mu_0 I^2 l}{4\pi d}$



$$F_b = F_{ba} + F_{bc} = \frac{\mu_0 I^2 l}{2\pi d} + \frac{\mu_0 I^2 l}{2\pi d} = \frac{\mu_0 I^2 l}{\pi d}$$

$$F_c = F_{ca} + F_{cb} = \frac{\mu_0 I^2 l}{2\pi(2d)} + \frac{\mu_0 I^2 l}{2\pi d} = \frac{3\mu_0 I^2 l}{4\pi d}$$

On comparing values as F_a , F_b and F_c , we get

$$F_b > F_c > F_a$$

29 (c) $F_1 = \frac{\mu_0}{4\pi} \frac{I^2 \times l}{x}$, towards A and $F_2 = \frac{\mu_0}{4\pi} \frac{2I^2 \times l}{2x}$, towards A

$$\therefore F_1 = F_2$$

30 (c) Force on wire Q due to wire R,

$$F_R = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 20 \times 10}{0.02} \times 0.1$$

$$= 20 \times 10^{-5} \text{ N (towards right)}$$

Force on wire Q due to wire P,

$$F_P = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 30 \times 10}{0.1} \times 0.1$$

$$= 6 \times 10^{-5} \text{ N (towards right)}$$

Net force on Q,

$$F = F_R + F_P = 20 \times 10^{-5} + 6 \times 10^{-5} = 26 \times 10^{-5} \text{ N}$$

$$= 2.6 \times 10^{-4} \text{ N (towards right)}$$

31 (a) Force on the wire, $F = iLB \sin \theta$

$$\sin \theta = \frac{F}{iLB} = \frac{15}{10 \times 1.5 \times 2} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

$\therefore$ The angle between the magnetic field and direction of the current, $\theta = 30^\circ$.

32 (d) If charge q is positive, then due to electric field their velocity will be towards positive X-axis.

Now, $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

where, $\mathbf{v} \rightarrow$ towards $\hat{i}$

and $\mathbf{B} \rightarrow$ towards $\hat{k}$

$\therefore \mathbf{F}_m \rightarrow$ towards $-\hat{j}$ (i.e. - y-direction)

Similarly, we can see with $-q$ charge.

33 (c) For minimum value of B , it is colliding after one pitch. It will collide the target again, if pitch is halved or remains same.

$$p = (v \cos \theta) \left(\frac{2\pi m}{Bq} \right) \text{ or } p \propto \frac{v}{B}$$

For $p/p_0 = 1$ and $\frac{1}{2}$, both options (a) and (b) are correct.

34 (b) $\tau = \mathbf{M} \times \mathbf{B} = IAB$

Magnetic moment is in positive x-direction and the magnetic field in positive y-direction. So, the torque (IAB) must be in positive z-direction.

35 (c) We know that, current, $i = qf$

$$= 1.6 \times 10^{-19} \times 6.6 \times 10^{15} = 10.5 \times 10^{-4} \text{ A}$$

$$\therefore \text{Area, } A = \pi R^2 = 3.14 \times (0.528)^2 \times 10^{-20} \text{ m}^2$$

$$\therefore \text{Magnetic moment, } M = iA$$

$$= 10.5 \times 10^{-4} \times 3.14 \times (0.528)^2 \times 10^{-20}$$

$$\approx 10 \times 10^{-24} \text{ units} = 1 \times 10^{-23} \text{ units}$$

36 (a) All magnetic fields are cancelling each other.

37 (d) Here, diameter of circular path, $y = 2r = 2 \frac{mv_0}{qB_0}$

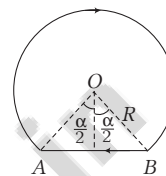
38 (a) Torque acting on the coil, $\tau = NiBA$

$$= 100 \times 2 \times 0.2 \times (0.08 \times 0.1) = 0.32 \text{ N-m}$$

According to Fleming's left hand rule, force on side AD is in upward direction out of the page and force on side BC is in downward direction into the page.

Hence, this torque rotates the side AD out of the page.

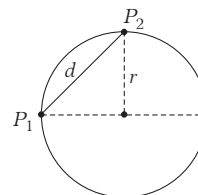
39 (c) From the figure,



$$AB = 2R \sin \frac{\alpha}{2}$$

$$\therefore \text{Force} = IB \left(2R \sin \frac{\alpha}{2} \right) = 2BIR \sin \frac{\alpha}{2}$$

40 (b) Given, time, $t = \frac{\pi m}{Bq} = \frac{T}{2}$ of proton



$$\text{Separation, } d = \sqrt{2}r = \frac{\sqrt{2}mv}{Bq}$$

41 (b) As, $dA = (2\pi r) dr$

$$\therefore dq = \sigma \cdot dA = (2\pi \alpha r^3) \cdot dr$$

$$\text{Current, } i = (dq) f = (dq) \frac{\omega}{2\pi} = (\alpha \omega r^3) dr$$

$$\text{Now, } dB = \frac{\mu_0 i}{2r} = \frac{\alpha \mu_0 \omega r^2 dr}{2}$$

$$\therefore B = \int_0^R dB = \frac{\mu_0 \alpha \omega R^3}{6}$$

42 (b) As, $(mg)(r) = MB_x = i(\pi r^2) B_x$

$$\therefore \text{Current, } i = \frac{mg}{\pi r B_x}$$

43 (b) At the centre of coil 1,

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{2\pi i_1}{r_1} \quad \dots(i)$$

At the centre of coil-2,

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{2\pi i_2}{r_2} \quad \dots(ii)$$

$$\text{But } B_1 = B_2$$

$$\therefore \frac{\mu_0}{4\pi} \frac{2\pi i_1}{r_1} = \frac{\mu_0}{4\pi} \frac{2\pi i_2}{r_2} \quad \text{or} \quad \frac{i_1}{r_1} = \frac{i_2}{r_2}$$

As, $r_1 = 2r_2$

$$\therefore \frac{i_1}{2r_2} = \frac{i_2}{r_2} \quad \text{or} \quad i_1 = 2i_2$$

Now, ratio of potential differences,

$$\frac{V_2}{V_1} = \frac{i_2 \times R_2}{i_1 \times R_1} = \frac{i_2 \times R_2}{2i_2 \times 2R_2} = \frac{1}{4}$$

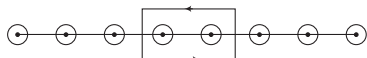
[$\because$ Resistance, $R \propto r$ (radius)]

$$\Rightarrow \frac{V_1}{V_2} = \frac{4}{1}$$

44 (d) Velocity, $v = \frac{E}{B}$ and radius of path,

$$r = \frac{mv}{Bq} = \frac{v}{B \cdot q/m} = \frac{E/B}{B \cdot S} = \frac{E}{B^2 S}$$

45 (b) Applying Ampere's circuital law in the closed loop as shown in figure,



$$2Bl = \mu_0 (\lambda \cdot l)$$

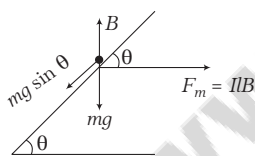
$$\therefore \text{Magnetic field induction, } B = \frac{\mu_0 \lambda}{2}$$

46 (a) We have, $\mathbf{F}_{CAD} = \mathbf{F}_{CD} = \mathbf{F}_{CED}$

$$\therefore \text{Net force on frame} = 3 \mathbf{F}_{CD} \\ = 3 (Bil) = 3 \times 4 \times 2 \times 1 = 24 \text{ N}$$

47 (c) According to the question,

$$mg \sin \theta = IlB \cos \theta$$



$$\therefore \text{Magnitude of magnetic field, } B = \frac{mg}{Il} \tan \theta$$

$$= \frac{mg \tan 60^\circ}{2 \times (2L)} = \frac{\sqrt{3}mg}{4L}$$

48 (a) Let the magnetic field,

$$\mathbf{B} = B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}$$

Applying $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$ two times, we have

$$q[-\hat{j} + \hat{k}] = q[(\hat{i} \times (B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}))] = q[B_2 \hat{k} - B_3 \hat{j}]$$

On comparing two sides, we get

$$B_2 = 1 \text{ and } B_3 = 1$$

$$\text{Further, } q[\hat{i} - \hat{k}] = q[(\hat{j} \times (B_1 \hat{i} + B_2 \hat{j} + B_3 \hat{k}))]$$

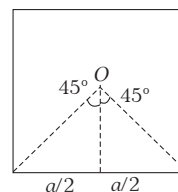
$$= q[-B_1 \hat{k} + B_3 \hat{i}]$$

Again comparing, we get

$$B_1 = 1 \text{ and } B_3 = 1$$

$$\therefore \mathbf{B} = (\hat{i} + \hat{j} + \hat{k}) \text{ Wb/m}^2$$

49 (b) According to question, $l = 2\pi R \Rightarrow R = \frac{l}{2\pi}$



$$\text{and } l = 4a \Rightarrow a = \frac{l}{4}$$

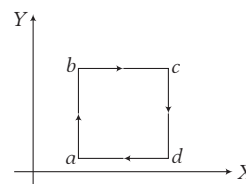
$$B' = \frac{\mu_0 i}{2R} = \frac{\mu_0 i}{2 \cdot \frac{l}{4}} = \frac{2\mu_0 \pi i}{l}$$

$$B = 4 \left[\frac{\mu_0}{4\pi} \cdot \frac{i}{a/2} (\sin 45^\circ + \sin 45^\circ) \right] \\ = \frac{2\sqrt{2} \mu_0 i}{\pi a} = \frac{8\sqrt{2} \mu_0 i}{\pi l}$$

$$\left[\because a = \frac{l}{4} \right]$$

$$\therefore \frac{B}{B'} = \frac{8\sqrt{2}}{\pi^2}$$

50 (c) Here, $\mathbf{F}_{bc} + \mathbf{F}_{da} = 0$



$$\text{Force, } \mathbf{F}_{ab} = IlB_0 \left(1 + \frac{x}{l} \right) = IB_0 l + IB_0 x \quad (\text{towards right})$$

$$\text{Force, } \mathbf{F}_{cd} = IlB_0 \left[1 + \frac{x+l}{l} \right] = 2IB_0 l + IB_0 x \quad (\text{towards left})$$

$$\therefore \mathbf{F}_{\text{net}} = \mathbf{F}_{cd} - \mathbf{F}_{ab} = IB_0 l \quad (\text{towards left})$$

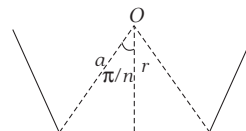
51 (d) At centre, $B = 0$.

As, $x \rightarrow a$, $B \rightarrow 0$ between centre and $x = a$, there will be a maximum value.

Along X -axis for $x > 0$, B_{net} is downwards.

Along X -axis for $x < 0$, B_{net} is upwards.

52 (b) Radius, $r = a \cos \frac{\pi}{n}$



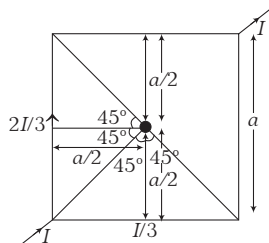
$\therefore$ Magnetic field,

$$B = n \left[\frac{\mu_0}{4\pi} \cdot \frac{i}{a \cos \pi/n} \left(\sin \frac{\pi}{n} + \sin \frac{\pi}{n} \right) \right] \\ = \frac{\mu_0 n i}{2\pi a} \tan \left(\frac{\pi}{n} \right)$$

53 (c) The magnitude of the force acting on the wire,

$$F_{\text{net}} = \sqrt{F^2 + F^2} = \sqrt{2} F = \sqrt{2} IlB$$

54 (b) Magnetic field at O ,



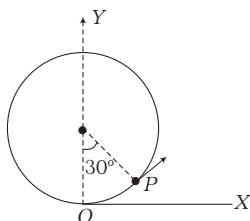
$$B = 2 \left[\frac{\mu_0}{4\pi} \frac{2I/3}{a/2} (\sin 45^\circ + \sin 45^\circ) - \frac{\mu_0}{4\pi} \frac{I/3}{a/2} (\sin 45^\circ + \sin 45^\circ) \right]$$

$$= \frac{\sqrt{2}\mu_0 I}{3\pi a}$$

55 (b) As, $kx_0 = mg = ILB = \left(\frac{E}{R}\right) \cdot LB$

$$\therefore \text{The magnetic field, } B = \frac{mgR}{EL}$$

56 (a) Motion of charged particle in magnetic field ($\mathbf{B} = -2\hat{\mathbf{k}}\text{T}$) is shown below



$$\text{Time period, } T = \frac{2\pi m}{Bq} = \frac{2\pi}{B \cdot \frac{q}{m}} = \frac{2\pi}{2 \cdot \pi} = 1\text{ s}$$

Since, the particle will be at point P after time, $t = \frac{1}{6}\text{ s} = \frac{T}{6}$

Hence, if it is deviated by angle θ , then $\theta = \frac{2\pi}{6} = 60^\circ$.

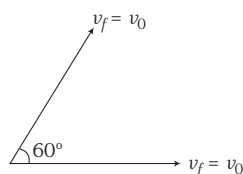
Therefore, velocity of the particle after $t = \frac{1}{6}\text{ s}$, is given by

$$\mathbf{v} = 10 (\cos 60^\circ \hat{\mathbf{i}} + \sin 60^\circ \hat{\mathbf{j}})$$

$$= 10 \left(\frac{\hat{\mathbf{i}}}{2} + \frac{\hat{\mathbf{j}}\sqrt{3}}{2} \right)$$

$$= (5\hat{\mathbf{i}} + 5\sqrt{3}\hat{\mathbf{j}})\text{ m/s}$$

57 (d) Deviation, $\theta = \sin^{-1} \left(\frac{x}{R} \right) = \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = 60^\circ$



Velocity of the particle when it comes out of the field,

$$|\Delta \mathbf{v}| = 2v_0 \sin \frac{\theta}{2}$$

$$= 2v_0 \sin 30^\circ = v_0$$

58 (c) As proton is moving with constant velocity, so acceleration is zero.

When $E \neq 0$, $B = 0$, then proton will move with a acceleration. Hence, this region of space for the motion proton in given condition does not satisfy.

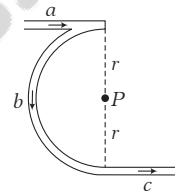
So, $E = 0$ and $B = 0$

When $E = 0$ and $B \neq 0$ but proton is moving in parallel to the direction of magnetic field, then there will be no net force acting on proton.

When, $E \neq 0$ and $B \neq 0$ but electric force and magnetic force cancel each other, then also velocity will remain unchanged.

59 (a) For first figure,

Magnetic field at P due to straight part (a), $B_a = \frac{\mu_0}{4\pi} \cdot \frac{i}{r}$



Magnetic field at P due to circular part (b), $B_b = \frac{\mu_0}{4\pi} \frac{\pi i}{r}$

Magnetic field at P due to straight part (c), $B_c = \frac{\mu_0}{4\pi} \frac{i}{r}$

So, net magnetic field at the centre of first figure,

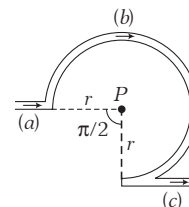
$$B_1 = B_b + B_c - B_a = \frac{\mu_0}{4\pi} \frac{\pi i}{r} \quad \dots(i)$$

For second figure, magnetic field at P due to two straight and one circular part,

$$B_2 = \frac{\mu_0}{4\pi} \frac{\pi i}{r} \quad \dots(ii)$$

For third figure, magnetic field at P due to straight part (a),

$$B_a = 0$$



Magnetic field at P due to circular part (b),

$$B_b = \frac{\mu_0}{4\pi} \frac{(2\pi - \pi/2)i}{r} = \frac{\mu_0}{4\pi} \cdot \frac{3\pi}{2r} \cdot i$$

Magnetic field at C due to straight part (c), $B_c = \frac{\mu_0}{4\pi} \frac{i}{r}$

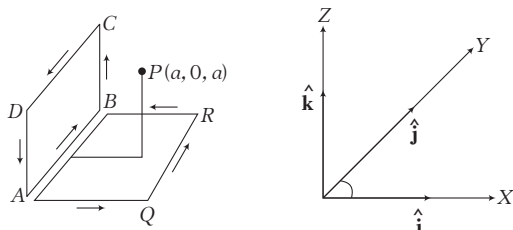
So, net magnetic field at centre of third figure,

$$B_3 = \frac{\mu_0}{4\pi} \cdot \frac{i}{r} \left(\frac{3\pi}{2} - 1 \right) \quad \dots(iii)$$

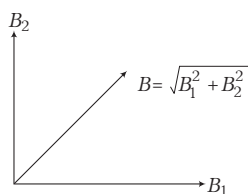
From Eqs. (i), (ii) and (iii), we get

$$B_1 : B_2 : B_3 = \pi : \pi : \left(\frac{3\pi}{2} - 1 \right) \\ = -\frac{\pi}{2} : \frac{\pi}{2} : \left(\frac{3\pi}{4} - \frac{1}{2} \right)$$

- 60 (d)** We can consider the non-planar loop as made by two loops ABCDA and AQRBA as shown in figure.



Magnetic field due to loop ABCDA will be along X-axis and due to loop AQRBA along Z-axis. Magnitude of magnetic field due to both loops will be equal.



Therefore, direction of resultant magnetic field at P will be along $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$.

- 61 (a)** Area vector, $\mathbf{A} = \mathbf{CO} \times \mathbf{OA}$

$$= (-b \cos 37^\circ \hat{i} - b \sin 37^\circ \hat{k}) \times (a \hat{j}) \\ = -\frac{ab}{5} [4(\hat{i} \times \hat{j}) + 3(\hat{k} \times \hat{j})] \\ \left[\because \cos 37^\circ = \frac{4}{5} \text{ and } \sin 37^\circ = \frac{3}{5} \right] \\ = -\frac{ab}{5} [4\hat{k} - 3\hat{i}] \\ = \frac{ab}{5} [3\hat{i} - 4\hat{k}]$$

Magnetic moment, $\mathbf{M} = Ni_0 \mathbf{A}$

$$= \frac{Ni_0 ab}{5} (3\hat{i} - 4\hat{k})$$

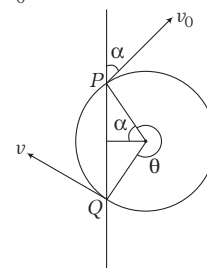
$$\mathbf{B} = B_0 \hat{i}$$

$$\therefore \text{Torque, } \boldsymbol{\tau} = \mathbf{M} \times \mathbf{B} = \frac{Ni_0 ab}{5} (3\hat{i} - 4\hat{k}) \times B_0 \hat{i} \\ = \frac{Ni_0 ab B_0}{5} [3(\hat{i} \times \hat{i}) - 4(\hat{k} \times \hat{i})] \\ = -\frac{4Ni_0 ab B_0}{5} \hat{j}$$

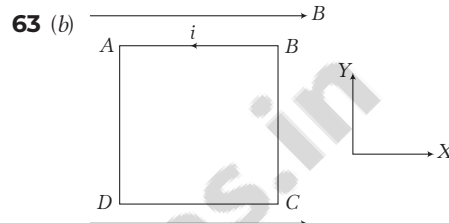
- 62 (d)** From the question, $PQ = 2r \sin \alpha = \frac{2mv_0}{Bq} \sin \alpha$

$$\therefore \theta = (2\pi - 2\alpha)$$

Here, $\alpha = \beta$ and $v = v_0$



$$\therefore t = \frac{\theta}{\omega} = \frac{2(\pi - \alpha)}{(Bq/m)} = \frac{2m(\pi - \alpha)}{Bq}$$



Deflecting torque due to current,

$$\tau_1 = (niA)B \sin 90^\circ = niL^2 B$$

Restoring torque,

$$\tau_2 = \text{force} \times \text{perpendicular distance} = Mg \times \frac{L}{2}$$

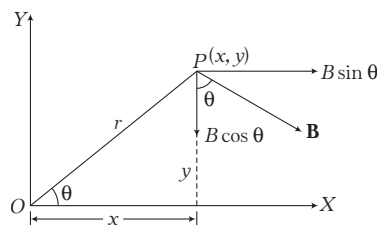
$\Rightarrow$

$$\tau_1 \geq \tau_2 \\ niL^2 B \geq Mg \frac{L}{2} \Rightarrow B \geq \frac{Mg}{2niL}$$

$$\therefore B_{\min} = \frac{Mg}{2niL}$$

Note $\mathbf{M}$ is along + Z-axis and $\mathbf{B}$ is along + X-axis. The magnetic moment vector $\mathbf{M}$ tries to align along $\mathbf{B}$ through smaller angle to have minimum potential energy, so coil rotates clockwise about line BC.

- 64 (a)** In the $z = 0$ plane, the situation is as follows



Here, $P(x, y)$ is the point and $r = \sqrt{x^2 + y^2}$

Magnetic field at P is perpendicular to OP, as it is shown in figure.

$$\text{So, } \mathbf{B} = B \sin \theta \hat{i} - B \cos \theta \hat{j}$$

$$\text{Here, } B = \frac{\mu_0 I}{2\pi r}$$

$$\sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}$$

$$\therefore \mathbf{B} = \frac{\mu_0 I}{2\pi r} \left(\frac{y}{r} \hat{i} - \frac{x}{r} \hat{j} \right) = \frac{\mu_0 I}{2\pi} \frac{(y \hat{i} - x \hat{j})}{(x^2 + y^2)}$$

(B) Medical entrance special format question

• Assertion and reason

1 (b) To the right of B , both fields are downwards and to the left of A , both fields are upwards. At mid-point between wires A and B , magnetic field due to current carrying wire A is equal and opposite to the magnetic field due to current carrying wire B . Hence, they cancel to each other.

2 (d) Velocity will change but speed will remain constant. Hence, momentum will change.

3 (d) As, $r = \frac{\sqrt{2Km}}{Bq}$ or $r \propto \frac{\sqrt{m}}{q}$

$$\left(\frac{\sqrt{m}}{q}\right)_\alpha = \frac{\sqrt{4}}{2} = 1 \text{ and } \left(\frac{\sqrt{m}}{q}\right)_d = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\therefore r_d > r_\alpha$$

Further, $\left(\frac{q}{m}\right)_\alpha = \frac{2}{4} = \frac{1}{2}$ and $\left(\frac{q}{m}\right)_d = \frac{1}{2}$

$$\therefore \left(\frac{q}{m}\right)_\alpha = \left(\frac{q}{m}\right)_d$$

4 (c) Acceleration of charged particle in uniform field is non-zero but speed remains constant. In given condition, particle will rotate in a circular path in xy -plane. Therefore, if velocity is $v_0 \hat{i}$ at some instant, then it can have the value $v_0 \hat{j}$ at other instant also.

5 (c) Magnetic force can change the velocity but not the speed. Speeds at two instants must be same.

6 (c) Magnetic force of attraction on lower wire is upwards while its weight is downwards. At a certain distance x , these two forces are equal. If the lower wire is displaced upwards from this position, then magnetic force will increase but weight will remain same. Therefore, net force is upwards or equilibrium is unstable.

• Statement based questions

1 (d) The magnetic force on an electron, projected with uniform velocity along the axis of a long current carrying solenoid, $F = evB \sin 180^\circ = 0$ or $F = evB \sin 0^\circ = 0$ as magnetic field and velocity are parallel. So, the electron will continue to move with uniform velocity along the axis of the solenoid.

2 (c) Radius, $r = \frac{\sqrt{2mk}}{qB}$

$$\therefore r \propto \frac{\sqrt{m}}{q}$$

$$\therefore r_{H^+} : r_{He^+} : r_{O^{2+}} = \frac{\sqrt{1}}{1} : \frac{\sqrt{4}}{1} : \frac{\sqrt{16}}{2} = 1 : 2 : 2$$

3 (b) Here, $N = 1$, $A = 4 \times 10^{-2} \text{ m}^2$

$$B = 5 \times 10^{-2} \text{ Wb/m}^2$$

$$i = 5 \times 10^{-3} \text{ A}, \theta = 0.2 \text{ rad}, K = ?, I_s = ?$$

We have, $i = \frac{K}{NAB} \theta$

$$\Rightarrow K = \frac{NABi}{\theta} = \frac{1 \times 4 \times 10^{-2} \times 5 \times 10^{-2} \times 5 \times 10^{-3}}{0.2} = 5 \times 10^{-5} \text{ N-m rad}^{-1}$$

Current sensitivity,

$$I_s = \frac{NBA}{K} = \frac{1 \times 5 \times 10^{-2} \times 4 \times 10^{-2}}{5 \times 10^{-5}} = 40 \text{ rad A}^{-1}$$

4 (b) It may or may not take a circular shape depending upon the direction of $\mathbf{B}$.

5 (a) If a positive charged particle is rotating in a circle in clockwise direction, then equivalent current is also clockwise. Therefore, $\mathbf{B}$ at centre and $\mathbf{M}$ produced by motion of charged particle, both are inwards. Hence, they are parallel. But at some other point they may not be parallel.

• Match the columns

1 (b) As, $r = \frac{\sqrt{2Km}}{Bq} \Rightarrow r \propto \frac{\sqrt{m}}{q}$

$\Rightarrow r_d > r_\alpha$ (towards left) and $r_e < r_{Cl^-}$ (towards right)
Hence, $A \rightarrow q$; $B \rightarrow p$; $C \rightarrow s$; $D \rightarrow r$.

2 (a) As, $r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2Km}}{Bq} \Rightarrow \omega = \frac{Bq}{m}$

Hence, $A \rightarrow p$; $B \rightarrow q$; $C \rightarrow p$; $D \rightarrow q$.

3 (c) Magnetic field over the loop from the straight wire is perpendicular to paper inwards.

Force, $\mathbf{F} = i(d\mathbf{l} \times \mathbf{B})$

Hence, $A \rightarrow p$; $B \rightarrow q$; $C \rightarrow q,s$; $D \rightarrow q,s$.

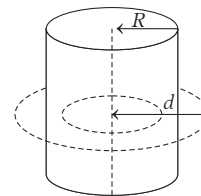
(C) Medical entrances' gallery

1 (d) Given, $l = 50 \text{ cm} = 0.5 \text{ m}$, $N = 100$ turns and $I = 2.5 \text{ A}$

$\therefore$ Magnetic field at the centre of solenoid,

$$B = \mu_0 n I = \mu_0 \left(\frac{N}{l}\right) \cdot I = 4\pi \times 10^{-7} \times \frac{100}{0.5} \times 2.5 = 6.28 \times 10^{-4} \text{ T}$$

2 (b) The cylinder can be considered to be made from concentric circles of radius R .



(i) The magnetic field at a point outside cylinder, i.e. $d > R$.

From Ampere's circuital law, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$

$$\Rightarrow B \int dl = \mu_0 I$$

$$B(2\pi d) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi d} \Rightarrow B \propto \frac{1}{d}$$

where, μ_0 = permeability of free space.

(ii) The magnetic field at surface, i.e. $d = R$,

$$B = \frac{\mu_0 I}{2\pi R}$$

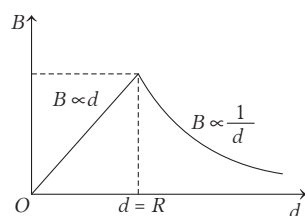
(iii) The current for a point inside the cylinder is given by
 I' = current per unit cross-sectional area of cylinder $\times$
 cross-section of loop

$$= \frac{I}{\pi R^2} \pi d^2 = \frac{Id^2}{R^2}$$

$\therefore$ Magnetic field at a point inside cylinder,

$$B = \frac{\mu_0 I'}{2\pi d} = \frac{\mu_0 Id^2}{2\pi R^2 d} = \frac{\mu_0 I}{2\pi R^2} d \Rightarrow B \propto d$$

So, the variation of magnetic field can be plotted as



3 (a) The magnetic field within the turns of toroid, $B = \frac{\mu_0 NI}{2\pi r}$

where, N = number of turns,

I = current in loops

and r = radius of each turn.

Given, $N_1 = 200$, $N_2 = 100$, $r_1 = 40$ cm, $r_2 = 20$ cm and current I is same, i.e. $I_1 = I_2 = I$, then

$$\frac{B_1}{B_2} = \frac{\mu_0 N_1 I}{2\pi r_1} \times \frac{2\pi r_2}{\mu_0 N_2 I}$$

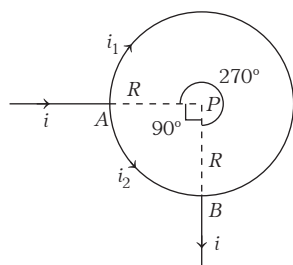
Substituting the given values in the above equation, we get

$$\begin{aligned} \frac{B_1}{B_2} &= \left(\frac{N_1}{N_2} \right) \left(\frac{r_2}{r_1} \right) \\ &= \left(\frac{200}{100} \right) \left(\frac{20}{40} \right) = 2 \times \frac{1}{2} = 1 \end{aligned}$$

$\therefore B_1 : B_2 = 1 : 1$

4 (a) The magnetic field at the centre of an arc subtended at an angle θ is given by

$$B = \frac{\mu_0 i}{2R} \times \frac{\theta}{2\pi}$$



Then, the magnetic field due to larger arc AB,

$$B_1 = \frac{\mu_0 i}{2R} \times \frac{270}{2\pi} \quad \dots(i)$$

which acts in inward direction as per right hand thumb rule.

And magnetic field due to smaller arc AB,

$$B_2 = \frac{\mu_0 i}{2R} \times \frac{90}{2\pi} \quad \dots(ii)$$

which acts in outward direction.

The resultant magnetic field, $B_R = B_1 + B_2$

$$= -\frac{\mu_0 i \times 270}{4\pi R} + \frac{\mu_0 i \times 90}{4\pi R} \quad \dots(iii)$$

[from Eqs. (i) and (ii)]

which acts in inward direction as $B_1 > B_2$.

Two arcs can also be seen as the two resistances in parallel combination.

So, the potential across them will be same, i.e.

$$\begin{aligned} V_1 &= V_2 \\ i_1 R_1 &= i_2 R_2 \quad \dots(iv) \end{aligned}$$

where, R_1 and R_2 = resistance of respective segments.

The wire is uniform,

$$\therefore \frac{R_1}{R_2} = \frac{L_1}{L_2} = \frac{R \times 270}{R \times 90} \quad [\because \text{length of arc} = \text{radius} \times \text{angle}]$$

$$\Rightarrow \frac{R_1}{R_2} = 3 \Rightarrow \frac{R_2}{R_1} = \frac{1}{3} \quad \dots(v)$$

From Eq. (iv), we get

$$\Rightarrow \frac{i_1}{i_2} = \frac{R_2}{R_1} = \frac{1}{3} \quad \text{or} \quad 3i_1 = i_2 \quad \dots(vi)$$

From Eqs. (iii) and (v), we get

$$\begin{aligned} B_R &= \frac{\mu_0}{4\pi R} (-270i_1 + 90i_2) = \frac{\mu_0}{4\pi R} [-270i_1 + 90(3i_1)] \\ &= \frac{\mu_0}{4\pi R} (-270i_1 + 270i_1) = 0 \end{aligned}$$

5 (d) The centripetal force required for circular motion is provided by magnetic force.

$$\Rightarrow \frac{mv_p^2}{r} = Bqv_p$$

$$\Rightarrow r = \frac{mv_p}{qB} \quad \dots(i)$$

where, v_p = perpendicular velocity of particle and q = charge on particle.

As, momentum, $p = mv_p$

$$\therefore r = \frac{p}{qB} \quad [\text{from Eq. (i)}]$$

According to the question, momenta of both particles are same.

$$\Rightarrow r \propto \frac{1}{q}$$

For ionised hydrogen atom, $q_H = e$

and for α -particle, $q_\alpha = 2e$

$$\Rightarrow \frac{r_H}{r_\alpha} = \frac{q_\alpha}{q_H} = \frac{2e}{e} = \frac{2}{1} \quad \text{or} \quad 2:1$$

- 6 (d) When a charged particle of charge (q) is released from rest ($u = 0$) in the uniform magnetic field (B), then magnetic force on the charged particle, $F = Bqu \sin \theta = 0$ because $u = 0$. Therefore, the charge particle will not move in circular path. In the magnetic field, force on charged particle always acts in perpendicular direction to the direction of velocity of charged particle, therefore, work done by magnetic field on charge particle is zero. Hence, both Assertion and Reason are incorrect.

- 7 (c) Given, radius of identical circular loops,

$$R = 10 \text{ cm} = 10^{-1} \text{ m}$$

$$\text{Current, } I = \frac{7}{2} \text{ A}$$

Distance from both circular loops at point P ,

$$x_1 = x_2 = x = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

From figure, according to Maxwell's right hand thumb rule, it is clear that magnetic field will be in same direction by both the coils,

$$\begin{aligned} \text{i.e. } B &= B_1 + B_2 = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}} + \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}} \\ &= \frac{\mu_0 IR^2}{(R^2 + x^2)^{3/2}} = \frac{\mu_0 \times 7/2 \times (10^{-1})^2}{[(0.1)^2 + (0.05)^2]^{3/2}} \\ &= \frac{56}{\sqrt{5}} \mu_0 \text{ T} \end{aligned}$$

- 8 (a) Given, velocity of proton, $\mathbf{v} = 2\hat{\mathbf{i}}$

Magnetic field, $\mathbf{B} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \mu\text{T}$

Electric field, $\mathbf{E} = 10\hat{\mathbf{i}} \mu\text{Vm}^{-1}$

Applied Lorentz force on the proton,

$$\begin{aligned} \mathbf{F} &= q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})] \\ &= 1.6 \times 10^{-19} [10 \times 10^{-6} \hat{\mathbf{i}} + 2\hat{\mathbf{i}} \times (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \times 10^{-6}] \\ &= 1.6 \times 10^{-19} \times 10^{-6} [10\hat{\mathbf{i}} + 6\hat{\mathbf{k}} - 8\hat{\mathbf{j}}] \\ \mathbf{F} &= 1.6 \times 10^{-25} [10\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 6\hat{\mathbf{k}}] \text{ N} \end{aligned}$$

$\therefore$ Acceleration of proton,

$$\begin{aligned} \mathbf{a} &= \frac{\mathbf{F}}{m_p} \quad (\because m_p = 1.6 \times 10^{-27} \text{ kg}) \\ &= \frac{1.6 \times 10^{-25} [10\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 6\hat{\mathbf{k}}]}{1.6 \times 10^{-27}} \end{aligned}$$

$$\mathbf{a} = 100 [10\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 6\hat{\mathbf{k}}]$$

$$\begin{aligned} \therefore a &= |\mathbf{a}| = 100 \sqrt{10^2 + 8^2 + 6^2} \\ &= 100 \times 14.14 = 1414 \text{ ms}^{-2} \approx 1400 \text{ ms}^{-2} \end{aligned}$$

- 9 (a) When an electron enters into a uniform magnetic field in a direction perpendicular to the direction of magnetic field, then a magnetic force acts on the electron in perpendicular direction of both direction of magnetic field and direction of velocity of electron and direction of force can be determined by Fleming's left hand rule.

We know that, when direction of force on a particle is in perpendicular direction to the direction of velocity, then

particle moves in uniform circular motion. Therefore, electron moving perpendicular to magnetic field ($\mathbf{B}$) will perform circular motion.

Hence, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

- 10 (a) Given, speed of proton, $v = 4.5 \times 10^5 \text{ ms}^{-1}$

Electrostatic force between two protons,

$$F_e = k \frac{e^2}{r^2} \quad \dots(i)$$

Magnetic field produced due to moving proton of speed v ,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{ev}{r^2} \quad \dots(ii)$$

$\therefore$ Magnetic force on the proton,

$$F_m = B ev = \frac{\mu_0}{4\pi} \cdot \frac{ev}{r^2} \cdot ev \quad [\text{From Eq. (ii)}]$$

$$F_m = \frac{\mu_0}{4\pi} \cdot \frac{e^2 v^2}{r^2} \quad \dots(iii)$$

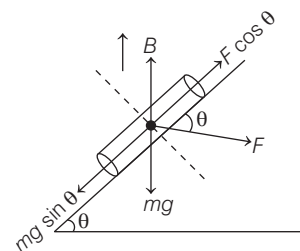
From Eqs. (i) and (iii), we get

$$\begin{aligned} \frac{F_e}{F_m} &= \frac{\frac{ke^2}{r^2}}{\frac{\mu_0}{4\pi} \cdot \frac{e^2 v^2}{r^2}} = \frac{k \cdot 4\pi}{v^2 \cdot \mu_0} \\ \frac{F_e}{F_m} &= \frac{9 \times 10^9 \times 4 \times 3.14}{(4.5 \times 10^5)^2 \times 4\pi \times 10^{-7}} = 4.4 \times 10^5 \end{aligned}$$

- 11 (d) As, the system is in equilibrium,

$$\therefore \Sigma F_x = 0$$

According to the question,



$$mg \sin \theta = F \cos \theta \quad \dots(i)$$

where, F is the magnitude of force experienced by the rod when placed in a magnetic field and current I is flowing through it.

But the force experienced by the given rod in a uniform magnetic field,

$$F = ILB$$

$\therefore$ Eq. (i) becomes

$$mg \sin \theta = ILB \cos \theta$$

$$\Rightarrow I = \frac{mg \sin \theta}{LB \cos \theta} = \frac{mg}{LB} \tan \theta$$

$$I = \left(\frac{m}{L} \right) \frac{g \tan \theta}{B} \quad \dots(ii)$$

$$\text{Here, } \frac{m}{L} = 0.5 \text{ kg m}^{-1},$$

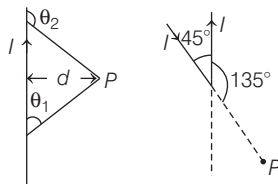
$$g = 9.8 \text{ ms}^{-2}, \theta = 30^\circ,$$

$$B = 0.25 \text{ T}$$

Substituting the given values in Eq. (ii), we get

$$I = \frac{0.5 \times 9.8}{0.25} \tan 30^\circ = \frac{0.5 \times 9.8}{0.25} \times \frac{1}{\sqrt{3}} = 11.32 \text{ A}$$

$$12 \text{ (a) } \therefore B \text{ (at } P) = \frac{\mu_0 I}{4\pi d} (\cos \theta_1 - \cos \theta_2)$$



In given case, $d = R \sin 45^\circ = R/\sqrt{2}$

$$\theta_1 = 135^\circ, \theta_2 = 180^\circ$$

$$\therefore B \text{ (at } P) = \frac{\mu_0 I}{4\pi(R/\sqrt{2})} [\cos 135^\circ - \cos 180^\circ]$$

$$= \frac{\mu_0 I}{4\pi R} \sqrt{2} \left[\frac{-1}{\sqrt{2}} - (-1) \right]$$

$$= \frac{\mu_0 I}{4\pi R} \sqrt{2} \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)$$

$$\Rightarrow B \text{ (at } P) = \frac{\mu_0 I}{4\pi R} (\sqrt{2} - 1) \text{ T}$$

$$13 \text{ (b) Here, } dl = dx = 1 \text{ cm} = 10^{-2} \text{ m,}$$

$$i = 10 \text{ A, } r = 0.5 \text{ m}$$

$$\therefore dB = \frac{\mu_0}{4\pi} \cdot \frac{i(dl \times r)}{r^3}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{idl}{r^2} (\hat{i} \times \hat{j})$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{idl}{r^2} \hat{k}$$

$$= \frac{10^{-7} \times 10 \times 10^{-2}}{(0.5)^2} \hat{k} = 4 \times 10^{-8} \hat{k} \text{ T}$$

$$14 \text{ (a) Magnetic moment,}$$

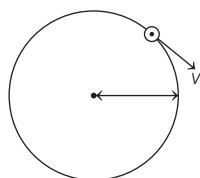
$$M = NiA$$

where, N = number of turns in the current loop

and i = current.

Since, the orbiting electron behaves as a current loop of current i , we can write

$$i = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$



where, A = area of the loop = πr^2

$$\Rightarrow M = (1) \left(\frac{ev}{2\pi r} \right) (\pi r^2) = \frac{evr}{2}$$

$$15 \text{ (a) The torque acting on a current carrying loop is given by}$$

$$\tau = MB \cos \theta$$

$$= NiAB \cos \theta$$

$$[\because M = NiA]$$

where, N = number of turns,

i = current of loop,

A = area of loop

and B = magnetic field.

Thus, torque does not depend on shape of loop.

$$16 \text{ (a) According to Biot-Savart's law, the magnetic induction at a point to a current carrying element } i\delta l \text{ is given by}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{i\delta l \sin \theta}{r^2}$$

Directed normal to plane containing δl and r , θ being angle between δl and r .

Field due to semicircular arc

Now, angle between a current element δl of semicircular arc and the radius vector of the element to point c is $\pi/2$.

Therefore, the magnitude of magnetic induction B at O due to this element,

$$\delta B = \frac{\mu_0}{4\pi} \cdot \frac{i\delta l \sin \pi/2}{r^2} = \frac{\mu_0 i \delta l}{4\pi r^2}$$

Hence, magnetic induction due to whole semicircular loop is

$$B = \Sigma \delta B = \Sigma \frac{\mu_0}{4\pi} \cdot \frac{i\delta l}{r^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{i}{r^2} \Sigma \delta l = \frac{\mu_0 i}{4\pi r^2} (\pi r) = \frac{\mu_0 i}{4r}$$

The magnetic field due to ab and de is zero, because $\theta = 0^\circ$ or 180° , so net magnetic field is

$$B = \frac{\mu_0 i}{4r}$$

$$17 \text{ (c) Magnetic field due to straight wire above } O \text{ is zero,}$$

i.e. $B_1 = 0$ (since, $\theta = 0^\circ$)

The magnetic field due to semicircular part,

$$B_2 = \frac{1}{2} \times \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{4r}$$

The magnetic field due to lower straight portion,

$$B_3 = \frac{\mu_0 I}{4\pi r} (\sin 0^\circ + \sin 90^\circ) = \frac{\mu_0 I}{4\pi r} \quad (\text{upward})$$

Net magnetic field,

$$B = B_1 + B_2 + B_3 = 0 + \frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}$$

$$= \frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}$$

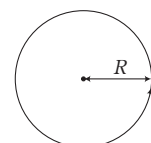
(upwards)

$$18 \text{ (b) Magnetic field at the centre of the circular loop of one turn, } B = \frac{\mu_0 i}{2R}$$

Now, the wire is bent into n circular coils.

Thus, new radius can be determined as

$$n \times 2\pi r = 2\pi R$$



$$\Rightarrow r = \frac{R}{n}$$

∴ Magnetic field at the centre of these loops,

$$B' = n \frac{\mu_0 i}{2r} = n \frac{\mu_0 i}{2 \left(\frac{R}{n} \right)} = n^2 \left(\frac{\mu_0 i}{2R} \right) = n^2 B$$

- 19** (a) The radius of a charged particle of mass m in a magnetic field B is given by

$$r = \frac{mv}{qB} \quad \dots(i)$$

where, q = charge on the particle.

and v = speed of the particle

∴ The time taken to complete the circle,

$$T = \frac{2\pi r}{v} \Rightarrow \frac{T}{2\pi} = \frac{m}{qB} \quad [\text{from Eq. (i)}]$$

$$\therefore \omega = \frac{2\pi}{T} = \frac{qB}{m}$$

$$\because q = e \text{ and } \frac{e}{m} = 1.76 \times 10^{11} \text{ C/kg}$$

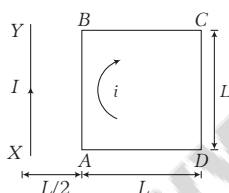
$$B = 3.57 \times 10^{-2} \text{ T}$$

$$\therefore \omega = \frac{eB}{m}$$

Now, frequency of revolution,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \frac{e}{m} B = \frac{1}{2\pi} \times 1.76 \times 10^{11} \times 3.57 \times 10^{-2} \\ = 1.0 \times 10^9 \text{ Hz} = 1 \text{ GHz}$$

- 20** (d) The given figure is shown below



From this figure, it can be seen that the direction of currents in the long straight conductor XY and arm AB of a square loop $ABCD$ are in the same direction. So, there exists a force of attraction between the two which will be experienced by BA ,

$$F_{BA} = \frac{\mu_0 i I L}{2\pi \left(\frac{L}{2} \right)}$$

In the case of XY and arm CD , the direction of currents are in the opposite direction. So, there exists a force of repulsion which will be experienced by CD . i.e.

$$F_{CD} = \frac{\mu_0 i I L}{2\pi \left(\frac{3L}{2} \right)}$$

Therefore, net force on the loop $ABCD$,

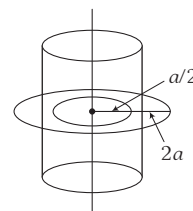
$$F_{\text{loop}} = F_{BA} - F_{CD} = \frac{\mu_0 i I L}{2\pi} \left[\frac{1}{(L/2)} - \frac{1}{(3L/2)} \right]$$

$$F_{\text{loop}} = \frac{2\mu_0 i I}{3\pi}$$

- 21** (b) Consider two Amperian loops of radius $\frac{a}{2}$ and $2a$ as shown in the figure.

Applying Ampere's circuital law for these loops, we get

$$\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{\text{enclosed}}$$



For the smaller loop,

$$\Rightarrow \mathbf{B} \times 2\pi \frac{a}{2} = \mu_0 \times \frac{I}{\pi a^2} \times \pi \left(\frac{a}{2} \right)^2 = \mu_0 I \times \frac{1}{4} = \frac{\mu_0 I}{4}$$

$$\Rightarrow \mathbf{B} = \frac{\mu_0 I}{4\pi a} \text{ at distance } \frac{a}{2} \text{ from the axis of the wire.}$$

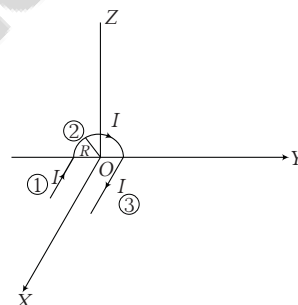
Similarly, for bigger Amperian loop,

$$\mathbf{B}' \times 2\pi (2a) = \mu_0 I \quad (\text{total current enclosed by Amperian loop is } I)$$

$$\Rightarrow \mathbf{B}' = \frac{\mu_0 I}{4\pi a} \text{ at distance } 2a \text{ from the axis of the wire.}$$

$$\text{So, ratio of } \frac{\mathbf{B}}{\mathbf{B}'} = \frac{\mu_0 I}{4\pi a} \times \frac{4\pi a}{\mu_0 I} = 1$$

- 22** (c) The magnetic fields in the different regions are given by



$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{R} (-\hat{k}), B_3 = \frac{\mu_0 I}{4\pi R} (-\hat{k})$$

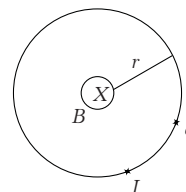
$$\text{and } B_2 = \frac{\mu_0 I}{4R} (-\hat{i})$$

Net magnetic field at the centre O ,

$$B = B_1 + B_2 + B_3 = \frac{\mu_0 I}{4\pi R} (-2\hat{k}) + \frac{\mu_0 I}{4R} (-\hat{i}) = -\frac{\mu_0 I}{4\pi R} (2\hat{k} + \pi\hat{i})$$

- 23** (d) Current, $I = \frac{q}{t}$

So, for an electron revolving in a circular orbit of radius r ,



$$q = e \text{ and } t = T$$

$$\Rightarrow I = \frac{e}{T} = \frac{e}{2\pi/\omega} = \frac{\omega e}{2\pi} = \frac{2\pi n e}{2\pi} = n e$$

The magnetic field produced at the centre,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 n e}{2r}$$

- 24** (b) The angle subtended by circular part of the conductor is $3\pi/2$ or 270° .

Net magnetic field at point O , $B_{\text{net}} = B_1 + B_2$

where, B_1 = magnetic field due to arc II and B_2 = magnetic field due to arc I.

$$B_{\text{net}} = \frac{\mu_0 i}{4\pi \times 3R} \times \frac{\pi}{2} + \frac{\mu_0 i}{4\pi R} \times 3 \times \frac{\pi}{2} = \frac{5\mu_0 i}{12R} \quad (\text{downward})$$

- 25** (c) Given, speed of proton, $v = 3 \times 10^6$ m/s

Magnetic field, $B = 2 \times 10^{-3}$ T

Now, force (magnetic) on proton,

$$\begin{aligned} F &= qvB \sin \theta \\ &= 1.6 \times 10^{-19} \times 3 \times 10^6 \times 2 \times 10^{-3} \times \sin 90^\circ \\ &= 3.2 \times 3 \times 10^{-16} = 9.6 \times 10^{-16} \text{ N} \end{aligned}$$

$\therefore$ Acceleration of proton,

$$\begin{aligned} a &= \frac{F}{m} = \frac{9.6 \times 10^{-16} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = \frac{9.6}{1.67} \times 10^{11} \text{ m/s}^2 \\ &= 5.74 \times 10^{11} \text{ m/s}^2 \approx 5.8 \times 10^{11} \text{ m/s}^2 \end{aligned}$$

- 26** (c) Magnetic field at the centre of circular coil,

$$B = \frac{\mu_0 I}{2r} \quad \dots(i)$$

Now magnetic field at the centre of smaller circular coil,

$$B' = \frac{\mu_0 ni}{2r/n} \Rightarrow B' = n^2 \frac{\mu_0 i}{2r}$$

From Eq. (i), we get

$$B' = n^2 B \Rightarrow \frac{B'}{B} = \frac{n^2}{1}$$

- 27** (a) Two infinitely long and straight parallel wires carrying equal currents in same direction will attract each other.
- 28** (b) Points A and A' are the inflection points, where the sign of curvature changes.

At these points (i.e. A and A'), field is constant, so

$$\frac{dB}{dt} \text{ as well as } \frac{d^2B}{dt^2} \text{ are zero.}$$

- 29** (b) Let the masses of two particles are m_1 and m_2 .
As the charges are of the same magnitude and being accelerated through same potential, so these charges enter into the magnetic field with the same speeds (let v). Now, radii of the circular paths followed by two charges is given by

$$R_1 = \frac{m_1 v}{qB} \text{ and } R_2 = \frac{m_2 v}{qB} \Rightarrow \frac{R_1}{R_2} = \frac{m_1}{m_2}$$

- 30** (c) The magnetic field produced by the ring at its centre,

$$B = \frac{\mu_0 I}{2r} \quad \dots(ii)$$

where, I = current through the ring

$$= \frac{Q}{T} = Q / (2\pi / \omega) = \frac{Q\omega}{2\pi} \quad \dots(iii)$$

Here, Q = total charge on the ring

and T = time period of the ring.

According to the question, $\lambda = \frac{Q}{2\pi r} \Rightarrow Q = \lambda \times 2\pi r \quad \dots(iii)$

From Eqs. (i), (ii) and (iii), we get

$$B = \frac{\mu_0 \lambda \omega}{2}$$

$$\mathbf{31} \text{ (c) Since, } r = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{2mV}{qB^2}} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{m_1}{m_2} = \frac{r_1^2}{r_2^2}$$

$$\text{Hence, } \frac{m_1}{m_2} = \frac{(2)^2}{(3)^2} = \frac{4}{9}$$

- 32** (a) Given, $B = 10^{-4}$ Wb/m²,

$$\frac{q}{m} = 10^{11} \text{ C/kg and } v = 10^9 \text{ m/s}$$

Radius of the circle described,

$$\begin{aligned} r &= \frac{mv}{qB} \Rightarrow r = \frac{v}{\left(\frac{q}{m}\right)B} \\ r &= \frac{10^9}{(10^{11})(10^{-4})} \Rightarrow r = 100 \text{ m} \end{aligned}$$

- 33** (d) As, electrons cannot be accelerated in a cyclotron. A large increase in their energy increases their velocity to a very large extent. This throws the electrons out of step with the oscillating field while neutron, being electrically neutral, cannot be accelerated in a cyclotron.

So, cyclotron is used to accelerate only positively charged particles.

- 34** (c) Two parallel beams of positron moving in the same direction set up two parallel currents flowing in the same direction. Hence, they attract each other.

- 35** (a) Given, $I_1 = 3$ A, $I_2 = 4$ A, $R = 2\pi$ cm = $2\pi \times 10^{-2}$ m

Magnetic field at the centre of a coil, $B = \frac{\mu_0 I}{2R}$

$$\begin{aligned} \text{Now, } B_1 &= \frac{\mu_0 I_1}{2R} = \frac{\mu_0}{2} \times \frac{3}{2\pi \times 10^{-2}} = \frac{\mu_0}{4\pi} \times \frac{3}{10^{-2}} \\ &= 10^{-7} \times \frac{3}{10^{-2}} = 3 \times 10^{-5} \text{ T} \end{aligned}$$

$$\begin{aligned} \text{Similarly, } B_2 &= \frac{\mu_0 I_2}{2R} = \frac{\mu_0}{2} \times \frac{4}{2\pi \times 10^{-2}} = \frac{\mu_0}{4\pi} \times \frac{4}{10^{-2}} \\ &= 10^{-7} \times \frac{4}{10^{-2}} = 4 \times 10^{-5} \text{ T} \end{aligned}$$

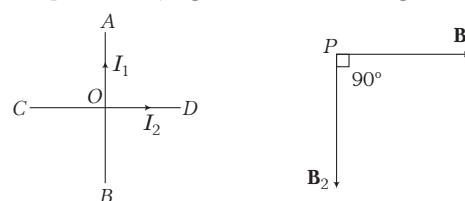
Now, net magnetic field at centre of the coils, $B = \sqrt{B_1^2 + B_2^2}$

$$\Rightarrow B = \sqrt{(3 \times 10^{-5})^2 + (4 \times 10^{-5})^2}$$

$$\Rightarrow B = \sqrt{10^{-10} (9 + 16)}$$

$$\Rightarrow B = 5 \times 10^{-5} \text{ T} = 5 \times 10^{-5} \text{ Wb/m}^2$$

- 36** (d) The point P is lying at a distance d along the Z -axis.



$$|B_1| = \frac{\mu_0 I_1}{2\pi d} \text{ and } |B_2| = \frac{\mu_0 I_2}{2\pi d}$$

$$B_{\text{net}} = \sqrt{B_1^2 + B_2^2}$$

$$B_{\text{net}} = \frac{\mu_0}{2\pi d} (I_1^2 + I_2^2)^{1/2}$$

37 (b) The magnetic field inside the solenoid is given by

$$B = \mu_0 n I$$

where, n = number of turns per unit length

and I = current in coil

$$\text{Now, } n = \frac{N}{L} = \frac{400}{0.4 \times 10^{-2}} = 10^5$$

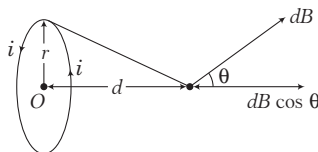
$$\text{and } \mu_0 = 4\pi \times 10^{-7} \text{ T-m/A}$$

$$\therefore B = 4 \times 3.14 \times 10^5 \times 5 \times 10^{-7} \\ = 62800 \times 10^{-5} = 6.28 \times 10^{-1} \text{ T}$$

38 (b) Magnetic field B at any point in the open space inside the toroid is zero because the amperian loop encloses net current equal to zero.

39 (b) The magnetic field at axis of current carrying loop is given by

$$B = \frac{\mu_0 i r^2}{2(r^2 + d^2)^{3/2}} \quad \dots(i)$$



where, i = current in loop, r = radius of loop

d = distance on axis

and B = magnetic field of current carrying loop.

Let B_1 be the magnetic field at centre ($d = 0$).

$$\text{Now, } B_1 = \frac{\mu_0 i r^2}{2(r^2 + 0^2)^{3/2}} = \frac{\mu_0 i r^2}{2r^3} = \frac{\mu_0 i}{2r} \quad \dots(ii)$$

On dividing Eqs. (i) and (ii), we get

$$\frac{B}{B_1} = \frac{\mu_0 i r^2 \times 2r}{2(r^2 + d^2)^{3/2} \times \mu_0 i} \\ \Rightarrow \frac{B}{B_1} = \frac{r^3}{(r^2 + d^2)^{3/2}} = \frac{3^3}{(3^2 + 4^2)^{3/2}}$$

$$\Rightarrow \frac{B}{B_1} = \frac{27}{(25)^{3/2}} \Rightarrow \frac{54 \mu\text{T}}{B_1} = \frac{27}{5^3}$$

$$B_1 = \frac{54 \times 125}{27}$$

$$\Rightarrow B_1 = 250 \mu\text{T}$$

40 (b) Given, kinetic energy = E

$$\text{Mass} = m$$

$$\text{Magnetic field} = B$$

$$\text{and charge} = q$$

$$\text{Magnetic force, } F = qvB \sin \theta$$

$$\text{If } \theta = 90^\circ, \text{ then } F = qvB$$

...(i)

Now, centripetal force,

$$F = \frac{mv^2}{r} \quad \dots(ii)$$

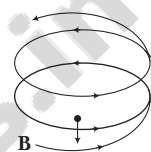
From Eqs. (i) and (ii), we get

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$

$$\therefore r = \frac{m \sqrt{\frac{2E}{m}}}{qB} \quad \left(\because E = \frac{1}{2} mv^2, v = \sqrt{\frac{2E}{m}} \right)$$

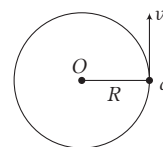
$$\Rightarrow r = \frac{\sqrt{2Em}}{qB}$$

41 (d) Let the velocity v of the particle entering the field B , instead of being perpendicular to B makes an angle with it.



Then, v may be resolved into two components: $v_{\parallel} = v \cos \theta$ parallel to B and $v_{\perp} = v \sin \theta$ perpendicular to B . The component $v_{\parallel}$ gives a linear path and the component $v_{\perp}$ gives a circular path to the particle. The resultant of these two is a helical path whose axis is parallel to the magnetic field.

42 (c) The number of revolutions per second by the charge q is given as

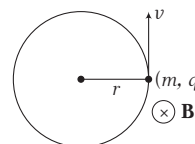


$$N = \frac{1}{T} = \frac{v}{2\pi R}$$

$$\text{Current through circular path, } i = q(I/T) = vq/2\pi R$$

$$\text{Magnetic moment, } \mu = iA = \frac{vq}{2\pi R} (\pi R^2) \Rightarrow \mu = \frac{1}{2} vqR$$

43 (d) Consider a charged particle of mass m and charge q moving along a circular path in anti-clockwise direction as shown in figure.



The magnetic field B is assumed to be into plane of paper and of constant magnitude.

Clearly, centripetal force required for the circular motion of the charge will be provided by magnetic Lorentz force $q(v \times B)$.

$$\text{Hence, } \frac{mv^2}{r} = qvB \sin 90^\circ$$

$$\Rightarrow \frac{mv^2}{r} = qvB$$

$$\Rightarrow \text{Speed, } v = \frac{qBr}{m}$$

$$\Rightarrow r = \frac{mv}{qB} \quad \dots(i)$$

Let T be the time period of the periodic motion, i.e. time taken to complete one revolution.

$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{v/r} = \frac{2\pi r}{v} = \frac{2\pi}{v} \left(\frac{mv}{qB} \right) \quad [\text{from Eq. (i)}]$$

$$= \frac{2\pi m}{qB}$$

44 (b) Magnetic moment,

$$M = \text{Current} \times \text{Area of enclosed by loop}$$

$$= I \times A \quad \dots(i)$$

Magnetic induction at the centre of circular loop,

$$B = \frac{\mu_0 I}{2R} \Rightarrow I = \frac{2BR}{\mu_0} \quad \dots(ii)$$

$$\text{Here, } A = \pi R^2 \quad \dots(iii)$$

Substituting Eqs. (ii) and (iii) in Eq. (i), we get

$$M = \frac{2BR}{\mu_0} \times \pi R^2 = \frac{2\pi BR^3}{\mu_0}$$

45 (c) In cyclotron, force on charge = centripetal force

$$\therefore qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{Bq} \Rightarrow r \propto v$$

46 (a) When a magnetic field is applied on a stationary electron, then it remains stationary.

$$\text{Because, } \mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

$$\text{If } v = 0 \Rightarrow F = 0$$

47 (d) Given, $r = 0.05 \text{ nm} = 0.05 \times 10^{-9} \text{ m}$, ($\because 1 \text{ nm} = 10^{-9} \text{ m}$)

$$n = 10^{16} \text{ rev/s, } e = 1.6 \times 10^{-19} \text{ C}$$

Magnetic moment, $M = Ai$

$$M = \pi r^2 \times ne \quad (\because A = \pi r^2)$$

$$= 3.14 \times (0.05 \times 10^{-9})^2 \times 10^{16} \times 1.6 \times 10^{-19}$$

$$\text{or } = 1.26 \times 10^{-23} \text{ A-m}^2$$

48 (c) Given, radius of coil, $r = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$

Number of turns, $N = 100$ turns, current in coil, $i = 1 \text{ A}$

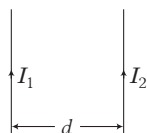
Magnetic moment, $M = ?$

Now, $M = NiA$, where $A = \pi r^2$

$$M = N i \pi r^2 = 100 \times 1 \times 3.142 \times (10 \times 10^{-2})^2$$

$$= 100 \times 1 \times 3.142 \times 100 \times 10^{-4} = 3.142 \text{ A-m}^2$$

49 (c) The given situation is shown in the figure.



Since, given wires are very large, hence magnetic field due to current in the first wire,

$$B_1 = \left[\frac{\mu_0}{2\pi d} I_1 \right]$$

The field is perpendicularly inward.

Now, force on the second wire due to first wire,

$$\mathbf{F}_{21} = I_2(\mathbf{l} \times \mathbf{B}_1) \quad (\text{towards left})$$

$$F_{21} = I_2 B_1 \sin 90^\circ$$

$$F_{21} = I_2 B_1$$

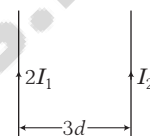
where, l is the length of the wire and d is separation between the two wires.

$$F_{21} = I_2 \left(\frac{\mu_0 I_1}{2\pi d} \right) = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

According to question, $|\mathbf{F}_{21}| = F$

$$\Rightarrow F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Now, consider the situation, when the current in first wire is doubled and reversed, we can write



$$F' = \frac{\mu_0 (2I_1)(I_2)l}{2\pi d \times 3}$$

Here, force on second wire will be towards right.

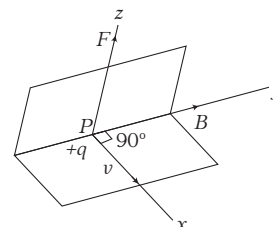
$$\text{So, } \frac{F'}{F} = \left[\frac{\mu_0 (2I_1)(I_2)l}{2\pi d \times 3} \right] \left[\frac{2\pi d}{\mu_0 I_1 I_2 l} \right]$$

$$\frac{F'}{F} = \left[\frac{2I_1 I_2}{3I_1 I_2} \right] = \frac{2}{3}$$

$$\Rightarrow F' = -\frac{2F}{3}$$

Here, negative sign is due to opposite directions of F' and F .

50 (a) From Fleming's left-hand rule, when a particle is in motion and magnetic field is perpendicular to the velocity.



51 (d) Angular momentum, $L = mvr \quad \dots(i)$

The orbital motion of electron is equivalent to a current.

$$\therefore I = e (1/T)$$

$$\text{Period of revolution of electron, } T = \frac{2\pi r}{v}$$

$$\therefore I = e \left(\frac{1}{2\pi r / v} \right) = \frac{ev}{2\pi r}$$

Area of electron orbit, $A = \pi r^2$

Magnetic dipole moment of the atom,

$$M = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

Using Eq. (i), we have

$$M = \left(\frac{e}{2m} \right) L$$

$$\Rightarrow \frac{M}{L} = \frac{e}{2m}$$

- 52 (c)** Circumference of circle = Length of wire

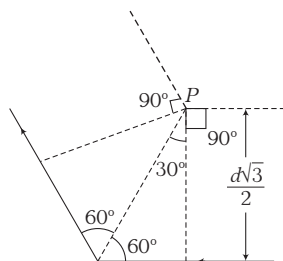
$$\therefore 2\pi R = L$$

$$\Rightarrow R = \left(\frac{L}{2\pi} \right)$$

$$\text{Area, } A = \pi R^2 = \frac{\pi L^2}{4\pi^2} = \frac{L^2}{4\pi}$$

$$\text{Magnetic moment, } M = IA = \frac{IL^2}{4\pi}$$

- 53 (d)** The given figure can be drawn as follows.



The magnetic field at point P,

$$\begin{aligned} B_{\text{net}} &= 2 \left[\frac{\mu_0 I}{4\pi r} (\sin \theta_1 + \sin \theta_2) \right] \\ &= 2 \left[\frac{\mu_0}{4\pi} \times \frac{I}{\frac{d\sqrt{3}}{2}} \times (\sin 90^\circ + \sin 30^\circ) \right] \\ &= 2 \left[\frac{\mu_0}{4\pi} \times \frac{2I}{d\sqrt{3}} \times \left(1 + \frac{1}{2} \right) \right] \\ &= 2 \left[\frac{\mu_0}{4\pi} \times \frac{2I}{d\sqrt{3}} \times \frac{3}{2} \right] = \frac{\sqrt{3}\mu_0 I}{2\pi d} \end{aligned}$$

- 55 (d)** If the direction of dipole moment of the loop is parallel to the direction of magnetic field, then loop is in stable equilibrium and if the direction of dipole moment of the loop is anti-parallel to the direction of magnetic field, then it is in unstable equilibrium.

- 56 (a)** The radius of the circular path of a charged particle in magnetic field,

$$r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB}$$

Here, kinetic energy for proton and helium is same and both are moving in the same magnetic field.

$$\therefore r \propto \frac{\sqrt{m}}{q}$$

$$\begin{aligned} \text{So, } \frac{r_P}{r_{\text{He}}} &= \frac{\frac{q_P}{\sqrt{m_P}}}{\frac{q_{\text{He}}}{\sqrt{m_{\text{He}}}}} = \sqrt{\frac{m_P}{m_{\text{He}}}} \times \frac{q_{\text{He}}}{q_P} \\ &= \sqrt{\frac{m}{4m}} \cdot \frac{2q}{q} = \frac{1}{1} \end{aligned}$$

- 57 (a)** Time period, $T = \frac{2\pi m}{Bq}$

$$\Rightarrow T = \frac{m}{q}$$

$$\text{or } \frac{T_1}{T_2} = \frac{m_1}{m_2} \times \frac{q_2}{q_1} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8} \text{ or } 3 : 8$$

- 58 (c)** The magnetic field at centre of a coil, $B = \frac{\mu_0 Ni}{2R}$

Given, $i = 2 \text{ A}$, $N = 1$

$$\text{So, } R = \frac{\mu_0 Ni}{2B}$$

$$R = \frac{4\pi \times 10^{-7} \times 1 \times 2}{2 \times 4\pi \times 10^{-6}}$$

$$\Rightarrow R = \frac{1}{10} = 0.1 \text{ m}$$

- 59 (b)** Magnetic field, $B = \frac{\mu_0 I}{2\pi r}$

Given, $\mu = 4\pi \times 10^{-7} \text{ N/A}^2$, $r = 8 \text{ cm} = 8 \times 10^{-2} \text{ m}$, $I = 12 \text{ A}$

$$B = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{12}{8 \times 10^{-2}}$$

$$\Rightarrow B = 3 \times 10^{-5} \text{ Wb/m}^2$$

- 60 (a)** Given, $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$, $n = 500 \text{ turns/m}$, $I = 0.8 \text{ A}$

We know that, $B = \mu_0 nI$

$$B = 5.024 \times 10^{-4} \text{ Wb/m}^2$$

- 61 (d)** Given that, the net magnetic field at the centre O is zero. Therefore, magnetic field at O due to circular coil and straight conductor must be equal and opposite in direction.

$$\therefore \frac{\mu_0 I_1}{2R} = \frac{\mu_0 I_2}{2\pi(2R)} \Rightarrow \frac{I_1}{I_2} = \frac{1}{\pi}$$

- 62 (a)** Force between wire is

$$\begin{aligned} F &= \frac{\mu_0}{2\pi} \frac{I_1 I_2}{r} l \\ &= \frac{2 \times 10^{-7} \times 30 \times 40 \times (0.1)}{0.02} = 1.2 \times 10^{-3} \text{ N} \end{aligned}$$

- 63 (c)** The charged particle goes undeflected through both the fields, therefore force experience by charged particle due magnetic field must be equal to the force experienced by the charge particle due to electric field, i.e. $F_m = F_e$

$$\text{or } evB \sin \theta = eE$$

Given, $v = 2 \times 10^3 \text{ ms}^{-1}$

$$B = 1.5 \text{ T and } \theta = 90^\circ$$

Hence, $E = vB \sin \theta = 2 \times 10^3 \times 1.5 \times \sin 90^\circ$

$$= 3 \times 10^3 \text{ V/m or N/C}$$

- 64 (a)** The magnetic field (B) at the centre of circular current carrying coil of radius R and current

$$I, B = \frac{\mu_0 I}{2R}$$

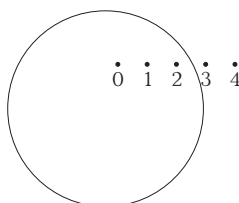
Similarly, if current $= 2I$, then

$$\text{Magnetic field} = \frac{\mu_0 2I}{2R} = 2B$$

So, resultant magnetic field

$$\begin{aligned} &= \sqrt{B^2 + (2B)^2} \\ &= \sqrt{5B^2} = \sqrt{5}B \\ &= \frac{\mu_0 I \sqrt{5}}{2R} \end{aligned}$$

- 65 (d)** The given figure is shown below



Magnetic field outside the long rod,

$$B = \frac{\mu_0 i}{2\pi r} \quad [\text{for rod 3 and rod 4, } r_3 < r_4]$$

$$\therefore B_3 > B_4 \neq 0$$

- 66 (d)** The magnetic field at the centre of new loop

$$B' = n^2 B$$

where, n is the number of turns in the loop.

$$\therefore B' = (4)^2 B \text{ or } B' = 16B$$

- 67 (c)** Under uniform magnetic field, force evB acts on proton and

provides the necessary centripetal force $\frac{mv^2}{a}$.

$$\therefore \frac{mv^2}{a} = evB \Rightarrow v = \frac{aeB}{m}$$

Now, angular momentum,

$$L = r \times p = a \times aeB = a^2 eB$$

- 68 (c)** Given, $B = 8.35 \times 10^{-2} \hat{i} \text{ T}$, $v = (2 \times 10^5 \hat{i} + 4 \times 10^5 \hat{j}) \text{ m/s}$

The distance covered by proton, $d = T(v) = 2\pi \frac{m}{qB}(v)$

$$\begin{aligned} &= 2 \times 3.14 \times \frac{1.67 \times 10^{-27}}{1.6 \times 10^{-19} \times 8.35 \times 10^{-2} \hat{i}} \\ &\quad \times (2 \times 10^5 \hat{i} + 4 \times 10^5 \hat{j}) \end{aligned}$$

$$\Rightarrow d = 0.157 \text{ m}$$

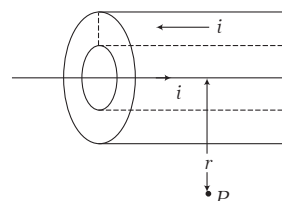
- 69 (d)** Given, $n = 12$ turns, $I = 15 \text{ A}$, $B = 0.2 \hat{i}$

$$A = -0.04 \hat{i} \text{ m}^2$$

$\therefore$ Potential energy, $U = nIAB$

$$= 12 \times 15 \times (-0.04) \times 0.2 = -1.44 \text{ J}$$

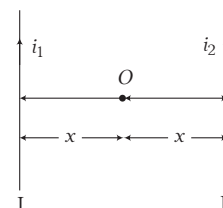
- 70 (b)** The described condition can be shown as



The magnetic field at P due to inner and outer conductors are equal and opposite. Hence, the net magnetic field at P will be zero.

- 71 (b)** When the currents in the wires are in same direction. Magnetic field at mid-point O due to I and II wires are respectively

$$B_I = \frac{\mu_0}{4\pi} \frac{2i_1}{x} \otimes \quad \text{and} \quad B_{II} = \frac{\mu_0}{4\pi} \frac{2i_2}{x} \odot$$



So, the net magnetic field at O ,

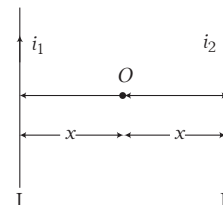
$$B_{\text{net}} = \frac{\mu_0}{4\pi} \cdot \frac{2}{x} (i_1 - i_2)$$

$$\Rightarrow 10 \times 10^{-6} = \frac{\mu_0}{4\pi} \cdot \frac{2}{x} (i_1 - i_2) \quad \dots(i)$$

When the direction of i_2 is reversed,

$$B_I = \frac{\mu_0}{4\pi} \cdot \frac{2i_1}{x} \otimes$$

$$\text{and} \quad B_{II} = \frac{\mu_0}{4\pi} \cdot \frac{2i_2}{x} \otimes$$



So, net magnetic field at O ,

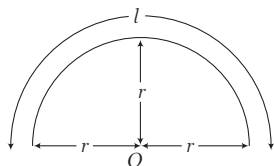
$$B_{\text{net}} = \frac{\mu_0}{4\pi} \cdot \frac{2}{x} (i_1 + i_2)$$

$$\Rightarrow 40 \times 10^{-6} = \frac{\mu_0}{4\pi} \cdot \frac{2}{x} (i_1 + i_2) \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{i_1 + i_2}{i_1 - i_2} = \frac{4}{1} \Rightarrow \frac{i_1}{i_2} = \frac{5}{3}$$

- 73** (c) When wire is bent in the form of semicircular arc, then
 $l = \pi r$



$\therefore$ The radius of semicircular arc, $r = l/\pi$

Distance between two end points of semicircular wire

$$= 2r = \frac{2l}{\pi}$$

$\therefore$ Magnetic moment of semicircular wire

$$= m \times 2r = m \times \frac{2l}{\pi} = \frac{2}{\pi} ml$$

But ml is the magnetic moment of straight wire, i.e. $ml = M$

$\therefore$ New magnetic moment $= \frac{2}{\pi} M$

- 74** (b) Given, $\theta = 23^\circ$, $B = 2.6 \text{ mT} = 2.6 \times 10^{-3} \text{ T}$

and $F = 6.5 \times 10^{-17} \text{ N}$

We know that, $F = qvB \sin \theta$

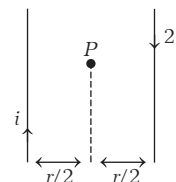
$$6.5 \times 10^{-17} = 1.6 \times 10^{-19} \times v \times 2.6 \times 10^{-3} \times \sin 23^\circ$$

$$v = \frac{6.5 \times 10^{-17}}{2.6 \times 10^{-3} \times 1.6 \times 10^{-19} \times 0.39} = 4 \times 10^5 \text{ ms}^{-1}$$

- 75** (c) Radius, $r = \frac{mv}{qB}$

$$\text{or } B = \frac{mv}{qr} = \frac{9.1 \times 10^{-31} \times 1.3 \times 10^6}{1.6 \times 10^{-19} \times 0.35} = 2.1 \times 10^{-5} \text{ T}$$

- 76** (a) The magnetic field induction at P due to currents through both the wires,



$$B = \frac{\mu_0}{4\pi} \frac{2i}{r/2} + \frac{\mu_0}{4\pi} \frac{2(2i)}{r/2} = \frac{\mu_0}{4\pi} \cdot \frac{12i}{r}$$

It is acting perpendicular to plane of wire inwards. Now, B and v are acting in the same direction, i.e. $\theta = 0^\circ$.

$\therefore$ Force on charged particle is $F = qvB \sin \theta = qvB \times 0 = 0$.

- 77** (b) We have, $M = NIA$, $B = \mu_0 nI$

Torque, $\tau = MB$

Here, $\tau = (n_1 I_1 A)(\mu_0 n_2 I_2)$

$$= \left(10 \times \frac{21}{44} \times 10^{-6}\right) \left(4 \times \frac{22}{7} \times 10^{-7} \times 10^3 \times 2.5\right) = 1.5 \times 10^{-8} \text{ N-m}$$

CHAPTER 05

Magnetism and Matter

A Greek philosopher, Thales of Miletus had observed in 600 BC that a naturally occurring ore of iron attracted small pieces of iron towards it. This ore was found in the district of Magnesia in Asia Minor in Greece. Hence, the ore was named magnetite. Later, William Gilbert first suggested that earth behaves as a huge magnet which causes the alignment of compass needles, then oersted discovered that moving charges are the sources of magnetic field. So, the science of magnetism was known long before the 19th century.

The phenomenon of attraction of small bits of iron, steel, cobalt, nickel, etc., towards the ore was called **magnetism**. Matters (*i.e.* solids, liquids or gases) show magnetism and hence they are classified into different categories according to their magnetic properties. In this chapter, we shall learn about magnetism and magnetisation of matter followed by knowledge of magnets magnetics dipole and their magnetic fields. We will also discuss about the earth's magnetism.

MAGNET

A magnet is a material or object that exhibits a strong magnetic field and has a property to attract some specific materials like iron towards it. The magnetic field is invisible but is responsible for properties of a magnet.

Magnets are of two types

- (i) Natural magnets
- (ii) Artificial magnets

Natural magnets are generally irregular in shape and weaker in strength. On the other hand, artificial magnets may have desired shape and strength. A bar magnet, a horse shoe magnet, compass needle, etc., all are examples of artificial magnet.

Bar magnet

A bar magnet consists of two equal and non-separable magnetic poles. One pole is designated as north pole (N) and the other as south pole (S). These poles are separated at a small distance but they are not exactly at the ends. The distance between two poles of a bar magnet is known as **magnetic length of a magnet**. Its direction is from S -pole of the magnet to N -pole and is represented by $2l$. This length is sometimes also known as **effective length** (L_e) of the magnet and is less than its geometric length (L_g).

Inside

1 Magnet

- Magnetic field lines
- Magnetic dipole
- Coulomb's law for magnetism
- Magnetic field strength at a point due to magnetic dipole or bar magnet
- Current carrying loop as a magnetic dipole
- Bar magnet in a uniform magnetic field

2 Earth's magnetism

- Elements of earth's magnetism
- Neutral points
- Vibration magnetometer

3 Magnetic induction and magnetic materials

- Classification of substances on the basis of magnetic behaviour
- Curie's law
- Atomic model of magnetism
- Hysteresis

For a bar magnet, $L_e = \left(\frac{5}{6}\right)L_g$

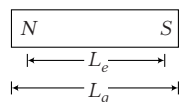


Fig. 5.1 Bar magnet

General properties of magnet

Few properties of magnet are discussed below

Directional property

When a magnet is suspended freely, then it points in the earth's *N-S* direction (in magnetic meridian).

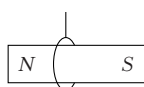


Fig. 5.2 Freely suspended bar magnet

Monopole non-existence

If a magnet is broken into number of pieces, then each piece behaves as an individual magnet rather than isolated poles. This means that monopoles do not exist.

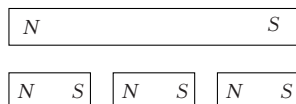


Fig. 5.3 If a bar magnet is broken, each piece behaves as a small magnet

Attractive/Repulsive properties

Like magnetic poles repel each other and unlike magnetic poles attract each other.

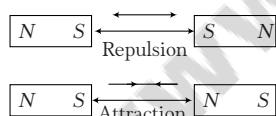


Fig. 5.4 Like poles repel and unlike poles attract

Note Repulsion is a sure test of magnetism.

Magnetic field lines

The magnetic field lines of a magnetic field are the imaginary lines which continuously represent the direction of that magnetic field.

Properties of magnetic field lines

- (i) The magnetic field lines of a magnet form closed continuous loops. This property is unlike electric field lines, which begin from a positive charge and terminate at the negative charge or escape to infinity.

- (ii) At any point, tangent to the magnetic field line represents the direction of net magnetic field (**B**) at that point.
- (iii) Larger the number of field lines crossing per unit area, stronger is the magnitude of the magnetic field *B*.
- (iv) Magnetic field lines do not intersect each other, for if they did, the direction of the magnetic field would not be unique at the point of intersection.
- (v) The direction of field lines is from *N* to *S*, if they are outside the magnet and from *S* to *N*, if they are inside the magnet.
- (vi) Fig. 5.5 (a) shows the uniform magnetic field lines and Fig. 5.5 (b) shows non-uniform magnetic field lines.

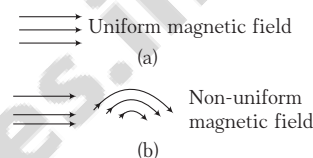


Fig. 5.5

- (vii) Magnetic field lines come out and go into a magnetic material at any angle.

Different patterns of magnetic field lines

Few patterns of magnetic field lines around a magnet or a pair of magnets are shown below

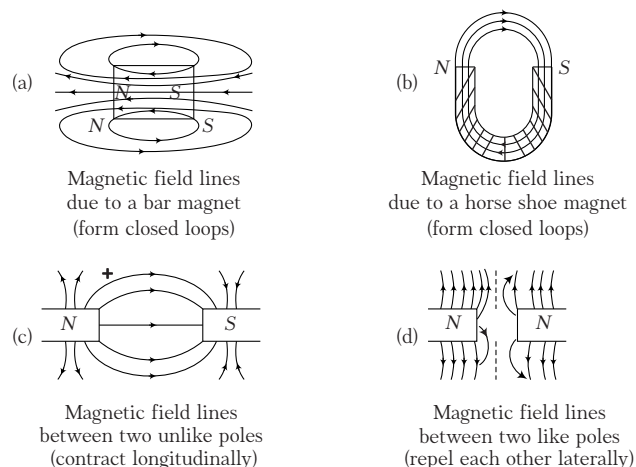


Fig. 5.6 Different patterns of magnetic field lines

Magnetic dipole

A magnetic dipole is an arrangement which consists of two magnetic poles of equal and opposite strengths separated at a small distance. A bar magnet, a compass needle, etc., are the examples of magnetic dipoles.

Pole strength

Like electric charge, we have equivalent analogous in magnetism which is known as pole strength. Thus, **pole strength can be defined as the strength of a magnetic pole to attract magnetic materials towards itself.** It is a scalar quantity and its SI unit is ampere-metre (A-m). The strength of N -pole and S -pole of a magnet is conventionally represented by $+m$ and $-m$, respectively. It depends on the nature of material and area of cross-section of the magnet.

Strength of N -pole and S -pole of a magnet is always equal and opposite ($+m$ and $-m$).

It is the product of the strength of either of the pole strength and the magnetic length of the magnet. It is represented by $\mathbf{M}$. It is a vector quantity.

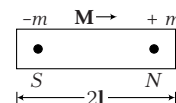


Fig. 5.7 Bar magnet

The direction of magnetic dipole moment is same as that of $2l$. Therefore, $\mathbf{M} = m(2l)$

Its SI unit is ampere-metre² (A-m²).

Magnetic dipole moment

Pole strength and magnetic dipole moment in special cases

Special cases	Figure	Effect on pole strength	Formula for new magnetic dipole moment
If bar magnet is cut into two equal pieces such that the length of each piece becomes half		Remains unchanged	$M' = m \cdot \frac{2l}{2} = \frac{M}{2}$ (becomes half)
If bar magnet is cut into two equal pieces such that the width of each piece becomes half		Pole strength of each piece becomes half	$M' = \left(\frac{m}{2}\right)(2l) = \frac{M}{2}$ (becomes half)
If a bar magnet is bent in the form of semi-circle		Remains unchanged	$M' = m(2r) [\because 2l = \pi r]$ $M' = m \times 2 \left(\frac{2l}{\pi}\right) = \frac{2M}{\pi}$ (becomes $\frac{2}{\pi}$ times)
When two identical bar magnets are joined perpendicular to each other		Remains unchanged	$M = \sqrt{M_1^2 + M_2^2} = \sqrt{2}M$
When two bar magnets are inclined at an angle θ		Remains unchanged	Resultant magnetic moment, $M' = \sqrt{M_1^2 + M_2^2 + 2M_1M_2 \cos \theta}$ Angle made by resultant magnetic moment (M') with M_1 is given by $\tan \phi = \frac{M_2 \sin \theta}{M_1 + M_2 \cos \theta}$

Example 5.1 Consider a short magnetic dipole of magnetic length 10 cm. Find its geometric length.

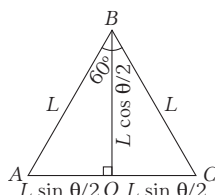
Sol. Geometric length of a magnet is $\frac{6}{5}$ times its magnetic length.

$$\therefore \text{Geometric length} = \frac{6}{5} \times 10 = 12 \text{ cm}$$

Example 5.2 A thin bar magnet of length $2L$ is bent at the mid-point, so that the angle between them is 60° . Find the new length of the magnet.

Sol. On bending the magnet, the length of the magnet,

$$\begin{aligned} AC &= AO + OC = L \sin\left(\frac{\theta}{2}\right) + L \sin\left(\frac{\theta}{2}\right) \\ &= 2L \sin 30^\circ = 2L \times \frac{1}{2} = L \end{aligned}$$



Example 5.3 The length of a magnetised steel wire is l and its magnetic moment is M . It is bent into the shape of L with two sides equal. What will be the new magnetic moment?

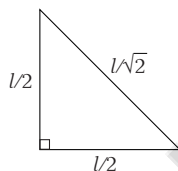
Sol. If m is strength of each pole, then magnetic moment, $M = m \times l$

When the wire is bent into L shape, effective distance between the poles

$$= \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{l}{\sqrt{2}}$$

$\therefore$ New magnetic moment,

$$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}} \quad (m \text{ will remain unchanged})$$



Coulomb's law for magnetism

This law states that the force of attraction or repulsion between two magnetic poles is directly proportional to the product of their pole strength (m) and inversely proportional to the square of the distance between them.



Fig. 5.8 Two magnetic poles separated by a distance r

Let pole strength of either pole be m , then magnetic force between these two isolated poles kept at separation r will be

$$F \propto \frac{m_1 \times m_2}{r^2} \Rightarrow F = \frac{\mu_0}{4\pi} \frac{m_1 \times m_2}{r^2}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7} \text{ N/A}^2$

Important points about the Coulomb's law for magnetism are as follows

- Force will be attractive, if one pole is north and other pole is south, i.e. opposite poles.
- Force will be repulsive, if both poles are of same type (i.e. north-north or south-south).

Example 5.4 Two magnetic poles, one of which is four times stronger than the other, exert a force of 10 gf on each other when placed at a distance of 20 cm. Find the strength of each pole.

Sol. Let the pole strength of the two dipoles be m and $4m$.

$$\text{Here, } F = 10 \text{ gf} = 10 \times 10^{-3} \text{ kg-f} = 10 \times 10^{-3} \times 9.8 \text{ N}$$

$$\text{and } r = 20 \text{ cm} = 0.2 \text{ m}$$

$$\text{Using Coulomb's law of magnetism, } F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}$$

$$\text{Substituting the values, } 10 \times 10^{-3} \times 9.8 = \frac{10^{-7} \times m \times 4m}{(0.2)^2}$$

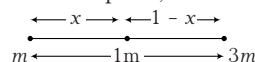
$$\text{or } m^2 = \frac{10 \times 9.8 \times (0.2)^2 \times 10^4}{4} = 9800$$

$$\text{or } m_1 = m = 98.9 \text{ A-m}$$

$$\text{and } m_2 = 4m = 4 \times 98.9 = 396 \text{ A-m}$$

Example 5.5 Two similar magnetic poles, having pole strengths in the ratio 1 : 3 are placed 1 m apart. Find the point where a unit pole experiences no net force due to these two poles.

Sol. Let the pole strengths of the two magnetic poles be m and $3m$. Suppose the required point is located at distance x from the first pole. Then at this point,



Force on unit pole due to first pole

= Force on unit pole due to second pole

$$\text{or } \frac{\mu_0}{4\pi} \cdot \frac{m \times 1}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{3m \times 1}{(1-x)^2}$$

$$\text{or } 3x^2 = (1-x)^2 \text{ or } \sqrt{3}x = 1-x$$

$$\text{or } x = \frac{1}{1+\sqrt{3}} = 0.366 \text{ m}$$

Magnetic field strength at a point due to magnetic dipole or bar magnet

The strength of a magnetic field at any point is defined as the force experienced by a hypothetical unit strength north pole placed at that point.

i.e. $\mathbf{B} = \frac{\mathbf{F}}{m}$, where m is the pole strength of hypothetical

north pole. Magnetic field strength is a vector quantity.

The direction of magnetic field $\mathbf{B}$ is the direction along which hypothetical north pole would tend to move, if free to do so. We have used the word hypothetical north pole in the above discussion because an isolated magnetic pole does not exist.

Let us now calculate the magnetic field strength at different points of magnetic dipole (or bar magnet).

1. When point lies on axial line of a bar magnet

Let $2l$ be the magnetic length of a bar magnet with its centre at O . The magnetic dipole moment of the magnet is M , where $M = m \times 2l$. The distance of the observation point P on the axial line from the centre of the magnet be $OP = r$.

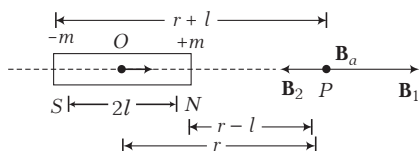


Fig. 5.9 Magnetic field of bar magnet on axial line

If m is the strength of each pole, then magnetic field strength at P due to N -pole of magnet is given by

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{m}{(NP)^2} = \frac{\mu_0}{4\pi} \frac{m}{(r-l)^2}, \text{ along } OP$$

Magnetic field strength at P due to S -pole of magnet,

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{m}{(SP)^2} = \frac{\mu_0}{4\pi} \frac{m}{(r+l)^2}, \text{ along } PO$$

$\therefore$ Magnetic field strength at P due to the bar magnet,

$$B_a = B_1 - B_2 = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2}$$

(Here - ve sign due to opposite direction)

When the magnet is short, $l^2 \ll r^2$, such that l^2 is neglected, then,

$$B_a = \frac{\mu_0}{4\pi} \frac{2Mr}{r^4} \Rightarrow B_a = \frac{\mu_0}{4\pi} \frac{2M}{r^3} \quad \dots(i)$$

The direction of B_a is along OP .

2. When point lies on equatorial line of a bar magnet

In figure given below, the point P is shown on equatorial line of the same bar magnet, where $OP = r$.

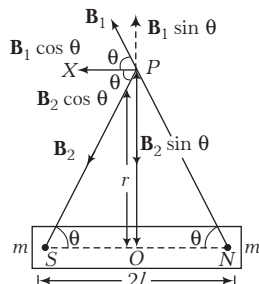


Fig. 5.10 Magnetic field of bar magnet on equatorial line

Magnetic field strength at P due to N -pole of magnet,

$$B_1 = \frac{\mu_0}{4\pi} \frac{m}{(NP)^2} = \frac{\mu_0}{4\pi} \frac{m}{(r^2 + l^2)}, \text{ from } N\text{-pole to point } P$$

Magnetic field strength at P due to S -pole of magnet,

$$B_2 = \frac{\mu_0}{4\pi} \frac{m}{(SP)^2} = \frac{\mu_0}{4\pi} \frac{m}{(r^2 + l^2)}, \text{ from point } P \text{ to } S\text{-pole}$$

As, $B_1 = B_2$ in magnitude, their vertical components $B_1 \sin \theta$ along OP produced and $B_2 \sin \theta$ along PO will cancel out. However, horizontal components along PX will add. Therefore, magnetic field strength at P due to the bar magnet,

$$B_e = B_1 \cos \theta + B_2 \cos \theta = 2 B_2 \cos \theta, \text{ along } PX$$

$$\therefore B_e = \frac{\mu_0}{4\pi} \frac{m \times 2l}{(r^2 + l^2)^{3/2}} \Rightarrow B_e = \frac{\mu_0}{4\pi} \frac{M}{(r^2 + l^2)^{3/2}}$$

If the magnet is short, $l^2 \ll r^2$, such that l^2 is neglected.

$$\therefore B_e = \frac{\mu_0}{4\pi} \frac{M}{(r^2)^{3/2}} \Rightarrow B_e = \frac{\mu_0}{4\pi} \frac{M}{r^3} \quad \dots(ii)$$

The direction of B_e is along PX , a line parallel to line joining N -pole to S -pole.

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{B_a}{B_e} = 2 \Rightarrow B_a = 2B_e$$

Hence, magnitude of magnetic field due to a short bar magnet at any point on the axial line of magnet is twice the magnetic field at a point at the same distance on the equatorial line of the magnet.

3. When point makes angle θ with axis of a bar magnet

At an angle θ with the axis of magnet, the magnetic field at point P at a distance r from centre of magnet is

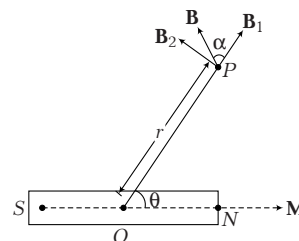


Fig. 5.11

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \sqrt{1 + 3 \cos^2 \theta}$$

and

$$\tan \alpha = \frac{1}{2} \tan \theta$$

For axial position of point P , $\theta = 0^\circ$ and for equatorial position, $\theta = 90^\circ$.

Note The magnetic potential due to a magnetic dipole at distance r ,

$$V = \frac{\mu_0}{4\pi} \cdot \frac{M \cos \theta}{r^2}$$

Now, (i) on the axis of magnet, $\theta = 0^\circ$,

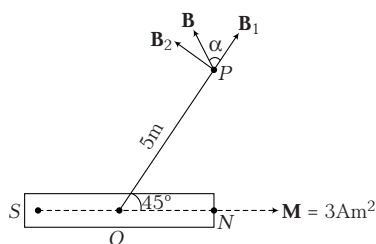
$$\therefore V = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^2}$$

(ii) On the neutral axis, $\theta = 90^\circ$,

$$\therefore V = 0$$

Example 5.6 Find the magnetic field due to a dipole of magnetic moment 3 A m^2 at a point 5 m away from it in the direction making angle of 45° with the dipole axis.

Sol. The condition given in the figure can be drawn as



So, the magnetic field at point P ,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \sqrt{1 + 3 \cos^2 \theta}$$

Here, $M = 3 \text{ A m}^2$, $r = 5 \text{ m}$, $\theta = 45^\circ$

$$\begin{aligned} \therefore B &= 10^{-7} \times \frac{3}{(5)^3} \sqrt{1 + 3 \cos^2(45^\circ)} \\ &= 10^{-7} \times \frac{3}{125} \times \sqrt{1 + 1.5} \\ &= 10^{-7} \times \frac{3}{125} \times 1.58 \\ &= 3.79 \times 10^{-9} \text{ T} \end{aligned}$$

Example 5.7 A bar magnet of length 0.1 m has pole strength of 50 A-m . Calculate the magnetic field at a distance of 0.2 m from its centre on

(i) its axial line (ii) and its equatorial line.

Sol. Here, $m = 50 \text{ A-m}$, $r = 0.2 \text{ m}$, $2l = 0.1 \text{ m}$ or $l = 0.05 \text{ m}$

$\therefore$ Magnetic dipole moment,

$$M = m(2l) = 50 \times 0.1 = 5 \text{ A-m}^2$$

$$\begin{aligned} \text{(i) } B_{\text{axial}} &= \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l^2)^2} = \frac{10^{-7} \times 2 \times 5 \times 0.2}{(0.2^2 - 0.05^2)^2} \\ &= 1.42 \times 10^{-4} \text{ T} \end{aligned}$$

$$\begin{aligned} \text{(ii) } B_{\text{equi}} &= \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + l^2)^{3/2}} = \frac{10^{-7} \times 5}{(0.2^2 + 0.05^2)^{3/2}} \\ &= 5.71 \times 10^{-5} \text{ T} \end{aligned}$$

Formula with $(r^2 - l^2)$ and $(r^2 + l^2)$ are used because here $r \gg l$ does not apply.

Example 5.8 Calculate the magnetic induction at a point $1 \text{ \AA}$ away from a proton, measured along its axis of spin. The magnetic moment of the proton is $1.4 \times 10^{-26} \text{ A-m}^2$.

Sol. On the axis of a magnetic dipole, magnetic induction is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

Substituting the values, we get

$$B = \frac{(10^{-7})(2)(1.4 \times 10^{-26})}{(10^{-10})^3} = 2.8 \times 10^{-3} \text{ T} = 2.8 \text{ mT}$$

Example 5.9 A short bar magnet has a magnetic moment of 0.48 JT^{-1} . Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of magnet on (i) the axial and (ii) the equatorial lines (normal bisector) of the magnet.

Sol. (i) When the point lies on the axial line, then let B_1 be the magnetic field at P .

$$r = 10 \text{ cm} = 0.1 \text{ m}$$

$$\begin{aligned} \therefore B_1 &= \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3} = 10^{-7} \times \frac{2 \times 0.48}{(0.1)^3} \\ &= 0.96 \times 10^{-4} \text{ T from S-pole to N-pole} \end{aligned}$$

(ii) Let B_2 be the magnetic field at point P on the equatorial line.

$$\begin{aligned} \therefore B_2 &= \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} = 10^{-7} \times \frac{0.48}{(0.1)^3} \\ &= 0.48 \times 10^{-4} \text{ T} = 0.48 \text{ G along from N-pole to S-pole.} \end{aligned}$$

Magnetic field due to a hypothetical monopole

At any point, magnetic field due to hypothetical monopole is given by

$$B = \frac{F_m}{m_0}$$

So, at a point P situated at distance r from a monopole, magnetic field is given by

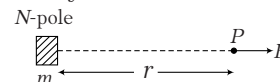


Fig. 5.12 Magnetic field due to a monopole (N-pole)

$$B = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^2}$$

Important points about magnetic field due to monopole

(i) It is away from pole, if it is N-pole as shown in Fig. 5.12.

(ii) It is towards pole, if it is S-pole, as shown in Fig. 5.13.

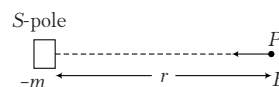


Fig. 5.13 Magnetic field due to S-pole

Current carrying loop as a magnetic dipole

Consider a planar loop of wire carrying current as shown in Fig. 5.14. Looking from the upper face, current is anti-clockwise. Therefore, it has a north polarity ($\curvearrowright$).

Looking from the lower face of the loop, current is clockwise ($\curvearrowleft$), therefore it has a south polarity. Thus, the

current carrying loop behaves as a system of two equal and opposite magnetic poles and can be considered as a magnetic dipole.

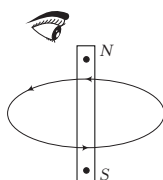


Fig. 5.14 Current loop with north and south polarity

Magnetic dipole moment of current carrying loop

The magnetic dipole moment of the current carrying loop (M) is directly proportional to

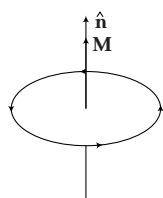


Fig. 5.15 Magnetic dipole moment of current loop

- (i) strength of current (I) through the loop and
- (ii) area (A) enclosed by the loop.

$$\text{i.e. } M \propto I \text{ and } M \propto A$$

$$\therefore M = kIA \quad \dots (i)$$

where, k is proportionality constant.

If we define unit magnetic dipole moment as that of a small single turn loop of unit area carrying unit current, then from Eq. (i), we get

$$1 = k \times 1 \times 1 \text{ or } k = 1$$

$$\therefore \text{From Eq. (i), } M = IA$$

$$\text{For } N \text{ turns, } M = NIA$$

$$\text{In vector form, } \mathbf{M} = NIA\hat{n}$$

where, $\hat{n}$ is unit vector perpendicular to the plane of the loop.

Here, the factor NI is called **ampere turns of current loop**. So, magnetic dipole moment of current loop = ampere turns $\times$ loop area

Note Direction of magnetic moment is given by right hand rule.

Example 5.10 A current of $6A$ is flowing through a 20 turns circular coil of radius 5 cm. The coil lies in the XY -plane. What is the magnitude and direction of the magnetic dipole moment associated with it?

Sol. Magnetic dipole moment is given by

$$M = NIA$$

$$\text{Here, } N = 20, I = 6A \text{ and } r = 5 \text{ cm} = 0.05 \text{ m}$$

$$\therefore M = (20) \times (6) \times \frac{22}{7} \times (0.05)^2$$

$$= 0.94 \text{ A-m}^2$$

The direction of magnetic dipole moment is perpendicular to the plane of the coil. Hence, it is along Z -axis.

Magnetic dipole moment of a revolving electron in an atom

The circular motion of an electron around the positively charged nucleus of an atom can be treated as a current loop producing a magnetic field. Hence, it behaves like a magnetic dipole.

The (negatively charged) electron is revolving anti-clockwise and so the current is clockwise. Hence, according to the right-hand rule, the dipole moment $\mathbf{M}$ is perpendicular to the plane of the current loop and is directed downwards.

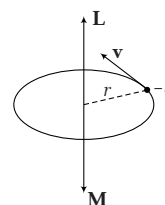


Fig. 5.16 Orbital magnetic moment of a revolving electron

Let m_e be the mass and $-e$ is the charge of an electron revolving with speed v in a circular orbit of radius r .

The magnitude of the magnetic dipole moment $\mathbf{M}$ associated with the revolving electron,

$$M = IA = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2} \quad \left(\because I = \frac{e}{T} = \frac{ev}{2\pi r} \right) \dots (i)$$

The magnitude of the orbital angular momentum $\mathbf{L}$ of electron,

$$L = m_e vr \quad \dots (ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{M}{L} = \frac{e}{2m_e} \quad \dots (iii)$$

Thus, the ratio of the magnitude of the magnetic dipole moment to the magnitude of the angular momentum of the revolving electron is a constant. This ratio is called the gyromagnetic ratio. Its value for an electron is $8.8 \times 10^{10} \text{ C/kg}$.

The vector form of Eq. (iii) can be written as

$$\mathbf{M} = - \left(\frac{e}{2m_e} \right) \mathbf{L}$$

Here, – ve sign means that $\mathbf{M}$ is directed opposite to $\mathbf{L}$.

Now, according to Bohr's quantisation principle, the angular momentum of a revolving electron assumes discrete values only. That means, it is an integral multiple of $\frac{h}{2\pi}$, where h is a fundamental constant of quantum

mechanics named after Max Planck, known as **Planck's constant** having value 6.626×10^{-34} J-s. Therefore, angular momentum can be written as

$$L = m_e v r = \frac{nh}{2\pi} \quad \dots(\text{iv})$$

where, n is an integer of values $n > 0$ and is also known as **principal quantum number**.

Now, from Eqs. (iii) and (iv), we get

$$\begin{aligned} \frac{M}{nh/2\pi} &= \frac{e}{2m_e} \\ \Rightarrow M &= \frac{neh}{4\pi m_e} \quad \dots(\text{v}) \end{aligned}$$

When $n = 1$, $M = \mu$ (the elementary magnetic dipole moment), thus

$$\therefore \mu = \frac{eh}{4\pi m_e} \quad \dots(\text{vi})$$

The elementary magnetic moment of a revolving electron is also known as **Bohr magneton** (μ).

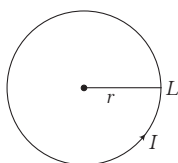
Now, substituting $e = 1.6 \times 10^{-19}$ C, $h = 6.626 \times 10^{-34}$ J-s, $\pi = 3.14$ and $m_e = 9.1 \times 10^{-31}$ kg in Eq. (vi), we get

$$\begin{aligned} \mu &= \frac{1.6 \times 10^{-19} \times 6.626 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} \text{ A-m}^2 \\ &= 9.27 \times 10^{-24} \text{ A-m}^2 \end{aligned}$$

$$\therefore 1 \text{ Bohr magneton} = 9.27 \times 10^{-24} \text{ A-m}^2$$

Example 5.11 A current I flows in a conducting wire of length L . If we bent it in a circular form, then calculate its magnetic dipole moment.

Sol. Let a wire of length L is bent in a circular form of radius r , then

$$2\pi r = L \Rightarrow r = \frac{L}{2\pi} \quad \dots(\text{i})$$


The magnetic dipole moment of a circular ring,

$$M = IA \quad (\text{where, } A \text{ is area of the ring})$$

$$\text{or } M = I\pi r^2 \quad \dots(\text{ii})$$

On putting the value of r from Eq. (i) in Eq. (ii), we get

$$M = I\pi \left(\frac{L}{2\pi} \right)^2$$

$$\Rightarrow M = I\pi \times \frac{L^2}{4\pi^2}$$

$$\Rightarrow M = \frac{IL^2}{4\pi} \text{ A-m}^2$$

Example 5.12 The electron in hydrogen atom moves with a speed of 2.2×10^6 m/s in an orbit of radius 5.3×10^{-11} cm. Find the magnetic moment of the orbiting electron.

Sol. Frequency of revolution, $f = \frac{v}{2\pi r}$

The moving charge is equivalent to a current loop, given by

$$I = f \times e \quad \text{or} \quad I = \frac{ev}{2\pi r}$$

If A be the area of the orbit, then the magnetic moment of the orbiting electron,

$$M = IA = \left(\frac{ev}{2\pi r} \right) (\pi r^2) = \frac{evr}{2}$$

Putting the values, we get

$$\begin{aligned} M &= \frac{(1.6 \times 10^{-19})(2.2 \times 10^6)(5.3 \times 10^{-11} \times 10^{-2})}{2} \\ &= 9.3 \times 10^{-26} \text{ A-m}^2 \end{aligned}$$

Bar magnet as an equivalent solenoid

The magnetic field lines for a bar magnet and a current carrying solenoid resemble very closely. Therefore, a bar magnet can be thought as a large number of circulating currents in analogy with a solenoid. The expression for the magnetic field due to a solenoid,

$$B = \frac{\mu_0}{4\pi} \frac{2m}{r^3}$$

where, r is the distance between the centre of solenoid to the required point.

This expression is same as the expression for magnetic field of a bar magnet at its axial point. Thus, a bar magnet and a solenoid produce similar magnetic fields.

Therefore magnetic field lines associated with a solenoid are also similar to a bar magnet as shown in the figure.

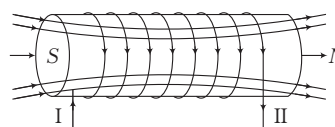


Fig. 5.17 Magnetic field lines due to a solenoid carrying current

Example 5.13 A closely wound solenoid of 800 turns and area of cross-section $2.5 \times 10^{-4} \text{ m}^2$ carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

Sol. It is given that, number of turns, $N = 800$

and area of cross-section, $A = 2.5 \times 10^{-4} \text{ m}^2$

Now, magnetic moment of the solenoid,

$$M = NIA = 800 \times 3.0 \times 2.5 \times 10^{-4} \\ = 0.60 \text{ JT}^{-1}$$

which acts along the axis of the solenoid in the direction related to the sense of flow of current according to right handed screw rule.

Bar magnet in a uniform magnetic field

When a bar magnet is placed in a uniform magnetic field, torque acts on it. Also, magnetic potential energy is associated with the magnet due to its orientation, which is discussed in following sections.

Torque on bar magnet in a uniform magnetic field

Let us consider a bar magnet of length $2l$ placed in a uniform magnetic field B . Let the magnetic axis of the bar magnet makes angle θ with the field B , as shown in the figure.

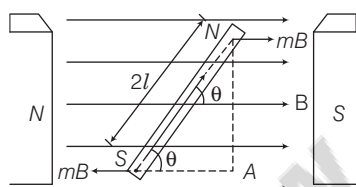


Fig. 5.18 Torque on a bar magnet

Then, force on N-pole = mB , along B

Force on S-pole = mB , opposite to B

where, m = strength of each pole

and B = strength of magnetic field.

These forces being equal and opposite form a couple, which tends to rotate the magnet clockwise, so as to align it along B . So, the moment of couple or torque is given by

$$\tau = \text{Force} \times \text{Perpendicular distance}$$

$$\Rightarrow \tau = mB \times NA \quad \dots(i)$$

$$\text{In } \triangle NAS, \quad \sin\theta = \frac{NA}{NS} = \frac{NA}{2l}$$

$$\therefore NA = 2l \sin\theta$$

$$\text{Eq. (i) becomes, } \tau = mB \times 2l \sin\theta$$

$$\text{Also, } M = m \times 2l$$

$$\Rightarrow \tau = MB \sin\theta \quad \dots(ii)$$

In vector form, we can rewrite this equation as $\tau = \mathbf{M} \times \mathbf{B}$

The direction of τ is perpendicular to the plane containing $\mathbf{M}$ and $\mathbf{B}$ and is given by right handed screw rule.

Work done or potential energy of a magnetic dipole (bar magnet) in a magnetic field

The torque acting on the dipole tends to align it in the direction of the field. Work has to be done in rotating the dipole against the action of the magnetic torque. This work done is stored in the form of potential energy of the dipole.

Now, a small amount of work done in rotating the dipole through a small angle $d\theta$ is given by

$$dW = \tau d\theta = MB \sin\theta \cdot d\theta \quad [\text{from Eq. (ii)}]$$

Total work done in rotating the dipole from $\theta = \theta_0$ to $\theta = \theta$ can be obtained by integrating the above expression.

$\therefore$ The potential energy of the dipole is given by

$$U = W = -MB(\cos\theta - \cos\theta_0)$$

Let us assume that, $\theta_0 = 90^\circ$

$$\text{So, } U = W = -MB(\cos\theta - \cos 90^\circ)$$

$$\text{Therefore, } U = -MB \cos\theta$$

In vector notation, we may rewrite this equation as

$$U = -\mathbf{M} \cdot \mathbf{B}$$

Special cases

(i) When dipole is perpendicular to magnetic field its potential energy is zero, i.e. when $\theta = 90^\circ$

$$\Rightarrow U = -MB \cos\theta = -MB \cos 90^\circ = 0$$

(ii) When the magnetic dipole is aligned along the magnetic field, then it is in stable equilibrium having minimum potential energy, i.e. when $\theta = 0^\circ$

$$\Rightarrow U = -MB \cos\theta = -MB \cos 0^\circ = -MB$$

(iii) When $\theta = 180^\circ$

$$U = -MB \cos\theta \\ = -MB \cos 180^\circ = MB$$

In this condition, potential energy is maximum.

Example 5.14 A magnet of magnetic moment $50\hat{i} \text{ A-m}^2$ is placed along the X-axis in a magnet field $\mathbf{B} = (0.5\hat{i} + 3\hat{j}) \text{ T}$. Find the torque acting on the magnet.

Sol. $\therefore$ Torque, $\tau = \mathbf{M} \times \mathbf{B}$

where, M is magnetic moment and B is magnetic field.

$$\text{Given, } M = 50\hat{i} \text{ A-m}^2, B = 0.5\hat{i} + 3\hat{j} \text{ T}$$

$$\therefore \tau = 50\hat{i} \times (0.5\hat{i} + 3\hat{j})$$

$$\tau = 150(\hat{i} \times \hat{j})$$

Using $\hat{i} \times \hat{j} = \hat{k}$, we have

$$\tau = 150 \hat{k} \text{ N-m}$$

Example 5.15 A bar magnet when placed at an angle of 30° to the direction of magnetic field of 5×10^{-2} T, experiences a moment of couple 2.5×10^{-6} N-m. If the length of the magnet is 5 cm, then what will be its pole strength?

Sol. Here, $\theta = 30^\circ$, $B = 5 \times 10^{-2}$ T,
 $\tau = 2.5 \times 10^{-6}$ N-m, and $2l = 5\text{cm} = 0.05$ m
 $\therefore$ Torque $\tau = MB \sin \theta = m(2l)B \sin \theta$
 $\Rightarrow m = \frac{\tau}{B(2l) \sin \theta} = \frac{2.5 \times 10^{-6}}{5 \times 10^{-2}(0.05) \sin 30^\circ}$
 $\therefore m = 2 \times 10^{-3}$ A-m

Example 5.16 The work done in turning a magnet of magnetic moment M by an angle 90° from the meridian is n times the corresponding work done to turn it through an angle of 60° . What is the value of n ?

Sol. Here, $W_1 = MB(\cos 0^\circ - \cos 90^\circ) = MB(1 - 0) = MB$
 Similarly, $W_2 = MB(\cos 0^\circ - \cos 60^\circ) = MB\left(1 - \frac{1}{2}\right) = \frac{MB}{2}$
 $\therefore W_1 = 2W_2 \Rightarrow n = 2$

Example 5.17 A bar magnet of magnetic moment 2.0 A-m^2 is free to rotate about a vertical axis through its centre. The magnet is released from rest from the east-west position. Find the kinetic energy of the magnet as it takes the north-south position. The horizontal component of the earth's magnetic field is $B = 25 \mu\text{T}$. Earth's magnetic field is from south to north.

Sol. Gain in kinetic energy = Loss in potential energy
 Thus, $\text{KE} = U_i - U_f$
 As, $U = -MB \cos \theta$
 Initially, $\theta_i = \frac{\pi}{2}$ (for east-west direction)
 and finally, $\theta_f = 0^\circ$ (for north-south direction)
 $\therefore \text{KE} = -MB \cos\left(\frac{\pi}{2}\right) - (-MB \cos 0^\circ) = -0 + MB = MB$
 Substituting the values, we have
 $\text{KE} = (2.0)(25 \times 10^{-6})\text{J} = 50 \mu\text{J}$

Example 5.18 A short bar magnet of magnetic moment $m = 0.32 \text{ JT}^{-1}$ is placed in a uniform magnetic field 0.15 T . If the bar magnet is free to rotate in the plane of the field, which orientation would correspond to its (i) stable and (ii) unstable equilibrium? What is the potential energy of the magnet in each case?

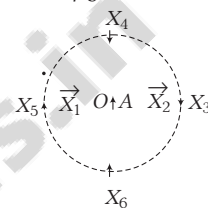
Sol. (i) When $\mathbf{m}$ is parallel to $\mathbf{B}$, then the magnet is in stable equilibrium, i.e. $\theta = 0^\circ$
 $\therefore$ PE in this case is given by
 $U = -\mathbf{m} \cdot \mathbf{B} = -mB \cos \theta$
 $= -0.32 \times 0.15 \times 1 = -0.048 \text{ J}$
 (ii) When $\mathbf{m}$ is anti-parallel to $\mathbf{B}$, then the magnet will be in unstable equilibrium, i.e. $\theta = 180^\circ$

Thus potential energy in this case is given by,

$$U = -\mathbf{m} \cdot \mathbf{B} = -mB \cos 180^\circ \\ = -0.32 \times 0.15 \times (-1) = +0.048 \text{ J}$$

Example 5.19 Consider the situation shown in the diagram, where a small magnetised needle A is placed at a centre marked as O. The direction of its magnetic moment is indicated by arrow. The other arrows show different positions (and orientations of the magnetic moment) of another identical magnetised needle X.

- In which configuration, the system is not in equilibrium?
- In which configuration is the system in (a) stable and (b) unstable equilibrium?
- Which configuration corresponds to the lowest potential energy among all the configurations shown?



Sol. Potential energy arises due to the position of dipole X in magnetic field of A.

Magnetic field due to A,

On end position, $\mathbf{B}_A = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$, along $\mathbf{M}$

On broadside position, $\mathbf{B}'_P = \frac{\mu_0}{4\pi} \cdot \frac{\mathbf{M}}{r^3}$, opposite to $\mathbf{M}$

The PE, $U = -\mathbf{M} \cdot \mathbf{B} = -MB \cos \theta$

For stable equilibrium, U is minimum, i.e. $\theta = 0^\circ$

For unstable equilibrium, U is maximum, i.e. $\theta = 180^\circ$

- In X_1 and X_2 , system is not in equilibrium.
- (a) In X_5 and X_6 , the system is in stable equilibrium and (b) in X_4 and X_3 , the system is in unstable equilibrium.
- X_6 corresponds to lowest PE,

$$U = -MB \cos \theta \\ = -M \cdot \frac{\mu_0}{4\pi} \frac{2M}{r^3} \cos 0^\circ = -\frac{\mu_0 \cdot 2M^2}{4\pi r^3} = U_{\min}$$

Oscillations of a freely suspended magnet

When a small bar magnet of magnetic moment $\mathbf{M}$ is placed in a uniform magnetic field $\mathbf{B}$ such as, it is free to vibrate in a horizontal plane of magnetic field $\mathbf{B}$ about a vertical axis passing through its centre of mass, then this bar magnet will oscillates. The restoring torque in this case will be

$$\tau = -MB\theta \quad (\because \text{For small oscillation, } \sin \theta \approx \theta)$$

The deflecting torque on the magnet,

$$\tau = I\alpha = I \frac{d^2\theta}{dt^2}$$

where, I is the moment of inertia of the magnet about the axis of rotation and $\frac{d^2\theta}{dt^2}$ is the angular acceleration.

In equilibrium, deflecting torque = restoring torque

$$\text{or } \frac{d^2\theta}{dt^2} = \frac{-MB\theta}{I} = -\omega^2\theta, \text{ where } \omega = \sqrt{\frac{MB}{I}}$$

i.e. angular acceleration $\frac{d^2\theta}{dt^2} \propto$ angular displacement θ

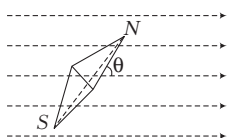
So, the oscillations of a freely suspended magnet in a uniform magnetic field are simple harmonic. The period of vibration is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB}}$$

Magnetic field B can be calculated from above equation and is given as

$$B = \frac{4\pi^2 I}{MT^2}$$

Example 5.20 A magnetic needle is free to oscillate in a uniform magnetic field as shown in figure. The magnetic moment of magnetic needle is 7.2 Am^2 and moment of inertia $I = 6.5 \times 10^{-6} \text{ kg-m}^2$. The number of oscillations performed in 5 s is 10. Calculate the magnitude of magnetic field.



Sol. Here, $T = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{5}{10} = 0.5 \text{ s}$

$$M = 7.2 \text{ Am}^2 \text{ and } I = 6.5 \times 10^{-6} \text{ kg-m}^2$$

$$\text{As, } T = 2\pi\sqrt{\frac{I}{MB}} \text{ or } T^2 = 4\pi^2 \frac{I}{MB}$$

The magnitude of the magnetic field,

$$B = \frac{4\pi^2 I}{MT^2} = \frac{4 \times (3.14)^2 \times 6.5 \times 10^{-6}}{7.2 \times (0.5)^2} = 1.42 \times 10^{-4} \text{ T}$$

Comparison between an electric dipole and a magnetic dipole: The electrostatic analogue

The behaviour of a magnetic dipole (may be a bar magnet also) is similar to the behaviour of an electric dipole. The only difference is that the electric dipole moment $\mathbf{p}$ is replaced by magnetic dipole moment $\mathbf{M}$ and the constant

$\frac{1}{4\pi\epsilon_0}$ is replaced by $\frac{\mu_0}{4\pi}$.

The table given below gives a comparison between an electric dipole and a magnetic dipole

Physical quantity to be compared	Electric dipole	Magnetic dipole
Dipole moment	$p = q(2l)$	$M = m(2l)$
Direction of dipole moment	From negative charge to the positive charge	From south pole to north pole
Net force in uniform field	0	0
Net torque in uniform field	$\tau = \mathbf{p} \times \mathbf{E}$	$\tau = \mathbf{M} \times \mathbf{B}$
Field at far away point on the axis	$\frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}$ (along $\mathbf{p}$)	$\frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$ (along $\mathbf{M}$)
Field at far away point on perpendicular bisector	$\frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$ (opposite to $\mathbf{p}$)	$\frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$ (opposite to $\mathbf{M}$)
Potential energy	$U_\theta = -\mathbf{p} \cdot \mathbf{E}$ $= -pE \cos\theta$	$U_\theta = -\mathbf{M} \cdot \mathbf{B}$ $= -MB \cos\theta$
Work done in rotating the dipole	$W_{\theta_1-\theta_2} = pE (\cos\theta_1 - \cos\theta_2)$	$W_{\theta_1-\theta_2} = MB (\cos\theta_1 - \cos\theta_2)$

Note In the table, θ is the angle between field ($\mathbf{E}$ or $\mathbf{B}$) and dipole moment ($\mathbf{p}$ or $\mathbf{M}$).

Magnetism and Gauss's law

The Gauss's law of magnetism states that, "the surface integral of a magnetic field over a closed surface is zero, i.e. the net magnetic flux through any closed surface is always zero".

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

Consequences of Gauss's law

- Consider a Gaussian surface (any closed surface) enclosing one of the poles (say south) of the magnet is shown in figure. Here, the number of field lines entering the Gaussian surface is same as the number of lines leaving it. i.e. Net magnetic flux for the whole Gaussian surface is zero.

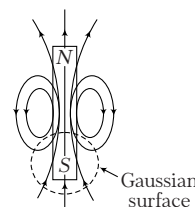


Fig. 5.19 Magnetic field lines are continuous curves

- The Gauss's theorem in magnetism implies that isolated magnetic poles do not exist.
- In case of open surface, $\int_S \mathbf{B} \cdot d\mathbf{S} = \phi_B$ (magnetic flux associated with the surface)
- The magnetic poles always exist as unlike pairs of equal strengths.

CHECK POINT 5.1

- Magnetic length is
 - less than geometric length
 - equal to geometric length
 - greater than geometric length
 - None of the above
- Magnetic lines of force due to a bar magnet do not intersect because
 - a point always has a single net magnetic field
 - the lines have similar charges and so repel each other
 - the lines always diverge from a single point
 - None of the above
- The unit of pole strength is
 - A-m
 - Am^{-1}
 - Am^{-2}
 - A-m^2
- A bar magnet of magnetic moment M_1 is axially cut into two equal parts. If these two pieces are arranged perpendicular to each other, the resultant magnetic moment is M_2 . Then, the value of M_1 / M_2 is
 - $\frac{1}{\sqrt{2}}$
 - 1
 - $\frac{1}{\sqrt{2}}$
 - $\sqrt{2}$
- At a point on the right bisector of a magnetic dipole, the magnetic
 - potential varies as $1/r^2$
 - potential is zero at all points on the right bisector
 - field varies as r^2
 - field is perpendicular to the axis of dipole
- The ratio of the magnetic fields due to small bar magnet in end on position to broadside on position is (at equal distance from the magnet)
 - 1/4
 - 1/2
 - 1
 - 2
- Two solenoids acting as short bar magnets P and Q are arranged such that their centres are on the X -axis and are separated by a large distance. The magnetic axes of P and Q are along X and Y -axes, respectively. At a point R , mid-way between their centres, if B is the magnitude of induction due to Q , then the magnitude of total induction at R due to the both magnets is
 - $3B$
 - $\sqrt{5}B$
 - $\frac{\sqrt{5}}{2}B$
 - B
- The intensity of magnetic field due to an isolated pole of strength m at a point distant r from it will be proportional to
 - $\frac{m}{r^2}$
 - mr^2
 - $\frac{r^2}{m}$
 - $\frac{m}{r}$
- A particle of charge q and mass m moves in a circular orbit of radius r with angular speed ω . The ratio of the magnitude of its magnetic moment to that of its angular momentum is
 - $-\frac{q}{2m}$
 - $\frac{q\omega r^2}{2}$
 - $\frac{q\omega}{2mr^2}$
 - $\frac{q\omega r^2}{2m}$
- A bar magnet of magnetic moment $\mathbf{M}$, is placed in a magnetic field of induction $\mathbf{B}$. The torque exerted on it is
 - $\mathbf{M} \times \mathbf{B}$
 - $-\mathbf{B} \cdot \mathbf{M}$
 - $\mathbf{M} \cdot \mathbf{B}$
 - $\mathbf{M} + \mathbf{B}$
- The couple acting on a magnet of length 10 cm and pole strength 15 A-m, kept in a field of $B = 2 \times 10^{-5}$ T, at an angle of 30° is
 - 1.5×10^{-5} N-m
 - 1.5×10^{-3} N-m
 - 1.5×10^{-2} N-m
 - 1.5×10^{-6} N-m
- A bar magnet is held at right angle to a uniform magnetic field. The couple acting on the magnet is to be halved by rotating it from this position. The angle of rotation is
 - 60°
 - 45°
 - 30°
 - 75°
- If a bar magnet of magnetic moment M is freely suspended in a uniform magnetic field of strength B , then the work done in rotating the magnet through an angle θ is
 - $MB(1 - \sin \theta)$
 - $MB \sin \theta$
 - $MB \cos \theta$
 - $MB(1 - \cos \theta)$
- The effect due to uniform magnetic field on a freely suspended magnetic needle is
 - Both torque and net force are present
 - torque is present but no net force
 - Both torque and net force are absent
 - net force is present but not torque
- The net magnetic flux through any closed surface kept in a magnetic field is
 - zero
 - $\frac{\mu_0}{4\pi}$
 - $4\pi\mu_0$
 - $\frac{4\mu_0}{\pi}$

EARTH'S MAGNETISM

Our earth behaves as a huge powerful magnet. The value of magnetic field on the surface of earth is a few tenths of a gauss ($1 \text{ G} = 10^{-4} \text{ T}$) and its strength varies from place to place on the earth's surface.

The earth's magnetic south pole is located near the geographic north pole and the earth's magnetic north pole is located near the geographic south pole.

In fact, the configuration of the earth's magnetic field is very much similar to the one that would be achieved by burying a gigantic bar magnet deep in the interior of the

earth. The axis of earth's magnet makes an angle of 11.5° with the earth's rotational axis. The magnetic lines of force around the earth are shown in figure.

Some definitions related to earth's magnetism are

(i) **Geographic axis** The straight line passing through the geographical north and south poles of the earth is called its geographic axis. It is the axis of rotation of the earth.

(ii) **Magnetic axis** The straight line passing through the magnetic north and south poles of the earth is called the magnetic axis.

- (iii) **Magnetic equator** It is the greatest circle on the earth perpendicular to the magnetic axis.
- (iv) **Magnetic meridian** The vertical plane in the direction of B is called magnetic meridian.
- (v) **Geographic meridian** The vertical plane passing through the line joining the geographical north and south poles is called the geographic meridian.

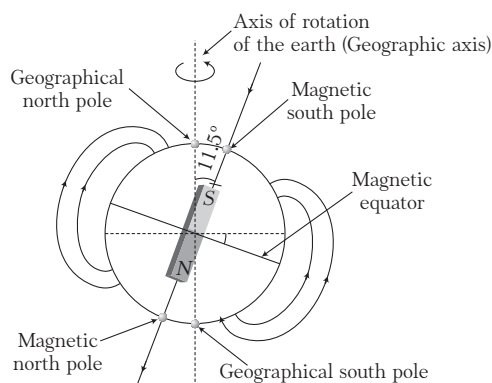


Fig. 5.20 Magnetic field of the earth

Note Earth's magnetic field varies irregularly from place-to-place and at a place it varies with time also.

Elements of earth's magnetism

The earth's magnetic field at a place can be completely described by three parameters which are called elements of earth's magnetic field. These three elements are

(i) Magnetic declination or Angle of declination (α)

At any place, the acute angle between the magnetic meridian and the geographical meridian is called **angle of declination α** . The value of α is small in India. It is $0^\circ 41' E$ for Delhi and $0^\circ 58' W$ for Mumbai. This means that at this place compass needle tells true north very accurately.

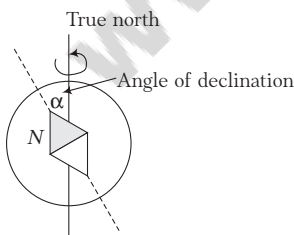


Fig. 5.21 Determining angle of declination

(ii) Magnetic inclination or Angle of dip (θ)

The angle of dip (θ) at a place is the angle between the direction of earth's magnetic field (B) and horizontal line in the magnetic meridian. At earth's magnetic poles, the

magnetic field of earth is vertical, i.e. angle of dip is 90° , the freely suspended magnetic needle is vertical there. At magnetic equator, field is horizontal or angle of dip is 0° .

(iii) Horizontal and vertical components of earth's magnetic field

Let B_e be the net magnetic field at some point and H & V be the horizontal and vertical components of B_e . Let θ is the angle of dip at the same place, then we can see that

$$H = B_e \cos \theta \quad \dots(i)$$

and

$$V = B_e \sin \theta \quad \dots(ii)$$

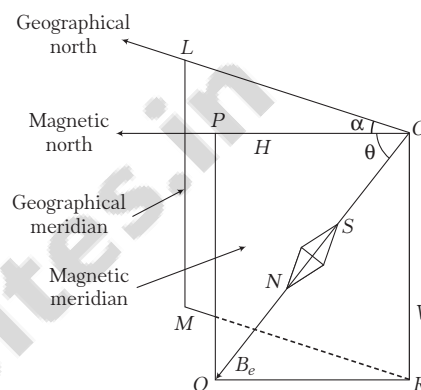


Fig. 5.22 Elements of earth's magnetic field

Squaring and adding Eqs. (i) and (ii), we get

$$B_e = \sqrt{H^2 + V^2}$$

Further, dividing Eq. (ii) by Eq. (i), we get

$$\theta = \tan^{-1} \left(\frac{V}{H} \right)$$

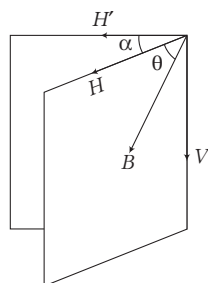
By knowing H and θ at some place, we can find B_e and V at that place.

Note

- (i) At magnetic equator, $H = B_e \cos 0^\circ = B_e$ and at poles, $H = B_e \cos 90^\circ = 0$. Similarly, at magnetic equator, $V = B_e \sin 0^\circ = 0$ and at poles, $V = B_e \sin 90^\circ = B_e$.
- (ii) $\tan \theta = 2 \tan \phi$, where ϕ is magnetic latitude of place.
- (iii) Magnetic maps show variation of magnetic elements from place-to-place. Some important lines drawn on magnetic maps are
 - (a) **Isoclinic lines** These are the lines joining points of equal dip or inclination. A line joining places of zero dip is called **aclinic line** or **magnetic equator**.
 - (b) **Isogonic lines** These are the lines joining places of equal declination. The line joining places of zero declination is called **agonic line**.
 - (c) **Isodynamic lines** These are the lines joining places having the same value of horizontal component of earth's magnetic field.
- (iv) In a vertical plane making an angle α to the magnetic meridian

$$H' = H \cos \alpha, V' = V$$

So, angle of dip in vertical plane making an angle α with magnetic meridian, $\tan \theta' = \frac{\tan \theta}{\cos \alpha}$



Example 5.21 A compass needle of magnetic moment $60 \text{ A}\cdot\text{m}^2$ is pointing geographical north at a certain place. It experiences a torque of $1.2 \times 10^{-3} \text{ N}\cdot\text{m}$. The horizontal component of earth's magnetic field at that place is $40 \mu\text{Wb}/\text{m}^2$. What is the angle of declination at that place?

Sol. A compass needle in stable equilibrium position points towards magnetic north, i.e. along the horizontal component H of earth's magnetic field. When it is turned through the angle of declination α , so as to point geographical north, then it experiences a torque of magnitude $MH \sin \alpha$.

$$\therefore MH \sin \alpha = 1.2 \times 10^{-3} \text{ N}\cdot\text{m} \text{ (given)}$$

$$\text{Here, } M = 60 \text{ A}\cdot\text{m}^2,$$

$$H = 40 \times 10^{-6} \text{ Wb}/\text{m}^2$$

$$\therefore \sin \alpha = \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = 0.5$$

$$\therefore \alpha = 30^\circ$$

Example 5.22 In the magnetic meridian of a certain place, the horizontal component of earth's magnetic field is 0.26 G and the dip angle is 60° . Find

- vertical component of earth's magnetic field
- and the net magnetic field at this place.

Sol. Given, $H = 0.26 \text{ G}$ and $\theta = 60^\circ$

$$\text{(i) As, } \tan \theta = \frac{V}{H}$$

$$\therefore V = H \tan \theta = (0.26) \tan 60^\circ = 0.45 \text{ G}$$

$$\text{(ii) As, } H = B_e \cos \theta$$

$$\therefore B_e = \frac{H}{\cos \theta} = \frac{0.26}{\cos 60^\circ} = 0.52 \text{ G}$$

Example 5.23 The horizontal and vertical components of earth's field at a place are 0.22 G and 0.38 G , respectively. Calculate the angle of dip and resultant intensity of earth's field.

Sol. Here, $H = 0.22 \text{ G}$ and $V = 0.38 \text{ G}$

$$\text{Now, } \tan \theta = \frac{V}{H} = \frac{0.38}{0.22} = 1.7272$$

$$\therefore \text{Angle of dip, } \theta = 59^\circ 56'$$

Resultant magnetic field of the earth,

$$B = \sqrt{H^2 + V^2} = \sqrt{0.22^2 + 0.38^2} = 0.439 \text{ G}$$

Example 5.24 At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G . Specify the direction and magnitude of the earth's field at the location.

Sol. Using the relation, $H = B_e \cos \theta$, we get the magnitude of B

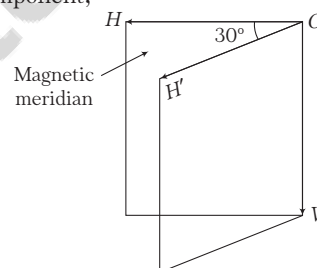
$$\text{given by } B_e = \frac{H}{\cos \theta} = \frac{0.16}{\cos 60^\circ} = \frac{0.16}{\left(\frac{1}{2}\right)} = 0.16 \times 2 = 0.32 \text{ G}$$

$$\text{or } B = 0.32 \times 10^{-4} \text{ T} \quad [\because 1 \text{ G} = 10^{-4} \text{ T}]$$

Direction of B The earth's field lies in a vertical plane 12° west of geographic meridian at an angle of 60° above the horizontal line.

Example 5.25 A magnetic needle suspended in a vertical plane at 30° from the magnetic meridian makes an angle of 45° with the horizontal. Find the true angle of dip.

Sol. In a vertical plane at 30° from the magnetic meridian, the horizontal component,



$$H' = H \cos 30^\circ$$

While vertical component is still V , therefore apparent dip

$$\text{will be given by } \tan \theta' = \frac{V}{H'} = \frac{V}{H \cos 30^\circ}$$

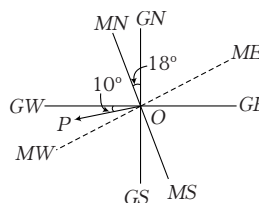
$$\text{But } \frac{V}{H} = \tan \theta'$$

$$\therefore \tan \theta' = \frac{\tan \theta}{\cos 30^\circ} \quad (\text{where, } \theta = \text{true angle of dip})$$

$$\therefore \theta = \tan^{-1} [\tan \theta' \cos 30^\circ] \\ = \tan^{-1} [(\tan 45^\circ) (\cos 30^\circ)] = \tan^{-1} \left(1 \times \frac{\sqrt{3}}{2} \right) \approx 41^\circ$$

Example 5.26 A ship is to reach a place 10° south of west. In which direction should it be steered, if the declination at the place is 18° west of north?

Sol. As the ship has to reach a place 10° south of west, i.e. along OP , so it should be steered west of magnetic north at angle of $90^\circ - 18^\circ + 10^\circ = 82^\circ$.



Example 5.27 A dip circle shows an apparent dip of 45° at a place where the true dip is 30° . If the dip circle is rotated through 90° , what apparent dip will it show?

Sol. Let θ_1 and θ_2 are the angles of dip in two arbitrary planes which are perpendicular to each other.

Here, $\theta_1 = 45^\circ$ and $\theta = 30^\circ$

As, $\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2$

where, θ is true dip.

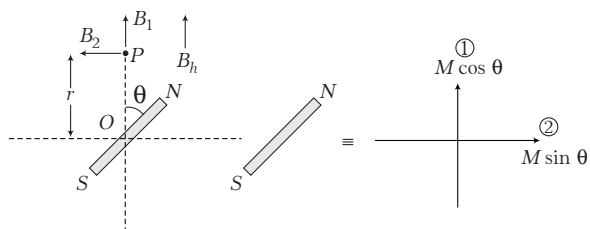
$$\therefore \cot^2 30^\circ = \cot^2 45^\circ + \cot^2 \theta_2$$

$$\cot^2 \theta_2 = 3 - 1 = 2 \Rightarrow \cot \theta_2 = 1.414$$

$$\therefore \theta_2 = 35.2^\circ$$

Example 5.28 A short magnet ($M = 4 \times 10^{-2} \text{ A-m}^2$) lying in a horizontal plane with its north-pole points 37° east of north. Find the net horizontal field at a point 0.1 m away from the magnet (Here, $B_h = 11 \mu \text{ T}$) ($\sin 37^\circ = 3/5$, $\cos 37^\circ = 4/5$).

Sol.



Magnetic field due to magnet at P,

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M \cos \theta}{r^3}$$

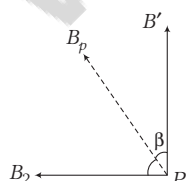
$$= \frac{10^{-7} \times 2 \times 4 \times 10^{-2} \times \frac{4}{5}}{(0.1)^3} = 6.4 \times 10^{-6} \text{ T}$$

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M \sin \theta}{r^3} = \frac{10^{-7} \times 4 \times 10^{-2} \times \frac{3}{5}}{(0.1)^3} = 2.4 \times 10^{-6} \text{ T}$$

Since, B_1 and B_h are in same direction,

$$\therefore B' = B_1 + B_h = 6.4 \times 10^{-6} + 11 \times 10^{-6}$$

$$= 17.4 \times 10^{-6} \text{ T}$$



$$\therefore B_P = \sqrt{B'^2 + B_2^2} = \sqrt{(17.4)^2 + (2.4)^2} \times 10^{-6}$$

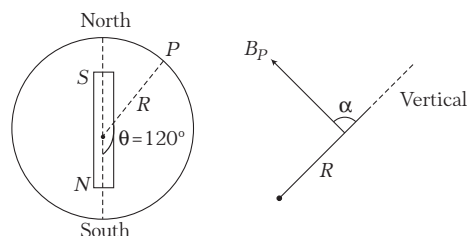
$$= \sqrt{308.52} \times 10^{-6} \text{ T}$$

$$= 17.5 \times 10^{-6} \text{ T}$$

$$\text{and } \tan \beta = \frac{B_2}{B'} = \frac{2.4 \times 10^{-6}}{17.5 \times 10^{-6}} = 0.14$$

Example 5.29 The earth's magnetic field at geomagnetic poles has a magnitude $8 \times 10^{-5} \text{ T}$. Find the magnitude and the direction of the field at a point on the earth's surface where the radius makes an angle of 120° with the axis of the earth's assumed magnetic dipole. What is the inclination (dip) at this point?

Sol.



The geomagnetic poles are in end-on-position.

The magnetic field at geomagnetic poles,

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{2M}{R^3} = 8 \times 10^{-5} \text{ T} \quad (\text{given})$$

The magnetic field at point P,

$$B_P = \frac{\mu_0 M}{4\pi R^3} (1 + 3 \cos^2 \theta)^{1/2}$$

$$= \frac{B'}{2} (1 + 3 \cos^2 \theta)^{1/2}$$

$$= \frac{8 \times 10^{-5}}{2} [1 + 3 \cos^2 120^\circ]^{1/2}$$

$$= 4 \times 10^{-5} \left[1 + \frac{3}{4} \right]^{1/2}$$

$$\text{Hence, } \tan \alpha = \frac{1}{2} \tan \theta = \frac{1}{2} \tan 120^\circ$$

$$= \frac{1}{2} \times (-\sqrt{3}) = -\frac{\sqrt{3}}{2}$$

$$\alpha = \tan^{-1} \left(-\frac{\sqrt{3}}{2} \right)$$

Dip ϕ is an angle made by the earth's magnetic field with the horizontal plane,

$$\phi = \alpha - 90^\circ = \tan^{-1} \left(-\frac{\sqrt{3}}{2} \right) - 90^\circ$$

Neutral points

When we trace magnetic lines of force around a magnet using a compass needle, we obtain the resultant of the magnetic field of magnet and that of the earth. In the plot of the resultant magnetic field, we come across points at which field (B) due to the magnet becomes equal and opposite to the horizontal component (H) of the earth's field, i.e. $B = H$. Therefore, the net magnetic field at these points will be zero.

So, the points where net magnetic field due to the magnet and due to the earth's horizontal component is zero are called **neutral points**.

A small compass needle placed at a neutral point shall experience no force/torque. Therefore, it can set itself in any direction, which may be different from usual *N-S* direction.

- (i) When a magnet is placed with its north pole towards geographic south, then neutral points lie on axial line of the magnet.

At each neutral point,

$$B_1 = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - l^2)^2} = H$$

For a short magnet ($l \ll r$), $H = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$

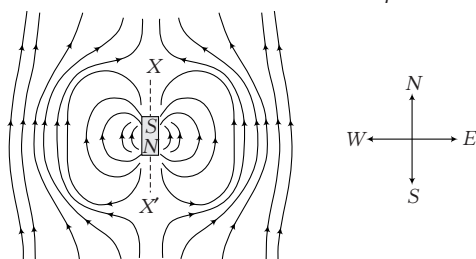


Fig. 5.23 Neutral points on axial line

- (ii) When a magnet is placed with its north pole towards geographic north, neutral points lie on equatorial line of the magnet.

At each neutral point, $B_2 = \frac{\mu_0}{4\pi} \frac{M}{(r^2 + l^2)^{3/2}} = H$

For a short magnet ($l \ll r$), $H = \frac{\mu_0}{4\pi} \frac{M}{r^3}$

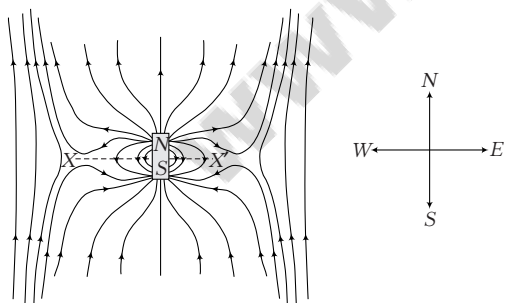


Fig. 5.24 Neutral points on equatorial line

Note When the magnet is held vertically on the board, there will be only one neutral point on a horizontal board. In the other orientations, there will be two neutral points.

Example 5.30 A bar magnet 30 cm long is placed in the magnetic meridian with its north pole pointing south. The neutral point is observed at a distance of 30 cm from its one end. Calculate the pole strength of the magnet. (Given, horizontal component of earth's field = 0.34 G)

Sol. Here, $2l = 30$ cm or $l = 15$ cm = 0.15 m,

$$r = 30$$
 cm = 0.30 m and $H = 0.34$ G = 0.34×10^{-4} T

When magnet is placed with its north pole pointing south, then neutral point is obtained on its axial line.

$$\therefore B_{\text{axial}} = H \quad \text{or} \quad \frac{\mu_0}{4\pi} \times \frac{2Mr}{(r^2 - l^2)^2} = H$$

$$\begin{aligned} \text{or} \quad M &= \frac{4\pi}{\mu_0} \times \frac{H(r^2 - l^2)^2}{2r} \\ &= \frac{1}{10^{-7}} \times \frac{0.34 \times 10^{-4} \times (0.30^2 - 0.15^2)^2}{2 \times 0.30} \\ &= \frac{0.34 \times 10^{-4} \times (0.0675)^2}{10^{-7} \times 2 \times 0.30} = 2.582 \text{ A-m}^2 \end{aligned}$$

The pole strength of the magnet, $m = \frac{M}{2l} = \frac{2.582}{0.30} = 8.607$ A-m

Example 5.31 A short bar magnet is placed with its north pole pointing north. The neutral point is 10 cm away from the centre of the magnet. If $H = 0.4$ G, calculate the magnetic moment of the magnet.

Sol. When north pole of the magnet points towards magnetic north, null point is obtained on perpendicular bisector of the magnet. Simultaneously, magnetic field due to the bar magnet should be equal to the horizontal component of earth's magnetic field H .

$$\text{Thus,} \quad H = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \quad \text{or} \quad M = \frac{Hr^3}{(\mu_0/4\pi)}$$

Substituting the values, we get

$$M = \frac{(0.4 \times 10^{-4}) (10 \times 10^{-2})^3}{10^{-7}} = 0.4 \text{ A-m}^2$$

Tangent galvanometer

It is a device used to measure very small current. It is a moving magnet type galvanometer and works on the principle of tangent law.

Tangent law

It states that, if a magnet is placed in two magnetic fields right angle to each other, then it will be acted upon by two couples tending to rotate it in opposite directions.

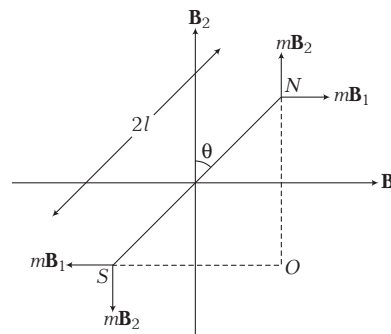


Fig. 5.25 Tangent law

The magnet will be deflected through an angle θ , such that the two couples balance each other and the tangent of the angle of deflection gives the ratio of the two fields as

$$\tan \theta = \frac{B_1}{B_2}$$

where, θ is angle between magnet and magnetic field B_2 .

In case of tangent galvanometer, a magnetic compass needle is placed horizontally at the centre of a vertically fixed current-carrying coil whose plane is in the magnetic meridian.

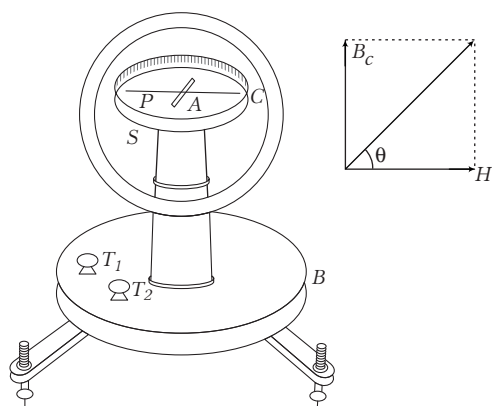


Fig. 5.26 Tangent galvanometer

As shown in figure, pointer P is rigidly attached to compass needle and perpendicular to it. The compass needle together with pointer can rotate freely about the vertical axis.

When no current flows through the coil, its plane is parallel to compass needle and coincides with magnetic meridian. When current passes through the coil, it produces magnetic field at the centre which deflects the compass needle and pointer deflects through the same angle. The magnetic field produced at centre due to a coil,

$$B_c = \frac{\mu_0 NI}{2r}$$

where, N is number of turns in the coil, r is radius of coil and I is current in the coil.

Field B_c is perpendicular to the plane of coil, i.e. perpendicular to magnetic meridian or perpendicular to the horizontal component of earth's magnetic field.

The resultant horizontal magnetic field,

$$B = \sqrt{B_c^2 + H^2}$$

$$\Rightarrow \tan \theta = \frac{B_c}{H} \quad (\text{tangent law of perpendicular field})$$

The needle will stay in equilibrium when its length is parallel to B .

The deflection of the needle from its original position θ is given by

$$H \tan \theta = B_c = \frac{\mu_0 NI}{2r} \Rightarrow I = \frac{2rH}{\mu_0 N} \tan \theta = k \tan \theta \quad \dots(i)$$

where, $k = \frac{2rH}{\mu_0 N}$ is constant for a given galvanometer at

given place and is called **reduction factor of galvanometer**.

Sensitivity

Sensitivity is the measure of change in the deflection produced by a unit current.

$$\text{It is given by } \frac{d\theta}{dI} = \frac{1}{k \sec^2 \theta} = \frac{1}{k \left(1 + \frac{I^2}{k^2} \right)} \quad [\text{using Eq. (i) and } \sec^2 \theta = 1 + \tan^2 \theta]$$

For good sensitivity, the change in deflection should be large for a given fractional change of current.

Note The tangent galvanometer is most sensitive when deflection is around 45° .

Example 5.32 In a tangent galvanometer, when a current of 10 mA is passed, the deflection is 31° . By what percentage, the current has to be increased, so as to produce a deflection of 42° ?

Sol. Here, $I_1 = 10 \text{ mA}$, $\theta_1 = 31^\circ$, $I_2 = ?$, $\theta_2 = 42^\circ$

$$\text{As, } \frac{I_2}{I_1} = \frac{\tan \theta_2}{\tan \theta_1}$$

$$\therefore I_2 = I_1 \frac{\tan \theta_2}{\tan \theta_1} = 10 \times \frac{\tan 42^\circ}{\tan 31^\circ} = \frac{10 \times 0.9}{0.6} = 15 \text{ mA}$$

Percentage increase in current,

$$\frac{(I_2 - I_1)}{I_1} \times 100 = \frac{(15 - 10)}{10} \times 100 = 50\%$$

Example 5.33 The coil of a tangent galvanometer of radius 12 cm is having 200 turns. If the horizontal component of earth's magnetic field is $25 \mu\text{T}$, find the current which gives a deflection of 60° .

Sol. Given, r = radius of coil = 12 cm = 0.12 m,

N = number of turns of coil = 200,

$H = 25 \mu\text{T} = 25 \times 10^{-6} \text{ T}$ and $\phi = 60^\circ$

As we know, $I = k \tan \theta$

$$\text{where, } k = \frac{2rH}{\mu_0 N}$$

$$\therefore I = \frac{2rH}{\mu_0 N} \tan \theta = \frac{2 \times 0.12 \times 25 \times 10^{-6} \times \tan 60^\circ}{4\pi \times 10^{-7} \times 200} = 0.042 \text{ A}$$

Vibration magnetometer

It is an instrument which is used to find magnetic moment of bar magnet and to compare magnetic fields of two magnets. This device works on the principle that whenever a freely suspended magnet in a uniform magnetic field is disturbed from its equilibrium position, it starts vibrating about the mean position.

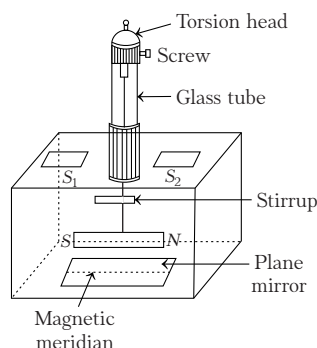


Fig. 5.27 Vibration magnetometer

Time period of oscillation of experimental bar magnet (with magnetic moment M) in earth's magnetic field (H) is given by the formula, $T = 2\pi\sqrt{\frac{I}{MH}}$, where I = moment of inertia of short bar magnet.

- (i) **Comparison of horizontal components of earth's magnetic field at two places**

$$T = 2\pi\sqrt{\frac{I}{MH}}, \text{ since } I \text{ and } M \text{ of the magnet are constants.}$$

$$\text{So, } T^2 \propto \frac{1}{H} \Rightarrow \frac{H_1}{H_2} = \frac{T_2^2}{T_1^2}$$

- (ii) **Determination of magnetic moment of a magnet**

The experimental (given) magnet is put into vibration magnetometer and its time period T is determined.

$$\text{Now, } T = 2\pi\sqrt{\frac{I}{MH}} \Rightarrow M = \frac{4\pi^2 I}{H \cdot T^2}$$

- (iii) **Comparison of magnetic moment of two magnets of same size and mass**

$$\text{As, } T = 2\pi\sqrt{\frac{I}{MH}}, \text{ here } I \text{ and } H \text{ are constants.}$$

$$\text{So, } M \propto \frac{1}{T^2} \Rightarrow \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

- (iv) **Comparison of magnetic moments of magnets of different sizes**

Following methods are used for the respective positions of magnets

Sum position This method can be used when magnets are placed with their poles in same direction.

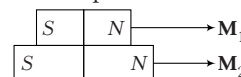


Fig. 5.28

Net magnetic moment, $M_s = M_1 + M_2$

Net moment of inertia, $I_s = I_1 + I_2$

Time period of oscillation of this pair in earth's magnetic field H ,

$$T_s = 2\pi\sqrt{\frac{I_s}{M_s H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)H}} \dots(i)$$

$$\therefore \text{Frequency, } \nu_s = \frac{1}{2\pi} \sqrt{\frac{(M_1 + M_2)H}{I_s}}$$

Difference position This method can be used when magnets are placed with their poles in opposite direction.



Fig. 5.29

Net magnetic moment, $M_d = M_1 - M_2$

Net moment of inertia, $I_d = I_1 + I_2$

$$\text{and } T_d = 2\pi\sqrt{\frac{I_d}{M_d H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)H}} \dots(ii)$$

$$\therefore \text{Frequency, } \nu_d = \frac{1}{2\pi} \sqrt{\frac{(M_1 - M_2)H}{I_1 + I_2}}$$

So, from Eqs. (i) and (ii), we get

$$\frac{T_s}{T_d} = \sqrt{\frac{(M_1 - M_2)}{(M_1 + M_2)}} \Rightarrow \frac{M_1}{M_2} = \frac{T_s^2 + T_d^2}{T_d^2 - T_s^2}$$

- (ii) **Comparison of two magnetic fields** Suppose we wish to compare the magnetic fields B_1 and B_2 at some point P due to two magnets. For this, vibration magnetometer is so placed that the centre of its magnet lies on P .

Now one of the given magnets is placed at some known distance from P in the magnetic meridian, such that point P lies on its axial line and its north pole points north. In this position, the fields B_1 at P produced by the magnet will be in the direction of H .

Hence, the magnet suspended in the magnetometer will vibrate in the resultant magnetic field $(H + B_1)$. Its period of vibration is noted, say it is T_1 , then

$$T_1 = 2\pi\sqrt{\frac{I}{M(H + B_1)}}$$

Now the first magnet is replaced by the second magnet and the second magnet is placed in the same position and again the time period is noted.

If the field produced at P due to this magnet be B_2 and the new time period be T_2 , then $T_2 = 2\pi \sqrt{\frac{I}{M(H+B_2)}}$.

Finally, the time period of the magnetometer under the influence of the earth's magnetic field alone is determined. Let it be T , then $T = 2\pi \sqrt{\frac{I}{MH}}$.

Solving above three equations for T , T_1 and T_2 , we can show that

$$\frac{B_1}{B_2} = \frac{(T^2 - T_1^2) T_2^2}{(T^2 - T_2^2) T_1^2}$$

Example 5.34 The time period of vibration of two magnets in sum position (magnets placed with similar poles on one side one above the other) is 3 s. When polarity of weaker magnet is reversed the combination makes 12 oscillations per minute. What is the ratio of magnetic moments of two magnets?

Sol. Here, $T_1 = 3\text{ s}$ and $T_2 = \frac{1}{12} \text{ min} = \frac{60\text{ s}}{12} = 5\text{ s}$

As, we know that

$$\begin{aligned} \frac{M_1}{M_2} &= \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} \\ \Rightarrow \frac{M_1}{M_2} &= \frac{5^2 + 3^2}{5^2 - 3^2} = \frac{34}{16} = \frac{17}{8} \end{aligned}$$

Example 5.35 A bar magnet of length 5 cm, width 3 cm and height 2 cm takes 5 s to complete an oscillation in vibration magnetometer placed in a horizontal magnetic field of $20 \mu\text{T}$. The mass of this bar magnet is 250 g. (i) Find the magnetic moment of the magnet. (ii) If the magnet is put in the magnetometer with its 0.5 cm edge horizontal, what would be the new time period?

Sol. (i) Moment of inertia of magnet is given by

$$I = \frac{m(l^2 + b^2)}{12}$$

where m = mass of magnet.

$$\begin{aligned} \therefore I &= \frac{250(5^2 + 3^2) \times 10^{-4} \times 10^{-3}}{12} \\ &= 7.08 \times 10^{-5} \text{ kg-m}^2 \end{aligned}$$

$$\text{Also, } T = 2\pi \sqrt{\frac{I}{MH}}$$

$$\Rightarrow M = \frac{4\pi^2 I}{HT^2}$$

On putting values, we get

$$M = \frac{4 \times (3.14)^2 \times (7.08 \times 10^{-5})}{20 \times 10^{-6} \times 5 \times 5} = 5.58 \text{ Am}^2$$

(ii) New moment of inertia is given by

$$I' = \frac{m_0(l^2 + h^2)}{12}$$

$$\begin{aligned} \therefore T' &= 2\pi \sqrt{\frac{I'}{MH}} \\ \Rightarrow \frac{T'}{T} &= \sqrt{\frac{I'}{I}} = \sqrt{\frac{l^2 + h^2}{l^2 + b^2}} = \sqrt{\frac{5^2 + (0.5)^2}{5^2 + 3^2}} \\ \Rightarrow \frac{T'}{T} &= 0.86 \\ \therefore T' &= T \times 0.86 = 5 \times 0.86 = 4.30 \text{ s} \end{aligned}$$

Example 5.36 A magnetic needle performs 20 oscillations per minute in a horizontal plane. If the angle of dip be 30° , then how many oscillations per minute will this needle perform in vertical north-south plane and in vertical east-west plane?

Sol. In horizontal plane, the magnetic needle oscillates earth's horizontal component H .

$$\therefore T = 2\pi \sqrt{\frac{I}{MH}}$$

In the vertical north-south plane (magnetic meridian), the needle oscillates in the total earth's magnetic field B_e and in vertical east-west plane (plane perpendicular to the magnetic meridian) it oscillates only in earth's vertical component V . If its time period be T_1 and T_2 , then

$$\begin{aligned} T_1 &= 2\pi \sqrt{\frac{I}{MB_e}} \\ \text{and } T_2 &= 2\pi \sqrt{\frac{I}{MV}} \end{aligned}$$

From above equations, we can find,

$$\frac{T_1^2}{T^2} = \frac{H}{B_e} \text{ or } \frac{n_1^2}{n^2} = \frac{B_e}{H}$$

$$\text{Similarly, } \frac{n_2^2}{n^2} = \frac{V}{H}$$

$$\text{Further, } \frac{B_e}{H} = \sec \theta = \sec 30^\circ = \frac{2}{\sqrt{3}}$$

$$\text{and } \frac{V}{H} = \tan \theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\therefore n_1^2 = (n)^2 \left(\frac{B_e}{H} \right) = (20)^2 \left(\frac{2}{\sqrt{3}} \right)$$

$$\text{or } n_1 = 21.5 \text{ oscillations/min}$$

$$\text{and } n_2^2 = (n)^2 \left(\frac{V}{H} \right) = (20)^2 \left(\frac{1}{\sqrt{3}} \right)$$

$$\therefore n_2 = 15.2 \text{ oscillations/min}$$

Example 5.37 A magnet performs 15 oscillations per minute in a horizontal plane, where angle of dip is 60° and earth's total field is 0.5 G. At another place, where total field is 0.6 G, the magnet performs 20 oscillations per minute. What is the angle of dip at this place?

Sol. As, $H = B \cos \theta$

where, H = horizontal component of earth's magnetic field,
 B = total earth's magnetic field

and θ = angle of dip

Then, $H_1 = B_1 \cos \theta_1$ and $H_2 = B_2 \cos \theta_2$

$$\text{Further, } T_1 = 2\pi \sqrt{\frac{I}{MH_1}} = 2\pi \sqrt{\frac{T}{MB_1 \cos \theta_1}}$$

$$\text{and } T_2 = 2\pi \sqrt{\frac{I}{MH_2}} = 2\pi \sqrt{\frac{I}{MB_2 \cos \theta_2}}$$

$$\therefore \frac{T_1^2}{T_2^2} = \frac{B_2 \cos \theta_2}{B_1 \cos \theta_1}$$

$$\begin{aligned} \therefore \cos \theta_2 &= \frac{B_1}{B_2} \times \frac{T_1^2}{T_2^2} \cos \theta_1 \\ &= \frac{B_1}{B_2} \times \left(\frac{v_2}{v_1}\right)^2 \cos \theta_1 \\ &= \left(\frac{0.5}{0.6}\right) \left(\frac{20}{15}\right)^2 \cos 60^\circ = 0.74 \end{aligned}$$

$$\therefore \theta_2 = \cos^{-1}(0.74) = 42.2^\circ$$

Example 5.38 A small bar magnet having a magnetic moment of $9 \times 10^{-9} \text{ A-m}^2$ is suspended at its centre of gravity by a light torsionless string at a distance of 10^{-2} m vertically above a long, straight horizontal wire carrying a current of 1.0 A from east to west. Find the frequency of oscillation of the magnet about its equilibrium position. The moment of inertia of the magnet is $6 \times 10^{-9} \text{ kg-m}^2$. (Take, $H = 3 \times 10^{-5} \text{ T}$)

Sol. The magnetic moment of the bar magnet,

$$M = 9 \times 10^{-9} \text{ A-m}^2$$

The magnitude of the magnetic field at the location of the magnet due to current carrying wire is,

$$\begin{aligned} B &= \frac{\mu_0}{2\pi} \frac{i}{r} = \frac{(2 \times 10^{-7}) (1.0)}{10^{-2}} \\ &= 2 \times 10^{-5} \text{ Wb/m}^2 \quad (\text{S to N}) \end{aligned}$$

The earth's horizontal magnetic field,

$$\begin{aligned} H &= 3 \times 10^{-5} \text{ T} \\ &= 3 \times 10^{-5} \text{ Wb/m}^2 \end{aligned}$$

$$\therefore B + H = 5 \times 10^{-5} \text{ Wb/m}^2$$

The frequency of oscillation will be

$$\begin{aligned} v &= \frac{1}{2\pi} \sqrt{\frac{M(B+H)}{I}} \quad [\text{here, } I = \text{moment of inertia}] \\ &= \frac{1}{2\pi} \sqrt{\frac{(9 \times 10^{-9}) \times (5 \times 10^{-5})}{6 \times 10^{-9}}} \quad [\text{S to N}] \\ &= 1.38 \times 10^{-3} \text{ Hz} \end{aligned}$$

Example 5.39 A thin rectangular magnet suspended freely has a period of oscillation equal to T . Now, it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T' , then find the ratio T'/T .

Sol. When magnet is divided into two equal parts, the magnetic dipole moment,

$$M' = \text{pole strength} \times \frac{l}{2} = \frac{M}{2} \quad [\text{pole strength remain same}]$$

Also, the mass of magnet becomes half, i.e. $m' = \frac{m}{2}$.

Moment of inertia of magnet, $I = \frac{ml^2}{12}$.

$$\text{New moment of inertia, } I' = \frac{1}{12} \left(\frac{m}{2}\right) \left(\frac{l}{2}\right)^2 = \frac{ml^2}{12 \times 8}$$

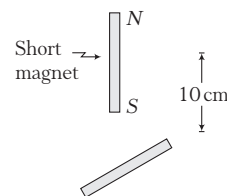
$$\Rightarrow I' = \frac{I}{8}$$

$$\text{Now, } T = 2\pi \sqrt{\frac{I}{MB}}$$

$$\text{and } T' = 2\pi \sqrt{\frac{I'}{M'B}} = 2\pi \sqrt{\frac{I/8}{MB/2}}$$

$$T' = \frac{T}{2} \Rightarrow \frac{T'}{T} = \frac{1}{2}$$

Example 5.40 The time period of the magnet in an oscillation magnetometer in the earth's magnetic field is 2 s . A short bar magnet is placed to the north of the magnetometer, at a separation 10 cm from the oscillating magnet, with its north pole pointing towards north. The time period becomes half. Calculate the magnetic moment of this short magnet. (Take, $B_h = 12 \mu\text{T}$)



Sol. Time period, $T = 2\pi \sqrt{\frac{I}{MB}}$

$$\Rightarrow T \propto \frac{1}{\sqrt{B}} \Rightarrow B \propto \frac{1}{T^2}$$

Let M is magnetic moment and B' is the magnetic field due to short magnet, along south to north.

Given, $T_1 = 2 \text{ s}$, $B_h = 12 \mu\text{T}$

$$T_2 = 1 \text{ s}, B_h = B + B' = 12 + B'$$

$$\frac{(B + B')}{B} = \frac{T_1^2}{T_2^2}$$

$$\frac{12 + B'}{12} = \left(\frac{2}{1}\right)^2 = 4$$

$$\Rightarrow B' = 36 \mu\text{T}$$

$$B' = \frac{\mu_0}{4\pi} \frac{2M}{r^3} \Rightarrow 36 \times 10^{-6} = 10^{-7} \times \frac{2M}{(0.10)^3}$$

$$M = 0.18 \text{ A-m}^2$$

CHECK POINT 5.2

- The earth's magnetic field is
(a) 10^{-4}T (b) 10^{-5}T
(c) 10^{-6}T (d) None of these
- Magnetic meridian is a
(a) point (b) horizontal plane
(c) vertical plane (d) line along N-S
- The angle between the magnetic meridian and geographical meridian is called
(a) angle of dip (b) angle of declination
(c) magnetic moment (d) power of magnetic field
- Angle of dip at the equator is
(a) 0° (b) 30°
(c) 60° (d) 90°
- Earth's magnetic field always has a horizontal component except at
(a) magnetic equator (b) magnetic pole
(c) geographical north pole (d) everywhere
- If $H = \frac{1}{\sqrt{3}}V$, then find angle of dip. (where, symbols have their usual meanings)
(a) 60° (b) 30° (c) 45° (d) 90°
- Let V and H be the vertical and horizontal components of earth's magnetic field at any point on earth. Near the north pole
(a) $V \gg H$ (b) $V \ll H$
(c) $V = H$ (d) $V = H = 0$
- If a magnet is suspended at angle 30° to the magnetic meridian, then the dip needle makes angle of 45° with the horizontal. The real dip is
(a) $\tan^{-1}(\sqrt{3}/2)$ (b) $\tan^{-1}(\sqrt{3})$
(c) $\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$ (d) $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$
- The dip at a place is δ . For measuring it, the axis of the dip needle is perpendicular to the magnetic meridian. If the axis of the dip needle makes an angle θ with the magnetic meridian, then the apparent dip will be given by
(a) $\tan \delta \operatorname{cosec} \theta$ (b) $\tan \delta \sin \theta$
(c) $\tan \delta \cos \theta$ (d) $\tan \delta \sec \theta$
- At a neutral point
(a) field of magnet is zero
(b) field of earth is zero
(c) field of magnet is perpendicular to field to earth
(d) None of the above
- Tangent galvanometer measures
(a) capacitance (b) current
(c) resistance (d) potential difference
- Two tangent galvanometers A and B are identical except in their number of turns. They are connected in series. On passing a current through them, deflections of 60° and 30° are produced. The ratio of the number of turns in A and B is
(a) 1 : 3 (b) 3 : 1
(c) 1 : 2 (d) 2 : 1
- Vibration magnetometer is used for comparing
(a) magnetic fields (b) earth's field
(c) magnetic moments (d) All of these
- The time period of a freely suspended bar magnet in a field is 2 s. It is cut into two equal parts along its axis, then the time period is
(a) 4 s (b) 0.5 s
(c) 2 s (d) 0.25 s
- A bar magnet suspended freely in a uniform magnetic field is vibrating with a time period of 3 s. If the field strength is increased to 4 times of the earlier field strength, then the time period (in second) will be
(a) 12 (b) 6 (c) 1.5 (d) 0.75

MAGNETIC INDUCTION AND MAGNETIC MATERIALS

Several experiments have been done which showed that magnetic lines get modified due to the presence of certain materials in the magnetic field. Few substances such as O_2 , air, platinum, aluminium, etc. show a very small increase in the magnetic flux passing through them, when placed in a magnetic field. Such substances are called **paramagnetic substances**.

Few other substances such as H_2 , H_2O , Cu, Zn, Sb, Bi, etc., show a very small decrease in magnetic flux and are said to be **diamagnetic substances**. There are other substances like Fe, Co, etc., through which the magnetic flux increases to a larger value and are known as **ferromagnetic substances**.

Some important terms used in magnetism

Magnetic materials have different properties, so some important terms which are used to define them, are given below

(i) Magnetic induction (B)

The number of magnetic lines of force inside a magnetic substance crossing per unit area normal to their direction is called the **magnetic induction** or **magnetic flux density** inside the substance. It is denoted by **B**.

The SI unit of **B** is tesla (T) or weber/metre² (Wb/m^2). The CGS unit is gauss (G).

$$1 \text{ Wb/m}^2 = 1 \text{ T} = 10^4 \text{ G}$$

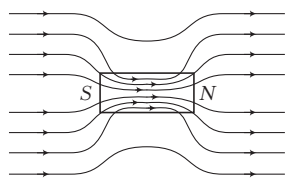


Fig. 5.30 Magnetic field lines of a magnetic material

(ii) Intensity of magnetisation (I)

It is defined as the magnetic moment per unit volume of the magnetised substance. This basically represents the extent to which the substance is magnetised.

Thus,
$$\mathbf{I} = \frac{\mathbf{M}}{V}$$

The SI unit of $\mathbf{I}$ is ampere/metre (A/m). Its dimensional formula is $[M^0 L^{-1} T^0 A]$.

Note For bar magnet, $I = m / A$.

(iii) Magnetic intensity or Magnetic field strength (H)

The capability of the magnetising field to magnetise the substance is expressed by means of a vector $\mathbf{H}$, called the magnetic intensity of the field. It is defined through the vector relation,

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{I}$$

The SI unit of $\mathbf{H}$ is same as that of $\mathbf{I}$, i.e. ampere/metre (A/m). The CGS unit is oersted.

Note

- (i) In vacuum, $I = 0 \therefore \mathbf{H} = \frac{\mathbf{B}}{\mu_0}$.
- (ii) The magnetic field intensity of solenoid is $H = nI$. It is independent of the material of the core of a solenoid.
- (iii) When magnetic material is kept inside the solenoid, then $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_M$
where, $\mathbf{B}_0$ = magnetic field due to magnetising field
and $\mathbf{B}_M$ = magnetic field due to magnetisation of the material.

(iv) Magnetic permeability (μ)

It is defined as the ratio of the magnetic induction $\mathbf{B}$ inside the magnetised substance to the magnetic intensity $\mathbf{H}$ of the magnetising field.

i.e.
$$\mu = \frac{\mathbf{B}}{\mathbf{H}}$$

It is basically a measure of conduction of magnetic lines of force through a substance. The SI unit of magnetic permeability is weber/ampere-metre (Wb/A-m) or tesla metre-ampere⁻¹ (TmA⁻¹).

(v) Relative magnetic permeability (μ_r)

It is the ratio of the magnetic permeability μ of the substance to the permeability of free space.

Thus,
$$\mu_r = \frac{\mu}{\mu_0}$$

μ_r is a pure ratio, hence dimensionless. For vacuum, its value is 1.

μ_r can also be defined as the ratio of the magnetic field B in the substance, when placed in magnetic field B_0 . Thus,

$$\mu_r = \frac{B}{B_0}$$

For paramagnetic substance, $\mu_r > 1$.

For diamagnetic substance, $\mu_r < 1$

and for ferromagnetic substance, $\mu_r \gg 1$.

(vi) Magnetic susceptibility (χ_m)

It is the property of the substance which shows how easily a substance can be magnetised when kept in a magnetising field. It can also be defined as the ratio of intensity of magnetisation (I) in a substance to the magnetic intensity

(H) applied to the substance, i.e. $\chi_m = \frac{I}{H}$.

It is a scalar quantity with no units and dimensions.

For paramagnetic substance, χ is small and positive.

For diamagnetic substance, χ is small and negative.

(vii) Relation between μ_r and χ_m

We have, $\mathbf{B} = \mu_0 (\mathbf{I} + \mathbf{H})$

or $B = \mu_0 H \left(\frac{I}{H} + 1 \right)$ or $B = B_0 (\chi_m + 1) \Rightarrow \frac{B}{B_0} = \chi_m + 1$

But
$$\frac{B}{B_0} = \frac{\mu}{\mu_0} = \mu_r = \text{relative permeability}$$

$$\therefore \mu_r = \chi_m + 1$$

Example 5.41 The magnetic moment of a magnet ($15 \text{ cm} \times 2 \text{ cm} \times 1 \text{ cm}$) is 1.2 A-m^2 . Calculate its intensity of magnetisation.

Sol. Given, magnetic moment, $M = 1.2 \text{ A-m}^2$

Volume, $V = (15 \times 2 \times 1) \times 10^{-6} \text{ m}^3 = 30 \times 10^{-6} \text{ m}^3$

$\therefore$ Intensity of magnetisation,

$$I = \frac{M}{V} = \frac{1.2}{30 \times 10^{-6}} = 4 \times 10^4 \text{ Am}^{-1}$$

Example 5.42 Relative permeability of iron is 5500, what will be its magnetic susceptibility?

Sol. As, we know that, $\mu_r = 1 + \chi_m \Rightarrow 5500 = 1 + \chi_m$

$$\Rightarrow \chi_m = 5500 - 1 = 5499$$

Example 5.43 The magnetising field of 20 CGS units produces a flux of 2400 CGS units in a bar of iron of cross-section 0.2 cm^2 . Calculate the (i) permeability and (ii) susceptibility of the bar.

Sol. Given, $H = 20$ oersted $= \frac{20 \times 10^3}{4\pi} \text{ A/m}$

$$\phi = 2400 \text{ Maxwell} = 2400 \times 10^{-8} \text{ Wb}$$

$$\text{and } A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$$

$$\therefore \text{Magnetic field, } B = \frac{\phi}{A} = \frac{2400 \times 10^{-8}}{0.2 \times 10^{-4}} = 1.20 \text{ Wb/m}^2$$

(i) The permeability of the bar material,

$$\mu = \frac{B}{H} = \frac{1.20}{(20/4\pi) \times 10^3} = 7.54 \times 10^{-4} \text{ H/m}$$

(ii) The magnetic susceptibility and permeability of a material are related with each other as,

$$\mu = \mu_0 (1 + \chi_m) \text{ or } \chi_m = \frac{\mu}{\mu_0} - 1 = \frac{7.54 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 599$$

Example 5.44 A solenoid having 2000 turns/m has a core of a material with relative permeability 220. The area of core is 4 cm^2 and carries a current of 5A. Calculate (i) magnetic intensity, (ii) magnetic field (iii) and magnetisation of the core. Also, calculate the pole strength developed.

Sol. Given, $n = 2000$ turns/m, $i = 5\text{A}$,

$$\mu_r = 220, A = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}$$

(i) Magnetic intensity, $H = ni = 2000 \times 5 = 10000 \text{ A/m}$

(ii) Magnetic field, $B = \mu H = \mu_0 \mu_r H$
 $= 4\pi \times 10^{-7} \times 220 \times 10000 = 88\pi \times 10^{-2} \text{ T}$

(iii) As we know, $B = \mu_0 (H + I)$
 $88\pi \times 10^{-2} = 4\pi \times 10^{-7} (10000 + I)$

$$\Rightarrow 2.20 \times 10^6 = 10^4 + I$$

$$\therefore I = (2.20 \times 10^6 - 10^4) = 2.19 \times 10^6 \text{ A/m}$$

$$\text{Pole strength, } m = IA = (2.19 \times 10^6)(4 \times 10^{-4}) = 876 \text{ A-m}$$

Example 5.45 The space within a current carrying solenoid is filled with magnesium having magnetic susceptibility, $\chi_{Mg} = 1.2 \times 10^{-5}$. What will be the percentage increase in magnetic field?

Sol. Magnetic field without magnesium, $B_0 = \mu_0 H$

With magnesium, $B = \mu H = \mu_0 (1 + \chi) H$

$$\therefore \% \text{ increase} = \left(\frac{B}{B_0} - 1 \right) \times 100 = \chi_{Mg} \times 100$$

$$= 1.2 \times 10^{-5} \times 100 = 1.2 \times 10^{-3}$$

Example 5.46 Consider a bar magnet having pole strength 2 A-m, magnetic length 4 cm and area of cross-section 1 cm^2 . Find

(i) the magnetisation I , (ii) the magnetic intensity H
 (iii) and the magnetic field at the centre of magnet.

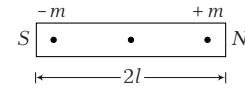
Sol. Given, $m = 2 \text{ A-m}$, $2l = 4 \text{ cm}$

$$\Rightarrow l = 2 \times 10^{-2} \text{ m}, A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$$

$$(i) \text{ Magnetisation, } I = \frac{M}{V} = \frac{m \cdot 2l}{A \cdot 2l} = \frac{m}{A} = \frac{2}{1 \times 10^{-4}} = 2 \times 10^4 \text{ A/m}$$

The direction will be from S to N-pole.

$$(ii) \text{ At centre, } H_N = \frac{\mu_0 m}{4\pi l^2} = \frac{m}{4\pi l^2}, \text{ along N-pole}$$



$$H_S = \frac{m}{4\pi l^2}, \text{ along S-pole}$$

$$H = H_N + H_S = \frac{m}{2\pi l^2} = \frac{2}{2\pi(2 \times 10^{-2})^2}$$

$$= \frac{1}{4\pi} \times 10^4 \text{ A/m, along S-pole}$$

(iii) Magnetic field at the centre of magnet, $B = \mu_0 (H + I)$

$$= 4\pi \times 10^{-7} \left[-\frac{1}{4\pi} \times 10^4 + 2 \times 10^4 \right]$$

$$= -10^{-3} + 8\pi \times 10^{-3} = (8\pi - 1) \times 10^{-3}$$

$$= 2.4 \times 10^{-2} \text{ T, along N-pole}$$

Classification of substances on the basis of magnetic behaviour

All substances (whether solid, liquid or gases) may be classified into three categories in terms of their magnetic behaviour (i) paramagnetic, (ii) diamagnetic and (iii) ferromagnetic. In the following section we will discuss them in details

1. Diamagnetic substances

Diamagnetic substances are those which are repelled by a magnetic field. Examples of such substances are bismuth, antimony, gold, quartz, water, alcohol, etc.

They have the following properties

- (i) These substances when placed in a magnetic field, acquire feeble magnetisation in a direction opposite to that of the applied field. Thus, the magnetic lines of induction inside the substance is smaller than that outside to it.

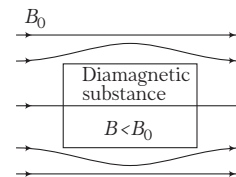


Fig. 5.31 Magnetic field inside a diamagnetic substance

- (ii) In a non-uniform magnetic field, these substances move from stronger to weaker parts of the external magnetic field. As, they acquire magnetisation in opposite direction, hence these are weakly repelled by the magnetic field.

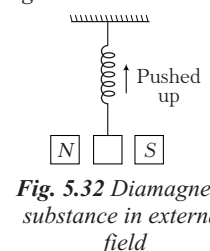


Fig. 5.32 Diamagnetic substance in external field

- (iii) If a diamagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole of an electromagnet, then the level of liquid depresses when the field is switched ON.
- (iv) The relative permeability μ_r is slightly less than 1, i.e. $\mu_r < 1$.
- (v) The susceptibility χ_m of such substances is always small and negative. It is constant and does not vary with field or the temperature as shown in figure

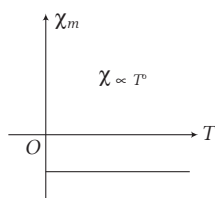


Fig. 5.33 χ_m versus T graph for diamagnetic substance

- (vi) Here, intensity of magnetisation varies inversely with magnetic field strength, i.e. $-I \propto H$ as shown in figure.

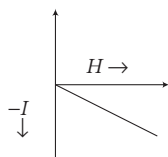


Fig. 5.34 I versus H graph for diamagnetic substance

Note Diamagnetism is a universal property, but in most of the substances, the effect is so weak that they behave like paramagnetic or ferromagnetic, etc.

2. Paramagnetic substances

Paramagnetic substances are those which are attracted by a magnetic field. Examples of such substances are platinum, aluminium, chromium, manganese, CuSO_4 solution, etc.

They have the following properties

- (i) These substances when placed in a magnetic field, acquire a feeble magnetisation in the same sense as the applied field.

Thus, the magnetic lines of induction inside the substance is slightly greater than outside to it.

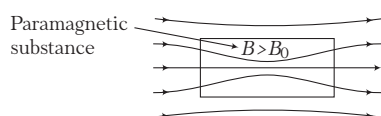


Fig. 5.35 Magnetic field inside a paramagnetic substance

- (ii) These substances are attracted towards regions of stronger magnetic field when placed in a non-uniform magnetic field.

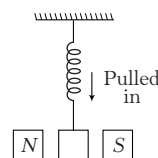


Fig. 5.36 Paramagnetic substance in external field

Thus, they move from weaker to stronger parts of the field.

- (iii) If a paramagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole pieces of an electromagnet such that the level of the liquid is in line with the field, then the liquid will rise in the limb as the field is switched ON.

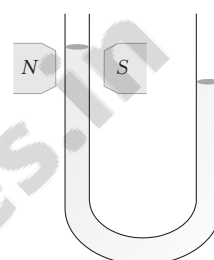


Fig. 5.37

- (iv) For paramagnetic substances, the relative permeability μ_r is slightly greater than one.
- (v) Magnetic susceptibility is positive.
- (vi) I - H curve can be given as

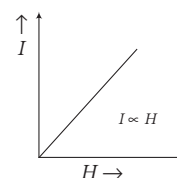


Fig. 5.38 I versus H graph for paramagnetic substance

Curie's law

According to this law, the magnetic susceptibility of paramagnetic substances is inversely proportional to its absolute temperature, i.e.

$$\chi_m \propto \frac{1}{T} \Rightarrow \chi_m = \frac{C}{T}$$

where, C is Curie constant and T = absolute temperature in kelvin. On increasing temperature, the magnetic susceptibility of paramagnetic material decreases and vice-versa as shown in figure. This variation can be shown as

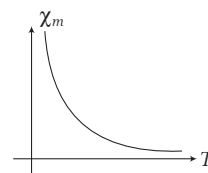


Fig. 5.39 χ_m versus T graph for paramagnetic substance

Example 5.47 The magnetic susceptibility of a paramagnetic material at -73°C is 0.0075. Find its value at -173°C .

Sol. Magnetic susceptibility,

$$\chi_{m_1} = 0.0075, T_1 = -73^\circ\text{C} = (-73 + 273)\text{K} = 200\text{K}$$

$$\chi_{m_2} = ?, T_2 = -173^\circ\text{C} = (-173 + 273)\text{K} = 100\text{K}$$

According to Curie's law, $\chi_m \propto \frac{1}{T}$

$$\therefore \text{Ratio of magnetic susceptibilities, } \frac{\chi_{m_2}}{\chi_{m_1}} = \frac{T_1}{T_2} = \frac{200}{100} = 2$$

$$\Rightarrow \chi_{m_2} = 2\chi_{m_1} = 2 \times 0.0075 = 0.015$$

Example 5.48 A solenoid having 5000 turns/m carries a current of 2A. An aluminium ring at temperature 300K inside the solenoid provides the core. (i) If the magnetisation I is $2 \times 10^{-2}\text{ A/m}$, find the susceptibility of aluminium at 300K. (ii) If temperature of the aluminium ring is 320K, what will be the magnetisation?

Sol. (i) Here, $H = ni = 5000 \times 2 = 10^4\text{ A/m}$

$$\therefore I = \chi H$$

$$\therefore \chi = \frac{I}{H} = \frac{2 \times 10^{-2}}{10^4} = 2 \times 10^{-6}$$

(where, I = intensity of magnetisation)

(ii) According to Curie law,

$$\chi = \frac{C}{T} \Rightarrow \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2}$$

$$\chi_2 = \frac{T_1}{T_2} \chi_1 = \frac{300}{320} \times 2 \times 10^{-6} = 1.875 \times 10^{-6}$$

Magnetisation at 320 K,

$$I = \chi_2 H = 1.875 \times 10^{-6} \times 10^4 = 1.875 \times 10^{-2}\text{ A/m}$$

Curie temperature, the ferromagnetic substances become paramagnetic. It is 1000°C for iron, 770°C for steel, 360°C for nickel and 1150°C for cobalt.

(vi) Its I - H curve can be drawn as

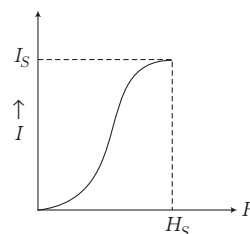


Fig. 5.41 I versus H curve for ferromagnetic substance

Curie-Weiss law

When temperature increases above the Curie temperature, a ferromagnetic substance becomes an ordinary paramagnetic substance whose magnetic susceptibility obeys the Curie-Weiss law as given below

$$\chi_m = \frac{C'}{T - T_C}$$

where, T_C = Curie temperature and C' = constant.

Variation of χ_m versus T is shown below in the figure

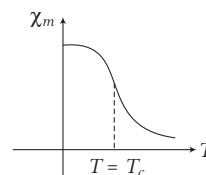


Fig. 5.42 χ_m versus T graph for a magnetic substance

3. Ferromagnetic substances

Ferromagnetic substances are those which are strongly attracted by a magnetic field. Examples of such substances are iron, nickel, steel, cobalt and their alloys. These substances resemble to a higher degree with paramagnetic substances as regard with their behaviour.

They have the following properties

- These substances are strongly magnetised by even a weak magnetic field.
- The relative permeability is very large and is of the order of hundreds and thousands. Here, $B \gg B_0$.



Fig. 5.40 Magnetic field inside ferromagnetic substance

- The susceptibility is positive and very large.
- Magnetic susceptibility of ferromagnetic substance does not change according to Curie law.
- Susceptibility decreases steadily with the rise in temperature. Above a certain temperature known as

Atomic model of magnetism

We are familiar with the model of an atom which has a nucleus that contains the protons and neutrons and electron in the orbit around the nucleus. Within the atom, the electrons behave as a magnetic dipole having permanent dipole moment, hence atoms are like a tiny bar magnet.

The three properties of atoms that give rise to magnetic dipole moment are discussed below.

- The electrons moving around the nucleus in the orbits act as small current loop and contribute magnetic moments.
- The spinning electron has an intrinsic magnetic dipole moment.
- The nucleus contribute to magnetic moment due to the motion of charge within the nucleus.

The magnitude of nuclear moments is about 10^{-3} times that of electronic moments or the spin magnetic moments as the later two are of the same order. Major contribution in magnetic moment of an atom is produced by electron spin as the net contribution of the orbital revolution is very small.

This is because most of the electrons pair are in such a way that they produce equal and opposite orbital magnetic moment and so, they cancel out. Although, these electrons also try to pair up with their opposite spins but in case of spin motion of an electron, it is not always possible to form equal and opposite pairs.

Explanation of magnetism in magnetic substances

On the basis of the above discussed properties, we can explain the origin of magnetism in the magnetic materials as follow

(i) Diamagnetism

The property of diamagnetism is generally found in those substances whose atoms (or molecules) have even number of electrons in outermost orbit which form pairs. "The net magnetic moment of an atom of a diamagnetic substance is thus zero." When a diamagnetic substance is placed in an external magnetic field, the spin motion of electrons is so modified that the electrons which produce the magnetic moments in the direction of external field slow down while the electrons which produce magnetic moments in opposite direction get accelerated.

Thus, a net magnetic moment is induced in the opposite direction of applied magnetic field. Hence, the substance is magnetised opposite to the external field.

Note Diamagnetism is temperature independent.

(ii) Paramagnetism

The property of paramagnetism is found in those substances, whose atoms or molecules have an excess of electrons, spinning in the same direction.

Hence, atoms of paramagnetic substances have a permanent magnetic moment and behave like tiny bar magnets. In the absence of external magnetic field, the tiny bar magnets are randomly oriented and net magnetic moment is, thus zero.

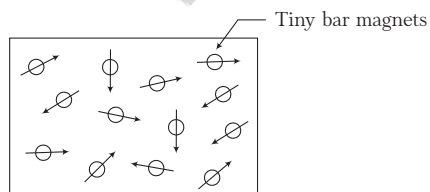


Fig. 5.43 In the absence of external magnetic field

When paramagnetic substance is placed in an external magnetic field, then each atomic magnet experiences a torque which tends to turn the magnet in the direction of the field. The atomic magnets are thus, aligned in the direction of the field.

Thus, the whole substance is magnetised in the direction of the external magnetic field.

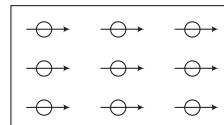


Fig. 5.44 In presence of external magnetic field

As the temperature of substance is increased, the thermal agitation disturbs the magnetic alignment of the atoms. Thus, we can say that paramagnetism is temperature dependent.

(iii) Ferromagnetism

Iron like elements and their alloys are known as ferromagnetic substances. The susceptibility of these substances is in several thousands.

Like paramagnetic substances, atoms of ferromagnetic substances have a permanent magnetic moment and behave like tiny magnets. But in ferromagnetic substances, the atoms form innumerable small effective regions called **domains**.

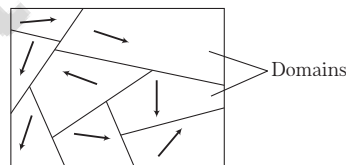


Fig. 5.45 Unmagnetised material

The size of the domain vary from about 10^{-6} cm^3 to 10^{-2} cm^3 . Each domain has 10^{17} to 10^{21} atoms whose magnetic moments are aligned in the same direction. In an unmagnetised ferromagnetic specimen, the domains are oriented randomly, so that their resultant magnetic moment is zero.

When the specimen is placed in a external magnetic field, then the resultant magnetisation may increase in two different ways.

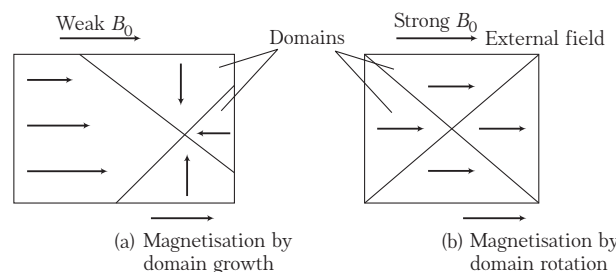
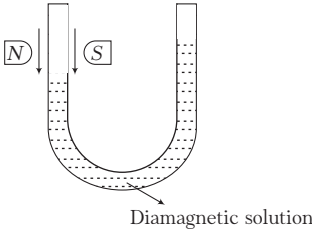
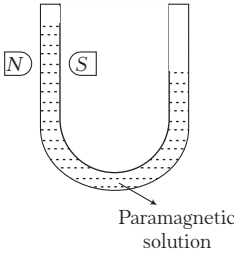
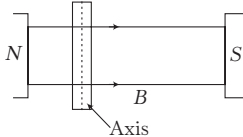
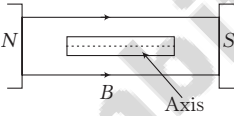
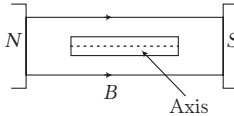
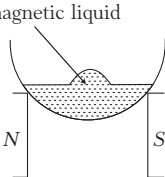
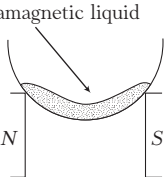


Fig. 5.46 Ferromagnetic substance in (a) weak B_0 and (b) strong B_0

- The domains which are oriented favourably with respect to the field will increase in size, whereas those oriented opposite to the external magnetic field will reduce.
- The domains rotate towards the direction of magnetic field.

Comparative study of magnetic materials

S.No.	Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
(i)	These substances when placed in a magnetic field, acquire feeble magnetism opposite to the direction of the magnetic field.	These substances when placed in a magnetic field, acquire feeble magnetism in the direction of the magnetic field.	These substances when placed in a magnetic field are strongly magnetised in the direction of the field.
(ii)	These substances are repelled by a magnet.	These substances are feebly attracted by a magnet.	These substances are strongly attracted by a magnet.
(iii)	When a diamagnetic solution is poured into a U-tube and one arm is placed between the poles of strong magnet, then the level of solution in that arm is lowered. 	The level of the paramagnetic solution in that arm rises. 	No liquid is ferromagnetic.
(iv)	If a rod of diamagnetic material is suspended freely between two magnetic poles, then its axis becomes perpendicular to the magnetic field. 	In case of paramagnetic rod, its axis becomes parallel to the magnetic field. 	Also, in case of ferromagnetic rod, its axis becomes parallel to the magnetic field. 
(v)	In non-uniform magnetic field, the diamagnetic substances are attracted towards the weaker fields, i.e. they move from stronger to weaker magnetic field.	In non-uniform magnetic field, they move from weaker to stronger part of the magnetic field slowly.	In non-uniform magnetic field, they move from weaker to stronger magnetic field rapidly.
(vi)	Their permeability is less than one ($\mu < 1$). $0 < \mu_r < 1, \mu < \mu_0$	Their permeability is slightly greater than one ($\mu > 1$). $1 < \mu_r < 1 + \epsilon, \mu > \mu_0$	Their permeability is much greater than one ($\mu \gg 1$). $\mu_r \gg 1, \mu \gg \mu_0$
(vii)	Their susceptibility is small and negative [$-1 < \chi_m < 0$]. It is independent of temperature.	Their susceptibility is small and positive ($0 < \chi_m < \epsilon$). It is inversely proportional to absolute temperature which is Curie's law, i.e. $\chi \propto \frac{1}{T}$	Their susceptibility is large and positive ($\chi_m \gg 1$). They also follow Curie's law, i.e. $\chi \propto \frac{1}{T - T_C}$. At Curie temperature, ferromagnetic substances change into paramagnetic substances.
(viii)	Shape of diamagnetic liquid in a glass crucible and kept over two magnetic poles 	Shape of paramagnetic liquid in a glass crucible and kept over two magnetic poles 	No liquid is ferromagnetic.
(ix)	In these substances, the magnetic lines of force are farther than in air.	In these substances, the magnetic lines of force are closer than in air.	In these substances, magnetic lines of force are much closer than in air.
(x)	The resultant magnetic moment of these substances is zero.	These substances have a permanent magnetic moment.	These substances also have a permanent magnetic moment.

Hysteresis

The word hysteresis means delayed. The phenomenon of the lagging of magnetic induction behind the magnetising field is called **hysteresis**.

- (i) The plot of intensity of magnetisation (I) of ferromagnetic substances *versus* magnetic intensity (H) for a complete cycle of magnetisation and demagnetisation is called **hysteresis loop**.

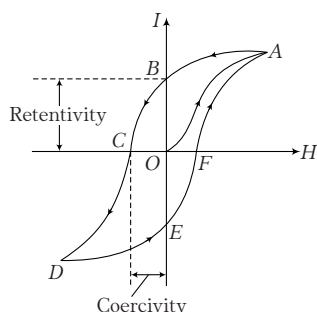


Fig. 5.47 Hysteresis loop for a ferromagnetic substance

- (ii) It is clear from the above figure that if intensity of magnetising field (H) is increased, the intensity of magnetisation also increases. This is because, more and more domains are aligned in the direction of applied field.
- (iii) When all domains are aligned, material is magnetically saturated. Beyond this, if intensity of magnetising field (H) is increased, then intensity of magnetisation (I) does not increase.
- (iv) When intensity of magnetising field (H) is decreased intensity of magnetisation also decreases but it lags behind H . Therefore, H becomes zero but I does not reduce to zero, i.e. the curve does not retrace itself.
- (v) Now this value of intensity of magnetisation (I) which is left in the material at $H = 0$, is called **retentivity** or **remanence** (BO). This occurs because all domains do not dealign even, if $H = 0$.
- (vi) When the magnetising field is applied in reverse direction and its intensity H is increased. The material starts demagnetising. The value of reverse magnetising field needed to reduce magnetisation to zero is called **coercivity** (OC).
- (vii) If reverse magnetising field is increased, further the material becomes saturated. Now, if the magnetising field is reduced after attaining the reverse saturation, the cycle repeats itself.

- (viii) The area enclosed by the loop represents loss of energy during a cycle of magnetisation and demagnetisation.

- (ix) Unit of area of I - H loop is $\text{A}^2 \text{m}^{-2}$.

- (x) Unit of area of BH loop
 $= \mu_0 \times \text{unit of area of } I\text{-}H \text{ loop}$

- (xi) Area under B - H loop is equal to the energy loss per cycle per unit volume.

Example 5.49 The hysteresis loss for a specimen of iron weighing 15 kg is equivalent to $300 \text{ Jm}^{-3} \text{ cycle}^{-1}$. Find the loss of energy per hour at 25 cycle s^{-1} . Density of iron is 7500 kg m^{-3} .

Sol. Let Q be the energy dissipated per unit volume per hysteresis cycle in the given sample, then the total energy lost by the volume V of the sample in time t will be

$$W = Q \times V \times v \times t$$

where, v is the number of hysteresis cycles per second.

Here, $Q = 300 \text{ Jm}^{-3} \text{ cycle}^{-1}$, $v = 25 \text{ cycle s}^{-1}$, $t = 1 \text{ h} = 3600 \text{ s}$

$$\text{Volume, } V = \frac{\text{Mass}}{\text{Density}} = \frac{15}{7500} \text{ m}^3$$

$$\therefore \text{Hysteresis loss, } W = \left(300 \times \frac{15}{7500} \times 25 \times 3600 \right) \text{ J} = 54000 \text{ J}$$

Example 5.50 The coercivity of a certain permanent magnet is $4.0 \times 10^4 \text{ Am}^{-1}$. The magnet is placed inside a solenoid 20 cm long and having 700 turns and a current is passed in the solenoid to demagnetise it completely. Find the value of current.

Sol. The coercivity of $4 \times 10^4 \text{ Am}^{-1}$ of the permanent magnet implies that a magnetic intensity $H = 4 \times 10^4 \text{ Am}^{-1}$ is required to be applied in opposite direction to demagnetise the magnet.

$$\text{Here, } n = \frac{700}{20 \text{ cm}} = \frac{700}{20 \times 10^{-2} \text{ m}} = 3500 \text{ turns/m}$$

$$\text{As, } H = ni$$

$$\therefore \text{Current, } i = \frac{H}{n} = \frac{4 \times 10^4}{3500} = 11.43 \approx 11.5 \text{ A}$$

Magnetic properties of soft Iron and steel

A comparison of the magnetic properties of ferromagnetic substances can be made by the comparison of the shapes and sizes of their hysteresis loops.

There are two types of ferromagnetic materials

- (i) **Soft magnetic materials** These are those materials which have low retentivity, coercivity and small hysteresis loss. e.g. Soft iron.

- (ii) **Hard magnetic materials** These are those materials which have high retentivity, coercivity and large hysteresis loss. *e.g.* Steel.

Hysteresis loop for soft iron and steel can be drawn as below

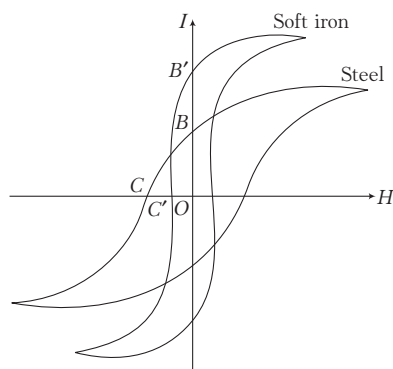


Fig. 5.48 Hysteresis loop for soft iron and steel

Following three conclusions can be drawn from their hysteresis loops

- Retentivity of soft iron is greater than the retentivity of steel.
- Coercivity of soft iron is less than the coercivity of steel.
- Area of hysteresis loop (*i.e.* hysteresis loss) in soft iron is smaller than that in steel.

Choice of magnetic materials

The choice of a magnetic material for different uses is decided from the hysteresis curve of a specimen of the material.

- (i) **Permanent magnets** The materials for a permanent magnet should have

- high retentivity (so that the magnet is strong) and
- high coercivity (so that the magnetisation is not wiped out by stray magnetic fields).

As the material in this case is never put to cyclic changes of magnetisation, hence hysteresis is immaterial. From the point of view of these facts **steel** is more suitable for the construction of permanent magnets than soft iron.

Modern permanent magnets are made of **cobalt-steel**, alloys like **ticonal**.

- (ii) **Electromagnets** The materials for the construction of electromagnets should have

- high initial permeability
- and low hysteresis loss.

From the view point of these facts, **soft iron** is an ideal material for this purpose.

- (iii) **Transformer cores and telephone diaphragms** As, the magnetic material used in these cases is subjected to cyclic changes. Thus, the essential requirements for the selection of the material are

- high initial permeability
- and low hysteresis loss to prevent the breakdown.

Electromagnet

As we know that, a current carrying solenoid behaves like a bar magnet. If we place a soft iron rod in the solenoid, then the magnetism of the solenoid increases hundreds of times and the solenoid is called an electromagnet. It is a temporary magnet.

An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft iron straight rod or a horse shoe rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.

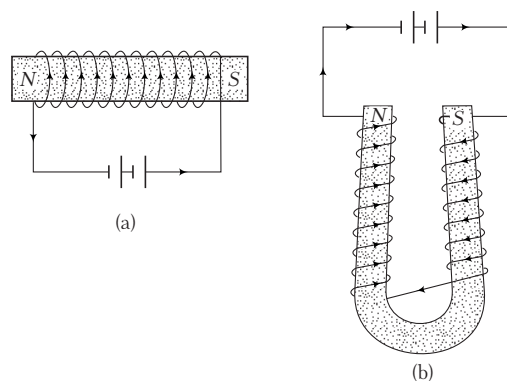


Fig. 5.49 Electromagnets

Applications of electromagnets

- Electromagnets are used in electric bell, transformer, telephone diaphragms, etc.
- In medical field, they are used in extracting bullets from the human body.
- Large electromagnets are used in cranes for lifting and transferring big machines and their parts.

CHECK POINT 5.3

1. Which one of the following is a non-magnetic substance?
(a) Iron (b) Nickel
(c) Cobalt (d) Brass
2. The SI unit of magnetic permeability is
(a) A m^{-1}
(b) A-m
(c) Henry m^{-1}
(d) No unit, it is a dimensionless number
3. The unit of magnetic susceptibility is
(a) H (b) Wb/m
(c) A/m (d) None of these
4. The relation between B , H and I in SI system is
(a) $B = H + I$ (b) $B = H - I$
(c) $B = \mu_0 (H + I)$ (d) $B = \mu_0 (H - I)$
5. An example of a diamagnetic substance is
(a) aluminium (b) copper
(c) iron (d) nickel
6. The universal property of all substances is
(a) diamagnetism
(b) ferromagnetism
(c) paramagnetism
(d) All of the above
7. Which magnetic materials have negative susceptibility?
(a) Diamagnetic materials (b) Paramagnetic materials
(c) Ferromagnetic materials (d) All of these
8. Identify the paramagnetic substance.
(a) Iron (b) Aluminium
(c) Nickel (d) Hydrogen
9. Which of the following is true?
(a) Diamagnetism is temperature dependent
(b) Paramagnetism is temperature dependent
(c) Paramagnetism is temperature independent
(d) None of the above
10. Magnetic permeability is maximum for
(a) diamagnetic substance (b) paramagnetic substance
(c) ferromagnetic substance (d) All of these
11. The temperature at which a ferromagnetic material becomes paramagnetic is called a
(a) neutral temperature (b) Curie temperature
(c) inversion temperature (d) critical temperature
12. Substances in which the magnetic moment of a single atom is not zero, is known as
(a) diamagnetism (b) ferromagnetism
(c) paramagnetism (d) ferrimagnetism
13. Liquid oxygen remains suspended between two pole faces of a magnet because it is
(a) diamagnetic (b) paramagnetic
(c) ferromagnetic (d) anti-ferromagnetic
14. The only property possessed by ferromagnetic substance is
(a) hysteresis
(b) susceptibility
(c) directional property
(d) attracting magnetic substances
15. The permanent magnet is made from which one of the following substances?
(a) Diamagnetic (b) Paramagnetic
(c) Ferromagnetic (d) Electromagnetic

Chapter Exercises

(A) Taking it together

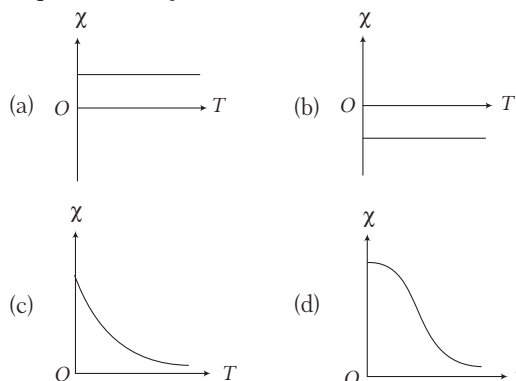
Assorted questions of the chapter for advanced level practice

- 1** A magnet is placed in iron powder and then taken out, then maximum iron powder is at
 - (a) some distance away from north pole
 - (b) some distance away from south pole
 - (c) the middle of the magnet
 - (d) the end of the magnet
- 2** A permanent magnet
 - (a) attracts all substances
 - (b) attracts only magnetic substances
 - (c) attracts magnetic substances and repels all non-magnetic substances
 - (d) attracts non-magnetic substances and repels magnetic substances
- 3** Magnetic field is measured by
 - (a) pyrometer
 - (b) hydrometer
 - (c) thermometer
 - (d) fluxmeter
- 4** Lines which represent places of constant angle of dip are called
 - (a) isobaric lines
 - (b) isogonic lines
 - (c) isoclinic lines
 - (d) isodynamic lines
- 5** A line passing through places having zero value of magnetic dip is called
 - (a) isoclinic line
 - (b) agonic line
 - (c) isogonic line
 - (d) aclinic line
- 6** Aclinic lines are the lines joining places of
 - (a) zero dip
 - (b) equal dip
 - (c) zero declination
 - (d) equal declination
- 7** The arms of a deflection magnetometer in the tan B position are placed
 - (a) east-west
 - (b) north-south
 - (c) north-east
 - (d) south-west
- 8** If the current is doubled, the deflection is also doubled in
 - (a) a tangent galvanometer
 - (b) a moving coil galvanometer
 - (c) Both (a) and (b)
 - (d) None of the above
- 9** Which of the following is diamagnetic?
 - (a) Aluminium
 - (b) Quartz
 - (c) Nickel
 - (d) Bismuth
- 10** The permeability of paramagnetic substance is
 - (a) slightly more than vacuum
 - (b) slightly less than vacuum
 - (c) much more than vacuum
 - (d) None of the above
- 11** Unit of magnetic flux density (or magnetic induction) is
 - (a) tesla
 - (b) weber/metre²
 - (c) newton/ampere-metre
 - (d) All of these
- 12** Magnetic field intensity is defined as
 - (a) magnetic moment per unit volume
 - (b) magnetic induction force acting on a unit magnetic pole
 - (c) number of lines of force crossing per unit area
 - (d) number of lines of force crossing per unit volume
- 13** Permeability is defined as the ratio between
 - (a) magnetic induction and susceptibility
 - (b) magnetic induction and magnetising field
 - (c) magnetising field and magnetic induction
 - (d) magnetising field and susceptibility
- 14** Hysteresis loss for steel is that for iron.
 - (a) lesser than
 - (b) equal to
 - (c) greater than
 - (d) Either (b) or (c)
- 15** Hysteresis is exhibited by a substance.
 - (a) paramagnetic
 - (b) ferromagnetic
 - (c) diamagnetic
 - (d) All of the above
- 16** Which of the following materials has got the maximum retentivity?
 - (a) Copper
 - (b) Zinc
 - (c) Soft iron
 - (d) Hard iron
- 17** The area enclosed by a hysteresis loop is a measure of
 - (a) retentivity
 - (b) susceptibility
 - (c) permeability
 - (d) energy loss per cycle
- 18** Which of the following is the most suitable material for making permanent magnet?
 - (a) Steel
 - (b) Soft iron
 - (c) Copper
 - (d) Nickel
- 19** The materials suitable for making electromagnets should have
 - (a) high retentivity and high coercivity
 - (b) low retentivity and low coercivity
 - (c) high retentivity and low coercivity
 - (d) low retentivity and high coercivity
- 20** Which of the following is most suitable for the core of electromagnets?
 - (a) Iron
 - (b) Steel
 - (c) Soft iron
 - (d) Cu-Ni alloy

- 21** A magnetic needle kept in a non-uniform magnetic field experiences

(a) a force and torque
(b) a force but not a torque
(c) a torque but not a force
(d) Neither a torque nor a force

- 22** The variation of magnetic susceptibility (χ) with temperature for a diamagnetic substance is best represented by



- 23** The angle between the earth's magnetic axis and the earth's geographic axis is

(a) zero (b) 11.5°
(c) 23° (d) None of these

- 24** If a magnet is hanged with its magnetic axis, then it stops in

(a) magnetic meridian (b) geometric meridian
(c) angle of dip (d) None of these

- 25** A dip needle in a plane perpendicular to magnetic meridian will remain

(a) vertical
(b) horizontal
(c) in any direction
(d) at an angle of dip to the horizontal

- 26** A dip circle is at right angles to the magnetic meridian. What will be the apparent dip?

(a) 0° (b) 30°
(c) 60° (d) 90°

- 27** A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It will

(a) stay in north-south direction only
(b) stay in east-west direction only
(c) become rigid showing no movement
(d) stay in any position

- 28** When the N -pole of a bar magnet points towards the south and S -pole towards the north, the null points are at the

(a) magnetic axis
(b) magnetic centre
(c) perpendicular divider of magnetic axis
(d) N and S poles

- 29** Due to earth's magnetic field, charged cosmic ray particles

(a) require greater kinetic energy to reach the equator than pole
(b) require less kinetic energy to reach the equator than pole
(c) can never reach the pole
(d) can never reach the equator

- 30** A magnetic needle, suspended horizontally by an unspun silk fibre, oscillates in the horizontal plane, because of a restoring force originating mainly from

(a) the torsion of the silk fibre
(b) the force of gravity
(c) the horizontal component of earth's magnetic field
(d) All the above factors

- 31** An electron moving around the nucleus with an angular momentum L has a magnetic moment

(a) $\frac{e}{m} L$ (b) $\frac{e}{2m} L$ (c) $\frac{2e}{m} L$ (d) $\frac{e}{2\pi m} L$

- 32** A vibration magnetometer is placed at the south pole, then the time period will be

(a) zero
(b) infinity
(c) same as at magnetic equator
(d) same as at any other place on earth

- 33** Which of the following statements are true about the magnetic susceptibility χ_m of paramagnetic substance?

(a) Value of χ_m is inversely proportional to the absolute temperature of the sample.
(b) χ_m is zero at all temperature.
(c) χ_m is negative at all temperature.
(d) χ_m does not depend on the temperature of the sample.

- 34** Resultant force acting on a diamagnetic material in a magnetic field is in direction

(a) from stronger to the weaker part of the magnetic field
(b) from weaker to the stronger part of the magnetic field
(c) perpendicular to the magnetic field
(d) in the direction making 60° to the magnetic field

- 35** The mathematical equation for magnetic field lines of force is

(a) $\nabla \cdot \mathbf{B} = 0$ (b) $\nabla \cdot \mathbf{B} \neq 0$ (c) $\nabla \cdot \mathbf{B} > 0$ (d) $\nabla \cdot \mathbf{B} < 0$

- 36** Two lines of force due to a bar magnet

(a) intersect at the neutral point
(b) intersect near the poles of the magnet
(c) intersect on the equatorial axis of the magnet
(d) do not intersect at all

- 37** What happens to the force between magnetic poles when their pole strength and the distance between them both gets doubled?

(a) Force increases to two times the previous value
(b) No change
(c) Force decreases to half the previous value
(d) Force increases to four times the previous value

38 If a magnet of pole strength m is divided into four parts such that the length and width of each part is half that of initial one, then the pole strength of each part will be

- (a) $m/4$ (b) $m/2$ (c) $m/8$ (d) $4m$

39 Two magnets have the same length and the same pole strength. But one of the magnets has a small hole at its centre. Then

- (a) both have equal magnetic moment
(b) one with hole has smaller magnetic moment
(c) one with hole has larger magnetic moment
(d) one with hole loses magnetism through the hole

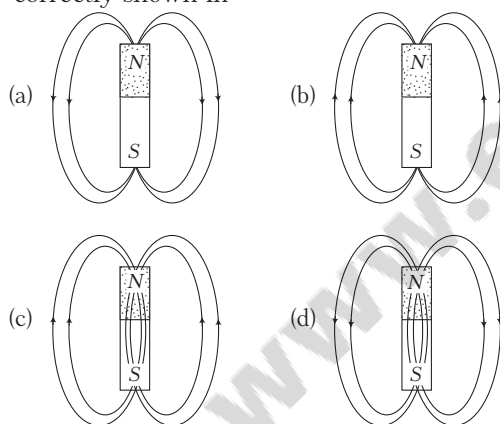
40 The magnetic field at a distance d from a short bar magnet in longitudinal and transverse positions are in the ratio

- (a) 1:1 (b) 2:3 (c) 2:1 (d) 3:2

41 When a diamagnetic substance is brought near north or south pole of a bar magnet, it is

- (a) attracted by the poles
(b) repelled by the poles
(c) attracted by the north pole and repelled by the south pole
(d) repelled by the north pole and attracted by the south pole

42 The magnetic field lines due to a bar magnet are correctly shown in



43 Susceptibility is positive and large for a

- (a) paramagnetic substance
(b) ferromagnetic substance
(c) non-magnetic substance
(d) diamagnetic substance

44 Torques τ_1 and τ_2 are required for a magnetic needle to remain perpendicular to the magnetic fields B_1 and B_2 at two different places. The ratio of B_1/B_2 is

- (a) $\frac{\tau_2}{\tau_1}$ (b) $\frac{\tau_1}{\tau_2}$
(c) $\frac{\tau_1 + \tau_2}{\tau_1 - \tau_2}$ (d) $\frac{\tau_1 - \tau_2}{\tau_1 + \tau_2}$

45 A dip circle is taken to geomagnetic equator. The needle is allowed to move in a vertical plane perpendicular to the magnetic meridian. The needle will stay in

- (a) horizontal direction only
(b) vertical direction only
(c) any direction except vertical and horizontal
(d) None of the above

46 At the magnetic north pole of the earth, the values of the horizontal component H and the angle of dip θ are

- (a) $H = 0, \theta = 45^\circ$ (b) $H = B_e, \theta = 0^\circ$
(c) $H = 0, \theta = 90^\circ$ (d) $H = B_e, \theta = 90^\circ$

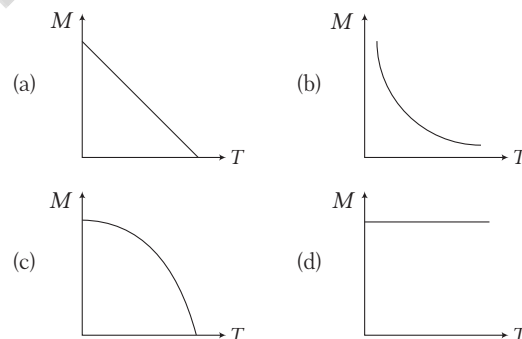
47 When the magnetic inclination (dip) was measured at various places on earth, in one of the following countries it was found to be zero. Which one was it?

- (a) Pakistan (b) Brazil (c) Scotland (d) Canada

48 In a deflection magnetometer, the needle is short and the pointer is long, because the

- (a) needle cannot be made long
(b) circular scale cannot be made short
(c) needle must be in a uniform field
(d) pointer must be in a non-uniform field

49 A curve between magnetic moment and temperature of magnet is



50 The tangents deflection produced in $\tan A$ and $\tan B$ positions by a short magnet at equal distances are in the ratio

- (a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) None of these

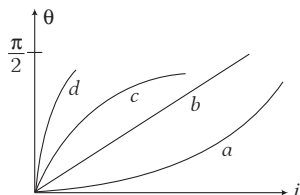
51 The relative permeability is represented by μ_r and the susceptibility is denoted by χ for a magnetic substance. Then, for a paramagnetic substance

- (a) $\mu_r < 1, \chi < 0$ (b) $\mu_r < 1, \chi > 0$
(c) $\mu_r > 1, \chi < 0$ (d) $\mu_r > 1, \chi > 0$

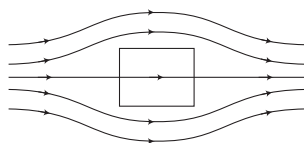
52 When a piece of a ferromagnetic substance is put in a uniform magnetic field, the flux density inside it is four times the flux density away from the piece. The magnetic permeability of the material is

- (a) 1 (b) 2 (c) 3 (d) 4

- 53 Which of the four graphs may best represent the current- deflection relation in a tangent galvanometer?



- (a) *d* (b) *a* (c) *c* (d) *b*
- 54 If a diamagnetic solution is poured into a U-tube and one arm of this U-tube placed between the poles of a strong magnet with the meniscus in a line with the field, then the level of the solution will
(a) rise (b) fall
(c) oscillate slowly (d) remain as such
- 55 A paramagnetic liquid is filled in a U-tube, of which one limb is placed between the pole pieces of an electromagnet. When the field is switched on, the liquid in the limb which is placed between the field will
(a) rise (b) fall
(c) remain stationary (d) first rise and then fall
- 56 The magnetic moment of a magnet of length 10 cm and pole strength 4.0 A-m will be
(a) 0.4 A-m² (b) 1.6 A-m² (c) 20 A-m² (d) 8.0 A-m²
- 57 All the magnetic materials lose their magnetic properties when
(a) dipped in water
(b) dipped in oil
(c) brought near a piece of iron
(d) strongly heated
- 58 A ferromagnetic material is heated above its Curie temperature. Which one is a correct statement?
(a) Ferromagnetic domains are perfectly arranged.
(b) Ferromagnetic domains become random.
(c) Ferromagnetic domains are not influenced.
(d) Ferromagnetic material changes into diamagnetic material.
- 59 Above the Curie temperature, the susceptibility of a ferromagnetic substance varies
(a) directly as the absolute temperature
(b) inversely as the absolute temperature
(c) inversely as the square of absolute temperature
(d) directly as the square of absolute temperature
- 60 The given figure represents a material which is



- (a) paramagnetic (b) diamagnetic
(c) ferromagnetic (d) None of these

- 61 A magnet of magnetic moment M and pole strength m is cut in two equal parts along the axis of magnet, then magnetic moment of each part will be
(a) M (b) $M/2$ (c) $M/4$ (d) $2M$
- 62 Two identical thin bar magnets each of length l and pole strength m are placed at right angles to each other with north pole of one touching south pole of the other. Magnetic moment of the system is
(a) ml (b) $2ml$ (c) $\sqrt{2}ml$ (d) $\frac{1}{2}ml$
- 63 A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.16 T experiences a torque of magnitude 0.032 J. The magnetic moment of the bar magnet will be
(a) 0.23 JT⁻¹ (b) 0.40 JT⁻¹
(c) 0.80 JT⁻¹ (d) zero
- 64 A bar magnet when placed at an angle of 30° to the direction of magnetic field induction of 5×10^{-2} T, experiences a moment of couple 25×10^{-6} N-m. If the length of the magnet is 5 cm, its pole strength is
(a) 2×10^{-2} A-m (b) 5×10^{-2} A-m
(c) 2 A-m (d) 5 A-m
- 65 A bar magnet of magnetic moment 3 A-m^2 is placed in a uniform magnetic field induction of 2×10^{-5} T. If each pole of the magnet experiences a force of 6×10^{-4} N, the length of the magnet is
(a) 0.5 m (b) 0.3 m (c) 0.2 m (d) 0.1 m
- 66 A toroid of n turns, mean radius R and cross-sectional radius a carries current I . It is placed on a horizontal table taken as XY-plane. Its magnetic moment $\mathbf{m}$ [NCERT Exemplar]
(a) is non-zero and points in the z -direction by symmetry
(b) points along the axis of the toroid ($\mathbf{m} = m_\phi$)
(c) is zero, otherwise there would be a field falling as $\frac{1}{r^3}$ at large distances outside the toroid
(d) is pointing radially outwards
- 67 A magnet of magnetic moment M is situated with its axis along the direction of a magnetic field of strength B . The work done in rotating it by an angle of 180° will be
(a) $-MB$ (b) $+MB$ (c) zero (d) $+2MB$
- 68 A magnet of magnetic moment 2 J/T is aligned in the direction of magnetic field of 0.1 T. What is the net work done to bring the magnet normal to the magnetic field?
(a) 0.1 J (b) 0.2 J (c) 1 J (d) 2 J

- 69** A planar coil having 15 turns carries 20 A current. The coil is oriented with respect to the uniform magnetic field $\mathbf{B} = 0.5\hat{i}$ T such that its directed area is $\mathbf{A} = -0.04\hat{i}$ m². The potential energy of the coil in the given orientation is

(a) 0 (b) +0.72 J
(c) 6 J (d) -1.44 J

- 70.** The magnetic field of the earth can be modelled by that of a point dipole placed at the centre of the earth. The dipole axis makes an angle of 11.3° with the axis of the earth. At Mumbai, declination is nearly zero. Then, [NCERT Exemplar]

(a) the declination varies between 11.3° W to 11.3° E
(b) the least declination is 0°
(c) the plane defined by dipole axis and the earth axis passes through Greenwich
(d) declination averaged over the earth must be always negative

- 71** The magnetic field on the axis of a short bar magnet at a distance of 10 cm is 0.2 oersted. What will be the field at a point, distant 5 cm on the line perpendicular to the axis and passing through the magnet?

(a) 0.025 oersted (b) 0.2 oersted
(c) 0.4 oersted (d) 0.8 oersted

- 72** If the angles of dip at two places are 30° and 45° respectively, then the ratio of horizontal components of earth's magnetic field at the two places will be

(a) $\sqrt{3} : \sqrt{2}$ (b) $1 : \sqrt{2}$
(c) $1 : \sqrt{3}$ (d) $1 : 2$

- 73** The earth's magnetic field at a certain place has a horizontal component 0.3 G and the total strength 0.5 G. The angle of dip is

(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$
(c) $\tan^{-1}\left(\frac{4}{3}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right)$

- 74** At a certain place, the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.50 oersted. The earth's total magnetic field (in oersted) is

(a) $\sqrt{3}$ (b) 1
(c) $1/\sqrt{3}$ (d) $\frac{1}{2}$

- 75** In a permanent magnet at room temperature,

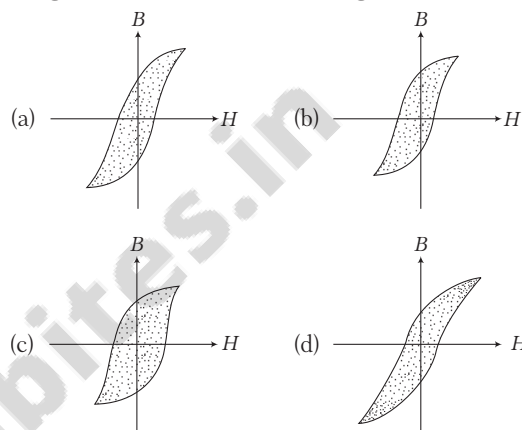
[NCERT Exemplar]

(a) magnetic moment of each molecule is zero
(b) the individual molecules have non-zero magnetic moment which are all perfectly aligned
(c) domains are partially aligned
(d) domains are all perfectly aligned

- 76** A dip needle lies initially in the magnetic meridian when it shows an angle of dip θ at a place. The dip circle is rotated through an angle x in the horizontal plane and then it shows an angle of dip θ' . Then $\frac{\tan \theta'}{\tan \theta}$ is

(a) $1/\cos x$ (b) $1/\sin x$ (c) $1/\tan x$ (d) $\cos x$

- 77** For different substances hysteresis (B - H) curves are given as shown in figure. For making temporary magnet which of the following is best?



- 78** A bar magnet is oscillating in the earth's magnetic field with time period T . If its mass is increased four times, then its time period will be

(a) $4T$ (b) $2T$ (c) T (d) $T/2$

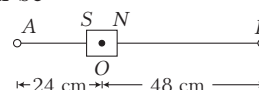
- 79** When 2A current is passed through a tangent galvanometer, it gives a deflection of 30° . For 60° deflection, the current must be

(a) 1 A (b) $2\sqrt{3}$ A
(c) 4 A (d) 6 A

- 80.** Two tangent galvanometers having coils of the same radius are connected in series. A current flowing in them produces deflections of 60° and 45° , respectively. The ratio of the number of turns in the coils is

(a) $4\sqrt{3}$ (b) $(\sqrt{3} + 1)/1$
(c) $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ (d) $\frac{\sqrt{3}}{1}$

- 81** A bar magnet of length 3 cm has points A and B along its axis at distances of 24 cm and 48 cm on the opposite sides. Ratio of magnetic fields at these points will be



(a) 8 (b) $\frac{1}{2\sqrt{2}}$ (c) 3 (d) 4

- 82** The magnetic moment produced in a substance of 1g is $6 \times 10^{-7} \text{ Am}^2$. If its density is 5 g/cm^3 , then the intensity of magnetisation in A/m will be

(a) 8.3×10^6 (b) 3.0
(c) 1.2×10^{-7} (d) 3×10^{-6}

- 83** A short bar magnet is arranged with its north pole pointing geographical north. It is found that, the horizontal component of earth's magnetic induction (B_H) is balanced by the magnetic induction of the magnet at a point which is at a distance of 20 cm from its centre. The magnetic moment of the magnet is (if $H = 4 \times 10^{-5} \text{ Wbm}^{-2}$)

(a) 3.2 Am^2 (b) 1.6 Am^2 (c) 6.4 Am^2 (d) 0.8 Am^2

- 84** A long magnetic needle of length $2L$, magnetic moment M and pole strength m units is broken into two at the middle. The magnetic moment and pole strength of each piece will be

(a) $M/2, m/2$ (b) $M, m/2$
(c) $M/2, m$ (d) M, m

- 85** Consider the two idealised systems (i) a parallel plate capacitor with large plates with small separation and (ii) a long solenoid of length $L \gg R$, radius of cross-section. In (i) $\mathbf{E}$ is ideally treated as a constant between plates and zero outside. In (ii) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however contradict fundamental laws as below [NCERT Exemplar]

(a) Case (i) contradicts Gauss's law for electrostatic fields
(b) Case (ii) contradicts Gauss's law for magnetic fields
(c) Case (i) agrees with $\oint \mathbf{E} \cdot d\mathbf{l} = 0$
(d) Case (ii) contradicts $\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{em}}$

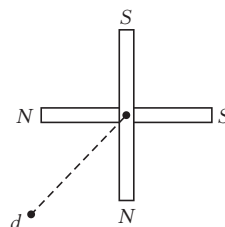
- 86** Due to a small magnet, intensity at a distance x in the end on position is 9 G. What will be the intensity at a distance $\frac{x}{2}$ on broadside on position?

(a) 9 G (b) 4 G (c) 36 G (d) 4.5 G

- 87** A magnet oscillating in a horizontal plane has a time period of 2 s at a place where the angle of dip is 30° and 3 s at another place where the angle of dip is 60° . The ratio of resultant magnetic fields at the two places is

(a) $\frac{4\sqrt{3}}{7}$ (b) $\frac{4}{9\sqrt{3}}$
(c) $\frac{9}{4\sqrt{3}}$ (d) $\frac{9}{\sqrt{3}}$

- 88** Two short magnets of equal dipole moments M are fastened perpendicularly at their centres (figure). The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is



(a) $\frac{\mu_0}{4\pi} \frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi} \frac{\sqrt{2} M}{d^3}$
(c) $\frac{\mu_0}{4\pi} \frac{2\sqrt{2} M}{d^3}$ (d) $\frac{\mu_0}{4\pi} \frac{2M}{d^3}$

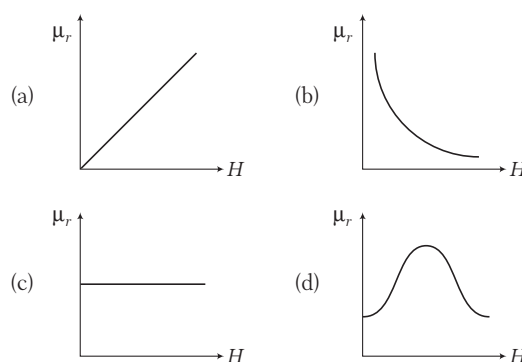
- 89** Two bar magnets of the same mass, length and breadth having magnetic moments M and $2M$ are joined together pole-to-pole and suspended in a vibration magnetometer. The time period of oscillation is 3 s. If the polarity of one of the magnets is reversed, the time period of oscillation will be

(a) $\sqrt{3} \text{ s}$ (b) $3\sqrt{3} \text{ s}$ (c) 3 s (d) 6 s

- 90.** A thin rectangular magnet suspended freely has a period of oscillation 4 s. If it is broken into two halves each having half their initial length, then when suspended similarly, the time period of oscillation of each part will be

(a) 4 s (b) 2 s (c) 1 s (d) $2\sqrt{2} \text{ s}$

- 91** For ferromagnetic material, the relative permeability (μ_r) versus magnetic intensity (H) has the following shape



- 92** Two magnets of same size and mass make respectively 10 and 15 oscillations per minute at certain place. The ratio of their magnetic moments is

(a) 4 : 9 (b) 9 : 4 (c) 2 : 3 (d) 3 : 2

- 93** Two like magnetic poles of strength 10 and 40 SI units are separated by a distance 30 cm. The intensity of magnetic field is zero on the line joining them

(a) at a point 10 cm from the stronger pole
(b) at a point 20 cm from the stronger pole
(c) at the mid-point
(d) at infinity

- 94** A magnet makes 40 oscillations per minute at a place having magnetic field intensity of 0.1×10^{-5} T. At another place, it takes 2.5 s to complete one vibration. The value of earth's horizontal field at that place is

(a) 0.25×10^{-6} T (b) 0.36×10^{-6} T
(c) 0.66×10^{-8} T (d) 1.2×10^{-6} T

- 95** A circular coil of radius 20 cm and 20 turns of wire is mounted vertically with its plane in magnetic meridian. A small magnetic needle is placed at the centre of the coil and it is deflected through 45° when a current is passed through the coil. Horizontal component of earth's field is 0.37×10^{-4} T. The current in coil is

(a) 0.6 A (b) 6 A
(c) 6×10^{-3} A (d) 0.06 A

- 96** A dip circle is so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 40° . Now, the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position, the needle will dip by an angle

(a) 40° (b) 30°
(c) more than 40° (d) less than 40°

- 97** An iron rod of 0.2 cm^2 cross-sectional area is subjected to a magnetising field of 1200 Am^{-1} . The susceptibility of iron is 599. The magnetic flux produced is

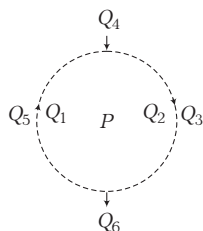
(a) 0.904 Wb (b) 1.81×10^{-5} Wb
(c) 0.904×10^{-5} Wb (d) 5.43×10^{-5} Wb

- 98** A paramagnetic sample shows a net magnetisation of 8 Am^{-1} , when placed in an external magnetic field of 0.6 T at a temperature of 4 K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16 K, the magnetisation will be

[NCERT Exemplar]

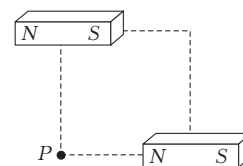
(a) $\frac{32}{3} \text{ Am}^{-1}$ (b) $\frac{2}{3} \text{ Am}^{-1}$
(c) 6 Am^{-1} (d) 2.4 Am^{-1}

- 99** The figure shows the various positions (labelled by subscripts) of small magnetised needles P and Q. The arrows show the direction of their magnetic moment. Which configuration corresponds to the lowest potential energy of all the configurations shown?



(a) PQ_3 (b) PQ_4 (c) PQ_5 (d) PQ_6

- 100** Two short magnets of magnetic moment 1000 Am^2 are placed as shown at the corners of a square of side 10 cm. The net magnetic induction at P is



(a) 0.1 T (b) 0.2 T
(c) 0.3 T (d) 0.4 T

- 101** Two magnets are held together in a vibration magnetometer and are allowed to oscillate in the earth's magnetic field. With like poles together 12 oscillations per minute are made but for unlike poles together only 4 oscillations per minute are executed. The ratio of their magnetic moments is

(a) 3 : 1 (b) 1 : 3 (c) 3 : 5 (d) 5 : 4

- 102** A magnet is suspended in such a way that it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per minute at a place where dip angle is 60° . The ratio of earth's magnetic fields at two places is

(a) $3\sqrt{3} : 8$ (b) $16 : 9\sqrt{3}$
(c) 4 : 9 (d) $2\sqrt{2} : 3$

- 103** Two identical short bar magnets, each having magnetic moment M , are placed a distance of $2d$ apart with axes perpendicular to each other in a horizontal plane. The magnetic induction at a point mid-way between them is

(a) $\frac{\mu_0}{4\pi} (\sqrt{2}) \frac{M}{d^3}$
(b) $\frac{\mu_0}{4\pi} (\sqrt{3}) \frac{M}{d^3}$
(c) $\left(\frac{2\mu_0}{\pi} \right) \frac{M}{d^3}$
(d) $\frac{\mu_0}{4\pi} (\sqrt{5}) \frac{M}{d^3}$

- 104** A short magnet oscillating in vibration magnetometer with a frequency 10 Hz. A downward current of 15 A is established in a long vertical wire placed 20 cm to the west of the magnet. The new frequency of the short magnet is (the horizontal component of earth's magnetic field is 12μ)

(a) 4 Hz (b) 25 Hz
(c) 9 Hz (d) 15 Hz

- 105** Two bar magnets having same geometry with magnetic moments M and $2M$ are placed in such a way that their similar poles are on the same side,

then its time period of oscillation is T_1 . Now, if the polarity of one of the magnets is reversed, then time period of oscillation is T_2 , then

- (a) $T_1 < T_2$ (b) $T_1 > T_2$
(c) $T_1 = T_2$ (d) $T_1 = \infty, T_2 = 0$

106 The length of a magnet is large compared to its width and breadth. The time period of its oscillation

in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be

- (a) 2 s (b) $\frac{2}{3}$ s (c) $2\sqrt{3}$ s (d) $\frac{2}{\sqrt{3}}$ s

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) *These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.*

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

1 Assertion The poles of a magnet cannot be separated by breaking it into two pieces.

Reason The magnetic moment will be reduced to half when a magnet is broken into two equal pieces.

2 Assertion The net magnetic flux coming out of a closed surface is always zero.

Reason Unlike poles of equal strength exist together.

3 Assertion Relative magnetic permeability has no units and no dimensions.

Reason $\mu_r = \mu / \mu_0$, where the symbols have their standard meaning.

4 Assertion $\chi_m - T$ graph for a diamagnetic material is a straight line parallel to T -axis.

Reason This is because susceptibility of a diamagnetic material is not affected by temperature.

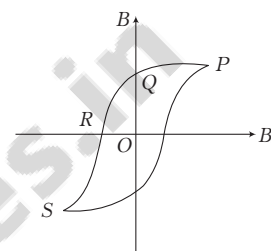
5 Assertion A magnet suspended freely in a uniform magnetic field experiences no net force, but a torque that tends to align the magnet along the field.

Reason Net force, $mB - mB = 0$. But the forces on north and south poles being equal, unlike and parallel, make up a couple that tends to align the magnet, along the field.

Statement based questions

1 The figure illustrates how B , the flux density inside a sample of unmagnetised ferromagnetic material varies with B_0 , the magnetic flux density in which the sample is kept. For the sample to be suitable for

making a permanent magnet, then which of the following statement(s) is/are correct?



- (a) OQ should be large and OR should be small.
(b) OQ and OR should both be large.
(c) OQ should be small and OR should be large.
(d) OQ and OR should both be small.

2 I. When radius of a circular wire carrying current is doubled, its magnetic moment becomes four times.
II. Magnetic moment is directly proportional to area of the loop.

Which of the following statement(s) is/are correct?

- (a) Only I (b) Only II
(c) Both I and II (d) None of these

3 I. The properties of paramagnetic and ferromagnetic substances are not affected by heating.
II. As temperature rises, the alignment of molecular magnets gradually decreases.

Which of the following statement(s) is/are correct?

- (a) Only I (b) Only II
(c) Both I and II (d) None of these

4 I. The ferromagnetic substances do not obey Curie's law.
II. At Curie point, ferromagnetic substances start behaving as a paramagnetic substances.

Which of the following statement(s) is/are correct?

- (a) Only I (b) Only II
(c) Both I and II (d) None of these

5 I. A soft iron core is used in a moving coil galvanometer to increase the strength of magnetic field.
II. From soft iron, more number of the magnetic lines of force passes.

Which of the following statement(s) is/are correct?

- (a) Only I (b) Only II
(c) Both I and II (d) None of these

Match the columns

- 1 With reference to magnetic dipole, match the terms of Column I with the terms of Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Dipole moment	(p) $-\mathbf{M} \cdot \mathbf{B}$
(B) Equatorial field for a short dipole	(q) $\mathbf{M} \times \mathbf{B}$
(C) Axial field for a short dipole	(r) $-\mu_0 \mathbf{M} / 4\pi r^3$
(D) External field : Torque	(s) $\mathbf{M}$
(E) External field : Energy	(t) $\mu_0 2\mathbf{M} / 4\pi r^3$

Codes

- | | | | | |
|-------|---|---|---|---|
| A | B | C | D | E |
| (a) s | t | r | q | p |
| (b) s | r | t | q | p |
| (c) s | p | q | r | t |
| (d) s | t | r | q | p |

- 2 Consider the expression for magnetic potential energy $U_m = \mathbf{m} \cdot \mathbf{B}$, match the terms of Column I with the terms of Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Potential energy at $\theta = 90^\circ$	(p) Minimum
(B) Potential energy at $\theta = 0^\circ$	(q) Maximum
(C) Potential energy at $\theta = 180^\circ$	(r) Zero

Codes

- | | | | | | |
|-------|---|---|-------|---|---|
| A | B | C | A | B | C |
| (a) p | r | q | (b) r | q | p |
| (c) r | p | q | (d) p | q | r |

- 3 Match the terms of Column I with the items of Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Negative susceptibility	(p) Ferromagnetic
(B) Positive and small susceptibility	(q) Diamagnetic
(C) Positive and large susceptibility	(r) Paramagnetic

Codes

- | | | | | | |
|-------|---|---|-------|---|---|
| A | B | C | A | B | C |
| (a) p | r | q | (b) p | q | r |
| (c) q | p | r | (d) q | r | p |

- 4 Match the terms of Column I with the items of Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Diamagnetic	(p) $\mu \gg \mu_0, \mu_r \gg 1$ and $\chi \gg 1$
(B) Paramagnetic	(q) $-1 \leq \chi < 0, 0 \leq \mu_r < 1$ and $\mu < \mu_0$
(C) Ferromagnetic	(r) $0 < \chi < \epsilon, 1 < \mu_r < 1 + \epsilon$ and $\mu > \mu_0$

Codes

- | | | | | | |
|-------|---|---|-------|---|---|
| A | B | C | A | B | C |
| (a) q | r | p | (b) r | p | q |
| (c) p | q | r | (d) q | r | p |

(C) Medical entrances' gallery

Collection of questions asked in NEET & various medical entrance exams

- 1 An iron rod of susceptibility 599 is subjected to a magnetising field of 1200 Am^{-1} . The permeability of the material of the rod is
(Take, $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$) [NEET 2020]
(a) $8.0 \times 10^{-5} \text{ T m A}^{-1}$ (b) $2.4\pi \times 10^{-5} \text{ T m A}^{-1}$
(c) $2.4\pi \times 10^{-7} \text{ T m A}^{-1}$ (d) $2.4\pi \times 10^{-4} \text{ T m A}^{-1}$
- 2 A wire of length L metre carrying a current of I ampere is bent in the form of a circle. Its magnetic moment is [NEET 2020]
(a) $(IL^2 / 4) \text{ A-m}^2$ (b) $(I\pi L^2 / 4) \text{ A-m}^2$
(c) $(2IL^2 / \pi) \text{ A-m}^2$ (d) $(IL^2 / 4\pi) \text{ A-m}^2$
- 3 At a point A on the earth's surface, the angle of dip δ is 25° . At a point B on the earth's surface, the angle of dip δ is -25° . [NEET 2019]

We can interpret that

- (a) A is located in the southern hemisphere and B is located in the northern hemisphere
(b) A is located in the northern hemisphere and B is located in the southern hemisphere
(c) A and B are both located in the southern hemisphere
(d) A and B are both located in the northern hemisphere
- 4 The relations amongst the three elements of earth's magnetic field, namely horizontal component H , vertical component, V and dip angle δ are (B_E = total magnetic field) [NEET (Odisha) 2019]
(a) $V = B_E \tan \delta, H = B_E$
(b) $V = B_E \sin \delta, H = B_E \cos \delta$
(c) $V = B_E \cos \delta, H = B_E \sin \delta$
(d) $V = B_E, H = B_E \tan \delta$

- 5 Assertion** Paramagnetic substances get poorly attracted in magnetic field.

Reason Because magnetic dipoles are aligned along external magnetic field weakly. [AIIMS 2019]

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is true but Reason is incorrect.
 (d) Both Assertion and Reason are incorrect.

- 6** Coercivity and retentivity of soft iron is [JIPMER 2019]

- (a) high coercivity, high retentivity
 (b) low coercivity, high retentivity
 (c) low coercivity, low retentivity
 (d) high coercivity, low retentivity

- 7** The horizontal component of the earth's magnetic field at any place is $0.36 \times 10^{-4} \text{ Wbm}^{-2}$. If the angle of dip at that place is 60° , then the value of vertical component of the earth's magnetic field will be (in Wbm^{-2}) [AIIMS 2018]

- (a) 0.12×10^{-4} (b) 0.24×10^{-4}
 (c) 0.40×10^{-4} (d) 0.622×10^{-4}

- 8 Assertion** The magnetism of magnet is due to the spin motion of electrons.

Reason Dipole moment of electron is smaller than that due to orbit motion around nucleus. [AIIMS 2017]

- (a) Both Assertion and Reason correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.

- 9** If θ_1 and θ_2 be the apparent angles of dip observed in two vertical planes at right angles to each other, then the true angle of dip θ is given by [JIPMER 2017]

- (a) $\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2$ (b) $\tan^2 \theta = \tan^2 \theta_1 + \tan^2 \theta_2$
 (c) $\cot^2 \theta = \cot^2 \theta_1 - \cot^2 \theta_2$ (d) $\tan^2 \theta = \tan^2 \theta_1 - \tan^2 \theta_2$

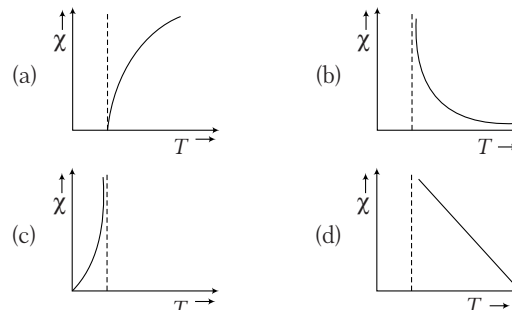
- 10.** A bar magnet is hung by a thin cotton thread in a uniform horizontal magnetic field and is in equilibrium state. The energy required to rotate it by 60° is W . Now the torque required to keep the magnet in this new position is [NEET 2016]

- (a) $\frac{W}{\sqrt{3}}$ (b) $\sqrt{3}W$ (c) $\frac{\sqrt{3}W}{2}$ (d) $\frac{2W}{\sqrt{3}}$

- 11** The magnetic susceptibility is negative for [AIPMT 2016]

- (a) paramagnetic material only
 (b) ferromagnetic material only
 (c) paramagnetic and ferromagnetic materials
 (d) diamagnetic material only

- 12** The variation of magnetic susceptibility with the temperature of a ferromagnetic material can be plotted as [AIIMS 2015]



- 13** If the magnetising field on a ferromagnetic material is increased, its permeability [AIIMS 2015]

- (a) decrease (b) increase
 (c) is unaffected (d) may be increase or decrease

- 14** If r be the distance of a point on the axis of a magnetic dipole from its centre, then the magnetic field at such a point is proportional to [UK PMT 2015]

- (a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$
 (c) $\frac{1}{r^3}$ (d) None of these

- 15** A bar magnet has a coercivity of $4 \times 10^3 \text{ Am}^{-1}$. It is placed inside a solenoid of length 12 cm having 60 turns in order to demagnetise it. The amount of current that should be passed through the solenoid is [EAMCET 2015]

- (a) 16A (b) 8A (c) 4A (d) 2A

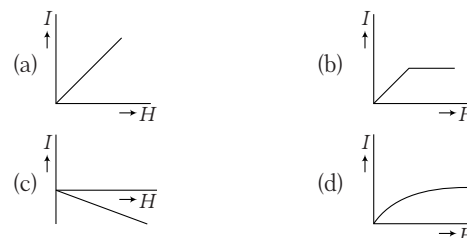
- 16** The effective length of magnet is 31.4 cm and its pole strength is 0.8 Am. The magnetic moment, if it is bent in the form of a semicircle is A-m^2 . [Guj. CET 2015]

- (a) 1.2 (b) 1.6 (c) 0.16 (d) 0.12

- 17** A tangent galvanometer has a coil of 50 turns and a radius of 20 cm. The horizontal component of earth's magnetic field is $B_H = 3 \times 10^{-5} \text{ T}$. What will be the current which gives a deflection of 45° ? [Guj. CET 2015]

- (a) 0.39 A (b) 0.29 A (c) 0.19 A (d) 0.09 A

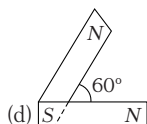
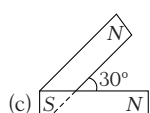
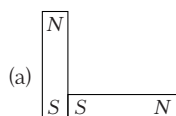
- 18** The correct curve between intensity of magnetisation (I) and magnetic field (H) for a ferromagnetic substance is given by [UP CPMT 2015]



- 19** A bar magnet with magnetic moment $2.5 \times 10^3 \text{ JT}^{-1}$ is rotating in horizontal plane in the space containing magnetic induction $B = 4 \times 10^{-5} \text{ T}$. The work done in rotating the magnet slowly from a direction parallel to the field to a direction 45° from the field, is (in joule) [UP CPMT 2015]
 (a) 0.0 (b) 0.2 (c) 0.03 (d) 0.02

- 20** Core of electromagnets are made of ferromagnetic material which has [KCET 2015]
 (a) low permeability and high retentivity
 (b) high permeability and low retentivity
 (c) low permeability and low retentivity
 (d) high permeability and high retentivity

- 21** Following figures show the arrangement of bar magnets in different configurations. Each magnet has magnetic dipole moment m . Which configuration has highest net magnetic dipole moment? [CBSE AIPMT 2014]



- 22** A bar magnet of moment M and pole strength m is cut into parts of equal lengths. The magnetic moment and pole strength of either part is [UK PMT 2014]
 (a) $\frac{M}{2}, \frac{m}{2}$ (b) $M, \frac{m}{2}$ (c) $\frac{M}{2}, m$ (d) M, m
- 23** A susceptibility of a certain magnetic material is 400. What is the class of the magnetic material? [KCET 2014]
 (a) Diamagnetic (b) Paramagnetic
 (c) Ferromagnetic (d) Ferroelectric

- 24** A paramagnetic sample shows a net magnetisation of 0.8 Am^{-1} , when placed in an external magnetic field of strength 0.8 T at a temperature 5K . When the same sample is placed in an external magnetic field of 0.4 T at a temperature of 20K . The magnetisation is [EAMCET 2014]
 (a) 0.8 Am^{-1} (b) 0.8 Am^2
 (c) 0.1 Am^{-2} (d) 0.1 Am^{-1}

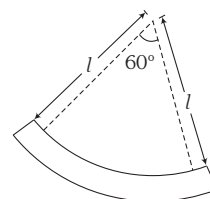
- 25** Nickel shows ferromagnetic property at room temperature. If the temperature is increased beyond Curie temperature, then it will show [UK PMT 2014]
 (a) paramagnetic (b) anti-ferromagnetism
 (c) diamagnetism (d) no magnetic property

- 26** The intensity of magnetisation of a bar magnet is $5 \times 10^4 \text{ Am}^{-1}$. The magnetic length and the area of cross-section of the magnet are 12 cm and 1 cm^2 , respectively. The magnitude of magnetic moment of this bar magnet (in SI unit) is [WB JEE 2014]
 (a) 0.6 (b) 1.3
 (c) 1.2 (d) 2.4

- 27** The magnetic susceptibility of a material of a rod is 299 and permeability of vacuum μ_0 is $4\pi \times 10^{-7} \text{ Hm}^{-1}$. Absolute permeability of the material of the rod is [EAMCET 2014]
 (a) $3771 \times 10^{-7} \text{ Hm}^{-1}$ (b) $3771 \times 10^{-5} \text{ Hm}^{-1}$
 (c) $3770 \times 10^{-6} \text{ Hm}^{-1}$ (d) $3771 \times 10^{-8} \text{ Hm}^{-1}$

- 28** An electron in a circular orbit of radius 0.05 nm performs 10^{16} revolutions per second. The magnetic moment due to this rotation of electron is (in A-m^2) [WB JEE 2014]
 (a) 2.16×10^{-23} (b) 3.21×10^{-22}
 (c) 3.21×10^{-24} (d) 1.26×10^{-23}

- 29** A bar magnet of length l and magnetic dipole moment M is bent in the form of an arc as shown in figure.



The new magnetic dipole moment will be [NEET 2013]
 (a) M (b) $\frac{3}{\pi} M$ (c) $\frac{2}{\pi} M$ (d) $\frac{M}{2}$

- 30** The horizontal and vertical components of earth's magnetic field at a place are 0.3 G and 0.52G . The earth's magnetic field and the angle of dip are [J&K CET 2013]
 (a) 0.3 G and $\delta = 30^\circ$ (b) 0.4 G and $\delta = 40^\circ$
 (c) 0.5 G and $\delta = 50^\circ$ (d) 0.6 G and $\delta = 60^\circ$

- 31** A bar magnet of pole strength 10 A-m is cut into two equal parts breadthwise. The pole strength of each magnet is [J&K CET 2013]
 (a) 5 A-m (b) 10 A-m
 (c) 15 A-m (d) 20 A-m

- 32** A short magnet of magnetic moment M is placed on a straight line. The ratio of magnetic induction fields B_1, B_2, B_3 values on this line at points which are at distance 30 cm , 60 cm and 90 cm respectively from the centre of the magnet is [EAMCET 2013]
 (a) $27:3.37:1$ (b) $373:1:27$
 (c) $27:8:3.37$ (d) $1:2:3$

- 33** A bar magnet of moment of inertia I is vibrated in a magnetic field of induction 0.4×10^{-4} T. The time period of vibration is 12 s. The magnetic moment of the magnet is 120 Am^2 . The moment of inertia of the magnet is (in kg-m^2) approximately [EAMCET 2013]
- (a) 172.8×10^{-4} (b) 2.1×10^{-2}
 (c) 1.57×10^{-2} (d) 1728×10^{-2}
- 34** On heating a ferromagnetic substance above curie temperature, it [MP PMT 2013]
- (a) becomes paramagnetic
 (b) becomes diamagnetic
 (c) remains ferromagnetic with constant magnetic susceptibility
 (d) becomes electromagnetic
- 35 Assertion** Susceptibility is defined as the ratio of intensity of magnetisation I to magnetic intensity H .
Reason Greater the value of susceptibility smaller the value of intensity of magnetisation I . [AIIMS 2012]
- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion
 (b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion
 (c) If Assertion is true but Reason is false
 (d) If both Assertion and Reason are false
- 36** A dip needle vibrates in a vertical plane perpendicular to magnetic meridian. The time of vibration is found to be 2s. The same needle is then allowed to vibrate in the horizontal plane and the time period is again found to be 2s, then the angle of dip is [UPCPMT 2012]
- (a) 0° (b) 30° (c) 45° (d) 90°
- 37** If the dipole moment of a short bar magnet is 1.25 A-m^2 , the magnetic field on its axis at a distance of 0.5 m from the centre of the magnet is [BCECE 2012]
- (a) $1 \times 10^{-4} \text{ NA}^{-1}\text{m}^{-1}$
 (b) $2 \times 10^{-6} \text{ NA}^{-1}\text{m}^{-1}$
 (c) $4 \times 10^{-2} \text{ NA}^{-1}\text{m}^{-1}$
 (d) $6.64 \times 10^{-8} \text{ NA}^{-1}\text{m}^{-1}$
- 38** The horizontal component of earth's magnetic field at a place is 3×10^{-4} T and the dip is $\tan^{-1}\left(\frac{4}{3}\right)$.
 A metal rod of length 0.25 m is placed in $N-S$ direction and is moved at a constant speed of 10 cm s^{-1} towards the east. The emf induced in the rod will be [BCECE Mains 2012]
- (a) $1 \mu\text{V}$ (b) $5 \mu\text{V}$
 (c) $7 \mu\text{V}$ (d) $10 \mu\text{V}$
- 39** If a magnet is suspended at an angle 30° to the magnetic meridian, the dip needle makes angle of 45° with the horizontal. The real dip is [Manipal 2012]
- (a) $\tan^{-1}(\sqrt{3}/2)$ (b) $\tan^{-1}(\sqrt{3})$
 (c) $\tan^{-1}(3/\sqrt{2})$ (d) $\tan^{-1}(2/\sqrt{3})$
- 40** If a steel wire of length l and magnetic moment M is bent into a semicircular arc, then the new magnetic moment is [JCECE 2012]
- (a) $M \times l$ (b) $\frac{M}{l}$ (c) $\frac{2M}{\pi}$ (d) M
- 41** An iron rod of volume 10^{-4} m^3 and relative permeability 1000 is placed inside a long solenoid having 5 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod is [BCECE 2012]
- (a) 20 Am^2 (b) 25 Am^2
 (c) 30 Am^2 (d) 35 Am^2
- 42** Two tangent galvanometers A and B have coils of radii 8 cm and 16 cm respectively and resistance 8Ω each. They are connected in parallel with a cell of emf 4V and negligible internal resistance. The deflections produced in the tangent galvanometers A and B are 30° and 60° , respectively. If A has 2 turns, then B must have [Manipal 2012]
- (a) 18 turns (b) 12 turns
 (c) 6 turns (d) 2 turns
- 43** There are four light-weight-rod samples A, B, C and D separately suspended by threads. A bar magnet is slowly brought near each sample and the following observations are noted. [CBSE AIPMT 2011]
- (i) A is feebly repelled.
 (ii) B is feebly attracted.
 (iii) C is strongly attracted.
 (iv) D remains unaffected.
- 44** At a place on earth, the vertical component of earth's magnetic field is $\sqrt{3}$ times its horizontal component. The angle of dip at this place is [J&K CET 2011]
- (a) 60° (b) 30°
 (c) 45° (d) 0°
- Which one of the following is true?
- (a) C is of diamagnetic material
 (b) D is of a ferromagnetic material
 (c) A is of a non-magnetic material
 (d) B is of a paramagnetic material
- 45** A magnetic wire of dipole moment $4\pi \text{ A-m}^2$ is bent in the form of semicircle. The new magnetic moment is
- (a) $4\pi \text{ A-m}^2$ (b) 8 A-m^2 [J&K CET 2011]
 (c) 4 A-m^2 (d) None of these

- 46** The magnetic field due to short bar magnet of magnetic dipole moment M and length $2l$, on the axis at a distance z (where, $z \gg l$) from the centre of the magnet is given by formula [J&K CET 2011]

(a) $\frac{\mu_0 M}{4\pi z^3} \hat{M}$ (b) $\frac{2\mu_0 M}{4\pi z^3} \hat{M}$
 (c) $\frac{4\mu_0 M}{\mu_0 z^3} \hat{M}$ (d) $\frac{\mu_0 M}{\pi z^3} \hat{M}$

- 47** The angle which the total magnetic field of earth makes with the surface of the earth is called [J&K CET 2011]

- (a) declination (b) magnetic meridian
 (c) geographic meridian (d) inclination

- 48** The plane of a dip circle is set in the geographic meridian and the apparent dip is δ_1 . It is then set in

a vertical plane perpendicular to the geographic meridian. The apparent dip angle is δ_2 . The declination θ at the plane is

(a) $\theta = \tan^{-1}(\tan \delta_1 \tan \delta_2)$ [Punjab PMET 2011]

(b) $\theta = \tan^{-1}(\tan \delta_1 + \tan \delta_2)$

(c) $\theta = \tan^{-1}\left(\frac{\tan \delta_1}{\tan \delta_2}\right)$

(d) $\theta = \tan^{-1}(\tan \delta_1 - \tan \delta_2)$

- 49** The magnetic moment produced in a substance of 1g is $6 \times 10^{-7} \text{ Am}^2$. If its density is 5 g cm^{-3} , then the intensity of magnetisation (in Am^{-1}) will be [Haryana PMT 2011]

- (a) 83×10^6 (b) 3.0
 (c) 1.2×10^{-7} (d) 3×10^{-6}

ANSWERS

CHECK POINT 5.1

1. (a)	2. (a)	3. (a)	4. (d)	5. (b)	6. (d)	7. (b)	8. (a)	9. (a)	10. (a)
11. (a)	12. (a)	13. (d)	14. (b)	15. (a)					

CHECK POINT 5.2

1. (d)	2. (c)	3. (b)	4. (a)	5. (b)	6. (a)	7. (a)	8. (d)	9. (a)	10. (d)
11. (b)	12. (b)	13. (d)	14. (c)	15. (c)					

CHECK POINT 5.3

1. (d)	2. (c)	3. (d)	4. (c)	5. (b)	6. (a)	7. (a)	8. (b)	9. (b)	10. (c)
11. (b)	12. (c)	13. (b)	14. (a)	15. (c)					

(A) Taking it together

1. (d)	2. (b)	3. (d)	4. (c)	5. (d)	6. (a)	7. (b)	8. (b)	9. (d)	10. (a)
11. (d)	12. (c)	13. (b)	14. (c)	15. (b)	16. (c)	17. (d)	18. (a)	19. (c)	20. (c)
21. (a)	22. (b)	23. (b)	24. (a)	25. (a)	26. (d)	27. (d)	28. (a)	29. (d)	30. (c)
31. (b)	32. (b)	33. (a)	34. (a)	35. (a)	36. (d)	37. (b)	38. (b)	39. (b)	40. (c)
41. (b)	42. (d)	43. (b)	44. (b)	45. (d)	46. (c)	47. (b)	48. (c)	49. (c)	50. (b)
51. (d)	52. (d)	53. (c)	54. (b)	55. (a)	56. (a)	57. (d)	58. (b)	59. (b)	60. (b)
61. (b)	62. (c)	63. (b)	64. (a)	65. (d)	66. (c)	67. (d)	68. (b)	69. (c)	70. (a)
71. (d)	72. (a)	73. (c)	74. (c)	75. (d)	76. (a)	77. (d)	78. (b)	79. (d)	80. (d)
81. (a)	82. (b)	83. (a)	84. (c)	85. (b)	86. (c)	87. (c)	88. (b)	89. (b)	90. (b)
91. (d)	92. (a)	93. (b)	94. (b)	95. (a)	96. (d)	97. (b)	98. (b)	99. (d)	100. (a)
101. (d)	102. (b)	103. (d)	104. (d)	105. (a)	106. (b)				

(B) Medical entrance special format questions• **Assertion and reason**

1. (b) 2. (a) 3. (a) 4. (a) 5. (a)

• **Statement based questions**

1. (b) 2. (c) 3. (b) 4. (c) 5. (c)

• **Match the columns**

1. (b) 2. (c) 3. (d) 4. (a)

(C) Medical entrances' gallery

1. (d)	2. (d)	3. (b)	4. (b)	5. (a)	6. (c)	7. (d)	8. (c)	9. (a)	10. (b)
11. (d)	12. (b)	13. (a)	14. (c)	15. (b)	16. (c)	17. (c)	18. (b)	19. (c)	20. (b)
21. (c)	22. (c)	23. (c)	24. (d)	25. (a)	26. (a)	27. (a)	28. (d)	29. (b)	30. (d)
31. (a)	32. (a)	33. (a)	34. (a)	35. (c)	36. (c)	37. (b)	38. (d)	39. (d)	40. (c)
41. (b)	42. (b)	43. (d)	44. (a)	45. (b)	46. (b)	47. (d)	48. (c)	49. (b)	

Hints & Explanations

• CHECK POINT 5.1

- 2** (a) Magnetic lines of force due to a bar magnet do not intersect because a point on magnetic lines always has a single net magnetic field. If they intersect then it means, there are two directions of magnetic field intensity at that point, which is impossible.

- 4** (d) Initial magnetic moment = M_1

When cut in two parts, magnetic moment of each part

$$M' = \frac{M_1}{2}$$

When these pieces are placed perpendicular, effective magnetic moment,

$$M_2 = \sqrt{2}M' = \sqrt{2} \times \frac{M_1}{2}$$

$$\therefore \frac{M_1}{M_2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

- 7** (b) Since, for magnet P , axis lies along X -axis and for magnet Q , axis is along Y -axis. The point R is along axial line w.r.t. magnet P and is along equatorial line w.r.t. magnet Q . Hence, magnetic field due to magnet Q ,

$$B_Q = \frac{\mu_0}{4\pi} \frac{M}{x^3} = B \quad [R \text{ at equatorial point}] \dots(i)$$

Magnetic field due to magnet P ,

$$B_P = \frac{\mu_0}{4\pi} \frac{2M}{x^3} = 2B \quad [R \text{ at axial point}] \dots(ii)$$

As, at point R magnetic field due to P and Q magnet are perpendicular to each other

and B_R = net magnetic field at R due to magnet P and Q , i.e.

$$B_R = \sqrt{B_P^2 + B_Q^2} = \sqrt{B^2 + (2B)^2} = \sqrt{5}B$$

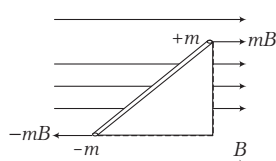
- 9** (a) The relation between magnetic moment (μ_l) and angular momentum (L) is

$$\mu_l = -\frac{q}{2m} L \Rightarrow \frac{\mu_l}{L} = -\frac{q}{2m}$$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment.

- 10** (a) When a bar magnet of magnetic moment $\mathbf{M}$ is placed in a magnetic field of induction $\mathbf{B}$, then net force on it is

$$F_R = mB + (-m)B = 0$$



While torque, $\tau = mB \times 2l \sin \theta = MB \sin \theta$ (as, $M = 2ml$)
i.e. $\tau = \mathbf{M} \times \mathbf{B}$

$$\begin{aligned} \text{11 (a) As, } \tau &= MB \sin \theta = (m \times 2l) \times 2 \times 10^{-5} \sin 30^\circ \\ &= 15 \times 10 \times 10^{-2} \times 2 \times 10^{-5} \times \frac{1}{2} \\ &= 1.5 \times 10^{-5} \text{ N-m} \end{aligned}$$

- 12** (a) As, $\tau = MB \sin \theta$

where, $\theta = 90^\circ \Rightarrow \tau = MB$

$$\text{Given, } \tau_2 = \frac{1}{2} \tau_1 \Rightarrow MB \sin \theta = \frac{1}{2} MB$$

$$\Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

$$\therefore \text{Angle of rotation} = 90^\circ - 30^\circ = 60^\circ$$

- 13** (d) Work done in rotating the magnet through an angle θ from initial position (i.e. $\theta_1 = 0^\circ$) is given by

$$W = MB(\cos \theta_1 - \cos \theta) = MB(\cos 0^\circ - \cos \theta) = MB(1 - \cos \theta)$$

- 15** (a) According to Gauss' theorem in magnetism, surface integral of magnetic field intensity over a surface (closed or open) is always zero, i.e. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$.

• CHECK POINT 5.2

- 1** (d) The strength of earth's magnetic field at the surface of the earth ranges from 0.25 G to 0.65 G.

- 6** (a) Magnetic dip or magnetic inclination is given by

$$\tan \theta = \frac{V}{H} \quad \dots(i)$$

where, V and H are vertical and horizontal components of earth's magnetic field, respectively.

$$\text{Given, } H = \frac{1}{\sqrt{3}} V \Rightarrow \frac{V}{H} = \sqrt{3} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

- 8** (d) As, $\tan \delta' = \frac{\tan \delta}{\cos \theta} = \frac{\tan 45^\circ}{\cos 30^\circ}$

$$\tan \delta' = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \delta' = \tan^{-1} \left(\frac{2}{\sqrt{3}} \right)$$

- 9** (a) As, $\tan \delta = \frac{V}{H}$ and $\tan \delta_1 = \frac{V}{H \cos(90^\circ - \theta)}$
$$= \frac{\tan \delta}{\cos(90^\circ - \theta)} = \tan \delta \operatorname{cosec} \theta$$

- 12** (b) As, the current and the other factors are same for both the galvanometers, so

$$N \propto \tan \theta \Rightarrow \frac{N_1}{N_2} = \frac{\tan 60^\circ}{\tan 30^\circ} \text{ or } \frac{N_1}{N_2} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3:1$$

14 (c) As, time period $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\therefore T = 2\pi\sqrt{\frac{I}{MB}}, M' = \frac{M}{2}$$

$$\Rightarrow T' = T = 2\pi\sqrt{\frac{I}{MB}}$$

15 (c) As, the time period, $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\text{or } T \propto \frac{1}{\sqrt{B}} \Rightarrow \frac{T_2}{T_1} = \sqrt{\frac{B_1}{B_2}}$$

$$\text{or } \frac{T_2}{3} = \sqrt{\frac{1}{4}} \Rightarrow T_2 = 1.5\text{ s}$$

• CHECK POINT 5.3

2 (c) Magnetic permeability,

$$\mu = \frac{B}{H}$$

$$\text{Its SI unit is } \frac{\text{Wb/m}^2}{\text{A/m}} = \frac{\text{Wb}}{\text{A}} \cdot \frac{1}{\text{m}} = \text{Henry m}^{-1} \left[\therefore \frac{\text{Wb}}{\text{A}} = \text{H (Henry)} \right]$$

14 (a) Some peculiar properties of ferromagnetic materials are commonly displayed by curve of **B** against **H**, which is called *B-H* curve or hysteresis loop. Diamagnetic and paramagnetic substances do not show these properties.

15 (c) If a magnet retains its attracting power for a long time it is said to be permanent, otherwise temporary. Permanent magnets are made of ferromagnetic substances.

(A) Taking it together

1 (d) Magnetic pole strength is stronger at end part of magnet, so maximum iron powder is collected at the end point of magnet.

2 (b) A permanent magnet has large retentivity and coercivity, so it attracts only magnetic substances.

5 (d) A line passing through places having zero value of magnetic dip is called aclinic line. At all places upon this line, a freely suspended magnet will remain horizontal.

11 (d) SI unit of magnetic induction is

$$B = \frac{\text{Unit of } \phi}{\text{Unit of } \Delta A} = \frac{\text{Wb}}{\text{m}^2} = \text{tesla}$$

Also, SI unit of *B* is weber/metre² and newton/ampere-metre.

12 (c) Magnetic moment per unit volume is intensity of magnetisation, and number of lines of force crossing per unit area is intensity of magnetic field. So correct option is (c).

18 (a) Steel has more retentivity and coercivity, so it is used for making permanent magnet.

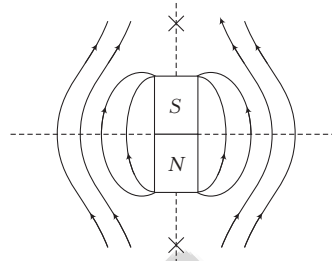
21 (a) In non-uniform magnetic field, a magnetic needle experiences a force and a torque.

22 (b) For a diamagnetic substance, χ is small, negative and independent of temperature.

24 (a) A freely hanged magnet stays with its magnetic axis parallel to magnetic meridian.

25 (a) A dip needle in a plane perpendicular to magnetic meridian will always remain vertical.

28 (a) If the *N*-pole of a bar magnet points south, the fields of the magnet and the earth will point in opposite directions along the axis of the magnet. So, two neutral points are obtained which are equidistant from the magnet on its axis.



31 (b) An electron moving around the nucleus has a magnetic moment given by $\mu = \frac{e}{2m} L$

where, *L* is the magnitude of the angular momentum of the circulating electron around the nucleus. The smallest value of μ is called the Bohr magneton μ_B and its value is $\mu_B = 9.27 \times 10^{-24} \text{ JT}^{-1}$.

35 (a) From Gauss's law $\oint \mathbf{B} \cdot d\mathbf{S} = 0$

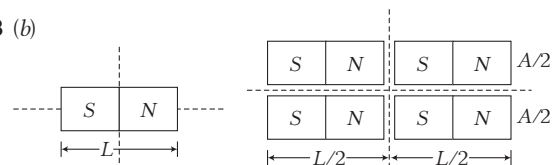
or divergence of $\mathbf{B} = \nabla \cdot \mathbf{B} = 0$

37 (b) Let pole strength be m_1 and m_2 and distance between them is *r*, then force between magnetic poles, $F \propto \frac{m_1 m_2}{r^2}$

$$\text{In second case, } F' \propto \frac{(2m_1)(2m_2)}{(2r)^2}, \propto \frac{m_1 m_2}{r^2}$$

$$\therefore F' = F$$

38 (b)



The situation is summarised in figure, we see that magnetic pole strength of each part becomes half, i.e. $m/2$.

39 (b) Due to hole, the effective length of magnet (*2l*) is decreased, so magnetic moment of one with hole is smaller.

40 (c) For longitudinal position, magnetic field will be $B_1 \propto \frac{2M}{d^3}$

For transverse position, magnetic field will be $B_2 \propto \frac{M}{d^3}$

$$\therefore \frac{B_1}{B_2} = 2:1$$

44 (b) Torque, $\tau = MB \sin \theta$

$$\therefore \tau_1 = MB_1 \sin 90^\circ = MB_1$$

$$\text{and } \tau_2 = MB_2 \sin 90^\circ = MB_2 \text{ or } \frac{MB_1}{MB_2} = \frac{\tau_1}{\tau_2}$$

$$\therefore \frac{B_1}{B_2} = \frac{\tau_1}{\tau_2}$$

- 46 (c) At the magnetic north pole, angle of dip, $\theta = 90^\circ$

$$\therefore H = B \cos \theta = B \cos 90^\circ = 0$$

- 49 (c) Magnetism of a magnet falls with rise of temperature and becomes practically zero above Curie temperature as shown in Fig. (c).

50 (b)
$$\frac{\tan A}{\tan B} = \frac{2d(d^2 + l^2)^{3/2}}{(d^2 - l^2)^2} = \frac{2d \times d^3 (l + l^2/d^2)^{3/2}}{d^4 \left(1 - \frac{l^2}{d^2}\right)^2}$$

$$\therefore l > \frac{l^2}{d^2} \Rightarrow \frac{\tan A}{\tan B} = 2:1$$

- 52 (d) Given, $B = 4B_0$

$$\therefore \frac{B}{B_0} = 4 \Rightarrow \mu_r = 4 \quad \left[\because \mu_r = \frac{B}{B_0} \right]$$

- 53 (c) Hint $i = K \tan \theta$, i.e. $i \propto \tan \theta$, at $\theta = 90^\circ$, $i = \infty$

So, the curve c represents this relation correctly.

- 56 (a) As, we know, $M = m(2l)$

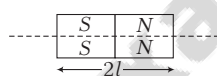
$$\therefore M = 4 \times 10 \times 10^{-2} = 0.4 \text{ A-m}^2$$

- 59 (b) Above the Curie temperature, ferromagnetic substance become paramagnetic in nature. So, its susceptibility varies inversely as the absolute temperature.

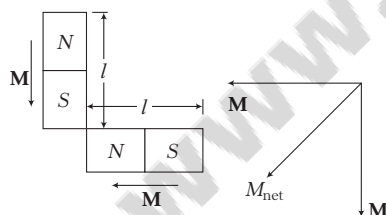
- 61 (b) If magnet is cut along the axis of magnet of length $2l$, then new pole strength becomes half, i.e. $m' = \frac{m}{2}$ and new length remains same, i.e. $2l' = 2l$.

$\therefore$ New magnetic moment,

$$M' = \frac{m}{2} \times 2l = \frac{m2l}{2} = \frac{M}{2}$$



- 62 (c) Net magnetic moment of the system is given by



$$M = \sqrt{M^2 + M^2} = \sqrt{2}M = \sqrt{2}ml$$

- 63 (b) Magnetic moment of the bar magnet is given by

$$M = \frac{\tau}{B \sin \theta} \quad (\because \tau = MB \sin \theta)$$

$$= \frac{0.032}{0.16 \times \sin 30^\circ} = \frac{0.032 \times 2}{0.16} = 0.4 \text{ J T}^{-1}$$

- 64 (a) As, $\tau = MB \sin \theta$

$$\therefore m = \frac{\tau}{(2l) B \sin \theta} = \frac{25 \times 10^{-6}}{(0.05)(5 \times 10^{-2}) \sin 30^\circ}$$

$$= 2 \times 10^{-2} \text{ A-m}$$

- 65 (d) Force experienced by each pole of bar magnet is given by

$$F = mB$$

where, m is pole strength of bar magnet.

$$\text{But } M = mL$$

$$\therefore F = \frac{M}{L} \times B \Rightarrow 6 \times 10^{-4} = \frac{3}{L} \times 2 \times 10^{-5}$$

$$\therefore L = 0.1 \text{ m}$$

- 66 (c) In case of toroid, the magnetic field is only confined inside the body of toroid in the form of concentric magnetic lines of force and there is no magnetic field outside the body of toroid. This is because, the loop encloses no current. Thus, the magnetic moment of toroid is zero.

In general, if we take r as a large distance outside the toroid, then $m \propto \frac{1}{r^3}$.

- 67 (d) Work done in rotating the magnet from angle θ_1 to angle θ_2 is given by

$$W = MB(\cos \theta_1 - \cos \theta_2)$$

$$\text{Here, } \theta_1 = 0^\circ \text{ and } \theta_2 = 180^\circ$$

$$W = MB(\cos 0^\circ - \cos 180^\circ) = 2MB$$

- 68 (b) Work done in rotating the magnet in uniform magnetic field is given by

$$W = MB(\cos \theta_1 - \cos \theta_2)$$

$$\text{Here, } \theta_1 = 0^\circ, \theta_2 = 90^\circ$$

$$\therefore W = 2 \times 0.1(\cos 0^\circ - \cos 90^\circ)$$

$$= 2 \times 0.1(1 - 0) = 0.2 \text{ J}$$

- 69 (c) Here, $B = 0.5 \text{ T}$; $I = 20 \text{ A}$; $N = 15$; $A = -0.04 \hat{i} \text{ m}^2$

Magnetic moment of the coil,

$$M = NIA$$

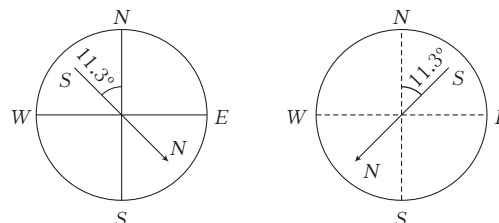
$$= (15)(20)(-0.04 \hat{i}) = -12 \hat{i} \text{ A m}^2$$

The potential energy of the coil in the given orientation,

$$U = -M \cdot B = -(-12 \hat{i}) \cdot (0.5 \hat{i}) = +6 \text{ J}$$

- 70 (a) For the earth's magnetism, the magnetic field lines of the earth resemble that of a hypothetical magnetic dipole located at the centre of the earth.

The axis of the dipole does not coincide with the axis of rotation of the earth but is presently tilted by approximately 11.3° with respect to the later. This results into two situations as given in the figures below



Hence, the declination varies between 11.3° W to 11.3° E .

- 71 (d) $\therefore \frac{B}{0.2} = \frac{M/(5)^3}{2M/(10)^3} = \frac{(10/5)^3}{2} = 4 \text{ G}$

$$\Rightarrow B = 4 \times 0.2 = 0.8 \text{ oersted}$$

$$72 \text{ (a)} \therefore \frac{H_1}{H_2} = \frac{B \cos \theta_1}{B \cos \theta_2} = \frac{\cos \theta_1}{\cos \theta_2} = \frac{\cos 30^\circ}{\cos 45^\circ} = \frac{\sqrt{3}}{2} \times \sqrt{2} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$73 \text{ (c)} \text{ Total strength of magnetic field, } B^2 = H^2 + V^2$$

$$\therefore V = \sqrt{B^2 - H^2} = \sqrt{(0.5)^2 - (0.3)^2} = 0.4 \text{ G}$$

$$\tan \phi = \frac{V}{H} = \frac{0.4}{0.3} = \frac{4}{3}$$

$$\therefore \phi = \tan^{-1} \left(\frac{4}{3} \right)$$

$$74 \text{ (c)} \text{ As, } H = B_e \cos \theta$$

$$\therefore \frac{1}{2} = B_e \frac{\sqrt{3}}{2}$$

$$B_e = \frac{1}{\sqrt{3}} \text{ oersted}$$

75 (d) As, we know a permanent magnet is a substance which at room temperature retain ferromagnetic property for a long period of time.

The individual atoms in a ferromagnetic material possess a dipole moment as in a paramagnetic material.

However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. Thus, we can say that in a permanent magnet at room temperature, domains are all perfectly aligned.

$$76 \text{ (a)} \text{ As, } \tan \theta = \frac{B_V}{B_H}$$

$$\tan \theta' = \frac{B_V}{B_H \cos x}$$

$$\therefore \frac{\tan \theta'}{\tan \theta} = \frac{1}{\cos x}$$

77 (d) For a temporary magnet, the hysteresis loop should be long and narrow.

$$78 \text{ (b)} T \propto \sqrt{I}$$

By increasing mass to four times, the moment of inertia will also increase four times. Then $T' = \sqrt{4I} = 2\sqrt{I} = 2T$

79 (d) For a tangent galvanometer, if I ampere current flows through coil, then this current is proportional to tangent of angle of deflection (of the needle), i.e.

$$I \propto \tan \theta$$

$$\therefore \frac{I_1}{I_2} = \frac{\tan \theta_1}{\tan \theta_2} \Rightarrow \frac{2}{I_2} = \frac{\tan 30^\circ}{\tan 60^\circ}$$

$$\therefore I_2 = 6A$$

80 (d) In series, current is same.

$$\Rightarrow \frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2} \text{ or } \frac{N_1}{N_2} = \frac{\tan 60^\circ}{\tan 45^\circ} = \sqrt{3}$$

81 (a) Magnetic field due to a bar magnet at a distance r from the centre of magnet on axial position is

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$$

$$\therefore \frac{B_1}{B_2} = \left(\frac{r_2}{r_1} \right)^3 = \left(\frac{48}{24} \right)^3 = \frac{8}{1} = 8$$

$$82 \text{ (b)} \text{ Intensity of magnetisation, } I = \frac{M}{V} = - \frac{M}{\text{mass/density}}$$

$$\text{Given, mass} = 1 \text{ g} = 10^{-3} \text{ kg}$$

$$\text{and density} = 5 \text{ g/cm}^3 = \frac{5 \times 10^{-3} \text{ kg}}{(10^{-2})^3} = 5 \times 10^3 \text{ kg/m}^3$$

$$\text{Hence, } I = \frac{6 \times 10^{-7} \times 5 \times 10^3}{10^{-3}} = 3 \text{ A/m}$$

$$83 \text{ (a)} \therefore H = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \Rightarrow 4 \times 10^{-5} = 10^{-7} \times \frac{M}{(20)^3 \times 10^{-6}}$$

$$M = \frac{(20)^3 \times 4 \times 10^{-5} \times 10^{-6}}{10^{-7}}$$

$$\therefore M = 3.2 \text{ A-m}^2$$

$$85 \text{ (b)} \text{ As, Gauss' law states that, } \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0} \text{ for electrostatic}$$

field. It does not contradicts for electrostatic fields as the electric field lines do not form continuous closed path.

According to Gauss' law in magnetism, $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$.

Which means that the number of magnetic field lines entering the Gaussian surface is equal to the number of field lines leaving it.

It contradicts for magnetic field, because there is a magnetic field inside the solenoid and no field outside the solenoid carrying current but the magnetic field lines form the closed paths.

$$86 \text{ (c)} \text{ In CGS, } B_{\text{axial}} = 9 = \frac{\mu_0}{4\pi} \frac{2M}{x^3} \quad \dots(i)$$

$$B_{\text{equatorial}} = \frac{\mu_0}{4\pi} \frac{M}{\left(\frac{x}{2}\right)^2} = \frac{\mu_0}{4\pi} \frac{8M}{x^3} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$B_{\text{equatorial}} = 4B_{\text{axial}} = 36 \text{ G}$$

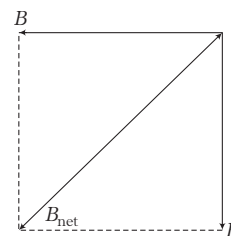
$$87 \text{ (c)} \text{ As, } T \propto \frac{1}{\sqrt{H}} \text{ and } H = B \cos \theta$$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{B_2 \cos \theta_2}{B_1 \cos \theta_1}}$$

$$\Rightarrow \frac{2}{3} = \sqrt{\frac{B_2 \cdot \cos 60^\circ}{B_1 \cos 30^\circ}} = \frac{B_2}{\sqrt{3}B_1}$$

$$\Rightarrow \frac{4}{9} = \frac{B_2}{\sqrt{3}B_1} \Rightarrow \frac{B_1}{B_2} = \frac{9}{4\sqrt{3}}$$

$$88 \text{ (b)} B = \frac{\mu_0}{4\pi} \frac{M}{d^3}$$



$$\therefore B_{\text{net}} = \sqrt{B^2 + B^2} = \sqrt{2}B = \frac{\mu_0}{4\pi} \frac{\sqrt{2}M}{d^3}$$

89 (b) $T \propto \frac{1}{\sqrt{M}}$
 $\Rightarrow M_1 = M + 2M = 3M$ and $M_2 = 2M - M = M$
 $\therefore T_2 = \sqrt{3} T_1 = 3\sqrt{3} \text{ s}$

90 (b) As, we know $T = 2\pi \sqrt{\frac{I}{MB}}$
 When magnet is broken into two equal parts, $M' = \frac{M}{2}$ and $l' = \frac{l}{2}$
 $\therefore \frac{T'}{T} = \sqrt{\frac{I'}{M'B}} \times \sqrt{\frac{MB}{I}} = \sqrt{\frac{I}{8} \times \frac{2}{M}} = \sqrt{\frac{M}{I}} = \frac{1}{2}$
 $\Rightarrow T' = \frac{T}{2} = \frac{4}{2} = 2 \text{ s}$

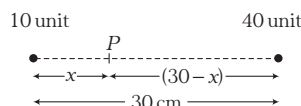
92 (a) Time period of vibrating magnet is given by

$$T = 2\pi \sqrt{\left(\frac{I}{MH}\right)}$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\left(\frac{M_2}{M_1}\right)}$$

$$\therefore \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2} = \frac{(60/15)^2}{(60/10)^2} = \frac{4}{9}$$

93 (b) Suppose magnetic field is zero at point P which lies at a distance x from 10 unit pole.



Hence, at P

$$\frac{\mu_0}{4\pi} \cdot \frac{10}{x^2} = \frac{\mu_0}{4\pi} \cdot \frac{40}{(30-x)^2} \Rightarrow x = 10 \text{ cm}$$

So, from stronger pole distance is 20 cm.

94 (b) Time period, $T = 2\pi \sqrt{\frac{I}{MB}}$

$$\therefore T \propto \frac{1}{\sqrt{B}}$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{B_2}{B_1}}$$

$$\Rightarrow \frac{60/40}{2.5} = \sqrt{\frac{B_2}{0.1 \times 10^{-5}}}$$

$$\therefore B_2 = 0.36 \times 10^{-6} \text{ T}$$

95 (a) The needle is deflected through 45° means magnetic field at the centre of circular coil is equal (in magnitude) to the horizontal component of earth's magnetic field.

$$\frac{\mu_0 NI}{2r} = H$$

$$\therefore I = \frac{2rH}{\mu_0 N} = \frac{2 \times 0.2 \times 0.37 \times 10^{-4}}{(4\pi \times 10^{-7}) (20)} \approx 0.6 \text{ A}$$

96 (d) We know that,

$$\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2$$

where, θ = true angle of dip,

θ_1 = apparent angle of dip

and θ_2 = angle of dip with magnetic meridian.

Given, $\theta_2 = 30^\circ$ and $\theta_1 = 40^\circ$

Hence, $\cot^2 \theta = \cot^2 40^\circ + \cot^2 30^\circ$

$$\Rightarrow \cot \theta = \sqrt{(1.19)^2 + (\sqrt{3})^2}$$

$$\Rightarrow \cot \theta = 2.1$$

$$\Rightarrow \theta = \cot^{-1}(2.1) = 25^\circ$$

Therefore, true angle of dip is less than 40° . So, the correct option is (d).

97 (b) As, $\mu = \mu_0(1 + \chi)$ or $\mu = 4\pi \times 10^{-7}(1 + 599)$

$$\text{or } \mu = 7.536 \times 10^{-4} \text{ TmA}^{-1}$$

$$B = \mu H = (7.536 \times 10^{-4} \times 1200) \text{ T}$$

$$\therefore \text{Hence, } \phi = BA = (7.536 \times 10^{-4} \times 1200 \times 0.2 \times 10^{-4}) \text{ Wb}$$

$$= 1.81 \times 10^{-5} \text{ Wb}$$

98 (b) From Curie's law, we can deduce a formula for the relation between magnetic field induction, temperature and magnetisation.

$$\text{i.e. } I (\text{magnetisation}) \propto \frac{B (\text{magnetic field induction})}{T (\text{temperature in kelvin})}$$

$$\Rightarrow \frac{I_2}{I_1} = \frac{B_2}{B_1} \times \frac{T_1}{T_2}$$

$$\text{Here } I_1 = 8 \text{ Am}^{-1}$$

$$B_1 = 0.6 \text{ T}, T_1 = 4 \text{ K}$$

$$B_2 = 0.2 \text{ T}, T_2 = 16 \text{ K}, I_2 = ?$$

$$\Rightarrow \frac{0.2}{0.6} \times \frac{4}{16} = \frac{I_2}{8}$$

$$\Rightarrow I_2 = 8 \times \frac{1}{12} = \frac{2}{3} \text{ Am}^{-1}$$

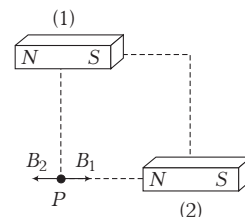
99 (d) As potential energy is given as

$$U = -MB(1 - \cos \theta) \Rightarrow U = -MB [\text{when } \theta = 0^\circ, U = \text{minimum}]$$

Hence, $\mathbf{M}$ and $\mathbf{B}$ should be parallel to each other for minimum potential energy.

Here, PQ_6 configuration provides the condition of lowest PE.

100 (a) Point P lies on equatorial line of magnet (1) and axial line of magnet (2) as shown



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$$

$$= 10^{-7} \times \frac{1000}{(0.1)^3} = 0.1 \text{ T}$$

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$$

$$= 10^{-7} \times \frac{2 \times 1000}{(0.1)^3} = 0.2 \text{ T}$$

$$\therefore B_{\text{net}} = B_2 - B_1 = 0.1 \text{ T}$$

$$101 \text{ (d)} \because \frac{v_1}{v_2} = \frac{12}{4} = 3$$

$$\therefore \frac{T_1}{T_2} = \frac{1}{3}$$

$$\text{Further, } T \propto \frac{1}{\sqrt{M}}$$

$$\therefore \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} = \frac{1}{3}$$

$$\text{This equation gives, } \frac{M_1}{M_2} = \frac{5}{4}$$

102 (b) Time period of suspended magnet,

$$T = 2\pi \sqrt{\frac{I}{MB \cos \delta}}$$

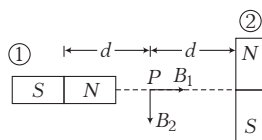
$$\Rightarrow v = \frac{1}{2\pi} \sqrt{\frac{MB \cos \delta}{I}}$$

$$v \propto \sqrt{B \cos \delta}$$

$$\text{or } B \propto \frac{v^2}{\cos \delta}$$

$$\therefore \frac{B_1}{B_2} = \frac{(20)^2}{\cos 30^\circ} \times \frac{\cos 60^\circ}{(15)^2} = \frac{16 \times 2}{9 \times \sqrt{3}} \times \frac{1}{2} = 16 : 9\sqrt{3}$$

103 (d) At point P, net magnetic field, $B_{\text{net}} = \sqrt{B_1^2 + B_2^2}$



$$\text{where, } B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$$

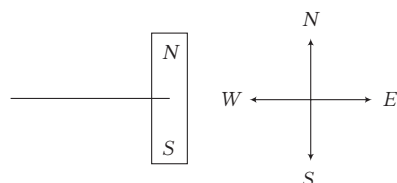
$$\text{and } B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$$

$$\Rightarrow B_{\text{net}} = \frac{\mu_0}{4\pi} \cdot \frac{\sqrt{5}M}{d^3}$$

104 (d) Frequency, $v = \frac{1}{2\pi} \sqrt{\frac{MH}{I}}$

$$\therefore v' = \frac{1}{2\pi} \sqrt{\frac{M(B+H)}{I}}$$

where, B = magnetic field due to downward conductor.



$$\text{Then, } B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}$$

$$\Rightarrow B = 10^{-7} \times \frac{2 \times 15}{20 \times 10^{-2}}$$

$$\text{So, } B = 15 \mu\text{T}$$

$$\therefore \frac{v'}{v} = \sqrt{\frac{B+BH}{B}}$$

$$\frac{v'}{10} = \sqrt{\frac{15+12}{12}} = 1.5$$

$$v' = 15 \text{ Hz}$$

105 (a) Using the formula for time period for magnetic system,

$$T = 2\pi \sqrt{\left(\frac{I}{MH}\right)} \Rightarrow T \propto \frac{1}{\sqrt{M}} \quad \dots(i)$$

When similar poles placed at same side, then

$$M_1 = M + 2M = 3M$$

$$\text{So, from Eq. (i), } T_1 \propto \frac{1}{\sqrt{3M}} \quad \dots(ii)$$

When the polarity of a magnet is reversed, then

$$M_2 = 2M - M = M$$

So, from Eq. (i)

$$T_2 \propto \frac{1}{\sqrt{M}} \quad \dots(iii)$$

Now, dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_1}{T_2} = \frac{\sqrt{M}}{\sqrt{3M}} = \frac{1}{\sqrt{3}} \Rightarrow T_2 = \sqrt{3} T_1$$

Hence,

$$T_1 < T_2$$

106 (b) Initially, the time period of the magnet,

$$T = 2\pi \sqrt{\left(\frac{I}{MB}\right)}$$

For each part of magnet, its moment of inertia is $\frac{I}{27}$ and its magnetic moment = $M/3$.

When magnet is cut along its length into three equal parts and placed on each other, then new moment of inertia of system,

$$I' = \frac{I}{27} \times 3 = \frac{I}{9}$$

and magnetic moment of system, $M' = \frac{M}{3} \times 3 = M$

$\therefore$ Time period of system,

$$T' = 2\pi \sqrt{\frac{I'}{M'B}} = \frac{1}{3} \times 2\pi \sqrt{\frac{I}{MB}} = \frac{T}{3} = \frac{2}{3} \text{ s}$$

(B) Medical entrance special format questions

• Assertion and reason

1 (b) When we break magnet into two pieces, its poles do not get separated as it again becomes a magnet. Also, when magnet is broken into two equal pieces, its magnetic moment gets reduced to half.

2 (a) According to Gauss's law in magnetism, $\oint \mathbf{B} \cdot d\mathbf{S} = 0$

It is because magnetic monopoles does not exist and unlike poles of equal strength exist together.

- 3 (a) Relative magnetic permeability is given by

$$\mu_r = \frac{\mu}{\mu_0}$$

It is a dimensionless quantity.

- 4 (a) The susceptibility of diamagnetic material is small and negative. It is independent of temperature. So, the χ_m - T graph is a straight line parallel to T -axis.
- 5 (a) A magnet does not experience any force since net force, $mB - mB = 0$. The forces on north and south poles are equal, unlike and parallel, makes a couple to align the magnet along the field.

• Statement based questions

- 2 (c) Magnetic moment of circular loop, $M = NIA$

where, $A = \pi r^2$, when $r' = 2r$

$$M' = NI(2\pi r)^2 = 4M$$

- 3 (b) The magnetic properties of a substance is due to alignment of molecular magnet in it. When these substances are heated, molecules acquire kinetic energy. Some of the molecules get back to the closed chain arrangement (leading to zero resultant). So, they lose their magnetic property. Hence, the properties of paramagnetic and ferromagnetic substances are affected by heating.
- 4 (c) The susceptibility of ferromagnetic substances varies inversely with temperature. At Curie point, ferromagnetic substance start behaving as a paramagnetic substance. After Curie point, the susceptibility of ferromagnetic substance varies inversely with its absolute temperature. These substances obey Curie's law only above its Curie point.
- 5 (c) Soft iron is a ferromagnetic material and hence when it is used as a core in moving coil galvanometer, more number of magnetic field lines passes through it. Thus, the strength of magnetic field and hence the sensitivity of galvanometer increases.

• Match the columns

- 2 (c) Since, the magnetic potential energy U_m is given by

$$U_m = \mathbf{m} \cdot \mathbf{B} = -mB \cos \theta$$

A. For $\theta = 90^\circ \Rightarrow U_m = -mB \cos 90^\circ = 0$

B. For $\theta = 0^\circ$, $U_m = -mB \cos 0^\circ = -mB$

The potential energy is minimum and hence the needle is in most stable position.

C. For $\theta = 180^\circ$, $U_m = -mB \cos 180^\circ = +mB$ (maximum)

- 3 (d) In terms of the susceptibility χ , a material is diamagnetic, if χ is negative; paramagnetic-if χ is positive & small and ferromagnetic-if χ is large & positive.

4 (a) Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \leq \chi < 0$	$0 < \chi < \epsilon$	$\chi \gg 1$
$0 \leq \mu_r < 1$	$1 < \mu_r < 1 + \epsilon$	$\mu_r \gg 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu \gg \mu_0$

(C) Medical entrances' gallery

- 1 (d) Given, susceptibility, $\chi = 599$

Magnetic field, $B = 1200 \text{ Am}^{-1}$

Permeability of the material of the rod = ?

$$\begin{aligned} \text{As, } \mu_m &= \mu_0(1 + \chi) \\ &= 4\pi \times 10^{-7}(1 + 599) \\ &= 2.4\pi \times 10^{-4} \text{ TmA}^{-1} \end{aligned}$$

- 2 (d) When wire of length L is bent in the form of a circle of radius R , then

$$L = 2\pi R$$

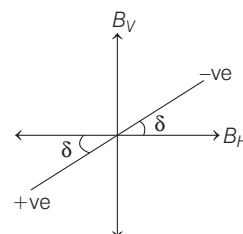
$$\Rightarrow R = \frac{L}{2\pi}$$

$\therefore$ Magnetic moment, $M = IA = I(\pi R^2)$

$$= I \cdot \pi \left(\frac{L}{2\pi} \right)^2 = \frac{I\pi L^2}{4\pi^2} = \frac{IL^2}{4\pi} \text{ A-m}^2$$

- 3 (b) The angle of dip δ is the angle between the horizontal component of earth's magnetic field and the total magnetic field of the earth. Its value is different at different places. It is zero at equator, as the dip needle becomes parallel to horizontal component. It varies from -90° in south pole to $+90^\circ$ in the north pole.

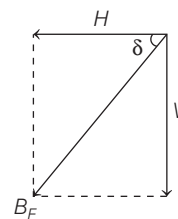
This means the values δ is positive in northern hemisphere and negative in southern hemisphere.



$\therefore$ For point A, $\delta = +25^\circ$, so A lies in the northern hemisphere.

Similarly, for B, $\delta = -25^\circ$, so B lies in the southern hemisphere.

- 4 (b) Let B_E be the net magnetic field at same point, H and V be the horizontal and vertical components of B_E . Let δ be the angle of dip, which is the angle between direction of earth's magnetic field B_E and horizontal line in the magnetic meridian.



Thus, from figure, we can see that

$$H = B_E \cos \delta$$

$$\text{and } V = B_E \sin \delta$$

- 5 (a) When paramagnetic substances are kept in an external magnetic field, then it develops feeble magnetisation in the direction of external applied magnetic field, *i.e.* magnetic dipoles are aligned along external magnetic field weakly.

So, they get poorly attracted in magnetic field.

- 6 (c) Coercivity and retentivity of soft iron is low, because its magnetisation disappears easily on the removal of external magnetising field.

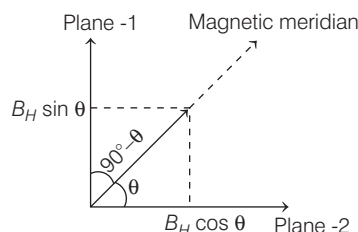
- 7 (d) Vertical component of earth's magnetic field,

$$\begin{aligned} B_V &= B_H \tan \delta = 0.36 \times 10^{-4} \times \tan 60^\circ \quad (\text{given}) \\ &= 0.36 \times 10^{-4} \times \sqrt{3} = 0.622 \times 10^{-4} \text{ T} \\ &= 0.622 \times 10^{-4} \text{ Wbm}^{-2} \end{aligned}$$

- 8 (c) We know that, the magnetism of the magnet is achieved due to the spin motion of electrons. Also, the spinning electrons possess magnetic dipole moment. This dipole moment is much greater than that due to orbital moment of electrons around the nucleus.

- 9 (a) Let the B_H and B_V be the horizontal and vertical components of earth's magnetic field B .

$$\tan \theta = \frac{B_V}{B_H} \Rightarrow \cot \theta = \frac{B_H}{B_V} \quad \dots(i)$$



Let planes 1 and 2 be mutually perpendicular planes making angles θ and $(90^\circ - \theta)$ with magnetic meridian. The vertical component of earth's magnetic field remain same in two planes but effective horizontal components in the two planes is given by

$$B_1 = B_H \cos \theta \quad \dots(ii)$$

$$\text{and } B_2 = B_H \sin \theta \quad \dots(iii)$$

$$\text{Then, } \tan \theta_1 = \frac{B_V}{B_1} = \frac{B_V}{B_H \cos \theta}$$

$$\cot \theta_1 = \frac{B_H \cos \theta}{B_V} \quad \dots(iv)$$

$$\text{Similarly, } \tan \theta_2 = \frac{B_V}{B_2} = \frac{B_V}{B_H \sin \theta}$$

$$\Rightarrow \cot \theta_2 = \frac{B_H \sin \theta}{B_V} \quad \dots(v)$$

From Eqs. (iv) and (v), we get

$$\Rightarrow \cot^2 \theta_1 + \cot^2 \theta_2 = \frac{B_H^2 \cos^2 \theta}{B_V^2} + \frac{B_H^2 \sin^2 \theta}{B_V^2}$$

$$\Rightarrow \cot^2 \theta_1 + \cot^2 \theta_2 = \frac{B_H^2}{B_V^2} (\cos^2 \theta + \sin^2 \theta)$$

$$\Rightarrow \cot^2 \theta_1 + \cot^2 \theta_2 = \cot^2 \theta \quad [\text{from Eq. (i)}]$$

- 10 (b) $\therefore$ Work done in rotating the magnet,

$$W = MB (\cos \theta_0 - \cos \theta)$$

where, M = magnetic moment of the magnet
and B = magnetic field.

$$\Rightarrow W = MB (\cos 0^\circ - \cos 60^\circ)$$

$$= MB \left(1 - \frac{1}{2} \right) = \frac{MB}{2}$$

$$\therefore MB = 2W \quad \dots(i)$$

Torque on a magnet in this position is given by

$$\tau = \mathbf{M} \times \mathbf{B} = MB \sin \theta = 2W \sin 60^\circ \quad [\text{from Eq. (i)}]$$

$$= 2W \frac{\sqrt{3}}{2} = W\sqrt{3}$$

- 11 (d) As we know, the relation between the magnetic permeability and susceptibility of material, $\mu_r = 1 + \chi_m$ $\dots(i)$

$\therefore$ For diamagnetic substances, $\mu_r < 1$

So, according to Eq. (i), the magnetic susceptibility (χ_m) of diamagnetic substance will be negative.

While in case of para and ferromagnetic substances susceptibility is positive.

- 12 (b) The magnetic susceptibility of a ferromagnetic substance decreases (in hyperbolic manner) with increase in temperature and above Curie's temperature, the substance behaves like paramagnetic one.

Hence, option (b) is correct.

- 13 (a) Magnetic permeability of substance,

$$\mu = \frac{B}{H} \Rightarrow \mu \propto \frac{1}{H}$$

where, H is the magnetising or magnetic field strength. As, the magnetising field of a ferromagnetic material is increasing thus its, permeability decreases.

- 14 (c) As, we know that, the magnetic field at a point due to the dipole is given by $B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3} \sqrt{1 + 3 \cos^2 \theta}$

where, M is magnetic dipole moment.

$$\Rightarrow B \propto \frac{1}{r^3}$$

- 15 (b) The coercivity of bar magnet = $4 \times 10^3 \text{ Am}^{-1}$,

Length of solenoid = 12 cm = $12 \times 10^{-2} \text{ m}$

and number of turns = 60

The coercivity of 12 cm length of solenoid

$$= 4 \times 10^3 \times 12 \times 10^{-2} = 480 \text{ Am}^{-1}$$

Now, current through each turn to demagnetise = $\frac{480}{60} = 8 \text{ A}$

- 16 (c) The effective length of magnet, $l = 31.4 \text{ cm} = 31.4 \times 10^{-2} \text{ m}$

Pole strength, $m = 0.8 \text{ Am}$

$$\text{Length of semi-circle} = \pi \frac{D}{2} = l$$

where, D = diameter of circle

$$\Rightarrow D = \frac{2l}{\pi} = \frac{2 \times 31.4 \times 10^{-2}}{3.14}$$

$$\Rightarrow D = 20 \times 10^{-2} \text{ m}$$

Now, the magnetic moment

$$= mD = 0.8 \times 20 \times 10^{-2}$$

$$= 16 \times 10^{-2} = 0.16 \text{ A-m}^2$$

- 17** (c) Given, $B_H = 3 \times 10^{-5} \text{ T}$, $\theta = 45^\circ$, $n = 50$

and $r = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$

$$\begin{aligned} \text{As, current, } I &= \frac{2rB_H \tan \theta}{\mu_0 n} \\ &= \frac{2 \times 20 \times 10^{-2} \times 3 \times 10^{-5} \times \tan 45^\circ}{4\pi \times 10^{-7} \times 50} \\ &= \frac{120 \times 10^{-7}}{4\pi \times 10^{-7} \times 50} = \frac{3}{5\pi} = \frac{3}{5 \times 3.14} = \frac{3}{15.7} = 0.19 \end{aligned}$$

A

- 18** (b) The intensity of magnetisation increases for a ferromagnetic substance with the applied magnetic field and become constant, when the domains get perfectly aligned with respect to the external magnetic fields.

- 19** (c) The work done (in rotating) by external agent

= change in potential energy

$$= -MB \cos \theta_2 - (-MB \cos \theta_1)$$

$$= -MB(\cos 45^\circ - \cos 0^\circ)$$

$$= \left(1 - \frac{1}{\sqrt{2}}\right) MB = \frac{0.41}{\sqrt{2}} \times \frac{1}{10} \approx 0.03 \text{ J}$$

- 21** (c) (a) $M_1 = m\sqrt{2}$

(b) $\xrightarrow[m]{m} \Rightarrow M_2 = 0$

(c) $M_3 = m\sqrt{(1 + \cos 30^\circ)2}$

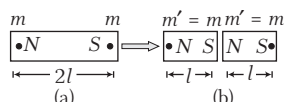
$$= m\sqrt{\left(1 + \frac{\sqrt{3}}{2}\right)2}$$

$$= m\sqrt{2 + \sqrt{3}}$$

(d) $M_4 = 2m \cos 30^\circ = m\sqrt{3}$

- 22** (c) The bar magnet is shown in the diagram.

For Figs. (a) and (b)



Magnetic moment for (a) is $M = m(2l)$

Magnetic moment for (b) is

$$M' = m'(l)$$

$$\text{Now, } \frac{M'}{M} = \frac{m'(l)}{m(2l)} = \frac{1}{2} \left(\frac{m'}{m} \right) \quad \dots(i)$$

When magnet is cut as shown in the diagram, pole strength will not be affected. Hence, $m' = m$

$$\Rightarrow \frac{m'}{m} = 1 \Rightarrow M' = \frac{M}{2} \quad [\text{from Eq. (i)}]$$

- 24** (d) For paramagnetic sample (Curie's law), $I \propto B/T$

where, $I_1 = 0.8 \text{ A/m}$, $B_1 = 0.8 \text{ T}$,

$$T_1 = 5 \text{ K}, B_2 = 0.4 \text{ T}$$

and $T_2 = 20 \text{ K}$

$$\Rightarrow \frac{I_1}{I_2} = \frac{B_1/T_1}{B_2/T_2}$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{B_1 \times T_2}{B_2 \times T_1}$$

$$\Rightarrow \frac{0.8}{I_2} = \frac{0.8 \times 20}{0.4 \times 5}$$

$$\Rightarrow I_2 = \frac{0.4 \times 5}{20} = 0.1 \text{ A-m}^{-1}$$

- 25** (a) When temperature increases beyond Curie temperature, the disorderness of the magnetic moments of the material increases, hence alignment of magnetic moments are disturbed, which caused material to behave as paramagnetic.

- 26** (a) We know that, intensity of magnetisation, $I = \frac{M}{V}$

where, M = magnetic moment and V = volume.

$$\text{So, } M = IV = 5 \times 10^4 \times \frac{12}{100} \times \frac{1}{(100)^2}$$

$$= 60 \times 10^4 \times 10^{-6}$$

$$= 0.6 \text{ A-m}^2 \text{ (SI unit)}$$

- 27** (a) Given, $\chi_m = 299$

and $\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$

We know that, $\mu = \mu_0(1 + \chi_m)$

$$\therefore \mu = 4\pi \times 10^{-7} (1 + 299)$$

$$\Rightarrow \mu = 4 \times \frac{22}{7} \times 10^{-7} \times 300$$

$$= \frac{26400}{7} \times 10^{-7} = 3771.4 \times 10^{-7} \text{ Hm}^{-1}$$

$$\Rightarrow \mu \approx 3771 \times 10^{-7} \text{ Hm}^{-1}$$

- 28** (d) Given, $r = 0.05 \text{ nm} = 0.05 \times 10^{-9} \text{ m}$ $[\because 1 \text{ nm} = 10^{-9} \text{ m}]$

$$n = 10^{16} \text{ revolutions/s}$$

and $e = 1.6 \times 10^{-19} \text{ C}$

We know that the magnetic moment, $M = AI$

$$M = \pi r^2 \times ne$$

$$= 3.14 \times (0.05 \times 10^{-9})^2 \times 10^{16} \times 1.6 \times 10^{-19}$$

or $M = 1.26 \times 10^{-23} \text{ A-m}^2$

- 29** (b) Let magnetic pole strength be m , then $M = ml$.

$$\therefore M' = m \left[2r \sin \left(\frac{60^\circ}{2} \right) \right], \text{ where } r \left(\frac{\pi}{3} \right) = l$$

$$\Rightarrow M' = 2m \left(\frac{3l}{\pi} \right) \left(\frac{1}{2} \right) = \frac{3ml}{\pi} = \frac{3M}{\pi}$$

- 30** (d) As, $B_E = \sqrt{B_H^2 + B_V^2}$ and $\tan \delta = \frac{B_V}{B_H}$

Given, $B_H = 0.3 \text{ G}$, $B_V = 0.52 \text{ G}$

$$\therefore B_E = \sqrt{(0.30)^2 + (0.52)^2}$$

$$= \sqrt{0.36} = 0.6 \text{ G}$$

$$\text{and } \delta = \tan^{-1} \frac{B_V}{B_H} = \tan^{-1} \frac{0.52}{0.3} = 60^\circ$$

31 (a) When magnet is cut into equal parts, area of cross-section becomes half. Hence, pole strength becomes half.

32 (a) We know that, the magnetic field induction at a point,

$$B = \frac{\mu_0}{4\pi} \frac{M}{r^3}$$

$$\text{According to question, } B_1 : B_2 : B_3 = \frac{1}{(0.3)^3} : \frac{1}{(0.6)^3} : \frac{1}{(0.9)^3}$$

$$= 27 : 3.37 : 1$$

33 (a) Given, $M = 120 \text{ Am}^2$, $T = 12 \text{ s}$ and $B = 0.4 \times 10^{-4} \text{ T}$

$$\text{We know that, } T = 2\pi \sqrt{\frac{I}{MB}} \text{ (squaring both sides)}$$

$$\text{So, } I = \frac{T^2 MB}{4\pi^2}$$

$$\text{or } I = \frac{(12)^2 \times (120) \times (0.4 \times 10^{-4})}{4 \times 3.14 \times 3.14}$$

$$I = 175.2 \times 10^{-4} \text{ kg-m}^2$$

In this question, the option (a) is approximately equivalent to 175.2×10^{-4} , so the answer is $I = 172.8 \times 10^{-4} \text{ kg-m}^2$.

35 (c) From the relation, susceptibility of the material,

$$\chi_m = \frac{I}{H} \Rightarrow \chi_m \propto I$$

Thus, it is obvious that greater the value of susceptibility of a material greater will be the value of intensity of magnetisation, i.e. more easily it can be magnetised.

36 (c) As, $T = 2\pi \sqrt{\frac{I}{MB}} \Rightarrow T \propto \frac{1}{\sqrt{B}}$, so $T_1 \propto \frac{1}{\sqrt{V}}$ and $T_2 \propto \frac{1}{\sqrt{H}}$

$$V = H \tan \delta \Rightarrow \frac{V}{H} = \tan \delta$$

$$\text{But, } \frac{V}{H} = 1 \quad [\because T_1 = T_2 \Rightarrow V = H]$$

$$\therefore \tan \delta = 1 \Rightarrow \delta = 45^\circ$$

37 (b) The magnetic field at a point on the axis of a bar magnet,

$$B = \frac{\mu_0}{4\pi} \frac{2M}{r^3} = 10^{-7} \times \frac{2 \times 1.25}{(0.5)^3} = 2 \times 10^{-6} \text{ NA}^{-1} \text{ m}^{-1}$$

38 (d) Rod is moving towards east, so induced emf across its end will be

$$B = B_V \nu l = (B_H \tan \theta) \nu l \quad [\because \tan \theta = B_V / B_H]$$

$$= 3 \times 10^{-4} \times \frac{4}{3} \times 10 \times 10^{-2} \times 0.25$$

$$= 10^{-5} \text{ V} = 10 \mu\text{V}$$

39 (d) $\therefore \tan \delta' = \frac{\tan \delta}{\cos \theta} = \frac{\tan 45^\circ}{\cos 30^\circ}$; $\tan \delta' = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$

$$\therefore \delta' = \tan^{-1} \left(\frac{2}{\sqrt{3}} \right)$$

40 (c) When wire is bent in the form of semicircular arc, then $l = \pi r$

$\therefore$ The radius of semicircular arc, $r = l/\pi$

Distance between two end points of semicircular wire

$$= 2r = \frac{2l}{\pi}$$

$\therefore$ Magnetic moment of semicircular wire

$$= m \times 2r = m \times \frac{2l}{\pi} = \frac{2}{\pi} ml$$

But ml is the magnetic moment of wire, i.e. $ml = M$

$$\therefore \text{New magnetic moment} = \frac{2}{\pi} M$$

41 (b) We know that, $B = \mu_0 H + \mu_0 I$

$$\therefore I = \frac{B - \mu_0 H}{\mu_0} = \frac{\mu H - \mu_0 H}{\mu_0} = \left(\frac{\mu}{\mu_0} - 1 \right) H$$

$$I = (\mu_r - 1) H$$

For a solenoid having n turns/length and current I' ,

$$H = nI'$$

$$I = (\mu_r - 1) nI' = (1000 - 1) 500 \times 0.5$$

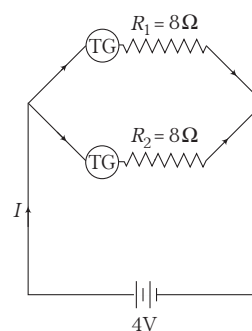
$$= 2.5 \times 10^5 \text{ Am}^{-1}$$

$\therefore$ Magnetic moment, $M = IV$

$$= 2.5 \times 10^5 \times 10^{-4} = 25 \text{ A-m}^2$$

42 (b) Current in tangent galvanometer,

$$I = \frac{2rH}{\mu_0 N} \tan \theta \quad \dots(i)$$



Here, R_1 and R_2 are in parallel

$$\therefore \frac{1}{R_{\text{net}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} = \frac{8 + 8}{8 \times 8}$$

$$R_{\text{net}} = 4 \Omega$$

$$\text{Hence, } I = \frac{V}{R} = \frac{4}{4} = 1 \text{ A}$$

From Eq. (i), we get

$$\frac{r \tan \theta}{N} = \frac{\mu_0 I}{2H}$$

$$\therefore \frac{r_A \tan \theta_A}{N_A} = \frac{r_B \tan \theta_B}{N_B} \quad \left(\because \frac{\mu_0 I}{2H} = \text{constant} \right)$$

$$\Rightarrow \frac{8 \times 1}{\sqrt{3} \times 2} = \frac{16 \times \sqrt{3}}{N_B} \quad (\because \theta_A = 30^\circ \text{ and } \theta_B = 60^\circ)$$

$$\therefore N_B = 12 \text{ turns}$$

- 43** (d) Paramagnetic materials will be feebly attracted, diamagnetic material will be feebly repelled and ferromagnetic material will be strongly attracted towards a magnet.

- 44** (a) As, vertical component of earth's magnetic field,

$$B_V = \sqrt{3}B_H \quad [\text{Given}]$$

Angle of dip at any place is given by

$$\tan \delta = \frac{B_V}{B_H} = \frac{\sqrt{3}B_H}{B_H} = \sqrt{3}$$

$$\Rightarrow \delta = \tan^{-1}(\sqrt{3}) = 60^\circ$$

- 45** (b) If length of wire is $2l$, then magnetic moment,

$$M = m \times 2l = 4\pi \text{ A} \cdot \text{m}^2 \text{ (given)}$$

As wire is bent in the form of semicircle, effective distance between the ends is $2r$.

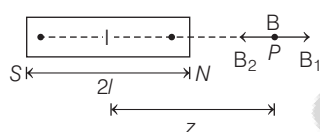
So, new dipole moment, $M' = m \times 2r$

$$\text{As, } \pi r = 2l$$

$$\Rightarrow r = 2l/\pi$$

$$\begin{aligned} \text{So, } M' &= m \times 2 \times \frac{2l}{\pi} = \frac{2}{\pi} (m \times 2l) \\ &= \frac{2}{\pi} M = \frac{2}{\pi} \frac{4\pi}{4\pi} = 8 \text{ A} \cdot \text{m}^2 \end{aligned}$$

- 46** (b) Consider a point P located on the axial line of a short bar magnet of magnetic length $2l$ and strength m . Let us find $\mathbf{B}$ at a point P which is at a distance z from the centre of magnet. Magnetic field at point P due to N -pole,



$$B_1 = \frac{\mu_0}{4\pi} \left[\frac{m}{(z-l)^2} \right] \text{ along } NP$$

Similarly, magnetic field at point P due to S -pole,

$$B_2 = \frac{\mu_0}{4\pi} \left[\frac{m}{(z+l)^2} \right] \text{ along } PS$$

Net magnetic field at P ,

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi} \left[\frac{m}{(z-l)^2} - \frac{m}{(z+l)^2} \right]$$

$$= \frac{\mu_0}{4\pi} \left[\frac{4mlz}{(z^2 - l^2)^2} \right]$$

$$= \frac{\mu_0}{4\pi} \left[\frac{m \times 2l}{(z^2 - l^2)^2} 2z \right]$$

$$B = \frac{\mu_0}{4\pi} \frac{2Mz}{(z^2 - l^2)^2} \quad [\because M = m(2l)]$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2Mz}{z^4} \quad (\because z > l)$$

Hence, in vector form, magnetic field at point P due to N -pole can be written as

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2M}{z^3} \hat{\mathbf{M}}$$

where, $\hat{\mathbf{M}}$ = unit vector along magnetic dipole moment.

48 (c) Here, $\tan \delta_1 = \frac{V}{H \cos \theta}$

$$\tan \delta_2 = \frac{V}{H \cos (90^\circ - \theta)} = \frac{V}{H \sin \theta}$$

$$\Rightarrow \frac{\tan \delta_1}{\tan \delta_2} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\text{or } \theta = \tan^{-1} \left(\frac{\tan \delta_1}{\tan \delta_2} \right)$$

- 49** (b) Intensity of magnetisation,

$$I = \frac{M}{V} = \frac{\text{Mass} \times \text{Density}}{\text{Mass} / \text{Density}}$$

Given, mass = $1 \text{ g} = 10^{-3} \text{ kg}$

$$\text{and density} = 5 \text{ g cm}^{-3} = \frac{5 \times 10^{-3}}{(10^{-2})^3}$$

$$= 5 \times 10^3 \text{ kg / m}^3$$

$$\text{Hence, } I = \frac{6 \times 10^{-7} \times 5 \times 10^3}{10^{-3}} = 3.0 \text{ Am}^{-1}$$

CHAPTER 06

Electromagnetic Induction

The phenomenon of generating current/emf in a conducting circuit by changing the strength, position or orientation of an associated external magnetic field is called **electromagnetic induction (EMI)**. The emf, so developed is called **induced emf**. If the conductor is in the form of a closed circuit, a current flows in the circuit. This current is called **induced current**. It is the reverse process of magnetic field produced by electric current. EMI was discovered by Michael Faraday in 1831. In this chapter, we will study about the Lenz's law, motional emf and inductances.

MAGNETIC FLUX

The total number of magnetic field lines crossing through any surface, normally when it is placed in a magnetic field is known as the magnetic flux of that surface.

Consider an element of area $d\mathbf{S}$ on an arbitrary shaped surface as shown in Fig. 6.1. If the magnetic field at this element is $\mathbf{B}$, then the magnetic flux through the element is

$$d\phi_B = \mathbf{B} \cdot d\mathbf{S} \Rightarrow d\phi_B = B dS \cos \theta$$

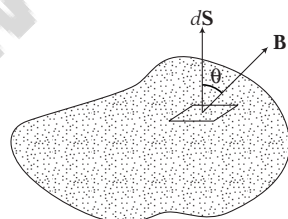


Fig. 6.1 Magnetic flux

Here, $d\mathbf{S}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dS and θ is the angle between $\mathbf{B}$ and $d\mathbf{S}$ at that element. In general, $d\phi_B$ varies from element to element.

The total magnetic flux through the surface is the sum of the contributions from the individual area elements.

$$\therefore \phi_B = \int B dS \cos \theta = \int \mathbf{B} \cdot d\mathbf{S}$$

Inside

1 Magnetic flux

- Faraday's laws of electromagnetic induction
- Lenz's law and conservation of energy

2 Motional electromotive force

- Induction of field

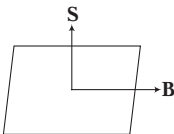
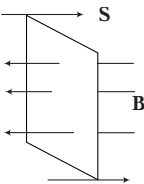
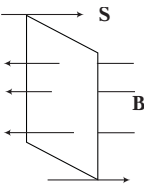
3 Self-induction

- Kirchhoff's second law with an inductor
- Self inductance of a coil
- Self inductance of a solenoid
- Energy stored in an inductor
- Combination of self-inductances

4 Mutual induction

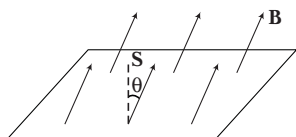
- Mutual inductances of some important coil configurations
- Coefficient of coupling
- Growth and decay of current in L-R circuit
- Application of EMI : eddy current

Specific conditions

For $\theta = 0^\circ$	For $\theta = 90^\circ$	For $\theta = 180^\circ$
		
$\mathbf{B}$ is parallel to surface vector, i.e. $\mathbf{S}$.	$\mathbf{B}$ is perpendicular to surface vector i.e. $\mathbf{S}$.	$\mathbf{B}$ is anti-parallel to surface vector, i.e. $\mathbf{S}$.
$\Rightarrow \phi_B = BS \cos 0^\circ$ $= BS$	$\Rightarrow \phi_B = BS \cos 90^\circ$ $= 0$	$\Rightarrow \phi_B = BS \cos 180^\circ$ $= BS (-1) = -BS$
$\Rightarrow \phi_B = \text{maximum}$		

Note

- (i) As magnetic lines of force are closed curves (i.e. monopoles do not exist), total magnetic flux linked with a closed surface is always zero.
- (ii) A special case, in which $\mathbf{B}$ is uniform over a plane surface with total area S , magnetic flux is given by $\phi_B = BS \cos \theta$.

Fig. 6.2 A plane of surface area S placed on uniform field B

Units and dimension of magnetic flux

- (i) Magnetic flux is a scalar quantity.
- (ii) The SI unit of magnetic flux is tesla-metre² (T-m^2).

This unit is also called **weber (Wb)**.

$$1 \text{ Wb} = 1 \text{ T-m}^2 = 1 \text{ N-m/A}$$

The CGS unit of flux is maxwell (Mx).

$$1 \text{ Mx} = 1 \text{ G-cm}^2$$

$$1 \text{ Wb} = 1 \text{ T} \times 1 \text{ m}^2 = 10^4 \text{ G} \times 10^4 \text{ cm}^2$$

$$\text{or } 1 \text{ Wb} = 10^8 \text{ Mx}$$

Thus, unit of magnetic field is also weber/m² (1 Wb/m^2)

$$\text{or } 1 \text{ T} = 1 \text{ Wb/m}^2$$

- (iii) The dimensional formula of magnetic flux is $[\text{ML}^2\text{T}^{-2}\text{A}^{-1}]$.

Example 6.1 A uniform magnetic field exists in the space $\mathbf{B} = B_1\hat{i} + B_2\hat{j} - B_3\hat{k}$. Find the magnetic flux through an area $\mathbf{S}$, if the area $\mathbf{S}$ is in YZ -plane.

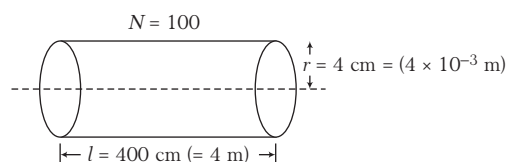
Sol. Since, the field is uniform, we can use formula, $\phi_B = \mathbf{B} \cdot \mathbf{S}$

Area $\mathbf{S}$ is in YZ -plane, it means $\mathbf{S} = S\hat{i}$

$$\text{Hence, } \phi_B = (B_1\hat{i} + B_2\hat{j} - B_3\hat{k}) \cdot (S\hat{i}) = B_1S.$$

Example 6.2 A long solenoid of radius 4 cm, length 400 cm carries a current of 3 A. The total number of turns is 100. Assuming ideal solenoid, find the flux passing through a circular surface having centre on axis of solenoid of radius 3 cm and is perpendicular to the axis of solenoid (i) inside and (ii) at the end of solenoid.

Sol.



Number of turns per unit length is given by

$$n = \frac{N}{l} = \frac{100}{4} = 25 \text{ turns/m}$$

- (i) Magnetic field of a solenoid at a point inside is

$$B = \mu_0 n i, \quad S = \pi r_1^2 \text{ and } \theta = 0^\circ$$

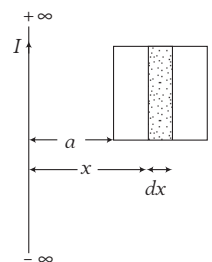
Magnetic flux

$$\begin{aligned} \phi_B &= BS \cos \theta = \mu_0 n i \pi r_1^2 \cos 0^\circ \\ &= 4\pi \times 10^{-7} \times 25 \times 3 \times \pi \times (3 \times 10^{-2})^2 \\ &= 0.27 \times 10^{-6} \text{ Wb} = 0.27 \mu \text{ Wb} \end{aligned}$$

- (ii) At the end, magnetic field of solenoid is given by

$$\begin{aligned} B &= \frac{1}{2} \mu_0 n i \\ \therefore \phi &= \frac{BS \cos \theta}{2} = \frac{\phi_B}{2} = \frac{0.27}{2} = 0.135 \mu \text{ Wb} \end{aligned}$$

Example 6.3 A long straight wire carrying current I and a square conducting wire loop of side l , at a distance 'a' from current wire as shown in the figure. Both the current wire and loop are in the plane of paper. Find the magnetic flux of current wire, passing through the loop.



Sol. As the field of current wire passing through the loop is same in direction (normally inward) but not uniform in magnitude. So, we will use integration method for finding the flux.

The small flux through a thin rectangular strip of length l and width dx , is given by

$$d\phi_B = \mathbf{B}_x \cdot d\mathbf{S} = B_x dS \cos 180^\circ$$

Magnetic field due to a long straight wire carrying current I is given by $B = \frac{\mu_0 I}{2\pi x}$ and area, $dS = l \times dx$.

$$\begin{aligned} \therefore \phi_B &= \int d\phi_B = - \int \frac{\mu_0 I}{2\pi x} \cdot l \cdot dx = - \frac{\mu_0 I l}{2\pi} [\log_e x]_{x=a}^{x=a+l} \\ &= - \frac{\mu_0}{2\pi} \cdot I l \log_e \frac{a+l}{a} \end{aligned}$$

Faraday's laws of electromagnetic induction

There are two laws of electromagnetic induction given by Faraday as below

Faraday's first law

Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit.

The actual number of magnetic lines passing through the circuit does not matter to the value of the induced emf. Induced emf is determined by the rate at which the magnetic flux changes.

Faraday's second law

The magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit. Mathematically, Faraday's second law can be expressed as,

Induced emf $\propto$ Rate of change of magnetic flux

$$e = \frac{-d\phi_B}{dt}$$

$$(\because \text{rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t_2 - t_1})$$

The negative sign in above relation indicates that the induced emf in the loop due to changing flux always opposes the change in the magnetic flux.

In the case of a closely wound coil of N turns, the change of flux associated with each turn, is same. Therefore, the expression for the total induced emf is given by

$$e = -N \frac{d\phi_B}{dt}$$

The induced emf can be increased by increasing the number of turns N of a closed coil.

The magnetic flux can be changed in the following ways

- The magnitude of $\mathbf{B}$ can change with time, *i.e.* time-varying magnetic field,
 $B = B(t)$
- The current producing the magnetic field can change with time, *i.e.* time-varying current, $i = i(t)$.
- The area enclosed by the loop can change with time. This can be done by pulling a loop inside (or outside) a magnetic field. By doing, so the area enclosed by loop (hatched area) can be changed.

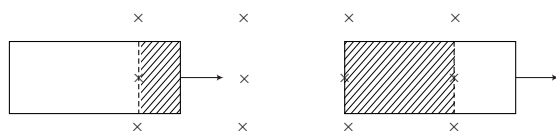


Fig. 6.3

- The angle θ between $\mathbf{B}$ and the normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.

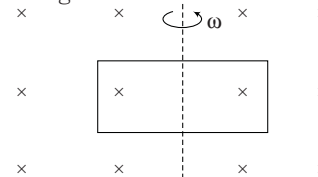


Fig. 6.4

- Any combination of the above can occur.

Example 6.4 A coil with an average diameter of 0.02 m is placed perpendicular to a magnetic field of 6000 T. If the induced emf is 11 V. When the magnetic field is changed to 1000 T in 4 s. What is the number of turns in the coil?

Sol. Given, radius of coil, $r = \frac{0.02}{2} = 0.01 \text{ m}$

$$B_1 = 6000 \text{ T}, B_2 = 1000 \text{ T}, t = 4 \text{ s}, e = 11 \text{ V}$$

$$\text{emf, } e = N \frac{(\phi_2 - \phi_1)}{t} = NA \frac{B_2 - B_1}{t} = N\pi r^2 \frac{B_2 - B_1}{t}$$

$$11 = N \cdot \frac{22}{7} \times (0.01)^2 \times \frac{(6000 - 1000)}{4}$$

$$\therefore \text{Number of turns, } N = \frac{11 \times 7 \times 4}{22 \times (0.01)^2 \times 5000} = 28$$

Induced current and induced charge

Induced current

When the magnetic flux passing through a loop is changed, an induced emf and hence an induced current is produced in the circuit.

If R is the resistance of the circuit, then induced current is given by

$$i = \frac{e}{R} = \frac{1}{R} \left(\frac{-d\phi_B}{dt} \right) \quad \dots(i)$$

If induced current is produced in a coil rotated in a uniform magnetic field, then $I = \frac{NBA \omega \sin \omega t}{R} = I_0 \sin \omega t$

where, $I_0 = \frac{NBA \omega}{R}$ = peak value of induced current,

N = number of turns in the coil,

B = magnetic field,

ω = angular velocity of rotation

and A = area of cross-section of the coil.

Induced charge

When the current starts flowing in the circuit, flow of charges also takes place. Charge flown in a circuit in time dt will be given by

$$dq = i dt = \frac{1}{R} (-d\phi_B) \quad [\text{from Eq. (i)}] \quad \dots(ii)$$

Thus, for a time interval dt we can write,

$$e = -\frac{d\phi_B}{dt}, i = \frac{1}{R} \left(-\frac{d\phi_B}{dt} \right) = -\frac{1}{R} \frac{BdS \cos\theta}{dt}$$

$$\text{and } dq = \frac{1}{R} (-d\phi_B) \quad \dots(\text{iii})$$

Direction of induced current

The direction of induced current in a loop may be obtained

$$\text{by using } e = -\frac{d\phi}{dt} \text{ and } i = \frac{1}{R} \left(-\frac{d\phi}{dt} \right) = \frac{1}{R} \frac{BdS \cos\theta}{dt}$$

So, the steps to decide the direction of induced current are as follows

- (i) Define a positive direction for the area vector $\mathbf{S}$.
- (ii) From the directions of $\mathbf{S}$ and the magnetic field $\mathbf{B}$, determine the sign of ϕ_B and its time rate of change, i.e. $\frac{d\phi_B}{dt}$.
- (iii) If $\frac{d\phi_B}{dt}$ is positive, i.e. flux is increasing, sign of induced current is negative and *vice-versa*.
In this way, we will determine the sign of the induced current.
- (iv) Finally, determine the direction of induced current using your right hand. Curl the fingers of your right hand around $\mathbf{S}$, with your right thumb in the direction of $\mathbf{S}$. If the induced current is positive, it is in the same direction as your curled fingers and if it is negative, it is in the opposite direction.

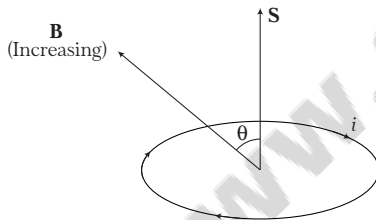


Fig. 6.5

$[\theta < 90^\circ, \phi_B > 0, \frac{d\phi_B}{dt} > 0, \text{ so induced current is negative}]$ [Fig. 6.5]

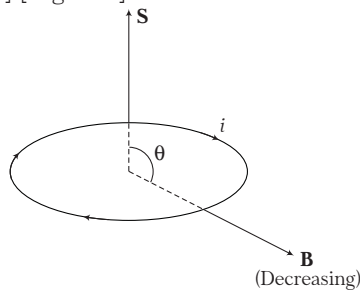


Fig. 6.6

$[\theta > 90^\circ, \phi_B < 0, \frac{d\phi_B}{dt} < 0, \text{ so induced current is positive}]$ [Fig. 6.6]

Example 6.5 A coil consists of 200 turns of wire having a total resistance of 2.0Ω . Each turn is a square of side 18 cm and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. If the field changes linearly from 0 to 0.5 T in 0.80 s, then what is the magnitude of induced emf and current in the coil while the field is changing?

Sol. Given, $N = 200$ turns, $R = 2.0 \Omega$, change in magnetic field, $\Delta B = 0.5 \text{ T}$, side of square, $l = 18 \text{ cm} = 18 \times 10^{-2} \text{ m}$ and $\Delta t = 0.80 \text{ s}$

From the Faraday's law,

$$\begin{aligned} \text{Induced emf, } |e| &= -\frac{Nd\phi_B}{dt} = (N \times S) \frac{\Delta B}{\Delta t} \\ [\because \phi_B &= \mathbf{B} \cdot \mathbf{S} \text{ and area, } S = l^2 = (18 \times 10^{-2})^2] \\ &= \frac{(200)(18 \times 10^{-2})^2 (0.5 - 0)}{0.8} \\ &= 4.05 \text{ V} \end{aligned}$$

$$\therefore \text{Further, induced current, } i = \frac{|e|}{R} = \frac{4.05}{2} \approx 2.0 \text{ A}$$

Example 6.6 The magnetic flux threading a metal ring varies with time t according to $\phi_B = 3(at^3 - bt^2) \text{ T-m}^2$ with $a = 2.00 \text{ s}^{-3}$ and $b = 6.00 \text{ s}^{-2}$. The resistance of the ring is 3.0Ω . Determine the maximum current induced in the ring during the interval from $t = 0$ to $t = 2.0 \text{ s}$.

Sol. Given, magnetic flux, $\phi_B = 3(at^3 - bt^2) \text{ T-m}^2$

where, $a = 2 \text{ s}^{-3}$, $b = 6 \text{ s}^{-2}$

$$\text{Induced emf, } |e| = \left| \frac{d\phi_B}{dt} \right| = 9at^2 - 6bt$$

$$\therefore \text{ Induced current, } i = \frac{|e|}{R} = \frac{9at^2 - 6bt}{3} = 3at^2 - 2bt \quad \dots(\text{i})$$

For current to be maximum, $\frac{di}{dt} = 0$

$$\therefore 6at - 2b = 0 \quad \text{or } t = \frac{b}{3a}$$

i.e. At $t = \frac{b}{3a}$, current is maximum.

Substituting the values in Eq. (i), the maximum current,

$$\begin{aligned} i_{\max} &= 3a \left(\frac{b}{3a} \right)^2 - 2b \left(\frac{b}{3a} \right) \\ &= \frac{b^2}{3a} - \frac{2b^2}{3a} = \frac{-b^2}{3a} \end{aligned}$$

Magnitude of this maximum current will be $i_{\max} = \frac{b^2}{3a}$.

Substituting the given values of a and b , we get

$$i_{\max} = \frac{(6)^2}{3(2)} = 6.0 \text{ A}$$

Example 6.7 A square loop of side 10 cm and resistance 0.5Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.10 T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70 s at a steady state. What is the magnitude of current in this time-interval?

Sol. Given, magnetic field, $B = 0.10 \text{ T}$

$$\text{and area of square loop} = 10 \times 10 = 100 \text{ cm}^2 \\ = 10^{-2} \text{ m}^2$$

As, the magnetic field is set up across the plane in the north-east direction, then $\theta = 45^\circ$.

The initial magnetic flux is given by

$$\phi_B = BS \cos \theta \\ \therefore \phi = \frac{0.1 \times 10^{-2}}{\sqrt{2}} \text{ Wb}$$

Final flux, $\phi_{\min} = 0$ (given)

The change in flux is brought about in 0.70 s , i.e. $\Delta t = 0.70 \text{ s}$.

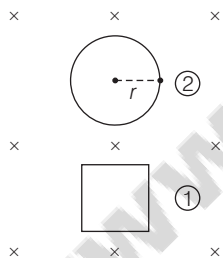
The magnitude of the induced emf is

$$e = \frac{\Delta \phi_B}{\Delta t} = \frac{|\phi - 0|}{\Delta t} = \frac{10^{-3}}{\sqrt{2} \times 0.7} \approx 1 \text{ mV}$$

The magnitude of induced current is $I = \frac{e}{R} = \frac{10^{-3}}{0.5} = 2 \text{ mA}$

Example 6.8 A wire of length l in the form of a square loop lies in a plane normal to a magnetic field B_0 . If this wire is converted into a circular loop in time t_0 , then find the average induced emf.

Sol.



Circumference of circle will be $l = 2\pi r \Rightarrow r = \frac{l}{2\pi}$

Side of square, $L = \frac{l}{4}$

Using the relation, $\phi_B = \mathbf{B} \cdot \mathbf{S} = BS \cos \theta$

Here, $\theta = 0$

$\Rightarrow \phi_B = BS$

Magnetic flux of square loop will be $\phi_1 = B_0 L^2 = \frac{B_0 l^2}{16}$

Magnetic flux of circular loop will be

$$\phi_2 = B_0 \pi r^2 = \frac{B_0 \pi l^2}{4\pi^2} = \frac{B_0 l^2}{4\pi}$$

Average induced emf is given by

$$\bar{e} = -\frac{\phi_2 - \phi_1}{\Delta t} = -\frac{B_0 l^2}{t_0} \left(\frac{1}{4\pi} - \frac{1}{16} \right) = \frac{B_0 l^2}{16\pi t_0} (4 - \pi)$$

Example 6.9 Suppose a coil of area 5 m^2 , resistance 10Ω and number of turns 200 held perpendicular to a uniform magnetic field of strength 0.4 T . The coil is now turned through 180° in time 1 s . What is

- average induced emf,
- average induced current
- and total charge that flows through a given cross-section of the coil?

Sol. Given, area of coil, $S = 5 \text{ m}^2$, resistance, $R = 10 \Omega$, number of turns, $N = 200$, magnetic field, $B = 0.4 \text{ T}$, $\Delta t = 1 \text{ s}$

When the plane of coil is perpendicular to the magnetic field, i.e. $\theta_i = 0^\circ$ and after it is rotated through 180° , then $\theta_f = 180^\circ$.

$\therefore$ Initial flux, $\phi_i = NBS \cos 0^\circ = NBS$

$$= 200 \times 0.4 \times 5 = 400 \text{ Wb}$$

and final flux $= NBS \cos 180^\circ = -NBS = -400 \text{ Wb}$

$$\text{Change in flux, } |\Delta \phi_B| \\ = NBS - (-NBS) = 2NBS = 800 \text{ Wb}$$

$$(i) \text{ Average induced emf } (\epsilon) = \frac{|\Delta \phi|}{\Delta t} = \frac{2NBA}{\Delta t} = 800 \text{ V}$$

$$(ii) \text{ Average current} = \frac{\epsilon}{R} = \frac{2NBA}{R\Delta t} = 80 \text{ A}$$

$$(iii) \text{ Total charge, } \Delta Q = \frac{2NBA}{R} = 80 \text{ C}$$

Example 6.10 Through a long solenoid of diameter 4.1 cm , having 100 turns per cm, a current $I = 1 \text{ A}$ is flowing. At its centre, a 60 turns closely packed coil of diameter 3.1 cm is placed such that the coil is co-axial with the long solenoid. The current in the solenoid is reduced to zero at a steady rate in 10 ms . What is the magnitude of emf induced in the coil while the current in the solenoid is changing?

Sol. Initially, magnetic flux passing through the coil (one turn),

$$\phi_1 = \mathbf{B} \cdot \mathbf{A} = BA \cos 0^\circ$$

Magnetic field at a point inside the solenoid is given by $B = \mu_0 nI$, where n is number of turns per metre.

$$\therefore \phi_1 = \mu_0 nI \times \frac{\pi d^2}{4}$$

Here, $n = 10000$ turns per m,

$I = 1 \text{ A}$ and $d = 3.1 \text{ cm}$

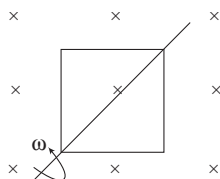
$$\therefore \phi_1 = \mu_0 nI \frac{\pi d^2}{4} \\ = 4\pi \times 10^{-7} \times 10000 \times 1 \times \pi \times \frac{(3.1 \times 10^{-2})^2}{4} \\ = 0.947 \times 10^{-5} \text{ Wb}$$

Finally, the flux becomes zero because the current reduces to zero.

$$\text{Thus, induced emf, } e = \frac{|\Delta \phi|}{\Delta t} = \frac{0.947 \times 10^{-5}}{10 \times 10^{-3}} = 9.47 \times 10^{-4} \text{ V}$$

$$\text{The total emf} = N \times e = 60 \times 9.47 \times 10^{-4} \\ = 568.2 \times 10^{-4} \text{ V}$$

Example 6.11 A square loop of edge b having M turns is rotated with a uniform angular velocity ω about one of its diagonals which is kept fixed in a horizontal position. A uniform magnetic field B_0 exists in the vertical direction.



- Find (i) the emf induced in the coil as a function of time t ,
 (ii) the maximum emf induced,
 (iii) the average emf induced in the loop over a long period,
 (iv) if resistance of loop is R , amount of charge flown in time $t = 0$ to $t = 2T$
 (v) and heat produced in time $t = 0$ to $t = 2T$.

Sol. (i) Initially, plane of loop is perpendicular to the field. Let at any time t , normal to loop makes an angle θ with magnetic field $\theta = \omega t$.

The flux passing through loop,

$$\phi_B = MB_0 S \cos \theta = MB_0 b^2 \cos \omega t$$

$$\Rightarrow e = -\frac{d\phi_B}{dt} = MB_0 b^2 \omega \sin \omega t = e_0 \sin \omega t,$$

where, $e_0 = MB_0 b^2 \omega$.

- (ii) For emf to be maximum, then $\sin \omega t$ should be equal to 1.

$$\Rightarrow e_{\max} = e_0 = MB_0 b^2 \omega$$

- (iii) Long period means one time period, i.e. $t \rightarrow 0$ to T .

We know that, average emf is given as

$$\bar{e} = \frac{\int_0^T e dt}{\int_0^T dt} = \frac{1}{T} \cdot e_0 \int_0^T \sin \omega t dt = 0 \quad \left(\because \int_0^T \sin \omega t dt = 0 \right)$$

- (iv) Using the relation, induced current,

$$i = \frac{e}{R} = \frac{e_0}{R} \sin \omega t = i_0 \sin \omega t$$

where, $i_0 = \frac{e_0}{R}$.

We know that, $q = \int i dt$

So, for time 0 to $2T$,

$$q = \int_0^{2T} i dt = i_0 \int_0^{2T} \sin \omega t dt = 0$$

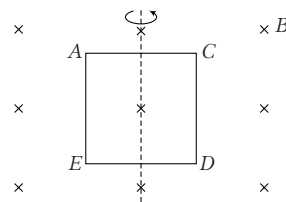
- (v) Heat (H) generated in a loop during time 0 to $2T$, is given as

$$\begin{aligned} H &= \int_0^{2T} \frac{e^2}{R} dt = \frac{e_0^2}{R} \int_0^{2T} \sin^2 \omega t dt \\ &= \frac{e_0^2}{R} \cdot 2 \cdot \frac{T}{2} = \frac{e_0^2}{R} \cdot \frac{2\pi}{\omega} = \frac{2\pi e_0^2}{\omega R} \quad \left(\because T = \frac{2\pi}{\omega} \right) \end{aligned}$$

Example 6.12 A square loop $ACDE$ of area 20 cm^2 and resistance 5Ω is rotated in a magnetic field $B = 2 \text{ T}$ through 180°

- (i) in 0.01 s (ii) and in 0.02 s .

Find the magnitude of e , i and Δq in both the cases.



Sol. Let us take the area vector $\mathbf{S}$ perpendicular to plane of loop inwards.

Hence, initial flux passing through the loop,

$$\begin{aligned} \phi_i &= BS \cos 0^\circ \\ &= (2) (20 \times 10^{-4}) (1) \\ &= 4 \times 10^{-3} \text{ Wb} \end{aligned}$$

Flux passing through the loop when it is rotated by 180° ,

$$\begin{aligned} \phi_f &= BS \cos 180^\circ \\ &= (2) (20 \times 10^{-4}) (-1) \\ &= -4 \times 10^{-3} \text{ Wb} \end{aligned}$$

Therefore, change in flux,

$$\begin{aligned} \Delta \phi_B &= \phi_f - \phi_i \\ &= -8 \times 10^{-3} \text{ Wb} \end{aligned}$$

- (i) Given at, $\Delta t = 0.01 \text{ s}$, $R = 5 \Omega$

Using the relation,

$$|e| = \left| -\frac{\Delta \phi_B}{\Delta t} \right| = \frac{8 \times 10^{-3}}{0.01} = 0.8 \text{ V}$$

$$\text{or induced current, } i = \frac{|e|}{R} = \frac{0.8}{5} = 0.16 \text{ A}$$

$$\text{and charge, } \Delta q = i \Delta t = 0.16 \times 0.01 = 1.6 \times 10^{-3} \text{ C}$$

- (ii) Similarly, given at, $\Delta t = 0.02 \text{ s}$, $R = 5 \Omega$

$$\therefore |e| = \left| -\frac{\Delta \phi_B}{\Delta t} \right| = \frac{8 \times 10^{-3}}{0.02} = 0.4 \text{ V}$$

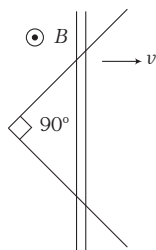
$$\therefore i = \frac{|e|}{R} = \frac{0.4}{5} = 0.08 \text{ A}$$

$$\begin{aligned} \text{and charge, } \Delta q &= i \Delta t = (0.08) (0.02) \\ &= 1.6 \times 10^{-3} \text{ C} \end{aligned}$$

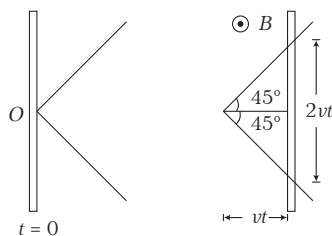
Example 6.13 The two conducting rails are placed perpendicular to each other, such that their ends are joined as shown in figure. A conducting bar is now placed over the rails and start moving with constant velocity v starting from the vertex at time $t = 0$.

- (i) Find the flux through the triangle (isosceles) by the rails and bar at $t = t_0$.
 (ii) Find the emf around the triangle at that time.

(iii) In what manner, does the emf around the triangle vary with time?



Sol.



(i) At time t , area of loop, $S = \frac{1}{2} \cdot vt \cdot 2vt = v^2 t^2$

($\because$ area of isosceles triangle $= \frac{1}{2} \times \text{base} \times \text{height}$)

Using the relation,

Magnetic flux, $\phi_B = BS = Bv^2 t^2$ and at $t = t_0$, $\phi = Bv^2 t_0^2$

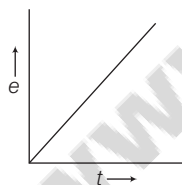
(ii) The emf developed around the triangle,

$$e = \left| \frac{d\phi_B}{dt} \right| = Bv^2 \cdot 2t = 2Bv^2 t$$

At $t = t_0$, $e = 2Bv^2 t_0$

(iii) According to the relation obtained for the emf,

$$\text{i.e. } e = 2Bv^2 t \Rightarrow e \propto t$$



Hence, emf varies linearly with time.

Lenz's law and conservation of energy

The negative sign in Faraday's equation of electromagnetic induction describes the direction in which induced emf drives the current. This direction is easily determined with the help of Lenz's law, an associated principle with Faraday's law.

Lenz's law give the same result as the sign rule we introduced in Faraday's law but is often easier to use. Lenz's law states that, the direction of any magnetic induction effect is such as to oppose the cause of effect.

This law is based upon law of conservation of energy. As the induced emf opposes the change in flux, work has to be done against the opposition offered by induced emf/current in changing the flux. The work done appears as electrical energy in the circuit.

Direction of induced current with the help of Lenz's law

Direction of induced current can be determined by checking whether the flux through a conducting loop or circuit is increasing or decreasing.

- If flux is decreasing, the magnetic field due to induced current will be along the existing magnetic field.
- If flux is increasing, the magnetic field due to induced current will be opposite to existing magnetic field.

Important points based on Lenz's law

The following are some important points that will explain the direction of induced current according to Lenz's law

- (i) Consider a conducting loop is being pulled inside a magnetic field, hence area of loop inside the magnetic field increases.

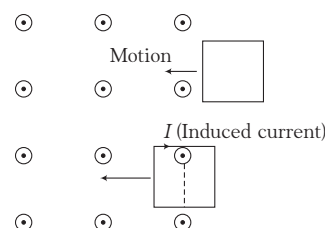


Fig. 6.7 Moving loop

From the above figure, it is clear that the magnetic flux through loop is increasing and hence magnetic field due to induced current will be opposite to the existing magnetic field, i.e. inside the plane of paper, thus direction of induced current will be clockwise.

- (ii) Consider a loop is being pulled out of magnetic field, area of loop inside field decreases and hence flux ($\phi = BS$) passing through loop also decreases.

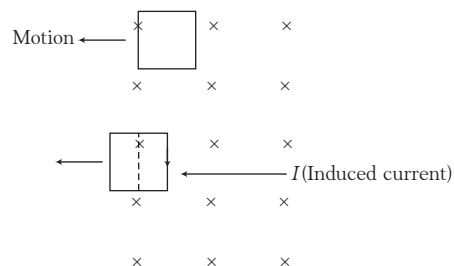


Fig. 6.8

So, magnetic field due to induced current will be along the existing magnetic field, *i.e.* inside the plane of paper, thus direction of induced current will be clockwise.

- (iii) Consider a loop is placed near a current carrying loop wire. The magnetic field due to long wire at distance x from it is given by

$$B = \frac{\mu_0 i}{2\pi x} \Rightarrow B \propto i \Rightarrow \phi \propto B \propto i$$

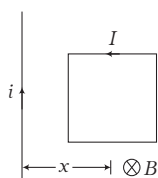


Fig. 6.9

The direction of magnetic field due to long wire will be $\otimes$.

If the current in long wire increases, then ϕ increases and the magnetic field due to induced current will be opposite to existing magnetic field, *i.e.* $\odot$. Hence, direction of induced current is anti-clockwise.

- (iv) If a conducting loop is brought away from a current carrying straight wire.

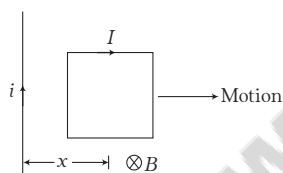


Fig. 6.10

As the loop is moving away, so flux through the loop will decrease and the magnetic field due to induced current will be along the existing magnetic field, *i.e.* $\otimes$. Hence, direction of induced current is clockwise.

- (v) Consider two conducting loops are placed in a magnetic field.

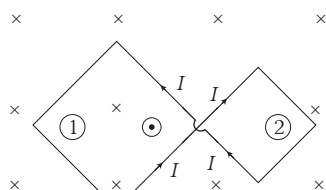


Fig. 6.11

In this case, if magnetic field is increasing, then flux through each loop will increase and the magnetic field due to induced current will be opposite to the existing

magnetic field, *i.e.* $\odot$. Hence, induced current will be anti-clockwise but single current will flow in the wire. The induced emf in loop 1 will be greater due to its bigger area, so single current will flow according to current in loop 1. Induced current in 1 will be anti-clockwise and in loop 2 will be clockwise.

- (vi) (a) When north pole moves towards ring, then flux will increase, induced current will oppose this, so north pole will be formed in loop as seen by observer.

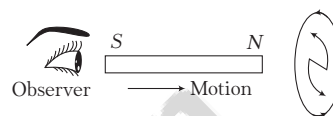


Fig. 6.12

Induced current will be anti-clockwise.

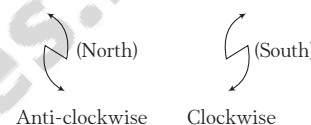


Fig. 6.13

- (b) When north pole moves away from ring, then flux will decrease, induced current will oppose this, so south pole will be formed in loop as seen by observer.

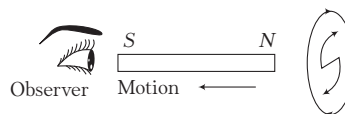


Fig. 6.14

Induced current will be clockwise.

- (c) Similar observations [as in case (a)] can be observed when south pole moves towards ring. So, induced current will be clockwise here.

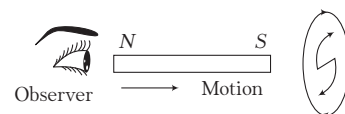


Fig. 6.15

- (d) Similar observations [as in case (b)] can be observed when south pole moves away from the ring. So, induced current in this case will be anti-clockwise.

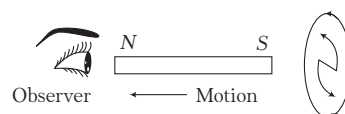


Fig. 6.16

- (vii) Two co-axial circular loops, identical to each other, carry equal amount of currents and circulating in the same direction.

- (a) When loops approaches each other, then the flux linked with each coil will increase.

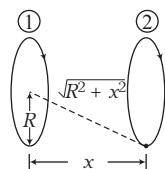


Fig. 6.17

Magnetic field due to loop ① at loop ② will be given by

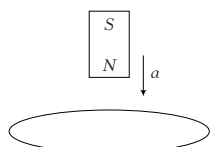
$$B = \frac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}}$$

So, x decreases, B will increase ϕ also increases.

The induced current in each coil will try to reduce flux and will be opposite to original current. So, current in each coil will decrease.

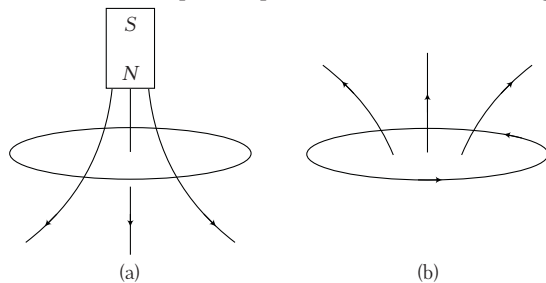
- (b) Now, loops move away from each other, then the flux linked with coil will decrease and so the induced current will try to increase the flux and hence current in each coil will increase.

Example 6.14 A bar magnet is freely falling along the axis of a circular loop as shown in figure. State whether its acceleration ' a ' is equal to, greater than or less than the acceleration due to gravity g .



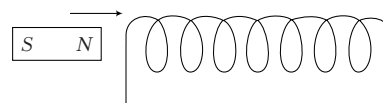
Sol. According to Lenz's law, whatever may be the direction of induced current, it will oppose the cause producing it. Here, the cause is the free fall of magnet and so the induced current will oppose it. So, the acceleration of magnet will be less than the acceleration due to gravity g .

Alternate method This can be understood in a different manner. When the magnet falls downwards with its north pole downwards. The magnetic field lines passing through the coil in the downward direction increase. Since, the induced current opposes this, the upper side of the coil will become north pole, so that field lines of coil's magnetic field are upwards. Now, like poles repel each other. Hence, $a < g$.

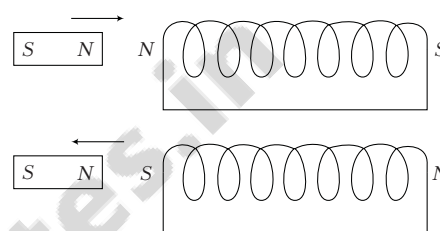


Note If this circular loop had a slot, an emf in that case will be induced but no current will flow, i.e. $a = g$.

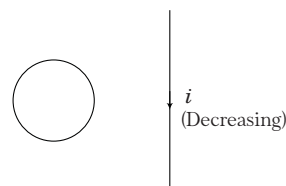
Example 6.15 A bar magnet is brought near a solenoid as shown in figure. Will the solenoid attract or repel the magnet?



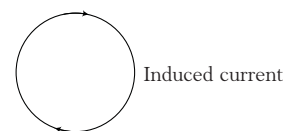
Sol. When the magnet is brought near the solenoid, then according to Lenz's law, both repel each other. On the other hand, if the magnet is moved away from the solenoid, then it attracts the magnet. When the magnet is brought near the solenoid, then the nearer side becomes the same pole and when it is moved away it becomes the opposite pole as shown in figure.



Example 6.16 A circular loop is placed near a current carrying conductor as shown in figure. Find the direction of induced current, if the current, in the wire is decreasing.

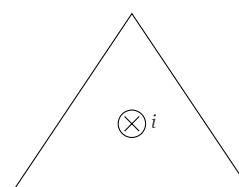


Sol. In this case, loop is placed to the right of current carrying wire (not to the left as it appears, because if you move in the direction of current, loop lies to the right).

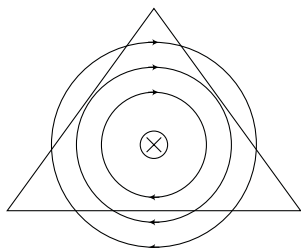


Now, the current is decreasing, therefore induced current in the loop is clockwise (S) as shown in above figure.

Example 6.17 A current carrying straight wire passes inside a triangular coil as shown in figure. The current in the wire is perpendicular to paper inwards. Find the direction of the induced current in the loop, if current in the wire is increased.

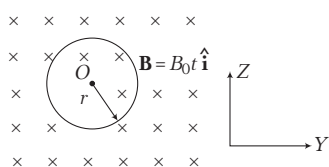


Sol. Magnetic field lines around the current carrying wire are as shown in figure below. Since, the magnetic lines are tangential to the loop ($\theta = 90^\circ$), the flux passing through the loop is zero, whether the current is increased or decreased. Hence, change in flux is zero. Therefore, induced current in the loop will be zero.



Example 6.18 A uniform magnetic field $\mathbf{B} = B_0 t \hat{i}$ in a region exists. A circular conducting loop of radius r and resistance R is placed with its plane in YZ -plane. Determine the current through the loop and sense of the current.

Sol.



We know that, magnetic flux is given as

$$\phi_B = \mathbf{B} \cdot \mathbf{S} = (B_0 t)(\pi r^2) \cos 0^\circ$$

$$\Rightarrow \phi_B = B_0 \pi r^2 t \quad \text{and} \quad \frac{d\phi}{dt} = B_0 \pi r^2$$

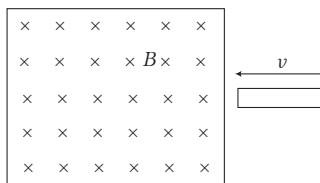
$$\text{Further, induced emf, } \epsilon = -\frac{d\phi}{dt} = -B_0 \pi r^2$$

$$\text{Therefore, current, } i = \frac{|\epsilon|}{R} = \frac{B_0 \pi r^2}{R}$$

By Lenz's law, i should be anti-clockwise, so that it can oppose the increase in magnetic field.

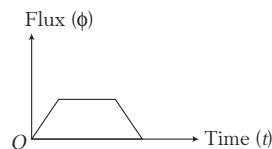
Example 6.19 Through a uniform B field, a small rectangular loop is moving towards left with constant velocity as shown in the figure. Counting of time t begins at the moment, the loop starts entering the field.

Plot the variation of flux through the loop with respect to time. Also, plot the variation of induced emf w.r.t. time t .

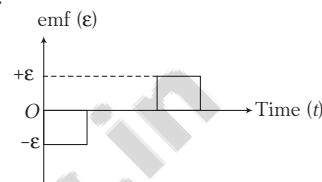


Sol. As the loop is moving with constant velocity, the flux will be increasing linearly, as long as the loop will be entering the B field. Once the loop has completely entered the B field the

flux will become constant. Now, when the loop will start coming out the flux will linearly decrease to zero value.

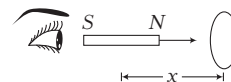


Since, emf induced, $\epsilon = -\frac{d\phi}{dt}$, it means emf is negative of the slope of the ϕ - t graph. So, emf is negative in the beginning, then it becomes zero and then it becomes positive and again it becomes zero.



Example 6.20 Through a conducting coil along its axis, a short bar magnet is rapidly pulled with uniform velocity with, which its north pole entering the coil first. Plot the variation of (i) flux, (ii) induced current and (iii) power dissipated in coil with time.

Sol. (i)

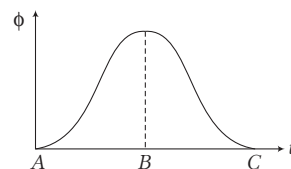


$$\text{Magnetic field due to magnet is given as } B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{x^3}$$

$$\Rightarrow B \propto \frac{1}{x^3} \Rightarrow \phi \propto \frac{1}{x^3}$$

When a bar magnet is pulled rapidly through a conducting coil, then flux first increases, reaching to a maximum value and then decreases.

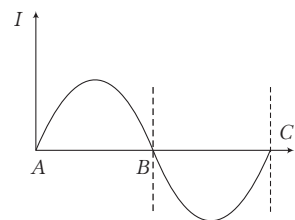
The variation of flux ϕ with time t is shown below



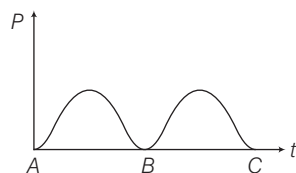
(ii) When flux increases, then north pole is formed and current is anti-clockwise.

When flux decreases, then south pole is formed and current is clockwise. (Taking anti-clockwise current as positive.)

The induced current I versus time t graph is shown

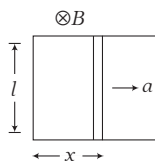


- (iii) As, power $\propto$ (induced current)². The corresponding power *versus* t graph can be drawn as



Example 6.21 In a uniform magnetic field, a π shaped metal frame is located perpendicular to the plane of the conductor and varying with time at the rate $(dB/dt) = 0.20 \text{ T/s}$. A conducting connector starts moving with an acceleration $a = 30 \text{ cm/s}^2$ along the parallel bars of the frame. The length of the connector is equal to $l = 44 \text{ cm}$. Find the emf induced in the loop at $t = 1 \text{ s}$ after the beginning of the motion, if at the moment $t = 0$, the loop area and the magnetic induction are equal to zero.

Sol.



We know, according to kinematic equation, distance travelled $x = ut + \frac{1}{2}at^2$. Here, initial velocity, $u = 0$, acceleration,

$$a = 30 \text{ cm/s}^2$$

So, in time t , distance travelled by conductor, $x = \frac{1}{2}bt^2$

$$\text{Flux passing through loop, } \phi = BS = Blx = \frac{Blbt^2}{2}$$

Here, B and t both are variable.

$$\begin{aligned} \text{So, induced emf, } e &= \left| \frac{d\phi}{dt} \right| = \frac{lb}{2} \left[\frac{dB}{dt} t^2 + B \cdot 2t \right] \\ &= \frac{0.44 \times 0.3}{2} (0.2 \times 1^2 + 2B) \\ &= 0.066 (0.2 + 2B) \end{aligned}$$

$$\text{Given, } \frac{dB}{dt} = 0.2 \text{ T/s} \Rightarrow dB = 0.2 dt$$

On integrating both sides, we get

$$\int_0^B dB = 0.2 \int_0^t dt \Rightarrow B = 0.2 t = 0.2 \times 1 = 0.2$$

$$\text{So, induced emf, } e = 0.066(0.2 + 0.2) = 0.0264 = 2.64 \times 10^{-2} \text{ V}$$

CHECK POINT 6.1

- The magnetic flux linked with a vector area $\mathbf{A}$ in a uniform magnetic field $\mathbf{B}$ is
(a) $\mathbf{B} \times \mathbf{A}$ (b) AB (c) $\mathbf{B} \cdot \mathbf{A}$ (d) $\frac{B}{A}$
- The unit of magnetic flux is
(a) Wbm^{-2} (b) Wb (c) H (d) Am^{-1}
- The dimensions of magnetic flux are
(a) $[\text{MLT}^{-2}\text{A}^{-2}]$ (b) $[\text{ML}^2\text{T}^{-2}\text{A}^{-2}]$
(c) $[\text{ML}^2\text{T}^{-1}\text{A}^{-2}]$ (d) $[\text{ML}^2\text{T}^{-2}\text{A}^{-1}]$
- The magnetic flux linked with a coil, in weber is given by the equation $\phi = 3t^2 + 4t + 9$. Then, the magnitude of induced emf at $t = 2 \text{ s}$ will be
(a) 2 V (b) 4 V (c) 8 V (d) 16 V
- A coil having an area A_0 is placed in a magnetic field which changes from B_0 to $4B_0$ in time interval t . The emf induced in the coil will be
(a) $3A_0B_0/t$ (b) $4A_0B_0/t$ (c) $3B_0/A_0t$ (d) $4B_0/A_0t$
- The magnetic flux ϕ (in weber) in a closed circuit of resistance 10Ω varies with time t (in second) according to equation $\phi = 6t^2 - 5t + 1$. The magnitude of induced current at $t = 0.25 \text{ s}$ is
(a) 1.2 A (b) 0.8 A (c) 0.6 A (d) 0.2 A
- The magnetic flux across a loop of resistance 10Ω is given by $\phi = (5t^2 - 4t + 1) \text{ Wb}$. How much current is induced in the loop after 0.2 s ?
(a) 0.4 A (b) 0.2 A
(c) 0.04 A (d) 0.02 A
- A circular ring of diameter 20 cm has a resistance 0.01Ω . How much charge will flow through the ring, if it is rotated from a position perpendicular to a uniform magnetic field of $B = 2 \text{ T}$ to a position parallel to field?
(a) 4 C (b) 6.28 C (c) 3.14 C (d) 2.5 C
- In a circuit with a coil of resistance 2Ω , the magnetic flux changes from 2.0 Wb to 10.0 Wb in 0.2 s . The charge that flows in the coil during this time is
(a) 5.0 C (b) 4.0 C (c) 1.0 C (d) 0.8 C
- The direction of induced emf during electromagnetic induction is given by
(a) Faraday's law (b) Lenz's law
(c) Maxwell's (d) Ampere's law
- Two different loops are concentric and lie in the same plane. The current in the outer loop is clockwise and increasing with time. The induced current in the inner loop, is
(a) clockwise
(b) zero
(c) counter-clockwise
(d) in a direction that depends on the ratio of the loop radii
- When the current through a solenoid increases at a constant rate, then the induced current
(a) is a constant and is in the direction of the inducing current
(b) is a constant and is opposite to the direction of the inducing current
(c) increase with time and is in the direction of inducing current
(d) increase with time and is opposite to the direction of inducing current

13. The north pole of a long horizontal bar magnet is being brought closer to a vertical conducting plane along the perpendicular direction. The direction of the induced current in the conducting plane will be
 (a) horizontal (b) vertical
 (c) clockwise (d) anti-clockwise
14. There is a uniform magnetic field directed perpendicular and into the plane of the paper. An irregular shaped

conducting loop is slowly changing into a circular loop in the plane of the paper. Then,

- (a) current is induced in the loop in the anti-clockwise direction
 (b) current is induced in the loop in the clockwise direction
 (c) AC is induced in the loop
 (d) No current is induced in the loop

MOTIONAL ELECTROMOTIVE FORCE

So far, we have considered the cases in which an emf is induced in a stationary circuit placed in a time-varying magnetic field. In this section, we will study about **motional emf**, i.e. the emf induced in a conductor when it is moving through a constant magnetic field. Now, let us consider, a straight conductor of length l as shown in Fig. 6.18 which is moving through a uniform magnetic field directed into the page. For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.

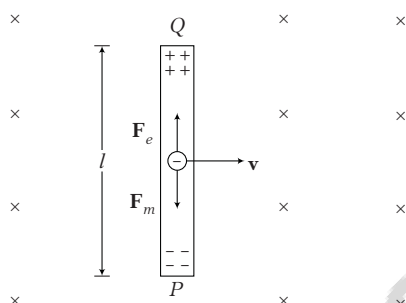


Fig. 6.18

Due to motion of conductor in magnetic field, a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field.

This potential difference is called **motional electromotive force**, which is denoted by e and is given by

$$e = Bvl \quad \dots(i)$$

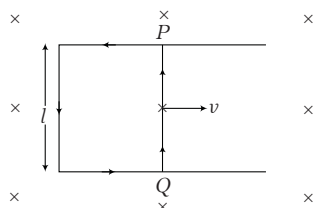


Fig. 6.19

If R is the resistance of the circuit, then current will be written as

$$\Rightarrow i = \frac{e}{R} = \frac{Bvl}{R} \quad \dots(ii)$$

General form of motional emf

To find motional emf for a conductor of any shape, moving in a uniform or non-uniform magnetic field, we will consider any small element $d\mathbf{l}$ of a conductor, then the contribution de to the emf is the magnitude of $d\mathbf{l}$ multiplied by the component of $(\mathbf{v} \times \mathbf{B})$ which is parallel to $d\mathbf{l}$.

$$i.e. \quad de = (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\Rightarrow e = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} \quad [\text{motional emf, closed conducting loop}]$$

Important points related to motional emf

Some important points related to motional emf are given below

- (i) For a semi-circular conducting loop of radius R with the centre at O and moving with velocity v , then the emf is given by

$$e = Bv(2R)$$

$$\Rightarrow V_P - V_Q = 2BvR$$

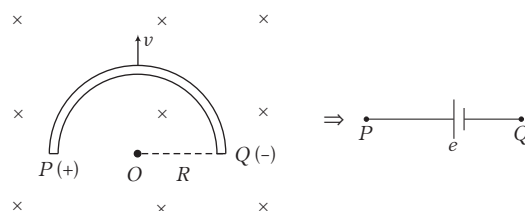


Fig. 6.20

- (ii) If a conducting rod AB moves with a velocity v in XY -plane, then we can resolve the velocity vector in two components as shown in Fig. 6.21.

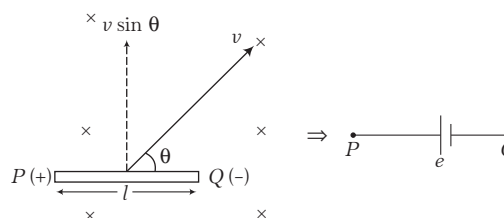


Fig. 6.21

No emf will be induced, due to $v \cos \theta$. So, the net effect of emf will be due to $v \sin \theta$, which is given by $e = B(v \sin \theta)l$.

- (iii) For a circular loop of radius R moving with velocity v , we can replace its two ends lying on opposite sides of diameter with a battery of emf e . Then, the emf is given by $e = Bv(2R) = 2BvR$

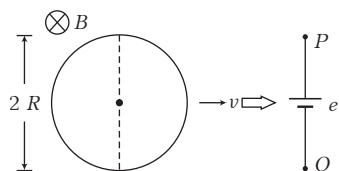


Fig. 6.22

- (iv) For an irregular shape, a conducting body of length l , we can assume it, as a straight conductor and replace it with a battery of emf e as shown in Fig. 6.23.

Then, $e = B(v \cos \theta)l$

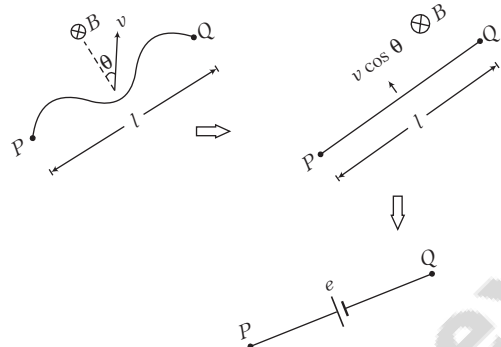


Fig. 6.23

- (v) When magnetic field varies at every point of conductor which is moving with speed v , then the induced emf will be given by

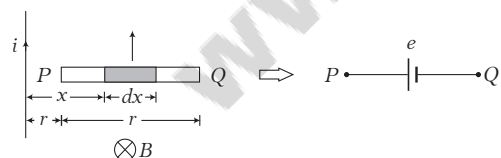


Fig. 6.24

$$de = Bv dx,$$

where, B is the magnetic field due to a long wire at distance x and is equal to $\frac{\mu_0 i}{2\pi x}$.

$$\begin{aligned} \text{Therefore, } V_P - V_Q = e &= \frac{\mu_0 i v}{2\pi} \int_r^{r+l} \frac{dx}{x} \\ &= \frac{\mu_0 i v}{2\pi} \ln \left(1 + \frac{l}{r} \right) \end{aligned}$$

Motional emf induced in a rotating bar

- (i) A conducting rod of length l rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field B is directed perpendicular to the plane of rotation as shown in Fig. 6.25.

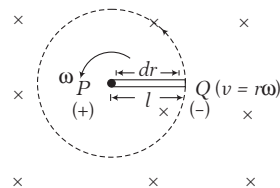


Fig. 6.25

Let us suppose an element of length dr at a distance r from P . The induced emf in this segment is

$$de = Bvdr = B(r\omega)dr$$

Because every segment of the rod is moving perpendicular to B , an emf de of same form is generated across each.

So, summing the emfs induced across the ends of the rod is

$$\begin{aligned} e &= \int_0^l de = \int_0^l Br\omega dr \\ &\Rightarrow e = \frac{1}{2} B\omega l^2 \\ &= Bl^2 \pi v = \frac{Bl^2 \pi}{T} \quad (\because 2\pi v = \omega) \end{aligned}$$

where, v = frequency (revolution per second)
and T = time period.

- (ii) If a conducting disc of radius r rotates with constant angular velocity ω about its axis in a uniform magnetic field parallel to its axis of rotation as shown in Fig. 6.26.

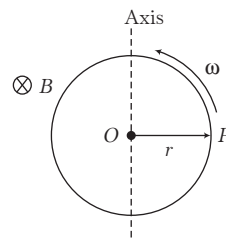


Fig. 6.26

Take a rod between centre and point P .

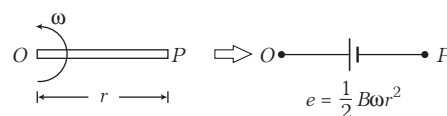


Fig. 6.27

As, we have already deduced the emf for a rod rotating in a constant magnetic field.

So, the induced emf is given by $e = \frac{1}{2} B \omega r^2$.

That means, disc is equivalent to rod.

Fleming's right hand thumb rule

The direction of motional emf or current can be given by Fleming's right hand thumb rule. If we stretch the thumb, the forefinger and the central finger of right hand in such a way that all are mutually perpendicular to each other and if thumb represents the direction of motion of the conductor, the forefinger represents the direction magnetic field, then central finger will represent the direction of induced current as shown in Fig. 6.28.

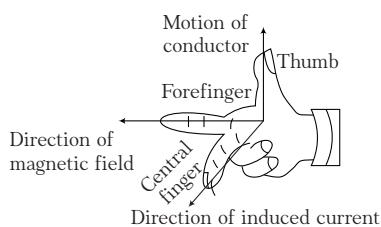


Fig. 6.28

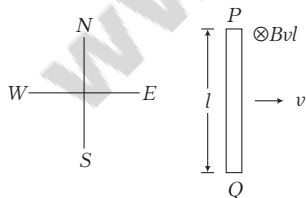
Example 6.22 A conducting rod of length 1 m moves with a velocity of 5 m/s in a direction perpendicular to its length and perpendicular to a uniform magnetic field of magnitude 0.4 T. Find the emf induced between the ends of the stick.

Sol. As, we know that, induced emf for a moving rod is given by

$$\Rightarrow e = Bvl = 0.4 \times 5 \times 1 = 2 \text{ V}$$

Example 6.23 The horizontal component of the earth's magnetic field at a place is $4.0 \times 10^{-4} \text{ T}$ and the dip is 45° . A metal rod of length 20 cm is placed in the north-south direction and is moved at a constant speed of 5 cm/s towards east. Calculate the emf induced in the rod.

Sol.



According to the question, the velocity v , length l and B_v (vertical component of earth magnetic field) are mutually perpendicular using the relation,

$$\begin{aligned} e &= B_v v l \\ &= B_h \tan \phi v l \\ &= 4 \times 10^{-4} \tan 45^\circ \times 0.05 \times 0.20 \\ &= 4 \times 10^{-6} \text{ V} \end{aligned} \quad \left(\because \tan \phi = \frac{B_v}{B_h} \right)$$

There will be no induced emf due to B_h (horizontal component of earth magnetic field) because it is parallel to length of rod.

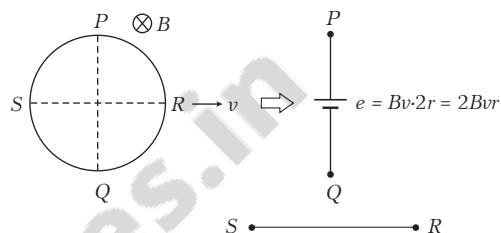
Example 6.24 A ring of radius 2 m translates in its plane with a constant velocity 5 m/s. A uniform magnetic field 0.1 T exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring.

- Between which pair of points is the emf maximum? What is the value of this maximum emf?
- What is the value of this minimum emf?

Sol. Given, radius, $r = 2 \text{ m}$, velocity, $v = 5 \text{ ms}^{-1}$

Constant magnetic field, $B = 0.1 \text{ T}$

According to the question,

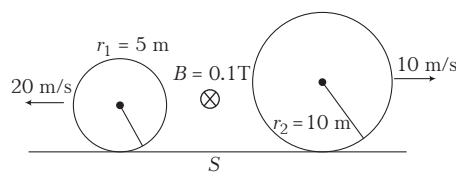


From figure, we can see that emf is maximum at PQ and minimum at RS, as PQ is perpendicular to motion and RS along the motion.

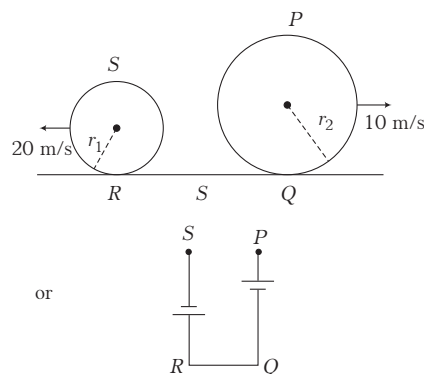
Using the relation for induced emf for a rod, $V_P - V_Q = 2Bvr$

$$\begin{aligned} e_{\max} &= 2Bvr \\ &= 2 \times 0.1 \times 5 \times 2 = 2 \text{ V} \\ V_S - V_R &= 0 \Rightarrow e_{\min} = 0 \end{aligned}$$

Example 6.25 Two rings of radii 5 m and 10 m move in opposite directions with velocity 20 m/s and 10 m/s respectively, on a conducting surface S. There is a uniform magnetic field of magnitude 0.1 T perpendicular to the plane of the rings. Find the potential difference between the highest points of the two rings.



Sol. The above situation can be drawn as



As, we know, induced emf in a rod is given by $e = Bvl$.

Potential at point P , $e = Bvl = 0.1 \times 10 \times 20 = 20 \text{ V}$

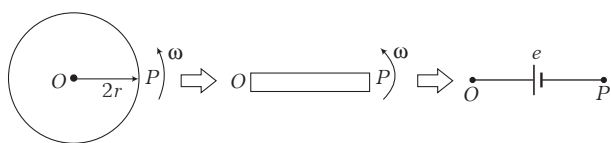
Potential at point S , $e = Bvl = 0.1 \times 20 \times 10 = 20 \text{ V}$

So, the net emf produced in above figure is given by

$$V_P - 20\text{V} - 20\text{V} = V_S \Rightarrow V_P - V_S = 40\text{V}$$

Example 6.26 A vertical disc of diameter 10 cm makes 20 revolutions per second about a horizontal axis passing through its centre. A uniform magnetic field 10^{-1} T acts perpendicular to the plane of the disc. Calculate the potential difference between its centre and rim in volts.

Sol. As, we know, disc is equivalent to a rod of length r . Divide disc into rods, each rod can be replaced by a battery and these batteries would be parallel in arrangement.



According to the question, radius of disc,

$$r = \frac{10}{2} = 5 \text{ cm} = 0.05 \text{ m}$$

Angular velocity, $\omega = 20 \times 2\pi = 40\pi \text{ rad/s}$

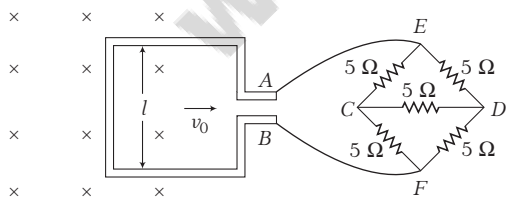
Magnetic field, $B = 10^{-1} \text{ T}$

The emf of a disc is given as

$$e = \frac{1}{2} B \omega r^2 = \frac{1}{2} (10^{-1}) (40\pi) (0.05)^2$$

$$\Rightarrow V_O - V_P = 1.57 \times 10^{-2} \text{ V}$$

Example 6.27 A square metal wire loop of side 20 cm and resistance 2Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 1 \text{ Wb/m}^2$ as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop. The loop is connected to a network of resistance each of value 5Ω . The resistances of the lead wires BF and AE are negligible. What should be the speed of the loop, so as to have a steady current of 2 mA in the loop? Give the direction of current in the loop.



Sol. From the figure, we see that, network $CEDF$ is balanced Wheatstone bridge, so no current will flow in branch CD .

So, the equivalent resistance of $CEDF$ network,

$$R_{eq} = \frac{10 \times 10}{10 + 10} = 5 \Omega$$

Resistance of loop = 2Ω

$$R_{total} = 2 + R_{eq} = 2 + 5 = 7 \Omega$$

We know that, induced emf, $e = Bv_0l$

$$\therefore \text{Induced current, } I = \frac{e}{R_{total}} = \frac{Bv_0l}{7}$$

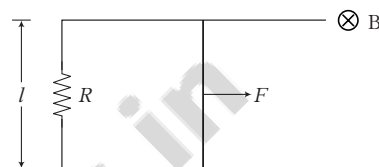
$$\Rightarrow 2 \times 10^{-3} = \frac{1 \times v_0 \times 0.2}{7}$$

$$\Rightarrow v_0 = 7 \times 10^{-2} \text{ m/s} = 7 \text{ cm/s}$$

As, flux is decreasing, so induced current I will be clockwise.

Example 6.28 Figure shows the top view of a rod that can slide without friction. The resistor is 6.0Ω and a 2.5 T magnetic field is directed perpendicularly downward into the paper.

Let $l = 1.20 \text{ m}$.



(i) Calculate the force F required to move the rod to the right at a constant speed of 2.0 ms^{-1} .

(ii) At what rate is energy delivered to the resistor?

(iii) Show that this rate is equal to the rate of work done by the applied force.

Sol. Given, resistance, $R = 6.0 \Omega$

Magnetic field, $B = 2.5 \text{ T}$

Length of the rod, $l = 1.20 \text{ m}$

The motional emf in the rod is given as, $e = Bvl$

$$\text{or } e = (2.5) (2.0) (1.2) \text{ V} = 6.0 \text{ V}$$

$$\text{The current in the circuit, } i = \frac{e}{R} = \frac{6.0}{6.0} = 1.0 \text{ A}$$

(i) The magnitude of force F required will be equal to the magnetic force acting on the rod, which opposes the motion.

$$\therefore F = F_m = qvB = i l B$$

$$\text{or } F = (1.0) (1.2) (2.5) \text{ N} = 3 \text{ N}$$

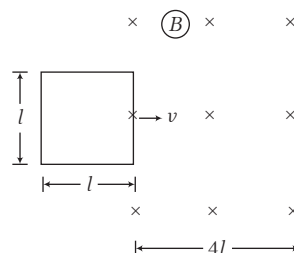
(ii) Rate by which energy is delivered to the resistor is

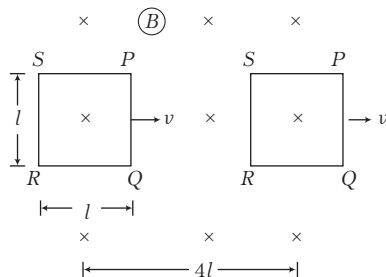
$$P_1 = i^2 R = (1)^2 (6.0) = 6 \text{ W}$$

(iii) The rate by which work is done by the applied force,

$$P_2 = Fv = (3) (2.0) = 6 \text{ W} \Rightarrow P_1 = P_2$$

Example 6.29 A square loop of side l being moved towards right at a constant speed v as shown in figure. The front edge enters the magnetic field B at $t = 0$. The width of field is $4l$. Sketch induced emf versus time graph.



Sol.

From $t = 0$ to $t = \frac{l}{v}$,

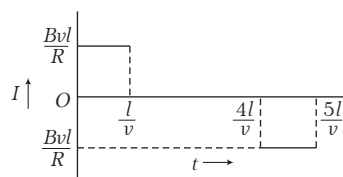
Induced emf, $e = Bvl$ and $I = \frac{e}{R} = \frac{Bvl}{R}$, which is anti-clockwise in direction.

Again from $t = \frac{l}{v}$ to $t = \frac{4l}{v}$, emf $e = 0$

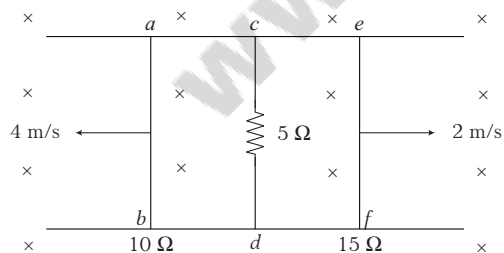
$t = \frac{4l}{v}$ to $t = \frac{5l}{v}$, emf $e = Bvl$

So, induced current $I = \frac{e}{R} = \frac{Bvl}{R}$, which is clockwise in direction now.

So, the plot of induced emf versus time is given as



Example 6.30 Two parallel rails with negligible resistance are 10 cm apart. They are connected by a $5\ \Omega$ resistor. The circuit also contains two metal rods having resistances of $10\ \Omega$ and $15\ \Omega$ along the rails. The rods are pulled away from the resistor at constant speeds 4.00 m/s and 2 m/s, respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the $5\ \Omega$ resistor.



Sol. Here, two conductors are moving in uniform magnetic field. So, we will use the motional approach. The rod ab will act as a source of emf,

$$e_1 = Bvl = (0.01)(4)(0.1) = 4 \times 10^{-3}\text{ V}$$

and internal resistance, $r_1 = 10\ \Omega$

Similarly, rod ef will also act as a source of emf,

$$e_2 = (0.01)(2)(0.1) = 2 \times 10^{-3}\text{ V}$$

and internal resistance, $r_2 = 15\ \Omega$

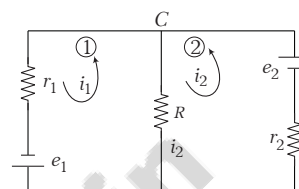
From right hand rule, we can see that,

$$V_b > V_a \text{ and } V_e > V_f$$

Now, either by applying Kirchhoff's laws (discussed in the chapter of current electricity) we can find current through $5\ \Omega$ resistor. We will here use the superposition principle. You solve it by using Kirchhoff's laws.

Now, in the figures, $R = 5\ \Omega$, $r_1 = 10\ \Omega$, $r_2 = 15\ \Omega$,

$$e_1 = 4 \times 10^{-3}\text{ V and } e_2 = 2 \times 10^{-3}\text{ V.}$$



KVL in loop 1,

$$e_1 - i_1 r_1 - (i_1 - i_2)R = 0$$

$$\Rightarrow e_1 = i_1(r_1 + R) - i_2 R \quad \dots(i)$$

Similarly, KVL in loop 2,

$$e_2 - i_2 r_2 - (i_2 - i_1)R = 0$$

$$\Rightarrow e_2 = -i_1 R + i_2(r_2 + R) \quad \dots(ii)$$

Now, putting values in Eqs. (i) and (ii), we get

$$4 \times 10^{-3} = 15i_1 - 5i_2 \quad \dots(iii)$$

$$\text{and } 2 \times 10^{-3} = -5i_1 + 20i_2 \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$i_2 = \frac{2}{11} = 0.18\text{ mA}$$

$$\text{and } i_1 = 0.33\text{ mA}$$

Hence, net current through $5\ \Omega$ resistor,

$$I = i_1 - i_2 = 0.33 - 0.18 = 0.15\text{ mA (from d to e)}$$

Induction of field

The modern view of electromagnetic induction states that induction occurs whether or not a conducting wire or material medium is present.

In general case, Faraday's law can be stated as, "An electric field is induced in any region of space in which a magnetic field is changing with time".

This law is also true, if magnetic and electric fields are interchanged. *i.e.*

"A magnetic field is induced in any region of space in which an electric field is changing with time."

In each case, the strength of the induced field is proportional to the rate of change of the inducing field. Induced electric and magnetic fields are at right angles to each other.

Induced electric field

As per Faraday's law, due to change in magnetic field, an electric field is induced.

Let us understand this by considering a ring of radius r . A uniform magnetic field B is perpendicular to its plane.

Let here, time rate of change of magnetic field is $\frac{dB}{dt}$, then the flux associated will be ϕ . So, the induced emf will be $e = \frac{-d\phi_B}{dt}$.

Hence, an induced current will flow in anti-clockwise direction, which means, if there is a current present in the ring, then there must be an electric field. And this electric field is produced by the changing magnetic field.

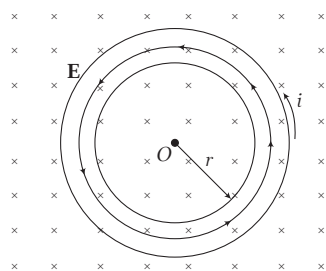


Fig. 6.29

Suppose a particle of charge q_0 is moving around the ring in a circular path. So, the work done by the induced electric field in one revolution is given by

$$W = q_0 e \quad \dots (i)$$

where, e is the induced emf.

$$\text{We also know, } W = \oint \mathbf{F} \cdot d\mathbf{l} = q_0 \cdot \oint \mathbf{E} \cdot d\mathbf{l} \quad \dots (ii)$$

Equating Eqs. (i) and (ii), we get

$$q_0 e = q_0 \oint \mathbf{E} \cdot d\mathbf{l}$$

$$\Rightarrow e = \oint \mathbf{E} \cdot d\mathbf{l} \quad \dots (iii)$$

Eq. (iii) is the relationship between induced emf and induced electric field.

$$\text{From the Faraday's law, } e = \frac{-d\phi_B}{dt} \quad \dots (iv)$$

From Eqs. (iii) and (iv), we get

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\phi_B}{dt}$$

This is known as integral form of Faraday's law of electromagnetic induction.

Note In case of electromagnetic induction, line integral of induced emf $\mathbf{E}$ around a close path is not zero, i.e. induced electric field is not conservative. In such a field, work done in moving a charge round or close path is not zero.

Example 6.31 A uniform magnetic field exists in a circular region of radius R centred at O . The field is perpendicular to the plane of paper and its strength varies with time as $B = B_0 t$. Find the induced electric field at a distance r from the centre for (i) $r < R$ (ii) and $r > R$. Also, plot a graph between $|E|$ and r for both the cases.

Sol. As we know, induced electric field is given by

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\phi_B}{dt}$$

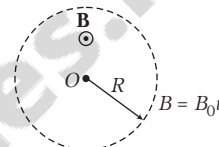
- (i) For $r < R$, consider a ring of radius $r (< R)$ centred at O . The induced field at the periphery of the ring is circular such that,

$$\oint \mathbf{E} \cdot d\mathbf{l} = E \times 2\pi r \quad \dots (i)$$

($\because dl$ = circumference of the ring = $2\pi r$)

As, magnetic flux, $\phi_B = \mathbf{B} \cdot \mathbf{S} = BS \cos \theta$

Here, $\theta = 0^\circ$, hence $\phi_B = (B_0 t) \pi r^2$ ($\because B = B_0 t$)



$$\therefore \frac{-d\phi_B}{dt} = -B_0 \pi r^2 \quad \dots (ii)$$

Equating Eqs. (i) and (ii), we get

$$\Rightarrow E \times 2\pi r = -B_0 \pi r^2$$

$$\Rightarrow E = \frac{-B_0 r}{2}$$

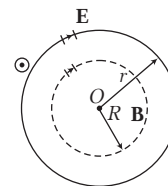
$$\Rightarrow E \propto r$$

(the sign comes, when clockwise electric field lines are developed)

- (ii) For $r > R$, consider a ring of radius $r (> R)$ centred at O . Again following the same procedure,

$$\oint \mathbf{E} \cdot d\mathbf{l} = E \times 2\pi r \quad \dots (iii)$$

Similarly, flux, $\phi_B = (B_0 t) \pi R^2$ (as field is present only in region $0 < r < R$).



$$\Rightarrow \frac{d\phi_B}{dt} = -B_0 \pi R^2 \quad \dots (iv)$$

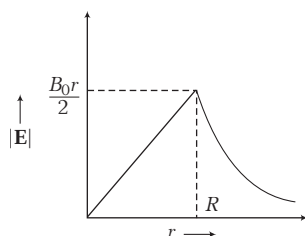
Equating Eqs. (iii) and (iv), we get

$$\Rightarrow E 2\pi r = -B_0 \pi R^2$$

$$\Rightarrow E = \frac{-B_0 R^2}{2r}$$

$$\Rightarrow E \propto \frac{1}{r}$$

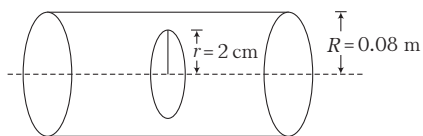
Hence, plot between $|E|$ and r will be



Example 6.32 The current in an ideal, long solenoid is varied at a uniform rate of 0.02 A/s . The solenoid has 1000 turns/m and its radius is 8 cm .

- Consider a circle of radius 2 cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in 4 s .
- Find the electric field induced at a point on the circumference of the circle.
- Find the electric field induced at a point outside the solenoid at a distance 9 cm from its axis.

Sol.



Given, for solenoid, $n = 1000 \text{ turns/m}$ and $\frac{di}{dt} = 0.02 \text{ A/s}$

$$\Rightarrow di = 0.02 dt \quad \dots(i)$$

- (i) Magnetic field due to solenoid is given as

$$B = \mu_0 n i$$

$$\text{Flux through circle of radius } r, \phi_B = B \pi r^2 \quad (\because \phi_B = \mathbf{B} \cdot \mathbf{S})$$

$$= \mu_0 n i \pi r^2$$

$$\Rightarrow d\phi_B = \mu_0 n \pi r^2 di$$

$$= \mu_0 n \pi r^2 (0.02) dt \quad [\because \text{using Eq. (i)}]$$

$$\Rightarrow \phi = \mu_0 n \pi r^2 (0.02) \int_0^4 dt$$

$$= 4\pi \times 10^{-7} \times 1000 \times \pi (0.02)^2 (0.02 \times 4)$$

$$= 1.26 \times 10^{-7} \text{ Wb}$$

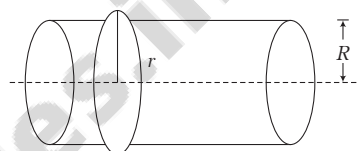
- (ii) Electric field induced is related to emf as $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{d\phi}{dt}$

$$\Rightarrow E \cdot 2\pi r = -\mu_0 n \pi r^2 \frac{di}{dt}$$

$$|E| = \frac{\mu_0 n r}{2} \frac{di}{dt} = \frac{4\pi \times 10^{-7} \times 1000 \times 0.02}{2} \times 0.02$$

$$= 8\pi \times 10^{-8} \text{ V/m}$$

- (iii)



As discussed in (i), similarly for radius outside the solenoid,

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} (\mu_0 n i \pi R^2)$$

$$\Rightarrow E \cdot 2\pi r = -\mu_0 n \frac{di}{dt} \pi R^2 \Rightarrow |E| = \frac{\mu_0 n R^2}{2r} \frac{di}{dt}$$

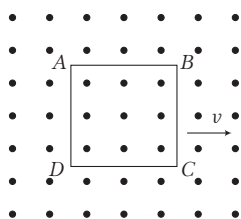
$$= \frac{4\pi \times 10^{-7} \times 1000 \times (0.08)^2 \times 0.02}{2 \times 0.09}$$

$$= 28\pi \times 10^{-8} \text{ V/m}$$

CHECK POINT 6.2

- A conducting rod of length l is falling with a constant velocity v perpendicular to a uniform horizontal magnetic field B . A potential difference between its two ends will be
(a) $2 Blv$ (b) Blv (c) $\frac{1}{2} Blv$ (d) $B^2 l^2 v^2$
- A wire of length 50 cm moves with a velocity of 300 m/min , perpendicular to a magnetic field. If the emf induced in the wire is 2 V , then the magnitude of the field in tesla is
(a) 2 (b) 5 (c) 0.8 (d) 2.5
- A 10 m wire kept in east-west direction is falling with velocity 5 m/s perpendicular to the field $0.3 \times 10^4 \text{ Wb/m}^2$. The induced emf across the terminal will be
(a) 0.15 V (b) 1.5 mV (c) 1.5 V (d) 15.0 V
- A boat is moving due east in a region, where the earth's magnetic field is $5.0 \times 10^{-5} \text{ NA}^{-1} \text{m}^{-1}$ due to north and horizontal. The boat carried a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1} , then the magnitude of the induced emf in the wire of aerial is
(a) 1 mV (b) 0.75 mV (c) 0.50 mV (d) 0.15 mV
- The magnitude of the earth's magnetic field at a place is B_0 and the angle of dip is δ . A horizontal conductor of length l lying along the magnetic north-south moves eastwards with a velocity v . The emf induced across the conductor is
(a) zero (b) $B_0 l v \sin \delta$
(c) $B_0 l v$ (d) $B_0 l v \cos \delta$
- A 0.1 m long conductor carrying a current of 50 A is held perpendicular to a magnetic field of 1.25 mT . The mechanical power required to move the conductor with a speed of 1 ms^{-1} is
(a) 62.5 mW (b) 625 mW
(c) 6.25 mW (d) 12.5 mW
- A conducting rod of length l is moving in a transverse magnetic field of strength B with velocity v . The resistance of the rod is R . The current in the rod is
(a) $\frac{Blv}{R}$ (b) Blv
(c) zero (d) $\frac{B^2 v^2 l^2}{R}$

8. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced



- (a) in AD but not in BC (b) in BC but not in AD
 (c) Neither in AD nor in BC (d) in both AD and BC
9. A coil of N turns and mean cross-sectional area A is rotating with uniform angular velocity ω about an axis at right angle to uniform magnetic field B . The induced emf E in the coil will be
 (a) $NBA \sin \omega t$ (b) $NB \omega \sin \omega t$
 (c) $NB / A \sin \omega t$ (d) $NBA \omega \sin \omega t$
10. A circular coil of mean radius of 7 cm and having 4000 turns is rotated at the rate of 1800 rev/min in the earth's magnetic field ($B = 0.5 \text{ G}$), the maximum emf induced in coil will be

- (a) 1.158 V (b) 0.57 V
 (c) 0.29 V (d) 5.8 V

11. A metal rod of length 2 m is rotating with an angular velocity of 100 rads^{-1} in a plane perpendicular to a uniform magnetic field of 0.3 T. The potential difference between the ends of the rod is

- (a) 30 V (b) 40 V
 (c) 60 V (d) 600 V

12. A rectangular coil rotates about an axis normal to the magnetic field. If E_m is the maximum value of the induced emf, then the instantaneous emf when the plane of the coil makes an angle of 45° with the magnetic field is

- (a) $\frac{1}{2} E_m$ (b) $\frac{1}{4} E_m$ (c) $\frac{1}{\sqrt{2}} E_m$ (d) E_m

13. Consider the following statements.

A. An emf can be induced by moving a conductor in a magnetic field.

B. A magnetic field can be produced by changing the electric field.

- (a) Both A and B are true (b) A is true but B is false
 (c) B is true but A is false (d) Both A and B are false

SELF-INDUCTION

Self-induction is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an emf in itself.

This induced emf is also called **back emf**. When the current in a coil is switched ON, then the self-induction opposes the growth of the current and when it is switched OFF, then the self-induction opposes the decay of the current. Hence, self-induction is also known as **inertia of electricity**.

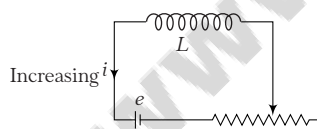


Fig. 6.30

A circuit or part of a circuit, that is designed to have a particular induction is called an **inductor**. The usual symbol for an inductor is shown in Fig. 6.31.



Fig. 6.31

Thus, an inductor is a circuit element which opposes the change in current through it. It may be a circular coil, solenoid, etc.

Coefficient of self-induction

It can be defined in two ways as discussed below

1. Definition using magnetic flux

If i is the strength of current flowing through a coil at any time and ϕ is the amount of magnetic flux linked with all the turns of the coil at that time, then it is found that

$$\phi \propto i \text{ or } \phi = Li \Rightarrow L = \frac{\phi}{i}$$

where, L is a constant of proportionality and is called **coefficient of self-induction** or **self-inductance of the coil**. The value of L depends on number of turns N , area of cross-section A and nature of material of the core on which coil is wound.

For N number of turns in coil, $L = \frac{N\phi}{i}$.

2. Definition using induced emf

If a current i is passed in a circuit and it is changed with a rate di/dt , then the induced emf e produced in the circuit is directly proportional to the time rate of change of

current. Thus, $e \propto \frac{di}{dt}$.

When the proportionality sign is removed, the same constant L again comes here.

Hence,
$$e = -L \frac{di}{dt}$$

The minus sign here is a reflection of Lenz's law. It says that the self-induced emf in a circuit opposes any change in the current in that circuit.

Average induced emf e is given by

$$e = -L \frac{di}{dt} = -\frac{L(i_2 - i_1)}{\Delta t}$$

From the above equation, we can write,
$$L = \left| \frac{-e}{di/dt} \right| \quad \dots(i)$$

If $\frac{di}{dt} = 1$, then $L = e$

So, when the coefficient of self-induction or self-inductance of a coil is equal to the emf induced in the coil, then time rate of change of current through the coil is unity. Inductance is a scalar quantity.

Some important points regarding coefficient of self-induction

- (i) The SI unit of L is henry.
- (ii) According to Eq. (i), we can write

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere/second}} = \frac{1 \text{ volt-second}}{\text{ampere}}$$

Self-inductance of a coil is said to be one henry (H) when a current change at the rate of 1 ampere/second through the coil induces an emf of 1 volt in the coil.

- (iii) Dimensions of self-inductance L are $[ML^2T^{-2}A^{-2}]$.

Kirchhoff's second law with an inductor

Let us consider a circuit as shown in Fig. 6.31.

According to Kirchhoff's second law (loop rule), when we go through an inductor in the same direction as the assumed current, we encounter a voltage drop equal to $L \frac{di}{dt}$, where di/dt is to be substituted with sign.

e.g. In the loop shown in figure, Kirchhoff's second law

gives the equation, $E - iR - L \frac{di}{dt} = 0$

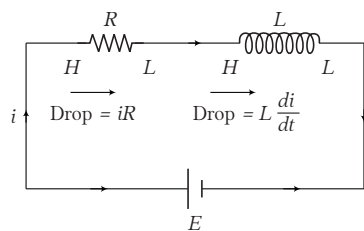
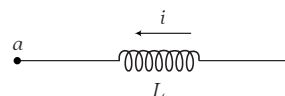


Fig. 6.32

Example 6.33 The inductor shown in figure has inductance 0.54 H and carries a current in the direction shown, which is decreasing at a uniform rate $\frac{di}{dt} = -0.03 \text{ A/s}$.



- (i) Find the back emf.
- (ii) Which end of the inductor a or b is at a higher potential?

Sol. (i) Given, inductance, $L = 0.54 \text{ H}$

and rate, $\frac{di}{dt} = -0.03$

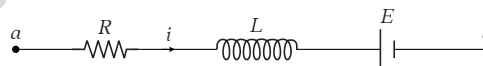
Self-induced emf (back emf) is given as

$$e = L \frac{di}{dt} = (0.54) (-0.03) \text{ V} = -1.62 \times 10^{-2} \text{ V}$$

- (ii) We know that, $V_{ba} = L \frac{di}{dt} = -1.62 \times 10^{-2} \text{ V}$

Since, $V_{ba} (= V_b - V_a)$ is negative. It implies that $V_a > V_b$ or a is at higher potential.

Example 6.34 In the circuit diagram shown in figure, $R = 10 \Omega$, $L = 5 \text{ H}$, $E = 20 \text{ V}$, $i = 2 \text{ A}$. This current is decreasing at a rate of -1.0 A/s , find V_{ab} at this instant.



Sol. Potential difference across inductor is given as

$$V_L = L \frac{di}{dt} = (5) (-1.0) = -5 \text{ V}$$

Now, using Kirchhoff's second law, $V_a - iR - V_L - E = V_b$

$$\begin{aligned} \therefore V_{ab} &= V_a - V_b = E + iR + V_L \\ &= 20 + (2)(10) - 5 = 35 \text{ V} \end{aligned}$$

Note In the above example, as the current is decreasing, the inductor can be replaced by a source of emf $e = \left| L \cdot \frac{di}{dt} \right| = 5 \text{ V}$ in such a

manner that this emf supports the decreasing current, or it sends the current in the circuit in the same direction as the existing current. So, positive terminal of this source is towards b. Thus, the given circuit can be drawn as

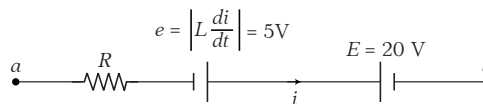
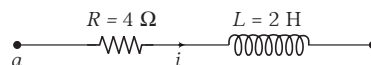


Fig. 6.33

Now, we can find V_{ab} .

Example 6.35 In the figure shown $i = 10 e^{-4t} \text{ A}$, find V_L and V_{ab} .



Sol. Potential difference across the inductor is given as

$$V_L = +L \frac{di}{dt} = (2) \frac{d}{dt} (10e^{-4t}) = -80 e^{-4t}$$

Further, by using Kirchhoff's second law, $V_a - iR - V_L = V_b$

$$\therefore V_a - V_b = iR + V_L$$

$$\text{or } V_{ab} = (10e^{-4t})(4) - 80e^{-4t} = -40e^{-4t}$$

Self-inductance of a coil

Consider a coil of radius r and current i is flowing through the coil. If number of turns in the coil is N , then magnetic field at the centre is given as

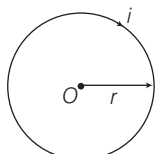


Fig. 6.34

$$\Rightarrow B_{\text{centre}} = \frac{\mu_0 Ni}{2r} \quad \text{and} \quad \phi_B = BS$$

where, S = area of cross-sectional of the coil $= \pi r^2$.

$$\text{So, } \phi_B = \frac{\mu_0 Ni}{2r} \times \pi r^2 \Rightarrow \phi_B = \frac{\mu_0 \pi N i r}{2}$$

Now, net flux for the coil of N number of turns,

$$\Rightarrow \phi = N\phi_B = \frac{N \times \mu_0 \pi N i r}{2} = \frac{\mu_0 \pi N^2 i r}{2}$$

$$\text{We know that, } L = \frac{\phi}{i} \Rightarrow L = \frac{\mu_0 N^2 \pi r}{2} \text{ henry}$$

Self-inductance of a solenoid

Let us find the inductance of a uniformly wound solenoid having N turns and length l . Assume that, l is much longer than the radius of the windings and that the core of the solenoid is air. We can assume that, the interior magnetic field due to a current i is uniform and given by equation,

$$\Rightarrow B = \mu_0 n i = \mu_0 \left(\frac{N}{l} \right) i$$



Fig. 6.35

where, $n = \frac{N}{l}$ is the number of turns per unit length.

The magnetic flux through each turn, $\phi_B = BS = \mu_0 \frac{NS}{l} i$

Here, S is the cross-sectional area of the solenoid.

$$\text{Now, as we know, } L = \frac{N\phi_B}{i} = \frac{N}{i} \left(\frac{\mu_0 NS i}{l} \right) = \frac{\mu_0 N^2 S}{l}$$

$$\Rightarrow L = \frac{\mu_0 N^2 S}{l}$$

This result shows that L depends on dimensions (S, l) and is proportional to the square of the number of turns.

$$\therefore L \propto N^2$$

Because $N = nl$, we can also express the result in the form,

$$L = \mu_0 \frac{(nl)^2}{l} S = \mu_0 n^2 S l = \mu_0 n^2 V \quad \text{or} \quad L = \mu_0 n^2 V$$

Here, $V = S l$ is the volume of the solenoid.

Note If the space inside the solenoid is filled with a material of relative permeability μ_r , then $L = \mu_0 \mu_r n^2 S l$.

Example 6.36

(i) Calculate the inductance of an air core solenoid containing 300 turns, if the length of the solenoid is 25 cm and its cross-sectional area is 4 cm^2 .

(ii) Calculate the self-induced emf in the solenoid, if the current through it is decreasing at the rate of 50 A/s.

Sol. (i) The inductance of a solenoid is given by $L = \frac{\mu_0 N^2 S}{l}$

where, S is the cross-sectional area of solenoid.

Substituting the values, we have

$$L = \frac{(4\pi \times 10^{-7}) (300)^2 (4 \times 10^{-4})}{(25 \times 10^{-2})} \text{ H}$$

$$= 1.81 \times 10^{-4} \text{ H}$$

(ii) The self-induced emf is given by $e = -L \frac{di}{dt}$

$$\text{Here, } \frac{di}{dt} = -50 \text{ A/s}^{-1}$$

$$\therefore e = - (1.81 \times 10^{-4}) (-50) = 9.05 \times 10^{-3} \text{ V}$$

or $e = 9.05 \text{ mV}$

Energy stored in an inductor

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.

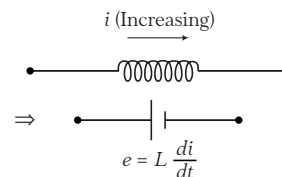


Fig. 6.36

An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power,

$$\text{thus } P = \frac{dW}{dt} = -ei = -Li \frac{di}{dt}$$

$$\text{From } dW = -dU \quad \text{or} \quad \frac{dW}{dt} = -\frac{dU}{dt}, \text{ we have}$$

$$\frac{dU}{dt} = Li \frac{di}{dt} \quad \text{or} \quad dU = Li \, di$$

The total energy U supplied while the current increases from zero to a final value i , $U = L \int_0^i i \, di = \frac{1}{2} Li^2$

$$\therefore \quad W = U = \frac{1}{2} Li^2$$

Energy stored per unit volume in magnetic field is known as **energy density**.

$$\therefore \quad \text{Energy density} = \frac{U}{V} = \frac{1}{2} \frac{B^2}{\mu_0}$$

Thus, if $i = 1 \text{ A}$, then $2W = L$.

Hence, the coefficient of self-inductance is equal to twice the work done in establishing a flow of one ampere current in the circuit.

Note Time constant in R-L circuit, $\tau = \frac{L}{R}$.

Example 6.37 What inductance would be needed to store 1 kWh of energy in a coil carrying a 200 A current? (Take, 1 kWh = $3.6 \times 10^6 \text{ J}$)

Sol. We have, $i = 200 \text{ A}$ and $U = 1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

$$\therefore \text{ Self-inductance, } L = \frac{2U}{i^2} = \frac{2(3.6 \times 10^6)}{(200)^2} = 180 \text{ H}$$

$$\left(\because U = \frac{1}{2} Li^2 \right)$$

Example 6.38 Two coils having self-inductances, $L_1 = 5 \text{ mH}$ and $L_2 = 1 \text{ mH}$. The current in the coil is increasing at same constant rate at a certain instant and the power supplied to the coils is also same. Find the ratio of

- induced voltage,
- current
- and energy stored in two coils at that instant.

Sol. Given, $L_1 = 5 \text{ mH}$ and $L_2 = 1 \text{ mH}$

$$(i) \text{ As we know, induced voltage is given by } e = \frac{L di}{dt}$$

$$\Rightarrow \quad \frac{e_1}{e_2} = \frac{L_1(di/dt)}{L_2(di/dt)} = \frac{L_1}{L_2} = \frac{5}{1} = 5:1 \quad \dots(i)$$

(ii) Power in the coil is given by $P = ei$

$$\text{Here,} \quad P_1 = P_2 \Rightarrow e_1 i_1 = e_2 i_2 \Rightarrow \frac{i_1}{i_2} = \frac{e_2}{e_1}$$

$$\text{Using Eq. (i), we can write, } \frac{i_1}{i_2} = \frac{1}{5} = 1:5$$

(iii) Energy stored in a coil is given by

$$U = \frac{1}{2} Li^2$$

$$\Rightarrow \quad \frac{U_1}{U_2} = \frac{(1/2)L_1 i_1^2}{(1/2)L_2 i_2^2} = \frac{L_1}{L_2} \left(\frac{e_2}{e_1} \right)^2 = \frac{5}{1} \left(\frac{1}{5} \right)^2 = 1:5$$

Example 6.39 Suppose a cube of volume 2 mm^3 is placed at the centre of a circular loop of radius 5 cm carrying current 2 A . Find the magnetic energy stored inside the cube.

Sol. Magnetic field at the centre of the circular loop is given by

$$B = \frac{\mu_0 i}{2R}$$

$$\text{We know that, energy density, } \mu = \frac{B^2}{2\mu_0}$$

and energy stored in the cube will be

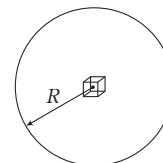
$$\text{given by} \quad U = \mu V_0 = \frac{B^2}{2\mu_0} V_0$$

where, V_0 is the volume of the cube.

Substituting the values in the above equation,

$$= \frac{1}{2\mu_0} \times \left(\frac{\mu_0 i}{2R} \right)^2 V_0 = \frac{\mu_0 i^2 V_0}{8R^2}$$

$$= \frac{4\pi \times 10^{-7} \times 2^2 \times 2 \times 10^{-9}}{8 \times (0.05)^2} = 16\pi \times 10^{-14} \text{ J}$$



Combination of self-inductances

(i) In series

If several inductances are connected in series and there is interactions through self-inductance only as shown in Fig. 6.37.

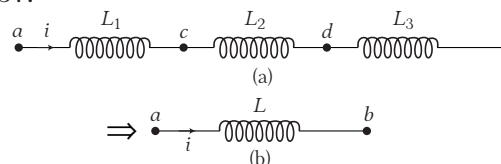


Fig. 6.37

Then, their equivalent inductance is given by

$$L = L_1 + L_2 + L_3$$

(ii) In parallel

When several inductances are connected in parallel as shown in the following Fig. 6.38.

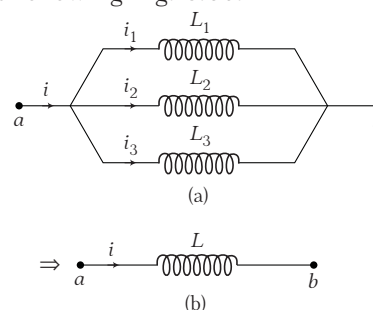


Fig. 6.38

Then, their equivalent inductance L is given by

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

CHECK POINT 6.3

- The SI unit of inductance, can be written as
 (a) weber/(ampere)² (b) volt-second/ampere
 (c) joule/(ampere)² (d) ohm-(second)²
- A long solenoid has 500 turns. When a current of 2 A is passed through it, then the resulting magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is
 (a) 1.0 H (b) 4.0 H (c) 2.5 H (d) 2.0 H
- If a current of 10 A changes in one second through a coil, and the induced emf is 10 V, then the self-inductance of the coil is
 (a) $\frac{2}{5}$ H (b) $\frac{4}{5}$ H (c) $\frac{5}{4}$ H (d) 1 H
- During a current change from 2 A to 4 A in 0.5 s, 8 V of emf is developed in a coil. The coefficient of self-induction is
 (a) 1 H (b) 2 H (c) 4 H (d) 8 H
- The current passing through a choke coil of 5 H is decreasing at the rate of 2 A/s. The emf developing across the coil is
 (a) 10 V (b) -10 V (c) 2.5 V (d) -2.5 V
- In a coil of self-inductance 0.5 H, the current varies at a constant rate from zero to 10 A in 2 s. The emf generated in the coil is
 (a) 10 V (b) 5 V (c) 2.5 V (d) 1.25 V
- Self-inductance of a coil is 50 mH. A current of 1 A passing through the coil reduces to zero at steady rate in 0.1 s, the self-induced emf is
 (a) 5 V (b) 0.05 V (c) 50 V (d) 0.5 V
- The self-inductance of a long solenoid cannot be increased by
 (a) increasing its area of cross-section
 (b) increasing its length
 (c) changing the medium with greater permeability
 (d) increasing the current through it
- The self-inductance of a coil is L . Keeping the length and area same, the number of turns in the coil is increased to four times. The self-inductance of the coil will now be
 (a) $\frac{1}{4}L$ (b) L (c) $4L$ (d) $16L$
- In circular coil, when number of turns is doubled and resistance becomes (1/4)th of initial, then inductance becomes
 (a) 4 times (b) 2 times
 (c) 8 times (d) No change
- The self-inductance of solenoid of length L , area of cross-section S and having N turns is
 (a) $\frac{\mu_0 N^2 S}{L}$ (b) $\frac{\mu_0 N S}{L}$ (c) $\mu_0 N^2 L S$ (d) $\mu_0 N L S$
- A solenoid has 2000 turns wound over a length of 0.30 m. The area of its cross-section is $1.2 \times 10^{-3} \text{ m}^2$. If an initial current of 2 A in the solenoid is reversed in 0.25 s, then the emf induced in the coil is
 (a) 6×10^{-4} V (b) 4.8×10^{-3} V
 (c) 6×10^{-2} V (d) 32.1×10^{-2} V
- A 50 mH coil carries a current of 2 A. The energy stored in joules is
 (a) 1 (b) 0.1 (c) 0.05 (d) 0.5
- In an inductor of inductance $L = 100$ mH, a current of $i = 10$ A is flowing. The energy stored in the inductor is
 (a) 5 J (b) 10 J (c) 100 J (d) 1000 J
- Two pure inductors each of self-inductance L are connected in parallel but are well separated from each other. The total inductance is
 (a) $2L$ (b) L (c) $\frac{L}{2}$ (d) $\frac{L}{4}$

MUTUAL INDUCTION

Consider two neighbouring coils of wire as shown in Fig. 6.39. A current flowing in coil 1 produces magnetic field and hence a magnetic flux through coil 2. If the current in coil 1 changes, then the flux through coil 2 changes as well. According to Faraday's law, this induces an emf in coil 2. In this way, a change in the current in one circuit can induce a current in a second circuit. This phenomenon is known as mutual induction.

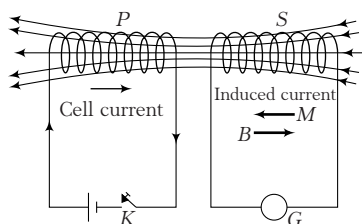


Fig. 6.39

In other words, the phenomena of the production of an electromotive force in a circuit by a change in the current in an adjacent circuit which is linked to the first by the flux lines of a magnetic field is called mutual induction.

Like the self-inductance (L) of single circuit (coil), two circuits has mutual inductance (M).

Coefficient of mutual inductance

It can be defined in two ways as discussed below

1. Definition using magnetic flux

Suppose the circuit 1 has a current i_1 flowing in it, then total flux $N_2\phi_{B_2}$ linked with circuit 2 is proportional to the current in circuit 1.

Thus, $N_2\phi_{B_2} \propto i_1$ or $N_2\phi_{B_2} = Mi_1$

Here, the proportionality constant M is known as **coefficient of mutual inductance** of the two circuits or coils.

$$\text{Thus, } M = \frac{N_2 \phi_{B_2}}{i_1} \quad \dots(i)$$

From this expression M can be defined as the total flux $N_2 \phi_{B_2}$ linked with circuit 2 per unit current in circuit 1.

2. Definition using induced emf

If we change the current in circuit 1 at a rate di_1/dt , an induced emf e_2 is developed in circuit 2, which is proportional to the rate di_1/dt .

$$\text{Thus, } e_2 \propto di_1/dt$$

$$\text{or } e_2 = -M di_1/dt \quad \dots(ii)$$

Here, the proportionality constant is again M . Minus sign indicates that e_2 is in such a direction that it opposes any change in the current in circuit 1.

$$\text{From the above equation, we can write } M = \left| \frac{-e_2}{di_1/dt} \right|.$$

If $\frac{di}{dt} = 1$, then $M = e$. Thus, coefficient of mutual induction

or mutual inductance of two coils is equal to the emf induced in one coil when rate time of change of current through the other coil is unity.

Some important points regarding the coefficient of mutual inductance

- (i) The SI unit of coefficient of mutual inductance is henry (H).

Coefficient of mutual induction or mutual inductance of two coils is said to be one henry, when a current change at the rate of one ampere/second in one coil induces an emf of one volt in the other coil.

- (ii) M depends upon closeness of the two circuits, their orientations and sizes and the number of turns, etc.

- (iii) Its dimensions are $[ML^2T^{-2}A^{-2}]$.

- (iv) **Reciprocity theorem** This theorem states that mutual inductance due to secondary coil on primary coil is equal to the mutual induction due to primary coil on secondary coil. i.e. $M_{21} = M_{12} = M$.

Using Eq. (ii),

$$e_2 = -M_{12}(di_1/dt)$$

$$\text{and } e_1 = -M_{21}(di_2/dt)$$

$$\text{Using Eq. (i), } M_{12} = \frac{N_2 \phi_{B_2}}{i_1} \quad \text{and} \quad M_{21} = \frac{N_1 \phi_{B_1}}{i_2}$$

- (v) Average induced emf, $\bar{e} = -M \frac{\Delta i}{\Delta t} = -M \frac{(i_2 - i_1)}{\Delta t}$.

- (vi) A good approach for calculating the mutual inductance of two circuits or coils consists of the following steps
- Assume anyone of the circuits as primary (first) and the other as secondary (second).
 - Suppose a current i_1 flows through the primary circuit.
 - Determine the magnetic field $\mathbf{B}$ produced by the current i_1 .
 - Obtain the magnetic flux (ϕ_{B_2}) linked with secondary circuit.
 - With the flux known, the mutual inductance can be found from

$$M = \frac{N_2 \phi_{B_2}}{i_1}$$

Mutual inductances of some important coil configurations

1. Mutual inductance for two concentric coils

Let us consider a coil having N_1 turns and of radius r is surrounded by another coil having N_2 turns and of radius R as shown in the Fig. 6.40 ($R \gg r$).

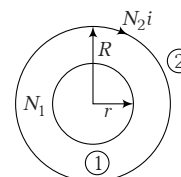


Fig. 6.40 Concentric coils

Magnetic field due to coil 2 at centre, $B_2 = \frac{\mu_0 N_2 i}{2R}$

Flux through coil 1, $\phi_{B_1} = N_1 B_2 S = \frac{N_1 \mu_0 N_2 i}{2R} \pi r^2 = Mi$

$\Rightarrow$

$$M = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}$$

2. Mutual inductance of two long co-axial solenoids

Fig. 6.41 shows two long co-axial solenoids, each of length l . Let n_1 be the number of turns per unit length of inner solenoid S_1 of radius r_1 and n_2 is number of turns per unit length of outer solenoid S_2 of radius r_2 .

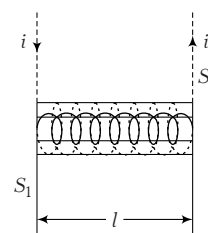


Fig. 6.41 Two co-axial solenoids

Magnetic field due to S_1 , $B_1 = \mu_0 n_1 i$

Flux through S_2 , $\phi_B = (n_2 l) B_1 \pi r_1^2$

$$= (n_2 l) \mu_0 n_1 i \pi r_1^2 = M i$$

$$\Rightarrow M = \mu_0 n_1 n_2 \pi r_1^2 l$$

$$M = \frac{\mu_0 N_1 N_2 \times S}{l}$$

where, S is the area of cross-section of inner solenoid.

Note We do not take the area of cross-section of outer solenoid because there is no magnetic field between the two solenoids.

3. Mutual inductance of a solenoid surrounded by a coil

Let N_1 be the number of turns of solenoid of radius r_1 and N_2 be the number of turns of coil of radius r_2 .

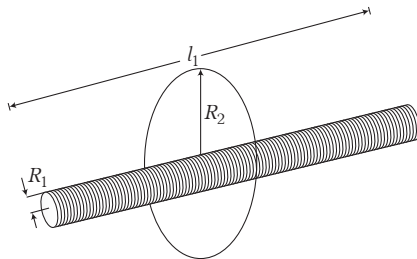


Fig. 6.42

Magnetic field due to solenoid,

$$B = \mu_0 (N_1 / l_1) i$$

Then, flux through coil,

$$\begin{aligned} \phi_B &= N_2 B S \\ &= N_2 \mu_0 \frac{N_1}{l} i \pi R_1^2 = M i \end{aligned}$$

$$\Rightarrow M = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l} \quad \left(\because M = \frac{N_2 \phi_B}{i} \right)$$

From the above relation, we see that, M is independent of radius R_2 . This is because, solenoid's magnetic field is confined to the interior coil.

Note Mutual inductance of a small circular loop and a rectangular loop ($r \ll a, b$),

$$M = \frac{\mu_0}{4\pi} \frac{8\sqrt{a^2 + b^2}}{ab} \pi r^2$$

Mutual inductance between two coils depends upon the geometry of two coils and their geometrical orientation with respect to each other. To find mutual inductance for a given arrangement, we assume a current flowing through one of the coils and find the flux through the other coil. Then, using the formula $\phi = M i$, mutual inductance M can be calculated.

Coefficient of coupling

The coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together. If L_1 and L_2 are the self-inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

It is also defined as

$$K = \frac{\text{magnetic flux linked in secondary coil}}{\text{magnetic flux linked in primary coil}}$$

where, $0 \leq K \leq 1$.

So, when $K = 0$, there will be no coupling,

$0 < K < 1$, for loose coupling

and $K = 1$, for maximum coupling.

It occurs when the two coils are wound over each other and over a ferromagnetic core.

The value of K is a number, depending on the geometry of the coils and their relative closeness.

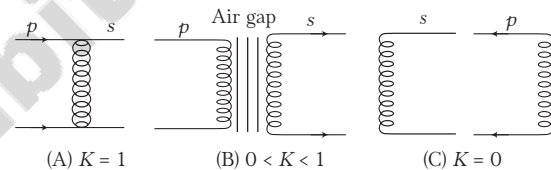


Fig. 6.43

Combination of mutual inductances

- (i) **In series** If two coils of self-inductances L_1 and L_2 having mutual inductance M are in series and situated close to each other, then net inductance will be

$$L_S = L_1 + L_2 \pm 2M$$

- (ii) **In parallel** If two coils of self-inductances L_1 and L_2 having mutual inductance M , are connected in parallel, situated close to each other, then

$$L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}$$

Example 6.40 Calculate the mutual inductance between two coils when a current $2A$ changes to $6A$ in $0.2 s$ and induces an emf of $20mV$ in the secondary coil.

Sol. We know that, emf is related to the mutual inductance by

$$e = M \frac{\Delta i}{\Delta t}$$

Substituting the given values, we get

$$20 \times 10^{-3} = \frac{M(6 - 2)}{0.2} = \frac{M \times 4}{0.2} = 20M$$

$$\Rightarrow M = 10^{-3} H = 1mH$$

Example 6.41 A straight solenoid has 50 turns per cm in primary and 200 turns in the secondary. The area of cross-section of the solenoid is 4 cm^2 . Calculate the mutual inductance.

Sol. The magnetic field at any point inside the straight solenoid of primary with n_1 turns per unit length carrying a current i_1 is given by the relation

$$B = \mu_0 n_1 i_1$$

The magnetic flux through the secondary of N_2 turns each of area S is given as

$$N_2 \phi_2 = N_2 (BS) = \mu_0 n_1 N_2 i_1 S$$

$$\therefore M = \frac{N_2 \phi_2}{i_1} = \mu_0 n_1 N_2 S$$

Substituting the values, we get

$$\begin{aligned} M &= (4\pi \times 10^{-7}) \left(\frac{50}{10^{-2}} \right) (200) (4 \times 10^{-4}) \\ &= 5.03 \times 10^{-4} \text{ H} \\ &\approx 5 \times 10^{-4} \text{ H} \end{aligned}$$

Growth and decay of current in L - R circuit

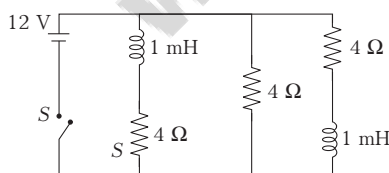
If a circuit containing a pure inductor L and a resistor R in series with a battery and a key is closed, then the current through the circuit rises exponentially and reaches upto a certain maximum value (steady state). If circuit is opened from its steady state condition, then current through the circuit decreases exponentially.

In case of growth of current at time $t = 0$, inductor offers infinite resistance and at time $t = \infty$ (steady state), inductor offers zero resistance. This type of circuit is called L - R circuit.

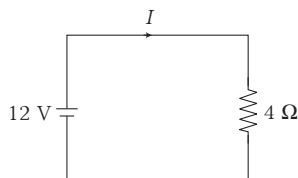
Example 6.42 Find current through the battery

(i) just after the switch is closed.

(ii) long after the switch has been closed.

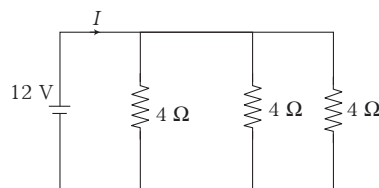


Sol. (i) Just after closing the switch, inductor offers infinite resistance, so the circuit will become as



So, the current will be $I = \frac{V}{R} = \frac{12}{4} = 3 \text{ A}$

(ii) After long time, inductor offers zero resistance, so the equivalent circuit will be



Net resistance, $R_{eq} = \frac{R}{3} = \frac{4}{3} = 1.33$

$\therefore$ Current, $I = \frac{V}{R_{eq}} = \frac{12}{1.33} \approx 9 \text{ A}$

Growth of current

Let us consider a circuit as shown in Fig. 6.43, when the switch K is closed, then the value of current at any instant of time t after closing the circuit (when current is rising) is given by

$$i = i_0 \left[1 - e^{-\frac{R}{L}t} \right] \quad \dots(i)$$

where, $i_0 = i_{\max} = \frac{E}{R}$ = steady state current at $t = \infty$

and time constant, $\tau_L = L/R$.

So, the Eq. (i) can be re-written as $i = i_0 (1 - e^{-t/\tau_L})$

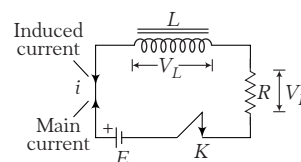


Fig. 6.44 Growth of current

Here, τ_L refers at time equal to one time constant in which the current has risen to $(1 - 1/e)$ or about 63% of its final value i_0 . The i - t graph will be shown as Fig. 6.45.

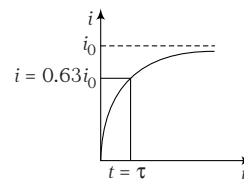


Fig. 6.45

Let us have an insight into the behaviour of an L - R circuit from energy considerations. The instantaneous rate at which the source delivers energy to the circuit ($P = Ei$) is equal to the instantaneous rate at which energy is

dissipated in the resistor ($i^2 R$) plus the rate at which energy is stored in the inductor ($i V_L = Li \frac{di}{dt}$).

Thus, $Ei = i^2 R + Li \frac{di}{dt}$

Note At any time from Fig. 6.44, $V_R + V_L = E$

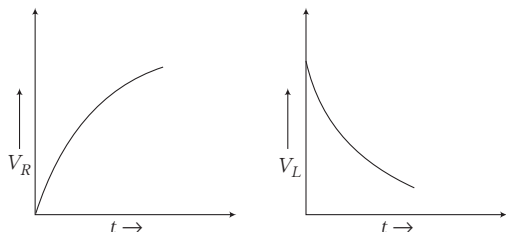


Fig. 6.46

Decay of current

Let us consider the Fig. 6.47, in which now the key K is open. Now, the current will decrease with time.

During the decaying of current, the value of current at any instant of time t after opening from the steady state

condition is given by $i = i_0 e^{-\frac{R}{L}t} = i_0 e^{-t/\tau_L}$

Here, τ_L is the time for current to decrease to $1/e$ or about 37% of its original value.

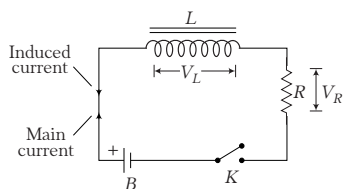


Fig. 6.47 Decay of current

The i - t graph is shown as Fig 6.48.

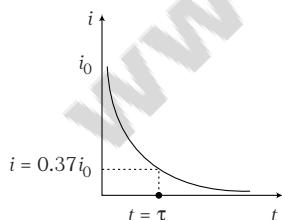


Fig. 6.48

The energy that is needed to maintain the current during this decay is provided by energy stored in the magnetic field. Thus, the rate at which energy is dissipated in the resistor = rate at which the stored energy decreases in magnetic field of inductor. This is given by

$$i^2 R = -\frac{dU}{dt} = -\frac{d}{dt} \left(\frac{1}{2} Li^2 \right) = Li \left(-\frac{di}{dt} \right)$$

or $i^2 R = Li \left(-\frac{di}{dt} \right)$

Note At any time, t , $V_R = V_L$

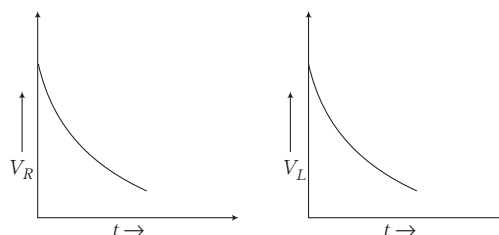


Fig. 6.49

Example 6.43 A coil of resistance 20Ω and inductance $0.5 H$ is switched to DC $200 V$ supply. Calculate the rate of increase of current

- at the instant of closing the switch and
- after one time constant.
- find the steady state current in the circuit.

Sol. (i) This is the case of growth of current in an L - R circuit.

Hence, current at time t is given by

$$i = i_0 (1 - e^{-t/\tau_L})$$

Rate of increase of current, $\frac{di}{dt} = \frac{i_0}{\tau_L} e^{-t/\tau_L}$

$$\text{At } t = 0, \quad \frac{di}{dt} = \frac{i_0}{\tau_L} = \frac{E/R}{L/R} = \frac{E}{L}$$

Substituting the value, we have

$$\frac{di}{dt} = \frac{200}{0.5} = 400 \text{ A/s}$$

$$(ii) \text{ At } t = \tau_L, \quad \frac{di}{dt} = (400)e^{-1} = (400)(0.37) = 148 \text{ A s}^{-1}$$

(iii) The steady state current in the circuit,

$$i_0 = \frac{E}{R} = \frac{200}{20} = 10 \text{ A}$$

Example 6.44 Self-inductance $0.8 \times 10^{-4} H$ of a uniformly wound solenoid, having resistance 3Ω is broken up into two identical coils. Those coils are connected in series across a $6 V$ battery of negligible resistance. Find time constant and steady state current.

Sol. For a solenoid, self-inductance is given by

$$L = \frac{\mu_0 N^2 \pi r^2}{l}$$

where, r = radius of solenoid, l = length of solenoid,

N = number of turns

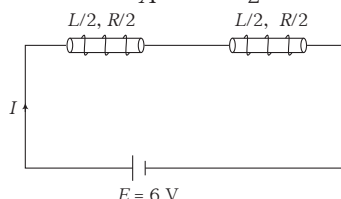
and we know that, resistance is given by

$$R = \rho \frac{l}{A} = \rho \frac{N 2 \pi r}{A}$$

So, on breaking it into two parts, now inductance and resistance is given as

$$N' = \frac{N}{2}, l' = \frac{l}{2} \Rightarrow L' = \frac{\mu_0 (N/2)^2 \pi r^2}{l/2} = \frac{L}{2}$$

$$R' = \frac{\rho (N/2) \times 2\pi r}{A} = \frac{R}{2}$$



So, the equivalent inductance, $L_{eq} = L_1 + L_2$

$$= L/2 + L/2 = L$$

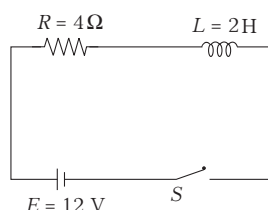
and equivalent resistance, $R_{eq} = R_1 + R_2 = R/2 + R/2 = R$

So, the time constant,

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{0.8 \times 10^{-4}}{3} = 0.267 \times 10^{-4} \text{ s}$$

$$\text{and current, } I = \frac{E}{R_{eq}} = \frac{E}{R} = \frac{6}{3} = 2 \text{ A}$$

Example 6.45



At $t = 0$, switch S is closed, calculate

- initial rate of increase of current, i.e. $\frac{di}{dt}$ at $t = 0$,
- $\frac{di}{dt}$ at time when current in the circuit is 0.5 A ,
- current at $t = 0.6 \text{ s}$,
- rate at which energy of magnetic field is increasing, rate of heat produced in resistance and rate at which energy is supplied by battery when $i = 0.5 \text{ A}$
- and energy stored in inductor in steady state.

Sol. We know that,

$$E = iR + L \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{E - iR}{L}$$

Current in the circuit at any time t ,

$$i = i_0 (1 - e^{-t/\tau_L})$$

where, $i_0 = E/R$ and $\tau_L = L/R$

- Rate of increase of current $\frac{di}{dt} = \frac{E - iR}{L}$, when $t = 0, i = 0$

$$\left. \frac{di}{dt} \right|_{t=0} = \frac{E}{L} = \frac{12}{2} = 6 \text{ A/s}$$

- At $i = 0.5 \text{ A}$, $\frac{di}{dt} = \frac{E - iR}{L} = \frac{12 - 0.5 \times 4}{2} = \frac{12 - 2}{2} = 5 \text{ A/s}$

$$\text{(iii) At } t = 0.6 \text{ s, current } i = \frac{E}{R} (1 - e^{-\frac{Rt}{L}})$$

$$= \frac{12}{4} (1 - e^{-\frac{4 \times 0.6}{2}}) = 3(1 - e^{-1.2}) \text{ A}$$

- Energy stored in inductor in steady state is given by

$$U = \frac{1}{2} Li^2$$

$$\Rightarrow \frac{dU}{dt} = Li \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{E - iR}{L} = \frac{12 - 0.5 \times 4}{2} = 5 \text{ A/s}$$

$$\frac{dU}{dt} = 2 \times 0.5 \times 5 = 5 \text{ J/s}$$

Power produced per second, $P = i^2 R = (0.5)^2 \times 4 = 1 \text{ J/s}$

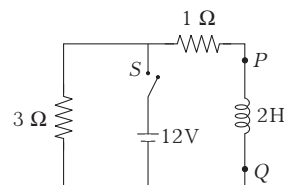
Power supplied by battery $= Ei = 0.5 \times 12 = 6 \text{ J/s}$

- We have, $i_0 = E/R = 12/4 = 3 \text{ A}$

Energy stored in inductor in steady state,

$$U = \frac{1}{2} Li_0^2 = \frac{1}{2} \times 2 \times (3)^2 = 9 \text{ J/s}$$

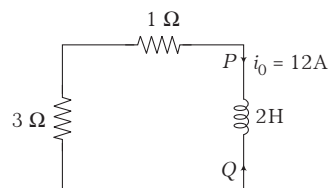
Example 6.46



The switch is closed for a long time and then opened at time $t = 0$. Find the initial voltage across L after $t = 0$, which end is at higher potential P or Q ?

Sol. Initially, current will be $i_0 = \frac{12}{1} = 12 \text{ A}$

So, the above circuit would be now as



Current at any time is given by $i = i_0 e^{-t/\tau_L}$ and $i_0 = 12 \text{ A}$

$$\therefore \tau_L = L/R = \frac{2}{(1+3)} = \frac{2}{4} = 0.5$$

Current, $i = 12e^{-t/0.5}$

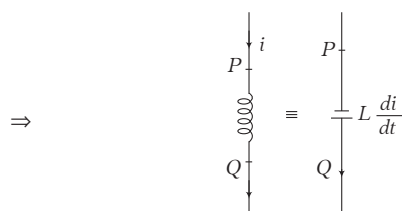
$$\Rightarrow \frac{di}{dt} = 12e^{-t/0.5} \left(-\frac{1}{0.5} \right)$$

$$\Rightarrow \left| \frac{di}{dt} \right| = 24e^{-2t}$$

$$\text{So, } V_L = L \frac{di}{dt} = 48e^{-2t}$$

$$\text{At, } t = 0, V_L = 48 \text{ V}$$

i.e. current is decreasing,



$$\Rightarrow V_Q > V_P$$

Applications of EMI: eddy current

When a changing magnetic flux is applied to a bulk piece of conducting material, then circulating currents induced in the body of the conductor, are called eddy currents. As the resistance of the bulk conductor is usually low, so the eddy currents often have large magnitudes and heat up the conductor.

Eddy currents are always produced in a plane perpendicular to the direction of magnetic field. This current shows both heating and magnetic effects.

The magnitude of eddy current is given by

$$i = \frac{\text{induced emf}}{\text{resistance}} = \frac{e}{R}$$

According to Faraday's law,

$$e = -\frac{d\phi}{dt}, \quad i = -\frac{d\phi/dt}{R}$$

The direction of eddy currents can be given by Lenz's law or by Fleming's right hand rule.

However, their flow patterns resemble swirling eddies in water. That is why, they are called eddy currents. These were discovered by Foucault in the year 1895 and hence, they are also named as **Foucault current**. *e.g.* When we move a metal plate out of a magnetic field, then the relative motion of the field and the conductor again induces a current in the conductor. The conducting electrons building up the induced current whirl around within the plate as, if they were caught in an eddy of water.

Effects of eddy current

Eddy currents are produced inside the iron cores of the rotating armatures of electric motors and dynamos and also in the cores of transformers, which experience flux changes,

when they are in use. The currents causes unnecessary heating and wastage of power and the heat produced may even damage the insulation of coils.

Methods to reduce eddy current

Eddy currents are minimised by using laminations of metal to make a metal core. The laminations are separated by an insulating material. The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths. This arrangement reduces the strength of eddy currents.

Applications of eddy current

Eddy currents are useful in many ways. Some of the important applications of eddy currents are given below

(i) Electromagnetic damping

In order to bring the moving coil of a galvanometer immediately to rest, we make the use of electromagnetic damping which uses eddy currents to bring the coil to rest. When the coil oscillates, then the eddy currents generated in the core oppose the motion and bring the coil to rest.

(ii) Induction furnace

In an induction furnace, high temperature can be produced by using eddy currents. We generally use induction furnace in preparation of alloys by melting the constituents of metal.

A coil is wound over the metal which needs to be melted and through the coil, we pass high frequency alternating current. The eddy current generated in the metal produces high temperature to melt the metal.

(iii) Electric power meter

Old electric power meters (analog type) had a metallic disc. The disc rotates due to generation of eddy currents which are produced due to sinusoidally varying current in the coil.

(iv) Magnetic braking in electric trains

Some electric powered trains make use of strong electromagnets which are situated above the rails. These electromagnets are used to produce eddy current in the rails which oppose the motion of the train and thus stop it. In this case, as there is no mechanical linkage, hence the braking effect is smooth.

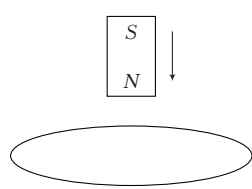
CHECK POINT 6.4

- Two coils X and Y are placed in a circuit such that when the current changes by 2 A in coil X . The magnetic flux changes by 0.4 Wb in Y . The value of mutual inductance of the coils is
 (a) 0.2 H (b) 5 H
 (c) 0.8 H (d) 4 H
- Two coils have a mutual inductance of 0.005 H. The current changes in the first coil according to equation $i = i_0 \sin \omega t$, where $i_0 = 10$ A and $\omega = 100\pi \text{ rads}^{-1}$. The maximum value of emf in the second coil is (in volt)
 (a) 2π (b) 5π
 (c) π (d) 4π
- The mutual inductance between a primary and secondary circuits is 0.5 H. The resistance of the primary and the secondary circuits are 20Ω and 5Ω , respectively. To generate a current of 0.4 A in the secondary, current in the primary must be changed at the rate of
 (a) 4.0 As^{-1} (b) 1.6 As^{-1} (c) 16.0 As^{-1} (d) 8.0 As^{-1}
- The coefficient of mutual induction between two circuits is equal to the emf produced in one circuit, when the current in the second circuit is
 (a) kept steady at 1 A
 (b) cut-off at 1 A level
 (c) changed at the rate of 1 As^{-1}
 (d) changed from 1 As^{-1} to 2 As^{-1}
- Two circuits have coefficient of mutual induction of 0.09 H. Average emf induced in the secondary by a change of current from 0 to 20 A in 0.006 s in the primary will be
 (a) 120 V (b) 80 V (c) 200 V (d) 300 V
- A current is varying at the rate of 3 A/s in a coil generates an emf of 8 mV in a nearby coil. The mutual inductance of the two coils is
 (a) 2.66 mH (b) $2.66 \times 10^{-3} \text{ mH}$
 (c) 2.66 H (d) 0.266 H
- A solenoid is placed inside another solenoid, the length of both being equal carrying same magnitude of current. The parameters like radius and number of turns are in the ratio 1: 2 for the two solenoids. The mutual inductance on each other would be
 (a) $M_{12} = M_{21}$ (b) $M_{12} = 2M_{21}$ (c) $2M_{12} = M_{21}$ (d) $M_{12} = 4M_{21}$
- Two coils of self-inductances L_1 and L_2 are placed closer to each other, so that total flux in one coil is completely linked with other. If M is mutual inductance between them, then
 (a) $M = L_1 L_2$ (b) $M = L_1 / L_2$
 (c) $M = \sqrt{L_1 L_2}$ (d) $M = (L_1 L_2)^2$
- Two coils of self-inductances 2 mH and 8 mH are placed, so close together that the effective flux in one coil is completely linked with the other. The mutual inductance between these coils is
 (a) 4 mH (b) 16 mH
 (c) 10 mH (d) 6 mH
- An ideal coil of 10 H is joined in series with a resistance of 5Ω and a battery of 5 V. After 2s of joining, the current flowing in ampere in the circuit will be
 (a) e^{-1} (b) $(1 - e^{-1})$ (c) $(1 - e)$ (d) e
- An L - R circuit has a cell of emf E , which is switched ON at time $t = 0$. The current in the circuit after a long time will be
 (a) zero (b) $\frac{E}{R}$ (c) $\frac{E}{L}$ (d) $\frac{E}{\sqrt{L^2 + R^2}}$
- During current growth in an L - R circuit, the time constant is the time in which the magnitude of current becomes
 (a) I_0 (b) $I_0/2$ (c) $0.63 I_0$ (d) $0.37 I_0$
- An L - R circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the connection?
 (a) Current in the circuit
 (b) Magnetic field energy
 (c) Power delivered by the battery
 (d) Emf induced in the inductor
- Eddy currents are produced when
 (a) a metal is kept in varying magnetic field
 (b) a metal is kept in a steady magnetic field
 (c) a circular coil is placed in a magnetic field
 (d) through a circular coil current is passed
- Which of the following is not an application of eddy currents?
 (a) Induction furnace (b) Galvanometer damping
 (c) Speedometer of automobiles (d) X-ray crystallography

Chapter Exercises

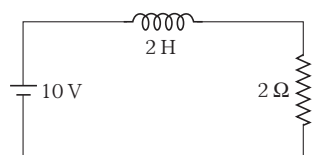
(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1 If L , C and R represent the physical quantities inductance, capacitance and resistance, respectively. Which of the following combination does not have the dimensions of frequency?
 (a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$
- 2 The self-inductance L of a solenoid of length l and area of cross-section A , with a fixed number of turns N increases as [NCERT Exemplar]
 (a) Both l and A increase
 (b) l decreases and A increases
 (c) l increases and A decreases
 (d) Both l and A decrease
- 3 In electromagnetic induction, the induced charge in a coil is independent of
 (a) change in the flux
 (b) time taken to change the flux
 (c) resistance in the circuit
 (d) None of the above
4. A coil and a bulb are connected in series with a DC source, a soft iron core is then inserted in the coil, then
 (a) intensity of the bulb remains the same
 (b) intensity of the bulb decreases
 (c) intensity of the bulb increases
 (d) the bulb ceases to glow
- 5 A magnet is brought towards a coil (i) speedily (ii) slowly, then the induced emf/induced charge will be respectively
 (a) more in first case/more in first case
 (b) more in first case/equal in both case
 (c) less in first case/more in second case
 (d) less in first case/equal in both case
- 6 In a coil of area 10 cm^2 and 10 turns, a magnetic field is directed perpendicular to the plane and is changing at the rate of 10^8 gauss/s . The resistance of the coil is 20Ω . The current in the coil will be
 (a) 5 A (b) 0.5 A
 (c) 0.05 A (d) $5 \times 10^8 \text{ A}$
- 7 Two identical circular loops of metal wires are lying on a table without touching each other. Loop A carries a current which increases with time. In response, the loop B
 (a) remains stationary
 (b) is attracted by loop A
 (c) is repelled by loop A
 (d) rotates about its centre of mass with CM fixed
- 8 A coil having 500 square loops each of side 10 cm is placed normal to a magnetic flux which increases at the rate of 1.0 Ts^{-1} . The induced emf in volts is
 (a) 0.1 (b) 0.5 (c) 1 (d) 5
- 9 A coil having an area 2 m^2 is placed in a magnetic field which changes from 1 Wbm^{-2} to 4 Wbm^{-2} in an interval of 2 s. The emf induced in the coil will be
 (a) 4 V (b) 3 V (c) 1.5 V (d) 2 V
- 10 The circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature. During this period, the two loops
 (a) attract each other
 (b) repel each other
 (c) do not exert any force on each other
 (d) attract or repel each other depending on the sense of the current
- 11 A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, then the current induced in the loop is
 (a) clockwise
 (b) anti-clockwise
 (c) zero
 (d) clockwise or anti-clockwise depending on whether the resistance is increased or decreased
- 12 The north pole of a magnet is brought near a metallic ring as shown in the figure. The direction of induced current in the ring will be


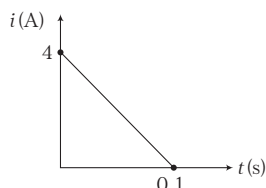
- (a) anti-clockwise
 (b) clockwise
 (c) first anti-clockwise and then clockwise
 (d) first clockwise and then anti-clockwise

13 In the figure, magnetic energy stored in the coil is

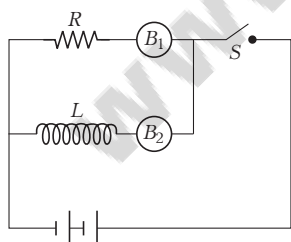


- (a) zero (b) infinite
 (c) 25 J (d) None of these
- 14** An emf of 15 V is applied in a circuit coil containing 5 H inductance and 10 Ω resistance. The ratio of the currents at time $t = \infty$ and $t = 1$ s is
- (a) $\frac{e^{1/2}}{e^{1/2} - 1}$ (b) $\frac{e^2}{e^2 - 1}$ (c) $1 - e^{-1}$ (d) e^{-1}

15 Some magnetic flux is changed from a coil of resistance 10 Ω. As a result an induced current is developed in it, which varies with time as shown in figure. The magnitude of change in flux through the coil in Wb is



- (a) 2 (b) 4
 (c) 6 (d) None of these
- 16** Figure shows two bulbs B_1 and B_2 , resistor R and inductor L . When the switch S is turned OFF



- (a) Both B_1 and B_2 die out promptly
 (b) Both B_1 and B_2 die out with some delay
 (c) B_2 dies out promptly but B_1 with some delay
 (d) B_1 dies out promptly but B_2 with some delay
- 17** A coil of 40 Ω resistance, 100 turns and radius 6 mm is connected to ammeter of resistance 160 Ω. Coil is placed perpendicular to the magnetic field. When coil is taken out of the field, 32 μC charge flows through it. The intensity of magnetic field will be
- (a) 6.55 T (b) 5.66 T (c) 2.55 T (d) 0.566 T

18 A coil has an area of 0.05 m² and it has 800 turns. It is placed perpendicularly in a magnetic field of strength 4×10^{-5} Wb m⁻², and is rotated through 90° in 0.1 s.

The average emf induced in the coil is

- (a) 0.056 V (b) 0.046 V
 (c) 0.026 V (d) 0.016 V

19 In a magnetic field of 0.05 T, area of a coil changes from 101 cm² to 100 cm² without changing the resistance which is 2 Ω. The amount of charge that flows during this period is

- (a) 2.5×10^{-6} C (b) 2×10^{-6} C
 (c) 10^{-6} C (d) 8×10^{-6} C

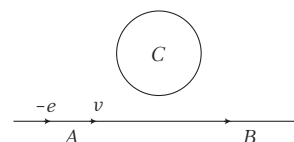
20 The resistance and inductance of series circuit are 5 Ω and 20 H, respectively. At the instant of closing the switch, the current is increasing at the rate 4 As⁻¹. The supply voltage is

- (a) 20 V (b) 80 V
 (c) 120 V (d) 100 V

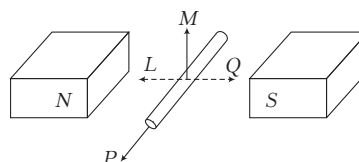
21 The number of turns in the coil of an AC generator is 5000 and the area of the coil is 0.25 m². The coil is rotated at the rate of 100 cycles/s in a magnetic field of 0.2 Wb m⁻². The peak value of the emf generated is nearly

- (a) 786 kV (b) 440 kV
 (c) 220 kV (d) 157 kV

22 An electron moves along the line AB which lies in the same plane as a circular loop of conducting wire as shown in figure. What will be the direction of the current induced (if any) in the loop?



- (a) No current will be induced
 (b) The current will be clockwise
 (c) The current will be anti-clockwise
 (d) The current will change direction as the electron passes through
- 23** An electric potential difference will be induced between ends of the conductor shown in the diagram, when the conductor moves in the direction

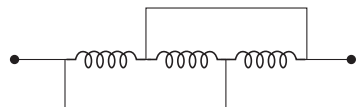


- (a) P (b) Q
 (c) L (d) M

- 24** A long horizontal metallic rod with length along the east-west direction is falling under gravity. The potential difference between its two ends will

(a) be zero
(b) be constant
(c) increase with time
(d) decrease with time

- 25** All the three inductors have inductance 3.0 H. The equivalent inductance of the circuit is



(a) 1 H
(b) 2 H
(c) 3 H
(d) 9 H

- 26** Two inductances connected in parallel are equivalent to a single inductance of 1.5 H and when connected in series are equivalent to a single inductance of 8 H. The difference in their inductance is

(a) 3 H (b) 7.5 H (c) 2 H (d) 4 H

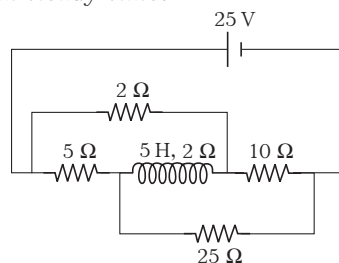
- 27** An inductance L and a resistance R are first connected to a battery. After some time, the battery is disconnected but L and R remain connected in a closed circuit. Then, the current reduces to 37% of its initial value in

(a) RL second (b) $\frac{R}{L}$ second
(c) $\frac{L}{R}$ second (d) $\frac{1}{LR}$ second

- 28** The time constant of an inductance coil is 2.0×10^{-3} s. When a 90Ω resistance is joined in series, then the time constant becomes 0.5×10^{-3} s. The inductance and resistance of the coil are

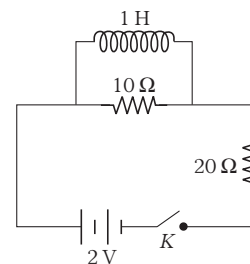
(a) 30 mH, 30Ω (b) 30 mH, 60Ω
(c) 60 mH, 30Ω (d) 60 mH, 60Ω

- 29** In the circuit shown, what is the energy stored in the coil at steady state?



(a) 21.3 J (b) 42.6 J
(c) Zero (d) 213 J

- 30** In the following figure, what is the final value of current in the 10Ω resistor when the plug of key K is inserted?



(a) $(3/10)$ A (b) $(3/20)$ A (c) $(3/11)$ A (d) Zero

- 31** A square loop of side l , resistance R is placed in a uniform magnetic field B acting normally to the plane of the loop. If we attempt to pull it out of the field with a constant velocity v , then the power needed is

(a) $BRlv$ (b) $\frac{B^2 l^2 v^2}{R}$ (c) $\frac{Bl^2 v^2}{R}$ (d) $\frac{Bvl}{R}$

- 32** A square of side L metres lies in the XY -plane in a region, where the magnetic field is given by $\mathbf{B} = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})$ T, where B_0 is constant. The magnitude of flux passing through the square is

[NCERT Exemplar]

(a) $2B_0L^2$ Wb (b) $3B_0L^2$ Wb (c) $4B_0L^2$ Wb (d) $\sqrt{29}B_0L^2$ Wb

- 33** A conducting loop of area 5.0 cm^2 is placed in a magnetic field which varies sinusoidally with time as $B = 0.2 \sin 300 t$. The normal to the coil makes an angle of 60° with the field. The emf induced at $t = (\pi / 900) \text{ s}$

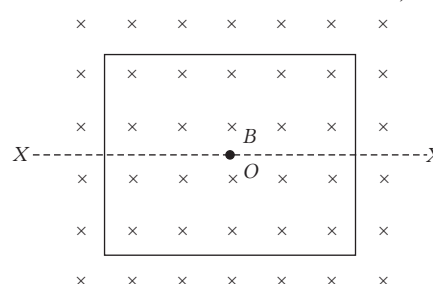
(a) 7.5×10^{-3} V (b) zero
(c) 15×10^{-3} V (d) 20×10^{-3} V

- 34** An infinitely long cylinder is kept parallel to a uniform magnetic field $\mathbf{B}$ directed along positive Z -axis. The direction of induced current as seen from the Z -axis will be

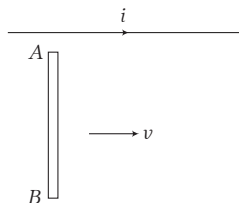
(a) clockwise of the positive Z -axis
(b) anti positive Z -axis clockwise of the positive z -axis
(c) zero
(d) along the magnetic field

- 35** A rectangular coil is placed in a region having a uniform magnetic field B perpendicular to the plane of the coil.

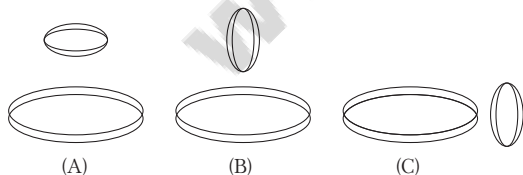
An emf will not be induced in the coil, if the



- (a) magnetic field is increased uniformly
 (b) magnetic field is switched OFF
 (c) coil is rotated about an axis XX'
 (d) coil is rotated about an axis perpendicular to the plane of the coil and passing through its centre O
- 36** If a coil of 40 turns and area 4.0 cm^2 is suddenly removed from a magnetic field, then it is observed that a charge of $2.0 \times 10^{-4} \text{ C}$ flows into the coil. If the resistance of the coil is 80Ω , then the magnetic flux density in Wb/m^2 is
- (a) 0.5 (b) 1.0
 (c) 1.5 (d) 2.0
- 37** A coil of wire of a certain radius has 600 turns and a self-inductance of 108 mH. The self-inductance of a 2nd similar coil of 500 turns will be
- (a) 74 mH (b) 75 mH
 (c) 76 mH (d) 77 mH
- 38** The current carrying wire and the rod AB are in the same plane. The rod moves parallel to the wire with a velocity v . Which one of the following statements is true about induced emf in the rod?



- (a) End A will be at lower potential with respect to B .
 (b) A and B will be at the same potential.
 (c) There will be no induced emf in the rod.
 (d) Potential at A will be higher than that at B .
- 39** Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be

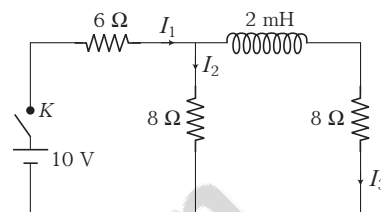


- (a) maximum in situation (A)
 (b) maximum in situation (B)
 (c) maximum in situation (C)
 (d) the same in all situations
- 40** A coil of inductance 300 mH and resistance 2Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in
- (a) 0.15 s (b) 0.3 s
 (c) 0.05 s (d) 0.1 s

- 41** An inductor of 2 H and a resistance of 10Ω are connected in series with a battery of 5 V. The initial rate of change in current is

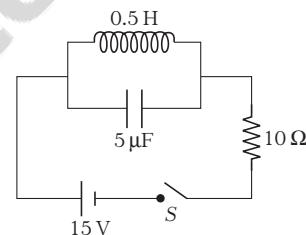
(a) 0.5 As^{-1} (b) 2.0 As^{-1}
 (c) 2.5 As^{-1} (d) 0.25 As^{-1}

- 42** In the circuit shown in the figure, what is the value of I_1 just after pressing the key K ?



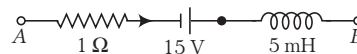
(a) $(5/7) \text{ A}$ (b) $(5/11) \text{ A}$
 (c) 1 A (d) None of these

- 43** For in the circuit shown in figure, current through the battery at $t = 0$ and $t = \infty$ is



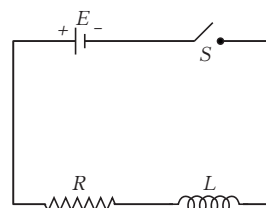
(a) 1.5 A, 1.5 A (b) 0, 0
 (c) 1, 1.5 A (d) 1.5 A, 0

- 44** The network shown in the figure is a part of a complete circuit. If at a certain instant the current i is 5 A and decreasing at the rate of 10^3 As^{-1} , then $V_B - V_A$ is

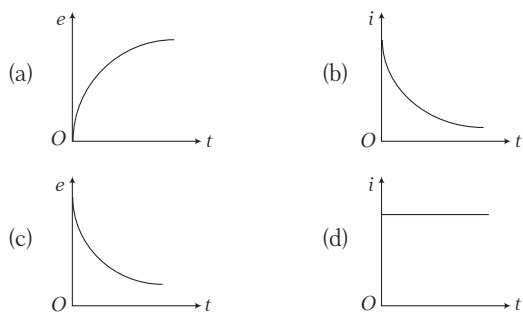


(a) 5 V (b) 10 V
 (c) 15 V (d) 20 V

- 45.** Switch S of the circuit shown in figure is closed at $t = 0$.



If emf in L is e and i is the current flowing through the circuit at time t , which of the following graphs is correct?



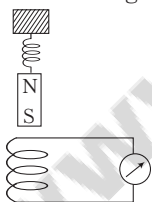
46 Coefficient of coupling between two coils of self-inductances L_1 and L_2 is unity. It means

- (a) 50% flux of L_1 is linked with L_2
- (b) 100% flux of L_1 is linked with L_2
- (c) $\sqrt{L_1}$ time of flux of L_1 is linked with L_2
- (d) None of the above

47 A rectangular, a square, a circular and an elliptical loop, all in the XY -plane, are moving out of a uniform magnetic field with a constant velocity $\mathbf{v} = u\hat{\mathbf{i}}$. The magnetic field is directed along the negative Z -axis direction. The induced emf during the passage of these loops, out of the field region, will not remain constant for

- (a) the rectangular, circular and elliptical loops
- (b) the circular and the elliptical loops
- (c) only the elliptical loop
- (d) any of the four loops

48 A magnet N - S is suspended from a spring and while it oscillates, the magnet moves in and out of the coil C . The coil is connected to a galvanometer G .



Then, as the magnet oscillates,

- (a) G shows deflection to the left and right with constant amplitude
- (b) G shows deflection on one side
- (c) G shows no deflection
- (d) G shows deflection to the left and right but the amplitude steadily decreases

49 A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating, then it is very difficult to stop. But, if an aluminium plate is placed near to the coil, then it stops. This is due to

- (a) development of air current when the plate is placed
- (b) induction of electrical charge on the plate

- (c) shielding of magnetic lines of force as aluminium is a paramagnetic material
- (d) electromagnetic induction in the aluminium plate give rise to electromagnetic damping

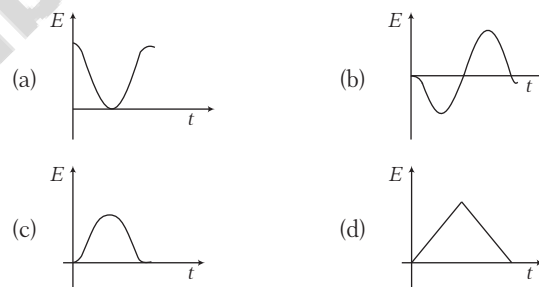
50 Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If then $R_1 \gg R_2$, then the mutual inductance M between them will be directly proportional to

- (a) R_1 / R_2
- (b) R_2 / R_1
- (c) R_1^2 / R_2
- (d) R_2^2 / R_1

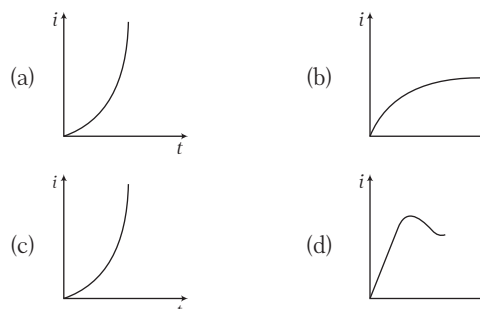
51 A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, then the electrical power dissipated would be

- (a) halved
- (b) the same
- (c) doubled
- (d) quadrupled

52 The variation of induced emf (E) with time (t) in a coil, if a short bar magnet is moved along its axis with a constant velocity is best represented as

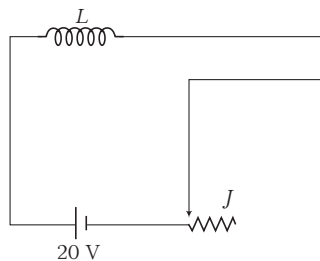


53 When a battery is connected across a series combination of self-inductance L and resistance R , then the variation in the current i with time t is best represented by



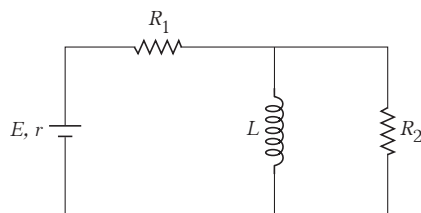
54 In the circuit shown in the figure, the jockey J is being pulled towards right, so that the resistance in the circuit is increasing. Its a value at some instant is $5\ \Omega$.

The current in the circuit at this instant will be



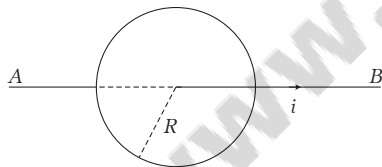
- (a) 4 A
(b) less than 4 A
(c) more than 4 A
(d) may be less than or more than 4 A depending on the value of L

55 The value of time constant for the given circuit is



- (a) $\frac{L}{R_1 + r + R_2}$
(b) $\frac{L}{(R_1 + r)}$
(c) $\frac{L(R_1 + R_2 + r)}{(R_1 + r)R_2}$
(d) None of these

56 An infinitely long conductor AB lies along the axis of a circular loop of radius R . If the current in the conductor AB varies at the rate of x ampere/second, then the induced emf in the loop is



- (a) $\frac{\mu_0 x R}{2}$
(b) $\frac{\mu_0 x R}{4}$
(c) $\frac{\mu_0 \pi x R}{2}$
(d) zero

57 A physicist works in a laboratory where the magnetic field is 2 T. She wears a necklace of enclosing area 0.01 m^2 in such a way that the plane of the necklace is normal to the field and is having a resistance $R = 0.01 \Omega$. Because of power failure, the field decays to 1 T in time 10^{-3} s . Then, what is the total heat produced in her necklace?

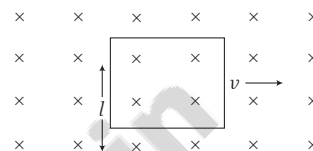
- (a) 10 J
(b) 20 J
(c) 30 J
(d) 40 J

58 An aeroplane in which the distance between the tips of the wings is 50 m is flying horizontally from east to west with the speed of 360 kmh^{-1} over a place where the vertical component of the earth's

magnetic field is $2.0 \times 10^{-4} \text{ Wbm}^{-2}$. The potential difference between the tips of the wings would be

- (a) 0.1 V
(b) 0.01 V
(c) 0.2 V
(d) 1.0 V

59 A conducting square loop of side l and resistance R is moving out of the plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop as shown in figure.



The current induced in the loop is

- (a) $\frac{Blv}{R}$ clockwise
(b) $\frac{Blv}{R}$ anti-clockwise
(c) $\frac{2Blv}{R}$ anti-clockwise
(d) zero

60 Two rails of a railway track, insulated from each other and the ground, are connected to a millivoltmetre. What is the reading of the millivoltmetre when a train travels at a speed of 20 ms^{-1} along the track? Given that, the vertical component of earth's magnetic field is $0.2 \times 10^{-4} \text{ Wbm}^{-2}$ and the rails are separated by 1 m.

- (a) 4 mV
(b) 0.4 mV
(c) 80 mV
(d) 10 mV

61 A pair of coils of turns n_1 and n_2 are kept close together. Current passing through the first is reduced at the rate r and emf 3 mV is developed across the other coil. If the second coil carries current which is then reduced at the rate $2r$, then the emf produced across the first coil will be

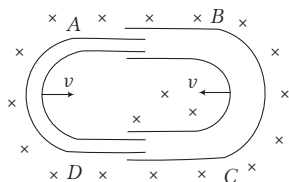
- (a) $\frac{6n_1}{n_2} \text{ mV}$
(b) $\frac{6n_2}{n_1} \text{ mV}$
(c) 6 mV
(d) $3/2 \text{ mV}$

62 A loop made of straight edges has six corners at $A(0, 0, 0)$, $B(L, 0, 0)$, $C(L, L, 0)$, $D(0, L, 0)$, $E(0, L, L)$ and $F(0, 0, L)$. A magnetic field $B = B_0(\hat{i} + \hat{k})$ T is present in the region. The flux passing through the loop $ABCDEF$ (in that order) is [NCERT Exemplar]

- (a) $B_0 L^2 \text{ Wb}$
(b) $2B_0 L^2 \text{ Wb}$
(c) $\sqrt{2}B_0 L^2 \text{ Wb}$
(d) $4B_0 L^2 \text{ Wb}$

63 One conducting U-tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v , then

the emf induced in the circuit in terms of B , l and v , where l is the width of each tube, will be

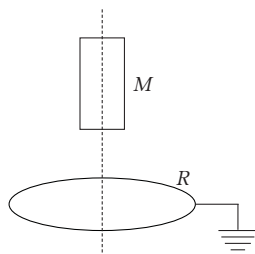


- (a) zero (b) $2Blv$ (c) Blv (d) $-Blv$

- 64** A magnetic field given by $B(t) = 0.2t - 0.05t^2$ tesla is directed perpendicular to the plane of a circular coil containing 25 turns of radius 1.8 cm and whose total resistance is 1.5Ω . The power dissipation at 3 s is approximately

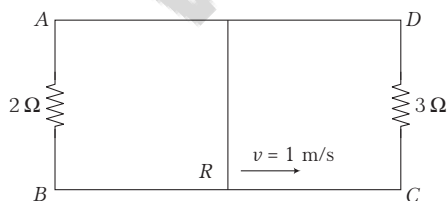
- (a) $1.37 \mu\text{W}$ (b) $7 \mu\text{W}$ (c) zero (d) $4 \mu\text{W}$

- 65** A small magnet M is allowed to fall through a fixed horizontal conducting ring R . Let g be the acceleration due to gravity. The acceleration of M , a will be



- (a) $< g$ when it is above R and moving towards R
 (b) $> g$ when it is above R and moving towards R
 (c) $> g$ when it is below R and moving away from R
 (d) None of the above

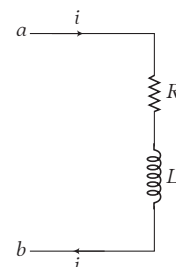
- 66** A rectangular loop with a sliding connector of length 10 cm is situated in uniform magnetic field perpendicular to plane of loop. The magnetic induction is 0.1 T and resistance of connector (R) is 1Ω . The sides AB and CD have resistances 2Ω and 3Ω , respectively. Find the current in the connector during its motion with constant velocity 1 ms^{-1} .



- (a) $\frac{1}{242} \text{ A}$ (b) $\frac{1}{220} \text{ A}$ (c) $\frac{1}{55} \text{ A}$ (d) $\frac{1}{440} \text{ A}$

- 67** When the current in the portion of the circuit shown in the figure is 2 A and increasing at the rate of 1 As^{-1} , then the measured potential difference $V_{ab} = 8 \text{ V}$. However, when the current is 2 A and decreasing at the rate of 1 As^{-1} , then the measured potential difference $V_{ab} = 4 \text{ V}$.

The values of R and L are

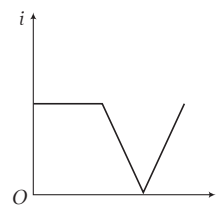


- (a) 3Ω and 2 H respectively
 (b) 3Ω and 3 H respectively
 (c) 2Ω and 1 H respectively
 (d) 3Ω and 1 H respectively

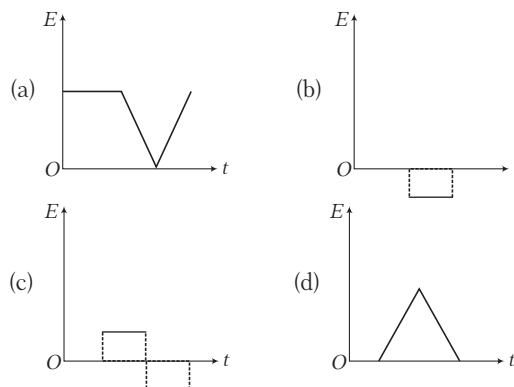
- 68** Two different coils have self-inductances $L_1 = 8 \text{ mH}$ and $L_2 = 2 \text{ mH}$. The current in one coil is increased at a constant rate. The current in second coil is also increased at the same constant rate. At certain instant of time, the power given to the coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1 , V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2 , V_2 and W_2 , respectively. Then, choose the wrong option.

- (a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$
 (c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_2}{V_1} = \frac{1}{4}$

- 69** The current i in an induction coil varies with time t according to the graph shown in figure.



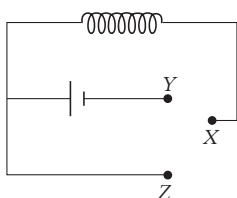
Which of the following graphs shows the induced emf (E) in the coil with time?



- 70** A coil of inductance $L = 50\mu\text{H}$ and resistance $= 0.5\ \Omega$ is connected to a battery of emf $= 5\text{ V}$. A resistance of $10\ \Omega$ is connected parallel to the coil. Now, at same instant the connection of the battery is switched OFF. Then, the amount of heat generated in the coil after switching OFF the battery is

- (a) 1.25 mJ (b) 2.5 mJ
(c) 0.65 mJ (d) 0.12 mJ

- 71** In the circuit shown, the coil has inductance and resistance. When X is joined to Y , the time constant is τ during growth of current. When the steady state is reached, then heat is produced in the coil at a rate P . If X is now joined to Z , then choose the correct statement.

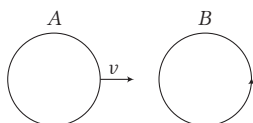


- (a) The total heat produced in the coil is $P\tau$.
(b) The total heat produced in the coil is $\frac{1}{2}P\tau$.
(c) The total heat produced in the coil is $2P\tau$.
(d) The data given is not sufficient to reach a conclusion.

- 72** A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through an ammeter A . Then,
[NCERT Exemplar]

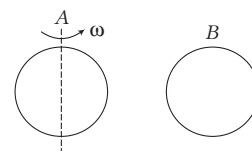
- (a) a direct current flows in the ammeter A
(b) no current flows through the ammeter A
(c) an alternating sinusoidal current flows through the ammeter A with a time period $T = \frac{2\pi}{\omega}$
(d) a time varying non-sinusoidal current flows through the ammeter A

- 73.** There are two coils A and B as shown in figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter-clockwise. B is kept stationary when A moves. We can infer that
[NCERT Exemplar]



- (a) there is a constant current in the clockwise direction in A
(b) there is a varying current in A
(c) there is no current in A
(d) there is a constant current in the counter-clockwise direction in A

- 74** Same as above problem except, the coil A is made to rotate about a vertical axis (figure). No current flows in B , if A is at rest. The current in coil A , when the current in B (at $t = 0$) is counter-clockwise and the coil A is as shown at this instant, $t = 0$, is
[NCERT Exemplar]



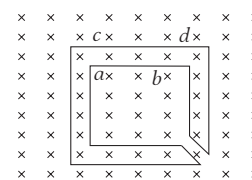
- (a) constant current clockwise
(b) varying current clockwise
(c) varying current counter-clockwise
(d) constant current counter-clockwise

- 75** A rectangular loop of sides 10 cm and 5 cm with a cut is stationary between the pole pieces of an electromagnet.

The magnetic field of the magnet is normal to the loop. The current feeding the electromagnet is reduced, so that the field decreased from its initial value of 0.2 T at the rate of 0.02 Ts^{-1} . If the cut is joined and the loop has a resistance of $2.0\ \Omega$, then the power dissipated by the loop as heat is

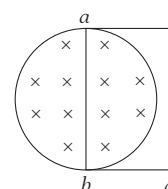
- (a) 5 nW (b) 4 nW (c) 3 nW (d) 2 nW

- 76** The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time, I_1 and I_2 are the currents in the segments ab and cd . Then,



- (a) $I_1 > I_2$
(b) $I_1 < I_2$
(c) I_1 is in the direction ba and I_2 is in the direction cd
(d) I_1 is in the direction ab and I_2 is in the direction dc

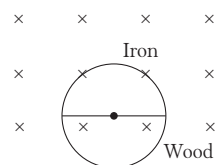
- 77** A uniform magnetic field B exists in a cylindrical region of radius 10 cm as shown in figure. A uniform wire of length 80 cm and resistance $4.0\ \Omega$ is bent into a square frame and is placed with one side along a diameter of the cylindrical region.



If the magnetic field increases at a constant rate of 0.01 T/s , then the current induced in the frame

- (a) $3.9 \times 10^{-5} \text{ A}$ (b) $4.0 \times 10^5 \text{ A}$
 (c) $4.1 \times 10^{-5} \text{ A}$ (d) $3.9 \times 10^{-4} \text{ A}$

- 78** Figure shows a circular wheel of radius 10.0 cm whose upper half, is made of iron and the lower half of wood as shown in figure. The two junctions are joined by an iron rod. A uniform magnetic field B of magnitude $2.0 \times 10^{-4} \text{ T}$ exists in the space above the central line as suggested by the figure. The wheel is set into pure rolling on the horizontal surface. If it takes 2.0 s for the iron part to come down and the wooden part to go up, then the average emf induced during this period is

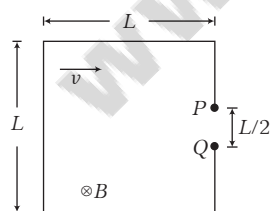


- (a) $1.57 \times 10^{-6} \text{ V}$ (b) $1.5 \times 10^5 \text{ V}$
 (c) $15.7 \times 10^{-6} \text{ V}$ (d) $1.55 \times 10^{-6} \text{ V}$

- 79** A wire of length 10 cm translates in a direction making an angle of 60° with its length. The plane of motion is perpendicular to a uniform magnetic field of 1.0 T that exists in the space. The emf induced between the ends of the rod, if the speed of translation is 20 cm/s is

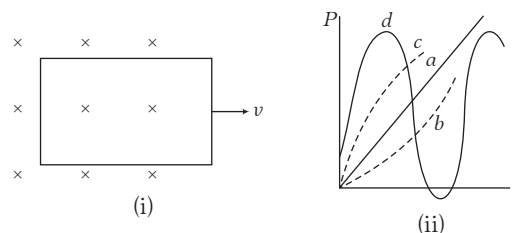
- (a) $1.7 \times 10^{-3} \text{ V}$ (b) $17 \times 10^{-3} \text{ V}$
 (c) $0.17 \times 10^{-3} \text{ V}$ (d) $1.7 \times 10^{-4} \text{ V}$

- 80** The loop shown moves with a velocity v in a uniform magnetic field of magnitude B , directed into the paper. The potential difference between P and Q is e



- (i) $e = BLv / 2$
 (ii) $e = BLv$
 (iii) P is positive with respect to Q
 (iv) Q is positive with respect to P
 (a) (ii), (iii) (b) (i), (iv) (c) (i), (iii) (d) (ii), (iv)

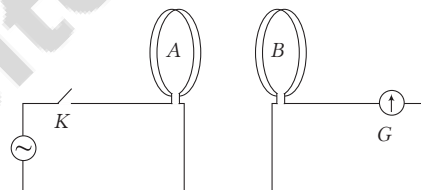
- 81** A conducting loop being pulled out of magnetic field with a speed v . Which of the following plots may represent the power delivered by the pulling agent as a function of the speed v ?



- (a) a (b) b
 (c) c (d) d

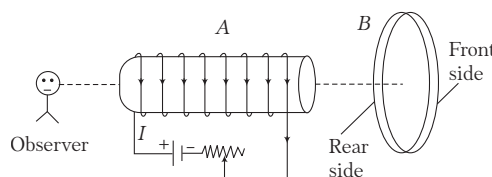
- 82** A square loop of wire with side length 10 cm is placed at angle of 45° with a magnetic field that changes uniformly from 0.1 T to zero in 0.7 s . The induced current in the loop (its resistance is 1Ω) is
- (a) 1.0 mA (b) 2.5 mA
 (c) 3.5 mA (d) 4.0 mA

- 83** The diagram below shows two coils A and B placed parallel to each other at a very small distance. Coil A is connected to an AC supply and G is a very sensitive galvanometer. When the key is closed



- (a) constant deflection will be observed in the galvanometer for 50 Hz supply
 (b) visible small variations will be observed in the galvanometer for 50 Hz input
 (c) oscillations in the galvanometer may be observed when the input AC voltage has a frequency of 1 to 2 Hz
 (d) No variation will be observed in the galvanometer even when the input AC voltage is 1 or 2 Hz

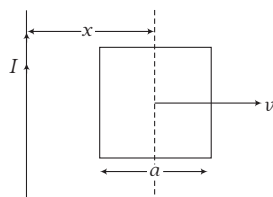
- 84** An aluminium ring B faces an electromagnet A . The current I through A can be altered



- (a) whether I increases or decreases, B will not experience any force
 (b) if I decreases, A will repel B
 (c) if I increases, A will attract B
 (d) if I increases, A will repel B

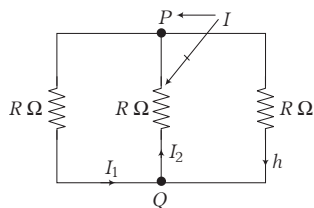
- 85** A conducting square frame of side a and a long straight wire carrying current I are located in the same plane as shown in the figure. The frame moves

to the right with a constant velocity v . The emf induced in the frame will be proportional to



- (a) $\frac{1}{(2x-a)^2}$ (b) $\frac{1}{(2x+a)^2}$
 (c) $\frac{1}{(2x-a)(2x+a)}$ (d) $1/x^2$

- 86** A rectangular loop has a sliding connector PQ of length l and resistance $R \Omega$ and it is moving with a speed v as shown. The set up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1 , I_2 and I are

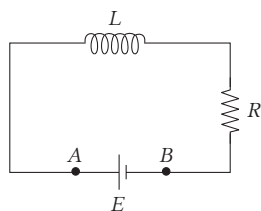


- (a) $I_1 = I_2 = \frac{Blv}{6R}$, $I = \frac{Blv}{3R}$ (b) $I_1 = -I_2 = \frac{Blv}{R}$, $I = \frac{2Blv}{R}$
 (c) $I_1 = I_2 = \frac{Blv}{3R}$, $I = \frac{2Blv}{3R}$ (d) $I_1 = I_2 = I = \frac{Blv}{R}$

- 87** Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 A and 4 A are the currents flowing in each coil, respectively. The magnetic induction in Wb/m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7} \text{ Wb/A-m}$)

- (a) 12×10^{-5} (b) 10^{-5} (c) 5×10^{-5} (d) 7×10^{-5}

- 88** An inductor ($L = 100 \text{ mH}$), a resistor ($R = 100 \Omega$) and a battery ($E = 100 \text{ V}$) are initially connected in series as shown in the figure. After a long time, the battery is disconnected after short-circuiting the points A and B. The current in the circuit 1 ms after the short-circuit is



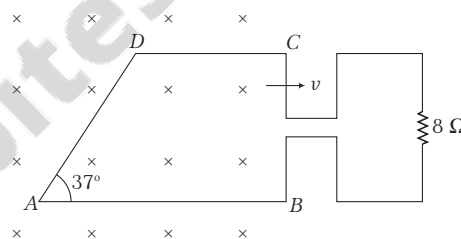
- (a) eA (b) $0.1A$ (c) $1A$ (d) $(1/e)A$

- 89.** Three solenoid coils of same dimension, same number of turns and same number of layers of windings are taken. Coil 1 with inductance L_1 was wound using a wire of resistance $11 \Omega/\text{m}$, coil 2 with inductance L_2 was wound using the similar wire but the direction of winding was reversed in each layer, coil 3 with inductance L_3 was wound using a superconducting wire. The self-inductance of the coils L_1, L_2 and L_3 are

- (a) $L_1 = L_2 = L_3$ (b) $L_1 = L_2, L_3 = 0$
 (c) $L_1 = L_3, L_2 = 0$ (d) $L_1 > L_2 > L_3$

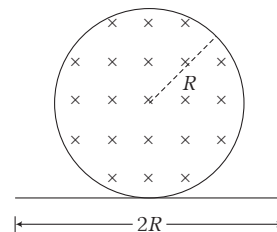
- 90** The loop $ABCD$ is moving with velocity v towards right. The magnetic field is 4 T. The loop is connected to a resistance of 8Ω . If steady current of 2 A flows in the loop, then value of v , if loop has resistance of 4Ω , is (Given, $AB = 30 \text{ cm}$,

$$AD = 30 \text{ cm}, \sin 37^\circ = \frac{3}{5})$$



- (a) $\frac{50}{3} \text{ ms}^{-1}$ (b) 20 ms^{-1} (c) 10 ms^{-1} (d) $\frac{100}{3} \text{ ms}^{-1}$

- 91** A uniform but time varying magnetic field is present in a circular region of radius R . The magnetic field is perpendicular and into the plane of the loop and the magnitude of field is increasing at a constant rate α . There is a straight conducting rod of length $2R$ placed as shown in figure.

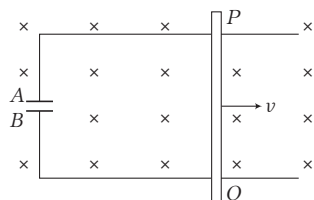


The magnitude of induced emf across the rod is

- (a) $\pi R^2 \alpha$ (b) $\frac{\pi R^2 \alpha}{2}$ (c) $\frac{R^2 \alpha}{\sqrt{2}}$ (d) $\frac{\pi R^2 \alpha}{4}$

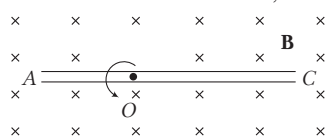
- 92** A conducting rod PQ of length, $L = 1.0 \text{ m}$ is moving with a uniform speed, $v = 2 \text{ ms}^{-1}$ in a uniform magnetic field, $B = 4.0 \text{ T}$ directed into the paper. A capacitor of capacity, $C = 10 \mu\text{F}$ is connected as shown in figure.

Then,



- (a) $q_A = +80 \mu\text{C}$ and $q_B = -80 \mu\text{C}$
 (b) $q_A = -80 \mu\text{C}$ and $q_B = +80 \mu\text{C}$
 (c) $q_A = 0 = q_B$
 (d) charge stored in the capacitor increases exponentially with time

- 93** A conducting rod AC of length $4l$ is rotated about a point O in a uniform magnetic field $\mathbf{B}$ directed into the paper. If $AO = l$ and $OC = 3l$, then

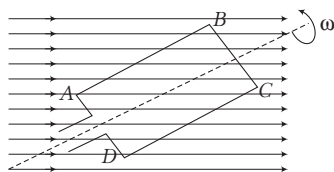


- (a) $V_A - V_O = \frac{B\omega l^2}{2}$
 (b) $V_O - V_C = \frac{7}{2}B\omega l^2$
 (c) $V_A - V_C = 4B\omega l^2$
 (d) $V_C - V_O = \frac{9}{2}B\omega l^2$

- 94** The current in an L - R circuit builds upto $\left(\frac{3}{4}\right)$ th of its steady state value in 4 s. The time constant of this circuit is

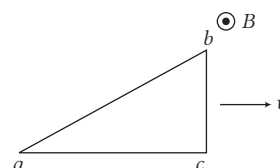
- (a) $\frac{1}{\ln 2}$ s (b) $\frac{2}{\ln 2}$ s (c) $\frac{3}{\ln 2}$ s (d) $\frac{4}{\ln 2}$ s

- 95** A rectangular coil $ABCD$ which is rotated at a constant angular velocity about an horizontal as shown in the figure. The axis of rotation of the coil as well as the magnetic field B are horizontal. Maximum current will flow in the circuit when the plane of the coil is



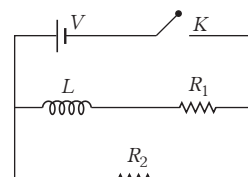
- (a) inclined at 30° to the magnetic field
 (b) perpendicular to the magnetic field
 (c) inclined at 45° to the magnetic field
 (d) parallel to the magnetic field

- 96** A right-angled triangle abc , made from a metallic wire, moves at a uniform speed v in its plane as shown in figure. A uniform magnetic field B exists in the perpendicular direction. The emf induced in the loop abc , in the segment bc , in the segment ac and in the segment ab are



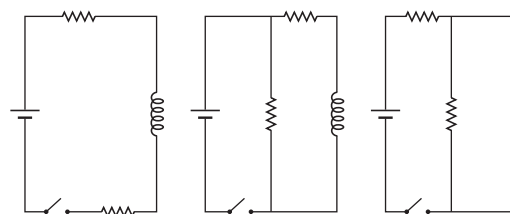
- (a) zero, $vB(bc)$, +ve at c , zero, $vB(bc)$, +ve at a
 (b) $vB(bc)$, +ve at c , zero, zero, $vB(bc)$, +ve at a
 (c) zero, zero, $vB(bc)$, +ve at c , $vB(bc)$, +ve at a
 (d) $vB(bc)$, +ve at c , $vB(bc)$, +ve at a , zero, zero

- 97** In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



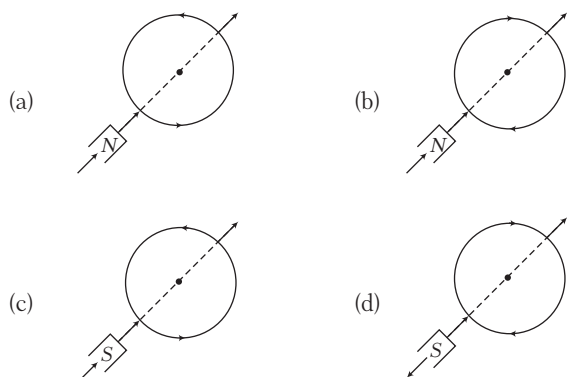
- (a) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$
 (b) $\frac{V(R_1 + R_2)}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$
 (c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$
 (d) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

- 98** The figure shows three circuits with identical batteries, inductors and resistors. Rank the circuits, in the decreasing order, according to the current through the battery (i) just after the switch is closed and (ii) a long time later.

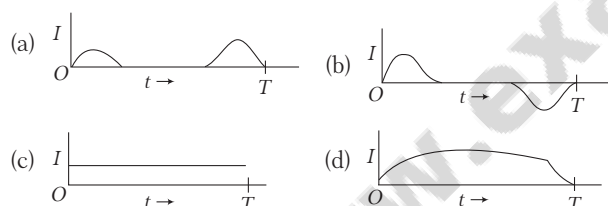


- (a) (i) $i_2 > i_3 > i_1$, ($i_1 = 0$) (ii) $i_2 > i_3 > i_1$
 (b) (i) $i_2 < i_3 < i_1$, ($i_1 \neq 0$) (ii) $i_2 > i_3 > i_1$
 (c) (i) $i_2 = i_3 = i_1$, ($i_1 = 0$) (ii) $i_2 < i_3 < i_1$
 (d) (i) $i_2 = i_3 > i_1$, ($i_1 \neq 0$) (ii) $i_2 > i_3 > i_1$

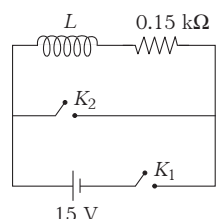
99 Which of the following figure correctly depicts the Lenz's law? The arrows show the movement of the labelled pole of a bar magnet into a closed circular loop and the arrows on the circle show the direction of the induced current.



100 A metallic ring is dropped down, keeping its plane perpendicular to a constant and horizontal magnetic field. The ring enters the region of magnetic field at $t = 0$ and completely emerges out at $t = T$ sec. The current in the ring varies as



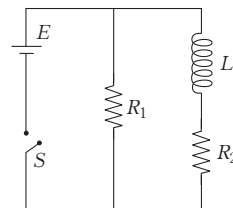
101 An inductor ($L = 0.03$ H) and a resistor ($R = 0.15$ k Ω) are connected in series to a battery of 15V emf in a circuit shown below. The key K_1 has been kept closed for a long time. Then, at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ m/s the current in the circuit will be ($e^5 \approx 150$)



- (a) 100 mA (b) 67 mA (c) 6.7 mA (d) 0.67 mA

102 An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf 12 V as shown in the figure. The

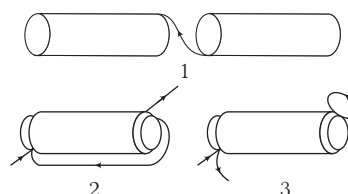
internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is



- (a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V (c) $6\left(1 - e^{\frac{-1}{0.2}}\right)$ V (d) $12e^{-5t}$ V

103 There are two solenoids of same length and inductance L but their diameters differ to the extent that one can just fit into the other. They are connected in three different ways in series.

- They are connected in series but separated by large distance.
- They are connected in series with one inside the other and senses of the turns coinciding.
- Both are connected in series with one inside the other with senses of the turns opposite, as depicted in Figures 1, 2 and 3, respectively. The total inductance of the solenoids in each of the case I, II and III are respectively.



- (a) 0, $4L_0$, $2L_0$ (b) $4L_0$, $2L_0$, 0
 (c) $2L_0$, 0, $4L_0$ (d) $2L_0$, $4L_0$, 0

104 A rectangular loop of length l and breadth b is placed at distance of x from infinitely long wire carrying current i such that the direction of current is parallel to breadth. If the loop moves away from the current wire in a direction perpendicular to it with a velocity v , then magnitude of the emf in the loop is (μ_0 = permeability of free space)

- (a) $\frac{\mu_0 iv}{2\pi x} \left(\frac{l+b}{b}\right)$ (b) $\frac{\mu_0 i^2 v}{4\pi^2 x} \log\left(\frac{b}{l}\right)$
 (c) $\frac{\mu_0 ilbv}{2\pi x(l+x)}$ (d) $\frac{\mu_0 ilbv}{2\pi} \log\left(\frac{x+l}{x}\right)$

105 A circular coil of one turn of radius 5.0 cm is rotated about a diameter with a constant angular speed of 80 revolutions per minute. A uniform magnetic field $B = 0.01$ T exists in a direction perpendicular to the

axis of rotation, the maximum emf induced, the average emf induced in the coil over a long period and the average of the squares of emf induced over a long period is

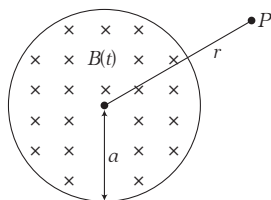
- (a) $6.4 \times 10^{-4} \text{ V}$, zero, $2.2 \times 10^{-7} \text{ V}^2$
 (b) $6.6 \times 10^{-4} \text{ V}$, zero, $2.0 \times 10^{-7} \text{ V}^2$
 (c) $6.8 \times 10^{-4} \text{ V}$, zero, $2.5 \times 10^{-7} \text{ V}^2$
 (d) $6.4 \times 10^{-4} \text{ V}$, zero, $2.0 \times 10^{-6} \text{ V}^2$

- 106** A non-conducting ring having charge q uniformly distributed over its circumference is placed on a rough horizontal surface. A vertical time varying magnetic field $B = 4t^2$ is switched ON at time $t = 0$. Mass of the ring is m and radius is R .

The ring starts rotating after 2 s, the coefficient of friction between the ring and the table is

- (a) $\frac{4qmR}{g}$ (b) $\frac{2qmR}{g}$ (c) $\frac{8qR}{mg}$ (d) $\frac{qR}{2mg}$

- 107** A uniform but time-varying magnetic field $B(t)$ exists in a cylindrical region of radius a and is directed into the plane of the paper as shown in figure. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region



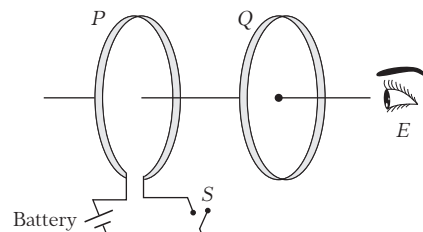
- (a) is zero (b) decreases as $\frac{1}{r}$
 (c) increases as r (d) decreases as $\frac{1}{r^2}$

- 108** As shown in the figure, P and Q are two co-axial conducting loops separated by some distance. When

the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q . The switch remains closed for a long time.

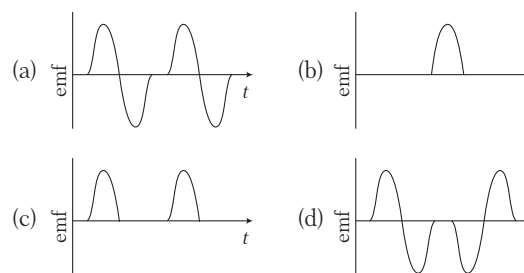
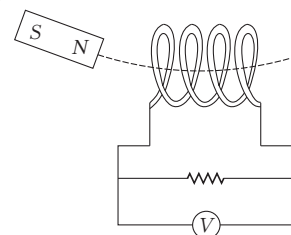
When S is opened, a current I_{Q2} flows in Q .

Then, the directions of I_{Q1} and I_{Q2} (as seen by E) are



- (a) respectively clockwise and anti-clockwise
 (b) both clockwise
 (c) both anti-clockwise
 (d) respectively anti-clockwise and clockwise

- 109** A magnet is made to oscillate with a particular frequency, passing through a coil as shown in figure. The time variation of the magnitude of emf generated across the coil during one cycle.



(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose anyone of the following four responses.

- (a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) If Assertion is true but Reason is false.
 (d) If Assertion is false but Reason is true.

1 Assertion Coefficient of self-induction of an inductor depends upon the rate of change of current passing through it.

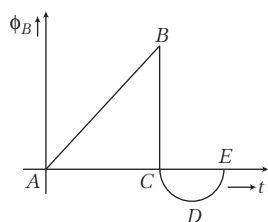
Reason From, $e = -L \frac{di}{dt}$

We can see that, $L = \frac{-e}{(di/dt)}$

or $L \propto \frac{1}{(di/dt)}$

- 5 Choose the correct statement. A magnet is dropped down an infinitely long vertical copper tube,
- the magnet moves with continuously increasing velocity and ultimately acquires a constant terminal velocity.
 - the magnet moves with continuously decreasing velocity and ultimately comes to rest.
 - the magnet moves with continuously increasing velocity but constant acceleration.
 - the magnet moves with continuously increasing velocity and acceleration.

- 6 The graph shows the variation in magnetic flux $\phi(t)$ with time through a coil. Which of the statements given below is incorrect?



- There is a change in the direction as well as magnitude of the induced emf between B and D.
- The magnitude of the induced emf is maximum between B and C.
- There is a change in the direction as well as magnitude of induced emf between A and C.
- The induced emf is not zero at B.

Match the columns

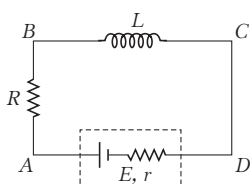
1. Match the Column I with Column II and mark the correct option from the codes given below.

Column I	Column II
(A) Tesla	(p) $[ML^2A^{-2}T^{-2}]$
(B) Weber	(q) $[MLA^{-2}T^{-1}]$
(C) Weber m^{-2}	(r) $[MA^{-1}T^{-2}]$
(D) Henry	(s) None

Codes

- | | | | | | | | |
|-------|---|---|---|-------|---|---|---|
| A | B | C | D | A | B | C | D |
| (a) s | p | r | q | (b) p | r | s | q |
| (c) r | s | r | p | (d) r | p | q | s |

2. In the circuit shown in figure, $E = 10\text{ V}$, $r = 1\ \Omega$, $R = 4\ \Omega$ and $L = 5\text{ H}$. The circuit is closed at time $t = 0$.



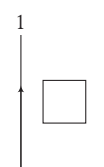
Match the Column I with Column II and mark the correct option from the codes given below.

Column I	Column II
(A) V_{AD} at $t = 0$	(p) 4 V
(B) V_{BC} at $t = 0$	(q) 8 V
(C) V_{AB} at $t = \infty$	(r) 0 V
(D) V_{AD} at $t = \infty$	(s) 10 V

Codes

- | | | | | | | | |
|-------|---|---|---|-------|---|---|---|
| A | B | C | D | A | B | C | D |
| (a) s | p | q | p | (b) p | q | q | r |
| (c) s | q | r | p | (d) s | s | q | q |

3. A square loop is symmetrically placed between two infinitely long current carrying wires in the same direction. Magnitude of currents in both the wires are same. Match the Column I with Column II and mark the correct option from the codes given below.

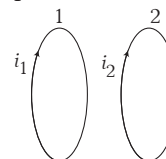


Column I	Column II
(A) Loop is moved towards the right	(p) Induced current in the loop is clockwise.
(B) Loop is moved towards the left	(q) Induced current in the loop is anti-clockwise.
(C) Wire-1 is moved towards the left	(r) Induced current in the loop is zero.
(D) Wire-2 is moved towards the right	(s) Induced current in the loop is non-zero.

Codes

- | | | | | | | | |
|---------|-----|-----|-----|---------|-----|-----|-----|
| A | B | C | D | A | B | C | D |
| (a) q,s | r,p | q,s | p,r | (b) p,s | q,s | p,s | q,s |
| (c) p,s | p,s | q,s | q,r | (d) p,s | q,s | q,p | p,s |

4. Two co-axial identical circular current carrying loops are shown in figure, currents in them are in the same directions. Match the Column I with Column II and mark the correct option from the codes given below.



Column I	Column II
(A) Current i_1 is increased	(p) Loops will attract each other
(B) Current i_2 is decreased	(q) Loops will repel each other
(C) Loop-1 is moved towards loop-2	(r) Current i_1 will increase
(D) Loop-2 is moved away from loop-1	(s) Current i_2 will increase

Codes

- | | | | |
|-------|-----|-----|-----|
| A | B | C | D |
| (a) q | p,p | q,r | p,s |
| (b) q | p,q | r,p | r,r |
| (c) p | q,s | r,q | p,q |
| (d) p | p,s | r,r | r,q |

5. Match the Column I with Column II and mark the correct option from the codes given below.

Column I (Planar loops of different shapes)	Column II (Direction of induced current)
(A)	(p) b a c b

Column I (Planar loops of different shapes)	Column II (Direction of induced current)
(B)	(q) c d a b c
(C)	(r) b c d a b

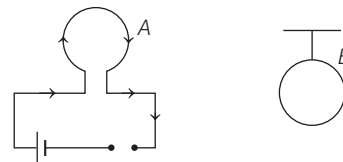
Codes

- | | | | | | |
|-------|---|---|-------|---|---|
| A | B | C | A | B | C |
| (a) p | q | r | (b) r | q | p |
| (c) p | r | q | (d) r | p | q |

(C) Medical entrances' gallery

Collection of questions asked in NEET and various medical entrance exams

- 1 The magnetic flux linked with a coil (in Wb) is given by the equation $\phi = 5t^2 + 3t + 16$.
The magnitude of induced emf in the coil at the fourth second will be [NEET 2020]
(a) 33 V (b) 43 V
(c) 108 V (d) 10 V
- 2 A wheel with 20 metallic spokes each 1 m long is rotated with a speed of 120 rpm in a plane perpendicular to a magnetic field of 0.4 G. The induced emf between the axle and rim of the wheel will be ($1 \text{ G} = 10^{-4} \text{ T}$) [NEET 2020]
(a) $2.51 \times 10^{-4} \text{ V}$ (b) $2.51 \times 10^{-5} \text{ V}$
(c) $4.0 \times 10^{-5} \text{ V}$ (d) 2.51 V
- 3 Lenz law is based on principle of conservation of [JIPMER 2020]
(a) linear momentum (b) energy
(c) charge (d) mass
- 4 A coil of 800 turns effective area 0.05 m^2 is kept perpendicular to a magnetic field $5 \times 10^{-5} \text{ T}$. When the plane of the coil is rotated by 90° around any of its co-planar axis in 0.1 s, the emf induced in the coil will be [NEET 2019]
(a) 0.2 V (b) $2 \times 10^{-3} \text{ V}$ (c) 0.02 V (d) 2 V
- 5 In which of the following devices, the eddy current effect is not used? [NEET 2019]
(a) Magnetic braking in train
(b) Electromagnet
(c) Electric heater
(d) Induction furnace
- 6 **Assertion** A metallic surface is moved in and out in magnetic field, then emf is induced in it.
Reason Eddy current will be produced in a metallic surface moving in and out of magnetic field. [AIIMS 2019]
(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Both Assertion and Reason are incorrect.
- 7 The magnetic potential energy stored in a certain inductor is 25 mJ, when the current in the inductor is 60 mA. This inductor is of inductance [NEET 2018]
(a) 1.389 H (b) 138.88 H
(c) 0.138 H (d) 13.89 H
- 8 A system S consists of two coils A and B. The coil A carries a steady current I while the coil B is suspended nearby as shown in figure. Now, if the system is heated, so as to raise the temperature of two coils steadily, then [AIIMS 2018]



- (a) the two coils shows attraction
(b) the two coils shows repulsion
(c) there is no change in the position of the two coils
(d) induced current are not possible in coil B

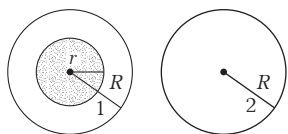
- 9** A long solenoid has 1000 turns. When a current of 4 A flows through it, then the magnetic flux linked with each turn of the solenoid is 4×10^{-3} Wb. The self-inductance of the solenoid is [NEET 2016]

(a) 3 H (b) 2 H
(c) 1 H (d) 4 H

- 10** A uniform magnetic field is restricted within a region of radius r . The magnetic field changes with time at a rate $\frac{dB}{dt}$. Loop 1 of radius $R > r$ encloses

the region r and loop 2 of radius R is outside the region of magnetic field as shown in the figure. Then, the emf generated is

[NEET 2016]



- (a) zero in loop 1 and zero in loop 2
(b) $-\frac{dB}{dt} \pi r^2$ in loop 1 and $-\frac{dB}{dt} \pi R^2$ in loop 2
(c) $-\frac{dB}{dt} \pi R^2$ in loop 1 and zero in loop 2
(d) $-\frac{dB}{dt} \pi r^2$ in loop 1 and zero in loop 2
- 11** The self-inductance of a coil having 500 turns is 50 mH. The magnetic flux through the cross-sectional area of the coil while current through it is 8 mA is found to be [AIIMS 2015]
- (a) 4×10^{-4} Wb (b) 0.04 Wb
(c) 4 μ Wb (d) 40 mWb
- 12** The phase difference between the flux linked with a coil rotating in a uniform magnetic field and induced emf produced in it is [AIIMS 2015]
- (a) $\pi/2$ (b) $\pi/3$
(c) $-\pi/6$ (d) π
- 13** A rectangular copper coil is placed in a uniform magnetic field of induction 40 mT with its plane perpendicular to the field. The area of the coil is shrinking at a constant rate of $0.5 \text{ m}^2 \text{ s}^{-1}$. The emf induced in the coil is [EAMCET 2015]
- (a) 10 mV (b) 20 mV
(c) 80 mV (d) 40 mV
- 14** Changing magnetic fields can set up current loops in nearby metal bodies and the currents are called as [Kerala CEE 2015]
- (a) eddy currents (b) flux currents
(c) alternating currents (d) leaking currents
(e) wattless currents

- 15** A rod of 10 cm length is moving perpendicular to uniform magnetic field of intensity $5 \times 10^{-4} \text{ Wb m}^{-2}$. If the acceleration of the rod is 5 m s^{-2} , then the rate of increase of induced emf is [Guj. CET 2015]

(a) $25 \times 10^{-4} \text{ Vs}^{-1}$ (b) $2.5 \times 10^{-4} \text{ Vs}^{-1}$
(c) $20 \times 10^{-4} \text{ Vs}^{-1}$ (d) $20 \times 10^{-4} \text{ Vs}^{-1}$

- 16** The identical loops of copper and aluminium are moving with the same speed in a magnetic field. Which of the following statements is true? [CG PMT 2015]

(a) Induced emf and induced current are same in both loops
(b) Induced emf remains same, induced current changes in both loops
(c) Induced emf changes but induced current remains same in both loops
(d) Induced emf will be more in aluminium loop

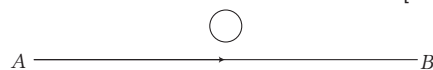
- 17** A straight conductor 0.1 m long moves in a uniform magnetic field 0.1 T. The velocity of the conductor is 15 ms^{-1} and is directed perpendicular to the field. The emf induced between the two ends of the conductor is [WB JEE 2015]

(a) 0.10 V (b) 0.15 V (c) 1.50 V (d) 15 V

- 18** The initial rate of increase of current, when a battery of emf 6 V is connected in series with an inductance of 2 H and resistance 12 Ω , is [CG PMT 2015]

(a) 0.5 As^{-1} (b) 1 As^{-1}
(c) 3 As^{-1} (d) 3.5 As^{-1}

- 19** The current flows from A to B as shown in the figure. What is the direction of current in circle? [UK PMT 2014]



(a) Clockwise (b) Anti-clockwise
(c) Straight line (d) None of these

- 20** A very small circular loop of radius a is initially (at $t = 0$) coplanar and concentric with a much larger fixed circular loop of radius b . A constant current I flows in the larger loop. The smaller loop is rotated with a constant angular speed ω about the common diameter. The emf induced in the smaller loop as a function of time t is [WB JEE 2014]

(a) $\frac{\pi a^2 \mu_0 I}{2b} \omega \cos \omega t$
(b) $\frac{\pi a^2 \mu_0 I}{2b} \omega \sin^2 t^2$
(c) $\frac{\pi a^2 \mu_0 I}{2b} \omega \sin \omega t$
(d) $\frac{\pi a^2 \mu_0 I}{2b} \omega \sin^2 \omega t$

- 21** A straight conductor of length 0.4 m is moved with a speed of 7 ms^{-1} perpendicular to a magnetic field of intensity 0.9 Wbm^{-2} . The induced emf across the conductor is [UK PMT 2014]

(a) 5.04 V (b) 1.26 V (c) 2.52 V (d) 25.2 V

- 22** The current in a self-inductance $L = 40 \text{ mH}$ is to be increased uniformly from 1 A to 11 A in 4 ms. The emf induced in the inductor during the process is [UK PMT 2014]

(a) 100 V (b) 0.4 V (c) 40 V (d) 440 V

- 23** The induced emf in a coil of 10 H inductance in which current varies from 9 A to 4 A in 0.2 s is [J & K CET 2013]

(a) 200 V (b) 250 V (c) 300 V (d) 350 V

- 24** A conductor of length 5 cm is moved parallel to itself with a speed of 2 m/s, perpendicular to a uniform magnetic field of 10^{-3} Wb/m^3 . The induced emf generated is [J & K CET 2013]

(a) $2 \times 10^{-3} \text{ V}$ (b) $1 \times 10^{-3} \text{ V}$ (c) $1 \times 10^{-4} \text{ V}$ (d) $2 \times 10^{-4} \text{ V}$

- 25** Two identical circular coils A and B are kept on a horizontal tube side by side without touching each other. If the current in the coil A increases with time, in response, the coil B [Karnataka CET 2013]

(a) is attracted by A (b) remains stationary
(c) is repelled (d) rotates

- 26** A rectangular coil of 100 turns and size $0.1 \text{ m} \times 0.05 \text{ m}$ is placed perpendicular to a magnetic field of 0.1 T. If the field drops to 0.05 T in 0.05 s, the magnitude of the emf induced in the coil is [Karnataka CET 2013]

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 0.5 (d) $\sqrt{6}$

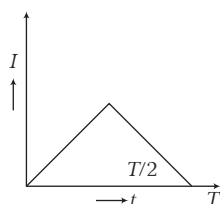
- 27** Electromagnetic induction is not used in [Kerala CET 2013]

(a) room heater (b) transformer
(c) AC generator (d) induction furnace
(e) speedometer

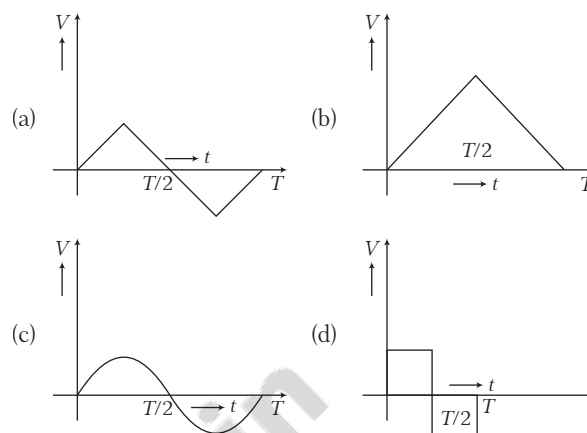
- 28** Two coils have the mutual inductance of 0.05 H. The current changes in the first coil as $I = I_0 \sin \omega t$, where $I_0 = 1 \text{ A}$ and $\omega = 100\pi \text{ rad/s}$. The maximum emf induced in secondary coil is [UP CPMT 2013]

(a) 2.5 V (b) 10 V
(c) $6\pi \text{ V}$ (d) $5\pi \text{ V}$

- 29** The current I in the inductance is varying with time according to the plot shown in figure.



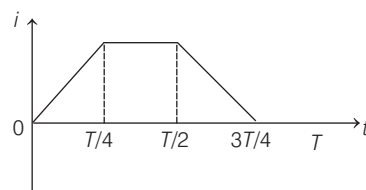
Which one of the following is the correct variation of voltage with time in the coil? [CBSE AIPMT 2012]



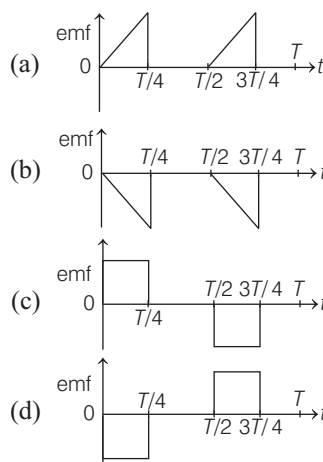
- 30** The magnetic flux linked with a coil satisfies the relation $\phi = (4t^2 + 6t + 9) \text{ Wb}$, where t is time in second. The emf induced in the coil at $t = 2 \text{ s}$ is [CBSE AIPMT 2012]

(a) 22 V (b) 18 V (c) 16 V (d) 40 V

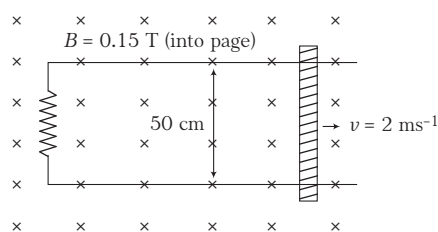
- 31** The current i in a coil varies with time as shown in the figure. The variation of induced emf with time would be



[AIIMS 2012]



- 32** As shown in the figure, a metal rod makes contact with a partial circuit and completes the circuit. The circuit area is perpendicular to a magnetic field with $B = 0.15 \text{ T}$. If the resistance of the total circuit is 3Ω , the force needed to move the rod as indicated with a constant speed of 2 ms^{-1} will be equal to [AMU 2012]



- (a) 3.75×10^{-3} N (b) 2.75×10^{-3} N
(c) 6.57×10^{-4} N (d) 4.36×10^{-4} N
- 33** If emf induced in a coil is 2 V by changing the current in it from 8 A to 6 A in 2×10^{-3} s, then the coefficient of self-induction is [AFMC 2012]
(a) 2×10^{-3} H (b) 10^{-3} H
(c) 0.5×10^{-3} H (d) 4×10^{-3} H
- 34** A wire of length 1m is moving at a speed of 2 ms^{-1} perpendicular to its length in a homogeneous magnetic field of 0.5T. If the ends of the wire are joined to a circuit of resistance 6Ω , then the rate at which work is being done to keep the wire moving at constant speed is [JCECE 2012]

- (a) 1W (b) $\frac{1}{3}$ W (c) $\frac{1}{6}$ W (d) $\frac{1}{12}$ W

- 35** The magnetic flux linked with a circuit of resistance 100Ω increases from 10 to 60 Wb. The amount of induced charge that flows in the circuit is (in coulomb) [UP CPMT 2012]
(a) 0.5 (b) 5
(c) 50 (d) 100
- 36** Two solenoids of equal number of turns have their lengths and the radii in the same ratio 1 : 2. The ratio of their self-inductances will be [Kerala CEE 2011]
(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 4
(e) 1 : 3
- 37** Two identical induction coils each of inductance L joined in series are placed very close to each other such that the winding direction of one is exactly opposite to that of the other. What is the inductance? [Kerala CEE 2011]
(a) L^2 (b) $2L$
(c) $L/2$ (d) L
(e) Zero

ANSWERS

CHECK POINT 6.1

1. (c)	2. (b)	3. (d)	4. (d)	5. (a)	6. (d)	7. (b)	8. (b)	9. (b)	10. (b)
11. (c)	12. (b)	13. (d)	14. (a)						

CHECK POINT 6.2

1. (b)	2. (c)	3. (b)	4. (d)	5. (b)	6. (c)	7. (c)	8. (d)	9. (d)	10. (b)
11. (c)	12. (c)	13. (a)							

CHECK POINT 6.3

1. (b)	2. (a)	3. (d)	4. (b)	5. (a)	6. (c)	7. (d)	8. (d)	9. (d)	10. (a)
11. (a)	12. (d)	13. (b)	14. (a)	15. (c)					

CHECK POINT 6.4

1. (a)	2. (b)	3. (a)	4. (c)	5. (d)	6. (a)	7. (a)	8. (c)	9. (a)	10. (b)
11. (b)	12. (c)	13. (a)	14. (a)	15. (d)					

(A) Taking it together

1. (d)	2. (b)	3. (b)	4. (b)	5. (b)	6. (a)	7. (c)	8. (d)	9. (b)	10. (a)
11. (c)	12. (a)	13. (c)	14. (b)	15. (a)	16. (d)	17. (d)	18. (d)	19. (a)	20. (b)
21. (d)	22. (d)	23. (d)	24. (c)	25. (a)	26. (d)	27. (c)	28. (c)	29. (c)	30. (d)
31. (b)	32. (c)	33. (a)	34. (c)	35. (d)	36. (b)	37. (b)	38. (d)	39. (a)	40. (d)
41. (c)	42. (a)	43. (a)	44. (c)	45. (c)	46. (b)	47. (b)	48. (d)	49. (d)	50. (d)

51. (b)	52. (b)	53. (b)	54. (c)	55. (c)	56. (d)	57. (a)	58. (d)	59. (a)	60. (b)
61. (c)	62. (b)	63. (b)	64. (d)	65. (a)	66. (b)	67. (a)	68. (b)	69. (c)	70. (d)
71. (b)	72. (b)	73. (d)	74. (a)	75. (a)	76. (d)	77. (a)	78. (a)	79. (b)	80. (c)
81. (b)	82. (a)	83. (c)	84. (d)	85. (c)	86. (c)	87. (c)	88. (d)	89. (a)	90. (d)
91. (d)	92. (a)	93. (c)	94. (b)	95. (d)	96. (a)	97. (c)	98. (a)	99. (a)	100. (b)
101. (d)	102. (d)	103. (d)	104. (c)	105. (b)	106. (c)	107. (b)	108. (d)	109. (a)	

(B) Medical entrance special format questions• **Assertion and reason**

1. (d) 2. (a) 3. (b) 4. (a) 5. (b)

• **Statement based questions**

1. (c) 2. (d) 3. (b) 4. (d) 5. (a) 6. (d)

• **Match the columns**

1. (c) 2. (d) 3. (b) 4. (a) 5. (d)

(C) Medical entrances' gallery

1. (d)	2. (a)	3. (b)	4. (c)	5. (c)	6. (a)	7. (d)	8. (a)	9. (c)	10. (d)
11. (a)	12. (a)	13. (b)	14. (a)	15. (b)	16. (b)	17. (b)	18. (c)	19. (d)	20. (c)
21. (c)	22. (a)	23. (b)	24. (c)	25. (c)	26. (c)	27. (a)	28. (d)	29. (d)	30. (a)
31. (d)	32. (a)	33. (a)	34. (c)	35. (a)	36. (a)	37. (b)			

Hints & Explanations

• CHECK POINT 6.1

- 4 (d) Magnitude of induced emf,

$$e = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}(3t^2 + 4t + 9) = -(6t + 4)$$

At $t = 2$ s, $e = -[6 \times (2) + 4] = -16$

$\therefore |e| = 16$ V

5 (a) As, $|e| = \frac{\Delta\phi}{\Delta t} = \frac{4B_0 A_0 - B_0 A_0}{t} = \frac{3B_0 A_0}{t}$

6 (d) Induced emf, $e = \left| \frac{d\phi}{dt} \right| = |12t - 5|$

At $t = 0.25$ s, $\Rightarrow e = 2$ V

$\therefore$ Induced current, $i = \frac{e}{R} = 0.2$ A

- 7 (b) Magnetic flux, $\phi = (5t^2 - 4t + 1)$ Wb

$\therefore \frac{d\phi}{dt} = (10t - 4)$ Wbs⁻¹

The induced emf, $e = \frac{-d\phi}{dt} = -(10t - 4)$

At $t = 0.2$ s, $e = -(10 \times 0.2 - 4) = 2$ V

The induced current, $I = \frac{e}{R} = \frac{2 \text{ V}}{10 \Omega} = 0.2$ A

8 (b) As, charge, $|\Delta q| = \frac{\Delta\phi}{R} = \frac{BA}{R} = \frac{(2)\left(\frac{\pi}{4}\right)(0.2)^2}{0.01} = 6.28$ C

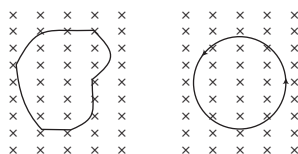
9 (b) As, charge, $\Delta Q = \frac{\Delta\phi}{R} = \frac{\phi_2 - \phi_1}{R} = \frac{(10 - 2)}{2} = 4$ C

- 11 (c) According to Lenz's law, the induced current will be in such a direction, so that it opposes the change due to which it is produced. So, the induced current in the inner loop will be in counter-clockwise direction.

- 13 (d) According to Lenz's law, the direction of induced current is such as to attain north polarity. So, it will be anti-clockwise.

- 14 (a) Due to change in the shape of the loop, the magnetic flux linked with the loop increases. Hence, current is induced in the loop in such a direction that it opposes the increase in flux.

Therefore, induced current flows in the anti-clockwise direction.



• CHECK POINT 6.2

- 2 (c) When a wire of length l moves with velocity v , perpendicular to a magnetic field B , then the emf is induced. The magnitude of induced emf is given by $|\epsilon| = Blv$.

Given, $l = 50$ cm = 0.5 m, $v = 300$ m/min = 5 m/s

$$|\epsilon| = 2 \text{ V or } B = \frac{|\epsilon|}{lv} = \frac{2}{0.5 \times 5} = 0.8 \text{ T}$$

3 (b) Induced emf $= Blv = 0.3 \times 10^{-4} \times 10 \times 5$
 $= 1.5 \times 10^{-3} \text{ V} = 1.5$ mV

4 (d) Induced emf $= B \times v \times l$
 $= 5.0 \times 10^{-5} \times 1.50 \times 2$
 $= 10.0 \times 10^{-5} \times 1.5$
 $= 1.5 \times 10^{-4} = 0.15$ mV

- 5 (b) When a conductor lying along the magnetic north-south, then moves eastwards, it will cut vertical component of B_0 . So, induced emf, $e = vB_v l = lv(B_0 \sin \delta) = B_0 lv \sin \delta$.

- 6 (c) Here, $l = 0.1$ m, $v = 1$ ms⁻¹

$$I = 50 \text{ A and } B = 1.25 \text{ mT} = 1.25 \times 10^{-3} \text{ T}$$

The induced emf, $\epsilon = Blv$

The mechanical power,

$$P = \epsilon I = BlvI = 1.25 \times 10^{-3} \times 0.1 \times 1 \times 50$$

$$= 6.25 \times 10^{-3} \text{ W} = 6.25 \text{ mW}$$

10 (b) Induced emf, $e_0 = \omega NBA = (2\pi\nu)NB(\pi r^2) = 2 \times \pi^2 \nu NBr^2$
 $= 2 \times (3.14)^2 \times \frac{1800}{60} \times 4000 \times 0.5 \times 10^{-4} \times (7 \times 10^{-2})^2 = 0.57$ V

- 11 (c) Induced emf, when rod rotates in a vertical plane perpendicular to magnetic field is given as

$$e = \frac{1}{2} Bl^2 \omega = \frac{1}{2} \times 0.3 \times 2^2 \times 100 = 60 \text{ V}$$

- 12 (c) $E = E_m \cos 45^\circ$

where, E_m is the maximum value of induced emf.

$$\therefore E = \frac{E_m}{\sqrt{2}}$$

• CHECK POINT 6.3

1 (b) $U = \frac{1}{2} Li^2$, $[L] = \left[\frac{U}{i^2} \right] = \frac{\text{joule}}{\text{ampere}^2}$

$$[L] = \left[\frac{e}{di/dt} \right] = \text{ohm-second}$$

$$[L] = [\phi_B / i] = \text{weber/ampere}$$

$$[L] = [\phi / i] = [edt/di] = \frac{\text{volt-second}}{\text{ampere}}$$

Hence, SI unit of inductance is volt-second/ampere.

- 2 (a) We have, $N\Phi_B = Li$

$$\text{Self-inductance, } L = \frac{N\Phi_B}{i} = \frac{500 \times 4 \times 10^{-3}}{2} = 1 \text{ H}$$

- 3 (d) Self-inductance of coil, $|e| = L \frac{di}{dt}$

$$\therefore 10 = L \times \frac{10}{1}$$

$\therefore$ Self-inductance, $L = 1 \text{ H}$

- 4 (b) Self-inductance, $L = \left| \frac{e}{di/dt} \right| = \frac{8}{(2/0.5)} = 2 \text{ H}$

- 5 (a) Given, $\frac{di}{dt} = -2 \text{ A/s}$ and $L = 5 \text{ H}$

$\therefore$ The emf developing across the coil,

$$e = -L \frac{di}{dt} = -5 \times (-2) = 10 \text{ V}$$

- 6 (c) We have, $\frac{\Delta i}{\Delta t} = \frac{10}{2} = 5 \text{ V}$

$$\text{emf, } e = L \frac{\Delta i}{\Delta t} = 0.5 \times 5 = 2.5 \text{ V}$$

- 7 (d) Here, $L = 50 \times 10^{-3} \text{ H}$

$$\text{and } \frac{di}{dt} = \frac{(1-0)}{0.1} = 10$$

$$\text{emf, } |e| = \frac{L \cdot dI}{dt} = 50 \times 10^{-3} \times 10 = 50 \times 10^{-2} = 0.5 \text{ V}$$

- 8 (d) The self-inductance of a long solenoid is given by

$$L = \frac{\mu_r \mu_0 n^2 A}{l}$$

It is clear that, the self-inductance of a long solenoid does not depend upon the current flowing through it.

- 9 (d) Self-inductance of coil is directly proportional to square of number of turns in the coil, i.e.

$$L \propto N^2$$

$$\therefore \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$$

$$\therefore \frac{L}{L_2} = \frac{N^2}{(4N)^2}$$

$$\therefore L_2 = 16L$$

- 10 (a) $L \propto N^2$

So, when number of turns is doubled, then L' will become 4 times of L , i.e. $L' = 4L$ or 4 times.

- 12 (d) Induced emf, $e = L \frac{di}{dt} = \frac{\mu_0 N^2 S}{l} \frac{di}{dt}$
- $$= \frac{4\pi \times 10^{-7} \times (2000)^2 \times 1.2 \times 10^{-3}}{0.30} \times \left(\frac{2 - (-2)}{0.25} \right)$$
- $$= 32.1 \times 10^{-2} \text{ V}$$

- 13 (b) Energy stored in the coil,

$$E = \frac{1}{2} Li^2 = \frac{1}{2} \times 50 \times 10^{-3} \times 4 = 0.1 \text{ J}$$

- 14 (a) $E = \frac{1}{2} Li^2 = \frac{1}{2} (100 \times 10^{-3})(10)^2 = 5 \text{ J}$

- 15 (c) Equivalent inductance, $\frac{1}{L_{eq}} = \frac{1}{L} + \frac{1}{L}$

$$\therefore \text{The total inductance, } L_{eq} = \frac{L}{2}$$

• CHECK POINT 6.4

- 1 (a) We have, $M = \frac{\Delta\Phi_2}{\Delta i_1} \Rightarrow M = \frac{0.4}{2}$

$\therefore$ Mutual inductance, $M = 0.2 \text{ H}$

- 2 (b) Value of induced emf in coil,

$$e = M \frac{di}{dt} = 0.005 \times \frac{d}{dt} (i_0 \sin \omega t)$$

$$= 0.005 \times i_0 \omega \cos \omega t \quad (\text{for } e_{\max}, \cos \omega t = 1)$$

$$\therefore e_{\max} = 0.005 \times 10 \times 100\pi = 5\pi$$

- 3 (a) emf, $e_2 = M \left(\frac{di_1}{dt} \right) = i_2 R_2$

$$\therefore \text{Current rate, } \frac{di_1}{dt} = \frac{i_2 R_2}{M} = \frac{(0.4)(5)}{0.5} = 4 \text{ As}^{-1}$$

- 5 (d) Average emf, $e = M \frac{di}{dt} = 0.09 \times \frac{20}{0.006} = 300 \text{ V}$

- 6 (a) As, $|e| = M \frac{di}{dt} \Rightarrow 8 \times 10^{-3} = M \times 3$

Mutual inductance, $M = 2.66 \text{ mH}$

- 7 (a) We have, $M = \frac{\mu_0 N_1 N_2 S}{L}$

Here, S is area of inner solenoid. So, N_1 , N_2 and S are same for both solenoids.

$$\Rightarrow M_{12} = M_{21}$$

- 8 (c) Mutual inductance between two coils,

$$M = - \frac{e_2}{(di_1/dt)} = - \frac{e_1}{(di_2/dt)}$$

$$\text{Also, } e_1 = -L_1 \frac{di_1}{dt} \text{ and } e_2 = -L_2 \frac{di_2}{dt}$$

$$\therefore M^2 = \frac{e_1 e_2}{\left(\frac{di_1}{dt} \right) \left(\frac{di_2}{dt} \right)} = L_1 L_2$$

$$\Rightarrow M = \sqrt{L_1 L_2}$$

- 9 (a) Mutual inductance between coils is

$$M = K \sqrt{L_1 L_2}$$

$$\Rightarrow M = 1 \sqrt{2 \times 10^{-3} \times 8 \times 10^{-3}} \quad (\because K = 1)$$

$$= 4 \times 10^{-3} = 4 \text{ mH}$$

- 10 (b) Growth of current in the circuit is given by

$$i = i_0 (1 - e^{-Rt/L})$$

where, i_0 is peak value of current and $i_0 = \frac{5}{5} = 1 \text{ A}$

$$\therefore i = 1(1 - e^{-5 \times 2/10}) = (1 - e^{-1}) \text{ A}$$

11 (b) In case of growth of current in an L - R circuit, the current in the circuit grows exponentially with time to the maximum value, $i_0 = E/R$.

12 (c) Time interval, during which the current in an inductive circuit rises to 63% of its maximum value is defined as time constant, i.e. $\tau = 0.63L/R$.

(A) Taking it together

1 (d) We have, $f = \frac{1}{\tau_C} = \frac{1}{RC}$, $f = \frac{1}{\tau_L} = \frac{R}{L}$ and $\omega = \frac{1}{\sqrt{LC}}$

$\therefore \frac{C}{L}$ does not have dimensions of frequency.

2 (b) The self-inductance of a long solenoid of cross-sectional area A and length l , having n turns per unit length, filled with a material of relative permeability (e.g. soft iron, which has a high value of relative permeability) inside the solenoid is given by

$$L = \mu_r \mu_0 n^2 A l = \frac{\mu_r \mu_0 N^2 A}{l}$$

3 (b) Induced emf, $e = -\frac{d\phi}{dt}$

But, $e = iR$ and $i = \frac{dq}{dt}$

$$\therefore \frac{dq}{dt} R = -\frac{d\phi}{dt} \quad \text{or} \quad dq = -\frac{d\phi}{R}$$

So, induced charge in a coil is independent of time taken to change the flux.

4 (b) The value of self induction of coil increases when soft iron core is inserted in the coil, hence current decreases due to increase of its reactance. So, intensity of bulb decreases.

5 (b) The magnitude of induced emf is directly proportional to the rate of change of magnetic flux. Also, induced charge is $dq = -\frac{d\phi}{R}$, which is independent of time.

$\therefore$ Induced emf and induced charge will be more in first case and equal in both case, respectively.

6 (a) Induced current is given by

$$i = \frac{e}{R} = -\frac{N(d\phi/dt)}{R} = -\frac{N}{R} \left(\frac{d\phi}{dt} \right) = -\frac{N}{R} \cdot \frac{dB}{dt} \cdot A$$

$$= \frac{10 \times 10^8 \times 10^{-4} \times 10^{-4} \times 10}{20} = 5 \text{ A}$$

7 (c) Applying Lenz's law, the loop B is repelled by loop A .

8 (d) Induced emf, $e = -N \frac{d\phi}{dt}$

$$|e| = N \frac{d\phi}{dt} = N \frac{d}{dt} (BA \cos \theta) = N \frac{dB}{dt} \cdot A \cos \theta$$

$$= 500 \times 1 \times (10 \times 10^{-2})^2 \cos 0^\circ = 5 \text{ V}$$

9 (b) Induced emf, $e = -\frac{d\phi}{dt}$

$$|e| = \frac{d}{dt} (BA) = A \frac{dB}{dt} = 2 \times \frac{(4-1)}{2} = 3 \text{ V}$$

11 (c) $\phi = \mathbf{B} \cdot \mathbf{A} = BA \cos 90^\circ = 0$. Therefore, $d\phi = 0$ and

$$i = \frac{dq}{dt} = \frac{1}{R} \frac{d\phi}{dt} = 0$$

12 (a) The magnet gets repelled. Therefore, north pole is formed on the top side and hence current will flow in anti-clockwise direction.

13 (c) Current in the circuit, $i = V/R = 10/2 = 5 \text{ A}$

$\therefore$ Magnetic energy stored in the coil,

$$U = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \times (5)^2 = 25 \text{ J}$$

14 (b) Time constant, $\tau_L = \frac{L}{R} = \frac{5}{10} = \frac{1}{2}$

We have, $i = i_0 (1 - e^{-t/\tau_L})$

$$\therefore \frac{i_\infty}{i_1} = \frac{i_0}{i_0 (1 - e^{-t/\tau_L})}$$

Substituting, $t = 1 \text{ s}$ and $\tau_L = \frac{1}{2} \text{ s}$, we get

$$\frac{i_\infty}{i_1} = \frac{e^2}{e^2 - 1}$$

15 (a) Charge induced in coil is given as

$$dq = \frac{d\phi_B}{R} = i dt = \text{Area under } i-t \text{ graph}$$

$$\therefore d\phi_B = (\text{Area under } i-t \text{ graph}) R$$

$$= \frac{1}{2} \times 4 \times 0.1 \times 10 = 2 \text{ Wb}$$

16 (d) Here, inductor L and bulb B_2 make a LR circuit, whereas resistor R and bulb B_1 make a simple R circuit.

So, B_1 dies out promptly but B_2 with some delay.

17 (d) We have, $\Delta q = -\frac{N}{R} \Delta\phi$ and $\Delta\phi = \Delta B \cdot A \cos \theta$

$$\therefore 32 \times 10^{-6} = -\frac{100}{(160 + 40)} (0 - B) \times \pi \times (6 \times 10^{-3})^2 \times \cos 0^\circ$$

$\therefore$ Intensity of magnetic field, $B = 0.566 \text{ T}$

18 (d) Induced emf, $e = \left| \frac{-NBA(\cos \theta_2 - \cos \theta_1)}{\Delta t} \right|$

$$= \left| \frac{800 \times 4 \times 10^{-5} \times 0.05(\cos 90^\circ - \cos 0^\circ)}{0.1} \right| = 0.016 \text{ V}$$

19 (a) Magnetic flux, $\phi = B \cdot A$

$\therefore$ Change in flux, $d\phi = B \cdot dA$

$$= 0.05 (101 - 100) \times 10^{-4} = 5 \times 10^{-6} \text{ Wb}$$

$\therefore$ Charge, $dq = \frac{d\phi}{R} = \frac{5 \times 10^{-6}}{2} = 2.5 \times 10^{-6} \text{ C}$

20 (b) For growth of current in L - R circuit, current is given by

$$i = i_0 (1 - e^{-Rt/L})$$

$$\therefore \frac{di}{dt} = -i_0 \left(-\frac{R}{L} \right) e^{-Rt/L} = \frac{i_0 R}{L} e^{-Rt/L}$$

At the instant of closing the switch, $\frac{di}{dt} = \frac{i_0 R}{L} = \frac{E}{L}$

$$\Rightarrow 4 = \frac{E}{20}$$

$$\therefore E = 20 \times 4 = 80 \text{ V}$$

21 (d) Peak value of emf generated in generator is given by

$$e_0 = \omega NBA = (2\pi\nu) NBA \\ = 2 \times 3.14 \times 100 \times 5000 \times 0.2 \times 0.25 = 157 \text{ kV}$$

22 (d) If electron is moving from left to right, the flux linked with the loop (which is into the page) will first increase and then decrease as the electron passes through it. So, the induced current in the loop will be first anti-clockwise and will change direction as the electron passes through.

23 (d) $\mathbf{B}$, $\mathbf{v}$ and $\mathbf{l}$ should be mutually perpendicular.

So, the conductor should move in the direction perpendicular to the plane of $\mathbf{l}$ and $\mathbf{B}$, i.e. in the direction of $\mathbf{M}$.

24 (c) Induced emf, $e = Bvl$

$$\therefore e \propto v$$

$$\text{Also, } v = 0 + gt$$

$$\therefore e \propto gt$$

So, potential difference between its two ends will increase with time.

25 (a) In the given circuit, three inductances are in parallel, their equivalent inductance is given by

$$\frac{1}{L_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \text{ or } L_{eq} = \frac{3}{3} = 1 \text{ H}$$

26 (d) $\therefore$ In parallel, $L_p = \frac{L_1 L_2}{L_1 + L_2}$

$$(L_1 + L_2) 1.5 = L_1 L_2 \quad \dots(i)$$

$$\text{and in series, } L_s = L_1 + L_2 = 8$$

$$\therefore L_1 L_2 = 12 \quad [\text{from Eq. (i)}]$$

$$(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1 L_2$$

$$L_D = \sqrt{(8)^2 - 4 \times 12}$$

$$L_D = 4 \text{ H}$$

27 (c) Time constant, $\tau = \frac{L}{R}$

It is the time interval during which the current after opening an inductive circuit falls to 37% of its maximum value.

28 (c) Time constant, $\tau = \frac{L}{R} = 2.0 \times 10^{-3} \text{ s}$

$$\text{and } \frac{L}{R + 90} = 0.5 \times 10^{-3} \text{ s}$$

Solving these two equations, we get

$$L = 60 \text{ mH and } R = 30 \Omega$$

29 (c) Four resistances form a balanced Wheatstone bridge. Therefore, energy stored in the coil is zero.

30 (d) The whole current will pass through the inductor, since $R = 0$. So, current in 10Ω resistor is zero.

31 (b) Power, $P = Fv = iBv = \left(\frac{Bv l}{R}\right) lBv = \frac{B^2 l^2 v^2}{R}$

32 (c) The magnetic flux linked with a surface of area A in uniform magnetic field is given by

$$\phi = \mathbf{B} \cdot \mathbf{A}$$

$$\text{Here, } A = L^2 \hat{\mathbf{k}} \text{ and } B = B_0 (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \text{ T}$$

$$\therefore \phi = \mathbf{B} \cdot \mathbf{A} = B_0 (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot L^2 \hat{\mathbf{k}} = 4B_0 L^2 \text{ Wb}$$

33 (a) $\therefore$ Magnetic flux,

$$\phi = BA \cos \theta = 0.2 \sin(300t) \times 5 \times 10^{-4} \times \cos 60^\circ$$

$$= 5 \times 10^{-5} \sin(300t)$$

$$\therefore \text{emf, } e = \frac{d\phi}{dt} = 5 \times 10^{-5} \times 300 \cos(300t)$$

$$= 1.5 \times 10^{-2} \cos\left(300 \times \frac{\pi}{900}\right) \quad \left(\because t = \frac{\pi}{900} \text{ s}\right)$$

$$= 1.5 \times 10^{-2} \cos(\pi/3) = 0.75 \times 10^{-2} = 7.5 \times 10^{-3} \text{ V}$$

34 (c) Since, the magnetic field is uniform, therefore there will be no change in flux, hence no current will be induced.

36 (b) Charge, $\Delta Q = \frac{\Delta\phi}{R} = \frac{N \times BA}{R}$

$\therefore$ Magnetic flux density,

$$B = \frac{\Delta Q \cdot R}{NA} = \frac{2 \times 10^{-4} \times 80}{40 \times 4 \times 10^{-4}} = 1 \text{ Wb/m}^2$$

37 (b) We have, $L \propto N^2$

$$\frac{L_B}{L_A} = \left(\frac{N_B}{N_A}\right)^2$$

$$\text{Self-inductance, } L_B = \left(\frac{500}{600}\right)^2 \times 108 = 75 \text{ mH}$$

39 (a) Mutual inductance between two coils depend on their degree of flux linkage, i.e. the fraction of flux linked with one coil when some current passes through the other coil. In Fig. (A), two coils with their planes are parallel. In this situation, maximum flux passes. So, mutual inductance will be maximum in this case.

40 (d) During growth of current in the coil,

$$i = i_0 (1 - e^{-Rt/L})$$

$$\text{For } i = \frac{i_0}{2}$$

$$\Rightarrow \frac{i_0}{2} = i_0 (1 - e^{-Rt/L})$$

$$\therefore t = 0.693 \frac{L}{R}$$

$$= 0.693 \times \frac{300 \times 10^{-3}}{2} = 0.1 \text{ s}$$

41 (c) Growth of current in the circuit is $i = i_0 (1 - e^{-Rt/L})$

$$\Rightarrow \frac{di}{dt} = \frac{d}{dt} i_0 - \frac{d}{dt} (i_0 e^{-Rt/L})$$

$$\therefore \frac{di}{dt} = 0 - i_0 \left(-\frac{R}{L}\right) e^{-Rt/L} = \frac{i_0 R}{L} e^{-Rt/L}$$

Initially, $t = 0$,

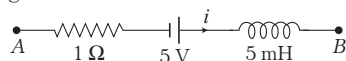
$$\therefore \frac{di}{dt} = \frac{i_0 R}{L} = \frac{E}{L} = \frac{5}{2} = 2.5 \text{ As}^{-1}$$

- 42** (a) Just after pressing the key K , the inductor offers infinite resistance. So, net resistance will be $14\ \Omega$.

$$\therefore I = V / R = \frac{10}{14} = (5/7) \text{ A}$$

- 43** (a) At $t = 0$, current will flow from the capacitor and at $t = \infty$ current will flow from the inductor. So, current in both cases will be 1.5 A.

- 44** (c) Applying Kirchhoff's second law to above circuit,



$$V_A - iR + 15 - L \frac{di}{dt} = V_B$$

$$\text{or } V_A - 5 \times 1 + 15 - (5 \times 10^{-3})(-10^3) = V_B$$

$$\therefore V_B - V_A = 15V$$

- 45** (c) At $t = 0$, induced emf e is maximum and is equal to applied emf, but current i is zero.

$$\text{As, } i = \frac{E}{R} (1 - e^{-tR/L}) \text{ and } V = \frac{L di}{dt} = L \cdot \frac{E}{R} \left(\frac{R}{L} \right) e^{-tR/L} = E e^{-tR/L}$$

So, after lapse of time, current through the circuit increases but induced emf decreases.

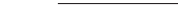
- 46** (b) Two coils are said to be magnetically coupled, if full or a part of the flux produced by one link with the other. Let L_1 and L_2 be the self-inductances of the coils and M be their mutual inductances, then coefficient of coupling $k = \frac{M}{\sqrt{L_1 L_2}}$.

When $k = 1$, then 100% flux produced by one coil links with the other. In this case, mutual inductance between the two is maximum and is given by $M = \sqrt{L_1 L_2}$

- 47** (b) Induced emf, $|e| = \frac{d\phi}{dt} = \frac{BdS}{dt}$

Now, as the square loop and rectangular loop move out of magnetic field, $\frac{BdS}{dt}$ is constant, therefore $|e|$ is constant. But

in case of circular and elliptical loops, $\frac{dS}{dt}$ changes. Therefore, emf does not remain constant.

- 49** (d) 

Eddy current induced in aluminium plate create damping effect in coil.

- 50** (d) Mutual inductance between two coils in the same plane with their centres coinciding is given by

$$M = \frac{\mu_0}{4\pi} \left(\frac{2\pi^2 R_2^2 N_1 N_2}{R_1} \right) \text{henry} \Rightarrow M \propto \frac{R_2^2}{R_1}$$

- 51** (b) Power, $P = \frac{e^2}{R}$, and $e = -\left(\frac{d\phi}{dt}\right)$, where $\phi = NBS$

$$\therefore e = -NS \left(\frac{dB}{dt} \right). \text{ Also, } R \propto \frac{l}{r^2}$$

where, R = resistance, r = radius, l = length

$$\therefore P \propto \frac{N^2 r^2}{l}$$

$$\frac{P_1}{P_2} = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{r_1}{r_2}\right)^2 \frac{l_2}{l_1} \quad \dots(i)$$

Given, $N_2 = 4N_1$

$$r_2 = \frac{r_1}{2}$$

Since, $V_1 = V_2$

$$\pi r_1^2 l_1 = \pi r_2^2 l_2$$

$$\Rightarrow \frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{l_2}{l_1} = \left(\frac{r_1}{r_2}\right)^2 \quad \dots(\text{ii})$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\begin{aligned}\frac{P_1}{P_2} &= \binom{N_1}{N_2}^2 \cdot \binom{r_1}{r_2}^2 \cdot \binom{r_1}{r_2}^2 \\ &= \binom{N_1}{N_2}^2 \cdot \binom{r_1}{r_2}^4 = \binom{N_1}{4N_1}^2 \left(\frac{r_1}{r_1/2} \right)^4 \\ &= \frac{1}{4^2} \times 2^4 = \frac{1}{16} \times 16 = 1\end{aligned}$$

$$\Rightarrow \frac{P_1}{P_2} = 1 \Rightarrow P_1 = P_2$$

- 52** (b) As the magnet moves towards the coil, the magnetic flux increases (non-linearly). Also, there is a change in polarity of induced emf when the magnet passes on to the other side of the coil. So, correct variation is shown in (b).

- 53** (b) $\therefore i = i_0 \left(1 - e^{-\frac{R}{L}t} \right)$

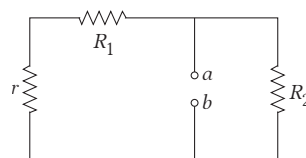
At $t = 0, i = 0$.

At $t \gg \tau (= L / R)$, $i = i_0$.

Thus, i increases exponentially upto i_0 and then become constant as shown in (b).

- 54** (c) Since, you are decreasing current in the circuit by increasing resistance of the circuit, then induced emf across inductor will support the battery. Hence, net emf of the circuit is greater than 20 V or current in the circuit is more than 4 A.

- 55** (c) After short-circuiting the battery.



$$\text{Resistance, } R_{ab} = \frac{R_2(R_1 + r)}{R_2 + R_1 + r}$$

$$\therefore \text{Time constant, } \tau_L = \frac{L}{R_{\text{net}}} = \frac{L(R_1 + R_2 + r)}{(R_1 + r)R_2}$$

- 56** (d) Magnetic lines on circular loop, due to current in wire AB are tangential to its plane.

$$\therefore \text{Magnetic flux, } \phi_B = 0 \text{ or induced emf, } \frac{d\phi_B}{dt} = 0$$

57 (a) Induced emf, $|e| = \frac{\Delta\phi}{\Delta t} = \frac{(2-1)(0.01)}{10^{-3}} = 10 \text{ V}$

$$\therefore \text{Current, } i = \frac{e}{R} = \frac{10}{0.01} = 10^3 \text{ A}$$

$$\text{Heat produced, } H = eit = (10)(10^3)(10^{-3}) = 10 \text{ J}$$

- 58** (d) As, potential difference,

$$V = Bvl = (2.0 \times 10^{-4}) \left(360 \times \frac{5}{18} \right) (50) = 1.0 \text{ V}$$

- 59** (a) If the loop remains inside the region of magnetic field, no current will be induced. But, if it moves out of the plane, then the induced current is given by $i = \frac{Blv}{R}$ and its direction will be clockwise (as per Lenz's law).

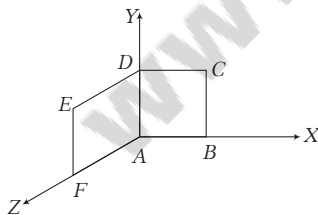
- 60** (b) $\therefore$ As, potential difference, $e = Bvl = (0.2 \times 10^{-4})(20)(1) \text{ V}$
 $= 0.4 \text{ mV}$

- 61** (c) $\therefore M_{12} = M_{21}$, when rate of change of current is doubled, induced emf also becomes two times.
 Thus, emf produced across the first coil is 6 mV.

- 62** (b) Here, loop $ABCD$ lies in XY -plane whose area vector $\mathbf{A}_1 = L^2 \hat{\mathbf{k}}$, whereas loop $ADEFA$ lies in YZ -plane whose area vector $\mathbf{A}_2 = L^2 \hat{\mathbf{i}}$.

Also, the magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$$\phi = \mathbf{B} \cdot \mathbf{A}$$



$$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 = (L^2 \hat{\mathbf{k}} + L^2 \hat{\mathbf{i}})$$

$$\text{and } \mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}}) \text{ T}$$

$$\text{Now, } \phi = \mathbf{B} \cdot \mathbf{A} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}}) \cdot (L^2 \hat{\mathbf{k}} + L^2 \hat{\mathbf{i}}) = 2B_0 L^2 \text{ Wb}$$

- 63** (b) Two emfs are induced in the closed circuit each of value Blv . These two emfs are additive, so $E_{\text{net}} = 2Blv$.

- 64** (d) Magnetic flux,

$$\phi = NAB = (25)(\pi)(1.8 \times 10^{-2})^2 [0.2t - 0.05t^2]$$

$$= 0.025(0.2t - 0.05t^2)$$

$$\text{Induced emf, } e = \left| \frac{d\phi}{dt} \right| = |0.025(0.2 - 2 \times 0.05t)|$$

$$\text{At } t = 3 \text{ s, } e = 0.0025 \text{ V}$$

$$i = \frac{e}{R} = 0.0016 \text{ A}$$

$$\therefore \text{Power dissipation, } P = ei = 4 \times 10^{-6} \text{ W} = 4 \mu\text{W}$$

- 65** (a) In the case (a), the induced current in the ring will be in such a direction that it repels the magnet.

i.e. $a < g$ and moving towards R .

In case (c), the induced current is such as to attract the magnet.

i.e. $a < g$ and moving away from R .

- 66** (b) Here, $e = Bvl = (0.1)(1)(0.1) = \frac{1}{100} \text{ V}$

$$\text{Net resistance} = 1 + \frac{2 \times 3}{2 + 3} = \frac{11}{5} \Omega$$

$$\therefore \text{Current, } i = \left(\frac{1/100}{11/5} \right) = \frac{1}{220} \text{ A}$$

- 67** (a) From the figure, $V_a - iR - L \frac{di}{dt} = V_b$

$$\therefore V_a - V_b = iR + L \frac{di}{dt}$$

According to given conditions,

$$8 = 2R + L \quad \dots(i)$$

$$4 = 2R - L \quad \dots(ii)$$

Solving these two equations, we get

$$R = 3 \Omega \text{ and } L = 2 \text{ H}$$

- 68** (b) $V = L \frac{di}{dt}$ or $V \propto L$ $\left(\therefore \frac{di}{dt} = \text{constant} \right)$

$$\therefore \frac{V_1}{V_2} = \frac{L_1}{L_2} = 4 \quad (\therefore P = Vi = \text{constant})$$

$$\therefore i \propto \frac{1}{V} \text{ or } \frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

$$\therefore \frac{W_1}{W_2} = \frac{L_1}{L_2} \left(\frac{i_1}{i_2} \right)^2 = (4) \left(\frac{1}{16} \right) = \frac{1}{4} \quad \left(\therefore W = \frac{1}{2} Li^2 \right)$$

$$\therefore \frac{i_1}{i_2} \neq 4$$

Thus, wrong option is (b).

- 69** (c) Induced emf, $E = -L \frac{di}{dt}$

or induced emf $\propto$ slope of i - t graph. So, graph (c) is correct.

70 (d) Energy, $U = \frac{1}{2} Li_0^2 = \frac{1}{2} L \left(\frac{V}{R} \right)^2$

$$= \frac{1}{2} \times 50 \times 10^{-6} \left(\frac{5}{0.5} \right)^2 \text{ J}$$

$$= 2.5 \text{ mJ}$$

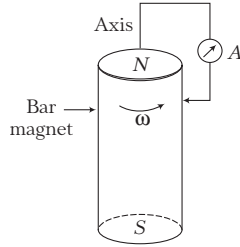
Now, after switching OFF the battery, this energy is dissipated in coil (resistance $= 0.5 \Omega$) and resistance (10Ω) in the ratio of their resistances ($H = i^2 Rt$ or $H \propto R$). Therefore, heat generated in the coil.

$$2.5 \times \left(\frac{0.5}{10 + 0.5} \right) = 0.12 \text{ mJ}$$

71 (b) Power, $P = (i_0)^2 R$, i.e., $(i_0)^2 = \frac{P}{R}$ and $\tau = \frac{L}{R}$

Total heat produced, $U = \frac{1}{2} L i_0^2 = \frac{1}{2} (\tau R) \left(\frac{P}{R} \right) = \frac{1}{2} P \tau$

- 72 (b) When cylindrical bar magnet is rotated about its axis, no change in flux linked with the circuit takes place, consequently no emf induces and hence no current flows through the ammeter A.



- 73 (d) When A stops moving, then current in B become zero, possible only if the current in A is constant in counter-clockwise direction. If the current in A would be variable, then there must be an induced emf (current) in B even, if A stops moving.

- 74 (a) When the current in B (at $t = 0$) is counter-clockwise and the coil A is considered above to it. The counter-clockwise flow of the current in B is equivalent to north pole of magnet and magnetic field lines are emanating upward to coil A. When coil A start rotating at $t = 0$, then the current in A is constant along clockwise direction by Lenz's rule.

- 75 (a) Here, area, $A = 10 \times 5 = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$

$$\frac{dB}{dt} = 0.02 \text{ Ts}^{-1} \text{ and } R = 2 \Omega$$

$$\therefore \text{emf, } e = \frac{d\Phi}{dt} = A \cdot \frac{dB}{dt} = 50 \times 10^{-4} \times 0.02 = 10^{-4} \text{ V}$$

Power dissipated in the form of heat

$$= \frac{e^2}{R} = \frac{10^{-4} \times 10^{-4}}{2} = 0.5 \times 10^{-8} \text{ W} = 5 \times 10^{-9} \text{ W} = 5 \text{ nW}$$

- 76 (d) The magnetic field $\otimes$ passing from the closed loop is increasing. Therefore, from Lenz's law, induced current will produce upward magnetic field. Hence, induced current is anti-clockwise, i.e. I_1 is in the direction ab and I_2 is in the direction dc .

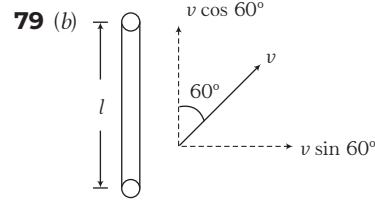
- 77 (a) Since, here only half area of ring will be considered.

$$\therefore \Phi_B = BS = B \cdot \frac{\pi r^2}{2}$$

$$\therefore e = \frac{d\Phi_B}{dt} = \frac{\pi r^2}{2} \cdot \frac{dB}{dt} = \frac{\pi (0.1)^2}{2} \times 0.01 = \frac{\pi}{2} \times 10^{-4} \text{ V}$$

$$\therefore i = \frac{e}{R} = \frac{\frac{\pi}{2} \times 10^{-4}}{4} = \frac{\pi}{8} \times 10^{-4} = 3.9 \times 10^{-5} \text{ A}$$

- 78 (a) $\therefore \text{emf, } |e| = \frac{\Delta\Phi}{\Delta t} = \frac{BS - 0}{\Delta t} = \frac{2 \times 10^{-4} \times \pi \times (0.1)^2}{2 \times 2}$
 $\Rightarrow |e| = 1.57 \times 10^{-6} \text{ V}$



Due to $v \cos 60^\circ$, $e = 0$

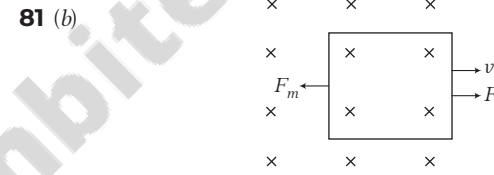
$$\begin{aligned} \text{Due to } v \sin 60^\circ, e &= B(v \sin 60^\circ)l \\ &= \frac{\sqrt{3}}{2} Bvl = \frac{\sqrt{3}}{2} \times 1 \times 0.2 \times 0.1 \\ &= 17.3 \times 10^{-3} \text{ V} \approx 17 \times 10^{-3} \text{ V} \end{aligned}$$

- 80 (c) As, the potential due to a straight rod of length L in magnetic field B is

$$e = BLv$$

$$\text{So, for } PQ = L/2, e = \frac{BLv}{2}$$

Hence, current is from P to Q , i.e. P is at positive potential w.r.t. Q .



Induced emf $= Bvl$

$$\text{Induced current, } i = \frac{Bvl}{R}$$

$$\text{Magnetic force on loop, } F_m = Bil = \frac{B^2 v l^2}{R}$$

Since, loop is moving with constant speed,

$$\Rightarrow F = F_m$$

$$\text{Input power, } P = \mathbf{F} \cdot \mathbf{v} = Fv = \frac{B^2 v^3 l^2}{R}$$

P versus v graph will be like,

$$y \propto x^2 \quad (\text{parabola, open upward})$$

- 82 (a) Area of square loop, $S = 10 \text{ cm} \times 10 \text{ cm}$

$$\Rightarrow S = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2 = 10^{-2} \text{ m}^2$$

Initial magnetic flux linked with loop,

$$\begin{aligned} \Phi_{B_1} &= B_1 S \cos \phi = 0.1 \times 10^{-2} \times \cos 45^\circ \\ &= \frac{0.1 \times 10^{-2} \times 1}{\sqrt{2}} = \frac{10^{-3}}{\sqrt{2}} \text{ Wb} \end{aligned}$$

Final magnetic flux linked with loop,

$$\Phi_{B_2} = 0 \text{ Wb} \quad (\because B_2 = 0)$$

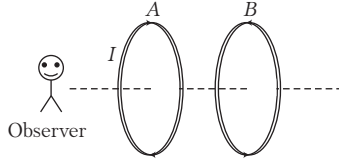
The induced emf in the loop,

$$\begin{aligned} \epsilon &= -\frac{d\Phi}{dt} = -\frac{(\Phi_2 - \Phi_1)}{t} \\ &= -\frac{\left(0 - \frac{10^{-3}}{\sqrt{2}}\right)}{0.7} = \frac{10^{-3}}{0.7 \times \sqrt{2}} \approx 10^{-3} \text{ V} \end{aligned}$$

The induced current in the loop,

$$I = \frac{\varepsilon}{R} = \frac{10^{-3}\text{V}}{1\Omega} = 10^{-3}\text{A} = 1.0\text{mA}$$

84 (d)



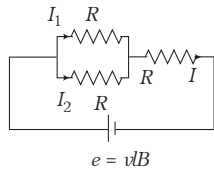
If current I through A increases, magnetic field linked with coil B increases. Hence, anti-clockwise current induces in coil B . As shown in figure, both the currents flowing in opposite directions produce repulsive effect.

85 (c) As, the emf induced,

$$= B_1 vl - B_2 vl = \frac{\mu_0 I}{2\pi(x-a/2)} vl - \frac{\mu_0 I}{2\pi(x+a/2)} vl$$

$$\Rightarrow e \propto \frac{1}{(2x-a)(2x+a)}$$

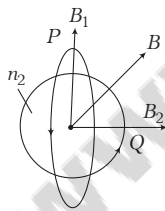
86 (c) Circuit can be reduced as



$$I = \frac{e}{3R/2} = \frac{2vB}{3R} \Rightarrow I_1 = I_2 = \frac{I}{2} = \frac{vB}{3R}$$

87 (c) Magnetic field at P , $B_1 = \frac{\mu_0 I_2}{2R}$

$$= \frac{4\pi \times 10^{-7} \times 4}{2 \times 0.02\pi} = 4 \times 10^{-5} \text{ Wb/m}^2$$



Magnetic field, at Q ,

$$B_2 = \frac{\mu_0 I_1}{2R} = \frac{4\pi \times 10^{-7} \times 3}{2 \times 0.02\pi} = 3 \times 10^{-5} \text{ Wb/m}^2$$

$$\therefore B = \sqrt{B_1^2 + B_2^2} = \sqrt{(4 \times 10^{-5})^2 + (3 \times 10^{-5})^2}$$

$$= 5 \times 10^{-5} \text{ Wb/m}^2$$

88 (d) During decay of current,

$$i = i_0 e^{-\frac{Rt}{L}} = \frac{E}{R} e^{-\frac{Rt}{L}} = \frac{100}{100} e^{-\frac{100 \times 10^{-3}}{100 \times 10^{-3}}} = \left(\frac{1}{e}\right) \text{A}$$

89 (a) The self-inductance of a solenoid, $L = \mu_0 n^2 S l$, where

μ_0 = permeability of air,

n = number of turns per unit length,

S = area of cross-section

and l = length of the solenoid.

This depends on the geometry of the inductor such as cross-sectional area, length and number of turns and not on the material, even if it is made of a superconducting material.

$$\therefore L_1 = L_2 = L_3$$

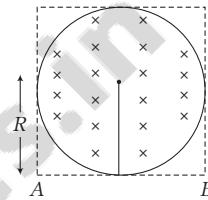
90 (d) Current, $I = \frac{\text{Potential difference}}{\text{Resistance}}$

$$\therefore \text{Potential difference} = 2 \times 12 = 24 \text{ V} = Bvl$$

$$\text{Here, } l = AD \sin 37^\circ = 0.3 \times \frac{3}{5} = 0.18 \text{ m}$$

$$\therefore \text{Velocity, } v = \frac{24}{Bl} = \frac{24}{4 \times 0.18} = \frac{100}{3} \text{ ms}^{-1}$$

91 (d) Induced emf, $V_{BA} = \frac{(E \cdot 8R)}{4} = \frac{1}{4} \left(\frac{d\phi}{dt} \right)$



$$= \frac{1}{4} (\pi R^2) \cdot \frac{dB}{dt} = \frac{\pi R^2 \alpha}{4} \left(\because \phi = B \cdot A \text{ and } \frac{dB}{dt} = \alpha \right)$$

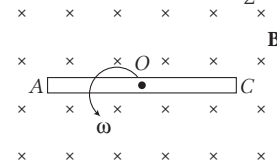
92 (a) According to Fleming's right hand rule, P is at higher potential and Q is at lower potential. Therefore, A is positively charged and B is negatively charged.

Also, charge, $Q = CV = C(Bvl)$

$$= 10 \times 10^{-6} \times 4 \times 2 \times 1 = 80 \mu\text{C}$$

$$\therefore q_A = 80 \mu\text{C} \text{ and } q_B = -80 \mu\text{C}$$

93 (c) For rotating rod, induced emf, $v = \frac{1}{2} Bl^2 \omega$



$$\text{For part AO, } V_{OA} = V_O - V_A = \frac{1}{2} Bl^2 \omega$$

$$\text{For part OC, } V_{OC} = V_O - V_C = \frac{1}{2} B(3l)^2 \omega$$

$$\therefore V_A - V_C = V_{OC} - V_{OA} = (V_O - V_C) - (V_O - V_A) = 4Bl^2 \omega$$

94 (b) In L - R circuit, for growth of current,

$$i = i_0 (1 - e^{-Rt/L}) \text{ or } \frac{3}{4} i_0 = i_0 (1 - e^{-t/\tau_L}) \text{ (given)}$$

$$\text{(where, } \tau_L = \frac{L}{R} = \text{time constant)}$$

$$\therefore e^{-t/\tau_L} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$e^{t/\tau_L} = 4 \text{ or } \frac{t}{\tau_L} = \ln 4$$

$$\tau_L = \frac{t}{\ln 4} = \frac{4}{2 \ln 2} \Rightarrow \tau_L = \frac{2}{\ln 2} \text{ s}$$

- 95** (d) As, the coil is rotated, angle θ (angle which is normal to the coil makes with $\mathbf{B}$ at any instant t) changes, therefore magnetic flux ϕ linked with the coil changes and hence an emf is induced in the coil. At this instant t , if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NSB \cos \omega t)$$

where, N is number of turns in the coil.

$$\text{or } e = -NSB \frac{d}{dt}(\cos \omega t) = -NSB(-\sin \omega t)\omega$$

$$\text{or } e = NSB\omega \sin \omega t \quad \dots(i)$$

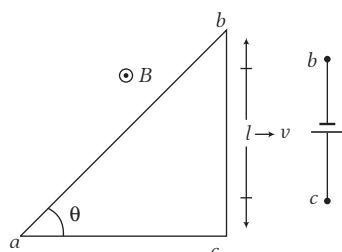
The induced emf will be maximum, when $\sin \omega t$ is maximum.

$$\therefore e_{\max} = e_0 = NSB\omega \times 1$$

$$\text{or } e = e_0 \sin \omega t$$

Therefore, e would be maximum, hence current is maximum (as $i_0 = e_0 / R$), when $\theta = 90^\circ$, i.e. normal of plane of coil is perpendicular to the field or plane of coil is parallel to magnetic field.

- 96** (a)

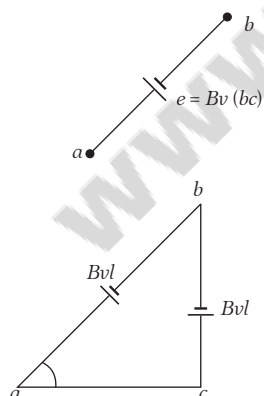


For segment bc , emf $e = Bvl \sin 90^\circ = Bv(bc)$

For segment ac , emf $ac: e = Bv(ac) \sin 0^\circ = 0$

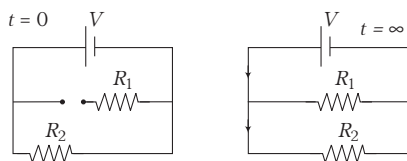
For segment ab , emf $ab: e = Bv(ab) \sin \theta = Bv(bc)$

Since, $(bc) = (ab) \sin \theta$



For whole loop, $e' = Bvl - Bvl = 0$

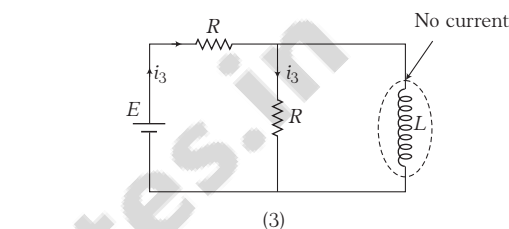
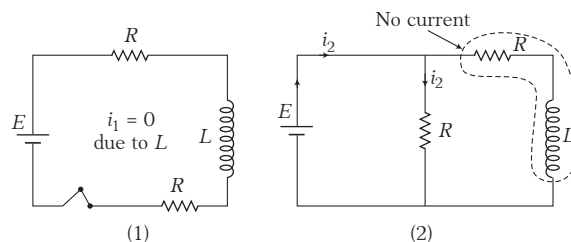
- 97** (c) At $t = 0$ inductor behaves as broken wire, then $i = \frac{V}{R_2}$.



At $t = \infty$, inductor behaves as a conducting wire,

$$i = \frac{V}{R_1 R_2 / (R_1 + R_2)} = \frac{V(R_1 + R_2)}{R_1 R_2}$$

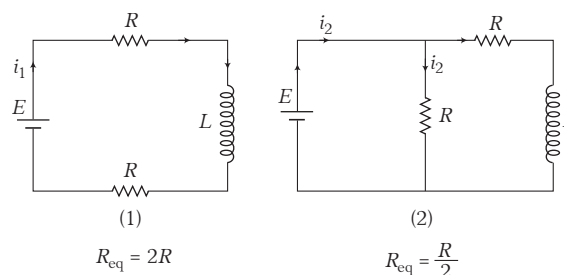
- 98** (a) (i) Just before closing the switch,



$$\text{As, } i_1 = 0, i_2 = \frac{E}{R}, i_3 = \frac{E}{2R},$$

$$\text{So, } i_2 > i_3 > i_1, [i_1 = 0].$$

- (ii) After a long time closing the switch,



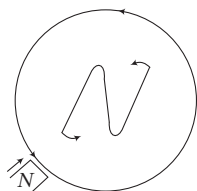
$$\text{As, } i = \frac{E}{R}$$

$$\Rightarrow i_1 = E/2R, i_2 = 2E/R, i_3 = E/R$$

Therefore, $i_2 > i_3 > i_1$.

- 99** (a) When the north pole of a bar magnet moves towards the coil, the induced current in the coil flows in a direction such that the coil presents its north pole to the bar magnet as shown in Fig. (a).

Therefore, the induced current flows in the coil in the anti-clockwise direction.



- 100** (b) When ring enters and leaves, then the field polarity of induced emf is opposite. Also, during the stay of ring completely in the field there is no induction.

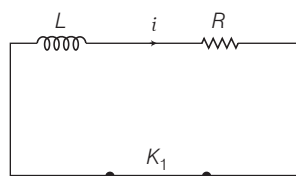
These variations are correctly shown in (b).

- 101** (d) When switch K_1 closed for long time current through

$$\text{inductor, } I = \frac{E}{R} = \frac{15}{0.15 \times 10^3} = 0.1 \text{ A}$$

When K_1 opened and K_2 is closed, then $i = Ie^{-t/\tau}$.

$$t = 1 \text{ ms} = 1 \times 10^{-3} \text{ s}$$



$$\therefore \tau_L = \frac{L}{R} = \frac{0.03}{0.15 \times 10^3} = \frac{10^{-3}}{5}$$

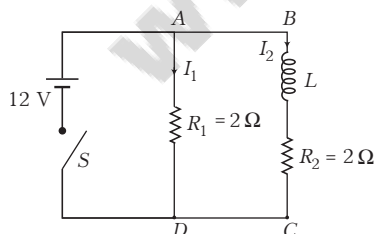
$$\therefore i = 0.1e^{\left(\frac{-1 \times 10^{-3}}{\frac{10^{-3}}{5}}\right)} = 0.1e^{-5}$$

$$i = \frac{0.1}{e^5} = \frac{0.1}{150} = \frac{1}{15} \times 10^{-2} \times 10^3 \text{ mA}$$

$$\text{Thus, } i = 0.67 \text{ mA}$$

- 102** (d) $E(\text{across } BC) = L \frac{dI_2}{dt} + R_2 I_2$ [using Kirchhoff's law] ... (i)

$$I_2 = I_0 (1 - e^{-t/t_0})$$



$$\text{At } t = 0, I_0 = \frac{E}{R_2} = \frac{12}{2} = 6 \text{ A}$$

$$\tau_L = t_0 = \frac{L}{R} = \frac{400 \times 10^{-3}}{2} = 0.2 \text{ s}$$

$$\therefore I_2 = 6(1 - e^{-t/0.2}) \quad [\text{using Eq. (i)}]$$

$$\text{Potential drop across } L = E - R_2 I_2 = 12 - 2 \times 6(1 - e^{-t/0.2})$$

$$= 12e^{-t/0.2} = 12e^{-5t} \text{ V}$$

- 103** (d) When two solenoids of inductance L_0 are connected in series at large distance and current i is passed through them, then the total flux linkage, $\Phi_{\text{total}} = L_0 i + L_0 i$.

If L be the equivalent inductance of the system, then

$$\Phi_{\text{total}} = Li$$

$$\therefore Li = L_0 i + L_0 i \quad \text{or } L = 2L_0$$

When solenoids are connected in series with one inside the other and senses of the turns coinciding, then there will be a mutual inductance L between them. In this case, the resultant induced emf in the coils is the sum of the emf's e_1 and e_2 in the respective coils, i.e.

$$e = e_1 + e_2$$

$$= \left(-L_0 \frac{di}{dt} \pm L_0 \frac{di}{dt} \right) + \left(-L_0 \frac{di}{dt} \pm L_0 \frac{di}{dt} \right)$$

where, (+) sign is for positive coupling and (−) sign for negative coupling.

$$\text{But, } e = -L \frac{di}{dt}$$

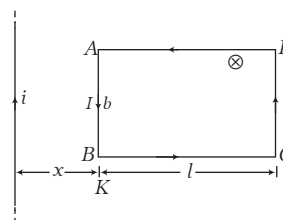
$$\therefore -L \frac{di}{dt} = -L_0 \frac{di}{dt} - L_0 \frac{di}{dt} \pm 2L_0 \frac{di}{dt}$$

$$\text{i.e. } L = L_0 + L_0 + 2L_0 = 4L_0 \quad (\text{for positive coupling})$$

When solenoids are connected in series with one inside the other with senses of the turn opposite, then there is a negative coupling.

$$\text{So, } L = L_0 + L_0 - 2L_0 = 0$$

- 104** (c) We can show the situation as



Since, loop is moving away from the wire, so the direction of current in the loop will be as shown in the figure.

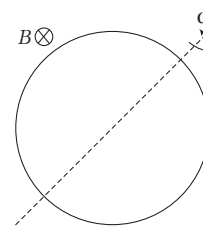
Net magnetic field on the loop due to wire,

$$B = \frac{\mu_0 i}{2\pi} \left(\frac{1}{x} - \frac{1}{l+x} \right) = \frac{\mu_0 i l}{2\pi x(l+x)}$$

So, the magnitude of the emf in the loop,

$$e = vBb = \frac{\mu_0 i l v b}{2\pi x(l+x)}$$

- 105** (b)



Let at any time, normal to loop makes an angle θ with magnetic field,

$$\phi_B = NBS \cos \theta = NBS \cos \omega t$$

$$e = -\frac{d\phi_B}{dt} = NBS \omega \sin \omega t = e_0 \sin \omega t$$

$$\therefore e_{\max} = e_0 = NBS\omega = 1 \times 0.01 \times \pi (5)^2 \times 10^{-4} \times \frac{80 \times 2\pi}{60}$$

$$= 6.6 \times 10^{-4} \text{ V}$$

Over a long period, i.e. one time period, average of induced emf,

$$\bar{e} = \frac{\int_0^T e dt}{T} = 0$$

and
$$\overline{e^2} = \frac{\int_0^T e^2 dt}{T} = \frac{e_0^2}{2} = 2.0 \times 10^{-7} \text{ V}^2$$

106 (c) As, $E l = \frac{d\phi_B}{dt}$

or $E (2\pi R) = \pi R^2 \cdot \frac{dB}{dt} = \pi R^2 (8t)$

$$\therefore E = 4Rt$$

$$F = qE = 4qRt$$

(tangential)

$$F = 4qR^2 t$$

$$f = (\mu mgR)$$

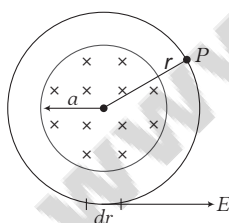
When $F > f$, ring will start rotating.

At $t = 2 \text{ s}$,

$$8qR^3 = \mu mgR$$

$\therefore$ Coefficient of friction, $\mu = \frac{8qR}{mg}$

107 (b)



Draw a concentric circle of radius r . The induced electric field E at any point on the circle is equal to that at P .

For this circle, induced emf,

$$e = \oint \mathbf{E} \cdot d\mathbf{l} = \left| \frac{d\phi}{dt} \right| = S \left| \frac{dB}{dt} \right|$$

$$\therefore e = E \int dl = \pi a^2 \left| \frac{dB}{dt} \right| \quad (\text{but } \oint dl = 2\pi r)$$

$$\therefore E \times 2\pi r = \pi a^2 \left| \frac{dB}{dt} \right|$$

$$\therefore E = \frac{a^2}{2r} \left| \frac{dB}{dt} \right|$$

$$\Rightarrow E \propto \frac{1}{r}$$

108 (d) When switch S is closed, then magnetic field lines passing through Q increases in the direction from right to left.

So, according to Lenz's law, induced current in Q , i.e. I_{Q_1} will flow in such a direction, so that the magnetic field lines due to I_{Q_1} passes from left to right through Q . This is possible when I_{Q_1} flows in anti-clockwise direction as seen by E .

Opposite is the case when switch S is opened, i.e. I_{Q_2} will be clockwise as seen by E .

109 (a) As the north pole approaches, a north pole is developed at the face, i.e. the current flows anti-clockwise. Finally, when it completes the oscillation, no emf is present.

Now, south pole approaches the other side, i.e. RHS, the current flows clockwise to repel the south pole. This means the current is anti-clockwise at the LHS of the coil as before. The break occurs, when the pendulum is at the extreme and momentarily stationary position.

(B) Medical entrance special format questions

• Assertion and reason

1 (d) Self-inductance does not depend upon current flowing or change in current flowing but it depends upon the number of turns N , area of cross-section (A) and permeability of medium (μ).

We have, $e = -L \frac{di}{dt} \Rightarrow L = \frac{-e}{(di/dt)}$ or $L \propto \frac{1}{(di/dt)}$

2 (a) If switch is closed in a circuit containing a pure inductor L and resistor R in series with a battery, then current through the circuit rises exponentially and reaches upto a certain maximum value. So, the corresponding potential drop across inductor is also maximum.

3 (b) If moving from left to right, current is increasing, then battery $\left(\text{induced emf} = L \cdot \frac{di}{dt} \right)$ will produce current from left to right current, i.e. its positive terminal is on left hand side or $V_A > V_B$.

Further $i = \text{constant} \Rightarrow \frac{di}{dt} = 0$ or $V_A - V_B = V_L$.

$$\Rightarrow V_A - V_B = 0 \Rightarrow V_A = V_B$$

4 (a) $V = L \frac{di}{dt} = \text{constant}$

$$\therefore L (di) = \text{constant}$$

$$\Rightarrow Li = \text{constant}$$

or $i \propto \frac{1}{L}$

5 (b) We know that, $\phi = BA \cos \omega t$

At $t = 0$, $\phi = \text{maximum}$

$$|e| = \left| \frac{d\phi}{dt} \right| = B \omega A \sin \omega t$$

For maximum value of $|e|$ is at, $\omega t = 90^\circ$, i.e. loop is rotated through 90° or it is parallel to magnetic field. At this instant, $\phi = 0$.

• Statement based questions

- 1 (c) In L - R circuit, current grows exponentially.
So, B_2 lights up earlier and finally B_1 shines brighter than B_2 .

- 2 (d) We have, $V = 2t$

$$\therefore V = L \cdot \frac{di}{dt} = 2t \text{ or } 2 \cdot \frac{di}{dt} = 2t \text{ or } \frac{di}{dt} = t$$

$$\text{or } (di) = t (dt)$$

$$\text{On integrating, we get, } i = \frac{t^2}{2}$$

i.e. i - t graph is a parabola.

$$\text{At } t = 2 \text{ s, } i = 2 \text{ A}$$

$$\therefore \text{Potential energy, } U = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \times 4 = 4 \text{ J}$$

$$\frac{dU}{dt} = Li \left(\frac{di}{dt} \right) = (2) \left(\frac{t^2}{2} \right) (t) = t^3$$

$$\text{At } t = 1,$$

$$\text{Potential energy, } \frac{dU}{dt} = 1 \text{ J s}^{-1}$$

- 3 (b) In position II, change in flux = 0, emf = 0.

In position I and III, $e = Bav$.

In position I, flux is increasing, hence anti-clockwise emf.

If position III, flux is decreasing, hence clockwise emf.

- 4 (d) $e = - \frac{d\phi}{dt} = -A \cdot \left(\frac{dB}{dt} \right) = -A (\text{slope of } B\text{-}t \text{ graph})$

At $t = 2$, slope is zero, so emf and current both are zero and it changes its sign.

- 5 (a) If bar magnet is falling vertically through the hollow region of long vertical copper tube, then the magnetic flux linked with the copper tube (due to non-uniform magnetic field of magnet) changes and eddy currents are generated in the body of the tube. The eddy currents oppose the falling of the magnet which, therefore experiences a retarding force. The retarding force increases with increasing velocity of the magnet and finally equals the weight of the magnet. The magnet, then attains a constant final terminal velocity, i.e. magnet ultimately falls with zero acceleration in the tube.

- 6 (d) At B , change of flux is sharp, so from $|e| = \frac{d\phi}{dt}$ at B , $|e| = 0$.

• Match the columns

- 1 (c) A. Tesla $\rightarrow [MT^{-2}A^{-1}]$
B. Weber $\rightarrow [ML^2T^{-2}A^{-1}]$
C. Weber/m² $\rightarrow [MA^{-1}T^{-2}]$
D. Henry $\rightarrow [ML^2A^{-2}T^{-2}]$

Hence, $A \rightarrow r$; $B \rightarrow s$; $C \rightarrow r$ and $D \rightarrow p$.

- 2 (d) V_{AD} (at $t = 0$) = $E = 10 \text{ V}$

$$V_{BC} \text{ (at } t = 0) = E = 10 \text{ V}$$

$$V_{AB} \text{ at } t = \infty$$

$$I = \frac{E}{R+r} = \frac{10}{4+1} = 2 \text{ A}$$

$$\therefore V_{AB} = V_A - V_B = I \times R = 2 \times 4 = 8 \text{ V}$$

$$V_{AD} \text{ at } t = \infty$$

$$V_{AD} = V_A - V_D = E - Ir = 10 - (2 \times 1) = 8 \text{ V}$$

Hence, $A \rightarrow s$; $B \rightarrow s$; $C \rightarrow q$; and $D \rightarrow q$.

- 3 (b) In all of the cases, the induced current in the loop is non-zero. The direction of induced current can be determined by Lenz's law. When loop is moved towards right, then induced current in the loop is clockwise. When loop is moved towards left, then induced current in the loop is anti-clockwise.

Similar observations are observed when wires 1 and 2 are moved in left and right sides, respectively.

Hence, $A \rightarrow p, s$; $B \rightarrow q, s$; $C \rightarrow p, s$ and $D \rightarrow q, s$.

- 4 (a) When current i_2 is decreased, then flux through 1 decreases. According to Lenz's law, attraction occurs between 1 and 2. Similarly, when current i_1 is increased, then loops will repel each other. Loops will repel each other and current i_1 will increase, when loop 1 is moved towards loop 2.

When loop 2 is moved away, loops will attract and i_2 will increase.

Hence, $A \rightarrow q$; $B \rightarrow p, p$; $C \rightarrow q, r$ and $D \rightarrow p, s$.

5. (d)

- A. The magnetic flux through the rectangular loop $abcd$ increases, due to the motion of the loop into the region of magnetic field. The induced current must flow along the path $bcadb$, so that it opposes the increasing flux.
B. Due to the outward motion, magnetic flux through the triangular loop abc decreases due to which the induced current flows along $bach$, so as to oppose the change in flux.
C. As the magnetic flux decreases due to motion of the irregular shaped loop $abcd$ out of the region of magnetic field, the induced current flows along $cdabc$, so as to oppose change in flux.

Hence, $A \rightarrow r$; $B \rightarrow p$ and $C \rightarrow q$.

(C) Medical entrances' gallery

- 1 (d) Magnetic flux linked with coil,

$$\phi = (5t^2 + 3t + 16) \text{ Wb}$$

$$\text{Magnitude of induced emf, } e = \frac{d\phi}{dt} = \frac{d}{dt} (5t^2 + 3t + 16) = 10t + 3$$

$$\text{At } t = 3 \text{ s,}$$

$$e_3 = 10 \times 3 + 3 = 33 \text{ V}$$

$$\text{At } t = 4 \text{ s}$$

$$e_4 = 10 \times 4 + 3 = 43 \text{ V}$$

$\therefore$ Induced emf in coil at the fourth second is given as

$$e = e_4 - e_3 = 43 - 33 = 10 \text{ V}$$

- 2 (a) Given, magnetic field, $B = 0.4 \text{ G} = 0.4 \times 10^{-4} \text{ T}$

$$l = 1 \text{ m}$$

$$\text{Frequency, } f = 120 \text{ rpm} = \frac{120}{60} \text{ rps} = 2 \text{ Hz}$$

Induced emf between the axle and rim of the wheel is given as

$$\begin{aligned} e &= \frac{1}{2} B \omega l^2 = \frac{1}{2} B (2\pi f) l^2 = \pi B f l^2 \\ &= 3.14 \times 0.4 \times 10^{-4} \times 2 \times 1^2 = 2.51 \times 10^{-4} \text{ V} \end{aligned}$$

- 4 (c) Given, area of coil, $A = 0.05 \text{ m}^2$,

magnetic field, $B = 5 \times 10^{-5} \text{ T}$

and number of turns, $N = 800$.

The magnetic flux linked with the coil,

$$\phi = N(\mathbf{B} \cdot \mathbf{A}) = NBA \cos \theta \quad \dots (i)$$

where, θ is the angle between $\mathbf{B}$ and $\mathbf{A}$.

The emf induced when the coil is rotated from $\theta_1 = 0^\circ$ to $\theta_2 = 90^\circ$,

$$\begin{aligned} e &= -\frac{\Delta \phi}{\Delta t} = -\frac{\Delta}{\Delta t} (NBA \cos \theta) \quad [\text{using Eq. (i)}] \\ &= -\frac{NBA}{\Delta t} (\cos \theta_2 - \cos \theta_1) \quad \dots (ii) \end{aligned}$$

Here, $\Delta t = 0.1 \text{ s}$

Thus, substituting the given values in Eq. (ii), we get

$$\begin{aligned} e &= -\frac{\left[800 \times 5 \times 10^{-5} \times 0.05 \right]}{0.1} \\ &\quad \times (\cos 90^\circ - \cos 0^\circ) \\ &= 2000 \times 10^{-5} = 0.02 \text{ V} \end{aligned}$$

- 5 (c) Electric heaters are not based on the eddy current effect. Its working is based on Joule's heating effect of current. According to this effect, the passage of an electric current through a resistor produces heat.

However, when a changing magnetic flux is applied to a bulk piece of conducting material, then circulating current is induced in the body of this conductor, which is usually known as eddy currents. This current shows both heating and magnetic effects.

Thus, it is the basic principle behind the working of magnetic braking in train, electromagnet and induction furnace.

- 6 (a) When a metallic surface is moved in and out in magnetic field, then magnetic flux linked with metallic surface is changed continuously, hence according to Faraday's law of electromagnetic induction, an induced emf is produced. Since, metallic plate behaves as a closed path, therefore an induced current (eddy current) starts flowing in the metallic surface.

- 7 (d) Given, magnetic potential energy stored in an inductor,

$$U = 25 \text{ mJ} = 25 \times 10^{-3} \text{ J}$$

Current in an inductor, $i_0 = 60 \text{ mA} = 60 \times 10^{-3} \text{ A}$

As, the expression for energy stored in an inductor is given as

$$U = \frac{1}{2} L i_0^2 \quad \dots (i)$$

where, L is the inductance of the inductor.

Substituting the given values in above Eq. (i), we get

$$(25 \times 10^{-3}) = \frac{1}{2} \times L \times (60 \times 10^{-3})^2$$

$$\Rightarrow L = \frac{2 \times 25 \times 10^{-3}}{3600 \times 10^{-6}} = \frac{500}{36}$$

$$\text{or } L = 13.89 \text{ H}$$

- 8 (a) Coil A carries a steady current with increase in temperature, its resistance increases and so current is decreasing at a constant rate, this induces an emf in coil B which opposes this change, i.e. current in coil B is in same direction of A, therefore they attract to each other.

- 9 (c) Given, number of turns of solenoid, $N = 1000$

Current, $I = 4 \text{ A}$

Magnetic flux, $\phi_B = 4 \times 10^{-3} \text{ Wb}$

$\therefore$ Self-inductance of solenoid is given by

$$L = \frac{\phi_B N}{I} \quad \dots (i)$$

Substituting the given values in Eq. (i), we get

$$L = \frac{4 \times 10^{-3} \times 1000}{4} = 1 \text{ H}$$

- 10 (d) Induced emf in the region is given by

$$e = -\frac{d\phi}{dt}$$

where, $\phi = \mathbf{B} \cdot \mathbf{A} = \pi r^2 \mathbf{B}$

$$\Rightarrow \frac{d\phi}{dt} = -\pi r^2 \frac{d\mathbf{B}}{dt}$$

Rate of change of magnetic flux associated with loop 1,

$$e_1 = -\frac{d\phi_1}{dt} = -\pi r^2 \frac{d\mathbf{B}}{dt}$$

Similarly, $e_2 =$ emf associated with loop 2

$$= -\frac{d\phi_2}{dt} = 0 \quad (\because \phi_2 = 0)$$

- 11 (a) Given, $n = 500$,

$$L = 50 \text{ mH} = 50 \times 10^{-3} \text{ H}$$

and $i = 8 \text{ mA} = 8 \times 10^{-3} \text{ A}$

The magnetic flux linked with the coil,

$$\begin{aligned} \phi &= Li = 50 \times 10^{-3} \times 8 \times 10^{-3} \\ &= 400 \times 10^{-6} = 4 \times 10^{-4} \text{ Wb} \end{aligned}$$

- 12 (a) The flux is given by

$$\phi = \mathbf{B} \cdot \mathbf{A}$$

$$\phi = |\mathbf{B}| |\mathbf{A}| \cos \omega t$$

$$\text{Also, induced emf, } e = -\frac{d\phi}{dt} = -\frac{d[|\mathbf{B}| |\mathbf{A}| \cos \omega t]}{dt}$$

$$= -|\mathbf{B}| |\mathbf{A}| \frac{d(\cos \omega t)}{dt} = |\mathbf{B}| |\mathbf{A}| \sin \omega t \cdot \omega$$

$$\Rightarrow e = BA \omega \cos (\omega t - \pi / 2)$$

- 13 (b) Given, $B = 40 \text{ mT} = 40 \times 10^{-3} \text{ T}$

Rate of shrinking of area $= 0.5 \text{ m}^2 \text{ s}^{-1}$

$$\Rightarrow \frac{dA}{dt} = 0.5 \text{ m}^2 \text{ s}^{-1}$$

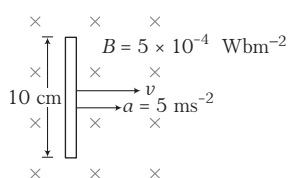
As, $\phi = BA \cos \theta = BA$ ($\because \cos \theta = \cos 0^\circ = 1$)

On differentiating with w.r.t. to t ,

$$\frac{d\phi}{dt} = B \cdot \frac{dA}{dt} = e$$

$$\begin{aligned} \Rightarrow e &= B \cdot \frac{dA}{dt} = 40 \times 10^{-3} \times 0.5 \\ &= 20 \times 10^{-3} \text{ V} \\ &= 20 \text{ mV} \end{aligned}$$

- 15 (b) Consider a rod of 10 cm length is moving perpendicular to uniform magnetic field as shown below



The emf induced in the rod, $e = vBl$

Differentiating with respect to time t , we get

$$\begin{aligned} \text{Rate of increase of induced emf} &= \frac{de}{dt} \\ &= \frac{dv}{dt} Bl = aBl \\ &= 5 \times 5 \times 10^{-4} \times 10 \times 10^{-2} \\ &= 250 \times 10^{-6} \\ &= 2.5 \times 10^{-4} \text{ Vs}^{-1} \end{aligned}$$

- 16 (b) Induced emf for both the loops will be given as

$$e = \frac{-d\phi}{dt} = B\omega \sin \omega t$$

Since, B and ω are same for both the cases, hence induced emf will be same.

However, induced current is given by

$$I = \frac{e}{R}$$

So, it will depend upon the value of resistance, lower the resistance higher will be the current. Therefore, induced current is different in both the loops.

- 17 (b) Given, length of conductor, $l = 0.1 \text{ m}$

Magnetic field, $B = 0.1 \text{ T}$

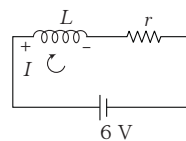
Velocity of conductor, $v = 15 \text{ ms}^{-1}$

The angle between v and B is 90° .

When v and B are mutually perpendicular, then emf (induced) is given by

$$\begin{aligned} e &= vBl = 15 \times 0.1 \times 0.1 \\ &= \frac{15}{100} = 0.15 \text{ V} \end{aligned}$$

- 18 (c) Applying KVL in the circuit, we can write



$$6 - L \frac{dI}{dt} - Ir = 0$$

At initial stage, $I = 0$

$$\Rightarrow 6 = L \frac{dI}{dt} \text{ or } \frac{dI}{dt} = \frac{6}{L} = \frac{6}{2} = 3 \text{ As}^{-1}$$

- 19 (d) As magnetic field through circle is constant because current through straight wire is constant. Magnetic flux through circle is not changing with time.

$$\frac{d\phi_B}{dt} = 0$$

$$\Rightarrow |e| = \left| \frac{d\phi_B}{dt} \right| = 0 \Rightarrow i = \frac{e}{R} = \frac{0}{R} = 0$$

Thus, no current flow in the circle.

- 20 (c) We know that, $e = NBA\omega \sin \omega t$

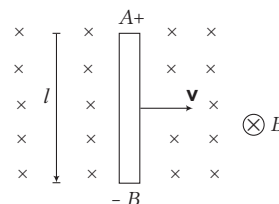
where, N = number of loops = 1

$$B = \frac{\mu_0 I}{2b} \text{ N/A-m} \Rightarrow A = \pi a^2 \text{ m}^2$$

$$\therefore e = \frac{\mu_0 I}{2b} (\pi a^2) \omega \sin \omega t$$

$$\Rightarrow = \frac{\pi a^2 \mu_0 I}{2b} \cdot \omega \sin \omega t$$

- 21 (c) Consider the diagram, where a conductor AB of length l moving perpendicular to an inward magnetic field B .



Value of induced emf,

$$|E_{\text{ind}}| = |l(\mathbf{v} \times \mathbf{B})|$$

$$\Rightarrow = lvB \sin 90^\circ \quad (\because v \perp B)$$

$$E_{\text{ind}} = vBl \quad \dots(i)$$

Given, $v = 7 \text{ ms}^{-1}$, $l = 0.4 \text{ m}$

and $B = 0.9 \text{ Wbm}^{-2}$

Now, from Eq. (i), we get

$$E_{\text{ind}} = 7(0.9)(0.4) \text{ V} = 2.52 \text{ V}$$

- 22** (a) Consider the inductor of inductance L and the current flowing through the inductor is i .

Now, we can write $\phi = Li$

where, ϕ is magnetic flux linked with the inductor.

$$\Rightarrow \frac{d\phi}{dt} = L \frac{di}{dt}$$

Given, $L = 40 \text{ mH}$

$$dt = \text{change in time} = t_2 - t_1 = 4 \text{ ms} = \Delta t$$

$$\text{and } di = i_2 - i_1 = 11 - 1 = 10 \text{ A} = \Delta i$$

$$\text{So, } \frac{d\phi}{dt} = \frac{\Delta\phi}{\Delta t} = L \frac{\Delta i}{\Delta t} = 40 \times \frac{10}{4}$$

$$= 10 \times 10 = 100 \quad \dots(i)$$

According to Faraday's law of electromagnetic induction,

$$\text{Emf induced, } e = -\frac{d\phi}{dt}$$

$$\Rightarrow |e| = \frac{\Delta\phi}{\Delta t} = 100 \text{ V} \quad [\text{from Eq. (i)}]$$

- 23** (b) $\therefore$ Induced emf, $e = -L \frac{di}{dt}$

$$\text{Given, } L = 10 \text{ H, } \Delta i = (9 - 4) \text{ A} = 5 \text{ A, } dt = 0.2 \text{ s}$$

$$\text{emf, } e = 10 \times \frac{5}{0.2} = 250 \text{ V}$$

- 24** (c) As, $e = Bvl$

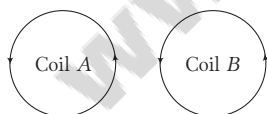
$$\text{Here, } B = 10^{-3} \text{ Wb/m}^2$$

$$v = 2 \text{ m/s and } l = 5 \text{ cm} = 0.05 \text{ m}$$

$$\therefore \text{emf, } e = (10^{-3}) (2) (0.05) = 1 \times 10^{-4} \text{ V}$$

- 25** (c) As the current in coil A changes, the magnetic flux linked with coil B changes. Direction of flux in B will be downwards, hence current induces in coil B is in anti-clockwise direction according to Lenz's law.

The direction of induced current in coil B is opposite to that of the direction of current in coil A as shown in figure



Due to currents in opposite directions in the near by sides of the coil, the coil B is repelled.

- 26** (c) Given, $n = 100$ turns, $A = (0.1 \times 0.05) \text{ m}^2$

$$B_1 = 0.1 \text{ T, } B_2 = 0.05 \text{ T, } dt = 0.05 \text{ s}$$

We know that,

$$e = \left| \frac{-d\phi}{dt} \right| = \left| \frac{d}{dt} (nBA \cos \theta) \right| = \left| \frac{nA dB \cos \theta}{dt} \right|$$

Here, $\theta = 0^\circ$

$$\therefore e = \left| \frac{nA dB}{dt} \right| = \left| \frac{100 \times 0.1 \times 0.05 \times (0.1 - 0.05)}{0.05} \right|$$

$$\Rightarrow e = 0.5 \text{ V}$$

- 27** (a) Electromagnetic induction is not used in room heater because it works on the principle of heating effect of electric current.

- 28** (d) Given, $M = 0.05 \text{ H}$ and $I = I_0 \sin \omega t$

$$\therefore \frac{dI}{dt} = I_0 \cos \omega t(\omega)$$

$$\Rightarrow \left(\frac{dI}{dt} \right)_{\max} = I_0(\omega) \times 1 = 1 \times 100 \pi \text{ A/s}$$

$$\begin{aligned} \text{So, maximum emf, } e_{\max} &= M \left(\frac{dI}{dt} \right)_{\max} \\ &= 0.05 \times 100\pi \\ &= 5\pi \text{ V} \end{aligned}$$

- 29** (d) For inductor, as we know induced voltage,

(for $t = 0$ to $t = T/2$)

$$V = L \frac{dI}{dt} = L \frac{d}{dt} \left(\frac{2I_0 t}{T} \right) = \text{constant}$$

(for $t = T/2$ to $t = T$)

$$V = \frac{L dI}{dt} \left(\frac{-2I_0 t}{T} \right) = -\text{constant}$$

Therefore, correct variation will be represented by graph (d).

- 30** (a) Given, $\phi = (4t^2 + 6t + 9) \text{ Wb}$ and $t = 2 \text{ s}$

$$\text{We know that, } |\mathcal{E}| = \left| \frac{d\phi}{dt} \right|$$

$$\text{Here, } \left| \frac{d\phi}{dt} \right| = |8t + 6|$$

$$\Rightarrow \left(\frac{d\phi}{dt} \right)_{t=2} = 8 \times 2 + 6$$

$$\therefore \text{Induced emf in the coil} = 22 \text{ V}$$

- 31** (d) Induced emf,

$$e = -L \frac{di}{dt}$$

During 0 to $\frac{T}{4}$, $\frac{di}{dt} = \text{constant and positive}$, so $e = \text{negative}$.

For $\frac{T}{4}$ to $\frac{T}{2}$, $\frac{di}{dt} = 0$, so $e = 0$.

For $T/2$ to $\frac{3T}{4}$, $\frac{di}{dt} = \text{constant and negative}$, so $e = \text{positive}$.

This condition is satisfied only by graph in option (d).

- 32** (a) Given, $B = 0.15 \text{ T, } l = 50 \text{ cm} = 50 \times 10^{-2} \text{ m}$

$$\text{and } v = 2 \text{ ms}^{-1}$$

$$\therefore \text{emf, } e = Bvl$$

$$e = 0.15 \times 2 \times 50 \times 10^{-2} = 0.15 \text{ V}$$

$$\text{Current, } i = \frac{e}{R} = \frac{0.15}{3} = 5 \times 10^{-2} \text{ A}$$

$$\begin{aligned} \text{Force, } F &= Bil = 0.15 \times 5 \times 10^{-2} \times 50 \times 10^{-2} \\ &= 3.75 \times 10^{-3} \text{ N} \end{aligned}$$

33 (a) Induced emf, $e = 2 \text{ V}$

$$i_1 = 8 \text{ A}, i_2 = 6 \text{ A and } \Delta t = 2 \times 10^{-3} \text{ s}$$

Coefficient of self-induction,

$$\begin{aligned} L &= \frac{e}{\Delta i / \Delta t} = \frac{-2}{(6 - 8) / 2 \times 10^{-3}} \\ &= \frac{-2 \times 2 \times 10^{-3}}{-2} \\ &= 2 \times 10^{-3} \text{ H} \end{aligned}$$

34 (c) Rate of doing work $= \frac{W}{t} = P$

$$\text{But, } P = Fv$$

$$\text{Also, } F = Bil = B \left(\frac{Bvl}{R} \right) l \left[\because \text{induced current, } i = \frac{Bvl}{R} \right]$$

$$\begin{aligned} \therefore \text{Power, } P &= B \left(\frac{Bvl}{R} \right) lv = \frac{B^2 v^2 l^2}{R} \\ &= \frac{(0.5)^2 \times (2)^2 \times (1)^2}{6} = \frac{1}{6} \text{ W} \end{aligned}$$

35 (a) Given, $\phi = 10 \text{ Wb}, \phi_2 = 60 \text{ Wb}$ and $R = 100 \Omega$

The amount of induced charge is given by

$$q = \frac{1}{R} \Delta \phi = \frac{1}{100} (60 - 10)$$

$$\therefore \text{Charge, } q = \frac{50}{100} = 0.5 \text{ C}$$

36 (a) Self-inductance, $L = \frac{\mu_0 N^2 \pi r^2}{l}$

$$\text{Given, } r_1 / r_2 = l_1 / l_2 = 1 / 2$$

$$\Rightarrow \frac{L_1}{L_2} = \left(\frac{r_1}{r_2} \right)^2 \left(\frac{l_2}{l_1} \right) = \left(\frac{1}{2} \right)^2 \times \left(\frac{2}{1} \right)$$

$$\Rightarrow \frac{L_1}{L_2} = \frac{1}{2} = 1 : 2$$

37 (b) For two such coils connected oppositely in series,

$$L_{\text{eq}} = L_1 + L_2 - 2M$$

$$\text{But } L_1 = L_2 = L \text{ and } M = 0$$

$$L_{\text{eq}} = L + L - 0 = 2L$$

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CHAPTER 07

Alternating Current

We have already studied in our previous chapters about electric and magnetic fields and how energy can be stored in capacitors and inductors. In this chapter, we will study how energy stored in one location can be transferred to another location, so that it can be put to use. Most of the electric power generated and used in the world is in the form of alternating current. Here, we will study about some alternating current system that transfers energy efficiently and we will also discuss some of the devices that make use of that energy.

TYPES OF CURRENT

There are two types of current which flows in any of the electrical appliances. These are as follow

Direct current (DC)

Current whose direction does not change with time through a load, is known as direct current (DC) and this voltage is known as direct voltage (DC voltage). The graph given below shows that the direction of direct current (I) or voltage (V) does not change with time (t).



Fig. 7.1 Voltage or current versus time graph for a DC

Alternating current (AC)

Current whose direction changes periodically through a load, is known as alternating current (AC) and the voltage is known as alternating voltage (AC voltage)

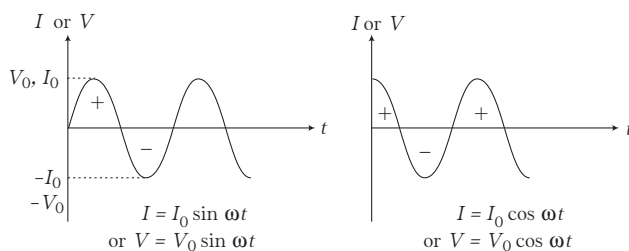


Fig. 7.2 Voltage or current versus time graph of an AC

Inside

1 Types of current

- Mean or average value of an alternating current
- Root mean square value of an alternating current
- Form factor
- Peak factor

2 Representation of I or V as rotating vectors

- Different types of alternating current circuits
- Inductor as low pass filter
- AC voltage applied to a series L-C-R circuit
- Parallel circuit (Rejector circuit)

3 Power in an AC circuit

- Half power points in series L-C-R circuit
- Wattless current
- L-C oscillations

4 Choke coil

- Transformer
- Electric generator or dynamo

Here, the magnitude of current or voltage increases from zero to a maximum value, then decreases to zero and reverses in direction, increases to a maximum value in this direction and then decreases to zero. The complete set of variations is known as a **cycle**. Thus, during one-half of the cycle the current or voltage flows in one direction, whereas in the next half-cycle, it flows in the opposite direction.

The instantaneous value of AC is given by

$$I = I_0 \sin \omega t$$

where, I = current at any instant t ,

I_0 = maximum or peak value of AC

and ω = angular frequency.

Alternating emf or voltage

The instantaneous value of alternating emf or voltage is given by

$$V = V_0 \sin \omega t$$

where, V = voltage at any time t ,

V_0 = maximum or peak value of alternating voltage

and ω = angular frequency.

Note Alternating current, alternating voltage or emf, flux, etc., all are sinusoidal waves.

Some definitions related to alternating current or voltage

- (i) **Time period** The time taken to complete one cycle of variations is called time period or periodic time. It is given by the formula,

$$T = \frac{2\pi}{\omega}$$

where, 2π = angular displacement in a complete cycle

and ω = angular frequency.

- (ii) **Frequency** The number of cycles completed per second by an alternating current is called its frequency and is denoted by f or ν .

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

The frequency of AC in India is 50 Hz, i.e.

$$f = 50 \text{ Hz} = 50 \text{ cps}$$

$$\text{or } \omega = 2\pi f \approx 314 \text{ rads}^{-1}$$

- (iii) **Peak value or amplitude** The maximum value of alternating current or alternating voltage is defined as peak value or amplitude. It is denoted by I_0 or V_0 .
- (iv) **Phase** Phase is a physical quantity representing both the instantaneous value and direction of alternating current at any instant. It is a dimensionless quantity and its unit is radian.

If $I = I_0 \sin(\omega t + \phi)$, then the argument of $\sin(\omega t + \phi)$ is called its phase.

- (v) **Phase difference** The difference between the phases of current and voltage is called phase difference. If alternating voltage and current is given by the relation,

$$V = V_0 \sin(\omega t + \phi_1)$$

$$\text{and } I = I_0 \sin(\omega t + \phi_2)$$

Then, phase difference, $\phi = \phi_2 - \phi_1$

(relative to voltage)

$$\text{and } \phi = \phi_1 - \phi_2 \quad (\text{relative to current})$$

Some important points related to alternating current

- An AC is produced by AC generator or AC dynamo and is represented by the symbol $\ominus$.
- The AC is converted into DC with the help of rectifier while DC is converted into AC with the help of inverter.
- It cannot produce electroplating or electrolysis.
- It is measured with the help of 'hot wire ammeter'.
- An AC can be transmitted over long distances without much power loss.
- In an AC (sinusoidal) current or voltage can have following four values
 - Instantaneous value
 - Peak value (I_0 or V_0)
 - rms value (I_{rms} or V_{rms})
 - Average value

Mean or average value of an alternating current

Average or mean value of AC is defined as the value of direct current which sends the same amount of charge in a circuit in the same time as is sent by the given alternating current in its half-cycle (i.e., $T/2$). It is denoted by I_{av} or I_m .

Let an alternating current be represented by $I = I_0 \sin \omega t$, where I_0 is the peak value. Clearly, the **mean value of the current over a complete cycle is zero**. It has no significance. Hence, the mean value of alternating current is defined as its average over half a cycle. For positive half cycle,

$$I_{\text{mean}} = \frac{1}{T/2} \int_0^{T/2} I dt$$

Putting $I = I_0 \sin \omega t$ and $T = (2\pi/\omega)$, we get

$$I_{\text{mean}} = \frac{2}{T} \int_0^{\pi/\omega} I_0 \sin \omega t \, dt = \frac{\omega}{\pi} \frac{I_0}{\omega} [-\cos \omega t]_0^{\pi/\omega}$$

$$= -\frac{I_0}{\pi} [\cos \pi - \cos 0] = -\frac{I_0}{\pi} [-1 - 1]$$

Thus, $I_{\text{mean}} = \frac{2}{\pi} I_0 = 0.637 I_0 = 63.7\% \text{ of } I_0$
 ...[Mean current over positive half cycle]
 In the same way, mean value of alternating emf,

$$E_{\text{mean}} = \frac{2E_0}{\pi} = 0.637 E_0 = 63.7\% \text{ of } E_0$$

Similarly, for negative half cycle the mean value of alternating current is $-2I_0/\pi$ and the average value of emf for negative half cycle is $-2E_0/\pi$.

Average values of some important expressions of current and voltage

- When $I = I_1 \sin \omega t + I_2 \cos \omega t$
 and $V = V_1 \sin \omega t + V_2 \cos \omega t$
 where I_1, I_2, V_1 and V_2 are constants,
 then $I_{\text{av}} = 0$ and $V_{\text{av}} = 0$
- Similarly, when $I = I_0 + I_1 \sin \omega t + I_2 \cos \omega t$
 and $V = V_0 + V_1 \sin \omega t + V_2 \cos \omega t$
 then $I_{\text{av}} = I_0$ and $V_{\text{av}} = V_0$
- When $I = I_0 + I_1 \sin \omega t + I_2 \cos 2\omega t$
 and $V = V_0 + V_1 \sin \omega t + V_2 \cos 2\omega t$
 then $I_{\text{av}} = I_0$ and $V_{\text{av}} = V_0$
- When $I = I_1 \sin \omega t + I_2 \sin 2\omega t + I_3 \sin 3\omega t + I_0$
 and $V = V_1 \sin \omega t + V_2 \sin 2\omega t + V_3 \sin 3\omega t + V_0$
 then $I_{\text{av}} = I_0$ and $V_{\text{av}} = V_0$

Example 7.1 If the peak value of a current in 50Hz AC circuit is 7.07 A. What is the mean value of current over half a cycle and the value of current 1/300 s after it was zero?

Sol. As, $I_m = \frac{2I_0}{\pi}$, where I_0 = peak value of current = 7.07A

$$\therefore I_m = 0.637 \times 7.07$$

$$= 4.5 \text{ A}$$

$$\text{Also, at } t = \frac{1}{300} \text{ s}$$

$$I = I_0 \sin \omega t = I_0 \sin 2\pi f t \quad (\because \omega = 2\pi f)$$

$$\therefore I = 7.07 \sin \left(2\pi \times 50 \times \frac{1}{300} \right) = 7.07 \sin \frac{\pi}{3}$$

$$= 7.07 \times \frac{\sqrt{3}}{2} = 6.12 \text{ A}$$

Example 7.2 Find the average value in the following cases

- $i = 4 + 3 \cos \omega t$
- $V = 5 \sin \omega t + 3 \cos \omega t$
- $i = \sin \omega t + 2 \sin 2\omega t + 3 \sin 3\omega t$
- $V = \cos \omega t + 3 \cos 2\omega t + 3 \cos 3\omega t + 2$

Sol. (i) Average value of current, $I_{\text{av}} = I_0 = 4$ unit
 (ii) Average value of voltage, $V_{\text{av}} = 0$
 (iii) Average value of current, $I_{\text{av}} = 0$
 (iv) Average value of voltage, $V_{\text{av}} = 2$ unit

Root mean square value of an alternating current

It is defined as that value of a direct current which produces the same amount of heating effect in a given resistor as is produced by the given alternating current when passed for the same time during a complete cycle. It is denoted by I_{rms} , I_V or I_{eff} .

It is also called virtual value or effective value of AC.

Let instantaneous value of alternating current,

$$I = I_0 \sin \omega t$$

If dH is small amount of heat produced in time dt in resistor R , then $dH = I^2 R \, dt$. In complete cycle, the total heat produced is

$$\int dH = \int_0^T I^2 R \, dt$$

$$H = \int_0^T (I_0 \sin \omega t)^2 R \, dt$$

On solving, we get

$$H = \frac{I_0^2 R T}{2} \quad \dots(i)$$

If I_{rms} is rms value of alternating current and H is the heat produced by rms current, then

$$H = I_{\text{rms}}^2 R T \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

or

$$I_{\text{rms}} = 0.707 I_0$$

or

$$I_{\text{rms}} = 70.7\% \text{ of } I_0$$

In the same way, rms value of alternating emf

$$E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = 0.707 E_0 = 70.7\% \text{ of } E_0$$

AC ammeter and AC voltmeter always measure the virtual value of AC or alternating voltage. Also, the power supply which we are getting in our homes as 220 V AC is the rms voltage and its voltage amplitude is $V_0 = \sqrt{2} V_{\text{rms}} = 311 \text{ V}$.

This is the reason why AC is more dangerous than DC as 311 V will cause more harm to the human body than 220 V DC. The different values I_0 , I_{av} and I_{rms} are shown in figure below,

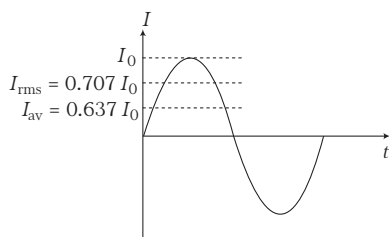


Fig. 7.3 Peak, rms and average value on the same graph

rms values for some important expressions of current and voltage

- (i) When $I = I_1 \sin \omega t + I_2 \cos \omega t$, where I_1 and I_2 are constant currents, then

$$I_{rms} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2}}$$

Similarly, when $V = V_1 \sin \omega t + V_2 \cos \omega t$, where V_1 and V_2 are constant voltages, then

$$V_{rms} = \sqrt{\frac{V_1^2}{2} + \frac{V_2^2}{2}}$$

- (ii) When $I = I_0 + I_1 \sin \omega t + I_2 \cos \omega t$
and $V = V_0 + V_1 \sin \omega t + V_2 \cos \omega t$

$$I_{rms} = \sqrt{I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2}}$$

$$\text{and } V_{rms} = \sqrt{V_0^2 + \frac{V_1^2}{2} + \frac{V_2^2}{2}}$$

- (iii) When $V = V_1 \sin \omega t + V_2 \sin 2\omega t + V_3 \sin 3\omega t + V_0$
and $I = I_1 \sin \omega t + I_2 \sin 2\omega t + I_3 \sin 3\omega t + I_0$, then

$$V_{rms} = \sqrt{\frac{V_1^2}{2} + \frac{V_2^2}{2} + \frac{V_3^2}{2} + V_0^2}$$

$$\text{and } I_{rms} = \sqrt{\frac{I_1^2}{2} + \frac{I_2^2}{2} + \frac{I_3^2}{2} + I_0^2}$$

Example 7.3

- (i) The peak voltage of an AC supply is 300 V. What is the rms voltage?
(ii) The rms value of current in an AC circuit is 10 A. What is the peak current?

Sol. (i) Here, peak value of AC supply, $V_0 = 300$ V

$$\therefore \text{Using the relation, } V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$\text{We get, } V_{rms} = \frac{300}{\sqrt{2}} = 212.1 \text{ V}$$

(ii) Let I_0 be the peak value of the current.

Here, I_{rms} = rms value of current = 10 A

$$\text{Then, } I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$\therefore I_0 = \sqrt{2} I_{rms} = 1.414 \times 10 = 14.14 \text{ A}$$

Example 7.4 If $V = 220\sqrt{2} \sin(314t - \phi)$, calculate (i) peak and rms value of the voltage, (ii) average voltage for half time-period and (iii) frequency of AC.

Sol. Given, $V = 220\sqrt{2} \sin(314t - \phi)$... (i)

Comparing with general equation, we get

(i) Peak voltage, $V_0 = 220\sqrt{2}$ V

$$\text{rms voltage, } V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{220\sqrt{2}}{\sqrt{2}} = 220 \text{ V}$$

$$(ii) \text{ Average voltage, } \bar{V} = \frac{2}{\pi} V_0 = \frac{2}{\pi} \times 220\sqrt{2} = \frac{440\sqrt{2}}{\pi} \text{ V}$$

(iii) As, $\omega = 314 \text{ rad s}^{-1}$

$$\Rightarrow \omega = 2\pi f = 314$$

$$\Rightarrow f = 50 \text{ Hz}$$

Example 7.5 If the current in an AC circuit is represented by the equation, $I = 5 \sin(300t - \pi/4)$

Here, t is in second and I is in ampere. Calculate,

- (i) peak and rms value of current,
(ii) frequency of AC,
(iii) and average current.

Sol. (i) Comparing the given equation with general equation, we have

$$I = I_0 \sin(\omega t \pm \phi)$$

$\therefore$ The peak value of current, $I_0 = 5$ A

$$\text{and } I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.535 \text{ A}$$

(ii) Angular frequency,

$$\omega = 300 \text{ rads}^{-1}$$

$$\therefore \nu = \frac{\omega}{2\pi} = \frac{300}{2\pi} \approx 47.75 \text{ Hz}$$

$$(iii) \text{ Average current, } I_{av} = \left(\frac{2}{\pi}\right) I_0 = \left(\frac{2}{\pi}\right) (5)$$

$$= 3.18 \text{ A}$$

Example 7.6 The voltage supplied to a circuit is given by $V = V_0 t^{3/2}$, where t is time in second. Find the rms value of voltage for the period, $t = 0$ to $t = 1$ s.

Sol. The mean square voltage,

$$\begin{aligned}\bar{V}^2 &= \frac{1}{t} \int_0^t V^2 dt = \frac{1}{t} \int_0^t V_0^2 t^3 dt \\ &= \frac{V_0^2}{t} \left[\frac{t^4}{4} \right]_0^t = \left[\frac{V_0^2 t^3}{4} \right]_0^t = \frac{V_0^2}{4}\end{aligned}$$

$$\therefore V_{\text{rms}} = \sqrt{\bar{V}^2} = \frac{V_0}{2}$$

Example 7.7 Calculate rms value of current and voltage for the following cases

(i) $I = 4 + 3 \sin \omega t$

(ii) $V = 5 + 2 \cos \omega t$

(iii) $I = 2 + 3 \sin \omega t + 2 \cos 2\omega t$

(iv) $V = \cos \omega t + 2 \cos 2\omega t + 3 \cos 3\omega t$

Sol. (i) rms value of current,

$$I_{\text{rms}} = \sqrt{I_0^2 + \frac{I_1^2}{2}} = \sqrt{4^2 + \frac{3^2}{2}} = 4.52 \text{ unit}$$

(ii) rms value of voltage,

$$\begin{aligned}V_{\text{rms}} &= \sqrt{V_0^2 + \frac{V_1^2}{2}} = \sqrt{5^2 + \frac{2^2}{2}} \\ &= 5.19 \text{ unit}\end{aligned}$$

(iii) rms value of current,

$$\begin{aligned}I_{\text{rms}} &= \sqrt{I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2}} \\ &= \sqrt{2^2 + \frac{3^2}{2} + \frac{2^2}{2}} = 3.24 \text{ unit}\end{aligned}$$

(iv) rms value of voltage,

$$\begin{aligned}V_{\text{rms}} &= \sqrt{\frac{V_1^2}{2} + \frac{V_2^2}{2} + \frac{V_3^2}{2}} \\ &= \sqrt{\frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2}} = 2.64 \text{ unit}\end{aligned}$$

Form factor

Form factor is defined as the ratio of rms value of AC to its average value during half-cycle, i.e.

$$\begin{aligned}R_f &= \frac{\text{rms value}}{\text{average value}} \\ &= \frac{V_0 / \sqrt{2}}{2V_0 / \pi} = \frac{\pi}{2\sqrt{2}} = 1.11\end{aligned}$$

Peak factor

The ratio of peak value and rms value is called peak factor. It is given by

$$R_p = \frac{\text{peak value}}{\text{rms value}} = \frac{V_0}{V_0 / \sqrt{2}} = \sqrt{2} = 1.41$$

Calculation of mean and rms value for some specific cases of alternating current

Sinusoidal waveform

(i) $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$

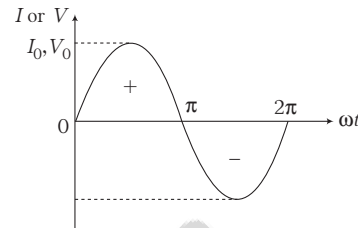


Fig. 7.4 Sinusoidal waveform

(ii) $I_{\text{av}} = \frac{2I_0}{\pi}$, for half-cycle

$I_{\text{av}} = 0$, for one complete cycle

(iii) $R_f = \frac{\pi}{2\sqrt{2}} = 1.11$

(iv) $R_p = \sqrt{2} = 1.41$

Square or rectangular waveform

(i) $I_{\text{rms}} = I_0$, for one complete cycle

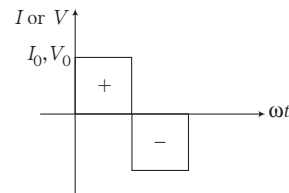


Fig. 7.5 Square or rectangular waveform

(ii) $I_{\text{av}} = I_0$, for half-cycle

$I_{\text{av}} = 0$, for one complete cycle

(iii) $R_f = \frac{I_0}{I_0} = 1$

(iv) $R_p = \frac{I_0}{I_0} = 1$

Half-wave rectified

(i) $I_{\text{rms}} = \frac{I_0}{2}$, for one complete cycle

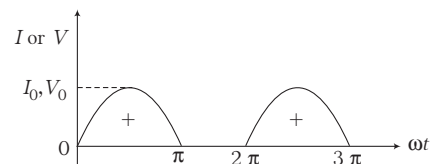


Fig. 7.6 Half-wave rectified waveform

$$(ii) I_{av} = \frac{I_0}{\pi}, \text{ for one complete cycle}$$

$$I_{av} = \frac{2I_0}{\pi}, \text{ for half-cycle}$$

$$(iii) R_f = \frac{I_0/2}{I_0/\pi} = \frac{\pi}{2} = 1.57$$

$$(iv) R_p = \frac{I_0}{I_0/2} = 2$$

Full wave rectified

$$(i) I_{rms} = \frac{I_0}{\sqrt{2}}, \text{ for one complete cycle}$$

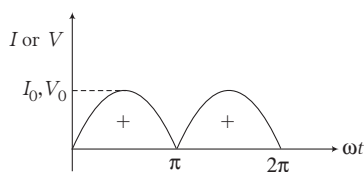


Fig. 7.7 Full wave rectified waveform

$$(ii) I_{av} = \frac{2I_0}{\pi}$$

$$(iii) R_f = \frac{I_0/\sqrt{2}}{2I_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$(iv) R_p = \frac{I_0}{I_0/\sqrt{2}} = \sqrt{2}$$

For a current varying wave as shown in the graph

$$(i) I_{rms} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt} \right]^{1/2} = \frac{I_0}{\sqrt{3}}$$

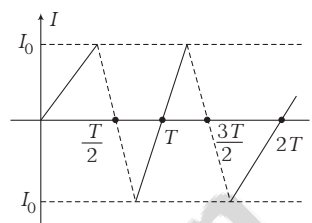


Fig. 7.8

$$(ii) I_{av} = \left[\frac{\int_0^T I dt}{\int_0^T dt} \right] = \frac{\text{Area of } I-t \text{ graph for } t = 0 \text{ to } t = T}{T} = \frac{I_0}{2}$$

$$(iii) R_f = \frac{I_0/\sqrt{3}}{I_0/2} = \frac{2}{\sqrt{3}} = 1.15$$

$$(iv) R_p = \frac{I_0}{I_0/\sqrt{3}} = \sqrt{3}$$

CHECK POINT 7.1

- The frequency of a sinusoidal wave $E = 0.40 \cos [2000t + 0.80]$ would be
(a) 1000π Hz (b) 2000 Hz
(c) 20 Hz (d) $1000/\pi$ Hz
- Frequency of AC mains in India is
(a) 30 cps (b) 50 cps (c) 60 cps (d) 120 cps
- A 220 V AC is more dangerous than 220 V DC because
(a) the AC attracts
(b) the DC repels
(c) the body offers less resistance to AC
(d) peak voltage for AC is much larger than 220 V
- Alternating current is transmitted to take places
(a) at high voltage and low current
(b) at high voltage and high current
(c) at low voltage and low current
(d) at low voltage and high current
- An AC voltage is given by $E = E_0 \sin \frac{2\pi t}{T}$
Then, the mean value of voltage calculated over any time interval of $T/2$ second
(a) is always zero (b) is never zero
(c) is always $(2E_0/\pi)$ (d) may be zero
- 220 V, 50 Hz, AC is applied to a resistor. The instantaneous value of voltage is
(a) $220\sqrt{2}\sin 100\pi t$ (b) $220\sin 100\pi t$
(c) $220\sqrt{2}\sin 50\pi t$ (d) $220\sin 50\pi t$
- The instantaneous current in an AC circuit is $I = \sqrt{2} \sin (50t + \pi/4)$. The rms value of current is
(a) $\sqrt{2}$ A (b) 50 A (c) 90 A (d) 1 A
- The peak value of an alternating current is 5 A and its frequency is 60 Hz. Find its rms value and time taken to reach the peak value of current starting from zero.
(a) 3.536 A, 4.167 ms (b) 3.536 A, 15 ms
(c) 6.07 A, 10 ms (d) 2.536 A, 4.167 ms
- If an alternating voltage is represented as $E = 141 \sin(628t)$, then the rms value of the voltage and the frequency are respectively
(a) 141 V, 628 Hz (b) 100 V, 50 Hz
(c) 100 V, 100 Hz (d) 141 V, 100 Hz
- An alternating current in a circuit is given by $I = 20 \sin (100\pi t + 0.05\pi)$ A. The rms value and the frequency of current respectively, are
(a) 10 A and 100 Hz (b) 10 A and 50 Hz
(c) $10\sqrt{2}$ A and 50 Hz (d) $10\sqrt{2}$ A and 100 Hz

REPRESENTATION OF I OR V AS ROTATING VECTORS

To simplify the study of AC circuit, alternating quantities (*i.e.* I or V) are represented as rotating vectors, with the angle between the vectors equal to the phase difference between the current and emf or voltage. These rotating vectors representing current and alternating voltage are called **phasors**.

Some important points regarding phasors are as follows

- The length of the vector represents the maximum or peak value, *i.e.* I_0 and V_0 .
- The projection of the vector on fixed axis gives the instantaneous value of alternating current and alternating emf. If $I = I_0 \sin \omega t$ and $V = V_0 \sin \omega t$, projection is taken on Y-axis. If $I = I_0 \cos \omega t$ and $V = V_0 \cos \omega t$, projection is taken on X-axis.
- A phasor is a geometric entity and not a real physical quantity that helps us to describe and analyse physical quantities that vary sinusoidally with time.
- A diagram representing alternating current and alternating emf (of same frequency) as rotating vectors (phasors) with the phase angle between them is called **phasor diagram**.

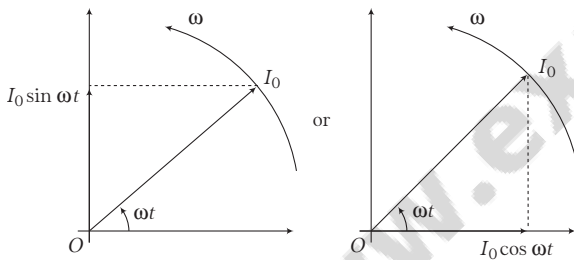


Fig. 7.9 Phasor diagram

Different types of alternating current circuits

In this section, we will derive voltage-current relations for individual as well as combined circuit elements carrying a sinusoidal current. We will consider resistors, inductors and capacitors.

1. Pure resistor in an AC circuit (Resistive circuit or R -circuit)

Suppose a resistor of resistance R is connected to an AC source of emf with instantaneous value (E) is given by

$$E = E_0 \sin \omega t \quad \dots(i)$$

Suppose E is the potential drop across resistance (R), then

$$E = IR \quad \dots(ii)$$

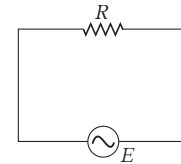


Fig. 7.10 AC voltage applied to a resistor

Instantaneous emf = Instantaneous value of potential drop

$\therefore$ From Eqs. (i) and (ii), we have

$$\begin{aligned} IR &= E = E_0 \sin \omega t \\ \text{or } I &= \frac{E}{R} = \frac{E_0 \sin \omega t}{R} \\ I &= I_0 \sin \omega t \quad \left(\because I_0 = \frac{E_0}{R} \right) \dots(iii) \end{aligned}$$

Comparing $I_0 = E_0 / R$ with Ohm's law, we find that resistors work equally well for both AC and DC voltages.

From Eqs. (i) and (iii), we get that there is zero phase difference between instantaneous alternating current and instantaneous alternating emf (*i.e.* they are in same phase).

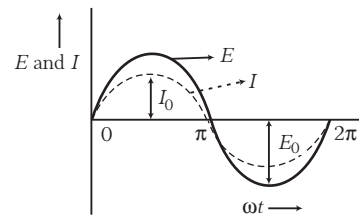


Fig. 7.11 E and I versus ωt graph of a resistive circuit

Phasor Diagram

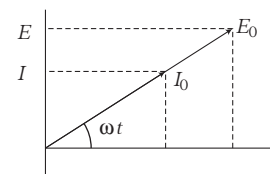


Fig. 7.12 Phasor diagram for a purely resistive circuit

Here, peak values E_0 and I_0 are represented by vectors rotating with angular velocity ω with respect to horizontal reference. Their projections on vertical axis give their instantaneous values.

Variation of resistance with frequency of source

Figure given below shows the graph of R as a function of angular frequency ω . The resistance R , does not depend upon angular frequency of the source.

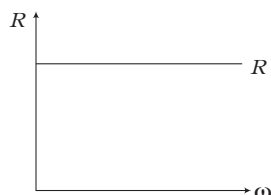


Fig. 7.13 R remains constant with ω or f

Example 7.8 A $200\ \Omega$ resistor is connected to a 220 V , 50 Hz AC supply. Calculate rms value of current in the circuit. Also, find phase difference between voltage and the current.

Sol. Given, $V_{\text{rms}} = 220\text{ V}$, $f = 50\text{ Hz}$, $R = 200\ \Omega$

Current in the purely resistive AC circuit, $I = \frac{V}{R}$

$\therefore$ rms value of current,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{220}{200} = 1.1\text{ A}$$

In purely resistive AC circuit, current and voltage are in phase. Therefore, phase difference is zero.

2. Pure capacitor in an AC circuit (Capacitive circuit or C-circuit)

Let us consider a capacitor with capacitance C be connected to an AC source with emf having instantaneous value.

$$E = E_0 \sin \omega t \quad \dots(i)$$

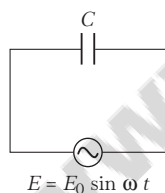


Fig. 7.14 An AC source connected to a capacitor

Due to this emf, charge will be produced and it will charge the plates of capacitor with positive and negative charge. If potential difference across the plates of capacitor is V , then

$$V = \frac{q}{C} \text{ or } q = CV$$

The instantaneous value of current in the circuit,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CE) \quad (\because V = E)$$

$$= \frac{d}{dt}(CE_0 \sin \omega t) \quad (\because E = E_0 \sin \omega t)$$

$$= CE_0 \cos \omega t \times \omega$$

$$= \frac{E_0}{1/\omega C} \cos \omega t$$

$$\Rightarrow I = \frac{E_0}{1/\omega C} \sin(\omega t + \pi/2)$$

$$[\because \cos \omega t = \sin(\pi/2 + \omega t)] \dots(ii)$$

I will be maximum when $\sin(\omega t + \pi/2) = 1$, so that $I = I_0$

where, peak value of current, $I_0 = \frac{E_0}{1/\omega C}$

$$\therefore I = I_0 \sin(\omega t + \pi/2) \quad \dots(iii)$$

From Eqs. (i) and (iii), it is clear that in a perfect capacitor, the current leads the voltage by a phase angle of $\pi/2$ (90°) or the voltage lags behind the current by a phase angle of $\pi/2$ (90°).

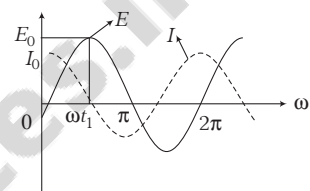


Fig. 7.15 E and I versus ωt graph of a capacitive circuit

Phasor Diagram

The phase representing peak emf E_0 makes an angle ωt in anti-clockwise direction with respect to horizontal axis. As current leads the voltage by 90° , the phasor representing I_0 is turned 90° anti-clockwise with the phasor representing E_0 . The projections of these phasors on the vertical axis give instantaneous values of E and I .

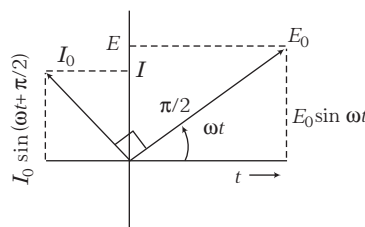


Fig. 7.16 Phasor diagram for purely capacitive circuit

Capacitive Reactance (X_C)

Instantaneous value of alternating current through a capacitor is given by

$$I = \frac{E_0}{1/\omega C} \sin(\omega t + \pi/2) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Comparing with Ohm's law, we get $I_0 = \frac{E_0}{1/\omega C}$

$$X_C = \frac{1}{\omega C}$$

where, X_C is called capacitive reactance.

The opposing nature of capacitor to the flow of alternating current is called **capacitive reactance**.

If f is the frequency of the alternating current, then

$$X_C = \frac{1}{2\pi f C}$$

The dimensions of capacitive reactance are same as that of resistance and its SI unit is ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit.

It is inversely proportional to the capacitance and frequency of the current. Thus, if frequency of AC increases, then its capacitive reactance decreases.

When capacitor is connected to DC source,

$$X_C = \frac{1}{\omega C} = \frac{1}{0} = \infty$$

($\because$ For DC, $\omega = 2\pi f = 0$, as $f = 0$)

Thus, capacitor blocks DC and acts as open circuit while passing AC of high frequency.

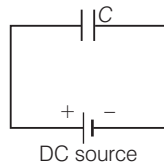


Fig. 7.17 Capacitor connected to a DC source

Variation of capacitive reactance with frequency

The graph of X_C as function of angular frequency ω can be drawn as

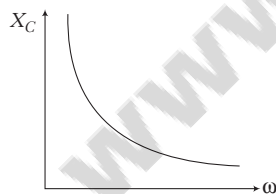


Fig. 7.18 Capacitive reactance X_C decreases with frequency

Capacitor as high pass filter

The capacitive reactance of a capacitor is inversely proportional to the capacitance C and to angular frequency ω . The greater the capacitance and higher the frequency, smaller is the capacitive reactance X_C .

Capacitors tend to pass high frequency current and block low frequency current, just the opposite of inductors. The device that passes signals of high frequency is called a **high pass filter**.

Example 7.9 A $60 \mu\text{F}$ capacitor is connected to a 110 V , 60 Hz AC supply. Determine the rms value of the current in the circuit.

Sol. As, $I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C}$

where, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

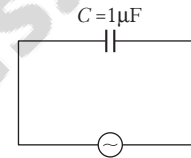
Here, $C = 60 \mu\text{F} = 60 \times 10^{-6} \text{ F}$, $V_{\text{rms}} = 110 \text{ V}$, $f = 60 \text{ Hz}$

So, $X_C = \frac{1}{2 \times \pi \times 60 \times 60 \times 10^{-6}} \Omega$

$\therefore I_{\text{rms}} = 110 \times 2 \times 3.14 \times 3600 \times 10^{-6} = 2.49 \text{ A}$

Example 7.10 An alternating voltage $V = 200\sqrt{2} \sin(100t) \text{ V}$ is connected to a $1 \mu\text{F}$ capacitor through an AC ammeter. What will be the reading of the ammeter?

Sol. Consider the purely capacitive circuit as shown below.



Given, $V = 200\sqrt{2} \sin(100t)$

Comparing it with general equation, $V = V_0 \sin(\omega t)$

we get, $V_0 = 200\sqrt{2} \text{ V}$, $\omega = 100 \text{ rad s}^{-1}$

$\therefore$ Reactance, $X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4 \Omega$

Peak value of current, $I_0 = \frac{V_0}{X_C} = \frac{200\sqrt{2}}{10^4} = 2\sqrt{2} \times 10^{-2} \text{ A}$

rms value of current, $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{2\sqrt{2} \times 10^{-2}}{\sqrt{2}}$
 $= 20 \times 10^{-3} \text{ A} = 20 \text{ mA}$

Reading of the ammeter will be same as, $I_{\text{rms}} = 20 \text{ mA}$.

3. Pure inductor in an AC circuit (Inductive circuit or L-circuit)

Suppose an inductor with self-inductance (L) is connected to AC source with instantaneous emf (E) is given by

$$E = E_0 \sin \omega t \quad \dots(i)$$

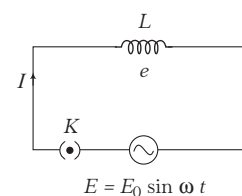


Fig. 7.19 An AC source connected to an inductor

When key is closed, then current begins to grow because magnetic flux linked with it changes and an emf is induced which opposes the applied emf.

According to Lenz's law,

$$e = -L \frac{dI}{dt}$$

where, e is induced emf and $\frac{dI}{dt}$ is the rate of change of current.

To maintain the flow of current in the circuit, applied voltage must be equal and opposite to the induced emf.

$$E = -e$$

$$\therefore E = -\left(-L \frac{dI}{dt}\right) = L \frac{dI}{dt} \quad \text{or} \quad dI = \frac{E}{L} dt$$

Integrating the above equation on both sides, we get

$$\int dI = \int \frac{E}{L} dt$$

$$I = \int \frac{E_0 \sin \omega t}{L} dt \quad [\because E = E_0 \sin \omega t]$$

$$\Rightarrow I = \frac{E_0}{L} \left[\frac{-\cos \omega t}{\omega} \right] = -\frac{E_0}{\omega L} \cos \omega t$$

$$\Rightarrow I = -\frac{E_0}{\omega L} \sin(\pi/2 - \omega t) \quad \left[\because \sin\left(\frac{\pi}{2} - \omega t\right) = \cos \omega t \right]$$

$$\Rightarrow I = \frac{E_0}{\omega L} \sin(\omega t - \pi/2) \quad \dots(ii)$$

If $\sin(\omega t - \pi/2) = \text{maximum} = 1$, then $I = I_0$

where, peak value of current, $I_0 = \frac{E_0}{\omega L}$

$$\Rightarrow I = I_0 \sin(\omega t - \pi/2) \quad \dots(iii)$$

From Eqs. (i) and (iii), it is clear that in a pure inductor, the current lags behind the voltage by a phase angle of $\pi/2$ (90°) or the voltage leads the current by a phase angle of $\pi/2$ (90°).

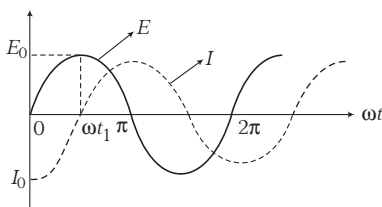


Fig. 7.20 E and I versus ωt graph of an inductive circuit

Phasor Diagram

The phasor representing peak emf E_0 makes an angle ωt in anti-clockwise direction from horizontal axis. As current lags behind the voltage by 90° , so the phasor representing I_0 is turned 90° clockwise with the direction of E_0 .

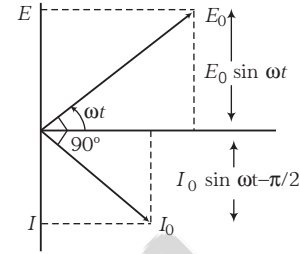


Fig. 7.21 Phasor diagram for purely inductive circuit

Inductive Reactance (X_L)

The opposing nature of inductor to the flow of current is called **inductive reactance**.

$$\text{As, } I = \frac{E_0}{\omega L} \sin(\omega t - \pi/2) \quad \text{or} \quad I_0 = E_0 / \omega L$$

Comparing with Ohm's law, $I_0 = \frac{E_0}{R}$. The quantity ωL is analogous to the resistance and is denoted by X_L .

So,

$$X_L = \omega L$$

X_L is called inductive reactance.

If f is the frequency of AC source, then

$$X_L = \omega L = 2\pi f L \quad \left(\because \omega = \frac{2\pi}{T} = 2\pi f \right)$$

The dimensions of inductive reactance are same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.

The inductive reactance is directly proportional to the inductance and to the frequency of the AC current.

Thus, if the frequency of AC increases, its inductive reactance also increases.

If inductor is connected to DC source, then for DC,

$$f = 0 \quad \left(\because f = \frac{1}{T} \right)$$

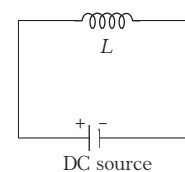


Fig. 7.22 Inductor connected to a DC source

Here, f is frequency.

$$\therefore X_L = 0$$

Therefore, inductor passes DC and blocks AC of very high frequency.

Variation of inductive reactance with frequency

The graph of X_L as function of angular frequency ω can be drawn as shown below. Clearly, X_L linearly increases with ω .

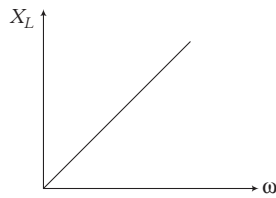


Fig. 7.23 X_L versus ω

Inductor as low pass filter

If an oscillating voltage of a given amplitude V_0 is applied across an inductor, the resulting current will have a smaller amplitude I_0 for larger value of ω . Since, X_L is proportional to frequency, a high frequency voltage applied to the inductor gives only a small current while a lower frequency voltage of the same amplitude gives rise to a larger current.

Inductors are used in some circuit applications, such as power supplies and radio interference filters to block high frequencies while permitting lower frequencies to pass through. A circuit device that uses an inductor for this purpose is called a **low pass filter**.

Example 7.11 An ideal inductor of inductance $50 \mu\text{H}$ is connected to an AC source of 220V , 50 Hz . Find the inductive reactance.

Sol. As inductive reactance, $X_L = \omega L = 2\pi fL$

where, L is inductance $= 50 \mu\text{H} = 50 \times 10^{-6} \text{ H}$

and f = source frequency $= 50 \text{ Hz}$

$$\begin{aligned} \therefore X_L &= 2 \times \pi \times 50 \times 50 \times 10^{-6} \\ &= 2\pi \times 25 \times 10^{-4} \Omega \\ &= 5\pi \times 10^{-3} \Omega \\ &= 5\pi \text{ m}\Omega \end{aligned}$$

Example 7.12 A 44 mH inductor is connected to 220 V , 50 Hz AC supply. Determine the rms value of the current in the circuit.

Sol. As, $I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L}$

where, $X_L = \omega L = 2\pi fL$ is the reactance of the inductor.

Here, $f = 50 \text{ Hz}$,

$$L = 44 \text{ mH} = 44 \times 10^{-3} \text{ H},$$

$$V_{\text{rms}} = 220 \text{ V}$$

$$\therefore X_L = 2\pi \times 50 \times 44 \times 10^{-3} = 13.82 \Omega$$

$$\therefore \text{rms value of current, } I_{\text{rms}} = \frac{220}{13.82} = 15.9 \text{ A}$$

AC voltage applied to a series L-C-R circuit

Suppose that an inductor (L), capacitor (C) and resistor (R) are connected in series to an AC source. I is the current passed through this circuit. As R, L, C are in series, therefore at any instant through the three elements, AC has the same amplitude and phase. Let it be represented by

$$I = I_0 \sin \omega t$$

However, voltage across each element bears a different phase relationship with the current.

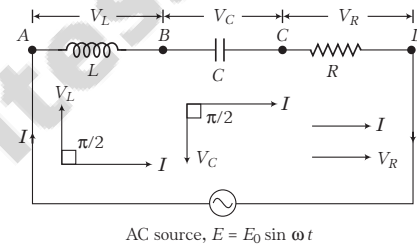


Fig. 7.24 An AC source connected to series L-C-R circuit

$$V_L = I_0 X_L \quad (V_L \text{ is maximum voltage across } L)$$

$$V_C = I_0 X_C \quad (V_C \text{ is maximum voltage across } C)$$

$$V_R = I_0 R \quad (V_R \text{ is maximum voltage across } R)$$

Inside phasor diagrams of each L, C and R are given. To form phasor diagram for series L-C-R circuit, combine all these phasor diagrams.

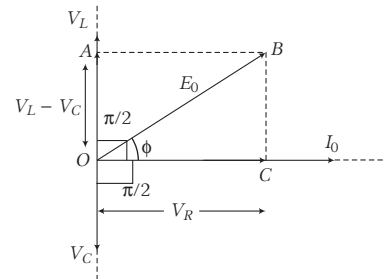


Fig. 7.25 Phasor diagram of a series L-C-R circuit

Since, voltage (V_L) is in upward direction and voltage (V_C) in downward direction, so net voltage upto point A is $(V_L - V_C)$ (assuming $V_L > V_C$) and net maximum voltage is E_0 .

From phasor diagram,

$$OB = \sqrt{(OC)^2 + (CB)^2} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$E_0 = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2} \quad [\because OB = E_0]$$

$$E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore Z = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

Here, Z is called **impedance**.

Impedance

It is the total resistance of a circuit, applied in the path of alternating current. It is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

From phasor diagram, it is clear that voltage leads the current by angle ϕ .

$\therefore$ From $\triangle OCB$,

$$\begin{aligned} \tan \phi &= \frac{CB}{OC} = \frac{V_L - V_C}{V_R} \\ &= \frac{I_0 X_L - I_0 X_C}{I_0 R} \\ \tan \phi &= \frac{X_L - X_C}{R} \end{aligned}$$

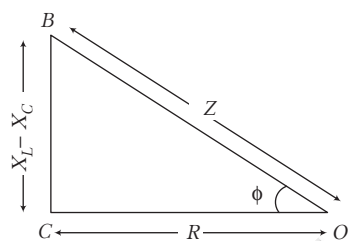


Fig. 7.26 Impedance triangle of an L-C-R circuit

$\therefore$ The alternating emf in the series L-C-R circuit would be represented by $E = E_0 \sin(\omega t + \phi)$.

Special Cases

(i) When $X_L = X_C$,

$$\therefore Z = R$$

$$\text{and } \tan \phi = 0$$

$$(\because \phi = 0^\circ)$$

Hence, voltage and current are in the same phase. The AC circuit is non-inductive or resistive circuit.

(ii) When $X_L > X_C$, then $\tan \phi$ is positive.

Hence, voltage leads the current by a phase angle ϕ . The AC circuit is inductive circuit.

(iii) When $X_C > X_L$, then $\tan \phi$ is negative.

Hence, voltage lags behind the current by a phase angle ϕ . The AC circuit is capacitive circuit.

The variation of Z with frequency ω

At $\omega = \omega_0$, $X_L = X_C$, $Z = R = \text{minimum}$.

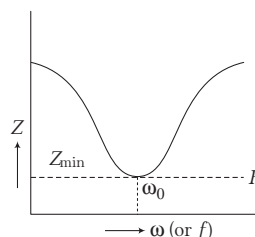


Fig. 7.27 Z versus ω for a series L-C-R circuit

Reduction of L-C-R circuit to L-C, R-C, L-R circuit

The general formula of L-C-R series circuit can be reduced to apply it on L-C, R-C, L-R, circuit.

$$\text{For L-C-R circuit, impedance } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

L-C circuit

For L-C circuit, there is no R , so, we can ignore terms related to R .

$$\therefore \text{For L-C circuit, } Z = \sqrt{(X_L - X_C)^2} \text{ or } (X_L - X_C)$$

- (i) If $X_L > X_C$, then circuit will behave like pure inductive circuit and voltage will lead the current by $\pi/2$.
- (ii) If $X_L < X_C$, then circuit will behave like pure capacitive circuit and voltage will lag behind the current by $\pi/2$.
- (iii) If $X_L = X_C$, then $Z = 0$ in this condition and current in the circuit will be maximum.

R-C circuit

For R-C circuit, there is no L , so we can ignore the term related to L .

$$\therefore \text{For R-C circuit, } Z = \sqrt{R^2 + X_C^2}$$

The phase difference between current and emf,

$$\tan \phi = \frac{X_C}{R}$$

Here, the current leads the voltage by ϕ .

$$\therefore I = I_0 \sin(\omega t + \phi)$$

L-R circuit

In L-R circuit, there is no C , so we can ignore the term related to C .

$$\therefore \text{For L-R circuit, } Z = \sqrt{R^2 + X_L^2}$$

The phase difference between current and emf, $\tan \phi = X_L/R$

Here, the emf leads the current by ϕ .

$$\therefore I = I_0 \sin(\omega t - \phi)$$

Resonance

In series L-C-R circuit, when phase (ϕ) between current and voltage is zero, the circuit is said to be in resonance.

As, applied frequency increases, then

$$X_L = \omega L, \quad X_L \text{ increases}$$

and $X_C = \frac{1}{\omega C}$, X_C decreases.

At some frequency (ω_0), $X_L = X_C$
 $\Rightarrow X_L = \omega_0 L$

and $X_C = \frac{1}{\omega_0 C}$

The frequency at which X_C and X_L becomes equal is called **resonant frequency**.

$$\therefore \omega_0 L = \frac{1}{\omega_0 C} \text{ or } \omega_0^2 = \frac{1}{LC} \text{ or } (2\pi f_0)^2 = \frac{1}{LC}$$

$$(\because \omega_0 = 2\pi f_0, \text{ where } f_0 \text{ is resonant frequency})$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}} \text{ or } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Resonant frequency is independent of the resistance of the circuit. At resonating frequency,

$$Z = R = \text{Minimum}$$

Since, Z is minimum, therefore I will be maximum.

$$\therefore I = \frac{V}{Z} = \text{Maximum}$$

These circuits are used for voltage amplification and as selector circuits in wireless telegraphy.

Response curves of series L - C - R circuit

The impedance of an L - C - R circuit depends on the frequency. The dependance is shown in figure. The frequency is taken on logarithmic scale because of its wide range.

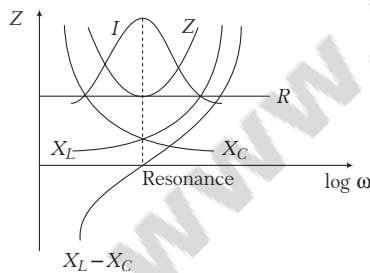


Fig. 7.28 Impedance (Z) versus $\log \omega$ graph for a series L - C - R circuit

From the figure, we can see that at resonance,

$$(i) X_L = X_C \text{ or } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$(ii) Z = Z_{\min} = R$$

(iii) and I is maximum.

Quality factor (Q -factor)

It is the measure of sharpness of the resonance of an L - C - R circuit. It is defined as, the ratio of voltage developed across the inductance or capacitance at

resonance to the impressed voltage, which is the voltage applied across R .

$$Q\text{-factor} = \frac{\text{Voltage across } L \text{ (or } C)}{\text{Voltage across } R}$$

$$Q\text{-factor} = \frac{V_L \text{ or } V_C}{V_R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

It is also defined as,

$$Q\text{-factor} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipation}}$$

Q is just a number having no dimensions, it can also be called **voltage multiplication factor** of the circuit.

From the above relation, it is clear that as R increases, (or current decreases) Q -factor of the circuit decreases which can be shown in graph below.

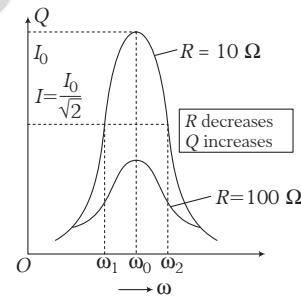


Fig. 7.29 Q versus ω graph of an L - C - R circuit

The electronic circuit with high Q values would respond to a very narrow range of frequencies and *vice-versa*. The higher the value of Q , the narrower and sharper is the resonance and greater is the current as shown below.

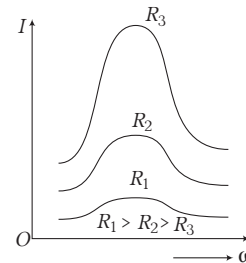


Fig. 7.30 I versus ω graph of an L - C - R circuit

$$\text{Also, } Q\text{-factor or } Q = \frac{\omega_0}{\omega_1 - \omega_2}$$

where, ω_1 and ω_2 are frequencies when current decreases to 0.707 times of the peak value of current (Fig. 7.29).

We may write, $\omega_1 = \omega_0 + \Delta\omega \Rightarrow \omega_2 = \omega_0 - \Delta\omega$

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is often called the **bandwidth of the circuit**.

The quantity $\left(Q = \frac{\omega_0}{2\Delta\omega}\right)$, is regarded as a measure of **sharpness of resonance**, i.e. Q -factor is defined as the ratio of resonant angular frequency to the bandwidth of the circuit.

The smaller the bandwidth ($\Delta\omega$), the sharper or narrower is the resonance.

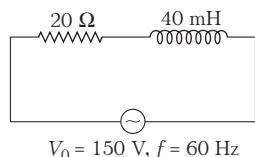
Note

- (i) Q -factor denotes the sharpness of tuning.
- (ii) High Q -factor indicates lower rate of energy loss.
- (iii) Higher value of Q -factor indicates sharper peak in the current.
- (iv) $R = 0$, Q -factor = infinity

Example 7.13 A sinusoidal voltage of frequency 60 Hz and peak value 150 V is applied to a series L - R circuit, where $R = 20 \Omega$ and $L = 40 \text{ mH}$.

- (i) Compute T , ω , X_L , Z and ϕ .
- (ii) Compute the amplitudes of current, V_R , V_L and V_0 .

Sol. (i) Consider a series L - R circuit as shown below



As $T = \frac{1}{f} = \frac{1}{60} \text{ s}$

$\therefore \omega = 2\pi f = 377 \text{ rad s}^{-1}$

$\therefore X_L = \omega L = (377)(0.040) = 15.08 \Omega$

$\Rightarrow Z = \sqrt{X_L^2 + R^2} = 25.05 \Omega$

Phase angle, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}(0.754) = 37^\circ$

(ii) Amplitudes (maximum value) are,

$$I_0 = \frac{V_0}{Z} = \frac{150}{25.05} \approx 6 \text{ A}$$

$\Rightarrow (V_0)_R = I_0 R = 120 \text{ V}$ and $(V_0)_L = I_0 X_L = 90.5 \text{ V}$

$\therefore V_0 = \sqrt{(V_0)_R^2 + (V_0)_L^2} = \sqrt{(120)^2 + (90.5)^2} = 150.3 \text{ V}$

Example 7.14 A 100Ω resistance is connected in series with a 4 H inductor. The voltage across the resistor is $V_R = (2.0 \text{ V}) \sin(10^3 \text{ rad s}^{-1})t$.

- (i) Find the expression of circuit current.
- (ii) Find the inductive reactance.
- (iii) Derive an expression for the voltage across the inductor.

Sol. (i) Current in the circuit, $I = \frac{V_R}{R} = \frac{(2.0 \text{ V}) \sin(10^3 \text{ rad s}^{-1})t}{100}$
 $= (2.0 \times 10^{-2} \text{ A}) \sin(10^3 \text{ rad s}^{-1})t$

(ii) Inductive reactance, $X_L = \omega L = (10^3 \text{ rad s}^{-1})(4 \text{ H})$
 $= 4.0 \times 10^3 \Omega$

(iii) The amplitude of voltage across inductor,
 $V_0 = I_0 X_L = (2.0 \times 10^{-2} \text{ A})(4.0 \times 10^3 \Omega) = 80 \text{ V}$

In an AC circuit, voltage across the inductor leads the current by 90° or $\frac{\pi}{2}$ rad. Hence,

$$V_L = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= (80 \text{ V}) \sin\left\{(10^3 \text{ rad s}^{-1})t + \frac{\pi}{2} \text{ rad}\right\}$$

Note The amplitude of voltage across the resistor ($= 2.0 \text{ V}$) is not same as the amplitude of the voltage across the inductor ($= 80 \text{ V}$), even though the amplitude of the current through both elements is the same.

Example 7.15 An alternating emf 200 virtual volts at 50 Hz is connected to a circuit of resistance 1Ω and inductance 0.01 H . What is the phase difference between the current and the emf in the circuit? Also find the virtual current in the circuit.

Sol. In case of an inductor, the voltage leads the current in phase by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Here, $X_L = \omega L = (2\pi f)L$
 $= (2\pi)(50)(0.01) = \pi \Omega$

and $R = 1 \Omega$

$\therefore \phi = \tan^{-1}(\pi) \approx 72.3^\circ$

Further, $I_{\text{rms}} = \frac{V_{\text{rms}}}{|Z|} = \frac{V_{\text{rms}}}{\sqrt{R^2 + X_L^2}}$

Substituting the values we have,

$$I_{\text{rms}} = \frac{200}{\sqrt{(1)^2 + (\pi)^2}} = 60.67 \text{ A}$$

Example 7.16 A resistance and inductance are connected in series across a voltage, $V = 283 \sin 314t$.

The current is found to be $I = 4 \sin\left(314t - \frac{\pi}{4}\right)$. Find the values of the inductance and resistance.

Sol. In L - R series circuit current lags the voltage by an angle,

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Here, $\phi = \frac{\pi}{4}$

$\therefore X_L = R$ or $\omega L = R$

$\therefore 314 L = R$ ($\because \omega = 314 \text{ rad s}^{-1}$) ... (i)

Further, $V_0 = I_0 |Z|$
 $\therefore 283 = 4 \sqrt{R^2 + X_L^2}$

$$\text{or } R^2 + (\omega L)^2 = \left(\frac{283}{4}\right)^2 = 5005.56$$

$$\text{or } 2R^2 = 5005.56 \quad (\text{as } \omega L = R)$$

$$\therefore R = 50 \, \Omega$$

and from Eq. (i), $L = 0.16 \, \text{H}$

Example 7.17 A long solenoid connected to a 12 V DC source passes a steady current of 2 A. When the solenoid is connected to an AC source of 12 V at 50 Hz, the current flowing is 1 A. Calculate inductance of the solenoid.

Sol. Remember that the solenoid mentioned here is not just a solenoid but has the resistance also due to its wire material. Now, in DC, the solenoid doesn't offer any reactance after a few milliseconds, however in the beginning it offers infinite resistance. So, the current by a DC source into an inductor is,

$$I = \frac{V}{R}$$

$$\text{or } R = \frac{12}{2} = 6 \, \Omega$$

But when an AC source is connected to the same inductor, the reactance will also come into picture.

$$\text{So, } I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\text{Here, } I = 1 \, \text{A}, V = 12 \, \text{V}, R = 6 \, \Omega$$

$$\omega = 2\pi \times 50 = 100\pi \, \text{rads}^{-1}, L = ?$$

$$\text{or } 1 = \frac{12}{\sqrt{6^2 + (100\pi L)^2}} \Rightarrow L = 33 \, \text{mH}$$

Example 7.18 A $100 \, \mu\text{F}$ capacitor in series with a $40 \, \Omega$ resistance is connected to 110 V, 60 Hz supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between the maximum current and the maximum voltage?

Sol. (a) Peak value of voltage,

$$E_0 = \sqrt{2} E_{\text{rms}} = \sqrt{2} \times 110 = 155.5 \, \text{V}$$

$$\text{and } \omega = 2\pi f = 2\pi \times 60 = 120\pi \, \text{rad/s}$$

Let I_0 = maximum current in the circuit.

$$\begin{aligned} \therefore I_0 &= \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \\ &= \frac{155.5}{\sqrt{1600 + 703.62}} = 3.23 \, \text{A} \end{aligned}$$

(b) In an R-C circuit, the voltage lags behind the current by the phase angle ϕ given by

$$\begin{aligned} \tan \phi &= \frac{X_C}{R} = \frac{1}{\omega CR} = \frac{1}{120\pi \times 10^{-4} \times 40} \\ &= 0.6631 = \tan 33.5^\circ \end{aligned}$$

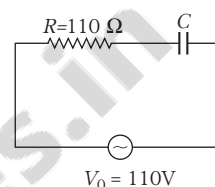
$$\therefore \phi = 33.5^\circ = 33.5 \times \frac{\pi}{180} \, \text{rad}$$

$\therefore$ Time lag between the maximum current and the maximum voltage is given by

$$\begin{aligned} t &= \frac{\phi}{\omega} = \frac{33.5 \pi}{120\pi} \quad (\because \phi = \omega t) \\ &= 1.55 \times 10^{-3} \, \text{s} \\ &= 1.55 \, \text{ms} \end{aligned}$$

Example 7.19 A circuit containing of a capacitor and an active resistance $R = 110 \, \Omega$ connected in series is fed an alternating voltage with amplitude $V_0 = 110 \, \text{V}$. In this case, the amplitude of current is equal to $I_0 = 0.50 \, \text{A}$. Find the phase difference between the current and the voltage fed.

Sol. Consider an RC circuit as shown below



Given, $I_0 = 0.50 \, \text{A}$

$$\therefore I_0 = \frac{V_0}{Z} \Rightarrow 0.5 = \frac{110}{Z} \Rightarrow Z = 220 \, \Omega$$

$$\therefore Z^2 = R^2 + X_C^2 \Rightarrow (220)^2 = (110)^2 + X_C^2$$

$$\Rightarrow X_C^2 = (220)^2 - (110)^2 = 330 \times 110$$

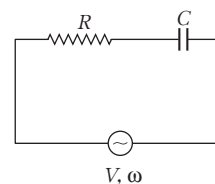
$$\Rightarrow X_C = 110\sqrt{3} \, \Omega$$

Now we can write,

$$\tan \phi = \frac{X_C}{R} = \frac{110\sqrt{3}}{110} = \sqrt{3} \Rightarrow \phi = 60^\circ$$

Example 7.20 An AC voltage source is applied across an R-C circuit. Angular frequency of the source is ω , resistance is R and capacitance is C . The current registered is I . If now the frequency of source is changed to $\omega/2$ (but maintaining the same voltage), the current in the circuit is found to be two third. Calculate the ratio of reactance to resistance at the original frequency ω .

Sol. Consider the R-C circuit as shown below.

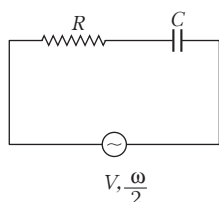


$$\therefore X_C = \frac{1}{\omega C}$$

$$\text{and } Z = \sqrt{R^2 + X_C^2}$$

$$\therefore I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_C^2}} \quad \dots(i)$$

When frequency is changed to $\frac{\omega}{2}$.



The value of new capacitive reactance is

$$X'_C = \frac{1}{(\omega/2)C} = \frac{2}{\omega C} = 2X_C$$

New impedance, $Z' = \sqrt{R^2 + X_C'^2} = \sqrt{R^2 + (2X_C)^2}$

$$\Rightarrow I' = \frac{V}{\sqrt{R^2 + 4X_C^2}}$$

It is given that, $I' = \frac{2}{3}I$

$$\Rightarrow \frac{2}{3} \times \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{\sqrt{R^2 + 4X_C^2}}$$

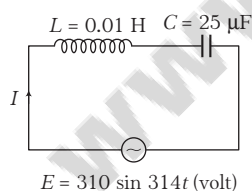
$$\Rightarrow 4R^2 + 16X_C^2 = 9R^2 + 9X_C^2$$

$$\Rightarrow 7X_C^2 = 5R^2$$

$$\Rightarrow \frac{X_C}{R} = \sqrt{\frac{5}{7}}$$

Example 7.21 A coil of inductance 0.01 H is connected in series with a capacitor of capacitance $25\text{ }\mu\text{F}$ with an AC source whose emf is given by $E = 310 \sin 314t\text{ V}$. What is the reactance of the circuit?

Sol. Consider a series L - C circuit as shown below



Given, $E = 310 \sin 314t$ (volt)

Comparing it with general equation, $E = E_0 \sin \omega t$

We get,

$$\omega = 2\pi f = 314$$

$$\text{or } f = \frac{314}{2\pi} = 50\text{ Hz}$$

$\therefore$ Inductive reactance,

$$X_L = \omega L = 2\pi f \cdot L \\ = 314 \times 0.01 = 3.14\text{ }\Omega$$

$$\text{Capacitive reactance, } X_C = \frac{1}{\omega C} = \frac{1}{314 \times 25 \times 10^{-6}} = 127.3\text{ }\Omega$$

$$\therefore \text{Net reactance, } X = X_C - X_L = (127.3 - 3.14)\text{ }\Omega = 124.16\text{ }\Omega$$

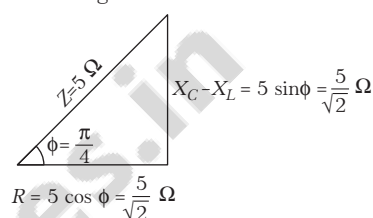
Example 7.22 A series L - C - R circuit is connected across a source of emf $E = 20 \sin \left(100\pi t - \frac{\pi}{6}\right)$. The current from the supply is $I = 4 \sin \left(100\pi t + \frac{\pi}{12}\right)$. Draw the impedance triangle for the circuit.

Sol. Given, $E_0 = 20\text{ V}$ and $I_0 = 4\text{ A}$

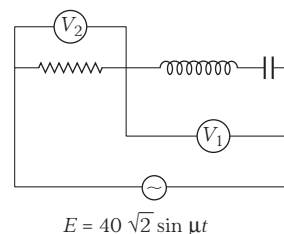
$$\therefore Z = \frac{E_0}{I_0} = 5\text{ }\Omega$$

The current leads emf by $\phi = \frac{\pi}{12} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{4}$

The impedance triangle can be drawn as



Example 7.23 If the reading of voltmeter V_1 is 30 V , what is the reading of voltmeter V_2 ?



Sol. AC voltmeter reads rms value of voltage.

$$\text{rms voltage applied} = \frac{V_0}{\sqrt{2}} = 40$$

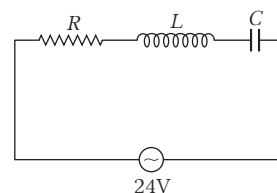
$$\text{If reading of voltmeter } V_2 \text{ is } X, \text{ then } V_{\text{rms}} = \sqrt{V_1^2 + V_2^2}$$

$$\text{or } \sqrt{X^2 + 30^2} = 40$$

$$\Rightarrow X = 26.45\text{ V}$$

Example 7.24 An inductor coil, a capacitor and an AC source of rms voltage 24 V are connected in series. When the frequency of the sources is varied, a maximum rms current of 6 A is observed. If this inductor coil is connected to a battery of emf 12 V and internal resistance $4.0\text{ }\Omega$. What will be the current?

Sol. Consider an L - C - R series circuit as shown below



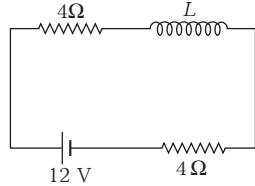
where, R is the reactance of the inductor.

Maximum current will occur at series resonance.

At series resonance, $Z_{\min} = R$ and $I_{\max} = 6\text{ A}$

$$\Rightarrow I_{\max} = \frac{V}{Z_{\min}} \Rightarrow 6 = \frac{24}{R} \Rightarrow R = 4\ \Omega$$

Now, this inductor coil is connected with a battery of emf 12 V and internal resistance $4\ \Omega$, so new circuit is



In DC circuit, inductor offers zero resistance,

$$\therefore \text{Current, } I = \frac{12}{4 + 4} = 1.5\text{ A}$$

Example 7.25 A coil of inductance 0.4 mH is connected to a capacitor of capacitance 400 pF . To what wavelength, is this circuit tuned?

Sol. Given, $L = 0.4\text{ mH} = 0.4 \times 10^{-3}\text{ H}$, $C = 400\text{ pF} = 4 \times 10^{-10}\text{ F}$

This is the condition of resonance.

$\therefore$ Resonant frequency,

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14} \times \frac{1}{\sqrt{0.4 \times 10^{-3} \times 4 \times 10^{-10}}} \\ = \frac{10^7}{6.28 \times 4}\text{ Hz}$$

If the speed of electromagnetic wave is v then for tuning the circuit, wavelength required would be

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8\text{ ms}^{-1}}{\frac{10^7}{6.28 \times 4}} = 753.6\text{ m}$$

Example 7.26 A 200 km telephone wire has capacity of $0.014\ \mu\text{F km}^{-1}$. If it carries an alternating current of frequency 50 kHz , what should be the value of an inductance required to be connected in series so that impedance is minimum?

Sol. Capacitance, $C = 0.014 \times 200\ \mu\text{F} = 2.8 \times 10^{-6}\text{ F}$

Frequency, $f = 50\text{ kHz} = 50 \times 10^3\text{ cycles s}^{-1}$

The impedance will be minimum at resonant frequency, when applied frequency will be same as resonant frequency ($f_0 = f$).

$$\text{The resonant frequency, } f_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{4\pi^2 f_0^2 C}$$

$$L = \frac{1}{4 \times \frac{22}{7} \times \frac{22}{7} \times (50 \times 10^3)^2 \times (2.8 \times 10^{-6})} = 0.36 \times 10^{-5}\text{ H}$$

Example 7.27 Find the voltage across the various elements, i.e. resistance, capacitance and inductance which are in series and having values $1000\ \Omega$, $1\ \mu\text{F}$ and 2.0 H , respectively.

Given emf as, $V = 100\sqrt{2} \sin 1000 t\text{ V}$

Sol. The rms value of voltage across the source, $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$

Comparing the given equation with general equation,

$$V_0 = 100\sqrt{2} \text{ and } \omega = 1000\text{ rad s}^{-1}$$

Also, $R = 1000\ \Omega$, $C = 1\ \mu\text{F} = 1 \times 10^{-6}\text{ F}$

and $L = 2\text{ H}$

$$\therefore V_{\text{rms}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100\text{ V}$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{|Z|} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \\ = \frac{100}{\sqrt{(1000)^2 + \left(1000 \times 2 - \frac{1}{1000 \times 1 \times 10^{-6}}\right)^2}} \\ = 0.0707\text{ A}$$

The current will be same everywhere in the circuit, therefore PD across resistor,

$$V_R = I_{\text{rms}} R = 0.0707 \times 1000 = 70.7\text{ V}$$

PD across inductor,

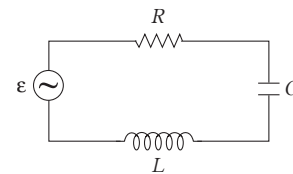
$$V_L = I_{\text{rms}} X_L = 0.0707 \times 1000 \times 2 = 141.4\text{ V}$$

and PD across capacitor,

$$V_C = I_{\text{rms}} X_C = 0.0707 \times \frac{1}{1 \times 1000 \times 10^{-6}} = 70.7\text{ V}$$

Note The rms voltages do not add directly as, $V_R + V_L + V_C = 282.8\text{ V}$, which is not the source voltage 100 V . The reason is that these voltages are not in phase and can be added by vector or by phasor algebra.

Example 7.28 Figure here, shows a series L - C - R circuit connected to a variable frequency source of 230 V . $L = 5.0\text{ H}$, $C = 80\ \mu\text{F}$ and $R = 40\ \Omega$.



- Determine the source frequency which drives the circuit in resonance.
- Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
- Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the L - C combination is zero at the resonating frequency.

Sol. (a) If ω_0 be the resonant angular frequency = source frequency at resonance, then

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50\text{ rad s}^{-1}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{50}{2 \times 3.142} = 7.96 \text{ Hz}$$

(b) At resonance, $Z = R = 40 \Omega$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{230}{40} = 5.75 \text{ A}$$

and
$$I_0 = \frac{V_0}{Z} = \frac{\sqrt{2} V_{\text{rms}}}{Z} = \frac{\sqrt{2} \times 230}{40} = 8.13 \text{ A}$$

(c) The rms potential drop across R is given by

$$V_{\text{rms}} = I_{\text{rms}} R = 5.75 \times 40 = 230 \text{ V}$$

The rms potential drop across L is given by

$$V_{L \text{ rms}} = I_{\text{rms}} X_L = I_{\text{rms}} (\omega_0 L) = 5.75 \times 50 \times 5 = 1437.5 \text{ V}$$

The rms potential drop across C is given by

$$V_{\text{rms}} = I_{\text{rms}} X_C = I_{\text{rms}} \times \frac{1}{\omega_0 C} = 5.75 \times \frac{1}{50 \times 80 \times 10^{-6}} = 1437.5 \text{ V}$$

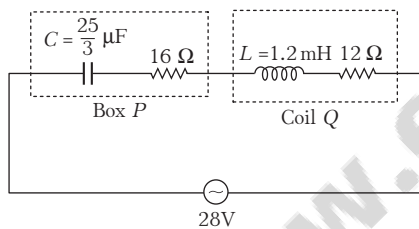
The rms potential drop across L - C is given by

$$V_{LC} = V_L - V_C = 1437.5 - 1437.5 = 0$$

Example 7.29 A box P and a coil Q are connected in series with an AC source of variable frequency. The emf of the source is constant at 28V. The frequency is so adjusted that the maximum current flows in P and Q . Find

(a) impedance of P and Q at this frequency,

(b) voltage across P and Q .



Sol. Current is maximum, i.e. case of series resonance,

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.2 \times 10^{-3} \times \frac{25}{3} \times 10^{-6}}} = 10^4 \text{ rad s}^{-1}$$

At resonance, $X_C = X_L$

$$X_C = \frac{1}{\omega_0 C} = \frac{1}{10^4 \times \frac{25}{3} \times 10^{-6}} = 12 \Omega = X_L$$

Resistance R_1 and R_2 are in series, so $R_{\text{net}} = R_1 + R_2$

$$\therefore I = \frac{V}{R_1 + R_2} = \frac{28}{16 + 12} = \frac{28}{28} = 1 \text{ A}$$

$$(a) Z_P = \sqrt{(16)^2 + (X_C)^2} = \sqrt{(16)^2 + (12)^2} = 20 \Omega$$

$$Z_Q = \sqrt{(12)^2 + (X_L)^2} = \sqrt{(12)^2 + (12)^2} = 12\sqrt{2} \Omega$$

$$(b) V_P = I Z_P = 1 \times 20 = 20 \text{ V}$$

$$V_Q = I Z_Q = 1 \times 12\sqrt{2} = 12\sqrt{2} \text{ V}$$

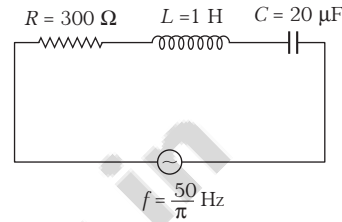
Example 7.30

(a) In a series L - C - R circuit with an AC source, $R = 300 \Omega$, $C = 20 \mu\text{F}$, $L = 1.0 \text{ H}$, $V_0 = 50\sqrt{2} \text{ V}$ and $f = 50/\pi \text{ Hz}$.

Find (i) the rms current in the circuit and (ii) the rms voltage across each element.

(b) Consider the situation of the previous part. Find the average electric field energy stored in the capacitor and the average magnetic field energy stored in the coil.

Sol. (a) Consider the series L - C - R circuit as shown below



It is given that, $V_0 = 50\sqrt{2} \text{ V}$

and
$$\omega = 2\pi f = 2\pi \times \frac{50}{\pi} = 100 \text{ rad s}^{-1}$$

$$\therefore X_L = \omega L = 100 \times 1 = 100 \Omega$$

$$\therefore X_C = \frac{1}{\omega C} = \frac{1}{100 \times 20 \times 10^{-6}} = 500 \Omega$$

Therefore,
$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(300)^2 + (500 - 100)^2} = 500 \Omega$$

$$\therefore \text{Peak value of current } I_0 = \frac{V_0}{Z} = \frac{50\sqrt{2}}{500} = 0.1\sqrt{2} \text{ A}$$

$$(i) \therefore I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{0.1\sqrt{2}}{\sqrt{2}} = 0.1 \text{ A}$$

(ii) rms voltage across each element is

$$V_R = I_{\text{rms}} R = 0.1 \times 300 = 30 \text{ V}$$

$$V_L = I_{\text{rms}} X_L = 0.1 \times 100 = 10 \text{ V}$$

$$\text{and } V_C = I_{\text{rms}} X_C = 0.1 \times 500 = 50 \text{ V}$$

(b) The average electric field energy stored in capacitor is given by

$$U_C = \frac{1}{2} C V_C^2 = \frac{1}{2} \times 20 \times 10^{-6} \times (50)^2 = 25 \text{ mJ}$$

Also, the average magnetic field energy stored in the coil is given by

$$U_L = \frac{1}{2} L I_{\text{rms}}^2 = \frac{1}{2} \times 1 \times (0.1)^2 = 5 \text{ mJ}$$

Example 7.31 A series L - C - R circuit containing a resistance of 120Ω has angular frequency $4 \times 10^5 \text{ rad s}^{-1}$. At resonance the voltages across resistance and inductance are 60 V and 40 V, respectively. Find the values of L and C . At what angular frequency the current lags the voltage by $\pi/4$?

Sol. At resonance, $X_L - X_C = 0$ and $Z = R = 120 \Omega$

$$\therefore I_{\text{rms}} = \frac{(V_R)_{\text{rms}}}{R} = \frac{60}{120} = \frac{1}{2} \text{ A}$$

Also, $I_{\text{rms}} = \frac{(V_L)_{\text{rms}}}{\omega L}$

$$\therefore L = \frac{(V_L)_{\text{rms}}}{\omega I_{\text{rms}}} = \frac{40}{(4 \times 10^5) \left(\frac{1}{2}\right)} = 2.0 \times 10^{-4} \text{ H} = 0.2 \text{ mH}$$

The resonance frequency is given by, $\omega = \frac{1}{\sqrt{LC}}$ or $C = \frac{1}{\omega^2 L}$.

Substituting the values, we have

$$C = \frac{1}{(4 \times 10^5)^2 (2.0 \times 10^{-4})} = 3.125 \times 10^{-8} \text{ F}$$

Current lags the voltage by $\pi/4$ or 45° , so

$$\tan 45^\circ = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Substituting the values of L , C , R and $\tan 45^\circ$, we get

$$\omega = 8 \times 10^5 \text{ rad s}^{-1}$$

Parallel circuit (Rejector circuit)

Let us consider an alternating source connected across an inductance L in parallel with a capacitor C .

The resistance in series with the inductance is R and with the capacitor is zero.

(Every inductor in real world has some non-zero internal resistance while in case of a capacitor, it can be neglected).

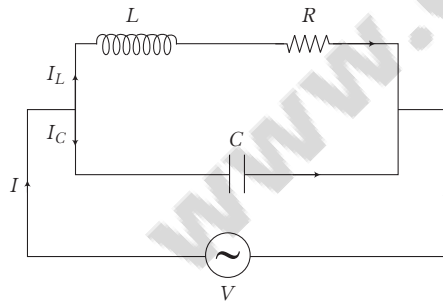


Fig. 7.31 A capacitor (C) is in parallel with series combination of R and L

Let the instantaneous value of emf applied be V and the corresponding currents are I , I_L and I_C . Then,

$$I = I_L + I_C \Rightarrow \frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as **admittance** (Y). Admittance is defined as the reciprocal of the impedance. Its unit is mho.

Therefore,

$$Y = \frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C = \frac{R + j(\omega CR^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

$\therefore$ The magnitude of the admittance,

$$|Y| = \left| \frac{1}{Z} \right| = \frac{\sqrt{R^2 + (\omega CR^2 + \omega^3 L^2 C - \omega L)^2}}{R^2 + \omega^2 L^2}$$

The admittance will be minimum, when

$$\omega CR^2 + \omega^3 L^2 C - \omega L = 0$$

or

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

It gives the condition of resonance and the corresponding frequency,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

is known as resonance frequency. At resonance frequency, admittance is minimum or the impedance is maximum. Thus, the parallel circuit does not allow this frequency from the source to pass in the circuit. Due to this reason the circuit with such a frequency is known as **rejector circuit**.

Note If $R = 0$, resonance frequency is $\frac{1}{2\pi\sqrt{LC}}$ same as resonance frequency in series L - C - R circuit or acceptor circuit.

At resonance, the reactive component of Y vanishes or Y is real. The reciprocal of the admittance is called the **parallel resistor** or the **dynamic resistance**. The dynamic resistance is thus, reciprocal of the real part of the admittance.

$$\therefore \text{Dynamic resistance} = \frac{R^2 + \omega^2 L^2}{R}$$

Substituting, $\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$ (resonance frequency)

We have, dynamic resistance = $\frac{L}{CR}$

$$\therefore \text{Peak current through the supply} = \frac{V_0}{L/CR} = \frac{V_0 CR}{L}$$

The peak current through capacitor = $\frac{V_0}{1/\omega C} = \omega CV_0$. The

ratio of the peak current through capacitor and through the supply is known as **Q-factor**.

Thus,
$$Q\text{-factor} = \frac{V_0 \omega C}{V_0 CR/L} = \frac{\omega L}{R}$$

This is basically the measure of current magnification. The rejector circuit at resonance exhibits current magnification of $\omega L/R$, similar to the voltage magnification of the same ratio exhibited by the series acceptor circuit at resonance.

Fig. (a) shows the variation of current I with angular frequency, ω in parallel LC circuit (i.e., $R = 0$) and Fig. (b) shows the variation of impedance Z with ω .

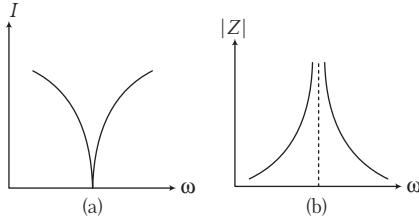


Fig. 7.32 For a parallel LC circuit (a) I versus ω (b) $|Z|$ versus ω

Note At resonance the current through the supply and voltage are in phase, while the current through the capacitor leads the voltage by 90° .

Example 7.32 A capacitor of capacitance 250 pF is connected in parallel with a choke coil having inductance of $1.6 \times 10^{-2} \text{ H}$ and resistance 20Ω . Calculate

- the resonance frequency and
- the circuit impedance at resonance.

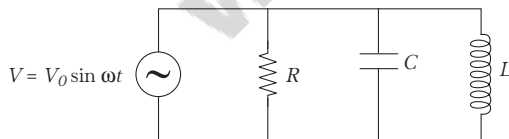
Sol. (a) The resonance frequency of a rejector L - C - R circuit is given by,

$$\begin{aligned} f &= \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \\ &= \frac{1}{2\pi} \sqrt{\frac{1}{(1.6 \times 10^{-2})(250 \times 10^{-12})} - \frac{(20)^2}{(1.6 \times 10^{-2})^2}} \\ &= 7.96 \times 10^4 \text{ Hz} \end{aligned}$$

(b) The circuit impedance at resonance is given by,

$$Z = \frac{L}{CR} = \frac{1.6 \times 10^{-2}}{(250 \times 10^{-12})(20)} = 3.2 \times 10^6 \Omega$$

Example 7.33 For the circuit shown in figure, find the instantaneous current through each element.



Sol. The three current equations are,

$$V = I_R R, V = L \frac{dI_L}{dt} \quad \text{and} \quad \frac{dV}{dt} = \frac{1}{C} I_C$$

For steady state, solutions of equations are,

$$I_R = \frac{V}{R} = \frac{V_0}{R} \sin \omega t = (I_0)_R \sin \omega t$$

$$I_L = \int \frac{V}{L} dt = -\frac{V_0}{\omega L} \cos \omega t$$

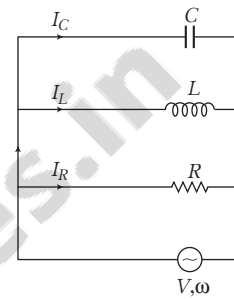
$$= -\frac{V_0}{X_L} \cos \omega t = -(I_0)_L \cos \omega t$$

$$\text{and } I_C = \frac{dV}{dt} = V_0 \omega C \cos \omega t$$

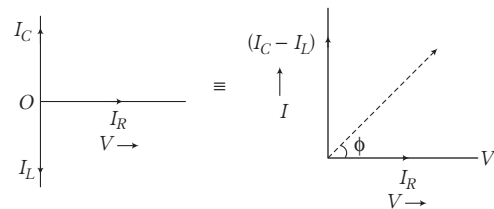
$$= \frac{V_0}{X_C} \cos \omega t = (I_0)_C \cos \omega t$$

where, the reactance X_L and X_C are as defined.

Example 7.34 AC voltage source (V, ω) is applied across a parallel LC circuit as shown in the figure. Find the impedance of the circuit and phase of current.



Sol. Current through the resistor, $I_R = V/R$, in phase with voltage, Current through the inductor, $I_L = V/X_L$, lags voltage by $\pi/2$, Current through the capacitor, $I_C = V/X_C$, leads voltage by $\pi/2$.



Assuming, resultant current, $I = [I_R^2 + (I_C - I_L)^2]^{1/2}$

$$\Rightarrow \frac{V}{Z} = \left[\left(\frac{V}{R} \right)^2 + \left(\frac{V}{X_C} - \frac{V}{X_L} \right)^2 \right]^{1/2}$$

$$\Rightarrow \frac{1}{Z} = \left[\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L} \right)^2 \right]^{1/2}$$

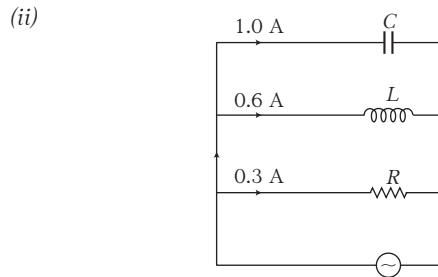
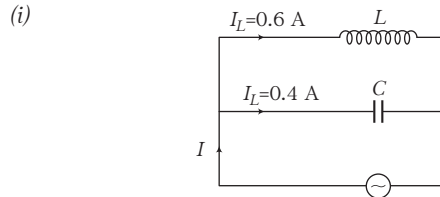
$$\text{Impedance, } Z = \left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L} \right)^2 \right]^{-1/2}$$

$$\text{Phase difference, } \tan \phi = \frac{I_C - I_L}{I_R} = \frac{\frac{1}{X_C} - \frac{1}{X_L}}{1/R}$$

$$\phi = \tan^{-1} \left[R \left(\omega C - \frac{1}{\omega L} \right) \right]$$

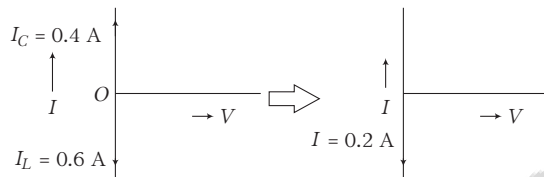
This means current leads the voltage.

Example 7.35 Find the current drawn from the source in each of the circuits as given below



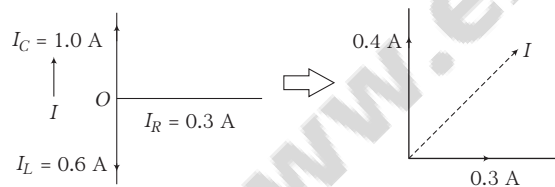
Sol. (i) Current in inductor lags the voltage by $\pi/2$.

Current in capacitor leads the voltage by $\pi/2$.
The phasor diagram can be drawn as



By vector algebra, current from source = 0.2 A

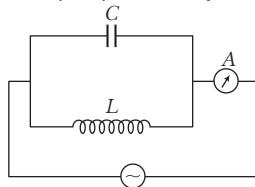
(ii) Current in resistor is in phase with voltage. The phasor diagram can be drawn as



So, net current drawn from the source is

$$i = \sqrt{(0.3)^2 + (0.4)^2} = 0.5 \text{ A}$$

Example 7.36 An LC circuit ($L = 0.01 \text{ H}$, $C = 1 \mu\text{F}$) is connected to an AC source of variable frequency. If the frequency is varied from 1 kHz to 2 kHz, then show the consequent variation of impedance by a rough sketch.



Sol. Impedance (Z) of the circuit is

$$\frac{1}{Z} = \frac{1}{X_C} - \frac{1}{X_L}$$

At resonance, $X_L = X_C$

$$\Rightarrow \omega L = \frac{1}{\omega C}$$

$$\Rightarrow \omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.01) \times 10^{-6}}}$$

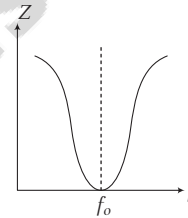
$$= 1592 \text{ Hz}$$

$$= 1.592 \text{ kHz}$$

At $f = f_0$, impedance is minimum and equal to zero.

At $f < f_0$, $X_C > X_L$, as frequency increases, impedance decreases.

When $f > f_0$, $X_C < X_L$, as frequency increases, impedance increases as shown below



Example 7.37 Inductance (L), capacitance (C) and resistance (R) are contained in a box. When 250 V DC is applied to the terminals of the box, a current of 1.0 A flows in the circuit. When an AC source of 250 V_{rms} at 2250 rad s^{-1} is connected, a current of 1.25 A_{rms} flows. It is observed that the current rises with frequency and becomes maximum at 4500 rad s^{-1} . Find the values of L , C and R . Draw the circuit diagram.

Sol. When DC source is applied, current = 1 A

In DC circuit, inductor offers zero resistance and capacitor offers infinite resistance. Hence, R and C cannot be in series.

$$I = \frac{V_{DC}}{R} \Rightarrow 1 = \frac{250}{R} \Rightarrow R = 250 \Omega$$

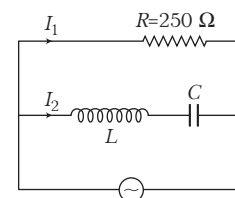
For AC source,

$$\text{impedance, } Z = \frac{V_{rms}}{I_{rms}} = \frac{250}{1.25} = 200 \Omega$$

As $Z < R$, L and C cannot be in series.

R cannot be in series with C as explained earlier.

As current is maximum at resonance and current increases with frequency, so R cannot be in series with L . In fact, L and C must be in series.



$$I_1 = \frac{250}{250} = 1 \text{ A, in phase with voltage}$$

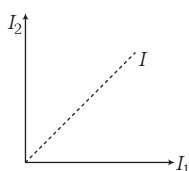
Impedance of lower branch,

$$Z' = \left| \left(\omega L - \frac{1}{\omega C} \right) \right|$$

$$\Rightarrow I_2 = \frac{250}{Z'}$$

The phase difference between I_2 and voltage is $\pi/2$.

The angle between I_1 and I_2 is 90° .



$$I_1^2 + I_2^2 = I^2 \Rightarrow (1)^2 + I_2^2 = (1.25)^2$$

$$\Rightarrow I^2 = (1.25)^2 - (1)^2 = 0.562$$

$$\Rightarrow I_2 = 0.75 \text{ A}$$

$$\Rightarrow I_2 = \frac{V}{Z'}$$

$$\therefore Z' = \frac{V}{I_2}$$

$$\Rightarrow \left| \omega L - \frac{1}{\omega C} \right| = \frac{250}{0.75} \Rightarrow \left| \frac{\omega^2 LC - 1}{\omega C} \right| = \frac{1000}{3}$$

$$\Rightarrow \left| \frac{(2250)^2 LC - 1}{2250 C} \right| = \frac{1000}{3} \quad \dots(i)$$

At resonance, I_2 is maximum, Z' is minimum

$$\Rightarrow Z' = \omega L - \frac{1}{\omega C} = 0$$

$$\Rightarrow \frac{1}{LC} = \omega^2 = \omega_r^2 = (4500)^2 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$\Rightarrow C = \frac{3/4}{2250} \times \frac{3}{1000} = 10^{-6} \text{ F} = 1 \mu\text{F} \Rightarrow LC = \frac{1}{(4500)^2}$$

$$\Rightarrow L = \frac{1}{(4500)^2 \times 10^{-6}} = \frac{100}{45 \times 45} = \frac{4}{81} \text{ H}$$

$$\therefore R = 250 \Omega, L = 4/81 \text{ H}, C = 1 \mu\text{F}$$

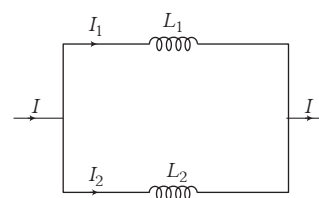
CHECK POINT 7.2

- Ohm's law expressed as $E = IR$
 - can never be applied to AC
 - applies to AC in the same manner as to DC
 - always applies to AC circuits when Z is substituted for R
 - tells us that $E_{\text{eff}} = 0.707(E_{\text{max}})$ for AC
- An alternating current of rms value 10 A is passed through a 12Ω resistor. The maximum potential difference across the resistor is
 - 20 V
 - 90 V
 - 169.70 V
 - None of these
- The reactance of a $25 \mu\text{F}$ capacitor at the AC frequency of 4000 Hz is
 - $\frac{5}{\pi} \Omega$
 - $\sqrt{\frac{5}{\pi}} \Omega$
 - 10Ω
 - $\sqrt{10} \Omega$
- The capacity of a pure capacitor is 1 F. In DC circuits, its effective resistance will be
 - zero
 - infinite
 - 1Ω
 - $1/2 \Omega$
- In an AC circuit, an alternating voltage $e = 200\sqrt{2}\sin 100t$ V is connected to a capacitor of capacity $1 \mu\text{F}$. The rms value of the current in the circuit is
 - 100 mA
 - 200 mA
 - 20 mA
 - 10 mA
- The reactance of a coil when used in the domestic AC power supply (220 V, 50 cycles per second) is 50Ω . The inductance of the coil is nearly
 - 2.2 H
 - 1.6 H
 - 0.22 H
 - 0.16 H
- An inductive coil has a resistance of 100Ω . When an AC signal of frequency 1000 Hz is applied to the coil, the

voltage leads the current by 45° . The inductance of the coil is

- $\frac{1}{10\pi}$
- $\frac{1}{20\pi}$
- $\frac{1}{40\pi}$
- $\frac{1}{60\pi}$

- Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown in figure. Then the ratio of currents I_1/I_2 at any time t is



- L_1/L_2
- L_2/L_1
- $\frac{L_1}{(l_1 + l_2)^2}$
- $\frac{L_2}{(l_1 + l_2)^2}$

- An inductance and a resistance are connected in series with an AC potential. In this circuit,
 - the current and the potential difference across the resistance lead the PD across the inductance by phase angle $\pi/2$
 - the current and the potential difference across the resistance lag behind PD across the inductance by an angle $\pi/2$
 - the current and the potential difference across the resistance lag behind the PD across the inductance by an angle π
 - the PD across the resistance lags behind the PD across the inductance by an angle $\pi/2$ but the current in the resistance leads the PD across inductance by $\pi/2$

10. In L - R circuit, resistance is $8\ \Omega$ and inductive reactance is $6\ \Omega$, then impedance is
 (a) $2\ \Omega$ (b) $14\ \Omega$ (c) $4\ \Omega$ (d) $10\ \Omega$
11. In an AC circuit, the current lags behind the voltage by $\frac{\pi}{3}$.
 The components in the circuit may be
 (a) R and L (b) L and C (c) R and C (d) only R
12. In an AC circuit, a resistance R is connected in series with an inductance L . If phase angle between voltage and current be 45° , the value of inductive reactance will be
 (a) $R/4$
 (b) $R/2$
 (c) R
 (d) cannot be found with the given data
13. In a circuit containing R and L , as the frequency of the impressed AC increases, the impedance of the circuit
 (a) decreases
 (b) increases
 (c) remains unchanged
 (d) first increases and then decreases
14. An AC voltage is applied to a resistance R and an inductor L in series. If R and the inductive reactance are both equal to $3\ \Omega$, the phase difference (in rad) between the applied voltage and the current in the circuit is
 (a) $\pi/4$ (b) $\pi/2$ (c) zero (d) $\pi/6$
15. In an L - R circuit, the value of L is $\left(\frac{0.4}{\pi}\right)$ H and the value of R is $30\ \Omega$. If in the circuit, an alternating emf of 200 V at 50 cycle s^{-1} is connected, the impedance of the circuit and current will be
 (a) $11.4\ \Omega, 17.5\text{ A}$ (b) $30.7\ \Omega, 6.5\text{ A}$
 (c) $40.4\ \Omega, 5\text{ A}$ (d) $50\ \Omega, 4\text{ A}$
16. The instantaneous values of current and voltage in an AC circuit are given by

$$I = 6\sin\left(100\pi t + \frac{\pi}{4}\right)$$

$$V = 5\sin\left(100\pi t - \frac{\pi}{4}\right), \text{ then}$$
 (a) current leads the voltage by 45°
 (b) voltage leads the current by 90°
 (c) current leads the voltage by 90°
 (d) voltage leads the current by 45°
17. In an L - C - R series circuit the AC voltage across R , L and C come out as 10 V , 10 V and 20 V , respectively. The voltage across the entire combination will be
 (a) 30 V (b) $10\sqrt{3}\text{ V}$ (c) 20 V (d) $10\sqrt{2}\text{ V}$
18. With increase in frequency of an AC supply, the impedance of an L - C - R series circuit
 (a) remains constant
 (b) increases
 (c) decreases
 (d) decreases at first, becomes minimum and then increases
19. A sinusoidal voltage of peak value 300 V and an angular frequency $\omega = 400\text{ rads}^{-1}$ is applied to series L - C - R circuit, in which $R = 3\ \Omega$, $L = 20\text{ mH}$ and $C = 625\ \mu\text{F}$. The peak value of current in the circuit is
 (a) $30\sqrt{2}\text{ A}$ (b) 60 A
 (c) 100 A (d) $60\sqrt{2}\text{ A}$
20. The value of current at resonance in a series L - C - R circuit is affected by the value of
 (a) R only (b) C only
 (c) L only (d) L, C and R
21. An L - C - R series circuit is connected to a source of alternating current. At resonance the applied voltage and current flowing through the circuit will have a phase difference of
 (a) zero (b) $\pi/4$
 (c) $\pi/2$ (d) π
22. A series L - C - R circuit is operated at resonance. Then
 (a) voltage across R is minimum
 (b) impedance is minimum
 (c) impedance is maximum
 (d) current amplitude is minimum
23. An L - C - R series circuit is under resonance. If I_m is current amplitude, V_m is voltage amplitude, R is the resistance, Z is the impedance, X_L is the inductive reactance and X_C is the capacitive reactance, then
 (a) $I_m = \frac{V_m}{Z}$ (b) $I_m = \frac{V_m}{X_L}$
 (c) $I_m = \frac{V_m}{X_C}$ (d) $I_m = \frac{V_m}{R}$
24. In an L - C - R series AC circuit, at resonance
 (a) the capacitive reactance is more than the inductive
 (b) the capacitive reactance equals the inductive reactance
 (c) the capacitive reactance is less than the inductive reactance
 (d) the current is minimum
25. An L - C - R series circuit connected to a source E , is at resonance. Then,
 (a) the voltage across R is zero
 (b) the voltage across R equals applied voltage
 (c) the voltage across C is zero
 (d) the voltage across C equals applied voltage

POWER IN AN AC CIRCUIT

For electric circuits power is defined as the product of voltage and current.

In AC circuit, both emf and current changes continuously with respect to time, so in AC circuit, we have to calculate average power in complete cycle ($0 \rightarrow T$).

Hence, average power is defined as the average of instantaneous power in an AC circuit over a full cycle. Its SI unit is watt.

Instantaneous power, $P = VI$

$$[\because V = V_0 \sin \omega t, I = I_0 \sin(\omega t + \phi)]$$

Here, V and I are instantaneous voltage and current, respectively.

Total work done or energy spent in maintaining current over one full cycle is given by

$$W = \frac{V_0 I_0 T}{2} \cos \phi$$

Thus, average power associated in AC circuit is given by

$$P_{av} = \frac{W}{T} = \frac{V_0 I_0}{2} \cos \phi$$

$$P_{av} = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$$

$$P_{av} = V_{rms} I_{rms} \cos \phi$$

Here, $\cos \phi$ is **power factor**.

Power factor is defined as cosine of the angle of lag or lead (i.e. $\cos \phi$). It is also defined as the ratio of resistance and impedance.

$$\cos \phi = \frac{P_{av}}{V_{rms} I_{rms}} = \frac{\text{True power}}{\text{Apparent power}}$$

It is said to be leading, if current leads voltage, lagging, if current lags voltage. Thus, a power factor of 0.5 lagging means current lags the voltage by 60° (as $\cos^{-1}(0.5) = 60^\circ$).

The product of V_{rms} and I_{rms} gives the **apparent power** (or **virtual power**). While the **true power** is obtained by multiplying the apparent power by the power factor $\cos \phi$.

Thus,

$$\text{Apparent power} = V_{rms} \times I_{rms}$$

$$\text{and true power} = \text{apparent power} \times \text{power factor}$$

Special cases

(i) **AC circuit containing R only**

$$\text{Here, } \phi = 0^\circ, P_{av} = V_{rms} I_{rms} \cos 0^\circ$$

$$\Rightarrow P_{av} = V_{rms} I_{rms} \quad [\text{maximum}]$$

(ii) **AC circuit containing L only**

$$\phi = \frac{\pi}{2} \text{ or } P_{av} = V_{rms} I_{rms} \cos \frac{\pi}{2} \Rightarrow P_{av} = 0$$

So, average power in L is zero.

(iii) **AC circuit containing C only**

$$\phi = \frac{\pi}{2} \text{ or } P_{av} = V_{rms} I_{rms} \cos \frac{\pi}{2} = 0$$

So, average power in C is zero.

(iv) **AC circuit containing L and R**

$$\tan \phi = \frac{\omega L}{R} \Rightarrow \cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\Rightarrow P_{av} = V_{rms} I_{rms} \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

(v) **AC circuit containing C and R**

$$\tan \phi = \frac{1/\omega C}{R}$$

$$\Rightarrow \cos \phi = \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

$$\Rightarrow P_{av} = V_{rms} I_{rms} \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

(vi) **AC circuit containing L, C and R**

$$\tan \phi = \frac{\omega L - 1/\omega C}{R}$$

$$\Rightarrow \cos \phi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$\Rightarrow P_{av} = V_{rms} I_{rms} \cdot \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Half power points in series L-C-R circuit

The values of ω at which the power input is half of its maximum value are called half power points.

At these points,

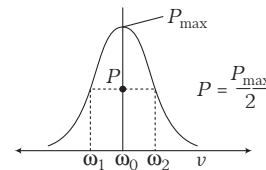
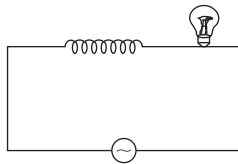


Fig. 7.33 Half power points in series L-C-R circuit

$$\omega = \omega_0 \pm \Delta\omega = \omega_0 \pm \frac{1}{2} \left(\frac{2\Delta\omega}{\omega_0} \right) \omega_0 = \omega_0 \left(1 \pm \frac{1}{2Q} \right)$$

The amplitude of the current falls by a factor of $\frac{1}{\sqrt{2}}$ and hence, the power ($P \propto I^2$) goes down by half. The separation between these two half power points is $2\Delta\omega = \frac{\omega_0}{Q}$. This separation is called the full-width at half maximum and is $\frac{1}{Q}$ th fraction of the resonant frequency.

Example 7.38 An iron cored coil is connected in series with an electric bulb, with an AC source, as shown in figure. As the iron piece is taken out of the coil, how will the brightness of bulb change?



Sol. As the iron rod is taken out of the coil, the self-inductance of the coil decreases, the impedance of circuit decreases, hence current increases. So, power consumed (and hence brightness of bulb) by bulb increases.

Example 7.39 A light bulb has the rating 200W, 220V. Find (i) resistance of the bulb filament and (ii) rms value of current flowing through the filament.

Sol. (i) Resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{220 \times 220}{200} = \frac{22 \times 22}{2} = 242 \Omega$$

$$(ii) \text{ The rms value of current } = \frac{P}{V} = \frac{200}{220} = \frac{10}{11} \text{ A} = 0.9 \text{ A}$$

Example 7.40 A series L-C-R circuit with $R = 20 \Omega$, $L = 1.5 \text{ H}$ and $C = 35 \mu\text{F}$ is connected to a variable frequency 200 V, AC supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Sol. When the frequency of the supply equals the natural frequency of the circuit, the circuit is said to be in resonance. At resonance,

$$Z = R = 20 \Omega$$

Since, the L-C-R circuit is resistive, so the phase angle between the current and voltage is zero.

$$i.e. \quad \phi = 0^\circ$$

$$\text{and } I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{200}{20} = 10 \text{ A}$$

P = average power transferred per cycle.

$$\begin{aligned} \therefore P &= I_{\text{rms}} E_{\text{rms}} \cos 0^\circ \\ &= 10 \times 200 = 2000 \text{ W} \\ &= 2.0 \text{ kW} \end{aligned}$$

Example 7.41 A 100Ω resistor is connected to a 220 V, 50 Hz AC supply.

(i) What is the rms value of current in the circuit?

(ii) What is the net power consumed over a full cycle?

Sol. (i) R_{rms} value of current in the circuit,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{220}{100} = 2.2 \text{ A}$$

(ii) We know that, the power dissipated in an AC circuit is given by $P = V_{\text{rms}} I_{\text{rms}} \cos \phi$

where, ϕ is the phase difference between current and voltage.

In a circuit containing resistor only, $\phi = 0^\circ$, thus $\cos \phi = \cos 0^\circ = 1$

$$\therefore P = V_{\text{rms}} I_{\text{rms}} = 220 \times 2.2 = 484 \text{ W}$$

Example 7.42 A series L-C-R circuit is connected across an AC source $E = 10 \sin \left[100\pi t - \frac{\pi}{6} \right]$. Current from the supply

is $I = 2 \sin \left[100\pi t + \frac{\pi}{12} \right]$, what is the average power dissipated?

Sol. Phase difference between voltage and current,

$$\phi = \frac{\pi}{12} - \left(-\frac{\pi}{6} \right) = \frac{\pi}{4}$$

$$\Rightarrow \text{Power factor} = \cos \phi = \frac{1}{\sqrt{2}}$$

Average power dissipated,

$$\begin{aligned} &= \frac{V_m I_m}{2} \cos \phi \\ &= \frac{10 \times 2}{2} \times \frac{1}{\sqrt{2}} = 5\sqrt{2} \text{ W} \end{aligned}$$

Example 7.43 An AC circuit containing 800 mH inductor and a $60 \mu\text{F}$ capacitor is in series with 15Ω resistance. They are connected to 230 V, 50 Hz AC supply. Obtain average power transferred to each element and total power absorbed.

Sol. Reactance of the inductor,

$$X_L = \omega L = 2\pi fL = 2 \times 3.14 \times 50 \times 800 \times 10^{-3} = 251.2 \Omega$$

Capacitive reactance,

$$\begin{aligned} X_C &= \frac{1}{\omega C} = \frac{1}{2\pi fC} \\ &= \frac{1}{2 \times 3.14 \times 50 \times 60 \times 10^{-6}} = 53.07 \Omega \end{aligned}$$

$$\begin{aligned} \therefore \text{ Impedance, } Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{15^2 + (251.2 - 53.07)^2} \end{aligned}$$

$$\Rightarrow Z = 198.7 \Omega$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{230}{198.7} = 1.157 \text{ A}$$

Note that the inductor and capacitor do not consume power over a cycle.

Now, average power dissipated in the resistor,

$$= I_{\text{rms}}^2 \cdot R = (1.157)^2 \times 15 = 20.07 \text{ W}$$

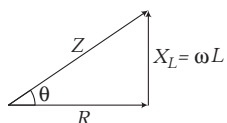
Total power consumed $= V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos \phi$

$$\begin{aligned} &= V_{\text{rms}} \cdot I_{\text{rms}} \cdot \frac{R}{Z} \\ &= \frac{230 \times 1.157 \times 15}{198.7} = 20.08 \text{ W} \end{aligned}$$

Note that the total power consumed always is same as that consumed in resistor.

Example 7.44 A 60 cycle AC circuit has a resistance of 200Ω and inductor of 100 mH . What is the power factor? What capacitance placed in the circuit will make the power factor unity?

Sol. $Z = \sqrt{R^2 + (\omega L)^2}$
 $= \sqrt{(200)^2 + (2\pi \times 60 \times 100 \times 10^{-3})^2} = 203.52 \Omega$



Power factor, $\cos \phi = \frac{R}{Z} = \frac{200}{203.52} = 0.983$

Now, for the power factor to be unity, $\phi = 0$ that means

$$X_L = X_C \text{ or } \omega L = \frac{1}{\omega C}$$

$$\therefore C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi \times 60)^2 \times 100 \times 10^{-3}}$$

$$\therefore C = 70.36 \mu\text{F}$$

Example 7.45 An L - C - R series circuit with 100Ω resistance is connected to an AC source of 200 V and angular frequency 300 rads^{-1} . When only the capacitance is removed, the current lags behind the voltage by 60° . When only the inductance is removed, the current leads the voltage by 60° . Calculate the current and the power dissipated in the L - C - R circuit.

Sol. When capacitance is removed,

$$\tan \phi = \frac{X_L}{R}$$

or $\tan 60^\circ = \frac{X_L}{R}$

$$\therefore X_L = \sqrt{3} R \quad \dots(i)$$

When inductance is removed,

$$\tan \phi = \frac{X_C}{R} \text{ or } \tan 60^\circ = \frac{X_C}{R}$$

$$\therefore X_C = \sqrt{3} R \quad \dots(ii)$$

From Eqs. (i) and (ii) we see that, $X_C = X_L$

So, the L - C - R circuit is in resonance.

Hence,

$$Z = R$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200}{100} = 2 \text{ A}$$

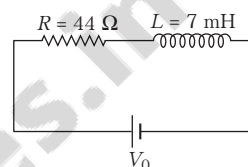
$$\text{Power dissipated } \langle P \rangle = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

At resonance, current and voltage are in phase or $\phi = 0^\circ$

$$\therefore \langle P \rangle = (200)(2)(1) = 400 \text{ W}$$

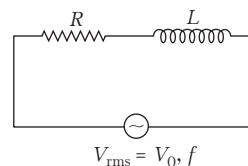
Example 7.46 A solenoid with inductance $L = 7 \text{ mH}$ and active resistance $R = 44 \Omega$ is first connected to a source of direct voltage V_0 and then to source of sinusoidal voltage with effective value $V = V_0$. At what frequency of the oscillator will the power consumed by the solenoid be $\eta = 5.0$ times less than in the former case?

Sol. When a solenoid is connected to a source of direct voltage, V_0



$$\text{Power consumed, } P_1 = \frac{V_0^2}{R}$$

When it is connected to a sinusoidal voltage, effective impedance comes into role.



$$\therefore Z = \sqrt{R^2 + X_L^2}$$

Now, power consumed will be

$$P_2 = V_{\text{rms}} I_{\text{rms}} \cos \phi = V_0 \cdot \frac{V_0}{Z} \cdot \frac{R}{Z} = \frac{V_0^2 R}{Z^2}$$

Given, $P_2 = \frac{P_1}{\eta}$

$$\Rightarrow \frac{V_0^2 R}{Z^2} = \frac{V_0^2}{\eta R}$$

$$\Rightarrow \eta R^2 = Z^2 = R^2 + X_L^2$$

$$\Rightarrow X_L^2 = (\eta - 1) R^2$$

$$\Rightarrow (2\pi f L)^2 = (\eta - 1) R^2$$

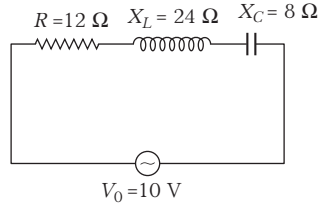
$$\therefore \text{Frequency of the oscillator, } f = \frac{R}{2\pi L} \sqrt{\eta - 1}$$

$$= \frac{44}{2 \times \frac{22}{7} \times 7 \times 10^{-3}} \sqrt{5 - 1}$$

$$= 2 \times 10^3 \text{ Hz} = 2 \text{ kHz}$$

Example 7.47 Consider the following R-L-C circuit in which $R = 12 \Omega$, $X_L = 24 \Omega$, $X_C = 8 \Omega$. The emf of source is given by $V = 10 \sin(100\pi t)V$.

- (i) Find the energy dissipated in 10 min.
 (ii) If resistance is removed from the circuit and value of inductance is doubled, express variation of current with time t in the new circuit.



Sol. (i) Impedance,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(12)^2 + (24 - 8)^2} = 20 \Omega$$

$$\cos \phi = \frac{R}{Z} = \frac{12}{20} = \frac{3}{5}$$

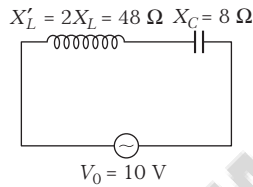
Power consumed,

$$P = \frac{1}{2} V_0 I_0 \cos \phi = \frac{1}{2} V_0 \cdot \frac{V_0}{Z} \cos \phi$$

$$= \frac{1}{2} \frac{(10)^2}{(20)} \times \frac{3}{5} = 1.5 \text{ W}$$

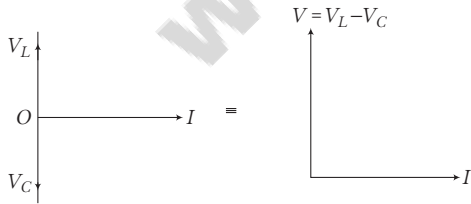
Energy dissipated, $E = Pt = (1.5)(10 \times 60) = 900 \text{ J}$

(ii)



Impedance, $Z = X'_L - X_C = 48 - 8 = 40 \Omega$

$$\therefore \text{Current, } I_0 = \frac{V_0}{Z} = \frac{10}{40} = \frac{1}{4} \text{ A}$$

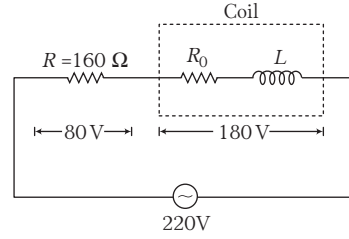


Current lags the voltage by $\pi/2$.

$$I = I_0 \sin(\omega t - \pi/2) = \frac{1}{4} \sin(100\pi t - \pi/2) \text{ A}$$

Example 7.48 A series circuit consisting of an inductance free resistance $R = 0.16 \text{ k} \Omega$ and a coil with active resistance is connected to the mains with effective voltage $V = 220 \text{ V}$. Find the heat power generated in the coil, if the effective voltage values across the resistance R and the coil are equal to $V_1 = 80 \text{ V}$ and $V_2 = 180 \text{ V}$, respectively.

Sol.



$$\text{Current in the circuit, } I_{\text{rms}} = \frac{V_r}{R} = \frac{80}{160} = \frac{1}{2} \text{ A}$$

$$\text{Now, } V_{R_0} = I_{\text{rms}} R_0 = \frac{R_0}{2}$$

$$\text{and } V_L = I_{\text{rms}} X_L = \frac{X_L}{2}$$

$$\Rightarrow (V_R + V_{R_0})^2 + V_L^2 = (220)^2$$

$$\Rightarrow \left(80 + \frac{R_0}{2}\right)^2 + \left(\frac{X_L}{2}\right)^2 = (220)^2 \quad \dots(i)$$

$$\text{Also, } V_{R_0}^2 + V_L^2 = (180)^2$$

$$\Rightarrow \left(\frac{R_0}{2}\right)^2 + \left(\frac{X_L}{2}\right)^2 = (180)^2 \quad \dots(ii)$$

Subtracting Eq. (ii) from Eq. (i), we get

$$\left(80 + \frac{R_0}{2}\right)^2 - \left(\frac{R_0}{2}\right)^2 = (220)^2 - (180)^2$$

$$\Rightarrow (80 + R_0)(80) = (400)(40)$$

$$\Rightarrow R_0 = 120 \Omega$$

Power consumed in coil,

$$P = I_{\text{rms}}^2 R_0 = \left(\frac{1}{2}\right)^2 \times 120 = 30 \text{ W}$$

Example 7.49 A current of 4 A flows in a coil when connected to a 12 V DC source. If the same coil is connected to a 12 V, 50 rads^{-1} AC source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit, if a 2500 μF capacitor is connected in series with the coil.

Sol. (i) A coil consists of an inductance (L) and a resistance (R).

In DC, only resistance is effective. Hence,

$$R = \frac{V}{I} = \frac{12}{4} = 3 \Omega$$

$$\text{In AC, } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\therefore L^2 = \frac{1}{\omega^2} \left[\left(\frac{V_{\text{rms}}}{I_{\text{rms}}} \right)^2 - R^2 \right]$$

$$\therefore L = \frac{1}{\omega} \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}} \right)^2 - R^2}$$

Substituting the values, we get

$$L = \frac{1}{50} \sqrt{\left(\frac{12}{2.4}\right)^2 - (3)^2} = 0.08 \text{ H}$$

(ii) When capacitor is connected to the circuit, the impedance is, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Here,

$$R = 3 \Omega$$

$$X_L = \omega L = (50)(0.08) = 4 \Omega$$

and

$$X_C = \frac{1}{\omega C} = \frac{1}{(50)(2500 \times 10^{-6})} = 8 \Omega$$

$\therefore$

$$Z = \sqrt{(3)^2 + (4 - 8)^2} = 5 \Omega$$

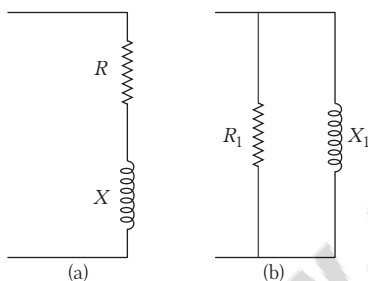
Now,

$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

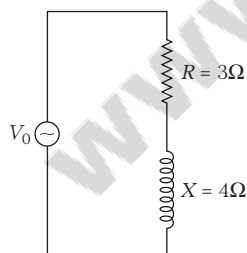
$$= V_{\text{rms}} \times \frac{V_{\text{rms}}}{Z} \times \frac{R}{Z} = \left(\frac{V_{\text{rms}}}{Z}\right)^2 \times R$$

$$\begin{aligned} \text{Substituting the values, we get } P &= \left(\frac{12}{5}\right)^2 \times 3 \\ &= 17.28 \text{ W} \end{aligned}$$

Example 7.50 The series and parallel circuits shown in figure have the same impedance and the same power factor. If $R = 3 \Omega$ and $X = 4 \Omega$, find the values of R_1 and X_1 . Also, find the impedance and power factor.



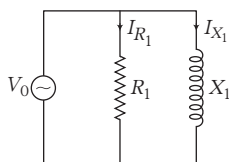
Sol.



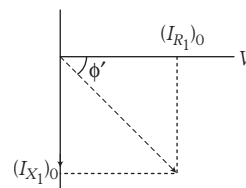
$$\text{Impedance, } Z = \sqrt{R^2 + X^2} = 5 \Omega$$

$$\text{Power factor, } \cos \phi = \frac{R}{Z} = \frac{3}{5} \Rightarrow \phi = \cos^{-1}(0.6)$$

$$\Rightarrow \tan \phi = \frac{X}{R} = \frac{4}{3}$$



$$\text{Now, we have, } (I_{R_1})_0 = \frac{V_0}{R_1}, (I_{X_1})_0 = \frac{V_0}{X_1}$$



$$\therefore I_0 = \sqrt{(I_{R_1})_0^2 + (I_{X_1})_0^2} = \sqrt{\left(\frac{V_0}{R_1}\right)^2 + \left(\frac{V_0}{X_1}\right)^2}$$

$$\Rightarrow \frac{V_0}{Z} = V_0 \sqrt{\frac{1}{R_1^2} + \frac{1}{X_1^2}} \Rightarrow Z = \frac{1}{\sqrt{\frac{1}{R_1^2} + \frac{1}{X_1^2}}} = 5 \quad \dots (i)$$

$$\tan \phi' = \frac{(I_{X_1})_0}{(I_{R_1})_0} = \frac{V_0/X_1}{V_0/R_1} = \frac{R_1}{X_1} = \tan \phi = \frac{4}{3} \quad \dots (ii)$$

Putting $X_1 = \frac{3}{4} R_1$ in Eq. (i)

$$\Rightarrow \frac{1}{R_1^2} + \frac{1}{\left(\frac{3}{4} R_1\right)^2} = \frac{1}{5^2}$$

$$\Rightarrow \frac{1}{R_1^2} \left(1 + \frac{16}{9}\right) = \frac{1}{25}$$

$$\Rightarrow \frac{1}{R_1^2} = \frac{9}{(25)^2}$$

$$\Rightarrow R_1 = \frac{25}{3} \Omega$$

$$\Rightarrow X_1 = \frac{3}{4} R_1 = \frac{25}{4} \Omega$$

Wattless current

The current which consumes no power for its maintenance in the circuit is called wattless current or idle current.

or

If the resistance in an AC circuit is zero, its power factor will be zero. Although the current flows in the circuit, yet the average power remains zero, i.e. there is no energy dissipation in the circuit. Such a circuit is called the wattless circuit and the current flowing is called the wattless current.

If the circuit contains either inductance or capacitance only, then phase difference between current and voltage is 90° , i.e. $\phi = 90^\circ$. The average power in such a circuit is

$$\begin{aligned} P_{\text{av}} &= V_{\text{rms}} \times I_{\text{rms}} \times \cos \phi \\ &= V_{\text{rms}} \times I_{\text{rms}} \times \cos 90^\circ = 0 \end{aligned}$$

The concept of wattless current is similar to that of a frictionless pendulum, where the total work done by gravity upon the pendulum in a cycle is zero.

L-C oscillations

When a charged capacitor is allowed to discharge through a non-resistive inductor, electrical oscillations of constant amplitude and frequency are produced. These oscillations are called *L-C oscillations*.

Working

When a capacitor is supplied with an AC current, it gets charged.

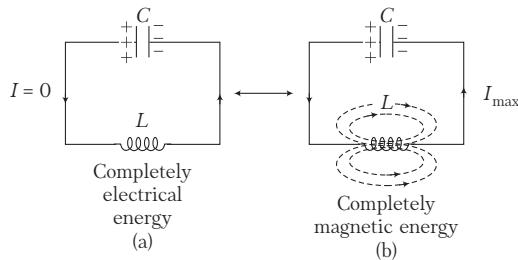


Fig. 7.34

When this charged capacitor is connected with an inductor, current flows through inductor, giving rise to magnetic flux, hence induced emf is produced in the circuit. Due to this, the charge (or energy) on the capacitor decreases and an equivalent amount of energy is stored in the inductor in the form of magnetic field. When the discharging of the capacitor completes, current and magnetic flux linked with L starts decreasing.

Therefore, an induced emf is produced which recharges the capacitor in opposite direction. This process of charging and discharging of capacitor is repeated and energy taken once from source keeps on oscillating between C and L .

The equation of LC oscillation is given by,

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

where, $q = q_0 \cos(\omega t + \phi)$

And the charge oscillates with a frequency,

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

The $L-C$ oscillations discussed above are not realistic for the two reasons.

- Every inductor has some resistance. The effect of this resistance will introduce a damping effect on the charge and current in the circuit and the oscillations finally die away.
- Even, if the resistance is zero, the total energy of the system would not remain constant. It is radiated away from the system in the form of electromagnetic waves. In fact, radio and TV transmitters depend on this radiation.

Example 7.51 A charged $30 \mu\text{F}$ capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Sol. Angular frequency of free oscillations of $L-C$ oscillations is

$$\begin{aligned}\omega &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} \\ &= 1.11 \times 10^3 \text{ rad s}^{-1}\end{aligned}$$

Example 7.52 A radio can tune over the frequency range of a portion of MW broadcast band (800 kHz to 1200 kHz). If its $L-C$ circuit has an effective inductance of $200 \mu\text{H}$, what must be the range of its variable capacitor?

Sol. For tuning, the natural frequency, i.e. the frequency of free oscillations of the LC circuit should be equal to the frequency of the radiowave. Now for

$$f = \frac{1}{2\pi\sqrt{LC}}, \text{ we get}$$

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} \text{ or } C_1 = \frac{1}{4\pi^2 L f_1^2}$$

$$\begin{aligned}\text{or } C_1 &= \frac{1}{4 \times 9.87 \times 2 \times 10^{-4} \times (8 \times 10^5)^2} \\ &= 197.8 \times 10^{-12} \text{ F} \\ &= 197.8 \text{ pF}\end{aligned}$$

$$\text{Similarly, } C_2 = \frac{1}{4\pi^2 L f_2^2} = \frac{1}{4 \times 9.87 \times 2 \times 10^{-4} \times (12 \times 10^5)^2}$$

$$\begin{aligned}\text{or } C_2 &= 87.95 \times 10^{-12} \text{ F} \\ &= 87.95 \text{ pF}\end{aligned}$$

Example 7.53 In an $L-C$ circuit $L = 33 \text{ H}$ and $C = 840 \text{ pF}$. At $t = 0$ charge on the capacitor is $105 \mu\text{C}$ and maximum. Compute the following quantities at $t = 2 \text{ ms}$.

- The energy stored in the capacitor.
- The total energy in the circuit.
- The energy stored in the inductor.

Also, write the equation for the variation of current in the inductor with time.

Sol. Given, $L = 33 \text{ H}$, $C = 840 \times 10^{-12} \text{ F}$

and $q_0 = 105 \times 10^{-6} \text{ C}$

The angular frequency of $L-C$ oscillations is,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{33 \times 840 \times 10^{-12}}} = 1.9 \times 10^4 \text{ rad s}^{-1}$$

Charge stored in the capacitor at time t , would be, $q = q_0 \cos \omega t$

(i) At $t = 2 \times 10^{-3} \text{ s}$

$$\begin{aligned}q &= (105 \times 10^{-6}) \cos(1.9 \times 10^4 \times 2 \times 10^{-3}) \\ &= 100.3 \times 10^{-6} \text{ C}\end{aligned}$$

$\therefore$ Energy stored in the capacitor,

$$U_C = \frac{1}{2} \frac{q^2}{C} = \frac{(100.3 \times 10^{-6})^2}{2 \times 840 \times 10^{-12}} = 6.0 \text{ J}$$

- (ii) Total energy in the circuit,

$$U = \frac{1}{2} \frac{q_0^2}{C} = \frac{(105 \times 10^{-6})^2}{2 \times 840 \times 10^{-12}} = 6.56 \text{ J}$$

- (iii) Energy stored in inductor in the given time

= total energy in circuit – energy stored in capacitor

$$= (6.56 - 6.0) \text{ J} = 0.56 \text{ J}$$

$$\text{Also, } I = I_0 \sin \omega t$$

$$\text{where, } I_0 = q_0 \times \omega$$

$$= 105 \times 10^{-6} \times 1.9 \times 10^4 = 2 \text{ A}$$

$$\therefore I = (2 \text{ A}) \sin \omega t$$

$$\left(\because I = \frac{q}{t} \right)$$

Example 7.54 An L - C circuit contains 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant, the circuit is closed be $t = 0$. What is the total energy stored initially? At what times is the total energy shared equally between the inductor and the capacitor?

Sol. Total energy stored initially means energy stored in the

$$\text{capacitor} = \frac{Q_0^2}{2C} = \frac{(10 \times 10^{-3})^2}{2 \times 50 \times 10^{-6}} = 1 \text{ J}$$

CHECK POINT 7.3

1. An electric heater rated 220 V and 550 W is connected to AC mains. The current drawn by it is
(a) 0.8 A (b) 2.5 A
(c) 0.4 A (d) 1.25 A

2. In an AC circuit, the potential difference V and current I are given respectively by

$$V = 100 \sin(100t) \text{ V}$$

$$\text{and } I = 100 \sin\left(100t + \frac{\pi}{3}\right) \text{ mA}$$

The power dissipated in the circuit will be

- (a) 10^4 W (b) 10 W (c) 2.5 W (d) 5 W

3. In an AC circuit, the power factor

- (a) is zero when the circuit contains an ideal resistance only
(b) is unity when the circuit contains an ideal resistance only
(c) is infinity when the circuit contains an ideal inductance only
(d) None of the above

4. The average power dissipated in a pure inductor of inductance L , when an AC current is passing through it, is

- (a) $\frac{1}{2} LI^2$ (b) $\frac{1}{4} LI^2$ (c) $2LI^2$ (d) zero

5. The average power dissipated in a pure capacitance C in an AC circuit is

- (a) CV (b) zero (c) $\frac{1}{CV^2}$ (d) $\frac{1}{4} CV^2$

Total energy is $\frac{Q_0^2}{2C}$. So, when the total energy is shared

equally between the capacitor and inductor, the energy stored in the capacitor that time will be half of the total energy

$$= \frac{1}{2} \left(\frac{Q_0^2}{2C} \right).$$

Let the charge at that time be q , then $\frac{q^2}{2C} = \frac{1}{2} \left(\frac{Q_0^2}{2C} \right)$.

$$\therefore q = \pm \frac{Q_0}{\sqrt{2}}$$

$$\text{Also, } q = Q_0 \cos \omega t \text{ or } \pm \frac{Q_0}{\sqrt{2}} = Q_0 \cos \omega t$$

$$\therefore \omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4} \text{ etc.}$$

$$\therefore t = \frac{\pi}{4\omega} = \frac{\pi}{4} \frac{T}{2\pi} = \frac{T}{8}$$

$$\text{or } t = \frac{3\pi}{4\omega} = \frac{3T}{8} \text{ and so on}$$

So, energy will be equally shared at $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}$ etc.

6. The impedance of a circuit consists of 3 Ω resistance and 4 Ω reactance. The power factor of the circuit is

- (a) 0.4 (b) 0.6
(c) 0.8 (d) 1.0

7. Power dissipated in an L - C - R series circuit connected to an AC source of emf ϵ is

$$(a) \frac{\epsilon^2 R}{\left[R^2 + \left(L\omega - \frac{1}{C\omega} \right)^2 \right]} \quad (b) \frac{\epsilon^2 \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega} \right)^2}}{R}$$

$$(c) \frac{\epsilon^2 \left[R^2 + \left(L\omega - \frac{1}{C\omega} \right)^2 \right]}{R} \quad (d) \frac{\epsilon^2 R}{\sqrt{R^2 + \left(L\omega + \frac{1}{C\omega} \right)^2}}$$

8. The energy stored in an inductor of self-inductance L henry carrying a current of I is

- (a) $\frac{1}{2} L^2 I$ (b) $\frac{1}{2} L I^2$
(c) $L I^2$ (d) $L^2 I$

9. In an inductor of inductance $L = 100 \text{ mH}$, a current of $I = 10 \text{ A}$ is flowing. The energy stored in the inductor is

- (a) 5 J (b) 10 J
(c) 100 J (d) 1000 J

CHOKE COIL

Choke coil is a device having high inductance and negligible resistance. It is used in AC circuits for the purpose of adjusting current to any required value in such a way that power loss in a circuit can be minimised. It is used in fluorescent tubes.

Principle

It is based on the principle of wattless current.

Construction

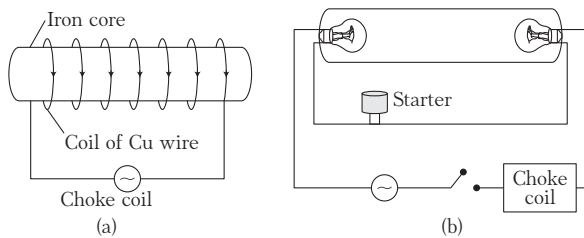


Fig. 7.35

It consists of a copper coil wound over a soft iron laminated core. Thick copper wire is used in order to reduce the resistance (R) of the circuit whereas to improve the inductance (L) of the coil, soft iron is used. The choke coil got its name because it is used to choke or reduce the current in the circuit.

Types of choke coil

Choke coil for different frequencies are made by using different substances in their core.

- (i) To reduce low frequency alternating currents, choke coils with laminated soft iron cores are used. These are called **af choke coils**.
- (ii) To reduce high frequency alternating currents, choke coils with air cores are used. These are called **rf choke coils**.

Theory

Let us consider a choke coil of large inductance L and low resistance R as shown in Fig. 7.36. The power factor for such a coil is given by

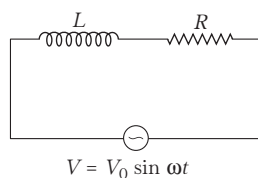


Fig. 7.36 AC voltage applied to a choke coil

$$\cos \phi = \frac{R}{\sqrt{R^2 + X_L^2}}$$

where, X_L is the inductive reactance or effective opposition of the choke coil. It is given by $X_L = L\omega$

$$\therefore \cos \phi \approx \frac{R}{L\omega} \approx 0 \quad (\text{as } R \ll L\omega)$$

As $R \ll L\omega$, $\cos \phi$ is very small. Thus, the power absorbed by the coil $V_{\text{rms}} I_{\text{rms}} \cos \phi$ is very small.

On account of its large impedance, $Z = \sqrt{R^2 + \omega^2 L^2}$, the current passing through the coil is very small. In actual practice choke coil is equivalent to a R - L circuit. For an ideal choke coil $R = 0$, therefore no electric energy is wasted, i.e. average power, $P = 0$.

The only loss of energy is due to hysteresis in the iron core, which is much less than the loss of energy in the resistance that can also reduce the current, if placed instead of the choke coil.

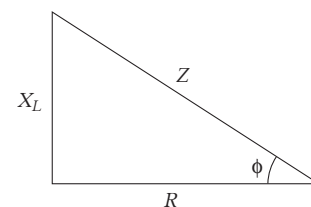
Example 7.55 An AC circuit consists of a $220 \, \Omega$ resistance and a $0.7 \, \text{H}$ choke. Find the power absorbed from $220 \, \text{V}$ and $50 \, \text{Hz}$ source connected in this circuit, if the resistance and choke are joined,

- (i) in series (ii) in parallel.

Sol. (i) In series the impedance of the circuit,

$$\begin{aligned} Z &= \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi fL)^2} \\ &= \sqrt{(220)^2 + (2 \times 3.14 \times 50 \times 0.7)^2} = 311 \, \Omega \end{aligned}$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{311} = 0.707 \, \text{A}$$



and from the impedance diagram drawn above,

$$\cos \phi = \frac{R}{Z} = \frac{220}{311} = 0.707$$

$\therefore$ The power absorbed in the circuit,

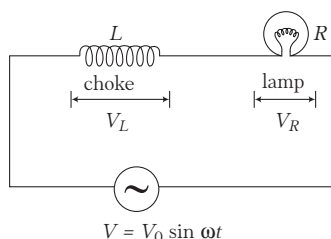
$$\begin{aligned} P &= V_{\text{rms}} I_{\text{rms}} \cos \phi \\ &= (220)(0.707)(0.707) \, \text{W} \\ &= 109.97 \, \text{W} \end{aligned}$$

- (ii) When the resistance and choke are in parallel, the entire power is absorbed in resistance, as the choke (having zero resistance) absorbs no power.

$$\therefore P = \frac{V_{\text{rms}}^2}{R} = \frac{(220)^2}{220} = 220 \, \text{W}$$

Example 7.56 A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The lamp has an effective resistance of $5\ \Omega$ when running at 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (DC), what additional resistance is required? Compare the power losses in both cases.

Sol. For lamp, $(V_{\text{rms}})_R = (i_{\text{rms}})(R) = 10 \times 5 = 50\ \text{V}$



In series,

$$\begin{aligned} (V_{\text{rms}})^2 &= (V_{\text{rms}})_R^2 + (V_{\text{rms}})_L^2 \\ \therefore (V_{\text{rms}})_L &= \sqrt{(V_{\text{rms}})^2 - (V_{\text{rms}})_R^2} \\ &= \sqrt{(160)^2 - (50)^2} = 152\ \text{V} \end{aligned}$$

As $(V_{\text{rms}})_L = (I_{\text{rms}})X_L = (I_{\text{rms}})(2\pi fL)$

$$\therefore L = \frac{(V_{\text{rms}})_L}{(2\pi f)(I_{\text{rms}})}$$

$$\begin{aligned} \text{Substituting the values, } L &= \frac{152}{(2\pi)(50)(10)} \\ &= 4.84 \times 10^{-2}\ \text{H} \end{aligned}$$

Now, when the lamp is operated at 160 V (DC) and instead of choke let an additional resistance R' is put in series with it then,

$$\begin{aligned} V &= I(R + R') \\ \text{or } 160 &= 10(5 + R') \\ \therefore R' &= 11\ \Omega \end{aligned}$$

In case of AC, as the choke has no resistance, power loss in choke is zero.

In case of DC, the power loss in additional resistance R' is,

$$P = I^2 R' = (10)^2 (11) = 1100\ \text{W}$$

Transformer

It is a device which is used to change the alternating voltage. The transformers are of the following types

Step-up transformer

It is used to increase the AC voltages.

Step-down transformer

It is used to decrease the AC voltages.

Principle

Transformer is based on the principle of mutual induction, i.e. whenever an amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.

Construction

It consists of two coils, primary coil (P) and secondary coil (S), insulated from each other wound on soft iron core. Often the primary coil is the input coil and secondary coil is the output coil. These soft iron cores are laminated to minimise eddy current loss.

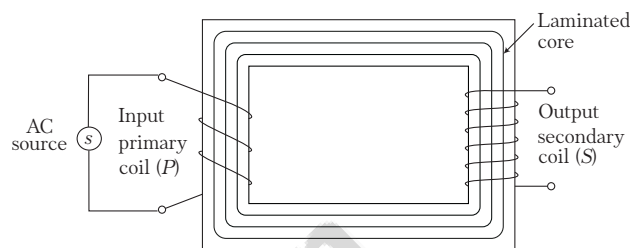


Fig. 7.37 Construction of a transformer

Theory and working

The value of the emf induced in secondary coil due to alternating voltage applied to primary depends on the number of turns in the secondary. We consider an **ideal transformer** in which the primary has negligible resistance and all the flux in the core links both primary and secondary windings.

Let ϕ be the flux in each turn in the core at time t , due to current in the primary, when a voltage V_P is applied to it. Then, the induced emf or voltage ϵ_S , in the secondary having N_S turns is

$$\epsilon_S = -N_S \frac{d\phi}{dt} \quad \dots(i)$$

The alternating flux ϕ also induces an emf, called back emf in the primary. This is

$$\epsilon_P = -N_P \frac{d\phi}{dt} \quad \dots(ii)$$

But $\epsilon_P = V_P$. If this were not so, the primary current would be infinite, since the primary has zero resistance (as considered). If the secondary is an open circuit or the current taken from it is small, then to a good approximation.

$$\epsilon_S = V_S$$

where, V_S is the voltage across the secondary.

Therefore, Eqs. (i) and (ii) can be written as,

$$V_S = -N_S \frac{d\phi}{dt} \quad \dots(iii)$$

$$V_P = -N_P \frac{d\phi}{dt} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we have

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \quad \dots(v)$$

The above relation is based on three assumptions

- (i) The primary resistance and current are small.
- (ii) The same flux links both the primary and the secondary as there is very little leakage of magnetic flux.
- (iii) The secondary current is small.

Efficiency of a transformer is defined as the ratio of output power to the input power.

$$i.e. \quad \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_S I_S}{V_P I_P}$$

If the transformer is assumed to be 100% efficient (no energy losses), the power input is equal to the power output, and since $P = IV$, $I_P V_P = I_S V_S$... (vi)

Although, some energy is always lost, still this is a good approximation, since a well designed transformer may have an efficiency of more than 95%.

Combining Eqs. (v) and (vi), we have

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P} \quad \dots(vii)$$

Since, I and V both oscillate with the same frequency as the AC source, Eq. (vii) also gives the ratio of the amplitudes or rms values of corresponding quantities.

Now, we can observe how a transformer affects the voltage and current. We have

$$V_S = \left(\frac{N_S}{N_P} \right) V_P \text{ and } I_S = \left(\frac{N_P}{N_S} \right) I_P \quad \dots(viii)$$

So, if the secondary coil has a greater number of turns than the primary (*i.e.* $N_S > N_P$), the voltage is stepped up ($V_S > V_P$). This type of arrangement is called a **step-up transformer**. However, in this arrangement, there is less current in the secondary than in the primary (*i.e.* $N_P/N_S < 1$ and $I_S < I_P$). If the secondary coil has less turns than the primary (*i.e.* $N_S < N_P$), we have a **step-down transformer**. In this case, $V_S < V_P$ and $I_S > I_P$, *i.e.*, the voltage is stepped down (or reduced) and the current is increased. The equations obtained above apply to ideal transformers (*i.e.* which have zero energy losses).

Note The ratio $k = \frac{N_S}{N_P}$, is also known as transformation ratio.

Energy losses in transformers

In actual transformers, small energy losses do occur due to the following reasons.

- (i) **Flux leakage** There is always some leakage of flux *i.e.*, not all of the flux that the primary passes through the secondary, due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

- (ii) **Resistance of the windings** The wire used for the windings has some resistance and so, energy is lost due to heat produced in the wire ($I^2 R$). In high current, low voltage windings, this is minimised by using thick wire.
- (iii) **Eddy currents** The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.
- (iv) **Hysteresis** The magnetisation of the core is repeatedly reversed by an alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.
- (v) **Magnetostriction** It is the humming noise of a transformer.

Uses of transformers

Transformers are used in almost all AC operations. Some of them are given below

- (i) In the induction furnaces.
- (ii) In voltage regulators for TV, computer, refrigerator etc.
- (iii) A step-down transformer is used for the purpose of weldings.
- (iv) In the transmission of AC over long distances.

Example 7.57 In a step-down transformer having primary to secondary turn ratio 10 : 1, the input voltage applied is 250V and output current is 10 A. Assuming 100% efficiency, calculate the

- (i) voltage across secondary coil,
- (ii) current in primary coil
- (iii) and power output.

Sol. Given, $\frac{N_S}{N_P} = \frac{1}{10}$, $V_P = 250V$

and $I_S = 10A$

$$(i) \quad \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$\Rightarrow V_S = \frac{N_S}{N_P} \times V_P = \frac{1}{10} \times 250 = 25V$$

$$(ii) \quad \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

$$\Rightarrow I_P = \frac{N_S}{N_P} I_S = \frac{1}{10} \times 10 = 1A$$

$$(iii) \text{ Power output } = V_S I_S = 25 \times 10 = 250W$$

Example 7.58 A 10 kW transformer has 20 turns in the primary and 100 turns in the secondary circuit. An AC voltage $V_1 = 600 \sin 314 t$ is applied to the primary. Find (i) the maximum value of flux and (ii) the maximum value of secondary voltage.

Sol. (i) Flux linked with each turn of primary,

$$\phi = BA \cos \omega t = \phi_0 \cos \omega t$$

Here, $\phi_0 = BA$ = maximum value of flux linked with each turn

$$\therefore V_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt} (\phi_0 \cos \omega t) = \omega N_1 \phi_0 \sin \omega t$$

$$\text{Peak value of } V_1 = V_0 = \omega N_1 \phi_0 \text{ or } \phi_0 = \frac{V_0}{\omega N_1}$$

$$\text{Given, } V_1 = 600 \sin 314 t = V_0 \sin \omega t$$

$$\therefore V_0 = 600 \text{ V, } \omega = 314 \text{ rad s}^{-1}$$

$$\text{Hence, } \phi_0 = \frac{600}{314 \times 20} = 0.0955 \text{ Wb}$$

$$(ii) \frac{V_2^0}{V_1^0} = \frac{N_2}{N_1}$$

$\therefore$ Maximum value of secondary voltage is,

$$V_2^0 = \frac{N_2}{N_1} V_1^0 = \frac{100}{20} \times 600 = 3000 \text{ V}$$

Example 7.59 (i) The primary of a transformer has 400 turns while the secondary has 2000 turns. If the power output from the secondary at 1100 V is 12.1 kW, calculate the primary voltage. (ii) If the resistance of the primary is 0.2Ω and that of the secondary is 2.0Ω and the efficiency of the transformer is 90 % calculate the heat losses in the primary and the secondary coils.

Sol. (i) Given, $N_1 = 400$, $N_2 = 2000$, $V_2 = 1100 \text{ V}$

$$\text{As, } V_1 = V_2 \cdot \frac{N_1}{N_2} = 1100 \times \frac{400}{2000} = 220 \text{ V}$$

(ii) Resistance of primary, $R_1 = 0.2 \Omega$

Resistance of secondary, $R_2 = 2.0 \Omega$

Output power, $P_2 = V_2 I_2 = 12.1 \text{ kW} = 12100 \text{ W}$

$\therefore$ Current in the secondary,

$$I_2 = \frac{P_2}{V_2} = \frac{12100}{1100} = 11 \text{ A}$$

As, efficiency = $\frac{\text{output power}}{\text{input power}}$

$$\Rightarrow \frac{90}{100} = \frac{12100 \text{ W}}{\text{Input power}}$$

$$\text{Input power, } P_1 = \frac{12100 \times 100}{90} = 13.44 \times 10^3 \text{ W}$$

Also, input power, $P_1 = V_1 I_1$

$$\therefore \text{Current in the primary, } I_1 = \frac{P_1}{V_1} = \frac{13.44 \times 10^3}{220} = 61.1 \text{ A}$$

Power loss in the primary,

$$= I_1^2 R_1 = (61.1)^2 \times 0.2 = 746.64 \text{ W}$$

Power loss in the secondary,

$$= I_2^2 R_2 = (11)^2 \times 2.0 = 242 \text{ W}$$

Electric generator or dynamo

Electric generator is a device which is used to convert mechanical energy to electrical energy. Generator provide nearly all of the power for electric power grids.

The reverse conversion of electrical energy to mechanical energy is done by electric motor.

Electric generators are of two types

1. AC generator
2. DC generator

1. AC generator or AC dynamo

It is used to produce alternating current energy from mechanical energy.

(i) Principle

It is based on the phenomenon of **electromagnetic induction**, i.e. whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the coil. The direction of induced current is given by Fleming's right-hand rule.

(ii) Construction

AC generator has following parts

- (a) **Armature** ABCD is a rectangular armature coil consisting of a large number of turns of insulated copper wire wound over a laminated soft iron core.
- (b) **Field magnets** N and S are the pole pieces of a strong electromagnet in which the armature coil is rotated. Axis of rotation is perpendicular to the magnetic field lines.
- (c) **Slip rings** R_1 and R_2 are two hollow metallic rings which are connected by the two ends of armature coil. These rings rotate with the rotation of the coil.

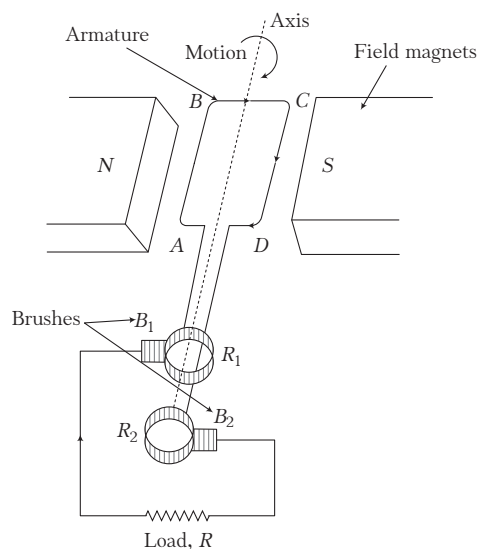


Fig. 7.38 Construction of an AC generators

- (d) **Brushes** B_1 and B_2 are two flexible graphite contacts. They are fixed and are kept in light contact with R_1 and R_2 , respectively. These are used to pass on current from the armature coil to the external load resistance R .
- (e) **Source of energy** The armature coil is rotated with the help of turbine or any other device connected to it. The rotational kinetic energy of the turbine is converted into electrical energy by the AC generator.

(iii) Theory and working

As the armature coil is rotated in the magnetic field, angle θ between the field and normal to the coil changes continuously. Hence, magnetic flux linked with the coil also changes and therefore emf is induced in the coil. Induced current in the external circuit changes direction after every half rotation of the coil.

Hence, the induced current is of alternating in nature. The emf, so induced is given by the formula, $E = E_0 \sin \omega t$.

where, $E_0 = E_{\max} = NAB\omega$,

ω = angular frequency of the coil,

N = number of turns in the coil,

A = area enclosed by each turn of the coil

and B = strength of magnetic field.

The variation of E with time in AC generator is shown in figure below

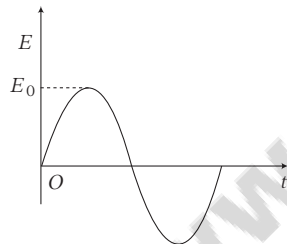


Fig. 7.39 Variation of emf in AC generator

Note If AC is produced by a generator having a large number of poles, then its frequency

$$v = \frac{\text{Number of poles} \times \text{rotation per second}}{2} = \frac{P \times n}{2}$$

where, P is the number of poles, n is the rotational frequency of the coil.

Example 7.60 An AC generator consists of a coil of 1000 turns each of area 100 cm^2 and rotating at an angular speed of 100 rpm in a uniform magnetic field of $3.6 \times 10^{-2} \text{ T}$. Find the peak and rms value of emf induced in the coil.

Sol. Given, number of turns, $N = 1000$,

$$\text{Area, } A = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2 = 10^{-2} \text{ m}^2$$

Frequency, $v = 100 \text{ rpm}$

$$= \frac{100}{60} \text{ rps} = \frac{5}{3} \text{ rps}$$

Magnetic field, $B = 3.6 \times 10^{-2} \text{ T}$

$\therefore$ Peak value of emf, $E_0 = NAB\omega$

$$= NAB(2\pi v)$$

$$= 1000 \times 10^{-2} \times 3.6 \times 10^{-2} \times 2 \times \pi \times \frac{5}{3}$$

$$= 3.77 \text{ V}$$

$$E_{\text{rms}} \text{ or } E_V = \frac{E_0}{\sqrt{2}}$$

$$= 2.67 \text{ V}$$

2. DC generator or DC dynamo

It is used for producing direct current energy from mechanical energy.

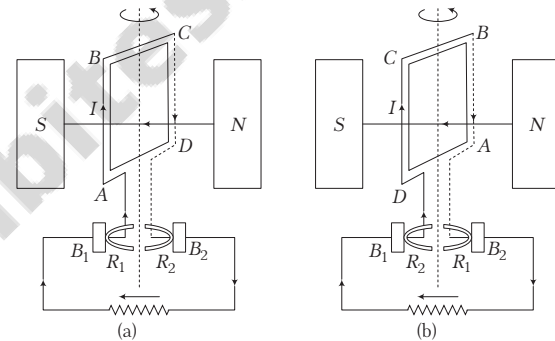


Fig. 7.40 (a) and (b) Constructions of DC generator

The principle and working of DC generator is same as that of AC generator. The only difference is that slip ring arrangement of AC generator is replaced by split ring arrangement or commutator arrangement in DC generator.

The magnitude of emf induced in DC generator is also

$$E = E_0 \sin \omega t$$

However, the direction of E is not reversed in the second half cycle because after half the rotation of the coil, R_1 goes in contact with B_2 and R_2 goes in contact with B_1 .

The variation of E with time in DC generator is shown in figure below.

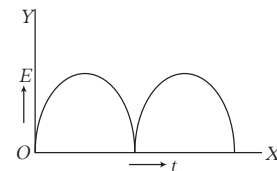


Fig. 7.41 Variation of emf in DC generator

Advantages of AC over DC

Advantages of AC over DC are given below.

- (i) The generation of AC is easy and economical.
- (ii) AC can easily be converted into DC with the help of rectifiers.
- (iii) In AC, energy loss is minimum, so it can be transmitted over long distances.
- (iv) Wide range of voltages are obtained by the use of transformer.
- (v) AC machines have low maintainance cost.

Disadvantages of AC over DC

Disadvantages of AC over DC are given below.

- (i) AC can be transmitted from surface of the conductor and hence need several strands of thin wire insulated from each other.
- (ii) AC shock is more attractive, while DC shock is repulsive. So, AC is more harmful.
- (iii) AC cannot be used in electroplating process because a constant current is needed in this, which is given by DC.

CHECK POINT 7.4

1. Choke coil is a device having
 - (a) low inductance and low resistance
 - (b) high inductance and high resistance
 - (c) low inductance and high resistance
 - (d) high inductance and low resistance
2. What is increased in step-down transformer?
 - (a) Voltage (b) Current (c) Power (d) Current density
3. A transformer works on the principle of
 - (a) self-induction
 - (b) electrical inertia
 - (c) mutual induction
 - (d) magnetic effect of the electrical current
4. Quantity that remains unchanged in a transformer is
 - (a) voltage (b) current
 - (c) frequency (d) None of these
5. The ratio of secondary to the primary turns in a transformer is 3 : 2. If the power output be P , then the input power neglecting all losses must be equal to
 - (a) $5P$ (b) $1.5P$ (c) P (d) $(2/3)P$
6. The transformation ratio in the step-up transformer is
 - (a) 1
 - (b) greater than one
 - (c) less than one
 - (d) the ratio greater or less than one depends on the other factors
7. In a transformer, the number of turns in primary and secondary are 500 and 2000, respectively. If current in primary is 48 A, the current in the secondary is
 - (a) 12 A (b) 24 A
 - (c) 48 A (d) 144 A
8. The core used in a transformer and other electromagnetic devices is laminated so that
 - (a) ratio of voltage in the primary and secondary may be increased
 - (b) energy loss due to eddy currents may be minimised
 - (c) the weight of the transformer may be reduced
 - (d) residual magnetism in the core may be reduced
9. Which of the following is constructed on the principle of electromagnetic induction?
 - (a) Galvanometer
 - (b) Electric motor
 - (c) Generator
 - (d) Voltmeter
10. When speed of a DC generator decreases, the armature current
 - (a) increases
 - (b) decreases
 - (c) does not change
 - (d) increases and decreases continuously

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- The resistance of a coil for DC is $5\ \Omega$. In case of AC, the resistance will
 - remain $5\ \Omega$
 - decrease
 - increase
 - be zero
- In non-resonant circuit, what will be the nature of the circuit for frequencies higher than the resonant frequency?
 - Resistive
 - Capacitive
 - Inductive
 - None of these
- A circuit contains a capacitor and inductance each with negligible resistance. The capacitor is initially charged and the charging battery is disconnected. At subsequent time, the charge on the capacitor will
 - increase exponentially
 - decrease exponentially
 - decrease sinusoidally
 - remain constant
- A choke is preferred to a resistance for limiting current in AC circuit because
 - choke is cheap
 - there is no wastage of power
 - choke is compact in size
 - choke is a good absorber of heat
- Which of the following curves correctly represent the variation of capacitive reactance (X_C) with frequency (f)?

(a)

(b)

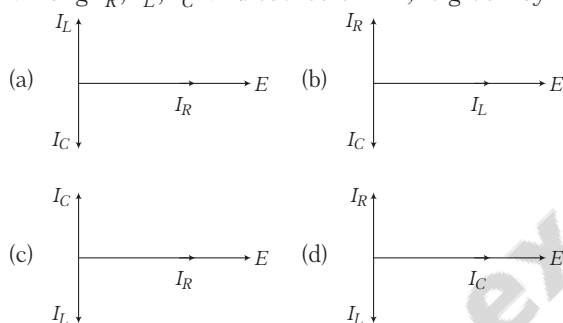
(c)

(d)
- An AC source is connected to a capacitor. The current in the circuit is I . Now a dielectric slab is inserted into the capacitor, then the new current is
 - equal to I
 - more than I
 - less than I
 - may be more than or less than I
- What will be the approximate resistance offered by a capacitor of $10\ \mu\text{F}$ and frequency $100\ \text{Hz}$?
 - $160\ \Omega$
 - $1600\ \Omega$
 - $16\ \Omega$
 - None of these
- The frequency for which a $5\ \mu\text{F}$ capacitor has a reactance of $\frac{1}{1000}\ \Omega$ is given by
 - $\frac{100}{\pi}\ \text{MHz}$
 - $\frac{100}{\pi}\ \text{Hz}$
 - $\frac{1}{1000}\ \text{Hz}$
 - $1000\ \text{Hz}$
- An alternating voltage is connected in series with a resistance R and an inductance L . If the potential drop across the resistance is $200\ \text{V}$ and across the inductance is $150\ \text{V}$, then the applied voltage is
 - $350\ \text{V}$
 - $250\ \text{V}$
 - $500\ \text{V}$
 - $300\ \text{V}$
- An L - R circuit has $R = 10\ \Omega$ and $L = 2\ \text{H}$. If $120\ \text{V}$, $60\ \text{Hz}$ AC voltage is applied, then current in the circuit will be
 - $0.32\ \text{A}$
 - $0.16\ \text{A}$
 - $0.48\ \text{A}$
 - $0.80\ \text{A}$
- A complex current wave is given by $i = (5 + 5 \sin 100\pi t)\ \text{A}$. Its average value over one time period is given as
 - $10\ \text{A}$
 - $5\ \text{A}$
 - $\sqrt{50}\ \text{A}$
 - 0
- If the rms current in a $50\ \text{Hz}$ AC circuit is $5\ \text{A}$, the value of the current $1/300\ \text{s}$ after its value becomes zero is [NCERT Exemplar]
 - $5\sqrt{2}\ \text{A}$
 - $5\sqrt{3}/2\ \text{A}$
 - $5/6\ \text{A}$
 - $5/\sqrt{2}\ \text{A}$
- The peak value of an alternating emf E given by $E = E_0 \cos \omega t$ is $10\ \text{V}$ and frequency is $50\ \text{Hz}$. At time $t = (1/600)\ \text{s}$, the instantaneous value of emf is
 - $10\ \text{V}$
 - $5\sqrt{3}\ \text{V}$
 - $5\ \text{V}$
 - $1\ \text{V}$
- Current and voltage in AC are $I = I_0 \sin(\omega t - \pi/4)$ and $V = V_0 \sin(\omega t + \pi/4)$. Then
 - $X_L > X_C$
 - $R = 0$
 - Both (a) and (b) are correct
 - Both are wrong

- 15** A $10\ \Omega$ resistance, 5 mH coil and $10\ \mu\text{F}$ capacitor are joined in series. When a suitable frequency alternating current source is joined to this combination, the circuit resonates. If the resistance is halved, the resonance frequency
- is halved
 - is doubled
 - remains unchanged
 - is quadrupled

- 16** The resonant frequency of a circuit is f . If the capacitance is made 4 times the initial value, then the resonant frequency will become
- $f/2$
 - $2f$
 - f
 - $f/4$

- 17** An alternating emf is applied across a parallel combination of a resistance R , capacitance C and an inductance L . If I_R , I_L and I_C are the currents through R , L and C respectively, then the diagram which correctly represents, the phase relationship among I_R , I_L , I_C and source emf E , is given by



- 18** An AC supply gives 30 V which passes through a $10\ \Omega$ resistance. The power dissipated in it is
- $90\sqrt{2}\text{ W}$
 - 90 W
 - $45\sqrt{2}\text{ W}$
 - 45 W
- 19** An alternating potential $V = V_0 \sin \omega t$ is applied across a circuit. As a result the current $I = I_0 \sin \left(\omega t - \frac{\pi}{2} \right)$ flows in it. The power consumed in the circuit per cycle is

- zero
 - $0.5 V_0 I_0$
 - $0.707 V_0 I_0$
 - $1.414 V_0 I_0$
- 20** A direct current of 2 A and an alternating current having a maximum value of 2 A flow through two identical resistances. The ratio of heat produced in the two resistances will be
- $1 : 1$
 - $1 : 2$
 - $2 : 1$
 - $4 : 1$

- 21** In a heating arrangement, an alternating current having a peak value of 28 A is used. To produce the same heat energy, if the constant current is used, its magnitude must be
- about 14 A
 - about 28 A
 - about 20 A
 - Cannot say

- 22** A lamp consumes only 50% of peak power in an AC circuit. What is the phase difference between the applied voltage and circuit current?
- $\frac{\pi}{6}$
 - $\frac{\pi}{3}$
 - $\frac{\pi}{4}$
 - $\frac{\pi}{2}$

- 23** 110 V is applied across a series circuit having resistance $11\ \Omega$ and impedance $22\ \Omega$. The power consumed is
- 275 W
 - 366 W
 - 550 W
 - 1100 W

- 24** A 20 V AC is applied to a circuit consisting of a resistance and a coil with a negligible resistance. If the voltage across the resistance is 12 V , the voltage across the coil is
- 16 V
 - 10 V
 - 8 V
 - 6 V

- 25** To reduce the resonant frequency in an L - C - R series circuit with a generator [NCERT Exemplar]
- the generator frequency should be reduced
 - another capacitor should be added in parallel to the first
 - the iron core of the inductor should be removed
 - dielectric in the capacitor should be removed

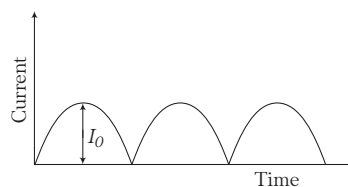
- 26** An alternating voltage is given by

$$e = e_1 \sin \omega t + e_2 \cos \omega t$$

Then, the root-mean-square value of voltage is given by

- $\sqrt{e_1^2 + e_2^2}$
- $\sqrt{e_1 e_2}$
- $\sqrt{\frac{e_1 e_2}{2}}$
- $\sqrt{\frac{e_1^2 + e_2^2}{2}}$

- 27** The output sinusoidal current *versus* time curve of a rectifier is shown in the figure. The average value of output current in this case is



- 0
- $\frac{I_0}{2}$
- $\frac{2I_0}{\pi}$
- I_0

- 28** Voltage and current in an AC circuit are given by
 $V = 5 \sin \left(100\pi t - \frac{\pi}{6} \right)$ and $I = 4 \sin \left(100\pi t + \frac{\pi}{6} \right)$

(a) voltage leads the current by 30°
 (b) current leads the voltage by 30°
 (c) current leads the voltage by 60°
 (d) voltage leads the current by 60°

- 29** An alternating voltage $E = 200\sqrt{2} \sin(100t)$ is connected to a $1 \mu\text{F}$ capacitor through an AC ammeter. The reading of the ammeter shall be
 (a) 10 mA (b) 20 mA (c) 40 mA (d) 80 mA

- 30** A choke coil and capacitor are connected in series and the current through the combination is maximum for AC of frequency n . If they are connected in parallel, at what frequency the current through the combination is minimum?

(a) n (b) $n/2$
 (c) $2n$ (d) None of these

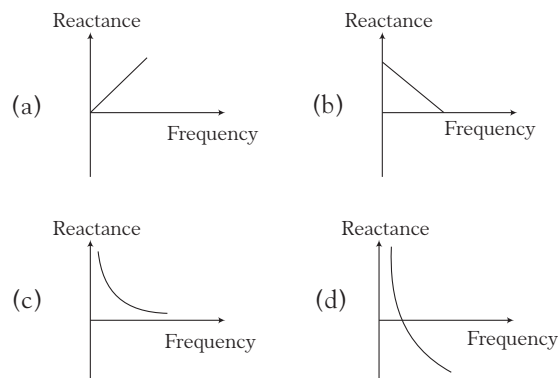
- 31** A coil having an inductance of $\frac{1}{\pi}$ H is connected in series with a resistance of 300Ω . If 20 V and a 200 Hz source are impressed across the combination, the value of the tangent of the phase angle between the voltage and the current is

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

- 32** A condenser of capacity $20 \mu\text{F}$ is first charged and then discharged through a 10 mH inductance. Neglecting the resistance of the coil, the frequency of the resulting vibrations will be
 (a) 356 cycles/s (b) 35.6 cycles/s
 (c) 356×10^3 cycles/s (d) 3.56 cycles/s

- 33** An electric current has both DC and AC components. DC component of 8A and AC component is given as $I = 6 \sin \omega t$. So, I_{rms} value of resultant current is
 (a) 8.05 A (b) 9.05 A (c) 11.58 A (d) 13.58 A

- 34** Which of the shown graphs may represent the reactance of a series L - C combination?



- 35** Two coils have a mutual inductance 0.005 H . The alternating current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10 \text{ A}$ and $\omega = 100\pi \text{ rads}^{-1}$. The maximum value of emf in the second coil is (in volt)

(a) 2π (b) 5π
 (c) π (d) 4π

- 36** Two identical electric heaters each marked 1000 W , 220 V are connected in series. This combination is connected to an AC supply of 220 V . What will be their combined rate of heating? (Assume that resistance of each heater remains constant)

(a) 2000 W (b) 1000 W
 (c) 500 W (d) 250 W

- 37** In an L - R circuit, the inductive reactance is equal to the resistance R of the circuit. An emf $E = E_0 \cos(\omega t)$ is applied to the circuit. The power consumed in the circuit is

(a) $\frac{E_0^2}{\sqrt{2}R}$ (b) $\frac{E_0^2}{4R}$ (c) $\frac{E_0^2}{2R}$ (d) $\frac{E_0^2}{8R}$

- 38** In a transformer, the coefficient of mutual inductance between the primary and the secondary coil is 0.2 H . When the current changes by 5 A/s in the primary, the induced emf in the secondary will be

(a) 5 V (b) 1 V
 (c) 25 V (d) 10 V

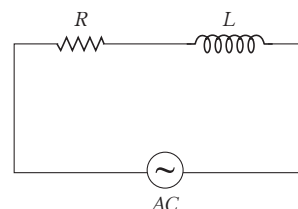
- 39** In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A , then that in the secondary is

(a) 4 A (b) 2 A
 (c) 6 A (d) 10 A

- 40** When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V . This means [NCERT Exemplar]

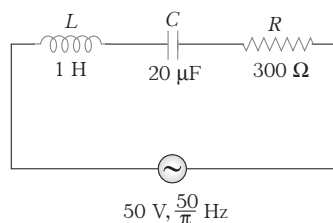
(a) input voltage cannot be AC voltage, but a DC voltage
 (b) maximum input voltage is 220 V
 (c) the meter does not read v but $\langle v^2 \rangle$ and is calibrated to read $\sqrt{\langle v^2 \rangle}$
 (d) the pointer of the meter is stuck by some mechanical defect

- 41** A circuit contains resistance R and an inductance L in series. An alternating voltage $V = V_0 \sin \omega t$ is applied across it. The currents in R and L respectively will be



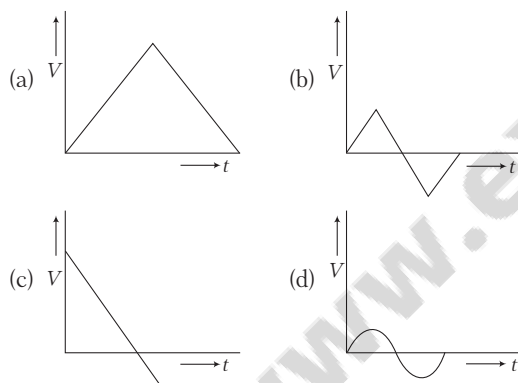
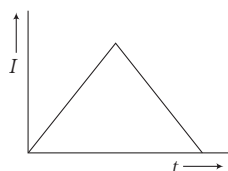
- (a) $I_R = I_0 \cos \omega t, I_L = I_0 \cos \omega t$
 (b) $I_R = -I_0 \sin \omega t, I_L = I_0 \cos \omega t$
 (c) $I_R = I_0 \sin \omega t, I_L = -I_0 \cos \omega t$
 (d) None of the above

42 In the series L - C - R circuit shown, the impedance is



- (a) 200Ω (b) 100Ω (c) 300Ω (d) 500Ω

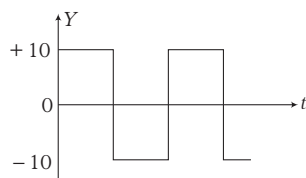
43 An alternating current I in an inductance coil varies with time according to the graph given below. Which one of the following graphs gives the variation of voltage with time?



44 A resistor and a capacitor are connected in series with an AC source. If the potential drop across the capacitor is 5 V and that across resistor is 12 V, then applied voltage is

- (a) 13 V (b) 17 V
 (c) 5 V (d) 12 V

45 The rms voltage of the waveform shown is

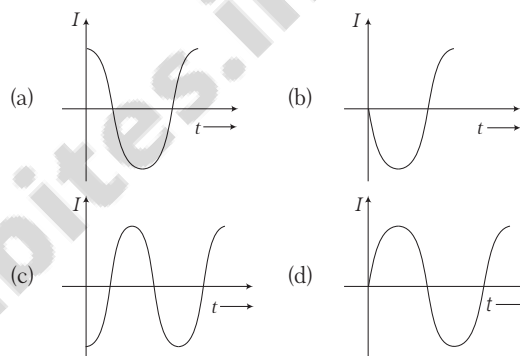
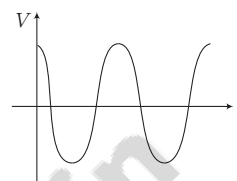


- (a) 10 V (b) 7 V
 (c) 6.37 V (d) None of these

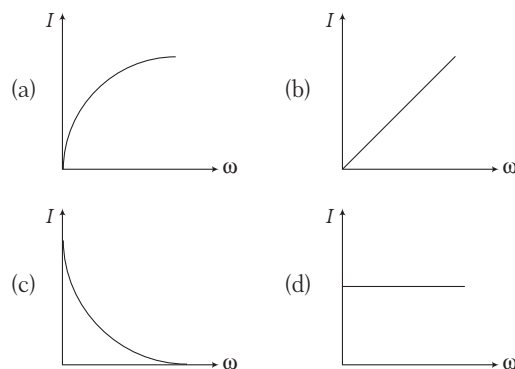
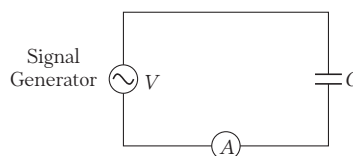
46 Using an AC voltmeter, the potential difference in the electrical line in a house is found to be 234 V. If the line frequency is known to be 50 cycles per second, the equation for the line voltage is

- (a) $165 \sin(200\pi t)$ (b) $234 \sin(100\pi t)$
 (c) $331 \sin(100\pi t)$ (d) $440 \sin(200\pi t)$

47 The voltage across a pure inductor is represented in figure. Which one of the following curves in the figure will represent the current?



48 A constant voltage at different frequencies is applied across a capacitance C as shown in the figure. Which of the following graphs correctly depicts the variation of current with frequency?



49 The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is [NCERT Exemplar]

- (a) $(1/\sqrt{2})$ A (b) $\sqrt{2}$ A (c) 2 A (d) $2\sqrt{2}$ A

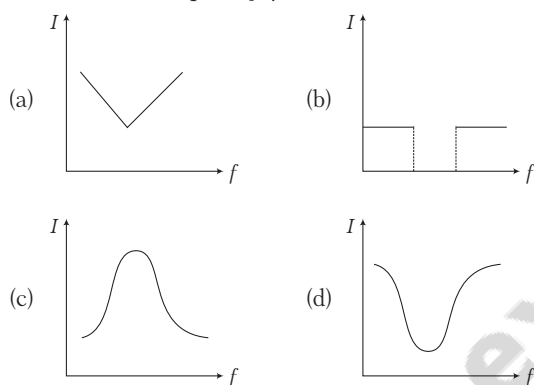
- 50** The rms value of an AC of 50 Hz is 10 A. The time taken by an alternating current in reaching from zero to maximum value and the peak value of current will be

(a) 2×10^{-2} s and 14.14 A
 (b) 1×10^{-2} s and 7.07 A
 (c) 5×10^{-3} s and 7.07 A
 (d) 5×10^{-3} s and 14.14 A

- 51** An inductance of 1 mH, a condenser of $10 \mu\text{F}$ and a resistance of 50Ω are connected in series. The reactances of inductor and condensers are same. The reactance of either of them will be

(a) 100Ω (b) 30Ω
 (c) 3.2Ω (d) 10Ω

- 52** An AC source of variable frequency f is connected to an L - C - R series circuit. Which one of the graphs in figure represents the variation of current I in the circuit with frequency f ?



- 53** The armature of DC motor has 20Ω resistance. It draws current of 1.5 A when run by 220 V DC supply. The value of back emf induced in it will be
- (a) 150 V (b) 170 V
 (c) 180 V (d) 190 V

- 54** A group of electric lamps having a total power rating of 1000 W is supplied by an AC voltage $E = 200 \sin(310t + 60^\circ)$, then the rms value of the circuit current is

(a) 10 A (b) $5\sqrt{2}$ A
 (c) 20 A (d) $10\sqrt{2}$ A

- 55** An alternating voltage $V = 30 \sin 50t + 40 \cos 50t$ is applied to a resistor of resistance 10Ω . The rms value of current through resistor is

(a) $\frac{5}{\sqrt{2}}$ A (b) $\frac{10}{\sqrt{2}}$ A
 (c) 5 A (d) None of these

- 56** An alternating voltage $V = 140 \sin 50t$ is applied to a resistor of 10Ω . This voltage produces ΔH heat in

the resistor in time Δt . To produce the same heat in the same time, required DC current is

(a) 14 A (b) about 20 A
 (c) about 10 A (d) None of these

- 57** In a certain circuit, current changes with time according to $i = 2\sqrt{t}$. Root mean square value of current between $t = 2$ to $t = 4$ s will be

(a) 3 A (b) $3\sqrt{3}$ A
 (c) $2\sqrt{3}$ A (d) $\sqrt{3}$ A

- 58** The power factor of an R - L circuit is $\frac{1}{\sqrt{2}}$. If the

frequency of AC is doubled, what will be the power factor?

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{\sqrt{11}}$

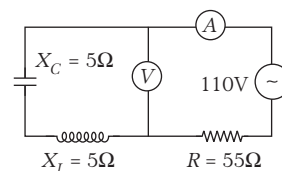
- 59** When a DC voltage of 200V is applied to a coil of self inductance $(2\sqrt{3} / \pi) \text{H}$, a current of 1A flows through it. But by replacing DC source with AC source of 200V, the current in the coil is reduced to 0.5A. Then, the frequency of AC supply is

(a) 30 Hz (b) 60 Hz
 (c) 75 Hz (d) 50 Hz

- 60** One 10 V, 60 W bulb is to be connected to 100 V line. The required self-inductance of induction coil will be ($f = 50 \text{ Hz}$)

(a) 0.052 H (b) 2.42 H
 (c) 16.2 H (d) 162 mH

- 61** The reading of ammeter in the circuit shown will be



(a) 2A (b) 2.4 A
 (c) zero (d) 1.7 A

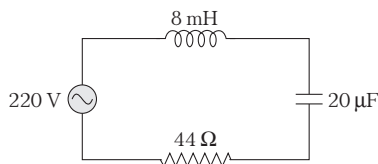
- 62** In series L - C - R circuit voltage drop across resistance is 8 V and across inductor is 6 V and across capacitor is 12 V. Then

(a) voltage of the source will be leading in the circuit
 (b) voltage drop across each element will be less than the applied voltage
 (c) power factor of the circuit will be $3/4$
 (d) None of the above

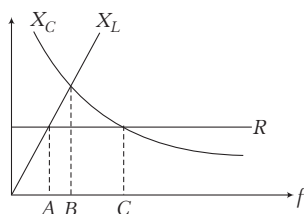
- 63** In a series L - C - R circuit, resistance $R = 10 \Omega$ and the impedance $Z = 10 \Omega$. The phase difference between the current and the voltage is

(a) 0° (b) 30°
 (c) 45° (d) 60°

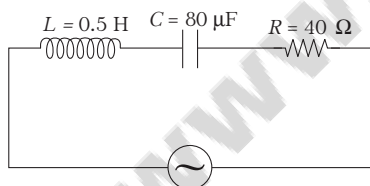
- 64** For the series L - C - R circuit shown in the figure, what is the resonance frequency and the current at the resonating frequency?



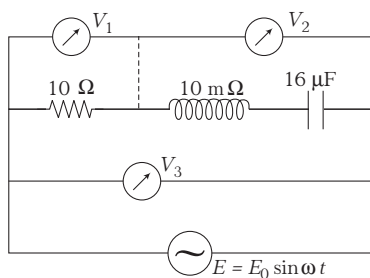
- (a) 2500 rad s^{-1} and $5\sqrt{2} \text{ A}$
 (b) 2500 rad s^{-1} and 5 A
 (c) 2500 rad s^{-1} and $\frac{5}{\sqrt{2}} \text{ A}$
 (d) 25 rad s^{-1} and $5\sqrt{2} \text{ A}$
- 65** The figure shows variation of R , X_L and X_C with frequency f in a series L , C , R circuit. Then, for what frequency point, the circuit is inductive?



- (a) A (b) B
 (c) C (d) All points
- 66** In the given figure, a series L - C - R circuit is connected to a variable frequency source of 230 V. The impedance and amplitude of the current at the resonating frequency will be



- (a) 20Ω and 4.2 A (b) 30Ω and 6.9 A
 (c) 25Ω and 5.8 A (d) 40Ω and 5.75 A
- 67** In figure, which voltmeter reads zero, when ω is equal to the resonant frequency of series L - C - R circuit?

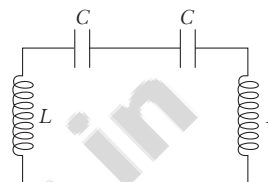


- (a) V_1 (b) V_2
 (c) V_3 (d) None of these

- 68** An R - L - C circuit containing a 52Ω resistor, a 230 mH inductor, and a $8.8 \mu\text{F}$ capacitor is driven by an AC voltage source that has an amplitude of 150 V and frequency $f = 80 \text{ Hz}$. How much average power is dissipated by this circuit?

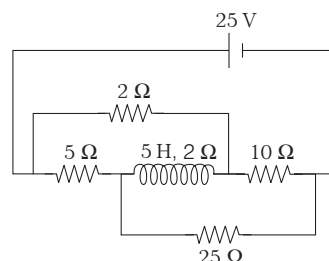
- (a) 78.6 W (b) Zero (c) 19.6 W (d) 24.8 W

- 69** The natural frequency of the circuit shown in the figure is



- (a) $\frac{1}{2\pi\sqrt{LC}}$ (b) $\frac{1}{2\pi\sqrt{2LC}}$
 (c) $\frac{2}{2\pi\sqrt{LC}}$ (d) None of these

- 70** In the circuit shown what is the energy stored in the coil at steady state?



- (a) 21.3 J (b) 42.6 J
 (c) Zero (d) 213 J

- 71** A loss-free transformer having 100 turns in primary is used to transmit 10 kW of power. The input voltage is 200 V and power is transmitted at 5 kV . The currents in the primary and secondary of the transformer are

- (a) 2 A and 50 A (b) 50 A and 2 A
 (c) 25 A and 4 A (d) 12.5 A and 8 A

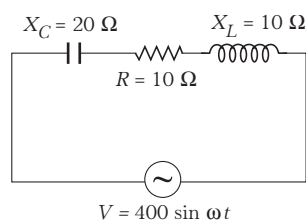
- 72** A pure resistive circuit element X when connected to an AC supply of peak voltage 200 V , gives a peak current of 5 A . A second circuit element Y , when connected to the same AC supply also gives the same value of peak current but the current lags behind by 90° . If the series combination of X and Y is connected to the same supply, what will be the rms value of current?

- (a) $\frac{10}{\sqrt{2}} \text{ A}$ (b) $\frac{5}{\sqrt{2}} \text{ A}$ (c) $\frac{5}{2} \text{ A}$ (d) 5 A

- 73** A coil, a capacitor and an AC source of rms voltage 24 V are connected in series. By varying the frequency of the source, a maximum rms current of 6 A is observed. If this coil is connected to a battery of emf 12 V and internal resistance $4\ \Omega$, the maximum current through it will be

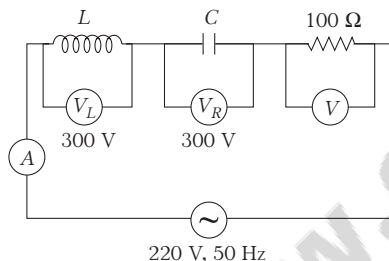
(a) 2.4 A (b) 1.8 A
(c) 1.5 A (d) 1.2 A

- 74** In the L - C - R circuit as shown in figure.



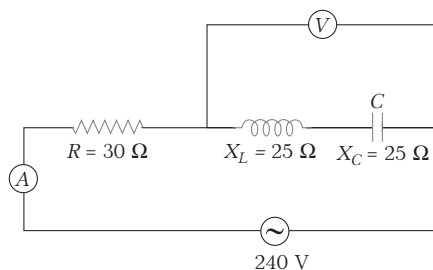
- (a) Current will lag the voltage
(b) Rms value of current is $20\sqrt{2}$ A
(c) Power factor of the circuit is $\frac{1}{2}$
(d) Voltage drop across resistance is 200 V

- 75** In the circuit shown below what will be the reading of the voltmeter and ammeter?



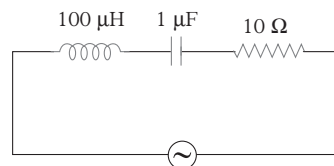
- (a) 200 V, 1 A (b) 800 V, 2 A
(c) 220 V, 2 A (d) 220 V, 2.2 A

- 76** In the circuit shown in figure neglecting source resistance, the voltmeter and ammeter readings will be respectively



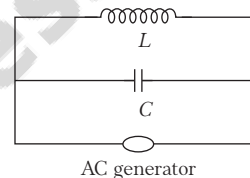
- (a) 0 V, 3 A (b) 150 V, 3 A
(c) 150 V, 6 A (d) 0 V, 8 A

- 77** The following series L - C - R circuit, when driven by an emf source of angular frequency 70 kilo-radians per second, the circuit effectively behaves like



- (a) purely resistive circuit
(b) series R - L circuit
(c) series R - C circuit
(d) series L - C circuit with $R = 0$

- 78** In the circuit shown in the figure, the alternating currents through inductor and capacitor are 1.2 A and 1.0 A, respectively. The current drawn from the generator is

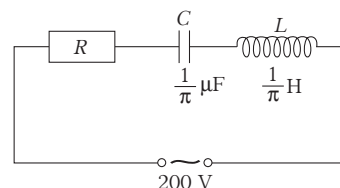


- (a) 0.4 A (b) 0.2 A (c) 1.0 A (d) 1.2 A

- 79** An alternating current generator has an internal resistance R_g and an internal reactance X_g . It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L . For maximum power to be delivered from the generator to the load, the value of X_L is equal to [NCERT Exemplar]

- (a) zero (b) X_g (c) $-X_g$ (d) R_g

- 80** In the circuit shown in figure, the supply has a constant rms value V but variable frequency f . The frequency at which the voltage drop across R is maximum is

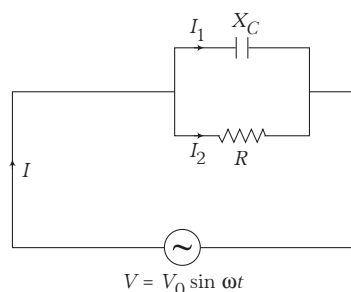


- (a) 100 Hz (b) 500 Hz
(c) 300 Hz (d) None of these

- 81** When an AC voltage, of variable frequency is applied to series L - C - R circuit, the current in the circuit is the same at 4 kHz and 9 kHz. The current in the circuit is maximum at

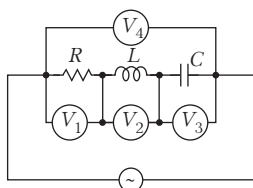
- (a) 5 kHz (b) 6.5 kHz
(c) 4.2 kHz (d) 6 kHz

82 In the given AC circuit,



- (a) current I_2 and V are in same phase
- (b) current I_2 leads I_1 by 90°
- (c) current I leads I_2 by $\theta < 90^\circ$
- (d) current I leads I_1 by $\theta < 90^\circ$

83 An ideal resistance R , ideal inductance L , ideal capacitance C and AC voltmeters V_1, V_2, V_3 and V_4 are connected to an AC source as shown. At resonance,

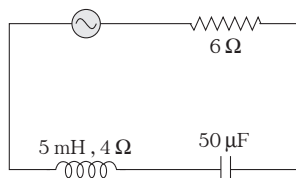


- (a) reading in V_3 = reading in V_1
- (b) reading in V_1 = reading in V_2
- (c) reading in V_2 = reading in V_4
- (d) reading in V_2 = reading in V_3

84 An AC voltage source of variable angular frequency ω and fixed amplitude V connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased,

- (a) the bulb glows dimmer
- (b) the bulb glows brighter
- (c) total impedance of the circuit is unchanged
- (d) total impedance of the circuit increases

85 In the circuit shown, the AC source has voltage $V = 20 \cos(\omega t)$ volt with $\omega = 2000 \text{ rads}^{-1}$. The amplitude of the current will be nearest to



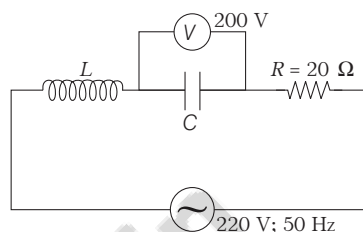
- (a) 2 A
- (b) 3.3 A
- (c) $2/\sqrt{5}$ A
- (d) $\sqrt{5}$ A

86 In a series L - C - R circuit the voltage across resistance, capacitance and inductance is 10V each.

If the capacitance is short circuited, the voltage across the inductance will be

- (a) $\frac{10}{\sqrt{2}}$ V
- (b) 10 V
- (c) $10\sqrt{2}$ V
- (d) 20 V

87 In the circuit shown, rms current is 11 A. The potential difference across the inductor is



- (a) 220 V
- (b) 0 V
- (c) 300 V
- (d) 200 V

88 An inductor of reactance 1Ω and a resistor of 2Ω are connected in series to the terminals of a 6V (rms) AC source. The power dissipated in the circuit is

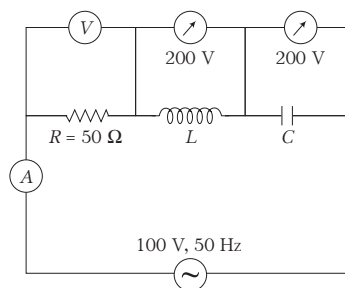
[NCERT Exemplar]

- (a) 8 W
- (b) 12 W
- (c) 14.4 W
- (d) 18 W

89 An AC circuit consists of a resistance and a choke coil in series. The resistance is of 220Ω and choke coils is of 0.7 H. The power absorbed from 220 V and 50 Hz, source connected with the circuit, is

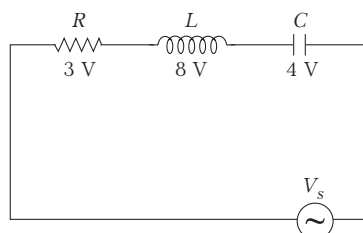
- (a) 55 W
- (b) 110 W
- (c) 220 W
- (d) 440 W

90 In the series L - C - R circuit, the voltmeter and ammeter readings are respectively

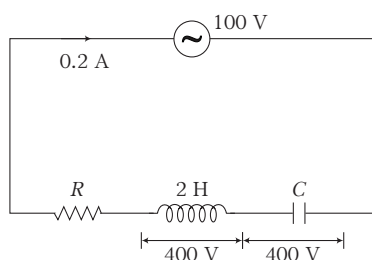


- (a) $V = 200 \text{ V}, I = 4 \text{ A}$
- (b) $V = 150 \text{ V}, I = 2 \text{ A}$
- (c) $V = 100 \text{ V}, I = 5 \text{ A}$
- (d) $V = 100 \text{ V}, I = 2 \text{ A}$

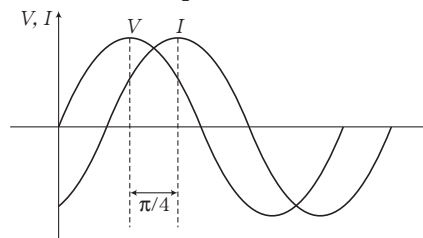
91 Current in resistance is 1 A, then



- (a) $V_s = 5 \text{ V}$
 (b) impedance of network is 5Ω
 (c) power factor of given circuit is (0.6) lagging (current is lagging)
 (d) All of the above
- 92** An L - C - R series circuit with a resistance of 100Ω is connected to an AC source of 200 V (rms) and angular frequency 300 rad s^{-1} . When only the capacitor is removed, the current lags behind the voltage by 60° . When only the inductor is removed the current leads the voltage by 60° . The average power dissipated in original L - C - R circuit is
 (a) 50 W (b) 100 W
 (c) 200 W (d) 400 W
- 93** A virtual current of 4 A and 50 Hz flows in an AC circuit containing a coil. The power consumed in the coil is 240 W . If the virtual voltage across the coil is 100 V , then its inductance will be
 (a) $\frac{1}{3\pi} \text{ H}$ (b) $\frac{1}{5\pi} \text{ H}$
 (c) $\frac{1}{7\pi} \text{ H}$ (d) $\frac{1}{9\pi} \text{ H}$
- 94** An inductor L , a capacitor of $20 \mu\text{F}$ and a resistor of 10Ω are connected in series with an AC source of frequency 50 Hz . If the current is in phase with the voltage, then the inductance of the inductor is
 (a) 2.00 H (b) 0.51 H
 (c) 1.5 H (d) 0.99 H
- 95** An L - C - R series circuit consists of a resistance of 10Ω , a capacitor of reactance 60Ω and an inductor coil. The circuit is found to resonate when put across a 300 V , 100 Hz supply. The inductance of coil is (take, $\pi = 3$)
 (a) 0.1 H (b) 0.01 H (c) 0.2 H (d) 0.02 H
- 96** A capacitor of capacitance $1 \mu\text{F}$ is charged to a potential of 1 V . It is connected in parallel to an inductor of inductance 10^{-3} H . The maximum current that will flow in the circuit has the value
 (a) $\sqrt{1000} \text{ mA}$ (b) 1 A
 (c) 1 mA (d) 1000 mA
- 97** Which of the following options is correct with respect to the circuit diagram given below?

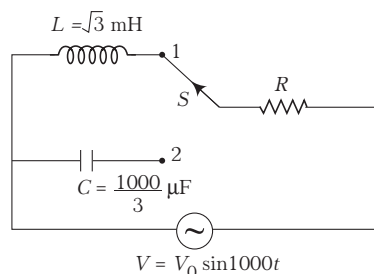


- (a) $R = 400 \Omega$, $C = 0.5 \mu\text{F}$
 (b) $R = 500 \Omega$, $C = 0.5 \mu\text{F}$
 (c) $R = 500 \Omega$, $C = 1 \mu\text{F}$
 (d) $R = 400 \Omega$, $C = 0.1 \mu\text{F}$
- 98** When 100 V DC is applied across a solenoid, a current of 1 A flows in it. When 100 V AC is applied across the same solenoid the current drops to 0.5 A . If the frequency of the AC source is 50 Hz , the impedance and inductance of the solenoid are
 (a) 200Ω and 0.55 H (b) 100Ω and 0.86 H
 (c) 200Ω and 1.0 H (d) 1100Ω and 0.93 H
- 99** An ideal choke takes a current of 8 A when connected to an AC source of 100 V and 50 Hz . A pure resistor under the same conditions takes a current of 10 A . If two are connected in series to an AC supply of 100 V and 40 Hz , then the current in the series combination of above resistor and inductor is
 (a) 10 A (b) 5 A (c) $5\sqrt{2} \text{ A}$ (d) $10\sqrt{2} \text{ A}$
- 100** An AC source is connected with a resistance (R) and an uncharged capacitance C , in series. The potential difference across the resistor is in phase with the initial potential difference across the capacitor for the first time at the instant (assume that at $t = 0$, emf is zero) (where, ω is the angular frequency)
 (a) $\frac{\pi}{4\omega}$ (b) $\frac{2\pi}{\omega}$
 (c) $\frac{\pi}{2\omega}$ (d) $\frac{3\pi}{2\omega}$
- 101** Current through an AC series L - C - R circuit is 2 A if operated at resonant frequency, and 1 A if operated at 50% less than resonant frequency. The current (in A) if the frequency is 100% more than the resonant frequency, is
 (a) $\sqrt{2}$ (b) 1
 (c) $\sqrt{3}$ (d) Data insufficient
- 102** An AC voltage $V = V_0 \sin 100 t$ is applied to the circuit, the phase difference between current and voltage is found to be $\frac{\pi}{4}$, then

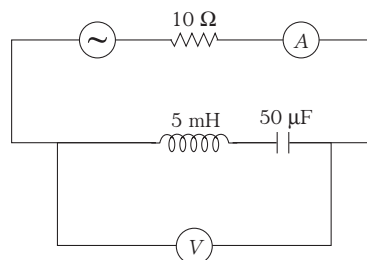


- (a) $R = 100 \Omega$, $C = 1 \mu\text{F}$ (b) $R = 1 \text{ k}\Omega$, $C = 10 \mu\text{F}$
 (c) $R = 10 \text{ k}\Omega$, $L = 1 \text{ H}$ (d) $R = 1 \text{ k}\Omega$, $L = 10 \text{ H}$

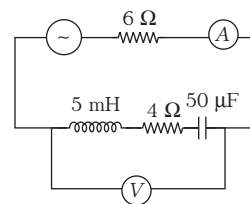
- 103** In the given AC circuit, when switch S is at position 1, the source emf leads current by $\pi/6$. Now, if the switch is at position 2, then



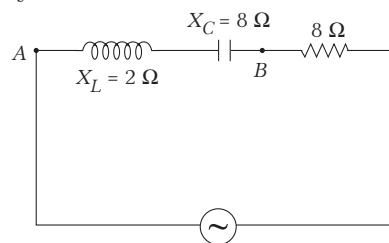
- (a) current leads source emf by $\frac{\pi}{4}$
 (b) current leads source emf by $\frac{\pi}{3}$
 (c) source emf leads current by $\frac{\pi}{4}$
 (d) source emf leads current by $\frac{\pi}{3}$
- 104** An L - C circuit contains a 20 mH inductor and a 50 μ F capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. At the instant the circuit is closed be $t = 0$. At what time is the energy stored completely magnetic?
- (a) $t = 0$ (b) $t = 1.57$ ms
 (c) $t = 3.14$ ms (d) $t = 6.28$ ms
- 105** When an alternating voltage of 220 V is applied across a device P , a current of 0.25 A flows through the circuit and it leads the applied voltage by an angle $\frac{\pi}{2}$ radian. When the same voltage source is connected across another device Q , the same current is observed in the circuit but in phase with the applied voltage.
- What is the current when the same source is connected across a series combination of P and Q ?
- (a) $\frac{1}{4\sqrt{2}}$ A lagging in phase by $\frac{\pi}{4}$ with voltage
 (b) $\frac{1}{4\sqrt{2}}$ A leading in phase by $\frac{\pi}{4}$ with voltage
 (c) $\frac{1}{\sqrt{2}}$ A leading in phase by $\frac{\pi}{4}$ with voltage
 (d) $\frac{1}{\sqrt{2}}$ A leading in phase by $\frac{\pi}{6}$ with voltage
- 106** In the circuit shown in figure, the AC source gives a voltage $V = 20 \cos(2000t)$. Neglecting source resistance, the voltmeter and ammeter readings will be (approximately)



- (a) 4 V, 2.0 A (b) 0 V, 2A
 (c) 0 V, $\sqrt{2}$ A (d) 8 V, 2.0 A
- 107** A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is
- (a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$ (c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$
- 108** A circuit draws 330 W from a 110 V, 60 Hz AC line. The power factor is 0.6 and the current lags the voltage. The capacitance of a series capacitor that will result in a power factor of unity is equal to
- (a) 31 μ F (b) 54 μ F (c) 151 μ F (d) 201 μ F
- 109** An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V(rms), 50 Hz AC supply, the series inductor needed for it to work is close to
- (a) 0.08 H (b) 0.044 H (c) 0.064 H (d) 80 H
- 110** In the circuit shown in the figure, the ac source gives a voltage $V = 20 \cos(2000t)$. Neglecting source resistance, the voltmeter and ammeter readings will be



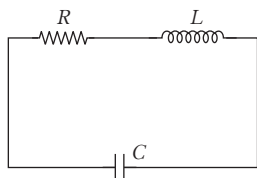
- (a) 0 V, 0.47 A (b) 1.68 V, 0.47 A
 (c) 0 V, 1.4 A (d) 5.6 V, 1.4 A
- 111** An inductor ($X_L = 2 \Omega$), a capacitor ($X_C = 8 \Omega$) and a resistance ($R = 8 \Omega$) are connected in series with an AC source. The voltage output of AC source is given by $V = 10 \cos(100\pi t)$.



The instantaneous potential difference between points A and B, when the applied voltage is $3/5$ th of the maximum value of applied voltage is

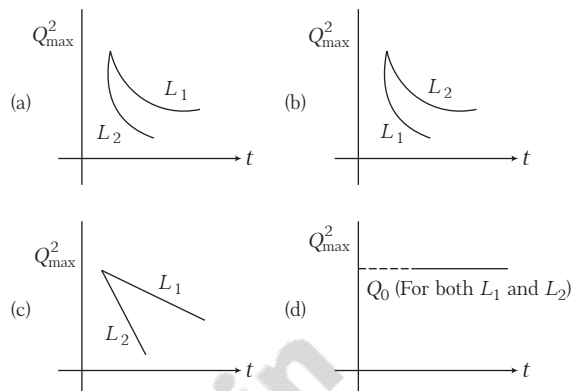
- (a) 0 V (b) 6 V
(c) 8 V (d) None of these

- 112** A L - C - R circuit is equivalent to a damped pendulum. In an L - C - R circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below.



If a student plots graphs of the square of maximum charge ($Q_{\max}^2$) on the capacitor with time (t) for two

different values L_1 and L_2 ($L_1 > L_2$) of L , then which of the following represents this graph correctly (plots are schematic and not drawn to scale)



(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-6) These questions consists of two statements each linked as Assertion and Reason. While answering these question you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

- 1 Assertion** If an inductor coil is connected to DC source, the current supplied by it is I_1 . If the same coil is connected with an AC source of same voltage. Then current is I_2 . Then $I_2 < I_1$.

Reason In AC circuit, inductor coil offers more resistance.

- 2 Assertion** In an AC, only capacitor circuit has instantaneous power equals to zero at any instant of time.

Reason Phase difference between current function and voltage function is 90° .

- 3 Assertion** When a ferromagnetic rod is inserted inside an inductor, then current in L - C - R , alternating circuit will decrease.

Reason By inserting the ferromagnetic rod inside the inductor, coefficient of self induction and hence the inductive reactance will increase.

- 4 Assertion** In L - C - R series AC circuit, $X_L = X_C = R$ at a given frequency. When frequency is doubled, the impedance of the circuit is $\frac{\sqrt{13}}{2} R$.

Reason The given frequency is resonance frequency.

- 5 Assertion** Average power in an AC circuit is given by $P = I_{\text{rms}}^2 R$.

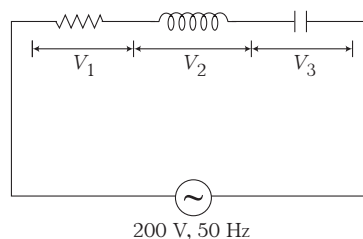
Reason In one full cycle, net power is dissipated only along a resistor.

- 6 Assertion** At resonance, power factor of series L - C - R circuit is zero.

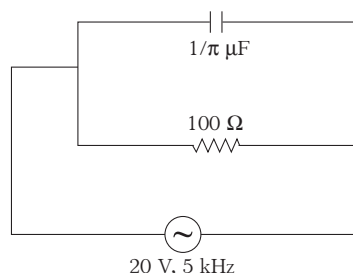
Reason At resonance, current function and voltage functions are in same phase.

Statement Based Questions

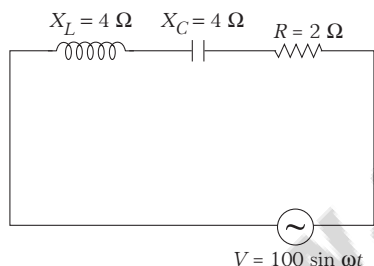
- 1** For series L - C - R AC circuit shown in figure, the readings of V_2 and V_3 are same and each equal to 100 V. Then, which of the following statement(s) is/are incorrect?



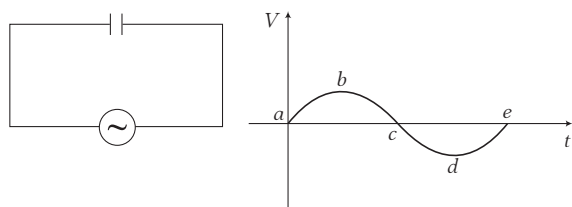
- (a) The reading V_1 is 200 V.
 (b) The reading of V_2 is 0.
 (c) The circuit is in resonant mode and resonant frequency is 50 Hz.
 (d) The inductive and capacitive reactance are equal.
- 2** A signal generator supplies a sine wave of 20 V, 5 kHz to the circuit shown in the figure. Then, choose the wrong statement.



- (a) The current in the resistive branch is 0.2 A.
 (b) The current in the capacitive branch is 0.126 A.
 (c) Total line current is ≈ 0.283 A.
 (d) Current in both the branches is same.
- 3** Which of the following statement is correct regarding the AC circuit shown in the adjacent figure?

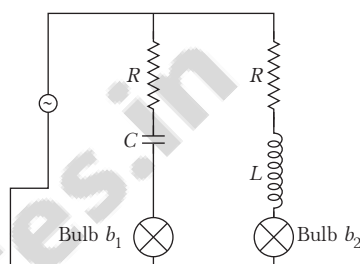


- (a) The rms value of current through the circuit is $i_{\text{rms}} = 5\sqrt{2}$ A.
 (b) The phase difference between source emf and current is $\phi = \cos^{-1}\left(\frac{1}{3}\right)$.
 (c) Average power dissipated in the circuit is 500 W.
 (d) None of the above
- 4** For an AC circuit containing capacitor only, the applied AC voltage waveform is shown in figure.

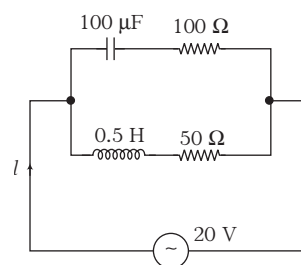


For this situation, mark the incorrect statement(s).

- (a) As V increases from a to b , the charging of capacitor takes place.
 (b) As V increases from a to b , the current in circuit decreases from maximum to zero value.
 (c) As V decreases from b to c , the capacitor discharges.
 (d) As V decreases from b to c charging of capacitor takes place.
- 5** Two identical incandescent light bulbs are connected as shown in figure. When the circuit is an AC voltage source of frequency f , which of the following statement(s) is/are correct?

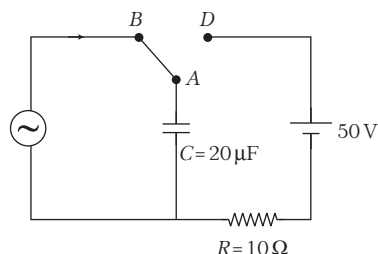


- (a) Both bulbs will glow alternatively.
 (b) Both bulbs will glow with same brightness provided $f = \frac{1}{2\pi} \sqrt{1/LC}$.
 (c) Bulb b_1 will light up initially and goes OFF, bulb b_2 will be ON constantly.
 (d) Bulb b_1 will blink and bulb b_2 will be ON constantly.
- 6** In the given circuit, the AC source has $\omega = 100 \text{ rad s}^{-1}$. Considering the inductor and capacitor to be ideal, which of the statement(s) is/are correct?

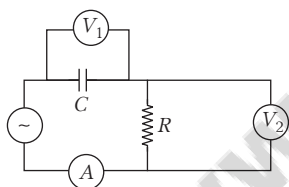


- (a) The current through the circuit, I is 0.6 A.
 (b) The current through the circuit, I is $0.3\sqrt{2}$ A.
 (c) The voltage across 100Ω resistor $= 10\sqrt{2}$ V.
 (d) The voltage across 50Ω resistor $= 10$ V.
- 7** At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key. Alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500 \text{ rads}^{-1}$ starts flowing in it with the initial

direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D . Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20 \mu\text{F}$, $R = 10 \Omega$ and the battery is ideal with emf of 50 V , identify the correct statement(s).



- (a) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is $1 \times 10^{-3} \text{ C}$.
- (b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.
- (c) Immediately after A is connected to D . The current in R is 20 A .
- (d) $Q = 2 \times 10^{-3} \text{ C}$
- 8 The diagram shows a capacitor C and a resistor R connected in series to an AC source, V_1 and V_2 are voltmeters and A is an ammeter.



Consider the following statements

- I. Readings in A and V_2 are always in phase.
 II. Reading in V_1 is ahead in phase with reading in V_2 .
 III. Readings in A and V_1 are always in phase.

Which of these statement(s) is/are correct?

- (a) I only (b) II only
 (c) I and II (d) II and III

Match the columns

- 1 Match the following two columns for L - C - R series AC circuit and choose the correct option from codes given below.

Column I	Column II
A. At resonance frequency	(p) Power factor = 0
B. No resistance in the circuit	(q) Power factor = 1
C. Only resistance in the circuit	(r) Circuit is capacitor circuit
D. Frequency greater than the resonance frequency	(s) Circuit is inductive

Codes

A	B	C	D	A	B	C	D
(a) r	s	p	q	(b) q	p	q	s
(c) p	s	p	q	(d) r	p	q	s

- 2 In an AC, series L - C - R circuit, $R = X_L = X_C$ and applied AC voltage is V . Then match the following two columns and choose the correct option from codes given below.

Column I	Column II
A. V_R	(p) zero
B. V_C	(q) V
C. V_{RL}	(r) $\sqrt{2} V$
D. V_{CL}	(s) $2 V$

Codes

A	B	C	D	A	B	C	D
(a) q	q	r	p	(b) p	s	r	r
(c) q	p	r	s	(d) s	p	r	s

- 3 In an AC series L - C - R circuit, applied voltage is

$$V = 100 \sqrt{2} \sin(\omega t + 45^\circ) \text{ V}$$

Given that, $R = 30 \Omega$, $X_L = 50 \Omega$ and $X_C = 10 \Omega$

Now match the following two columns and choose the correct option from codes given below.

Column I	Column II
A. Current in the circuit	(p) 120 SI units
B. Power dissipated in the circuit	(q) 60 SI units
C. Potential difference across resistance	(r) 2 SI units
D. Potential difference across inductance	(s) None

Codes

A	B	C	D	A	B	C	D
(a) p	q	r	s	(b) p	s	r	q
(c) r	s	p	q	(d) r	p	q	s

(C) Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

- 1 A series L - C - R circuit is connected to an AC voltage source. When L is removed from the circuit, the phase difference between current and voltage is $\frac{\pi}{3}$.

If instead C is removed from the circuit, the phase difference is again $\frac{\pi}{3}$ between current and voltage.

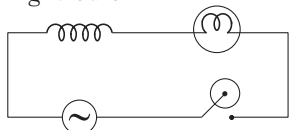
The power factor of the circuit is [NEET 2020]

- (a) 0.5 (b) 1 (c) -1.0 (d) zero

- 2 A $40\ \mu\text{F}$ capacitor is connected to a 200 V, 50 Hz AC supply. The rms value of the current in the circuit is, nearly [NEET 2020]

- (a) 2.05 A (b) 2.5 A (c) 25.1 A (d) 1.7 A

- 3 A light bulb and an inductor coil are connected to an AC source through a key as shown in the figure below. The key is closed and after sometime an iron rod is inserted into the interior of the inductor. The glow of the light bulb [NEET 2020]

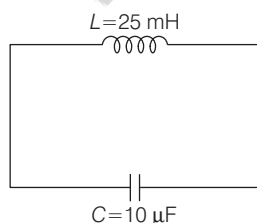


- (a) decreases (b) remains unchanged
(c) will fluctuate (d) increases

- 4 A circuit when connected to an AC source of 12 V gives a current of 0.2 A. The same circuit when connected to a DC source of 12 V, gives a current of 0.4 A. The circuit is [NEET 2019]

- (a) series L - R (b) series R - C
(c) series L - C (d) series L - C - R

- 5 If maximum energy is stored in a capacitor at $t = 0$, then find the time after which current in the circuit will be maximum. [AIIMS 2019]

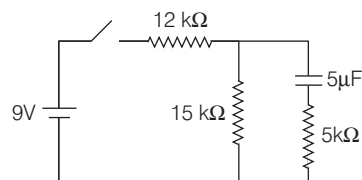


- (a) $\frac{\pi}{2}$ ms (b) $\frac{\pi}{4}$ ms (c) π ms (d) 2 ms

- 6 In an L - C - R series circuit, source voltage is 120 V and voltage in inductor is 50 V and in resistance is 40 V, then determine voltage in the capacitor. [AIIMS 2019]

- (a) $V_C = 10(5 - 8\sqrt{2})$ (b) $V_C = 10(5 + 8\sqrt{2})$
(c) $V_C = 20(5 + 8\sqrt{2})$ (d) $V_C = 10(5 + 7\sqrt{2})$

7



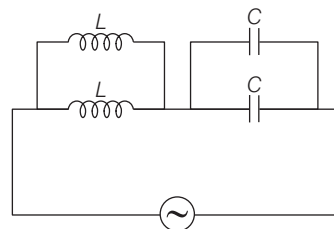
In the given circuit, find charge on capacitor after 1 s of opening the switch at $t = \infty$. [AIIMS 2019]

- (a) $20 e^{-10}\ \mu\text{C}$ (b) $25 e^{-10}\ \mu\text{C}$
(c) $30 e^{-10}\ \mu\text{C}$ (d) $35 e^{-10}\ \mu\text{C}$

- 8 A transformer with turns ratio $\frac{N_1}{N_2} = \frac{50}{1}$ is connected to a 120 V AC supply. If primary and secondary circuit resistances are $1.5\text{ k}\Omega$ and 1Ω respectively, then find out power of output. [AIIMS 2019]

- (a) 5.76 W (b) 11.4 W
(c) 2.89 W (d) 756 W

- 9 Find resonance frequency in the given circuit [JIPMER 2019]



- (a) $\frac{1}{\sqrt{LC}}$ (b) $\frac{2}{\sqrt{LC}}$ (c) $\frac{1}{2\sqrt{LC}}$ (d) $\frac{4}{\sqrt{LC}}$

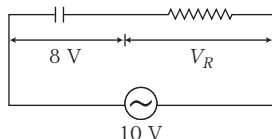
- 10 An inductor 20 mH, a capacitor $100\ \mu\text{F}$ and a resistor $50\ \Omega$ are connected in series across a source of emf $V = 10 \sin 314 t$. The power loss in the circuit is [AIIMS 2018]

- (a) 2.74 W (b) 0.43 W
(c) 0.79 W (d) 1.13 W

- 11 In a circuit L , C and R are connected in series with an alternating voltage source of frequency f . The current leads the voltage by 45° . The value of C is [AIIMS 2018]

- (a) $\frac{1}{2\pi f(2\pi/L + R)}$ (b) $\frac{1}{\pi f(2\pi/L + R)}$
(c) $\frac{1}{2\pi f(2\pi/L - R)}$ (d) $\frac{1}{\pi f(2\pi/L - R)}$

- 12** In a series R - C circuit shown in figure, the applied voltage is 10 V and the voltage across capacitor is found to be 8 V. Then, the voltage across R and the phase difference between current and the applied voltage will respectively be [AIIMS 2018]



- (a) 6 V, $\tan^{-1}\left(\frac{4}{3}\right)$ (b) 3 V, $\tan^{-1}\left(\frac{3}{4}\right)$
 (c) 6 V, $\tan^{-1}\left(\frac{5}{3}\right)$ (d) None of these
- 13** An ideal coil of 10 H is connected in series with a resistance of 5Ω and a battery of 5 V. After 2 s, after the connection is made, the current flowing (in ampere) in the circuit is [AIIMS 2018]
 (a) $(1 - e)$ (b) e
 (c) e^{-1} (d) $(1 - e^{-1})$
- 14** A series R - C circuit is connected to AC voltage source. Consider two cases; (A) When C is without a dielectric medium and (B) When C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in two cases. Which of the following is true? [AIIMS 2017]
 (a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$
 (c) $V_C^A < V_C^B$ (d) None of these
- 15** A filament bulb (500W, 100V) is to be used in a 230 V main supply. When a resistance R is connected in series, it works perfectly and the bulb consumes 500W. The value of R is [NEET 2016]
 (a) 230Ω (b) 46Ω (c) 26Ω (d) 13Ω
- 16** Which of the following combinations should be selected for better tuning of an L - C - R circuit used for communication? [NEET 2016]
 (a) $R = 20\Omega$, $L = 1.5\text{ H}$, $C = 35\mu\text{F}$
 (b) $R = 25\Omega$, $L = 2.5\text{ H}$, $C = 45\mu\text{F}$
 (c) $R = 15\Omega$, $L = 3.5\text{ H}$, $C = 30\mu\text{F}$
 (d) $R = 25\Omega$, $L = 1.5\text{ H}$, $C = 45\mu\text{F}$
- 17** The potential differences across the resistance, capacitance and inductance are 80 V, 40 V and 100 V respectively in an L - C - R circuit. The power factor of this circuit is [NEET 2016]
 (a) 0.4 (b) 0.5 (c) 0.8 (d) 1.0

- 18** A 100Ω resistance and a capacitor of 100Ω reactance are connected in series across a 220 V source. When the capacitor is 50% charged, the peak value of the displacement current is [NEET 2016]
 (a) 2.2 A (b) 11 A
 (c) 4.4 A (d) $11\sqrt{2}$ A
- 19** A small signal voltage $V(t) = V_0 \sin \omega t$ is applied across an ideal capacitor C [NEET 2016]
 (a) over a full cycle the capacitor C does not consume any energy from the voltage source
 (b) current $I(t)$ is in phase with voltage $V(t)$
 (c) current $I(t)$ leads voltage $V(t)$ by 180°
 (d) current $I(t)$, lags voltage $V(t)$ by 90°
- 20** An inductor 20 mH , a capacitor $50\mu\text{F}$ and a resistor 40Ω are connected in series across a source of emf $V = 10 \sin 340 t$. The power loss in AC circuit is [NEET 2016]
 (a) 0.67 W (b) 0.76 W
 (c) 0.89 W (d) 0.51 W
- 21** A resistance R draws power P when connected to an AC source. If an inductance is now placed in series with the resistance, such that the impedance of the circuit becomes Z , the power drawn will be [CBSE AIPMT 2015]
 (a) $P\left(\frac{R}{Z}\right)^2$ (b) $P\sqrt{\frac{R}{Z}}$
 (c) $P\left(\frac{R}{Z}\right)$ (d) P
- 22** A condenser of 250 pF is connected in parallel to a coil of inductance of 0.16 mH , while its effective resistance is 20Ω . Determine the resonant frequency. [AIIMS 2015]
 (a) $9 \times 10^4\text{ Hz}$ (b) $16 \times 10^7\text{ Hz}$
 (c) $8 \times 10^5\text{ Hz}$ (d) $9 \times 10^3\text{ Hz}$
- 23** An inductor coil is connected to a 12 V battery and drawing a current 24 A. This coil is connected to capacitor and an AC source of rms voltage rating 24 V in the series connection. The phase difference between current and emf is zero. The rms current through the circuit would found to be [UP CPMT 2015]
 (a) 48 A (b) 36 A
 (c) 12 A (d) 24 A
- 24** A transformer is used to light 100 W-110 V lamp from 220 V mains. If the main current is 0.5 A, the efficiency of the transformer is [KCET 2015]
 (a) 96% (b) 90%
 (c) 99% (d) 95%

- 25** An alternating voltage given as, $V = 100\sqrt{2} \sin 100t$ V is applied to a capacitor of $1 \mu\text{F}$. The current reading of the ammeter will be equal to mA. [Guj. CET 2015]
 (a) 20 (b) 10
 (c) 40 (d) 80
- 26** The electric current in AC circuit is given by the relation $i = 3 \sin \omega t + 4 \cos \omega t$. The rms value of the current in the circuit in ampere is [EAMCET 2015]
 (a) $\frac{5}{\sqrt{2}}$ (b) $5\sqrt{2}$
 (c) $\frac{\sqrt{2}}{5}$ (d) $\frac{1}{\sqrt{2}}$
- 27** In an L - C - R series resonant circuit, the capacitance is changed from C to $4C$. For the same resonant frequency, the inductance should be changed from L to [Kerala CEE 2015]
 (a) $2L$ (b) $\frac{L}{2}$ (c) $4L$ (d) $\frac{L}{4}$
 (e) $\frac{L}{8}$
- 28** The average power dissipated in a pure inductor is [KCET 2015]
 (a) $\frac{VI^2}{4}$ (b) $\frac{1}{2}VI$
 (c) zero (d) VI^2
- 29** A transformer having efficiency of 90% is working on 200 V and 3 kW power supply. If the current in the secondary coil is 6 A, then the voltage across the secondary coil and the current in the primary coil respectively are [CBSE AIPMT 2014]
 (a) 300 V, 15 A (b) 450 V, 15 A
 (c) 450 V, 13.5 A (d) 600 V, 15 A
- 30** A dynamo converts [Kerala CEE 2014]
 (a) mechanical energy into thermal energy
 (b) electrical energy into thermal energy
 (c) thermal energy into electrical energy
 (d) mechanical energy into electrical energy
 (e) electrical energy into mechanical energy
- 31** Transformer is used to [Kerala CEE 2014]
 (a) convert AC to DC voltage
 (b) convert DC to AC voltage
 (c) obtain desired DC power
 (d) obtain desired AC voltage and current
 (e) obtain desired DC voltage and current
- 32** A step-up transformer operates on a 230 V line and supplies a current of 2 A. The ratio of primary and secondary winding is 1 : 25. The primary current is [UK PMT 2014]
 (a) 12.5 A (b) 50 A
 (c) 8.8 A (d) 25 A
- 33** A step-down transformer has 50 turns on secondary and 1000 turns on primary winding. If a transformer is connected to 220 V, 1 A AC source, then what is output current of the transformer? [KCET 2014]
 (a) $\frac{1}{20}$ A (b) 20 A
 (c) 100 A (d) 2 A
- 34** In L - C - R series circuit, an alternating emf e and current i are given by the equations

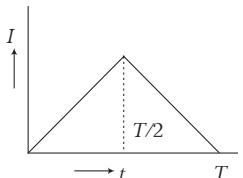
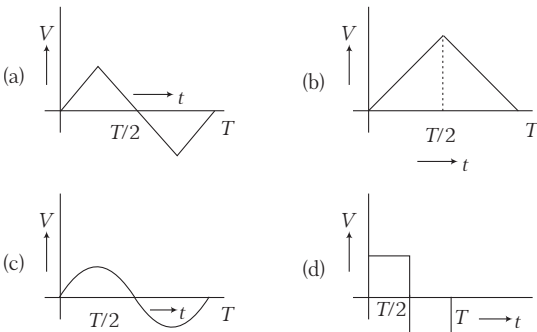
$$e = 100 \sin(100t) \text{ V}$$

$$i = 100 \sin\left(100t + \frac{\pi}{3}\right) \text{ mA}$$

 The average power dissipated in the circuit will be [MHT CET 2014]
 (a) 100 W (b) 10 W (c) 5 W (d) 2.5 W
- 35** The average power dissipated in AC circuit is 2 W. If a current flowing through a circuit is 2 A, impedance is 1Ω , then what is the power factor of the circuit? [KCET 2014]
 (a) 0.5 (b) 1 (c) Zero (d) $\frac{1}{\sqrt{2}}$
- 36** In an L - C - R series circuit, the potential difference between the terminals of the inductance is 60 V, between the terminals of the capacitor is 30 V and that across the resistance is 40 V. Then, the supply voltage will be equal to [UK PMT 2014]
 (a) 10 V (b) 50 V (c) 70 V (d) 130 V
- 37** An alternating emf given by equation

$$e = 300 \sin[(100\pi)t] \text{ V}$$

 is applied to a resistance 100Ω . The rms current through the circuit is (in amperes) [EAMCET 2014]
 (a) $\frac{3}{\sqrt{2}}$ (b) $\frac{9}{\sqrt{2}}$ (c) 3 (d) $\frac{6}{\sqrt{2}}$
- 38** A series L - C - R circuit contains inductance 5 mH, capacitance $2 \mu\text{F}$ and resistance 10Ω . If the frequency of AC source is varied, then what is the frequency at which maximum power is dissipated? [KCET 2014]
 (a) $\frac{10^5}{\pi}$ Hz (b) $\frac{10^{-5}}{\pi}$ Hz
 (c) $\frac{2}{\pi} \times 10^5$ Hz (d) $\frac{5}{\pi} \times 10^3$ Hz
- 39** A coil of self-inductance L is connected in series with a bulb B and an AC source. Brightness of the bulb decreases when [NEET 2013]
 (a) frequency of the AC source is decreased
 (b) number of turns in the coil is reduced
 (c) a capacitance of reactance $X_C = X_L$ is included in the same circuit
 (d) an iron rod is inserted in the coil

- 40** A 50 Hz AC signal is applied in a circuit of inductance of $(1/\pi)$ H and resistance $2100\ \Omega$. The impedance offered by the circuit is [J & K CET 2013]
 (a) $1500\ \Omega$ (b) $1700\ \Omega$ (c) $2102\ \Omega$ (d) $2500\ \Omega$
- 41** If the alternating current $I = I_1 \cos \omega t + I_2 \sin \omega t$, then the rms current is given by [J & K CET 2013]
 (a) $\frac{I_1 + I_2}{\sqrt{2}}$ (b) $\frac{|I_1 + I_2|}{\sqrt{2}}$ (c) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$ (d) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$
- 42** A 0.01 H inductor and $\sqrt{3}\pi\ \Omega$ resistance are connected in series with a 220 V , 50 Hz AC source. The phase difference between the current and emf is [EAMCET 2013]
 (a) $\frac{\pi}{2}\text{ rad}$ (b) $\frac{\pi}{6}\text{ rad}$ (c) $\frac{\pi}{3}\text{ rad}$ (d) $\frac{\pi}{4}\text{ rad}$
- 43** The alternating current in a circuit is given by $I = 50 \sin 314t$. The peak value and frequency of the current are [J & K CET 2013]
 (a) $I_0 = 25\text{ A}$ and $f = 100\text{ Hz}$
 (b) $I_0 = 50\text{ A}$ and $f = 50\text{ Hz}$
 (c) $I_0 = 50\text{ A}$ and $f = 100\text{ Hz}$
 (d) $I_0 = 25\text{ A}$ and $f = 50\text{ Hz}$
- 44** For a transformer, the turns ratio is 3 and its efficiency is 0.75. The current flowing in the primary coil is 2 A and the voltage applied to it is 100 V. Then the voltage and the current flowing in the secondary coil are, respectively. [Karnataka CET 2013]
 (a) 150 V, 1.5 A (b) 300 V, 0.5 A
 (c) 300 V, 1.5 A (d) 150 V, 0.5 A
- 45** In R - L - C series circuit, the potential difference across each element is 20 V. Now the value of the resistance alone is doubled, then PD across R , L and C respectively, [Karnataka CET 2013]
 (a) 20 V, 10 V, 10 V (b) 20 V, 20 V, 20 V
 (c) 20 V, 40 V, 40 V (d) 10 V, 20 V, 20 V
- 46** A series combination of resistor (R), capacitor (C) is connected to an AC source of angular frequency ω . Keeping the voltage same, if the frequency is changed to $\omega/3$, the current becomes half of the original current. Then, the ratio of the capacitive reactance and resistance at the former frequency is [Karnataka CET 2013]
 (a) $\sqrt{0.6}$ (b) $\sqrt{3}$
 (c) $\sqrt{2}$ (d) $\sqrt{6}$
- 47** If both the resistance and the inductance in an L - R AC series circuit are doubled the new impedance will be [Kerala CET 2013]
 (a) halved (b) fourfold
 (c) doubled (d) quadrupled
 (e) unchanged
- 48** A L - C - R circuit with $L = 1.00\text{ mH}$, $C = 1.0\ \mu\text{F}$ and $R = 50\ \Omega$, is driven with 5 V AC voltage. At resonance, the current through the circuit is [Kerala CET 2013]
 (a) 0.2 A (b) 0.25 A (c) 0.15 A (d) 0.1 A
 (e) 0.3 A
- 49** For a series L - C - R circuit, the rms values of voltage across various components are $V_L = 90\text{ V}$, $V_C = 60\text{ V}$ and $V_R = 40\text{ V}$. The rms value of the voltage applied to the circuit is [MP PMT 2013]
 (a) 190 V (b) 110 V (c) 70 V (d) 50 V
- 50** The power factor of an AC circuit having resistance R and inductance L connected in series to an AC source of angular frequency ω is [UP CPMT 2013]
 (a) zero (b) $\omega L/R$
 (c) $\frac{R}{\sqrt{R^2 + \omega^2 L^2}}$ (d) $R/\omega L$
- 51** A transformer of 100% efficiency has 200 turns in the primary coil and 40000 turns in secondary coil. It is connected to a 220 V main supply and secondary feeds to a $100\text{ k}\Omega$ resistance. The potential difference per turn is [AIIMS 2012]
 (a) 1.1 V (b) 25 V
 (c) 18 V (d) 11 V
- 52** A step down transformer is used on a 1000 V line to deliver 20 A at 120 V at the secondary coil. If the efficiency of the transformer is 80%, the current drawn from the line is [AIIMS 2012]
 (a) 3 A (b) 30 A (c) 0.3 A (d) 2.4 A
- 53** The current (I) in the inductance is varying with time according to the plot as shown in figure.
- 
- Which one of the following is the correct variation of voltage with time in the coil? [CBSE AIPMT (Screening) 2012]
- 

- 54** If in a L - R series circuit the power factor is $\frac{1}{2}$ and $R = 100\ \Omega$, then the value of L is, when AC mains is used [BCECE (Mains) 2012]

(a) π H (b) $\frac{\sqrt{3}}{\pi}$ H (c) $\frac{\pi}{\sqrt{3}}$ H (d) $\frac{\sqrt{2}}{\pi}$ H

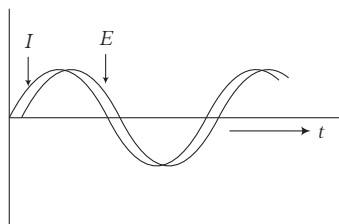
- 55** The self inductance of a choke coil is 10 mH. When it is connected with a 10 V DC source, then the loss of power is 20 W. When it is connected with 10 V AC source loss of power is 10 W. The frequency of AC source will be [BCECE (Mains) 2012]

(a) 80 Hz (b) 100 Hz (c) 120 Hz (d) 220 Hz

- 56** An electric motor runs on DC source of emf 200 V and draws a current of 10 A. If the efficiency be 40%, then the resistance of armature is [Manipal 2012]

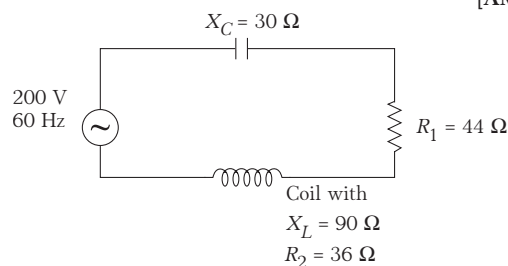
(a) 2 Ω (b) 8 Ω (c) 12 Ω (d) 16 Ω

- 57** When an AC source of emf $E = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf E and the current I is observed to be $\pi/4$, as shown in the figure. If the circuit consists possibly only of R - C or R - L in series, which of the following combinations is possible? [AMU 2012]



(a) $R = 1\text{ k}\Omega$, $C = 10\ \mu\text{F}$ (b) $R = 1\text{ k}\Omega$, $C = 1\ \mu\text{F}$
(c) $R = 1\text{ k}\Omega$, $L = 10\text{ H}$ (d) $R = 1\text{ k}\Omega$, $L = 1\text{ H}$

- 58** As given in the figure, a series circuit connected across a 200 V, 60 Hz line consists of a capacitor of capacitive reactance $30\ \Omega$, a non-inductive resistor of $44\ \Omega$ and a coil of inductive reactance $90\ \Omega$ and resistance $36\ \Omega$. the power dissipated in the coil is [AMU 2012]



(a) 320 W (b) 176 W (c) 144 W (d) 0

- 59** A generator at a utility company produces 100 A of current at 4000 V. The voltage is stepped up to 240000 V by a transformer before it is sent on a high voltage transmission line. The current in transmission line is [AFMC 2012]

(a) 3.67 A (b) 2.67 A
(c) 1.67 A (d) 2.40 A

- 60** The rms current in an AC circuit is 2 A. If the wattless current be $\sqrt{3}$ A, what is the power factor of the circuit? [JCECE 2012]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

- 61** An AC ammeter is used to measure current in a circuit. When a given direct current passes through the circuit, the AC ammeter reads 3A. When another alternating current passes through the circuit, the AC ammeter reads 4A. Then, the reading of this ammeter, if DC and AC flow through the circuit simultaneously, is [AIIMS 2011]

(a) 3 A (b) 4 A
(c) 7 A (d) 5 A

- 62** In an AC circuit, V and I are given by $V = 150 \sin(150t)$ volt and

$$I = 150 \sin \left[150t + \left(\frac{\pi}{3} \right) \right] \text{ ampere.}$$

The power dissipated in the circuit is [KCET 2011]

(a) 106 W (b) 150 W
(c) 5625 W (d) zero

- 63** An AC source is 120 V- 60 Hz. The value of voltage after $(1/720)$ s from start will be [BCECE 2011]

(a) 20.2 V (b) 42.4 V
(c) 84.8 V (d) 106.8 V

- 64** Alternating current is transmitted at far off places

(a) at high voltage and low current [JCECE 2011]
(b) at high voltage and high current
(c) at low voltage and low current
(d) at low voltage and high current

- 65** A transformer has 500 primary turns and 10 secondary turns. If the secondary has a resistive load of $15\ \Omega$, the currents in the primary and secondary respectively, are [DUMET 2011]

(a) 0.16A, 3.2×10^{-3} A
(b) 3.2×10^{-3} A, 0.16 A
(c) 0.16 A, 0.16 A
(d) 3.2×10^{-3} A, 3.2×10^{-3} A

ANSWERS

CHECK POINT 7.1

1. (d)	2. (b)	3. (d)	4. (a)	5. (d)	6. (a)	7. (d)	8. (a)	9. (c)	10. (c)
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CHECK POINT 7.2

1. (c)	2. (c)	3. (a)	4. (b)	5. (c)	6. (d)	7. (b)	8. (b)	9. (b)	10. (d)
11. (a)	12. (c)	13. (b)	14. (a)	15. (d)	16. (c)	17. (d)	18. (d)	19. (b)	20. (a)
21. (a)	22. (b)	23. (d)	24. (b)	25. (b)					

CHECK POINT 7.3

1. (b)	2. (c)	3. (b)	4. (d)	5. (b)	6. (b)	7. (a)	8. (b)	9. (a)	
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CHECK POINT 7.4

1. (d)	2. (b)	3. (c)	4. (c)	5. (c)	6. (b)	7. (a)	8. (b)	9. (c)	10. (b)
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(A) Taking it together

1. (c)	2. (c)	3. (c)	4. (b)	5. (c)	6. (b)	7. (a)	8. (b)	9. (b)	10. (b)
11. (b)	12. (b)	13. (b)	14. (c)	15. (c)	16. (a)	17. (c)	18. (b)	19. (a)	20. (c)
21. (c)	22. (b)	23. (a)	24. (a)	25. (b)	26. (d)	27. (c)	28. (c)	29. (b)	30. (a)
31. (d)	32. (a)	33. (b)	34. (d)	35. (b)	36. (c)	37. (b)	38. (b)	39. (b)	40. (c)
41. (d)	42. (d)	43. (c)	44. (a)	45. (a)	46. (c)	47. (d)	48. (b)	49. (a)	50. (d)
51. (d)	52. (c)	53. (d)	54. (b)	55. (a)	56. (c)	57. (c)	58. (b)	59. (d)	60. (a)
61. (a)	62. (d)	63. (a)	64. (b)	65. (c)	66. (d)	67. (b)	68. (a)	69. (a)	70. (c)
71. (b)	72. (c)	73. (c)	74. (d)	75. (d)	76. (d)	77. (b)	78. (b)	79. (c)	80. (b)
81. (d)	82. (a)	83. (d)	84. (b)	85. (a)	86. (a)	87. (d)	88. (c)	89. (b)	90. (d)
91. (d)	92. (d)	93. (b)	94. (b)	95. (a)	96. (a)	97. (b)	98. (a)	99. (c)	100. (d)
101. (b)	102. (b)	103. (a)	104. (b)	105. (b)	106. (c)	107. (b)	108. (b)	109. (c)	110. (d)
111. (b)	112. (a)								

(B) Medical entrance special format questions

• Assertion and reason

1. (a)	2. (d)	3. (d)	4. (b)	5. (b)	6. (d)
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• Statement based questions

1. (b)	2. (b)	3. (d)	4. (d)	5. (a)	6. (c)	7. (d)	8. (b)
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• Match the columns

1. (b)	2. (a)	3. (d)
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(C) Medical entrances' gallery

1. (b)	2. (b)	3. (a)	4. (a)	5. (b)	6. (a)	7. (b)	8. (a)	9. (a)	10. (c)
11. (c)	12. (a)	13. (d)	14. (b)	15. (c)	16. (c)	17. (c)	18. (a)	19. (a)	20. (d)
21. (a)	22. (c)	23. (a)	24. (b)	25. (b)	26. (a)	27. (d)	28. (c)	29. (b)	30. (d)
31. (d)	32. (b)	33. (b)	34. (d)	35. (a)	36. (b)	37. (a)	38. (d)	39. (d)	40. (c)
41. (c)	42. (b)	43. (b)	44. (b)	45. (a)	46. (a)	47. (c)	48. (d)	49. (d)	50. (c)
51. (a)	52. (a)	53. (d)	54. (b)	55. (a)	56. (c)	57. (c)	58. (a)	59. (c)	60. (a)
61. (d)	62. (c)	63. (c)	64. (a)	65. (b)					

Hints & Explanations

• CHECK POINT 7.1

1 (d) $2\pi f = 2000$

$$\therefore f = \frac{1000}{\pi} \text{ Hz}$$

6 (a) Here, $V_{\text{rms}} = 220\text{V}$, $f = 50 \text{ Hz}$

Peak value of voltage, $V_0 = \sqrt{2} V_{\text{rms}} = 220\sqrt{2} \text{ V}$

The instantaneous value of voltage is

$$\begin{aligned} V &= V_0 \sin 2\pi ft = 220\sqrt{2} \sin 2\pi \times 50t \\ &= 220\sqrt{2} \sin 100\pi t \end{aligned}$$

7 (d) $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1\text{A}$

8 (a) $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.536\text{A}$

Time taken, $T = \frac{1}{4 \times 60} = 4.167\text{ms}$

9 (c) $E = 141 \sin(628t)$

$$E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{141}{1.41} = 100\text{V and } 2\pi f = 628$$

$$\Rightarrow f = 100 \text{ Hz}$$

10 (c) $I = 20 \sin(100\pi t + 0.05\pi)$

$$\therefore I_{\text{rms}} = \frac{20}{\sqrt{2}} = 10\sqrt{2} \text{ A}$$

$$\omega = 100\pi \Rightarrow f = 50\text{Hz}$$

• CHECK POINT 7.2

2 (c) Maximum potential difference, $V_0 = I_0 R$

$$= (I_{\text{rms}} \sqrt{2}) R = (10\sqrt{2})(12) = 169.70 \text{ V}$$

3 (a) Capacitive reactance,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 4000 \times 25 \times 10^{-6}} = \frac{5}{\pi} \Omega$$

4 (b) In DC circuits, $f = 0$

$$\therefore X_C = \frac{1}{2\pi(0)C} = \frac{1}{0} = \infty$$

5 (c) Given, $e = 200\sqrt{2} \sin 100t$

and $C = 1\mu\text{F}$

Here, $E_{\text{rms}} = \frac{200\sqrt{2}}{\sqrt{2}} = 200 \text{ V}$

$$X_C = \frac{1}{\omega C} = \frac{1}{1 \times 10^{-6} \times 100} = 10^4 \Omega$$

$$\therefore i_{\text{rms}} = \frac{E_{\text{rms}}}{X_C} \Rightarrow i_{\text{rms}} = \frac{200}{10^4} = 2 \times 10^{-2} \text{ A} = 20 \text{ mA}$$

6 (d) $\therefore X_L = (2\pi f L)$

$$\therefore L = \frac{X_L}{2\pi f} = \frac{50}{(2\pi)(50)} = 0.16 \text{ H}$$

7 (b) As, $\tan \phi = \frac{X_L}{R}$

$$\therefore \tan 45^\circ = \frac{L\omega}{R}$$

$$\Rightarrow L = \frac{R}{\omega} = \frac{100}{2\pi \times 1000}$$

$$[\because \tan 45^\circ = 1]$$

$$\Rightarrow L = \frac{1}{20\pi}$$

8 (b) As, $I \propto \frac{1}{\omega L}$

$$\therefore \frac{I_1}{I_2} = \frac{\omega L_2}{\omega L_1} = \frac{L_2}{L_1}$$

10 (d) $Z = \sqrt{R^2 + X_L^2} = \sqrt{8^2 + 6^2} = 10 \Omega$

12 (c) $\tan 45^\circ = \frac{X_L}{R} \Rightarrow X_L = R$

13 (b) Impedance, $Z = \sqrt{R^2 + (2\pi f L)^2}$

As f increases, Z will increase.

14 (a) $\therefore \tan \phi = \frac{X_L}{R} = \frac{L\omega}{R}$

$$\therefore \tan \phi = \frac{3}{3} \Rightarrow \tan \phi = 1$$

$$\phi = \tan^{-1}(1) \Rightarrow \phi = 45^\circ$$

$$\phi = \frac{\pi}{4} \text{ rad}$$

15 (d) Given, $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \frac{0.4}{\pi} = 40 \Omega$

and $R = 30 \Omega$

$$\therefore Z = \sqrt{R^2 + X_L^2} = \sqrt{(30)^2 + (40)^2} = 50 \Omega$$

and $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200}{50} = 4 \text{ A}$

16 (c) The phase difference between instantaneous value of I and V is

$$\frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

Hence, current leads the voltage by 90° .

17 (d) $V = \sqrt{V_R^2 + (V_L - V_C)^2} = 10\sqrt{2} \text{ V}$

19 (b) The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here, $X_L = \omega L = 400 \times 20 \times 10^{-3} = 8 \text{ H}$

$$\text{and } X_C = \frac{1}{\omega C} = \frac{1}{400 \times 625 \times 10^{-6}} = 4\Omega$$

$$\therefore Z = \sqrt{(3)^2 + (8 - 4)^2} = 5\Omega$$

$$\therefore I = \frac{E}{Z} = \frac{300}{5} = 60\text{ A}$$

24 (b) At resonance (series resonant circuit),

$$X_L = X_C \Rightarrow Z_{\min} = R$$

Impedance is minimum and current is maximum.

25 (b) Equation of voltage, $E = \sqrt{V_R^2 + (V_L - V_C)^2}$

At resonance (series circuit),

$$V_L = V_C \Rightarrow E = V_R$$

i.e. Whole applied voltage appeared across the resistance.

• CHECK POINT 7.3

1 (b) $P = VI$

$$I = \frac{P}{V} = \frac{550}{220} = 2.5\text{ A}$$

2 (c) Voltage amplitude, $V_0 = 100\text{ V}$, current amplitude

$$I_0 = 100\text{ mA} = 100 \times 10^{-3}\text{ A} \text{ and phase difference between } I$$

$$\text{and } V \text{ is } \phi = \frac{\pi}{3} = 60^\circ.$$

Now power dissipated is given by

$$P = \frac{V_0 I_0}{2} \cos \phi$$

$$= \frac{100 \times 100 \times 10^{-3}}{2} \times \cos 60^\circ = 2.5\text{ W}$$

3 (b) Power factor of an AC circuit is given by

$$\cos \phi = \frac{R}{Z}$$

For circuit containing an ideal resistance only, $Z = R$

$$\therefore \cos \phi = 1$$

4 (d) In case of a pure inductance, $R = 0$

$$\therefore \text{Power factor} = \cos \phi = \frac{R}{Z} = \frac{0}{X} = 0$$

$$\text{So, } P_{\text{av}} = E_{\text{rms}} \cdot i_{\text{rms}} \times 0 = 0$$

6 (b) $Z = \sqrt{(3)^2 + (4)^2} = 5\Omega$

$$\cos \phi = \frac{R}{Z} = \frac{3}{5} = 0.6$$

8 (b) The energy stored in an inductor, $U = \frac{1}{2} LI^2$

$$\mathbf{9 (a)} U = \frac{1}{2} LI^2 = \frac{1}{2} (100 \times 10^{-3})(10)^2 = 5\text{ J}$$

• CHECK POINT 7.4

2 (b) In step-down transformer, $V_P > V_S$

$$\text{but, } \frac{V_P}{V_S} = \frac{I_S}{I_P}$$

$$\therefore I_S > I_P$$

So, current increases in a step-down transformer.

3 (c) A transformer works on the principle of mutual induction.

5 (c) If we neglect all losses, then input power = output power.

6 (b) Transformation ratio,

$$k = \frac{N_S}{N_P} = \frac{V_S}{V_P}$$

For step-up transformer,

$$V_S > V_P, \text{ i.e. } N_S > N_P$$

Hence, $k > 1$.

$$\mathbf{7 (a)} \text{ As, } \frac{N_1}{N_2} = \frac{I_2}{I_1} \Rightarrow I_2 = \left(\frac{N_1}{N_2} \right) I_1 = \left(\frac{500}{2000} \right) \times 48 = 12\text{ A}$$

(A) Taking it together

1 (c) In case of AC, $Z = \sqrt{R^2 + (\omega L)^2}$, while in case of DC, it is only R . So, in AC resistance increases.

2 (c) At frequencies higher than resonant frequency, $X_L > X_C$ i.e. circuit behaves as inductive circuit.

4 (b) The power factor of choke coil is given by

$$\cos \phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \approx \frac{R}{\omega L} \quad (\text{as } R \ll \omega L)$$

As $R \ll \omega L$, $\cos \phi$ is very small. Thus, the power absorbed by the coil is very small. The only loss of energy is due to hysteresis in the iron core, which is much less than the loss of energy in the resistance.

6 (b) By introducing the slab, C will increase. Therefore, X_C will decrease and new current will be more than I .

$$\mathbf{7 (a)} X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 10^{-5}} = 160\Omega$$

8 (b) Capacitive reactance, $X_C = \frac{1}{2\pi \nu C}$

$$\Rightarrow \frac{1}{1000} = \frac{1}{2\pi \times \nu \times 5 \times 10^{-6}}$$

$$\therefore \nu = \frac{100}{\pi} \text{ Hz}$$

9 (b) Applied voltage, $V = \sqrt{V_R^2 + V_L^2}$

$$\therefore V = \sqrt{(200)^2 + (150)^2} = 250\text{ V}$$

$$\mathbf{10 (b)} I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (2\pi f L)^2}}$$

$$= \frac{120}{\sqrt{(10)^2 + (2\pi \times 60 \times 2)^2}} = 0.16\text{ A}$$

$$\mathbf{11 (b)} < i >_T = < 5 >_R + < 5 \sin 100 \pi t >_I$$

$$= 5 + 0 = 5\text{ A}$$

12 (b) Given, $\nu = 50\text{ Hz}$, $I_{\text{rms}} = 5\text{ A}$

$$t = \frac{1}{300} \text{ s}$$

We have to find $I(t)$,

$$I_0 = \text{Peak value} = \sqrt{2} I_{\text{rms}} = \sqrt{2} \times 5$$

$$= 5\sqrt{2} \text{ A}$$

$$I = I_0 \sin \omega t = 5\sqrt{2} \sin 2\pi \nu t = 5\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{300}$$

$$= 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2}$$

$$= 5\sqrt{3/2} \text{ A}$$

$$\mathbf{13} \text{ (b) } E = 10 \cos(2\pi ft) = 10 \cos \left[2\pi \times 50 \times \frac{1}{600} \right]$$

$$= 10 \cos \frac{\pi}{6} = 5\sqrt{3} \text{ V}$$

14 (c) Phase difference between current and voltage is $\pi/2$, with voltage leading, so $X_L > X_C$ and $R = 0$.

$$\mathbf{15} \text{ (c) Resonant frequency, } f = \frac{1}{2\pi\sqrt{LC}}$$

It does not depend on resistance.

$$\mathbf{16} \text{ (a) Resonant frequency, } f = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore f \propto \frac{1}{\sqrt{C}} \quad \text{or} \quad \frac{f_1}{f_2} = \sqrt{\frac{C_2}{C_1}}$$

$$\Rightarrow \frac{f}{f_2} = \sqrt{\frac{4C}{C}} \Rightarrow f_2 = \frac{f}{2}$$

17 (c) Current in inductance I_L lags behind the voltage by $\pi/2$.

Current in resistance I_R is in phase with voltage, while current in capacitance I_C leads the voltage by a phase of $\pi/2$.

18 (b) Power dissipated in circuit,

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{(30)^2}{10} = 90 \text{ W}$$

19 (a) For complete cycle, power consumption is zero.

$$\mathbf{20} \text{ (c) } \frac{H_{\text{DC}}}{H_{\text{AC}}} = \left(\frac{i_{\text{DC}}}{i_{\text{rms}}} \right)^2 = \left(\frac{2}{\sqrt{2}} \right)^2 = 2:1 \quad (\because H \propto i^2)$$

$$\mathbf{21} \text{ (c) } I_0 = 28 \text{ A}$$

$$\therefore I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \approx 20 \text{ A}$$

$$\mathbf{22} \text{ (b) } P = \frac{1}{2} V_0 I_0 \cos \phi$$

$$\Rightarrow P = P_{\text{peak}} \cos \phi$$

$$\Rightarrow \frac{1}{2} (P_{\text{peak}}) = P_{\text{peak}} \cos \phi$$

$$\Rightarrow \cos \phi = \frac{1}{2}$$

$$\Rightarrow \phi = \frac{\pi}{3}$$

$$\mathbf{23} \text{ (a) } P = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$= V_{\text{rms}} \left(\frac{V_{\text{rms}}}{Z} \right) \left(\frac{R}{Z} \right)$$

$$= \frac{(110)^2 (11)}{(22)^2} = 275 \text{ W}$$

$$\mathbf{24} \text{ (a) Here, } V^2 = V_R^2 + V_L^2$$

$$\therefore V_L = \sqrt{V^2 - V_R^2} = \sqrt{(20)^2 - (12)^2} = 16 \text{ V}$$

25 (b) We know that, resonant frequency in an L - C - R circuit is given by

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

Now to reduce ν_0 either we can increase L or we can increase C . To increase capacitance, we must connect another capacitor parallel to the first.

$$\mathbf{26} \text{ (d) } e = \sqrt{e_1^2 + e_2^2} \sin(\omega t + \phi)$$

$$\text{where, } \cos \phi = \frac{e_1}{\sqrt{e_1^2 + e_2^2}} \text{ and } \sin \phi = \frac{e_2}{\sqrt{e_1^2 + e_2^2}}$$

$$\text{i.e., } e_0 = \sqrt{e_1^2 + e_2^2}$$

$$\text{or } e_{\text{rms}} = \frac{\sqrt{e_1^2 + e_2^2}}{\sqrt{2}}$$

27 (c) Average value of output current is given by

$$\begin{aligned} I_{\text{av}} &= \frac{\int_0^{T/2} I dt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin \omega t dt}{T/2} \\ &= \frac{2I_0}{T} \left[-\frac{\cos \omega t}{\omega} \right]_0^{T/2} \\ &= \frac{2I_0}{T} \left[-\frac{\cos(\omega T/2)}{\omega} + \frac{\cos 0^\circ}{\omega} \right] \\ &= \frac{2I_0}{\omega T} [-\cos \pi + \cos 0^\circ] = \frac{2I_0}{2\pi} [1 + 1] = \frac{2I_0}{\pi} \end{aligned}$$

28 (c) Phase different between current and voltage,

$$\Delta \phi = \phi_2 - \phi_1 = \frac{\pi}{6} - \left(-\frac{\pi}{6} \right) = \frac{\pi}{3} \text{ or } 60^\circ.$$

Hence, current leads voltage by 60° .

29 (b) Reading of ammeter = i_{rms}

$$\begin{aligned} i_{\text{rms}} &= \frac{V_{\text{rms}}}{X_C} = \frac{V_0}{\sqrt{2}} \times \omega C \\ &= \frac{200\sqrt{2} \times 100 \times (1 \times 10^{-6})}{\sqrt{2}} \\ &= 2 \times 10^{-2} \text{ A} = 20 \text{ mA} \end{aligned}$$

$$\mathbf{31} \text{ (d) } \tan \phi = \frac{X_L}{R} \quad (\text{In case of } L\text{-}R \text{ circuit})$$

$$= \frac{L\omega}{R} = \frac{L2\pi f}{R} = \frac{2 \times 200}{300} = \frac{4}{3}$$

$$\mathbf{32} \text{ (a) } \omega = \frac{1}{\sqrt{LC}}$$

$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{20 \times 10^{-6} \times 10 \times 10^{-3}}} = 356 \text{ Hz or cycles/s}$$

33 (b) $I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \sqrt{\langle (8 + 6 \sin \omega t)^2 \rangle}$

$$\Rightarrow I_{\text{rms}} = \sqrt{\langle (64 + 96 \sin \omega t + 36 \sin^2 \omega t) \rangle}$$

$$\Rightarrow I_{\text{rms}} = \sqrt{\langle 64 \rangle + 96 \langle \sin \omega t \rangle + 36 \langle \sin^2 \omega t \rangle}$$

$$\Rightarrow I_{\text{rms}} = \sqrt{64 + 0 + 36 \times 0.5} = 9.05 \text{ A}$$

Since, $\langle \sin^2 \omega t \rangle = 0.5$

34 (d) Reactance of series L - C circuit $= X_L + X_C$

$$= j \left(\omega L - \frac{1}{\omega C} \right)$$

So, its value is infinite at both zero and infinite frequency. It is shown in graph (d).

35 (b) Value of induced emf in coil,

$$e = M \frac{dI}{dt}$$

$$= 0.005 \times \frac{d}{dt} (I_0 \sin \omega t)$$

$$= 0.005 \times I_0 \omega \cos \omega t$$

$$\therefore e_{\text{max}} = 0.005 \times 10 \times 100\pi = 5\pi \text{ V}$$

36 (c) For only a resistance, it hardly matters, whether the source is DC or AC. In series,

$$P = \frac{P_1 P_2}{P_1 + P_2} = \frac{1000 \times 1000}{2000} = 500 \text{ W}$$

37 (b) $X_L = R$

$$\therefore Z = \sqrt{2}R$$

$$P = \left(\frac{V_{\text{rms}}}{Z} \right)^2 \cdot R = \left(\frac{E_0 / \sqrt{2}}{\sqrt{2}R} \right)^2 \cdot R = \frac{E_0^2}{4R}$$

38 (b) Induced emf in the circuit, $e = M \frac{di}{dt} = (0.2)(5) = 1 \text{ V}$

39 (b) $\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{i_S}{i_P}$

$$\therefore i_S = \frac{N_P i_P}{N_S} = \frac{4 \times 140}{280} = 2 \text{ A}$$

40 (c) The voltmeter connected to AC mains reads mean value ($\langle v^2 \rangle$) and is calibrated in such a way that it gives value of $\langle v^2 \rangle$, which is multiplied by form factor to give rms value, i.e. $\sqrt{\langle v^2 \rangle}$.

41 (d) Here, $I_R = I_L = I$

and they have a phase difference $0 < \phi < 90^\circ$, with applied voltage.

42 (d) $X_L = 2\pi fL = 2\pi \left(\frac{50}{\pi} \right) \times 1 = 100 \Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \left(\frac{50}{\pi} \right) 20 \times 10^{-6}} = 500 \Omega$$

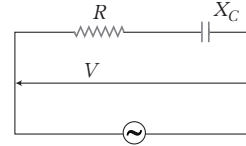
$$\text{Impedance, } Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\Rightarrow Z = \sqrt{(300)^2 + (400)^2} = 500 \Omega$$

43 (c) As, emf induced in coil is $e = -\frac{LdI}{dt}$

$\therefore$ Voltage *versus* time curve is a straight line with negative slope.

44 (a) Let the applied voltage be V volt



Here,

$$V_R = 12 \text{ V}, V_C = 5 \text{ V}$$

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25}$$

$$= \sqrt{169} = 13 \text{ V}$$

45 (a) $V_{\text{rms}} = \sqrt{V_{\text{av}}} = \sqrt{\frac{1}{T} \int_0^T V^2 dt} = \sqrt{\frac{1}{T} \int_0^T 10^2 dt} = 10 \text{ V}$

46 (c) Peak voltage, $V_0 = \sqrt{2} V_{\text{rms}} = 331 \text{ V}$

$$\text{and } \omega = 2\pi f = 100 \pi$$

$$\Rightarrow V_L = 331 \sin(100 \pi t)$$

where, V_L is line voltage.

47 (d) In pure inductor, voltage leads the current. So, correct graph is (d).

48 (b) For capacitive circuits, $X_C = \frac{1}{\omega C}$

$$\therefore I = \frac{V}{X_C} = V\omega C \Rightarrow I \propto \omega$$

49 (a) Secondary voltage, $V_S = 24 \text{ V}$

Power associated with secondary, $P_S = 12 \text{ W}$

$$I_S = \frac{P_S}{V_S} = \frac{12}{24} = \frac{1}{2} \text{ A} = 0.5 \text{ A}$$

Peak value of the current in the secondary,

$$I_0 = I_S \sqrt{2}$$

$$= (0.5)(1.414) = 0.707 = \frac{1}{\sqrt{2}} \text{ A}$$

50 (d) Time taken by the current to reach the maximum value.

$$t = \frac{T}{4} = \frac{1}{4f} = \frac{1}{4 \times 50} = 5 \times 10^{-3} \text{ s}$$

$$\text{and } I_0 = \sqrt{2} I_{\text{rms}} = 10\sqrt{2} = 14.14 \text{ A}$$

51 (d) According to problem, $X_L = X_C$

$$\omega L = \frac{1}{\omega C}$$

$$\therefore \omega^2 = \frac{1}{LC} = \frac{1}{\sqrt{10^{-3} \times 10 \times 10^{-6}}}$$

$$\omega = \frac{1}{\sqrt{10^{-8}}} = 10^4$$

$$\therefore X_L = \omega L = 10^4 \times 10^{-3} = 10 \Omega$$

52 (c) At resonance frequency, current is maximum.

53 (d) For DC motor, $I = \frac{V_S - V_B}{R}$

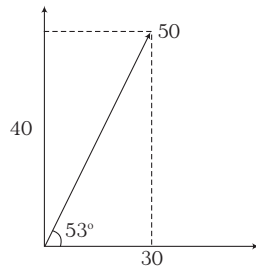
$$\therefore 1.5 = \frac{220 - V_B}{20}$$

$$\Rightarrow V_B = 190 \text{ V}$$

54 (b) $P = V_{\text{rms}} I_{\text{rms}} = \frac{V_0}{\sqrt{2}} I_{\text{rms}}$

$$\therefore I_{\text{rms}} = \frac{1000\sqrt{2}}{200} = 5\sqrt{2} \text{ A}$$

55 (a) $V = 30 \sin 50t + 40 \cos 50t = 50 \sin(50t + 53^\circ)$



$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{50/\sqrt{2}}{10} = \frac{5}{\sqrt{2}} \text{ A}$$

56 (c) $I_0 = \frac{140}{10} = 14 \text{ A}$

$$\therefore I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \approx 10 \text{ A}$$

$\therefore I_{\text{DC}}$ required will be approximately 10 A.

$$\begin{aligned} 57 \text{ (c) } i_{\text{rms}}^2 &= \frac{\int i^2 dt}{\int dt} \\ &= \frac{\int_2^4 (4t) dt}{\int_2^4 dt} = \frac{4 \int_2^4 t dt}{2} = 2 \left[\frac{t^2}{2} \right]_2^4 = 12 \text{ A}^2 \end{aligned}$$

$$\Rightarrow i_{\text{rms}} = 2\sqrt{3} \text{ A}$$

58 (b) Power factor, $\cos \phi = \frac{1}{\sqrt{2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

This gives, $\omega L = R$

When ω is doubled, we get

$$\cos \phi' = \frac{R}{\sqrt{R^2 + 4\omega^2 L^2}} = \frac{R}{\sqrt{R^2 + 4R^2}} = \frac{1}{\sqrt{5}}$$

59 (d) Resistance of coil (R) = $\frac{200}{1} = 200 \Omega$

$$\text{With AC source, } I = \frac{200}{\sqrt{R^2 + X_L^2}} \text{ or } 0.5 = \frac{200}{\sqrt{R^2 + X_L^2}}$$

$$\Rightarrow R^2 + (2\pi fL)^2 = (400)^2$$

$$\Rightarrow \left(2\pi f \times \frac{2\sqrt{3}}{\pi} \right)^2 = (400)^2 - (200)^2 = 200 \times 600$$

$$\Rightarrow 4\sqrt{3}f = \sqrt{12} \times 100 \Rightarrow f = 50 \text{ Hz}$$

60 (a) From, $P = VI = \frac{V^2}{R}$

$$\Rightarrow I = \frac{P}{V} = 6 \text{ A} \Rightarrow R = \frac{V^2}{P} = \frac{100}{60} = \frac{5}{3} \Omega$$

$$\text{Now, } I = \frac{V}{Z}$$

$$\Rightarrow 6 = \frac{100}{\sqrt{\left(\frac{5}{3}\right)^2 + (2\pi fL)^2}}$$

Solving this equation, we get

$$L = 0.052 \text{ H}$$

61 (a) Given, $X_L = X_C = 5 \Omega$, this is the condition of resonance.

So $V_L = V_C$, or net voltage across L and C combination will be zero.

$$\therefore Z = R = 55 \Omega$$

$$I = \frac{V}{Z} = \frac{110}{55} = 2 \text{ A}$$

62 (d) Since, $V_C > V_L$

$$\therefore X_C > X_L$$

Hence, current will lead the voltage.

$$\therefore V = \sqrt{V_R^2 + (V_C - V_L)^2} = 10 \text{ V} < V_C$$

$$\cos \phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{8}{10} = \frac{4}{5}$$

63 (a) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\therefore 10 = \sqrt{(10)^2 + (X_L - X_C)^2}$$

$$\Rightarrow 100 = 100 + (X_L - X_C)^2$$

$$\Rightarrow X_L - X_C = 0 \quad \dots(i)$$

Let ϕ is the phase difference between current and voltage,

$$\therefore \tan \phi = \frac{X_L - X_C}{R}$$

$$\therefore \tan \phi = \frac{0}{R} \quad [\text{from Eq. (i)}]$$

$$\phi = 0$$

64 (b) Resonance frequency, $\omega = \frac{1}{\sqrt{LC}}$

$$\begin{aligned} &= \frac{1}{\sqrt{8 \times 10^{-3} \times 20 \times 10^{-6}}} \\ &= 2500 \text{ rads}^{-1} \end{aligned}$$

$$\text{Resonance current, } I = \frac{V}{R} = \frac{220}{44} = 5 \text{ A}$$

65 (c) At point A, $X_C > X_L$

at point B, $X_C = X_L$

at point C, $X_C < X_L$

So at C, circuit is inductive.

66 (d) At resonance, $X_L = X_C$

$$\therefore Z = R = 40 \Omega$$

$$\text{and } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{230}{40} = 5.75 \text{ A}$$

- 67 (b) At resonance,

$$X_L = X_C$$

$$\therefore V_1 = V_3$$

Hence, V_2 will be zero.

$$68 (a) Z = \sqrt{(52)^2 + \left[230 \times 10^{-3} \times 2\pi \times 80 - \frac{1}{2\pi \times 80 \times 8.8 \times 10^{-6}} \right]^2}$$

$$= 122 \Omega$$

$$P = \left(\frac{V_{\text{rms}}}{Z} \right)^2 \cdot R = \left(\frac{150}{122} \right)^2 (52) = 78.6 \text{ W}$$

- 69 (a) $C_{\text{net}} = C/2$ (in series) and $L_{\text{net}} = 2L$ (in series)

$$\therefore \omega = \frac{1}{\sqrt{L_{\text{net}} C_{\text{net}}}}$$

$$\text{or } f = \frac{1}{2\pi \sqrt{(2L) \cdot \left(\frac{C}{2}\right)}} \text{ or } f = \frac{1}{2\pi \sqrt{LC}}$$

- 70 (c) Four resistances form a balanced Wheatstone bridge.

Current flowing in the coil will be zero and hence energy stored in the coil is zero.

- 71 (b) $N_p = 100$, $P_i = 10 \text{ kW}$, $V_i = 200 \text{ V}$

$$\therefore I_p = \frac{P_i}{V_i} = \frac{10000}{200} = 50 \text{ A}$$

Input power = Output power (Neglecting all losses)

$$\therefore I_s = \frac{P_i}{V_o} = \frac{10000}{5000} = 2 \text{ A}$$

- 72 (c) $R = X_L = \frac{V_o}{I_o} = \frac{200}{5} = 40 \Omega$

$$\therefore Z = \sqrt{R_f^2 + X_L^2} = 40\sqrt{2} \Omega$$

$$\Rightarrow I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{200 / \sqrt{2}}{40\sqrt{2}} = \frac{5}{2} \text{ A}$$

- 73 (c) Let R be the resistance of coil and capacitor. Then,

$$R = \frac{24}{6} = 4 \Omega$$

In the second case,

$$I_{\text{max}} = \frac{12}{4 + 4} = \frac{12}{8} = 1.5 \text{ A}$$

- 74 (d) $X_C > X_L$. Hence, current will lead the voltage.

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = 10\sqrt{2} \Omega$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{400 / \sqrt{2}}{10\sqrt{2}} = 20 \text{ A}$$

$$\cos \phi = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

$$V_R = I_{\text{rms}} R = (20)(10) = 200 \text{ V}$$

- 75 (d) Taking, $V^2 = V_R^2 + (V_L - V_C)^2$

According to the figure, we conclude that

$$V_L = V_C$$

$$\therefore V_R = V = 220 \text{ V}$$

Reading of voltmeter = 220 V

$$\text{Reading of ammeter, } I = \frac{220}{100} = 2.2 \text{ A}$$

- 76 (d) The voltages V_L and V_C are equal and opposite, so, voltmeter reading will be zero.

$$\text{Also, } R = 30 \Omega, X_L = X_C = 25 \Omega$$

$$\text{So, } I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{V}{R} = \frac{240}{30} = 8 \text{ A}$$

- 77 (b) Resonance frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100 \times 10^{-6} \times 1 \times 10^{-6}}} = 10^5 \text{ rads}^{-1}$$

Now, given $\omega = 70 \text{ kilo-rads}^{-1} = 70000 \text{ rads}^{-1}$

$$\Rightarrow \omega_0 > \omega$$

$$\text{As } X_L \propto \omega \text{ and } X_C \propto \frac{1}{\omega}$$

$$\therefore X_L > X_C$$

The circuit will be series R - L circuit.

- 78 (b) $I_L = I_{OL} \sin\left(\omega t - \frac{\pi}{2}\right) = -1.2 \cos \omega t$

$$I_C = I_{OC} \sin\left(\omega t + \frac{\pi}{2}\right) = 1.0 \cos \omega t$$

$$\therefore I = I_L + I_C = -0.2 \cos \omega t \Rightarrow |I| = 0.2 \text{ A}$$

- 79 (c) For delivering maximum power from the generator to the load, total internal reactance must be equal to conjugate of total external reactance.

$$\text{Hence, } X_{\text{int}} = X_{\text{ext}}^*$$

$$\Rightarrow X_g = (X_L)^* = -X_L \Rightarrow X_L = -X_g$$

- 80 (b) Voltage drop across R is maximum at resonance, i.e. at

$$\omega = \frac{1}{\sqrt{LC}} \text{ or } f = \frac{\omega}{2\pi} = \frac{1}{2\pi \sqrt{LC}}$$

$$= \frac{1}{2\pi \sqrt{\left(\frac{1}{\pi}\right) \left(\frac{1}{\pi} \times 10^{-6}\right)}} = 500 \text{ Hz}$$

- 81 (d) Given, $I_1 = I_2$

$$\therefore \omega_1 L - \frac{1}{\omega_1 C} = \frac{1}{\omega_2 C} - \omega_2 L$$

Solving this, we get

$$\frac{1}{\sqrt{LC}} = \text{resonance frequency} = \sqrt{\omega_1 \omega_2} = \sqrt{9 \times 4} = 6 \text{ kHz}$$

At resonance frequency, current will be maximum.

- 82 (a) The given AC circuit is the combination of two pure parallel circuits with the applied voltage. In this, I_2 is in phase with V and I_1 leads V by 90° .

- 83 (d) At resonance,

$$\text{Voltage across } L = \text{Voltage across } C$$

$$\therefore \text{Reading in } V_2 = \text{Reading in } V_3.$$

$$84 \text{ (b)} I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

When ω increases, I_{rms} increases, so the bulb glows brighter.

$$85 \text{ (a)} \text{ Total resistance of the circuit, } R = 6 + 4 = 10 \, \Omega$$

$$\text{Capacitive reactance, } X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10 \, \Omega$$

Inductive reactance,

$$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10 \, \Omega$$

$$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2} = 10 \, \Omega$$

$$\text{Amplitude of current, } I_0 = \frac{V_0}{Z} = \frac{20}{10} = 2 \text{ A}$$

$$86 \text{ (a)} \quad X_C = X_L = R$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\text{but } V_L = V_C$$

$$\therefore V = V_R = 10 \text{ V}$$

When capacitor is short circuited, $V_C = 0$

$$\text{and } V_R = V_L \quad (\text{as } R = X_L)$$

$$\therefore V = \sqrt{V_R^2 + V_L^2} \quad \text{or } 10 = \sqrt{2} V_L$$

$$\therefore V_L = \frac{10}{\sqrt{2}} \text{ V}$$

$$87 \text{ (d)} \quad V_C = 200 \text{ V} \quad [\text{given}]$$

$$V_R = (11)(20) = 220 \text{ V}$$

$$V = 220 \text{ V} \quad [\text{applied}]$$

$$\therefore V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow V_L = V_C = 200 \text{ V}$$

$$88 \text{ (c)} \text{ Given, } X_L = 1 \, \Omega, R = 2 \, \Omega$$

$$V_{\text{rms}} = 6 \text{ V, } P_{\text{av}} = ?$$

Average power dissipated in the circuit,

$$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi \quad \dots(i)$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{V_{\text{rms}}}{Z}$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{4 + 1} = \sqrt{5} \, \Omega$$

$$I_{\text{rms}} = \frac{6}{\sqrt{5}} \text{ A}$$

$$\cos \phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$

$$P_{\text{av}} = 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} \quad [\text{from Eq. (i)}]$$

$$= \frac{72}{\sqrt{5} \sqrt{5}} = \frac{72}{5} = 14.4 \text{ W}$$

$$89 \text{ (b)} X_L = (2\pi fL) = 2\pi \times 50 \times 0.7 = 220 \, \Omega$$

$$\therefore Z = \sqrt{R^2 + X_L^2} = 220\sqrt{2} \, \Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{220}{220\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ A}$$

$$\therefore \text{Power, } P = I_{\text{rms}}^2 \cdot R = \frac{1}{2} \times 220 = 110 \text{ W}$$

$$90 \text{ (d)} V_L = V_C \Rightarrow X_L = X_C \text{ or circuit shown is in resonance.}$$

$$\therefore V_{\text{applied}} = V_R = 100 \text{ V}$$

$$\text{and } I = \frac{V_R}{R} = 2 \text{ A}$$

$$91 \text{ (d)} V_S^2 = (3)^2 + (8 - 4)^2 \Rightarrow V_S = 5 \text{ V}$$

$$\text{Now, } Z = \frac{V_S}{I} = \frac{5}{1} = 5 \, \Omega$$

$$\text{Also, } V_R = IR \text{ or } R = \frac{3}{1} = 3 \, \Omega$$

$$\text{So, } \text{PF} = \frac{R}{Z} = \frac{3}{5} = 0.6; \text{ Also } V_L > V_C$$

As, I lags V , so PF is lagging in nature.

$$92 \text{ (d)} \text{ Phase angle, } \tan \phi = \frac{X_L}{R} = \frac{X_C}{R}$$

$$\Rightarrow \tan 60^\circ = \frac{X_L}{R} = \frac{X_C}{R} \Rightarrow X_L = X_C = \sqrt{3} R$$

$$\text{i.e., } Z = \sqrt{R^2 + (\sqrt{3} R - \sqrt{3} R)^2} \Rightarrow Z = R$$

$$\text{So, average power, } P = \frac{V^2}{R} = \frac{200 \times 200}{100} = 400 \text{ W}$$

$$93 \text{ (b)} \text{ Power consumed in the coil,}$$

$$P = I_{\text{rms}}^2 R$$

$$\therefore R = \frac{P}{I_{\text{rms}}^2} = \frac{240}{16} = 15 \, \Omega$$

$$\text{Impedance, } Z = \frac{V}{I} = \frac{100}{4} = 25 \, \Omega$$

$$\text{Now, } X_L = \sqrt{Z^2 - R^2} = \sqrt{(25)^2 - (15)^2} = 20 \, \Omega$$

$$\therefore 2\pi fL = 20$$

$$L = \frac{20}{2\pi \times 50} = \frac{1}{5\pi} \text{ H}$$

$$94 \text{ (b)} \text{ In an } L\text{-}C\text{-}R \text{ circuit, the current and the voltage are in phase } (\phi = 0), \text{ when}$$

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0 \quad \text{or} \quad \omega L = \frac{1}{\omega C}$$

$$\text{or } L = \frac{1}{\omega^2 C}$$

$$\text{Here, } \omega = 2\pi f = 2 \times 3.14 \times 50 \text{ s}^{-1} = 314 \text{ s}^{-1}$$

$$\text{and } C = 20 \, \mu\text{F} = 20 \times 10^{-6} \text{ F}$$

$$\therefore L = \frac{1}{(314)^2 \times (20 \times 10^{-6})} = 0.51 \text{ H}$$

$$95 \text{ (a)} \text{ Angular velocity,}$$

$$\omega_0 = 2\pi\nu = 2\pi \times 100$$

$$\omega_0 = 2 \times 3 \times 100$$

$$= 600 \text{ rads}^{-1}$$

$$[\because \pi = 3]$$

$$\text{Further, } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\dots(i)$$

$$\text{Also, } X_C = \frac{1}{C\omega_0} = 60 \, \Omega$$

$$\Rightarrow C = \frac{1}{\omega_0 \times 60} = \frac{1}{600 \times 60}$$

$$\Rightarrow C = \frac{1}{36 \times 10^3} \text{ F}$$

Putting values of C in Eq. (i), we get

$$600 = \frac{1}{\sqrt{L \left(\frac{1}{36 \times 10^3} \right)}}$$

$$\Rightarrow 36 \times 10^4 = \frac{36 \times 10^3}{L}$$

$$\Rightarrow L = \frac{36 \times 10^3}{36 \times 10^4} = \frac{1}{10} = 0.1 \text{ H}$$

96 (a) Charge on the capacitor,

$$q_0 = CV = 1 \times 10^{-6} \times 1 = 10^{-6} \text{ C}$$

Here, $q = q_0 \sin \omega t$

or $I_0 = \omega q_0 = \text{maximum current}$

$$\text{Now, } \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-9}}} = (10^9)^{1/2}$$

$$\therefore I_0 = (10^9)^{1/2} \times (1 \times 10^{-6}) = \sqrt{1000} \text{ mA}$$

97 (b) Circuit is in resonance,

$$\therefore V = V_R = IR$$

$$\text{or } R = \frac{V}{I} = \frac{100}{0.2} = 500 \, \Omega$$

$$X_L = \frac{V_L}{I} = \frac{400}{0.2} = 2000 \, \Omega = \omega L$$

$$\therefore \omega = 1000 \text{ rads}^{-1}$$

$$\text{Further, } X_L = X_C = 2000 \, \Omega = \frac{1}{\omega C}$$

$$\therefore C = \frac{1}{2000 \omega} = 0.5 \, \mu\text{F}$$

$$\textbf{98 (a) Impedance} = \frac{V_{AC}}{I_{AC}} = \frac{100}{0.5} = 200 \, \Omega = Z$$

$$R = \frac{V_{DC}}{I_{DC}} = \frac{100}{1} = 100 \, \Omega$$

$$X_L = \sqrt{Z^2 - R^2} = 100\sqrt{3} \, \Omega = (2\pi f L)$$

$$\therefore L = \frac{100\sqrt{3}}{2\pi f} = \frac{100 \times 1.732}{2 \times 3.14 \times 50} = 0.55 \text{ H}$$

99 (c) $X_L = \frac{100}{8} \, \Omega = 12.5 \, \Omega$ with 50 Hz frequency and with 40 Hz frequency.

$$X'_L = 12.5 \times \frac{40}{50} = 10 \, \Omega \quad [\text{as } X_L \propto \omega]$$

$$R = \frac{100}{10} = 10 \, \Omega$$

$$\therefore Z = \sqrt{R^2 + (X'_L)^2} = 10\sqrt{2} \, \Omega$$

$$I = \frac{100}{10\sqrt{2}} = 5\sqrt{2} \text{ A}$$

100 (d) Let $V = V_0 \sin \omega t$ [as $V = 0$ at $t = 0$]

Then $V_R = V_0 \sin \omega t$

$$\text{and } V_C = V_0 \sin (\omega t - \pi / 2)$$

V and V_R are in same phase. While V_C lags V (or V_R) by 90° .

Now V_R is in same phase with initial potential difference across the capacitor for the first time when,

$$\omega t = -\frac{\pi}{2} + 2\pi = \frac{3\pi}{2}$$

$$\therefore t = \frac{3\pi}{2\omega}$$

101 (b) At resonance, $X_C = X_L$

If ω is reduced to half,

$$X'_C = 2X_C$$

$$\left[\text{as } X_C \propto \frac{1}{\omega} \right]$$

$$\text{and } X'_L = \frac{X_L}{2}$$

$$[\text{as } X_L \propto \omega]$$

If ω is made two times,

$$X'_C = \frac{X_C}{2}$$

$$\text{and } X'_L = 2X_L$$

i.e. Value of Z will remain unchanged and current should be 1 A.

102 (b) In the given graph current is leading the voltage by 45° .

Therefore, circuit is C-R.

$$\tan \phi = \frac{X_C}{R} \text{ and } \phi = \frac{\pi}{4}$$

$$\therefore X_C = R$$

$$\Rightarrow \omega CR = 1$$

$$\Rightarrow CR = \frac{1}{\omega} = \frac{1}{100}$$

When $R = 1 \text{ k}\Omega = 10^3 \, \Omega$, then

$$C = \frac{1}{100 \times 10^3} = 10^{-5} \text{ F} = 10 \, \mu\text{F}$$

103 (a) In position-1,

$$\tan \frac{\pi}{6} = \frac{\omega L}{R} = \frac{(1000)(\sqrt{3} \times 10^{-3})}{R}$$

$$\text{or } \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{R}$$

$$\therefore R = 3 \, \Omega$$

In position-2,

$$\tan \phi = \frac{X_C}{R} = \frac{(1/\omega C)}{R}$$

$$= \frac{1}{\left(\frac{1000 \times \frac{1000}{3} \times 10^{-6}}{3} \right)} = 1$$

$$\therefore \phi = \frac{\pi}{4}; \text{ so current leads source emf.}$$

104 (b) For L-C, oscillator, $T = 2\pi\sqrt{LC}$

From one extreme to another; when electric field is maximum magnetic field is zero. And when electric field is zero, magnetic field is maximum.

At time $t = \frac{T}{4}$, energy stored is completely magnetic.

$$\therefore \text{Time, } t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4}$$

$$\Rightarrow t = \frac{\pi\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}}{2} = \frac{\pi\sqrt{1 \times 10^{-6}}}{2}$$

$$\Rightarrow t = 1.57 \text{ ms}$$

105 (b) P is a capacitor, $X_C = \frac{220}{0.25} = 880 \Omega$

Q is resistance, $R = \frac{220}{0.25} = 880 \Omega$

When P and Q are in series.

$$Z = \sqrt{R^2 + X_C^2}$$

$$= 880\sqrt{2} \Omega$$

$$\phi = \tan^{-1} \left(\frac{X_C}{R} \right) = \pi/4 \quad (\text{current leading})$$

$$\Rightarrow I = \frac{V}{Z} = \frac{1}{4\sqrt{2}} \text{ A}$$

106 (c) $X_L = \omega L = (2000)(5 \times 10^{-3}) = 10 \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{(2000)(50 \times 10^{-6})} = 10 \Omega$$

Since, $X_C = X_L$

$$\therefore I = \frac{V}{Z} = \frac{20/\sqrt{2}}{10} = \sqrt{2} \text{ A}$$

or ammeter reading $= \sqrt{2} \text{ A}$ and voltmeter reading $= 0$

107 (b) In L - C oscillation energy is transferred from C to L or L to C . Maximum energy in $L = \frac{1}{2}LI_{\max}^2$

and maximum energy in $C = \frac{q_{\max}^2}{2C}$.

Equal energy will be, when

$$\frac{1}{2}LI^2 = \frac{1}{2} \left(\frac{1}{2}LI_{\max}^2 \right)$$

$$I = \frac{1}{\sqrt{2}} I_{\max}$$

$$I = I_{\max} \sin \omega t = \frac{1}{\sqrt{2}} I_{\max}$$

$$\Rightarrow \omega t = \frac{\pi}{4}$$

$$\text{or } 2\frac{\pi}{T}t = \frac{\pi}{4} \quad \text{or } t = \frac{T}{8}$$

$$\Rightarrow t = \frac{1}{8} 2\pi\sqrt{LC} = \frac{\pi}{4}\sqrt{LC}$$

108 (b) Resistance of circuit,

$$R = \frac{V^2}{P} = \frac{110 \times 110}{330} = \frac{110}{3} \Omega$$

Ist case Power factor, $\cos \phi = 0.6$

Since, current lags the voltage, thus the circuit contains resistance and inductance.

$$\therefore \cos \phi = \frac{R}{\sqrt{R^2 + X_L^2}} = 0.6$$

$$\Rightarrow R^2 + X_L^2 = \left(\frac{R}{0.6} \right)^2$$

$$\Rightarrow X_L^2 = \frac{R^2}{(0.6)^2} - R^2$$

$$\Rightarrow X_L^2 = \frac{R^2 \times 0.64}{0.36}$$

$$\therefore X_L = \frac{0.8R}{0.6} = \frac{4R}{3} \quad \dots(i)$$

IInd case Now $\cos \phi = 1$

[given]

Therefore, circuit is purely resistive, i.e. it contains only resistance. This is the condition of resonance in which

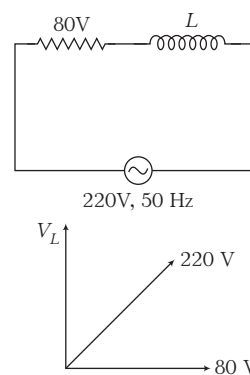
$$X_L = X_C$$

$$\therefore X_C = \frac{4R}{3} = \frac{4}{3} \times \frac{110}{3} = \frac{440}{9} \Omega \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \frac{1}{2\pi fC} = \frac{440}{9} \Omega$$

$$\therefore C = \frac{9}{2 \times 3.14 \times 60 \times 440} = 0.000054 \text{ F} = 54 \mu\text{F}$$

109 (c) $R = \frac{80}{10} = 8 \Omega$



$$\text{Also, } V_L^2 + 80^2 = 220^2$$

$$V_L^2 = (220 + 80)(220 - 80)$$

$$= 300 \times 140$$

$$\Rightarrow V_L = 204.9 \text{ V}$$

$$\Rightarrow I_{\text{rms}} X_L = 204.9$$

$$\Rightarrow \frac{220}{\sqrt{64 + X_L^2}} X_L = 204.9$$

$$\Rightarrow X_L = 20 \Rightarrow L = \frac{20}{2\pi f} = 0.064 \text{ H}$$

110 (d) $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$

$$R = 10 \Omega, X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}}$$

$$= 10 \Omega \Rightarrow Z = 10 \Omega$$

$$\text{Maximum current, } i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2 \text{ A}$$

$$\text{Hence, } i_{\text{rms}} = \frac{2}{\sqrt{2}} = 1.4 \text{ A}$$

$$\text{and } V_{\text{rms}} = 4 \times 1.4 = 5.6 \text{ V}$$

$$111 \text{ (b) } Z = \sqrt{R^2 + (X_C - X_L)^2} = 10 \Omega$$

$$\cos \phi = \frac{R}{Z} = \frac{4}{5}$$

$$\therefore \phi = 37^\circ$$

$$I_0 = \frac{V_0}{Z} = \frac{10}{10} = 1 \text{ A}$$

$$\therefore I = 1 \cos(100\pi t + 37^\circ)$$

$$\text{Given, } V = 10 \cos(100\pi t)$$

The applied voltage is $\frac{3}{5}$ th of the maximum applied voltage.

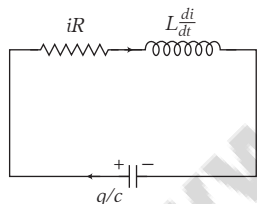
$$\text{Then, } 100\pi t = \cos^{-1}(3/5) = 53^\circ$$

$$\therefore I = 1 \cos(53^\circ + 37^\circ) = 0$$

$$\therefore V_R = 0$$

$$\therefore V_{AB} = V_{\text{applied}} = \frac{3}{5} \times 10 = 6 \text{ V}$$

$$112 \text{ (a) At any time } t \text{ apply KVL, } \frac{q}{C} - iR - L \frac{di}{dt} = 0$$



and

$$i = -\frac{dq}{dt}$$

$$\Rightarrow \frac{q}{C} + \frac{dq}{dt} R + \frac{L d^2 q}{dt^2} = 0$$

$$\Rightarrow \frac{d^2 q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{q}{LC} = 0$$

From damped harmonic oscillator, the amplitude is given by

$$A = A_0 e^{-\frac{bt}{2m}}, \text{ for general equation of double differential}$$

$$\text{equation } \frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

$$\Rightarrow Q_{\text{max}} = Q_0 e^{-\frac{Rt}{2L}}$$

$$\Rightarrow Q_{\text{max}}^2 = Q_0^2 e^{-\frac{Rt}{L}}$$

Lesser the self-inductance, faster will be damping, hence correct graph is in option (a).

(B) Medical entrance special format question

• Assertion and reason

$$1 \text{ (a) } Z_{\text{DC}} = r$$

$$Z_{\text{AC}} = \sqrt{r^2 + X_L^2}$$

Here, r = internal resistance of the coil.

$$\text{Also, } I \propto \frac{1}{Z}$$

$$\therefore I_1 > I_2$$

$$2 \text{ (d) If } V = V_0 \sin \omega t, \text{ then current function will lead the voltage function by } 90^\circ.$$

$$\therefore I = I_0 \cos \omega t$$

$\therefore$ Instantaneous power,

$$P = VI = V_0 I_0 \sin \omega t \cos \omega t$$

or

$$P \neq 0 \text{ at all times}$$

$$3 \text{ (d) } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

By inserting the ferromagnetic rod, L and therefore X_L will increase. But value of Z and therefore I may increase, decrease or remain same.

$$4 \text{ (b) At } X_L = X_C, \text{ the given frequency is resonance frequency.}$$

When frequency is doubled, $X_L = 2R$ [as $X_L \propto \omega$]

$$\text{and } X_C = \frac{R}{2} \quad (\text{as } X_C \propto 1/\omega)$$

$$\therefore Z = \sqrt{R^2 + \left(2R - \frac{R}{2}\right)^2} = \frac{\sqrt{13}}{2} R$$

$$5 \text{ (b) } P = V_{\text{rms}} I_{\text{rms}} \cos \phi = (I_{\text{rms}} Z) (I_{\text{rms}}) \left(\frac{R}{Z}\right) = I_{\text{rms}}^2 R$$

In one full cycle, the power is

$$P = P_R + P_L + P_C = P_R + 0 + 0 = P_R$$

i.e. Net power is dissipated only along resistor.

$$6 \text{ (d) At resonance, } \phi = 0$$

$$\therefore \cos \phi = 1 \text{ or power factor} = 1$$

Also, V and I are in same phase at resonance.

• Statement based questions

$$1 \text{ (b) At resonance, } V_L = V_C \text{ and } V = V_R.$$

$$\therefore V_1 = 200 \text{ V, } V_2 = V_3 = 100 \text{ V and } X_L = X_C$$

Here, resonant frequency is 50 Hz.

$$2 \text{ (b) } X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 5 \times 10^3 \times \frac{1}{\pi} \times 10^{-6}} = 100 \Omega$$

$$\text{Since, } X_C = R$$

$$\therefore I_C = I_R = \frac{20}{100} = 0.2 \text{ A}$$

$$\Rightarrow I = \sqrt{(0.2)^2 + (0.2)^2} = 0.283 \text{ A}$$

3 (d) $X_C = X_L$

$$\therefore Z = R = 2\Omega$$

$$\Rightarrow I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{100/\sqrt{2}}{2} = 25\sqrt{2} \text{ A}$$

Phase difference will be zero.

$$\begin{aligned} \text{Average power} &= V_{\text{rms}} I_{\text{rms}} \cos \phi = \frac{100}{\sqrt{2}} \times 25\sqrt{2} \times \cos 0^\circ \\ &= 2500 \text{ W} \end{aligned}$$

4 (d) $V = V_0 \sin \omega t$

$$q = CV = CV_0 \sin \omega t = q_0 \sin \omega t$$

$$i = \frac{dq}{dt} = q_0 \omega \cos \omega t = i_0 \cos \omega t$$

$\therefore$ As V increases from a to b , q increases and i decreases from maximum to zero value. Also, capacitor charges in this period. In b to c , the capacitor discharges.

5 (a) This is a parallel circuit and for oscillation, the energy in L and C will be alternately maximum. So, both bulbs will glow alternatively.

6 (c) $X_L = \omega L = 100 \times 0.5 = 50 \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100 \Omega$$



$$Z_1 = 100\sqrt{2} \Omega; I_1 = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}} \text{ A}$$

$$V \text{ across } 100\Omega = \frac{1}{5\sqrt{2}} \times 100 = \frac{20}{\sqrt{2}} = 10\sqrt{2} \text{ V}$$

Phase difference between I_1 and V ,

$$\cos \phi_1 = \frac{R_1}{Z_1} = \frac{100}{100\sqrt{2}}$$

$$\Rightarrow \phi = \pi/4$$

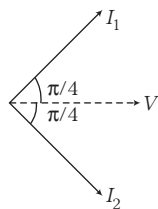
Therefore, I_1 leads V .



$$\text{Similarly, } Z_2 = 50\sqrt{2} \Omega; I_2 = \frac{20}{50\sqrt{2}} = \frac{2}{5\sqrt{2}} \text{ A}$$

$$V \text{ across } 50\Omega = \frac{2}{50\sqrt{2}} \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2} \text{ V and } \phi_2 = \pi/4$$

$\therefore I_2$ lags V by $\pi/4$.



$$\Rightarrow I_{\text{Net}} = \sqrt{I_1^2 + I_2^2}$$

$$I = \sqrt{\frac{1}{25 \times 2} + \frac{4}{25 \times 2}} = \sqrt{\frac{5}{50}} = \frac{1}{\sqrt{10}}$$

$$\Rightarrow I = 0.316 \text{ A}$$

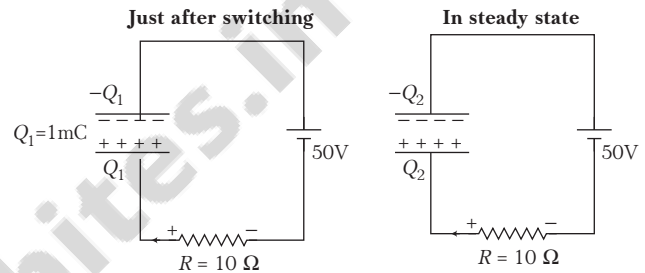
7 (d) Charge on capacitor will be maximum at $t = \frac{\pi}{2\omega}$.

$$Q_{\text{max}} = \int_0^{\pi/2\omega} \cos(500t) dt = \left(\frac{\sin 500t}{500} \right)_0^{\pi/2\omega} = \frac{1}{500} = 2 \times 10^{-3} \text{ C}$$

$$\text{Charge supplied by source from } t = 0 \text{ to } t = \frac{7\pi}{6\omega}$$

$$Q = \int_0^{\frac{7\pi}{6\omega}} \cos(500t) dt = \left[\frac{\sin 500t}{500} \right]_0^{\frac{7\pi}{6\omega}} = \frac{\sin \frac{7\pi}{6}}{500} = -1 \text{ mC}$$

i.e. Current in left part of the circuit is anti-clockwise.



Apply KVL just after switching,

$$50 + \frac{Q_1}{C} - IR = 0 \Rightarrow I = 10 \text{ A}$$

In steady state $Q_2 = 1 \text{ mC}$

Net charge flown from battery $Q = 2 \text{ mC}$ or $2 \times 10^{-3} \text{ C}$

8 (b) In RC series circuit, voltage across the capacitor (V_1) is ahead of the voltage across the resistance (V_2) by $\frac{\pi}{2}$.

• Match the columns

1 (b) Power factor, $\cos \phi = \frac{R}{Z}$

At resonance frequency, $X_L = X_C \Rightarrow \cos \phi = \frac{R}{R} = 1$

At frequency $>$ resonance frequency, $X_L > X_C$ as $X_L \propto \omega$.

2 (a) $Z = R$ [as $X_L = X_C$]

$$\therefore I = \frac{V}{Z} = \frac{V}{R}$$

Now,

$$V_R = IR = V$$

$$V_C = IX_C = V = V_L$$

$$V_{RL} = \sqrt{V^2 + V^2} = \sqrt{2} V$$

$$V_{CL} = V - V = 0$$

3 (d) As, $Z = \sqrt{R^2 + (X_L - X_C)^2} = 50 \Omega$

$$\therefore I = \frac{V_{\text{rms}}}{Z} = \frac{100\sqrt{2}/\sqrt{2}}{50} = \frac{100}{50} = 2 \text{ A}$$

$$P = I^2 R = 120 \text{ W}$$

$$V_R = IR = 60 \text{ V}$$

$$V_L = IX_L = 100 \text{ V}$$

(C) Medical entrances' gallery

- 1 (b) For L - C - R circuit, phase difference,

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

When L is removed, then $X_L = 0$.

$$\therefore \phi = \tan^{-1} \left(\frac{-X_C}{R} \right)$$

$$\Rightarrow \tan \phi = \left| \frac{X_C}{R} \right| \quad \dots(i)$$

When C is removed, then $X_C = 0$.

$$\therefore \phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

$$\Rightarrow \tan \phi = \frac{X_L}{R} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$X_L = X_C \quad (\because \phi = \pi/3)$$

$$\therefore \text{Impedance, } Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

$$\text{Power factor, } \cos \phi = \frac{R}{Z} = \frac{R}{R} = 1$$

- 2 (b) Given, $C = 40 \mu\text{F} = 40 \times 10^{-6} \text{ F}$

$$V_{\text{rms}} = 200 \text{ V}$$

$$v = 50 \text{ Hz}$$

$$\text{As, } X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 50 \times 40 \times 10^{-6}}$$

$$= \frac{10^6}{100\pi \times 40} = \frac{250}{\pi} \Omega$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{200}{\left(\frac{250}{\pi}\right)} = 2.5 \text{ A}$$

- 3 (a) When an iron rod is inserted into the interior of the inductor, then inductance (L) of the coil increases.

Hence, inductive reactance ($X_L = \omega L$) also increases.

$\therefore$ Current in the circuit is given as

$$I = \frac{V}{X_L}$$

Hence, when X_L increases, then current I decreases.

Therefore, glow of the light bulb will decrease.

- 4 (a) When the circuit is connected to AC source,

Voltage, $V = 12 \text{ V}$

Current, $I = 0.2 \text{ A}$

$$\therefore \text{Impedance, } Z = \frac{V}{I} = \frac{12}{0.2} = 60 \Omega$$

When it is connected to DC source,

Voltage, $V = 12 \text{ V}$

$$\text{Current, } I = 0.4 \text{ A} \Rightarrow \text{Resistance, } R = \frac{V}{I} = \frac{12}{0.4} = 30 \Omega$$

As in case of DC supply, the capacitor act as an open circuit and no current flows through the circuit. So, the given circuit will not have capacitor in series combination. Therefore, the circuit should be a series L - R circuit.

- 5 (b) Given, $L = 25 \text{ mH} = 25 \times 10^{-3} \text{ H}$

$$C = 10 \mu\text{F} = 10^{-5} \text{ F}$$

If T be the time period of L - C oscillation, then

$$T = 2\pi\sqrt{LC} = 2\pi\sqrt{25 \times 10^{-3} \times 10^{-5}}$$

$$= \pi \times 10^{-3} \text{ s} = \pi \text{ ms}$$

Current in the circuit will be maximum, when

$$t = \frac{T}{4} = \frac{\pi}{4} \text{ ms}$$

- 6 (a) Given, in L - C - R series circuit,

Source voltage (resultant voltage), $V = 120 \text{ V}$

Voltage across inductor, $V_L = 50 \text{ V}$

Voltage across resistor, $V_R = 40 \text{ V}$

In L - C - R series circuit,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\text{or } 120 = \sqrt{40^2 + (50 - V_C)^2}$$

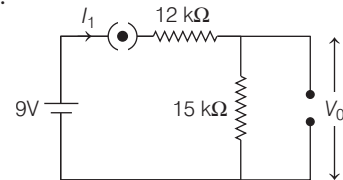
$$120^2 = 40^2 + (50 - V_C)^2$$

$$\Rightarrow (50 - V_C)^2 = 12800$$

$$\Rightarrow 50 - V_C = 80\sqrt{2}$$

$$\Rightarrow V_C = 10(5 - 8\sqrt{2})$$

- 7 (b) At $t \rightarrow \infty$, capacitor works as an open circuit as shown in the figure.



Therefore, current flow in the circuit,

$$I_1 = \frac{9}{(12 + 15) \times 10^3} = \frac{1}{3} \times 10^{-3} \text{ A}$$

$$\therefore V_0 = I_1 \times 15 \times 10^3$$

$$= \frac{1}{3} \times 10^{-3} \times 15 \times 10^3 = 5 \text{ V}$$

$\therefore$ Charge on capacitor of $5 \mu\text{F}$,

$$q_0 = CV_0 = 5 \times 10^{-6} \times 5$$

$$= 25 \times 10^{-6} \text{ C} = 25 \mu\text{C}$$

When switch is open, then capacitor starts discharging across $(5 + 15 = 20 \text{ k} \Omega)$ and instantaneous charge on capacitor is given by

$$q = q_0 e^{-t/RC}$$

$$= 25 e^{-\frac{1}{20 \times 10^3 \times 5 \times 10^{-6}}}$$

$$= 25 e^{-10} \mu\text{C}$$

$$[\because t = 1 \text{ s}]$$

- 8 (a) Given, turn ratio of a transformer,

$$\frac{N_1}{N_2} = \frac{50}{1}$$

$$\Rightarrow N_1 = 50N_2$$

Since, $\frac{N_1}{N_2} = \frac{V_1}{V_2}$

$$50 = \frac{120}{V_2}$$

$$[V_1 = 120 \text{ V (given)}]$$

$$\Rightarrow V_2 = \frac{12}{5} \text{ V}$$

Output power at secondary coil,

$$P_s = \frac{V_2^2}{R_2}$$

$$= \frac{\left(\frac{12}{5}\right)^2}{1} = \frac{144}{25}$$

$$= 5.76 \text{ W}$$

$$[R_2 = 1\Omega \text{ (given)}]$$

- 9 (a) In the given circuit diagram,

both inductors of inductance L are parallel to each other, hence

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} = \frac{L \cdot L}{L + L} = \frac{L}{2}$$

Similarly, $C_{eq} = C_1 + C_2 = C + C = 2C$

$\therefore$ Resonance frequency,

$$\omega_r = \frac{1}{\sqrt{L_{eq} \cdot C_{eq}}} = \sqrt{\frac{1}{\frac{L}{2} \cdot 2C}} = \sqrt{\frac{1}{LC}} = \frac{1}{\sqrt{LC}}$$

- 10 (c) Given, inductance, $L = 20 \text{ mH}$

$$= 20 \times 10^{-3} \text{ H}$$

Capacitance, $C = 100 \mu\text{F} = 100 \times 10^{-6} \text{ F}$

Resistance, $R = 50 \Omega$ emf, $V = 10 \sin 314t$... (i)

$\therefore$ The general equation of emf is given as $V = V_0 \sin \omega t$... (ii)

On comparing Eqs. (i) and (ii), we get

$$V_0 = 10 \text{ V}$$

$$\omega = 314 \text{ rad s}^{-1}$$

The power loss associated with the given AC circuit is given as

$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$= V_{\text{rms}} \left(\frac{V_{\text{rms}}}{Z} \right) \left(\frac{R}{Z} \right)$$

$$= \left(\frac{V_{\text{rms}}}{Z} \right)^2 R = \left(\frac{V_0}{\sqrt{2} \cdot Z} \right)^2 R \quad \dots \text{(iii)}$$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

Substituting the given values in the above equation, we get

$$Z = \sqrt{(50)^2 + \left[(314 \times 20 \times 10^{-3}) - \frac{1}{314 \times 10^{-4}} \right]^2}$$

$$= \sqrt{2500 + [6280 \times 10^{-3} - 0.00318 \times 10^4]^2}$$

$$= \sqrt{2500 + (-25.56)^2}$$

$$= 56.15 \Omega \approx 56 \Omega$$

Now, substituting this values in Eq. (iii), we get

$$P = \left(\frac{10}{\sqrt{2} \times 56} \right)^2 \times 50 = \frac{100}{2 \times 3136} \times 50 = 0.79 \text{ W}$$

Thus, the power loss in the circuit is 0.79 W.

- 11 (c) In an L - C - R circuit,

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

ϕ being the angle by which the current leads the voltage.

Given, $\phi = 45^\circ$

$$\therefore \tan 45^\circ = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow 1 = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow R = \omega L - \frac{1}{\omega C}$$

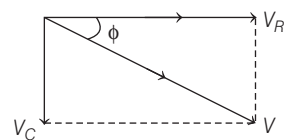
$$\Rightarrow \omega C = \frac{1}{\omega L - R}$$

$$\Rightarrow C = \frac{1}{\omega (\omega L - R)}$$

$$= \frac{1}{2\pi f (2\pi f L - R)}$$

- 12 (a) We know that, for series R - C circuit

$$V^2 = V_C^2 + V_R^2$$



$$100 = 64 + V_R^2$$

$$\Rightarrow V_R^2 = 36$$

$$\Rightarrow V_R = \sqrt{36} = 6 \text{ V}$$

Also, $\tan \phi = \frac{V_C}{V_R} \Rightarrow \tan \phi = \frac{8}{6}$

$$\tan \phi = \frac{4}{3}$$

$$\therefore \phi = \tan^{-1} \left(\frac{4}{3} \right)$$

- 13** (d) Rise of current in L - R circuit is given by

$$I = I_0(1 - e^{-t/\tau})$$

where, $I_0 = \frac{E}{R} = \frac{5}{5} = 1 \text{ A}$

Now, $\tau = \frac{L}{R} = \frac{10}{5} = 2 \text{ s}$

After 2 s, i.e. at $t = 2 \text{ s}$

Rise of current, $I = (1 - e^{-1}) \text{ A}$

- 14** (b) For circuit A,

Impedance, $Z_A = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

Current in circuit, $I_R^A = \frac{V}{\sqrt{R^2 + (1/\omega^2 C^2)}} \quad \dots(i)$

Potential difference across C,

$$V_C^A = I_R^A \times \frac{1}{\omega C} = \frac{V}{\sqrt{(R\omega C)^2 + 1}}$$

For circuit B,

Impedance, $Z_B = \sqrt{R^2 + \frac{1}{(4\omega)^2 C^2}}$

Current in circuit, $I_R^B = \frac{V}{\sqrt{R^2 + [1/(4\omega C)^2]}}$

Potential difference across C,

$$V_C^B = I_R^B \times \frac{1}{4\omega C} = \frac{V}{\sqrt{(4R\omega C)^2 + 1}} \quad \dots(ii)$$

Hence, from above Eqs. (i) and (ii), we conclude that as numerator of I_R^B is increased from I_R^A by a factor of 2.

So, $I_R^B > I_R^A$ and $V_C^A > V_C^B$.

- 15** (c) If a rated voltage and power are given,

then $P_{\text{rated}} = \frac{V_{\text{rated}}^2}{R} \quad \dots(i)$

$\therefore$ Current in the bulb, $I = \frac{P}{V} \quad [\because P = VI]$

$$I = \frac{500}{100} = 5 \text{ A}$$

$\therefore$ Resistance of bulb, $R_b = \frac{100 \times 100}{500} = 20 \Omega \quad [\text{from Eq. (i)}]$

$\therefore$ Resistance R is connected in series.

$\therefore$ Current, $I = \frac{E}{R_{\text{net}}} = \frac{230}{R + R_b}$

$$\Rightarrow R + 20 = \frac{230}{5} = 46$$

$\therefore R = 26 \Omega$

- 16** (c) For better tuning, peak of current growth must be sharp.

This is ensured by a high value of quality factor Q .

Now, quality factor is given by $Q = \frac{1}{R} \sqrt{\frac{L}{C}}$

From the given options highest value of Q is associated with $R = 15 \Omega$, $L = 3.5 \text{ H}$ and $C = 30 \mu\text{F}$.

- 17** (c) Power factor of the L - C - R circuit,

$$\begin{aligned} \cos \phi &= \frac{R}{Z} = \frac{IR}{IZ} = \frac{80}{I \sqrt{(X_L - X_C)^2 + R^2}} \\ &= \frac{80}{\sqrt{(IX_L - IX_C)^2 + (IR)^2}} \\ &= \frac{80}{\sqrt{(100 - 40)^2 + (80)^2}} \\ &= \frac{80}{\sqrt{(60)^2 + (80)^2}} = \frac{80}{100} = 0.8 \end{aligned}$$

- 18** (a) Impedance of the R - C circuit, $Z = \sqrt{R^2 + X_C^2}$

where, $R = 100 \Omega$ and $X_C = 100 \Omega$

$$\Rightarrow Z = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2} \Omega$$

Peak value of the current,

$$I_{\text{max}} = \frac{V_{\text{max}}}{Z} = \frac{V_{\text{rms}} \sqrt{2}}{Z} = \frac{220\sqrt{2}}{100\sqrt{2}} = 2.2 \text{ A}$$

- 19** (a) For an AC circuit containing capacitor only, the phase difference between current and voltage will be $\frac{\pi}{2}$ (i.e. 90°).

In this case, current is ahead of voltage by $\frac{\pi}{2}$.

Hence, power in this case is given by

$$P = VI \cos \phi$$

(ϕ = phase difference between voltage and current)

$$P = VI \cos 90^\circ = 0$$

- 20** (d) Given, inductance, $L = 20 \text{ mH}$

Capacitance, $C = 50 \mu\text{F}$

Resistance, $R = 40 \Omega$

$$\text{emf, } V = 10 \sin 340 t$$

$\therefore$ Power loss in AC circuit will be given as

$$\begin{aligned} P_{\text{av}} &= I_{\text{rms}}^2 R = \left[\frac{V_{\text{rms}}}{Z} \right]^2 \cdot R \\ &= \left(\frac{10}{\sqrt{2}} \right)^2 \cdot 40 \left[\frac{1}{40^2 + \left(340 \times 20 \times 10^{-3} - \frac{1}{340 \times 50 \times 10^{-6}} \right)^2} \right] \\ &= \frac{100}{2} \times 40 \times \frac{1}{1600 + (6.8 - 58.8)^2} \\ &= \frac{2000}{1600 + 2704} \approx 0.46 \text{ W} \approx 0.51 \text{ W} \end{aligned}$$

- 21** (a) When a resistor is connected to an AC source. The power drawn will be

$$P = V_{\text{rms}} \cdot I_{\text{rms}} = V_{\text{rms}} \cdot \frac{V_{\text{rms}}}{R} \Rightarrow V_{\text{rms}}^2 = PR$$

When an inductor is connected in series with the resistor, then the power drawn will be

$$P' = V_{\text{rms}} \cdot I_{\text{rms}} \cos \phi$$

where, ϕ = phase difference

$$\therefore P' = \frac{V_{\text{rms}}^2}{R} \cdot \frac{R^2}{Z^2} = P \cdot R \cdot \frac{R}{Z^2}$$

$$\Rightarrow P' = \frac{P \cdot R^2}{Z^2} = P \left(\frac{R}{Z} \right)^2$$

22 (c) The resonant frequency is given by,

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

where, R = resistance and L = inductance

$$\begin{aligned} \Rightarrow f_0 &= \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \\ &= \frac{1}{2 \times 3.14} \sqrt{\frac{1}{250 \times 10^{-12} \times 0.16 \times 10^{-3}} - \frac{20 \times 20}{(0.16 \times 10^{-3})^2}} \\ &= 8 \times 10^5 \text{ Hz} \end{aligned}$$

23 (a) The resistance of inductor (coil), $R = \frac{12}{24} = 0.5 \Omega$

If phase difference between current and emf is zero, then reactance of circuit must be zero.

The impedance of the circuit equals to resistance, i.e.

$$Z = 0.5 \Omega$$

$$\text{Now, rms current, } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{24}{0.5} = 48 \text{ A}$$

24 (b) Given, output power = 100 W

$$V = 220 \text{ V, } I = 0.5 \text{ A}$$

$$\text{We know that, efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

$$= \frac{100}{220 \times 0.5} \times 100 = 90\%$$

25 (b) Given, $V = 100\sqrt{2} \sin 100t \text{ V}$

$$C = 1 \mu\text{F}$$

The peak voltage, $V_0 = 100\sqrt{2} \text{ V}$ and $\omega = 100 \text{ rad/s}$

$$\therefore \text{rms voltage, } V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

$\therefore$ Current reading of the ammeter,

$$\begin{aligned} I &= \frac{V_{\text{rms}}}{X_C} = \frac{V_{\text{rms}}}{\left(\frac{1}{\omega C} \right)} = \frac{100 \text{ V}}{\frac{1}{100 \times 1 \times 10^{-6}}} \\ &= 100 \times 100 \times 10^{-6} = 10^{-2} \text{ A} \\ &= 10 \times 10^{-3} \text{ A} = 10 \text{ mA} \end{aligned}$$

26 (a) The current through the AC circuit is given by

$$i = 3 \sin \omega t + 4 \cos \omega t$$

The maximum value of current is

$$I_{\text{max}} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ A}$$

$$\text{The rms value of current, } I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ A}$$

27 (d) We know that,

$$\text{In } L\text{-C-R circuit, } f = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

For the same resonant frequency, the inductance should be changed from L to $\frac{L}{4}$.

28 (c) In the inductor circuit, phase difference (ϕ) = 90° or $\frac{\pi}{2}$

$$\begin{aligned} \text{We know that, } P_{\text{av}} &= V_{\text{rms}} I_{\text{rms}} \cos \phi \\ &= V_{\text{rms}} I_{\text{rms}} \cos \frac{\pi}{2} \end{aligned}$$

$$\therefore P_{\text{av}} = 0$$

29 (b) Initial power = 3 kW = 3000 W

As efficiency is 90%, then final power

$$= 3000 \times \frac{90}{100} = 2700 \text{ W}$$

$$\Rightarrow P_1 = V_1 I_1 = 3000 \text{ W} \quad \dots(i)$$

$$P_2 = V_2 I_2 = 2700 \text{ W} \quad \dots(ii)$$

$$\text{So, } V_2 = \frac{2700}{6} = \frac{900}{2} = 450 \text{ V}$$

$$\text{and } I_1 = \frac{3000}{200} = 15 \text{ A}$$

30 (d) A dynamo converts mechanical energy into electrical energy.

31 (d) Transformer is used to obtain desired AC voltage and current because a transformer is a device based on the principle of mutual induction, which is used for converting large AC at low voltage into small current at high voltage.

32 (b) As we know that, for transformer, $\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$

where all the symbols have their usual meanings.

(Suffix 1 is for primary and 2 is for the secondary)

$$\Rightarrow \frac{N_1}{N_2} = \frac{1}{25} = \frac{2}{I}$$

where, I = primary side current.

$$\therefore I = 50 \text{ A}$$

33 (b) We know that,

$$V_S \times N_P = V_P \times N_S$$

$$\Rightarrow \frac{N_S}{N_P} = \frac{V_S}{V_P}$$

where, V_S = voltage across secondary coil

V_P = voltage across primary coil = 220 V

N_P = number of turns in primary coil = 1000

and N_S = number of turns in secondary coil = 50

$$\Rightarrow \frac{50}{1000} = \frac{V_S}{220}$$

$$\Rightarrow V_S = \frac{50 \times 220}{1000} = 11 \text{ V}$$

Power in the secondary coil = Power in the primary coil

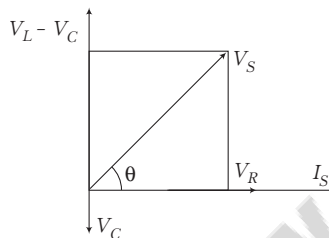
$$\begin{aligned} V_S I_S &= V_P I_P \\ \Rightarrow 11 \times I_S &= 220 \times 1 \\ \Rightarrow I_S &= \frac{220}{11} = 20 \text{ A} \end{aligned}$$

34 (d) Average power, $P_{av} = E_{rms} I_{rms} \cos \phi$

$$\begin{aligned} &= \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi \\ &= \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} \cos \frac{\pi}{3} \\ &= \frac{100 \times 100}{2} \times 10^{-3} \times \frac{1}{2} = 2.5 \text{ W} \end{aligned}$$

35 (a) As average power, $\bar{P} = 2 \text{ W}$, $I_{rms} = 2 \text{ A}$,
impedance, $Z = 1 \Omega$
Power factor ($\cos \phi$) = ?
We know that, $\bar{P} = V_{rms} \times I_{rms} \cdot \cos \phi$
 $\Rightarrow \bar{P} = I_{rms}^2 Z \cdot \cos \phi$ [$\because V_{rms} = I_{rms} \cdot Z$]
 $\cos \phi = \frac{\bar{P}}{I_{rms}^2 \cdot Z} = \frac{2}{(2)^2 \times 1} = \frac{2}{4} = \frac{1}{2} = 0.5$

36 (b) Given, $V_L = 60 \text{ V}$, $V_C = 30 \text{ V}$, $V_R = 40 \text{ V}$
In phasor diagram, V_C and V_L are in anti-phase to each other due to their 90° leading and lagging relationship with the circuit current I_S .



$$\begin{aligned} \text{So, } V_S &= \sqrt{(V_L - V_C)^2 + V_R^2} = \sqrt{(60 - 30)^2 + (40)^2} \\ &= \sqrt{(30)^2 + (40)^2} \\ &= \sqrt{900 + 1600} \\ &= \sqrt{2500} = 50 \text{ V} \end{aligned}$$

37 (a) The given equation, $e = 300 \sin[(100\pi)t] \text{ V}$... (i)
We know that, $e = e_0 \sin \omega t$... (ii)
On comparing Eqs. (i) and (ii), we get
 $e_0 = 300 \text{ V}$ and $R = 100 \Omega$ (given)
The rms current through the circuit,

$$I_{rms} = \frac{e_0}{\sqrt{2}R} = \frac{300}{\sqrt{2} \times 100} = \frac{3}{\sqrt{2}} \text{ A}$$

38 (d) We know that,
Resonant frequency is given by
 $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5 \times 10^{-3} \times 2 \times 10^{-6}}}$

$$\begin{aligned} \Rightarrow f_0 &= \frac{1}{2\pi\sqrt{10^{-8}}} = \frac{1}{2\pi \times 10^{-4}} \\ &= \frac{10^4}{2\pi} \text{ Hz} = \frac{5}{\pi} \times 10^3 \text{ Hz} \end{aligned}$$

39 (d) As, $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi f v L)^2}$

As $I = \frac{V}{Z}$ and $P = I^2 R$

When iron rod is inserted, the value of L increases,
i.e. $L \uparrow \Rightarrow Z \uparrow, I \downarrow$ and $P \downarrow$

40 (c) Impedance, $Z = \sqrt{R^2 + X_L^2}$, $X_L = \omega L = 2\pi f L$

Given, $R = 2100 \Omega$, $f = 50 \text{ Hz}$, $L = \frac{1}{\pi} \text{ H}$

$$\begin{aligned} \Rightarrow Z &= \sqrt{(2100)^2 + (2 \times 50)^2} \\ &= \sqrt{(2100)^2 + (100)^2} \\ \Rightarrow Z &= 2102 \Omega \end{aligned}$$

42 (b) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{2\pi v L}{R}\right)$
 $= \tan^{-1}\left(\frac{2\pi \times 50 \times 0.01}{\sqrt{3} \pi}\right)$
 $= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} \text{ rad}$

43 (b) From standard equation, we have

$$I = I_0 \sin \omega t \quad \dots (i)$$

Given, $I = 50 \sin 314 t$... (ii)

Comparing Eqs. (i) and (ii), we get

$$I_0 = 50 \text{ A}, \omega = 2\pi f = 314 \text{ rads}^{-1}$$

$$\Rightarrow f = \frac{314}{2 \times 3.14} = 50 \text{ Hz}$$

44 (b) $\frac{N_S}{N_P} = 3$

$$\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

$$V_S = V_P \cdot \frac{N_S}{N_P} = 100 \times 3 = 300 \text{ V}$$

Given, $\frac{V_S I_S}{V_P I_P} = 0.75$

$$\Rightarrow \frac{300 \times I_S}{100 \times 2} = 0.75$$

$$\Rightarrow I_S = 0.5 \text{ A}$$

45 (a) Given, the potential difference across each element is same, i.e. 20 V , so the circuit is at resonance.

We have, $V_R = V_L = V_C = 20 \text{ V}$

If the value of R is doubled, then value of I reduced to half

$$\therefore V_L = V_C = \frac{IX_L}{2} = \frac{IX_C}{2} = \frac{20}{2} = 10 \text{ V}$$

[$\because$ circuit is at resonance]

and $V_R = I \times 2R = 20 \text{ V}$

$$46 \text{ (a)} \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \quad \dots(i)$$

$$I'_{\text{rms}} = \frac{I_{\text{rms}}}{2} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \left[\left(\frac{1}{\omega C / 3}\right)\right]^2}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + \frac{9}{\omega^2 C^2}}} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$3R^2 = \frac{5}{\omega^2 C^2}$$

$$\frac{1}{\omega C \times R} = \sqrt{\frac{3}{5}}$$

or $\frac{X_C}{R} = \sqrt{\frac{3}{5}} \text{ or } \frac{X_C}{R} = \sqrt{0.6}$

$$47 \text{ (c)} \text{ For } L\text{-}R \text{ circuit, } Z = \sqrt{R^2 + X_L^2}$$

Here, $Z_1 = \sqrt{(R_1)^2 + (X_{L_1})^2}$

and $Z_2 = \sqrt{(R_2)^2 + (X_{L_2})^2}$

Given, $R_2 = 2R_1 \text{ and } X_{L_2} = 2X_{L_1}$
 $Z_2 = \sqrt{(2R_1)^2 + (2X_{L_1})^2} = 2Z_1$

$$48 \text{ (d)} \text{ Given, } L = 1.00 \text{ mH, } C = 1.0 \mu\text{F, } R = 50 \Omega \text{ and } V = 5 \text{ V}$$

At resonance, $I = \frac{V}{R} = \frac{5}{50} = 0.1 \text{ A}$

$$49 \text{ (d)} \text{ We know that for series } L\text{-}C\text{-}R \text{ circuit,}$$

$$V_{\text{rms}} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Given, $V_L = 90 \text{ V, } V_C = 60 \text{ V}$

and $V_R = 40 \text{ V}$

Now, $V_{\text{rms}} = \sqrt{40^2 + (90 - 60)^2} = 50 \text{ V}$

$$50 \text{ (c)} \text{ Power factor, } \cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$51 \text{ (a)} \text{ From transformer ratio,}$$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

$$\Rightarrow V_S = \frac{V_P \times N_S}{N_P} = \frac{220 \times 40000}{200} = 44000 \text{ V}$$

Potential difference per turn is $\frac{V_S}{N_S} = \frac{44000}{40000} = 1.1 \text{ V}$

$$52 \text{ (a)} \quad \eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\Rightarrow \frac{80}{100} = \frac{20 \times 120}{1000 \times I}$$

$$\Rightarrow I = \frac{20 \times 120 \times 100}{1000 \times 80} = 3 \text{ A}$$

$$53 \text{ (d)} \text{ For inductor, as we know, induced voltage leads the current.}$$

For $t = 0$ to $t = T/2$,

$$V = L \frac{dI}{dt} = L \frac{d}{dt} \left(\frac{2I_0 t}{T} \right) = \text{constant}$$

For $t = T/2$ to $t = T$,

$$V = \frac{L dI}{dt} \left(\frac{-2I_0 t}{T} \right) = -\text{constant}$$

Therefore, answer will be represented by graph (d).

$$54 \text{ (b)} \text{ Power factor, } \cos \phi = \frac{1}{2} \Rightarrow \phi = 60^\circ$$

$$\therefore \tan 60^\circ = \frac{\omega L}{R} \Rightarrow \sqrt{3} = \frac{2\pi v \times L}{100}$$

$$L = \frac{\sqrt{3} \times 100}{2\pi \times 50} = \frac{\sqrt{3}}{\pi} \text{ H}$$

$$55 \text{ (a)} \text{ With DC power, } P = \frac{V^2}{R}$$

$$R = \frac{(10)^2}{20} = 5 \Omega$$

With AC power, $P = \frac{V_{\text{rms}}^2 R}{Z^2}$

$$\Rightarrow Z^2 = \frac{(10)^2 \times 5}{10} = 50 \Omega$$

The impedance, $Z^2 = R^2 + 4\pi^2 v^2 L^2$

$$50 = (5)^2 + 4(3.14)^2 \times v^2 \times (10 \times 10^{-3})^2$$

$$v \approx 80 \text{ Hz}$$

$$56 \text{ (c)} \text{ Input power} = VI = 200 \times 10$$

$$= 2000 \text{ W}$$

Output power = Efficiency $\times$ Input power

$$= \frac{40}{100} \times 2000 = 800 \text{ W}$$

Power loss in heating the armature

$$= 2000 - 800 = 1200 \text{ W}$$

$$\therefore I^2 R = 1200$$

$$\text{or } R = \frac{1200}{I^2} = \frac{1200}{10 \times 10}$$

$$\text{or } R = 12 \Omega$$

$$57 \text{ (c)} \text{ Emf of AC source, } E = E_0 \sin(100t)$$

We have, $\omega = 100 \text{ rad/s}$ and $\phi = \frac{\pi}{4} = 45^\circ$

$$\tan \phi = \frac{\omega L}{R} \Rightarrow 1 = \frac{100 \times L}{R}$$

$$R = 100 L$$

If $L = 10 \text{ H}$, then $R = 1 \text{ k}\Omega$

$$58 \text{ (a)} \text{ We have, } X_L = 90 \Omega, X_C = 30 \Omega$$

and $R = R_1 + R_2 = 44 + 36 = 80 \Omega$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(80)^2 + (60)^2}$$

$$Z = \sqrt{6400 + 3600} = 100 \Omega$$

$$I = \frac{V}{Z} = \frac{200}{100} = 2 \text{ A}$$

$$P_{\text{av}} = I^2 R = (2)^2 \times 80 = 320 \text{ W}$$

- 59** (c) For step-up transformer, $V_S > V_P$ and $I_S < I_P$

For an ideal transformer,

$$V_S I_S = V_P I_P$$

$$\therefore 240000 I_S = 100 \times 4000$$

$$\text{or } I_S = 1.67 \text{ A}$$

- 60** (a) We know that, $I_{WL} = I_{rms} \sin \phi$

$$\Rightarrow \sqrt{3} = 2 \sin \phi \text{ or } \sin \phi = \frac{\sqrt{3}}{2} \Rightarrow \phi = 60^\circ$$

$$\text{So, power factor} = \cos \phi = \cos 60^\circ = \frac{1}{2}$$

- 61** (d) Quantity of heat liberated in the ammeter of resistance R

(i) due to direct current of 3 A = $(3)^2 R$ W

(ii) due to alternating current of 4 A = $(4)^2 R$ W

$\therefore$ Total heat produced per second

$$= (3)^2 R + (4)^2 R = 25R$$

Let the equivalent alternating current be I virtual ampere, then

$$I^2 R = 25R \Rightarrow I = 5 \text{ A}$$

- 62** (c) Given, $V = 150 \sin(150t)$ V

and $I = 150 \sin(150t + \pi/3)$ A

$$\therefore I_0 = 150 \text{ A and } V_0 = 150 \text{ V}$$

$$\text{Power, } P = \frac{1}{2} V_0 I_0 \cos \phi$$

$$= 0.5 \times 150 \times 150 \times \cos 60^\circ$$

$$= 5625 \text{ W}$$

- 63** (c) $V = V_0 \sin \omega t \Rightarrow V = V_{rms} \sqrt{2} \sin \omega t$

$$\text{After, } t = \frac{1}{720} \text{ s}$$

$$V = 120\sqrt{2} \sin 2\pi f t$$

$$= 120\sqrt{2} \sin \left(2\pi \times 60 \times \frac{1}{720} \right)$$

$$= 120\sqrt{2} \sin \frac{\pi}{6} = 120\sqrt{2} \times \frac{1}{2}$$

$$= 60\sqrt{2} = 84.8 \text{ V}$$

- 64** (a) Alternating current is transmitted at far off places at high voltage and low current.

- 65** (b) We have, $\frac{N_s}{N_p} = \frac{i_p}{i_s} \Rightarrow \frac{10}{500} = \frac{i_p}{i_s}$

$$\Rightarrow \frac{i_p}{i_s} = \frac{1}{50} \Rightarrow i_s = 50i_p$$

This condition is satisfied only when current in primary is 3.2×10^{-3} A and in secondary is 0.16 A.

CHAPTER 08

Electromagnetic Waves

We have learnt that an electric current produces magnetic field and a magnetic field changing with time gives rise to an electric field. This brought together the phenomena of electricity and magnetism into a coherent and unified theory. After this discovery, Maxwell in 1865, pointed out that a change in either electric or magnetic field with time produces the other field. From this Maxwell concluded that variation of electric and magnetic field vectors perpendicular to each other leads to the production of electromagnetic disturbances in space. These disturbances displayed properties of waves and can travel in space even without any material medium. These waves are called **electromagnetic waves**.

Maxwell also concluded that electromagnetic waves are transverse in nature and that light is an electromagnetic wave. In this chapter, we will study characteristics of these waves and electromagnetic spectrum followed by the concept of **displacement current** and **Maxwell's equations**.

DISPLACEMENT CURRENT

While applying the Ampere's circuital law to find magnetic field at a point outside a capacitor connected to a time varying current, Maxwell noticed an inconsistency in Ampere's circuital law. So, he suggested the existence of an additional current and called it **displacement current**.

It is defined as the, "current which is produced due to the rate of change of electric flux with respect to time". It is given by

$$I_d = \epsilon_0 \frac{d}{dt} (\phi_E)$$

To evaluate this relation of displacement current, consider a parallel plate capacitor which is being charged as shown in the Fig. 8.1.

According to Maxwell, contradiction in Ampere's circuital law is because of some missing term. This missing term must be such that one gets the same magnetic fields at point P , no matter what surface is used.

Inside

1 Displacement current

- Maxwell's equations

2 Electromagnetic waves

- Physical quantities associated with EM waves

3 Electromagnetic spectrum

Also, this missing term must be related to changing electric field which passes through area A_1 between the plates of capacitor as shown in the figure.

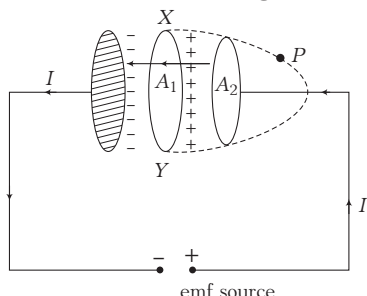


Fig. 8.1 A parallel plate capacitor with an emf source

If at an instant charge on the plates of capacitor is q , area of plate is A , then electric field between the plates is uniform, which is given by

$$E = \frac{q}{A\epsilon_0} \quad \dots(i)$$

(this field is perpendicular to area A_1)

Electric field outside the plates is zero.

The electric flux through area A_1 ,

$$\phi_E = E \times A_1 = \frac{q}{A\epsilon_0} \times A$$

($A_1 = A$, because flux will pass through area A)

$$\therefore \phi_E = \frac{q}{\epsilon_0} \quad \dots(ii)$$

If charge on the capacitor plate changes with time, then

$$I = \frac{dq}{dt}$$

So,

$$\frac{d\phi_E}{dt} = \frac{1}{\epsilon_0} \times \frac{dq}{dt}$$

This implies that, $\frac{dq}{dt} = \epsilon_0 \frac{d\phi_E}{dt} \Rightarrow I_d = \epsilon_0 \frac{d\phi_E}{dt}$

This current I_d (the missing term in the Ampere's circuital law) passes through the surface A_1 and is known as

Maxwell's displacement current.

Hence, Ampere's circuital law $\left(\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \right)$ was

modified to $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_c + I_d)$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_c + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

It is called **modified Ampere's circuital law** or **Ampere-Maxwell's circuital law.**

where, I_c = conduction current,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} = \text{displacement current}$$

and ϕ_E = flux of the electric field through the area bounded by closed curve.

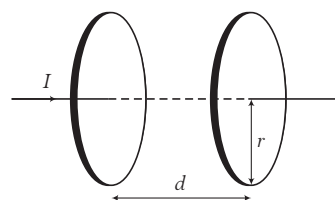
The inferences that can be drawn from the above discussion are as given below

- (i) The conduction and displacement currents are individually discontinuous but the currents together possess the property of continuity through any closed electric circuit.
- (ii) The displacement current is precisely equal to the conduction current when both are present in different parts of the circuit.
- (iii) The displacement current arises due to rate of change of electric flux (or electric field) between the two plates of the capacitor.
- (iv) Just as the conduction current, the displacement current is also the source of varying magnetic field.

Example 8.1 Figure shows a capacitor made of two circular plates each of radius 12 cm and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.

(i) Obtain the displacement current across the plates.

(ii) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.



Sol. (i) As displacement current across the plates,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

where, ϕ_E = electric flux across the loop.

$$\phi_E = EA = \frac{q}{\epsilon_0}$$

$$\therefore I_d = \epsilon_0 \frac{d}{dt} \left(\frac{q}{\epsilon_0} \right) = \frac{\epsilon_0}{\epsilon_0} \cdot \frac{dq}{dt} = I = 0.15 \text{ A}$$

i.e. displacement current is equal to conduction current.

(ii) Yes, Kirchhoff's first rule is valid at each plate of the capacitor. As, net current is equal to the sum of the conduction current and the displacement current.

Example 8.2 A parallel plate capacitor consists of two circular plates of radius $R = 0.1 \text{ m}$. They are separated by a short distance. If electric field between the capacitor plates changes as $\frac{dE}{dt} = 6 \times 10^{13} \frac{\text{V}}{\text{m} \times \text{s}}$. Find displacement current between the plates.

Sol. Given, area of plates, $A = \pi R^2 = 3.14 \times (0.1)^2 \text{ m}^2$

$$\text{and } \frac{dE}{dt} = 6 \times 10^{13} \frac{\text{V}}{\text{m} \times \text{s}}$$

Displacement current between the plates,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

$$I_d = \epsilon_0 A \frac{dE}{dt} \quad \left(\because \frac{d\phi_E}{dt} = A \frac{dE}{dt} \right)$$

$$= 8.85 \times 10^{-12} \times 3.14 \times (0.1)^2 \times 6 \times 10^{13}$$

$$\Rightarrow I_d = 16.67 \text{ A}$$

Example 8.3 Consider a parallel plate capacitor is charged by a constant current I , with plate area A and separation between the plates d . Consider a plane surface of area $A/4$ parallel to the plates and drawn symmetrically between the plates. Find the displacement current through this area.

Sol. Charge on the capacitor plates at time t is given by

$$q = It$$

Electric field between the plates of the capacitor at this instant,

$$E = \frac{q}{A\epsilon_0} = \frac{It}{A\epsilon_0}$$

So, electric flux through the given area,

$$\phi_E = \left(\frac{A}{4} \right) \cdot E = \frac{It}{4\epsilon_0}$$

Therefore, displacement current,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt} \left(\frac{It}{4\epsilon_0} \right) = \frac{I}{4}$$

Maxwell's equations

Maxwell's equations are the basic laws of electricity and magnetism. These equations give complete description of all electromagnetic interactions. Maxwell on the basis of his equations, predicted the existence of electromagnetic waves. These equations are as fundamental to electromagnetic phenomenon as Newton's law are for the study of mechanical phenomenon.

There are four Maxwell's equations which are explained as given below

1. Gauss's law in electrostatics

This law states that "the total electric flux through any closed surface is always equal to $\frac{1}{\epsilon_0}$ times the net charge

enclosed by that surface". This law relates electric field with a charge distribution.

It is given by

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$$

This equation is called **Maxwell's first equation**.

2. Gauss's law in magnetism

This law states that "the net magnetic flux through any closed surface is always zero".

It is given by

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

This equation is called **Maxwell's second equation**.

3. Faraday's law of electromagnetic induction

This law states that "the line integral of electric field along a closed path is equal to the rate of change of magnetic flux through the surface bounded by that closed path".

It is given by

$$\oint \mathbf{E} \cdot d\mathbf{l} = - \frac{d\phi_B}{dt}$$

This equation is called **Maxwell's third equation**.

4. Ampere-Maxwell's circuital law

This law states that "the line integral of the magnetic field along a closed path is equal to μ_0 times the total current (i.e. sum of conduction current and displacement current) threading the surface bounded by that closed path." It is given by

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_c + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

This equation is called **Maxwell's fourth equation**.

Example 8.4 A parallel plate capacitor of area 40 cm^2 and plate separation 3.0 mm is charged initially to $80 \mu\text{C}$. Due to radioactive source nearby, the medium between the plates gets slightly conducting and the plate loses charge initially at the rate of $2 \times 10^{-7} \text{ Cs}^{-1}$. What is the magnitude and direction of displacement current? What is the magnetic field between the plates?

Sol. Due to leaking, there is a flow of positive charge from the positive plate to the negative plate (or the flow of negative charge in the reverse direction). Thus, the conduction current within the plates is from the positive plate to the negative plate. Now, the displacement current,

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 A \frac{d}{dt} \left(\frac{q}{\epsilon_0 A} \right) = \epsilon_0 A \frac{1}{\epsilon_0 A} \frac{dq}{dt}$$

$$\text{or } I_d = \frac{dq}{dt} = 2 \times 10^{-7} \text{ Cs}^{-1}$$

The direction of displacement current is opposite to that of electric field E and hence, opposite to the conduction current. But its magnitude is same as that of the conduction current. The net current between the plates is zero.

Using Ampere-Maxwell's circuital law (I being replaced by $I = I_c + I_d = 0$),

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I = \mu_0 (I_c + I_d) = \mu_0 \times 0 = 0$$

So, the magnetic field within the plates is zero at all points.

CHECK POINT 8.1

- According to Maxwell's hypothesis, a changing electric field gives rise to
 - emf
 - electric current
 - magnetic field
 - pressure radiant
- Maxwell in his famous equation of electromagnetism introduced the concept of
 - AC current
 - DC current
 - displacement current
 - impedance
- Dimensions of $\epsilon_0 \frac{d\phi_E}{dt}$ are same as that of
 - charge
 - potential
 - capacitance
 - current
- According to Ampere-Maxwell's circuital law,
 - $\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\epsilon_0}$
 - $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_E}{dt}$
 - $\oint \mathbf{B} \cdot d\mathbf{S} = 0$
 - $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_c + \epsilon_0 \frac{d\phi_E}{dt} \right)$
- The conduction current in ideal case through a circuit is zero when charge on capacitor is
 - zero
 - maximum
 - any transient value
 - depends on capacitor used
- What is the displacement current between the square plate of side 1 cm of a capacitor, if electric field between the plates is changing at the rate of $3 \times 10^6 \text{ Vm}^{-1}\text{s}^{-1}$?
 - $2.7 \times 10^{-9} \text{ A}$
 - $3.2 \times 10^{-5} \text{ A}$
 - $4.2 \times 10^{-6} \text{ A}$
 - $4.0 \times 10^{-5} \text{ A}$
- A parallel plate capacitor of plate separation 2 mm is connected in an electric circuit having source voltage 400 V. What is the value of the displacement current for 10^{-6} s , if the plate area is 60 cm^2 ?
 - $1.062 \times 10^{-2} \text{ A}$
 - $2.062 \times 10^{-2} \text{ A}$
 - $3.062 \times 10^{-2} \text{ A}$
 - $5.062 \times 10^{-2} \text{ A}$

ELECTROMAGNETIC WAVES

Electromagnetic waves (or EM waves) are the waves which are composed of changing electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$, sinusoidally and propagating through space such that, the two fields are perpendicular to each other and perpendicular to the direction of wave propagation.

Sources of electromagnetic waves

These waves are produced by accelerating charged particles which can be related with following physical phenomena

- A charge oscillating harmonically is a source of EM waves of same frequency.

A simple L - C oscillator and energy source as shown in figure can produce EM waves of desired

$$\text{frequency} \left(\nu = \frac{1}{2\pi\sqrt{LC}} \right).$$

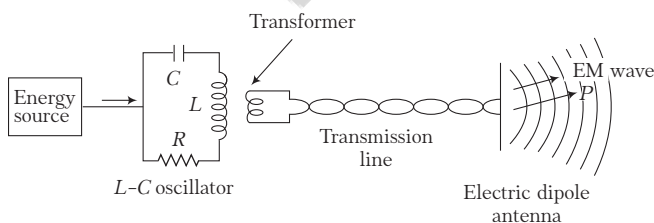


Fig. 8.2 An arrangement for generating electromagnetic wave

L - C oscillator produces a sinusoidal current in the antenna, which generates the wave. P is a distant point at which a detector can monitor the wave travelling past it.

- Electromagnetic waves are produced when high speed electron enters into target of high atomic weight.
- Electromagnetic waves are produced during de-excitation of nucleus in radioactivity.
- An electron in a stationary orbit orbiting around its nucleus does not emit EM waves. But during transition from higher energy orbit to lower energy orbit, it emits EM waves.

Graphical representation of propagation electromagnetic waves

During propagation of EM waves through a medium, electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ vary sinusoidally. Electric field and magnetic field become maximum at same position and time.

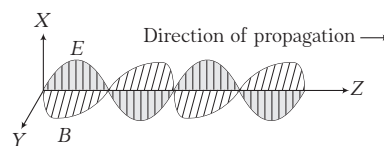


Fig. 8.3 A plane EM wave travelling along Z-axis

Also, $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other as well as to the direction of propagation of the wave ($\mathbf{E} \times \mathbf{B}$). In the given figure, electric field is along x -direction, magnetic field is along y -direction and the wave propagates along z -direction.

Therefore, $\mathbf{E}(x) \perp \mathbf{B}(y) \perp z$ (direction of propagation of the wave along z).

Equations of electromagnetic waves

The EM wave propagating in the positive z-direction may be represented by following equations

$$E = E_x = E_0 \sin(kz - \omega t)$$

$$B = B_y = B_0 \sin(kz - \omega t)$$

where, $k = 2\pi/\lambda$ (λ = wavelength),

$$\omega = 2\pi\nu \quad (\nu = \text{frequency}),$$

E_0 = amplitude of varying electric field

and B_0 = amplitude of varying magnetic field.

If the EM wave propagates in negative z-direction, negative sign is replaced by positive sign and equations become

$$E = E_x = E_0 \sin(kz + \omega t)$$

$$B = B_y = B_0 \sin(kz + \omega t)$$

Characteristics of electromagnetic waves

- (i) These waves do not require any material medium for propagation.
- (ii) These waves travel in free space with the speed of light ($\approx 3.0 \times 10^8$ m/s).
- (iii) Electromagnetic waves are transverse in nature.
- (iv) The oscillating electric and magnetic field vectors of an EM wave are oriented along mutually perpendicular directions and are in phase.
- (v) These waves are not deflected by electric field or magnetic field.
- (vi) EM waves are polarised.
- (vii) These waves carry energy and momentum.
- (viii) EM waves exert pressure called radiation pressure.
- (ix) Electric field vector of EM waves is responsible for its optical effects.

Physical quantities associated with EM waves

Important physical quantities associated with EM waves are explained as given below

(i) Speed

In free space, its speed, $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E_0}{B_0} = 3 \times 10^8$ m/s

In medium, $v = \frac{1}{\sqrt{\mu \epsilon}}$

where, μ_0 = absolute permeability,

ϵ_0 = absolute permittivity,

μ = permeability of medium

and ϵ = permittivity of medium.

(ii) Energy

The energy in EM waves is divided equally between the electric and magnetic fields.

Energy density (energy per unit volume) of electric field, $u_e = \frac{1}{2} \epsilon_0 E^2$, energy density of magnetic field, $u_m = \frac{1}{2} \frac{B^2}{\mu_0}$.

The total energy per unit volume,

$$u = u_e + u_m = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0} = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

Also, average energy density,

$$u_{av} = \frac{1}{4} \epsilon_0 E_0^2 + \frac{1}{4} \frac{B_0^2}{\mu_0} = \frac{1}{2} \epsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

(iii) Intensity

The energy crossing per unit area per unit time perpendicular to the direction of propagation of EM wave is called intensity of wave.

$$\begin{aligned} i.e. \quad I &= \frac{\text{Total EM wave energy (u)}}{\text{Surface area (A)} \times \text{Time (t)}} \\ &= \frac{\text{Total energy density} \times \text{Volume}}{\text{Surface area} \times \text{Time}} \end{aligned}$$

$$I = u_{av} \times c = \frac{1}{2} \epsilon_0 E_0^2 c = \frac{1}{2} \frac{B_0^2}{\mu_0} \cdot c \text{ Wm}^{-2}$$

Note Intensity due to a point source Consider a point source S, emanating electromagnetic waves (light) in all directions.

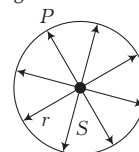


Fig. 8.4 A point source emitting light in all directions

If power of the source is P, then intensity of light at any point P is

$$I = \frac{u}{A \times t} = \frac{P}{A} = \left(\frac{P}{4\pi r^2} \right) \text{ W/m}^2$$

In the same way, intensity of a line source, $I \propto 1/r$ and for a plane source, intensity is independent of r.

(iv) Momentum

EM waves also carry momentum. If a portion of EM wave of energy u propagating with speed c , then

$$\text{linear momentum} = \frac{\text{energy (u)}}{\text{speed (c)}}$$

If wave is incident on a completely absorbing surface, then momentum delivered $p = \frac{u}{c}$. If wave is incident on a totally

reflecting surface, then momentum delivered $p = \frac{2u}{c}$.

(v) Poynting vector

The rate of flow of energy in an electromagnetic wave is described by the vector **S**, it is called the Poynting vector, which is defined by the expression,

$$\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$$

Its magnitude S is related to the rate at which energy is transported by a wave across a unit area at any instant.

Since, **E** and **B** are mutually perpendicular.

$$\text{Hence, } |\mathbf{E} \times \mathbf{B}| = E_0 B_0$$

$$\text{Thus, } S = \frac{E_0 B_0}{\mu_0}$$

The SI unit of Poynting vector **S** is Js^{-1}m^2 or Wm^{-2} .

Note The direction of the Poynting vector **S** of an EM wave at any point gives direction of the wave and the direction of energy transported at that point.

(vi) Radiation pressure

It is the mechanical pressure exerted upon any surface due to the exchange of momentum between the surface and the EM wave.

$$\text{For a perfectly reflecting surface, } p_r = \frac{2S}{c},$$

where S = Poynting vector and c = speed of light.

$$\text{For a perfectly absorbing surface, } p_a = \frac{S}{c}$$

Example 8.5 An L - C resonant circuit contains a 100 pF capacitor and a 400 μH inductor. It is set into oscillation coupled to an antenna. Find the frequency of the radiated electromagnetic waves.

Sol. Given, $C = 100 \text{ pF} = 100 \times 10^{-12} \text{ F}$,

$$L = 400 \mu\text{H} = 400 \times 10^{-6} \text{ H}$$

Frequency of the radiated EM waves,

$$\begin{aligned} v &= \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14 \sqrt{400 \times 10^{-6} \times 100 \times 10^{-12}}} \\ &= 0.0796 \times 10^7 \text{ Hz} \end{aligned}$$

Example 8.6 Electromagnetic waves travel in a medium with a speed of $2 \times 10^8 \text{ ms}^{-1}$. The relative magnetic permeability of the medium is 1. Find the relative electrical permittivity.

Sol. Given, $v = 2 \times 10^8 \text{ ms}^{-1}$ and $\mu_r = 1$

The speed of electromagnetic waves in a medium is given by

$$v = \frac{1}{\sqrt{\mu\epsilon}} \quad \dots(i)$$

where, μ and ϵ are permeability and permittivity of the medium, respectively.

$$\text{Now, } \mu = \mu_0 \mu_r \text{ and } \epsilon = \epsilon_0 \epsilon_r$$

$$\begin{aligned} \therefore \text{Eq. (i) becomes, } v &= \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \times \frac{1}{\sqrt{\mu_r \epsilon_r}} \\ \Rightarrow v &= \frac{c}{\sqrt{\mu_r \epsilon_r}} \quad \left(\because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \right) \end{aligned}$$

On squaring both sides, we get

$$\epsilon_r = \frac{c^2}{v^2 \mu_r} = \frac{(3 \times 10^8)^2}{(2 \times 10^8)^2 \times 1} = 2.25$$

Example 8.7 An electromagnetic wave of frequency 40 MHz travels in free space in the + x -direction.

(i) Determine the wavelength of the wave.

(ii) At some point and at some instant, the electric field has its maximum value of 750 NC^{-1} and is along the + y -axis.

Calculate the magnitude and direction of the magnetic field at this position and time.

Sol (i) Wavelength of the wave, $\lambda = \frac{c}{v} = \frac{3.0 \times 10^8}{40 \times 10^6} = 7.5 \text{ m}$

(ii) Given, maximum value of electric field, $E_0 = 750 \text{ NC}^{-1}$

$$\therefore \text{Magnetic field, } B_0 = \frac{E_0}{c} = \frac{750}{3 \times 10^8} = 2.5 \times 10^{-6} \text{ T}$$

Since, **E** and **B** are mutually perpendicular and they both are perpendicular to the propagation of wave.

$$\Rightarrow \mathbf{E} \hat{\mathbf{j}} \times \mathbf{B} = c \hat{\mathbf{i}}$$

From vector algebra, $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$

Thus, we concluded that magnetic field is in positive z -direction.

Example 8.8 Find the amplitude of electric and magnetic fields in a parallel beam of light of intensity 4.0 Wm^{-2} .

Sol. The intensity of plane electromagnetic wave,

$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$\therefore \text{Amplitude of electric field, } E_0 = \sqrt{\frac{2I}{\epsilon_0 c}}$$

$$= \sqrt{\frac{2 \times 4}{8.86 \times 10^{-12} \times 3 \times 10^8}} = 54.86 \text{ NC}^{-1}$$

Further, amplitude of magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{54.86}{3.0 \times 10^8} = 1.83 \times 10^{-7} \text{ T}$$

Example 8.9 The magnetic field in a plane electromagnetic wave is given by

$$B_y = (2 \times 10^{-7} \text{ T}) \sin (0.5 \times 10^3 x + 1.5 \times 10^{11} t)$$

(i) What is the wavelength and frequency of the wave?

(ii) Write an expression for the electric field.

Sol. (i) Comparing the given equation with the standard equation of electromagnetic wave,

$$B = B_0 \sin (kx + \omega t)$$

We have, $k = \frac{2\pi}{\lambda} = 0.5 \times 10^3$

$$\Rightarrow \lambda = \frac{2\pi}{0.5 \times 10^3} \text{ m} \\ = 0.0126 \text{ m} = 1.26 \text{ cm}$$

and $\omega = 2\pi\nu = 1.5 \times 10^{11}$

$$\Rightarrow v = \frac{1.5 \times 10^{11}}{2\pi} = 2.4 \times 10^{10} \text{ Hz}$$

(ii) Given, $B_0 = 2 \times 10^{-7} \text{ T}$

$$\therefore E_0 = B_0 c = 2 \times 10^{-7} \times 3 \times 10^8 = 60 \text{ NC}^{-1}$$

The electric field is perpendicular to both, the magnetic field and the direction of propagation of wave. Thus,

$$E = (60 \text{ NC}^{-1}) \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$$

Example 8.10 The magnetic field in a plane electromagnetic wave is given by

$$B = (200 \mu\text{T}) \sin \left[(4.0 \times 10^{15} \text{ s}^{-1}) \left(t - \frac{x}{c} \right) \right]. \text{ Find the}$$

average energy density corresponding to the electric field.

Sol. Average energy density corresponding to electric and magnetic field is equal and is given by

$$u_{\text{av}} = \frac{1}{2} \epsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

Substituting the values, we get

$$u_{\text{av}} = \frac{(200 \times 10^{-6})^2}{2(4\pi \times 10^{-7})} = 15.9 \times 10^{-3} \approx 16 \times 10^{-3} \text{ Jm}^{-3}$$

Example 8.11 The electric field of an electromagnetic wave is given by

$$E = (50 \text{ NC}^{-1}) \sin \omega (t - x/c)$$

Find the energy contained in a cylinder of cross-section 10 cm^2 and length 50 cm along the X-axis.

Sol. The average value of energy density (energy/volume) is given by

$$u_{\text{av}} = \frac{1}{2} \epsilon_0 E_0^2$$

Total volume of the cylinder, $V = A \cdot l$

$\therefore$ Total energy contained in the cylinder,

$$u = (u_{\text{av}})(V) = \left(\frac{1}{2} \epsilon_0 E_0^2 \right) (Al)$$

Substituting the values, we get

$$u = \frac{1}{2} \times (8.85 \times 10^{-12}) (50)^2 (10 \times 10^{-4}) (50 \times 10^{-2}) \text{ J} \\ = 5.5 \times 10^{-12} \text{ J}$$

Example 8.12 In a region of free space, the electric field at some instant of time is $\mathbf{E} = (80\hat{\mathbf{i}} + 32\hat{\mathbf{j}} - 64\hat{\mathbf{k}}) \text{Vm}^{-1}$ and the magnetic field is $\mathbf{B} = (0.2\hat{\mathbf{i}} + 0.08\hat{\mathbf{j}} + 0.29\hat{\mathbf{k}}) \mu\text{T}$.

(i) Show that these two fields are perpendicular to each other.

(ii) Determine the poynting vector for these fields.

Sol. (i) $\mathbf{E}$ will be perpendicular to $\mathbf{B}$, if $\mathbf{E} \cdot \mathbf{B} = 0$

Now,

$$\mathbf{E} \cdot \mathbf{B} = (80\hat{\mathbf{i}} + 32\hat{\mathbf{j}} - 64\hat{\mathbf{k}}) \cdot (0.2\hat{\mathbf{i}} + 0.08\hat{\mathbf{j}} + 0.29\hat{\mathbf{k}}) \\ = (80 \times 0.2 + 32 \times 0.08 - 64 \times 0.29) = 0$$

Thus, we can say that $\mathbf{E} \perp \mathbf{B}$.

(ii) The poynting vector, $\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$

$$= \frac{1}{4\pi \times 10^{-7}} \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 80 & 32 & -64 \\ 0.2 & 0.08 & 0.29 \end{vmatrix} \\ = [11.46\hat{\mathbf{i}} - 28.6\hat{\mathbf{j}}] \times 10^6 \text{ Wm}^{-2}$$

Example 8.13

(i) Find the energy stored in a 90 cm length of a laser beam operating at 6 mW .

(ii) Find the amplitude of electric field in a parallel beam of light of intensity 17.7 W/m^2 .

Sol. (i) The time taken by laser beam to travel a distance of 90 cm ,

$$t = \frac{90 \times 10^{-2}}{3 \times 10^8} = 3 \times 10^{-9} \text{ s}$$

Energy contained in 90 cm length,

$$u = Pt = 6 \times 10^{-3} \times 3 \times 10^{-9} = 18 \times 10^{-12} \text{ J}$$

(ii) Intensity of light, $I = \frac{1}{2} \epsilon_0 E_0^2 c$

$$\Rightarrow 17.7 = \frac{1}{2} (8.85 \times 10^{-12}) E_0^2 \times 3 \times 10^8$$

$$\Rightarrow E_0^2 = \frac{4}{3} \times 10^4$$

Therefore, the amplitude of the electric field in the parallel beam, $E_0 = \frac{2}{\sqrt{3}} \times 10^2 \text{ V/m}$.

Example 8.14 Light with an energy flux of 18 Wcm^{-2} falls on a non-reflecting surface at normal incidence. If the surface has an area of 20 cm^2 , then find the average force exerted on the surface during a span of 30 min .

Sol. Total energy falling on the surface,

$$u = (18 \times 10^4)(30 \times 60)(20 \times 10^{-4}) = 6.48 \times 10^5 \text{ J}$$

Therefore, the total momentum delivered,

$$p = \frac{u}{c} = \frac{6.48 \times 10^5}{3.0 \times 10^8} = 2.16 \times 10^{-3} \text{ kgms}^{-1}$$

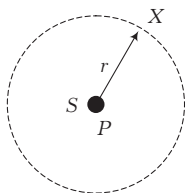
The average force exerted on the surface,

$$F = \frac{p}{t} = \frac{2.16 \times 10^{-3}}{30 \times 60} (\because F \cdot t = \text{change in momentum}) \\ = 1.2 \times 10^{-6} \text{ N}$$

Note If the surface is a perfect reflector, then the change in momentum will be twice the above value, so the force will be twice or $2.4 \times 10^{-6} \text{ N}$.

Example 8.15 Evaluate the amplitude of electric field and magnetic field produced by the radiation coming from a $20\pi W$ bulb at distance of $2m$. Assume that the efficiency of the bulb is 20% and it is a point source.

Sol. Consider the situation shown below



Intensity at distance r from a point source (bulb),

$$I = \frac{P}{4\pi r^2}$$

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Input}} = \frac{P}{P'}$$

$$\Rightarrow P = \eta P' = \left(\frac{20}{100}\right)(20\pi) = 4\pi W$$

$$I = \frac{P}{4\pi r^2} = \frac{4\pi}{4\pi(2)^2} = \frac{1}{4} = 0.25 \text{ W/m}^2$$

Also, intensity of EM wave is given by

$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$\Rightarrow \frac{1}{4} = \frac{1}{2} \epsilon_0 E_0^2 \times 3 \times 10^8$$

$$\Rightarrow E_0^2 = \frac{10^{-8}}{6\epsilon_0} = \frac{10^{-8}}{6 \times 8.85 \times 10^{-12}} = \frac{10^4}{6 \times 8.85}$$

$\therefore$ Amplitude of electric field,

$$E_0 = \frac{100}{\sqrt{6 \times 8.85}} = 13.73 \text{ V/m}$$

and amplitude of magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{13.73}{3 \times 10^8} = 4.57 \times 10^{-8} \text{ T}$$

ELECTROMAGNETIC SPECTRUM

The orderly arrangement of EM waves in increasing or decreasing order of wavelength λ or frequency ν is called **electromagnetic spectrum**. The range varies from 10^{-12} m to 10^4 m , i.e. from γ -rays to radio waves as shown in the figure given below.

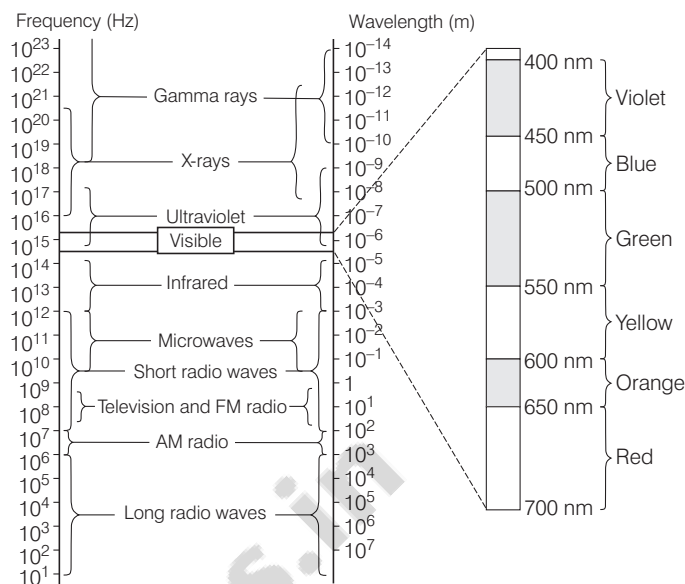


Fig. 8.5 Electromagnetic spectrum

The wavelength ranges, frequency ranges and uses of the various regions of electromagnetic spectrum are summarised below

Radio waves

These waves were first predicted by James Clerk Maxwell. Radio waves are produced due to oscillating charge particles between plates of capacitor in L - C circuit.

Uses of radio waves are given below

- These are used in AM (Amplitude Modulation) from 530 kHz to 1710 kHz. These are also used in ground wave propagation.
- These are used in TV waves ranging from 54 MHz to 890 MHz.
- These are used in FM (Frequency Modulation) ranging from 88 MHz to 108 MHz.
- UHF (Ultra High Frequency) waves are used in cellular phones.

Microwaves

These waves were also predicted by James Clerk Maxwell. Microwaves are called **short wavelength radio waves** which are produced by vacuum tubes.

Uses of microwaves are given below

- These are used in RADAR systems for aircraft navigation.
- These are used in microwave oven for cooking purpose.
- These are used in the study of atomic and molecular structures.
- These are used to measure the speed of vehicle, speed of cricket ball, etc.

Infrared waves

These waves were discovered by Herschel. Infrared waves are also called **heat waves**. These waves are produced by heat radiating bodies and molecules. They have high penetration power.

Uses of infrared waves are given below

- (i) These are used in heat therapy to heat muscular pain or sprain.
- (ii) These are used in satellite for army purpose.
- (iii) These are used in weather forecasting.
- (iv) These are used for producing dehydrated fruits.
- (v) These are used in solar water heater, solar cells and cooker.
- (vi) These are used in TV remote as signal carrier.
- (vii) These are used in night vision or infrared photography.

Visible rays

These rays were discovered by Sir Isaac Newton. It is that part of spectrum which is visible by human eye.

Visible rays are used by the optical organs of humans and animals for three primary purposes given below

- (i) To see things, avoid bumping from them and escape danger.
- (ii) To find stuff to eat.
- (iii) To find other living things with which to consort, so as to prolong the species.

Ultraviolet rays

These rays were discovered by Johann Ritter in 1801. UV-rays are produced by special lamps and very hot bodies. The sun is an important source of UV-rays but fortunately these rays are absorbed by ozone layer.

Uses of ultraviolet rays are given below

- (i) These are used in burglar alarm.
- (ii) These are used to study molecular structure and in detection of invisible writing.
- (iii) These are used to kill germs in minerals.
- (iv) These are used to sterilise surgical instruments.

X-rays

These rays were discovered by German professor Roentgen.

Uses of X-rays are given below

- (i) These are used in surgery to detect the fracture, diseased organs, stones in the body.
- (ii) These are used in engineering to detect fault, crack on bridges, testing of welds.
- (iii) These are used at metro station to detect metal or explosive material.
- (iv) These are used in scientific research.

Gamma (γ)-rays

These rays were discovered by Rutherford. They have high penetration power.

Uses of gamma (γ)-rays are given below

- (i) These are used to produce nuclear reaction.
- (ii) These are used in radiotherapy for the treatment of tumour and cancer.
- (iii) These are used in food industry to kill pathogenic microorganism.
- (iv) These are used to provide valuable information about the structure of atomic nucleus.

Production and detection of different types of electromagnetic waves

Type	Wavelength range	Frequency range	Production	Detection
Radio wave	> 0.1 m	$< 10^9$ Hz	Rapid acceleration and deceleration of electrons in aerials.	Receiver's aerials
Microwave	0.1 m to 1 mm	10^9 to 10^{11} Hz	Klystron valve or magnetron valve.	Point contact diodes
Infrared wave	1 mm to 700 nm	10^{11} to 10^{14} Hz	Vibration of atoms and molecules.	Thermopiles, bolometer, infrared photographic film
Visible rays	700 nm to 400 nm	4×10^{14} to 8×10^{14} Hz	Electrons in atoms emit light when they move from higher energy level to a lower energy level.	The eye, photocells, photographic film
Ultraviolet rays	400 nm to 1 nm	10^{14} to 10^{17} Hz	Inner shell electrons in atoms moving from higher energy level to a lower energy level.	Photocells, photographic film
X-rays	1 nm to 10^{-3} nm	10^{16} to 10^{20} Hz	X-ray tubes or inner shell electrons.	Photographic film, Geiger tubes, ionisation chamber
Gamma rays	$< 10^{-3}$ nm	$> 10^{22}$ Hz	Radioactive decay of the nucleus.	Photographic film, Geiger tubes, ionisation chamber

CHECK POINT 8.2

- An L - C resonant circuit contains a 200 pF capacitor and a 100 μ H inductor. It is set into oscillation coupled to an antenna. The wavelength of the radiated electromagnetic waves is
 (a) 377 mm (b) 266 m
 (c) 377 cm (d) 3.77 cm
- The velocity of electromagnetic wave is parallel to
 (a) $\mathbf{B} \times \mathbf{E}$ (b) $\mathbf{E} \times \mathbf{B}$ (c) $\mathbf{E}$ (d) $\mathbf{B}$
- An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$. Which of the following is independent of wavelength?
 (a) k (b) ω
 (c) k/ω (d) $k\omega$
- Which of the following statement is incorrect about electromagnetic waves?
 (a) These are transverse in nature.
 (b) These are produced by accelerating charges.
 (c) They travel with the same speed in all media.
 (d) They travel in free space with the speed of light.
- In electromagnetic waves, the phase difference between electric and magnetic field vectors $\mathbf{E}$ and $\mathbf{B}$ is
 (a) zero (b) $\pi/2$ (c) π (d) $\pi/4$
- The oscillating electric and magnetic field vectors of an electromagnetic waves are oriented along
 (a) the same direction but differ in phase by 90°
 (b) the same direction and are in phase
 (c) mutually perpendicular directions and are in phase
 (d) mutually perpendicular directions and differ in phase by 90°
- Amongst the following relations, which is correct?
 (a) $\sqrt{\epsilon_0} E_0 = \sqrt{\mu_0} B_0$
 (b) $\sqrt{\mu_0 \epsilon_0} = B_0/E_0$
 (c) $E_0 = \sqrt{\mu_0 \epsilon_0} B_0$
 (d) $\sqrt{\mu_0} E_0 = \sqrt{\epsilon_0} B_0$
- In an apparatus, the electric field was found to oscillate with amplitude of 18 Vm^{-1} . The magnitude of the oscillating magnetic field will be
 (a) $4 \times 10^{-6} \text{ T}$ (b) $6 \times 10^{-8} \text{ T}$
 (c) $9 \times 10^{-9} \text{ T}$ (d) $11 \times 10^{-11} \text{ T}$
- The rms value of the electric field of the light coming from the sun is 720 NC^{-1} . The average total energy density of the electromagnetic wave is
 (a) $6.37 \times 10^{-9} \text{ J m}^{-3}$ (b) $81.35 \times 10^{-12} \text{ J m}^{-3}$
 (c) $3.5 \times 10^{-3} \text{ J m}^{-3}$ (d) $4.58 \times 10^{-6} \text{ J m}^{-3}$
- The electric field of a plane electromagnetic wave varies with time of amplitude 2 Vm^{-1} propagating along Z -axis. The average density of the magnetic field is (in J m^{-3})
 (a) 13.29×10^{-12} (b) 8.86×10^{-12}
 (c) 17.71×10^{-12} (d) 4.43×10^{-12}
- In an electromagnetic wave, the electric and magnetic fields are 100 Vm^{-1} and 0.265 T , respectively. The maximum rate of energy flow is
 (a) $21 \times 10^6 \text{ Wm}^{-2}$ (b) $36.5 \times 10^6 \text{ Wm}^{-2}$
 (c) $46.7 \times 10^6 \text{ Wm}^{-2}$ (d) None of these
- Radiations of intensity 0.5 Wm^{-2} are striking a metal plate. The pressure on the plate is
 (a) $0.166 \times 10^{-8} \text{ Nm}^{-2}$ (b) $0.33 \times 10^{-8} \text{ Nm}^{-2}$
 (c) $0.111 \times 10^{-8} \text{ Nm}^{-2}$ (d) $0.083 \times 10^{-8} \text{ Nm}^{-2}$
- Which of the following waves have the maximum wavelength?
 (a) X-rays (b) Infrared waves
 (c) Ultraviolet (d) Radio waves
- Which of the following electromagnetic waves are used in weather forecasting?
 (a) Infrared waves (b) Radio waves
 (c) Microwaves (d) Visible rays
- Electromagnetic radiation of highest frequency is
 (a) infrared radiations (b) visible radiation
 (c) radio waves (d) γ -rays

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1 Which of the following wave cannot travel in vacuum?
(a) X-rays (b) Infrasonic waves
(c) Ultraviolet rays (d) Radio waves
- 2 Energy of electromagnetic waves is due to their
(a) wavelength
(b) frequency
(c) electric and magnetic fields
(d) None of the above
- 3 Which of the following represents an infrared wavelength?
(a) 10^{-4} cm (b) 10^{-5} cm (c) 10^{-6} cm (d) 10^{-7} cm
- 4 Which of the following has the shortest wavelength?
(a) X-rays (b) γ -rays
(c) Microwaves (d) Radio waves
- 5 The frequency of visible light is of the order of
(a) 10^{14} Hz (b) 10^{10} Hz (c) 10^6 Hz (d) 10^4 Hz
- 6 The structure of solids is investigated by using
(a) cosmic rays (b) X-rays
(c) γ -rays (d) infrared radiations
- 7 Energy stored in electromagnetic oscillations is in the form of
(a) electrical energy (b) magnetic energy
(c) Both (a) and (b) (d) None of these
- 8 The range of wavelength of the visible light is
(a) 10 Å to 100 Å (b) 4000 Å to 7000 Å
(c) 7000 Å to 10000 Å (d) 10000 Å to 15000 Å
- 9 Which of the following electromagnetic waves have the longest wavelength?
(a) Heat waves (b) Light waves
(c) Radio waves (d) Microwaves
- 10 Radio waves diffract around building although light waves do not. The reason is that radio waves
(a) travel with speed larger than c
(b) have longer wavelength than light
(c) carry news
(d) are not electromagnetic waves
- 11 Frequency of wave is 6×10^{15} Hz. The wave is
(a) radio wave (b) microwave
(c) X-ray (d) ultraviolet rays
- 12 The frequency 1057 MHz of radiation arising from two close energy levels in hydrogen belongs to
(a) radio waves (b) infrared waves
(c) microwaves (d) γ -rays
- 13 The part of the spectrum of the electromagnetic radiation used to cook food is
(a) ultraviolet rays (b) cosmic rays
(c) X-rays (d) microwaves
- 14 Which of the following shows greenhouse effect?
(a) Ultraviolet rays (b) Infrared rays
(c) X-rays (d) Visible rays
- 15 Which of the following waves are used in RADAR systems for aircraft navigation?
(a) X-rays (b) Infrared rays
(c) Ultraviolet rays (d) Microwaves
- 16 In an electromagnetic wave, the average energy density associated with electric field is
(a) $CV^2/2$ (b) $q^2/2C$ (c) $\epsilon_0^2/2E_0$ (d) $\epsilon_0 E^2/2$
- 17 Total energy density of electromagnetic waves in vacuum is given by the relation
(a) $\frac{1}{2} \cdot \frac{E^2}{\epsilon_0} + \frac{B^2}{2\mu_0}$ (b) $\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 B^2$
(c) $\frac{E^2 + B^2}{c}$ (d) $\frac{1}{2} \epsilon_0 E^2 + \frac{B^2}{2\mu_0}$
- 18 For a medium with permittivity ϵ and permeability μ , the velocity of light is given by
(a) $\sqrt{\mu/\epsilon}$ (b) $\sqrt{\mu\epsilon}$
(c) $1/\sqrt{\mu\epsilon}$ (d) $\sqrt{\epsilon/\mu}$
- 19 The speed of electromagnetic wave in vacuum
(a) increases as we move from γ -rays to radio waves
(b) decreases as we move from γ -rays to radio waves
(c) is same for all of them
(d) None of the above
- 20 In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2×10^{10} Hz and amplitude 48 Vm^{-1} . The wavelength of the wave is
(a) 24×10^{-10} m (b) 1.5×10^{-2} m
(c) 4.16×10^8 m (d) 3×10^8 m

- 21** The essential distinction between X-rays and γ -rays is that
 (a) γ -rays have smaller wavelength than X-rays
 (b) γ -rays emanate from nucleus while X-rays emanate from outer part of the atom
 (c) γ -rays have greater ionising power than X-rays
 (d) γ -rays are more penetrating than X-rays
- 22** The sun delivers 10^4 Wm^{-2} of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions $(10 \times 10) \text{ m}^2$ will be
 (a) 10^4 W (b) 10^5 W (c) 10^6 W (d) 10^7 W
- 23** If $\mathbf{E}$ and $\mathbf{B}$ represent electric and magnetic field vectors of the electromagnetic wave, then the direction of propagation of electromagnetic wave is along
 [NCERT Exemplar]
 (a) $\mathbf{E}$ (b) $\mathbf{B}$ (c) $\mathbf{B} \times \mathbf{E}$ (d) $\mathbf{E} \times \mathbf{B}$
- 24** An EM wave radiates outwards from a dipole antenna, with E_0 as the amplitude of its electric field vector. The electric field E_0 which transports significant energy from the source falls off as
 [NCERT Exemplar]
 (a) $\frac{1}{r^3}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r}$ (d) remains constant
- 25** The average electric field of electromagnetic waves in certain region of free space is $9 \times 10^{-4} \text{ NC}^{-1}$. Then, the average magnetic field in the same region is of the order of
 (a) $27 \times 10^{-4} \text{ T}$ (b) $3 \times 10^{-12} \text{ T}$
 (c) $\left(\frac{1}{3}\right) \times 10^{-12} \text{ T}$ (d) $3 \times 10^{12} \text{ T}$
- 26** The pressure exerted by an electromagnetic wave of intensity $I \text{ Wm}^{-2}$ on a non-reflecting surface is (c is the velocity of light)
 (a) Ic (b) Ic^2 (c) I/c (d) I/c^2
- 27** The electric and the magnetic fields associated with an electromagnetic wave, propagating along the Z-axis, can be represented by
 (a) $[E = E_0 \hat{k}, B = B_0 \hat{i}]$ (b) $[E = E_0 \hat{j}, B = B_0 \hat{j}]$
 (c) $[E = E_0 \hat{j}, B = B_0 \hat{k}]$ (d) $[E = E_0 \hat{i}, B = B_0 \hat{j}]$
- 28** A parallel plate capacitor is charged to $60 \mu\text{C}$. Due to a radioactive source, the plate loses charge at the rate of $1.8 \times 10^{-8} \text{ Cs}^{-1}$. The magnitude of displacement current is
 (a) $1.8 \times 10^{-8} \text{ Cs}^{-1}$ (b) $3.6 \times 10^{-8} \text{ Cs}^{-1}$
 (c) $4.1 \times 10^{-11} \text{ Cs}^{-1}$ (d) $5.7 \times 10^{-12} \text{ Cs}^{-1}$
- 29** Radiation of intensity 1 W/m^2 are striking on a metal plate. The pressure on the plate is
 (a) $0.66 \times 10^{-8} \text{ N/m}^2$ (b) 0.25 N/m^2
 (c) 2 N/m^2 (d) 4 N/m^2
- 30** An L - C resonant circuit contains a 200 pF capacitor and a $200 \mu\text{H}$ inductor. It is set into oscillation coupled to an antenna. The wavelength of the radiated electromagnetic waves is
 (a) 377 mm (b) 377 m
 (c) 377 cm (d) 3.77 cm
- 31** The magnetic field in a plane electromagnetic wave is given by $B_y = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$. This electromagnetic wave is
 (a) a visible light (b) an infrared wave
 (c) a microwave (d) a radio wave
- 32** An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$, $B = B_0 \sin(kx - \omega t)$. Which of the following equations is true?
 (a) $E_0 k = B_0 \omega$ (b) $E_0 \omega = B_0 k$
 (c) $E_0 B_0 = \omega k$ (d) None of these
- 33** A linearly polarised electromagnetic wave given as $\mathbf{E} = E_0 \hat{i} \cos(\hat{k}z - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as [NCERT Exemplar]
 (a) $\mathbf{E}_r = E_0 \hat{i} \cos(\hat{k}z - \omega t)$ (b) $\mathbf{E}_r = E_0 \hat{i} \cos(\hat{k}z + \omega t)$
 (c) $\mathbf{E}_r = -E_0 \hat{i} \cos(\hat{k}z + \omega t)$ (d) $\mathbf{E}_r = E_0 \hat{i} \sin(\hat{k}z - \omega t)$
- 34** A plane electromagnetic wave propagating in the x-direction has a wavelength 6.0 mm . The electric field is in the y-direction and its maximum magnitude is 33 Vm^{-1} . The equation for the electric field as a function of x and t is
 (a) $11 \sin \pi(t - x/c)$ (b) $33 \sin \pi \times 10^{11}(t - x/c)$
 (c) $33 \sin \pi(t - x/c)$ (d) $11 \sin \pi \times 10^{11}(t - x/c)$
- 35** An electromagnetic wave travels in vacuum along z-direction with $\mathbf{E} = (E_1 \hat{i} + E_2 \hat{j}) \cos(kz - \omega t)$. Choose the correct option from the following.
 [NCERT Exemplar]
 (a) The associated magnetic field is given as

$$\mathbf{B} = \frac{1}{c} (E_1 \hat{i} - E_2 \hat{j}) \cos(kz - \omega t)$$

 (b) The associated magnetic field is given as

$$\mathbf{B} = \frac{1}{c} (E_1 \hat{i} + E_2 \hat{j}) \cos(kz - \omega t)$$

 (c) The given electromagnetic field is circularly polarised
 (d) The given electromagnetic wave is plane polarised
- 36** A perfectly reflecting mirror has an area of 1 cm^2 . Light energy is allowed to fall on it for 1 h at the rate of 10 Wcm^{-2} . The force that acts on the mirror is
 (a) $3.35 \times 10^{-8} \text{ N}$ (b) $6.67 \times 10^{-8} \text{ N}$
 (c) $1.34 \times 10^{-7} \text{ N}$ (d) $2.4 \times 10^{-4} \text{ N}$

- 37** A plane electromagnetic wave is propagating along the z -direction. If the electric field component of this wave is in the direction $(\hat{i} + \hat{j})$, then which of the following is the direction of the magnetic field component?
- (a) $(-\hat{i} + \hat{j})$ (b) $(\hat{i} - \hat{j})$
 (c) $(-\hat{i} - \hat{j})$ (d) $(\hat{i} + \hat{j})$
- 38** One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in [NCERT Exemplar]
- (a) visible region (b) infrared region
 (c) ultraviolet region (d) microwave region
- 39** The electric field intensity produced by the radiations coming from 100 W bulb at a 3 m distance is E . The electric field intensity produced by the radiations coming from 50 W bulb at the same distance is [NCERT Exemplar]
- (a) $\frac{E}{2}$ (b) $2E$ (c) $\frac{E}{\sqrt{2}}$ (d) $\sqrt{2}E$
- 40** Light with an energy flux of 20 W/cm^2 falls on a non-reflecting surface at normal incidence. If the surface has an area of 30 cm^2 , then the total momentum delivered (for complete absorption) during 30 min is [NCERT Exemplar]
- (a) $36 \times 10^{-5} \text{ kg-m/s}$ (b) $36 \times 10^{-4} \text{ kg-m/s}$
 (c) $108 \times 10^4 \text{ kg-m/s}$ (d) $1.08 \times 10^7 \text{ kg-m/s}$
- 41** Magnitude of the electric and magnetic fields in an electromagnetic wave radiated by a 200 W bulb at a distance 2 m from it is (Assuming efficiency of bulb is 5% and it behaves like a point source.)
- (a) 12.27 N/C, $4.09 \times 10^{-8} \text{ T}$ (b) 10 N/C, 10^{-5} T
 (c) 11.7 N/C, $5 \times 10^{-6} \text{ T}$ (d) 5 N/C, 10^{-4} T

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-3) These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses :

- (a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) If Assertion is correct but Reason is incorrect.
 (d) If Assertion is incorrect but Reason is correct.

1 Assertion In electromagnetic wave, the direction of variations of electric and magnetic field are perpendicular to each other and also perpendicular to the direction of wave propagation.

Reason Electromagnetic waves are transverse in nature.

2 Assertion Ultraviolet radiations of higher frequency wave are dangerous to human being.

Reason Ultraviolet radiation are absorbed by the atmosphere.

3 Assertion The frequency of the electromagnetic wave is equal to the frequency of accelerating charge.

Reason The energy associated with the propagating wave comes at the expense of the energy of the source.

Statement based questions

- 1** Which one of the following is the property of a monochromatic, plane electromagnetic waves in free space?
- (a) Electric and magnetic fields have a phase difference of $\pi/2$.
 (b) The energy contribution of both electric and magnetic fields are equal.
 (c) The direction of propagation is in the direction of $\mathbf{B} \times \mathbf{E}$.
 (d) The pressure exerted by the wave is the product of its speed and energy density.
- 2** Which of the following statement is false about the properties of electromagnetic waves?
- (a) Both electric and magnetic field vectors attain the maxima and minima at the same place and same time.
 (b) The speed of the wave is E_0/B_0 .
 (c) Both electric and magnetic field vectors are parallel to each other and perpendicular to the direction of propagation of wave.
 (d) These waves do not require any material medium for propagation.
- 3** I. Microwaves are used in microwaves oven for cooking purpose.
 II. The frequency of microwaves is from 10^{14} to 10^{17} Hz .
 Which amongst the given statement(s) is/are correct?
- (a) Only I (b) Only II
 (c) Both I and II (d) Neither I nor II

- 4 I. Like light radiation, thermal radiation is also an electromagnetic radiation.
 II. The thermal radiations require no medium for propagation.
 Which amongst the given statement(s) is/are correct?
 (a) Only I (b) Only II
 (c) Both I and II (d) Neither I nor II

Match the columns

- 1 Match the Column I with Column II and mark the correct option from the codes given below.

Column I	Column II
(A) $\oint_S \mathbf{E} \cdot d\mathbf{S} = q/\epsilon_0$	(p) Faraday's law of EMI
(B) $\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	(q) Gauss's law in magnetism
(C) $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_B}{dt}$	(r) Maxwell-Ampere's circuital law
(D) $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left[I_c + \epsilon_0 \frac{d\phi}{dt} \right]$	(s) Gauss's law in electrostatics

Codes

A	B	C	D
(a) p	q	r	s
(b) s	q	p	r
(c) s	p	r	q
(d) q	s	r	p

- 2 Match the Column I (electromagnetic wave type) with Column II (its application) and mark the correct option from the codes given below.

Column I	Column II
(A) Infrared waves	(p) to treat muscular strain
(B) Radio waves	(q) for broadcasting
(C) X-rays	(r) to detect the fracture of bones
(D) Ultraviolet rays	(s) absorbed by the ozone layer of the atmosphere

Codes

A	B	C	D
(a) r	s	q	p
(b) s	r	q	p
(c) p	q	r	s
(d) p	q	s	r

(C) Medical entrances' gallery

Collection of questions asked in NEET & various medical entrance exams

- 1 The ratio of contributions made by the electric field and magnetic field components to the intensity of an electromagnetic wave is (where, c = speed of electromagnetic waves) [NEET 2020]
 (a) 1:1 (b) 1:c (c) 1:c² (d) c:1
- 2 The electromagnetic wave with shortest wavelength among the following is [NEET 2020]
 (a) UV-rays (b) X-rays
 (c) γ -rays (d) microwaves
- 3 The magnetic field in a plane electromagnetic wave is given by
 $B_y = 2 \times 10^{-7} \sin(\pi \times 10^3 x + 3\pi \times 10^{11} t) \text{ T}$
 Calculate the wavelength. [NEET 2020]
 (a) $\pi \times 10^{-3} \text{ m}$ (b) $2 \times 10^{-3} \text{ m}$
 (c) $2 \times 10^3 \text{ m}$ (d) $\pi \times 10^{-3} \text{ m}$
- 4 A parallel plate capacitor of capacitance $20 \mu\text{F}$ is being charged by a voltage source whose potential is changing at the rate of 3 V/s . The conduction current through the connecting wires and the displacement current through the plates of the capacitor would be respectively, [NEET 2019]
 (a) $60 \mu\text{A}$, $60 \mu\text{A}$ (b) $60 \mu\text{A}$, zero
 (c) zero, zero (d) zero, $60 \mu\text{A}$
- 5 An EM wave is propagating in a medium with a velocity $\mathbf{v} = v\hat{\mathbf{i}}$. The instantaneous oscillating electric field of this EM wave is along +Y-axis. Then, the direction of oscillating magnetic field of electromagnetic wave will be along [NEET 2018]
 (a) -y-direction (b) +z-direction
 (c) -z-direction (d) -x-direction
- 6 Out of the following options which one can be used to produce a propagating electromagnetic wave? [NEET 2016]
 (a) A stationary charge
 (b) A chargeless particle
 (c) An accelerating charge
 (d) A charge moving at constant velocity
- 7 The electric vector vibration of an electromagnetic wave is given by $E = (50 \text{ NC}^{-1}) \sin \omega \left(t - \frac{x}{c} \right)$.
 The intensity of the wave is [UP CPMT 2015]
 (a) 2.3 Wm^{-2} (b) 4.3 Wm^{-2}
 (c) 3.3 Wm^{-2} (d) 1.8 Wm^{-2}
- 8 The frequencies of X-rays, γ -rays and ultraviolet rays are respectively p , q and r , then [Guj. CET 2015]
 (a) $p > q$, $q > r$ (b) $p < q$, $q > r$
 (c) $p < q$, $q < r$ (d) $p > q$, $q < r$

- 9** Which radiations are used in treatment of muscles ache? [Manipal 2015]
 (a) Ultraviolet (b) Infrared
 (c) Microwave (d) X-rays
- 10** The ratio of amplitude of magnetic field to the amplitude of electric field for an electromagnetic wave propagating in vacuum is equal to [CBSE AIPMT 2014]
 (a) the speed of light in vacuum
 (b) reciprocal of speed of light in vacuum
 (c) the ratio of magnetic permeability to the electric susceptibility of vacuum
 (d) unity
- 11** In electromagnetic wave, according to Maxwell, changing electric field gives [MHT CET 2014]
 (a) stationary magnetic field
 (b) conduction current
 (c) eddy current
 (d) displacement current
- 12** The speed of light in an isotropic medium depends on [Kerala CEE 2014]
 (a) the nature of the source
 (b) its direction of propagation
 (c) its intensity
 (d) the motion of the source relative to the medium
 (e) None of the above
- 13** The energy of γ -ray photon is E_γ and that of an X-ray photon is E_X . If the visible light photon has an energy of E_v , then we can say that [WB JEE 2014]
 (a) $E_X > E_\gamma > E_v$ (b) $E_\gamma > E_v > E_X$
 (c) $E_\gamma > E_X > E_v$ (d) $E_X > E_v > E_\gamma$
- 14** The speed of electromagnetic waves in vacuum is equal to [UK PMT 2014]
 (a) $\mu_0 \epsilon_0$ (b) $\sqrt{\mu_0 \epsilon_0}$
 (c) $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ (d) $\frac{1}{\mu_0 \epsilon_0}$
- 15** A plane electromagnetic wave of frequency 20 MHz travels through a space along x-direction. If the electric field vector at a certain point in space is 6 V m^{-1} , then what is the magnetic field vector at that point? [KCET 2014]
 (a) $2 \times 10^{-8} \text{ T}$ (b) $\frac{1}{2} \times 10^{-8} \text{ T}$
 (c) 2 T (d) $\frac{1}{2} \text{ T}$
- 16** In electromagnetic spectrum, the frequencies of γ -rays and X-rays and ultraviolet rays are denoted by n_1, n_2 and n_3 respectively, then [MHT CET 2014]
 (a) $n_1 > n_2 > n_3$ (b) $n_1 < n_2 < n_3$
 (c) $n_1 > n_2 < n_3$ (d) $n_1 < n_2 > n_3$
- 17** The electromagnetic waves detected using a thermopile and used in physical therapy are [Kerala CEE 2014]
 (a) gamma radiations (b) X-rays
 (c) ultraviolet radiations (d) infrared radiations
 (e) microwave radiations
- 18.** The wavelength of X-rays is in the range [Guj. CET 2014]
 (a) 0.01 Å to 1 Å (b) 0.001 nm to 1 nm
 (c) 0.001 μm to 1 μm (d) 0.001 cm to 1 cm
- 19** The wavelength of the short radio waves, microwaves, ultraviolet waves are λ_1, λ_2 and λ_3 , respectively. Arrange them in decreasing order. [Guj. CET 2014]
 (a) $\lambda_1, \lambda_2, \lambda_3$ (b) $\lambda_1, \lambda_3, \lambda_2$ (c) $\lambda_3, \lambda_2, \lambda_1$ (d) $\lambda_2, \lambda_1, \lambda_3$
- 20** The condition under which a microwave oven heats up a food item containing water molecules most efficiently is [NEET 2013]
 (a) the frequency of the microwave must match the resonant frequency of the water molecules
 (b) the frequency of the microwave has no relation with natural frequency of water molecules
 (c) microwaves are heat waves, so always produce heating
 (d) infrared waves produce heating in a microwave oven
- 21.** Which component of electromagnetic spectrum have maximum wavelength? [J & K CET 2013]
 (a) Radio waves (b) Visible spectrum
 (c) Gamma rays (d) X-rays
- 22** The electric field associated with an electromagnetic wave in vacuum is given by $\mathbf{E} = \hat{i} 40 \cos(kz - 6 \times 10^8 t)$, where E , z and t are in volt m^{-1} , metre and second, respectively. The value of wave vector k is [CBSE AIPMT (Screening) 2012]
 (a) 2 m^{-1} (b) 0.5 m^{-1} (c) 6 m^{-1} (d) 3 m^{-1}
- 23** The wave function (in SI unit) for a light wave is given as $\psi(x, t) = 10^3 \sin \pi(3 \times 10^6 x - 9 \times 10^{14} t)$. The frequency of the wave is equal to [AMU 2012]
 (a) $4.5 \times 10^{14} \text{ Hz}$ (b) $3.5 \times 10^{14} \text{ Hz}$
 (c) $3.5 \times 10^{10} \text{ Hz}$ (d) $2.5 \times 10^{10} \text{ Hz}$

ANSWERS

CHECK POINT 8.1

1. (c)	2. (c)	3. (d)	4. (d)	5. (b)	6. (a)	7. (a)
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CHECK POINT 8.2

1. (b)	2. (b)	3. (c)	4. (c)	5. (a)	6. (c)	7. (b)	8. (b)	9. (d)	10. (c)
11. (a)	12. (b)	13. (d)	14. (a)	15. (d)					

(A) Taking it together

1. (b)	2. (c)	3. (a)	4. (b)	5. (a)	6. (b)	7. (c)	8. (b)	9. (c)	10. (b)
11. (d)	12. (a)	13. (d)	14. (b)	15. (d)	16. (d)	17. (d)	18. (c)	19. (c)	20. (b)
21. (b)	22. (c)	23. (d)	24. (c)	25. (b)	26. (c)	27. (d)	28. (a)	29. (a)	30. (b)
31. (c)	32. (a)	33. (b)	34. (b)	35. (d)	36. (b)	37. (a)	38. (c)	39. (c)	40. (b)
41. (a)									

(B) Medical entrance special format questions

• Assertion and reason

1. (a) 2. (b) 3. (b)

• Statement based questions

1. (b) 2. (c) 3. (a) 4. (c)

• Match the columns

1. (b) 2. (c)

(C) Medical entrances' gallery

1. (a)	2. (c)	3. (b)	4. (a)	5. (b)	6. (c)	7. (c)	8. (b)	9. (b)	10. (b)
11. (d)	12. (e)	13. (c)	14. (c)	15. (a)	16. (a)	17. (d)	18. (b)	19. (a)	20. (a)
21. (a)	22. (a)	23. (a)							

Hints & Explanations

CHECK POINT 8.1

3 (d) We have, displacement current, $I_d = \epsilon_0 \frac{d\phi_E}{dt}$

So, dimensions of $\epsilon_0 \frac{d\phi_E}{dt}$ are same as that of current.

4 (d) Ampere-Maxwell's circuital law is given by

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \cdot (I_c + I_d) = \mu_0 \left(I_c + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

5 (b) When capacitor is fully charged (i.e. charge on plates of capacitor is maximum) no current flows through the circuit. The potential of the plates became constant and there is no change in the electric field between the plates. So, both conduction current in the circuit and displacement current between the plates became zero.

6 (a) We know that, displacement current, $I_d = \epsilon_0 A \frac{dE}{dt}$

We have, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

Area, $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$ and $\frac{dE}{dt} = 3 \times 10^6 \text{ Vm}^{-1} \text{ s}^{-1}$

$\therefore I_d = 8.85 \times 10^{-12} \times 10^{-4} \times 3 \times 10^6 \approx 2.7 \times 10^{-9} \text{ A}$

7 (a) Displacement current, $I_d = \epsilon_0 \frac{d\phi_E}{dt}$

$$= \epsilon_0 \frac{EA}{t} = \frac{\epsilon_0 (V/d) \times A}{t} = \frac{\epsilon_0 VA}{td}$$

$$= \frac{8.85 \times 10^{-12} \times 400 \times (60 \times 10^{-4})}{10^{-6} \times (2 \times 10^{-3})}$$

$$= 1.062 \times 10^{-2} \text{ A}$$

CHECK POINT 8.2

1 (b) Frequency of L-C oscillations,

$$v = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14 \sqrt{100 \times 10^{-6} \times 200 \times 10^{-12}}}$$

$$= 1.126 \times 10^6 \text{ Hz}$$

So, wavelength, $\lambda = \frac{c}{v} = \frac{3 \times 10^8}{1.126 \times 10^6}$

$$= 266.4 \approx 266 \text{ m}$$

3 (c) The angular wave number, $k = \frac{2\pi}{\lambda}$, where λ is the wavelength. The angular frequency, $\omega = 2\pi v$

Ratio, $\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi v} = \frac{1}{v\lambda} = \frac{1}{c} = \text{constant}$

7 (b) As, $\frac{E_0}{B_0} = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \Rightarrow \sqrt{\mu_0 \epsilon_0} = \frac{B_0}{E_0}$

Here, c = speed of light

8 (b) As, $c = \frac{E_0}{B_0} \Rightarrow B_0 = \frac{E_0}{c} = \frac{18}{3 \times 10^8} = 6 \times 10^{-8} \text{ T}$

9 (d) Average energy density,

$$u_{av} = \frac{1}{2} \epsilon_0 E_0^2 = \frac{1}{2} \epsilon_0 (\sqrt{2} E_{rms})^2 = \epsilon_0 E_{rms}^2$$

$$= 8.85 \times 10^{-12} \times (720)^2 = 4.58 \times 10^{-6} \text{ Jm}^{-3}$$

10 (c) Amplitude of electric field and magnetic field are related by the relation, $\frac{E_0}{B_0} = c$

Average energy density of the magnetic field,

$$u_B = \frac{1}{2} \frac{B_0^2}{\mu_0} = \frac{1}{2} \frac{E_0^2}{\mu_0 c^2} \quad \left(\because B_0 = \frac{E_0}{c} \right)$$

$$= \frac{1}{2} \epsilon_0 E_0^2 \quad \left(\because c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \right)$$

$$= \frac{1}{2} \times 8.85 \times 10^{-12} \times 2^2$$

$$= 17.71 \times 10^{-12} \text{ Jm}^{-3}$$

11 (a) Here, $E_0 = 100 \text{ Vm}^{-1}$, $B_0 = 0.265 \text{ T}$

$\therefore$ Maximum rate of energy flow, $S = \frac{E_0 \times B_0}{\mu_0} = \frac{100 \times 0.265}{4\pi \times 10^{-7}}$

$$= 21 \times 10^6 \text{ Wm}^{-2}$$

12. (b) As metal is reflecting surface and for this type of surface radiation pressure is given as

$$p_r = \frac{2S}{c} = \frac{2 \times 0.5}{3 \times 10^8} = 0.33 \times 10^{-8} \text{ Nm}^{-2}$$

(A) Taking it together

1 (b) Infrasonic wave is not an electromagnetic wave. It is a mechanical wave which requires a material medium for propagation.

10 (b) Radio waves diffract around building although light waves do not. The reason is that radio waves have longer wavelength than light.

14 (b) Infrared radiations are reflected by lower clouds and keeps the earth warm. Hence, it shows greenhouse effect.

15 (d) $\lambda_{\text{Microwaves}} > \lambda_{\text{IR-rays}} > \lambda_{\text{UV-rays}} > \lambda_{\text{X-rays}}$

Hence, microwaves are less energetic electromagnetic waves, which are used in RADAR system for aircraft navigation.

- 17** (d) The energy in EM waves is divided equally between the electric and magnetic fields.

The total energy per unit volume, $u = u_e + u_m$

$$= \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2 \mu_0} B^2$$

- 18** (c) For a medium with permittivity ϵ and permeability μ , the velocity of light is given by $c = \frac{1}{\sqrt{\mu\epsilon}}$.

- 19** (c) Speed of electromagnetic waves in vacuum $= \frac{1}{\sqrt{\mu_0 \epsilon_0}} = v\lambda$
= constant

As, we go from γ -rays to radio waves frequency decreases and wavelength increases thereby maintaining the product constant.

- 20** (b) $\therefore$ Speed, $c = v\lambda$

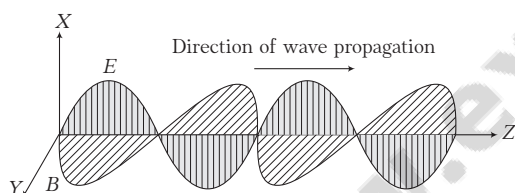
$$\therefore \text{Wavelength, } \lambda = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} \text{ m}$$

- 21** (b) The wavelength of the γ -rays is shorter. However, the essential distinction between them is the nature of emission.

- 22** (c) Total power $= 10^4 \times (10 \times 10) = 10^6 \text{ W}$

- 23** (d) The direction of propagation of electromagnetic wave is perpendicular to both electric field vector $\mathbf{E}$ and magnetic field vector $\mathbf{B}$, i.e. in the direction of $\mathbf{E} \times \mathbf{B}$.

This can be seen by the diagram given below



Here, electromagnetic wave is along the z-direction which is given by the cross product of $\mathbf{E}$ and $\mathbf{B}$.

- 24** (c) From a dipole antenna, the electromagnetic waves are radiated outwards.

The amplitude of electric field vector E_0 which transports significant energy from the source falls off inversely as the distance r from the antenna, i.e. $E_0 \propto \frac{1}{r}$.

- 25** (b) Given, $E = 9 \times 10^{-4} \text{ NC}^{-1}$

We know that, $c = \frac{E}{B}$

$$\Rightarrow B = \frac{9 \times 10^{-4}}{3 \times 10^8} = 3 \times 10^{-12} \text{ T}$$

- 26** (c) When a surface intercepts electromagnetic radiation, a force and a pressure are exerted on the surface. As the surface is non-reflecting, so it is completely absorbed and in such case, the force is

$$F = \frac{IA}{c}$$

The radiation pressure is the force per unit area,

$$p = \frac{F}{A} = \frac{I}{c}$$

- 28** (a) Displacement current is given by

$$I_d = \frac{dq}{dt} = 1.8 \times 10^{-8} \text{ Cs}^{-1}$$

- 29** (a) As metal is a reflecting surface and for reflecting surface, radiation pressure,

$$p_r = \frac{2I}{c} = \frac{2 \times 1}{3 \times 10^8} = 0.66 \times 10^{-8} \text{ Nm}^{-2}$$

- 30** (b) Frequency of L-C oscillations,

$$\nu = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14 \sqrt{200 \times 10^{-6} \times 200 \times 10^{-12}}}$$

$$= 0.0796 \times 10^7 \text{ Hz}$$

$$\text{So, wavelength, } \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{0.0796 \times 10^7} \approx 377 \text{ m}$$

- 31** (c) We have, $B_y = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$

Comparing with the standard equation, we get

$$B_y = B_0 \sin(kx + \omega t)$$

$$k = 0.5 \times 10^3$$

$$\text{Wavelength, } \lambda = \frac{2\pi}{0.5 \times 10^3} = 0.01256 \text{ m} \quad \left(\because \lambda = \frac{2\pi}{k} \right)$$

The wavelength range of microwaves is 10^{-3} m to 0.1 m . The wavelength of this wave lies between 10^{-3} m to 0.1 m , so the equation represents a microwave.

- 32** (a) From $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$, we get

$$\frac{k}{\omega} = \frac{\frac{2\pi}{\lambda}}{2\pi\nu} = \frac{1}{\nu\lambda} = \frac{1}{c}$$

$$\text{As, } \frac{E_0}{B_0} = c \Rightarrow \frac{k}{\omega} = \frac{B_0}{E_0}$$

$$\Rightarrow E_0 k = B_0 \omega$$

- 33** (b) When a wave is reflected from the denser medium, then the type of wave does not change but only its phase changes by 180° or π radian.

Thus, for the reflected wave $\mathbf{z} = -\mathbf{z}$, $\hat{\mathbf{i}} = -\hat{\mathbf{i}}$ and additional phase of π in the incident wave.

Here, the incident EM wave, $\mathbf{E} = E_0 \hat{\mathbf{i}} \cos(\mathbf{kz} - \omega t)$

$$\text{Hence, the reflected EM wave, } \mathbf{E}_r = E_0(-\hat{\mathbf{i}}) \cos[\mathbf{k}(-z) - \omega t + \pi]$$

$$= -E_0 \hat{\mathbf{i}} \cos[-(\mathbf{kz} + \omega t) + \pi]$$

$$= E_0 \hat{\mathbf{i}} \cos(\mathbf{kz} + \omega t)$$

- 34** (b) Angular frequency, $\omega = 2\pi\nu = \frac{2\pi c}{\lambda}$ ($\because \nu = c/\lambda$)

$$= \frac{2\pi \times 3 \times 10^8}{6 \times 10^{-3}} = \pi \times 10^{11} \text{ rads}^{-1}$$

The equation for the electric field along Y-axis in the electromagnetic wave,

$$E_y = E_0 \sin \omega \left(t - \frac{x}{c} \right) = 33 \sin \pi \times 10^{11} (t - x/c)$$

- 35** (d) Here, in electromagnetic wave, the electric field vector is given as, $\mathbf{E} = (E_1 \hat{\mathbf{i}} + E_2 \hat{\mathbf{j}}) \cos(kz - \omega t)$

In electromagnetic wave, the associated magnetic field vector,

$$\mathbf{B} = \frac{\mathbf{E}}{c} = \frac{E_1 \hat{\mathbf{i}} + E_2 \hat{\mathbf{j}}}{c} \cos(kz - \omega t)$$

Also, $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other and the propagation of electromagnetic wave is perpendicular to $\mathbf{E}$ as well as $\mathbf{B}$, so the given electromagnetic wave is plane polarised.

- 36** (b) Let E = energy falling on the surface per second = 10 W/cm^2

$$\text{Momentum of photons, } p = \frac{h}{\lambda} = \frac{h}{(c/v)} = \frac{hv}{c} = \frac{E}{c}$$

On reflection, change in momentum per second

$$= 2p = \frac{2E}{c} = \frac{2 \times 10}{3 \times 10^8} = 6.67 \times 10^{-8} \text{ N}$$

- 37** (a) The magnetic field component is perpendicular to the direction of propagation and the direction of electric field.

Using vector algebra $\mathbf{E} \times \mathbf{B}$ should be in z-direction.

$$\therefore (\hat{\mathbf{i}} + \hat{\mathbf{j}}) \times (-\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 2\hat{\mathbf{k}}$$

- 38** (c) Given, energy required to dissociate a carbon monoxide molecule into carbon and oxygen atoms, $E = 11 \text{ eV}$

We know that, $E = hv$

where, $h = 6.62 \times 10^{-34} \text{ J-s}$, v = frequency

$$\Rightarrow 11 \text{ eV} = hv$$

$$\begin{aligned} \text{or } v &= \frac{11 \times 1.6 \times 10^{-19}}{h} \quad (\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}) \\ &= \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} \\ &= 2.65 \times 10^{15} \text{ Hz} \end{aligned}$$

This frequency radiation belongs to ultraviolet region.

- 39** (c) We know that, $E_0 \propto \sqrt{P_{av}}$

$$\therefore \frac{(E_0)_1}{(E_0)_2} = \sqrt{\frac{(P_{av})_1}{(P_{av})_2}}$$

$$\Rightarrow \frac{E}{(E_0)_2} = \sqrt{\frac{100}{50}}$$

$$\Rightarrow (E_0)_2 = E / \sqrt{2}$$

- 40** (b) Given, energy flux, $\phi = 20 \text{ W/cm}^2$

Area, $A = 30 \text{ cm}^2$

Time, $t = 30 \text{ min} = 30 \times 60 \text{ s}$

Now, total energy falling on the surface in time t ,

$$u = \phi At = 20 \times 30 \times (30 \times 60) \text{ J}$$

$$\begin{aligned} \text{Momentum of the incident light} &= \frac{u}{c} \\ &= \frac{20 \times 30 \times (30 \times 60)}{3 \times 10^8} \\ &= 36 \times 10^{-4} \text{ kg-m/s} \end{aligned}$$

Momentum of the reflected light = 0

$$\therefore \text{Momentum delivered to the surface} = 36 \times 10^{-4} - 0$$

$$= 36 \times 10^{-4} \text{ kg-m/s}$$

- 41** (a) Effective power of the bulb, $P = \frac{5}{100} \times 200 = 10 \text{ W}$

Intensity at distance r is given by

$$\begin{aligned} I &= \frac{P}{4\pi r^2} = \frac{10}{4 \times 3.14 \times 2^2} \\ &= \frac{10}{16 \times 3.14} \\ &= 0.199 \approx 0.2 \text{ W/m}^2 \end{aligned}$$

$$\text{Also, } I = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$\Rightarrow E_0^2 = \frac{2I}{c \times \epsilon_0}$$

$$\begin{aligned} \text{Magnitude of electric field, } E_0 &= \sqrt{\frac{2I}{c \times \epsilon_0}} \\ &= \sqrt{\frac{2 \times 0.2}{3 \times 10^8 \times 8.85 \times 10^{-12}}} \\ &= 12.27 \text{ N/C} \end{aligned}$$

$$\begin{aligned} \therefore \text{Magnitude of magnetic field, } B_0 &= \frac{E_0}{c} = \frac{12.27}{3 \times 10^8} \\ &= \frac{12.27}{3} \times 10^{-8} \text{ T} \\ &= 4.09 \times 10^{-8} \text{ T} \end{aligned}$$

(B) Medical entrance special format questions

• Assertion and reason

1 (a) As per Maxwell's equations, electric and magnetic fields in an electromagnetic waves are perpendicular to each other and to the direction of wave propagation. Because, electromagnetic waves are transverse in nature.

2 (b) The wavelength of ultraviolet waves is very small. Due to smaller wavelength and higher frequency, they are absorbed by atmosphere. They also cause skin diseases and are harmful to eye which may cause permanent blindness.

3 (b) The oscillating electric and magnetic fields, thus regenerate each other, as the wave propagates through the space.

The frequency of the electromagnetic wave is equals to the frequency of accelerating charge. The energy associated with the propagating wave comes at the expense of the energy of the source or the accelerated charge.

• Statement based questions

- 2** (c) The time varying electric and magnetic fields are mutually perpendicular to each other and also perpendicular to the direction of propagation of the wave.
- 3** (a) The frequency range of microwave is from 10^9 to 10^{11} Hz.
- 4** (c) Light radiations and thermal radiations both belong to electromagnetic spectrum. Light radiation belongs to visible region while thermal radiation belongs to infrared region of EM spectrum.
- Also, EM radiations require no medium for propagation.

(C) Medical entrances' gallery

- 1** (a) Intensity in terms of electric field, $(I)_{\text{electric field}} = \frac{1}{2} \epsilon_0 E_0^2 \cdot c$

$$\text{Intensity in terms of magnetic field, } (I)_{\text{magnetic field}} = \frac{1}{2} \frac{B_0^2}{\mu_0} \cdot c$$

Now, taking the intensity in terms of electric field,

$$\begin{aligned} (I)_{\text{electric field}} &= \frac{1}{2} \epsilon_0 E_0^2 \cdot c \\ \Rightarrow &= \frac{1}{2} \epsilon_0 (cB_0)^2 \cdot c \quad (\because E_0 = cB_0) \\ &= \frac{1}{2} \epsilon_0 \times c^2 B_0^2 \cdot c \end{aligned}$$

$$\text{But, } c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\begin{aligned} \therefore (I)_{\text{electric field}} &= \frac{1}{2} \epsilon_0 \times \frac{1}{\mu_0 \epsilon_0} B_0^2 c \\ &= \frac{1}{2} \frac{B_0^2}{\mu_0} \cdot c \\ &= (I)_{\text{magnetic field}} \end{aligned}$$

Thus, the energy in electromagnetic wave is divided equally between electric field vector and magnetic field vector.

Therefore, the ratio of contributions by the electric field and magnetic field components to the intensity of an electromagnetic wave is 1 : 1.

- 2** (c) Gamma(γ)-rays has the shortest wavelength because it has higher frequency than UV-rays, microwaves and X-rays.
- 3** (b) Magnetic field in plane electromagnetic wave is given as

$$B_y = 2 \times 10^{-7} \sin(\pi \times 10^3 x + 3\pi \times 10^{11} t) \text{ T}$$

Comparing with $B = B_0 \sin(kx - \omega t)$, we get

$$\begin{aligned} k &= \pi \times 10^3 \\ \Rightarrow \frac{2\pi}{\lambda} &= \pi \times 10^3 \\ \Rightarrow \lambda &= \frac{2}{10^3} = 2 \times 10^{-3} \text{ m} \end{aligned}$$

- 4** (a) Given, $C = 20 \mu\text{F} = 20 \times 10^{-6} \text{ F}$

$$\text{and } \frac{dV}{dt} = 3 \text{ V/s}$$

The displacement current in a circuit is given by $I_d = \epsilon_0 \frac{d\phi}{dt}$

$$= \epsilon_0 \frac{d}{dt} (EA) \quad (\because \phi = EA)$$

$$= \epsilon_0 A \frac{d}{dt} \left(\frac{V}{d} \right) \quad (\because V = Ed)$$

$$= \frac{\epsilon_0 A}{d} \frac{dV}{dt}$$

As the capacitance of a parallel plate capacitor, $C = \frac{\epsilon_0 A}{d}$

$$\therefore I_d = C \frac{dV}{dt}$$

Substituting the given values in the above relation, we get

$$\begin{aligned} I_d &= 20 \times 10^{-6} \times 3 \\ &= 60 \times 10^{-6} \text{ A} = 60 \mu\text{A} \end{aligned}$$

As displacement current is in between the plates of capacitor and conduction current is in the connecting wires which are equal to each other. So,

$$I_c = I_d = 60 \mu\text{A}$$

- 5** (b) Here, velocity of EM wave, $\mathbf{v} = v\hat{\mathbf{i}}$

Instantaneous oscillating electric field, $\mathbf{E} = E\hat{\mathbf{j}}$

As we know that, during the propagation of electromagnetic waves through a medium, oscillating electric and magnetic field vectors are mutually perpendicular to each other and also to the direction of propagation of the wave.

$$\text{i.e. } \mathbf{E} \times \mathbf{B} = \mathbf{v}$$

$$\Rightarrow (E\hat{\mathbf{j}}) \times \mathbf{B} = v\hat{\mathbf{i}} \quad \dots(i)$$

As we know that, from vector algebra,

$$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$\mathbf{B} = B\hat{\mathbf{k}}$$

where, B (say) is the magnitude of magnetic field.

Thus, we can say that the direction of oscillating magnetic field of the EM wave will be along +z-direction.

- 6** (c) Electric charge at rest has electric field in the region around it, but no magnetic field. Whereas, a moving charge produces both the electric and magnetic fields. If a charge is moving with a constant velocity, then the electric and magnetic fields will not change with time, hence no EM wave will be produced. But, if the charge is moving with a non-zero acceleration, then both the electric and magnetic field will change with space and time, it then produces EM wave.

$$\begin{aligned} \text{7 (c) The intensity is, } I &= \frac{1}{2} \epsilon_0 E_0^2 c \\ &= \frac{1}{2} (8.85 \times 10^{-12}) \times (50)^2 \times 3 \times 10^8 \\ &= 3.3 \text{ Wm}^{-2} \end{aligned}$$

- 8** (b) Increasing order of wavelengths for the given EM waves are as given below

$\gamma\text{-rays} < \text{X-rays} < \text{Ultraviolet rays}$

Since, frequency $\propto \frac{1}{\text{wavelength}}$

$\therefore$ Frequency order, γ -rays > X-rays > Ultraviolet rays

$$\Rightarrow q > p > r$$

$$\text{Thus, } q > p$$

$$q > r$$

$$p > r$$

9 (b) Infrared rays are used for the treatment of muscles ache.

10 (b) The ratio of amplitude of magnetic field to the amplitude of electric field for an electromagnetic wave propagating in vacuum is equal to the reciprocal of speed of light in vacuum.

$$\text{i.e. } \frac{B_0}{E_0} = \frac{1}{c}$$

11 (d) According to Maxwell, time varying electric field produced displacement current.

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

12 (e) The speed of light in an isotropic medium depends on its wavelength, $v = \frac{c}{\lambda}$.

13 (c) $\because v_\gamma > v_X > v_v$

$\therefore$ We can say that, $E_\gamma > E_X > E_v$.

14 (c) According to Maxwell, it is found that the electromagnetic waves travel in free space (or vacuum) with a speed which is given by $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ ms}^{-1}$

where, $\mu_0 (= 4\pi \times 10^{-7} \text{ Wb A}^{-1} \text{ m}^{-1})$ and ϵ_0

($= 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$) are absolute permeability and permittivity of the free space, respectively.

15 (a) The magnetic field, $B_0 = \frac{E_0}{c}$, where $c = 3 \times 10^8 \text{ ms}^{-1}$

$$\Rightarrow B_0 = \frac{6}{3 \times 10^8} = 2 \times 10^{-8} \text{ T}$$

16 (a) Frequency (n_3) range of ultraviolet rays $\approx 10^{17}$ - 10^{14}

Frequency (n_2) range of X-rays $\approx 10^{20}$ - 10^{17}

Frequency (n_1) range of γ -rays $> 10^{20}$

$$\therefore n_1 > n_2 > n_3$$

17 (d) The electromagnetic waves detected using a thermopile and used in physical therapy are infrared radiations for treating muscular strain.

18 (b) X-rays wavelength range = $1 \times 10^{-12} \text{ m}$ to $1 \times 10^{-9} \text{ m}$
= 0.001 nm to 1 nm

19 (a) We know that

(i) Wavelength range of short radio waves,
= $1 \times 10^{-1} \text{ m}$ to $1 \times 10^4 \text{ m}$

(ii) Microwaves wavelength range
= $1 \times 10^{-3} \text{ m}$ to $3 \times 10^{-1} \text{ m}$

(iii) Ultraviolet waves wavelength range
= $1 \times 10^{-9} \text{ m}$ to $4 \times 10^{-7} \text{ m}$

So, decreasing order is $\lambda_1, \lambda_2, \lambda_3$.

21 (a) Radio waves have maximum wavelength in the range of 0.1 m to 10^4 m .

22 (a) Electromagnetic wave equation,

$$E = E_0 \cos(kz - \omega t) \quad \dots(i)$$

Speed of electromagnetic wave,

$$v = \frac{\omega}{k}$$

Given, equation,

$$E = \hat{i} 40 \cos(kz - 6 \times 10^8 t) \quad \dots(ii)$$

On comparing Eqs. (i) and (ii), we get

$$\omega = 6 \times 10^8$$

and $E_0 = 40$

Here, wave factor, $k = \frac{\omega}{v} = \frac{6 \times 10^8}{3 \times 10^8} = 2 \text{ m}^{-1}$

23 (a) The wave function

$$\psi(x, t) = 10^3 \sin \pi (3 \times 10^6 x - 9 \times 10^{14} t)$$

Here, angular frequency, $\omega = 9 \times 10^{14} \pi$

$$\omega = 2\pi v$$

$$\Rightarrow v = \frac{\omega}{2\pi} = \frac{9 \times 10^{14} \pi}{2\pi} = 4.5 \times 10^{14} \text{ Hz}$$

CHAPTER 09

Ray Optics

Light is a form of energy which produces sensation of vision to our eyes and makes objects visible. The branch of physics which deals with the study of nature of light, its properties, effects and propagation is known as optics. The subject of optics can be divided into two main branches—ray optics and wave optics.

- (i) **Ray or geometrical optics** This branch of optics deals with propagation of light in terms of rays which are valid, if sizes of obstacles are large in comparison with wavelength of light (in nm range). Ray optics concerns itself with image formation, reflection, refraction and dispersion of light by geometrical methods.
- (ii) **Wave or physical optics** This branch of optics deals with wave phenomenon like interference, diffraction and polarisation.

In this chapter, we are going to learn ray optics step by step followed by light sources as well as properties of light.

Light sources

We obtain light from different types of natural sources like sun, stars etc, and from artificial sources (man made) like-candle, electric bulb, etc. On the basis of this, light source can be categorised in two types as follows

- (i) **Luminous sources** These sources can produce their own light, *e.g.* sun, stars, electric bulb, etc.
- (ii) **Non-luminous sources** These sources cannot produce their own light but return the light coming from a luminous source, *e.g.* moon, glass, etc.

Properties of light

Following are the properties of light

- (i) Light is itself invisible but in its presence we can see objects due to scattering of light.
- (ii) Light is an electromagnetic wave which requires no medium for its propagation and its speed in vacuum is 3×10^8 m/s.
- (iii) The wavelength of visible light ranges from 400 nm to 700 nm.
- (iv) Light can be considered to propagate from one point to another, along a straight line joining them. The path is called a **ray of light** and a bundle of such rays constitutes a **beam of light**.
- (v) Light can pass through transparent mediums like glass, air, etc. However, it cannot pass through non-transparent mediums like wood, iron, etc.

Inside

1 Reflection of light

- Reflection by a plane mirror

2 Spherical mirrors

- Image formation by spherical mirrors
- Mirror formula
- Magnification
- Uses of spherical mirrors

3 Refraction of light

- Refractive index (μ or n)
- Image due to refraction at a plane surface
- Refraction through a glass slab
- Critical angle and Total Internal Reflection (TIR)

4 Refraction at spherical surfaces

- Lenses
- Image formation by lens

5 Prism

- Dispersion of light by a prism
- Combination of prisms

6 Optical instruments

7 Defects of vision

- Defects of images

- (vi) Speed of light is different in different mediums.
- (vii) Light can reflect from polished surfaces.
- (viii) When light travels from one medium to another medium, it deviates from the path.

REFLECTION OF LIGHT

When light is incident on a polished smooth surface, then most of the incident light returns to the same medium. This phenomenon of returning of light after striking a smooth polished surface is called **reflection of light**.

After reflection, velocity, wavelength and frequency of light remains same but intensity decreases.

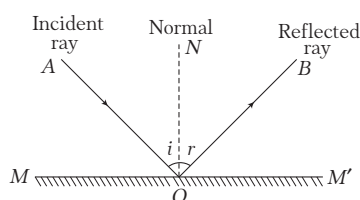


Fig. 9.1 Reflection of light (plane surface)

Some important terms related with reflection of light

- (i) **Reflecting surface** The surface, after striking which, light reflects is called reflecting surface (MM').
- (ii) **Incident ray** The incident ray (AO) on the reflecting surface is called incident ray.
- (iii) **Reflected ray** The reflected ray (OB) after striking the reflecting surface is called reflected ray.
- (iv) **Point of incidence** It is the point (O) on the reflecting surface at which incident light strikes.
- (v) **Normal** It is the normal (ON) to the surface drawn from point of incidence (O) on the reflecting surface.
- (vi) **Angle of incidence** It is the angle (i) between incident ray (AO) and the normal (ON). In the given figure, $\angle AON = i$.
- (vii) **Angle of reflection** It is the angle (r) between reflected ray (OB) and the normal (ON). In the given figure, $\angle NOB = r$.

Laws of reflection

There are two laws for reflection of light from a smooth reflecting surface.

- (i) **First law** The angle of incidence is equal to the angle of reflection, i.e. $\angle i = \angle r$.

- (ii) **Second law** The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

These laws are valid for plane as well as curved surfaces as shown in the figure given below.

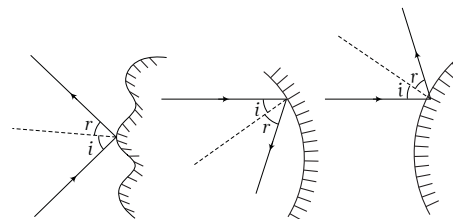


Fig. 9.2 Reflection of light from curved surfaces

Note

- (i) In geometrical optics angles are measured with normal.
- (ii) Also in geometrical optics, we deal with plane and spherical surfaces. In case of spherical surface, normal to surface will pass through centre of sphere.

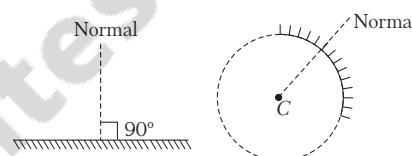
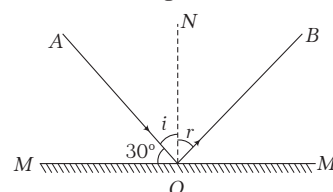


Fig. 9.3

Example 9.1 A ray of light is incident at an angle of 30° with the plane of a plane mirror. Find the angle of reflection.

Sol. Consider the situation shown in the figure. Angle made by the incident ray with the plane of the mirror MM' is $\angle AOM = 30^\circ$. Therefore, angle of incidence,



$$\angle AON = i = \angle NOM - \angle AOM = 90^\circ - 30^\circ = 60^\circ$$

Also, angle of reflection, $\angle BON = r = i = 60^\circ$

Image

Light rays emerging from an object (or a point) after reflection or refraction meets or appears to meet at a point is called image of object (or first point).

Images are of two types

(i) Real image

When light rays emerging from a point object really meets at a point, the later point is called real image of the first point. These types of images can be obtained on a screen.

(ii) Virtual image

When light rays emerging from a point object appears to meet at a point, the later point is called virtual image of

the first point. These types of images cannot be obtained on a screen.

Note If the incident rays diverge from a point object, the object is **real object**.

If rays are converging towards a mirror, the point where these rays would meet if there was no mirror is a **virtual object**.

Mirror

There are two types of mirror

- (i) **Plane mirror** The mirror whose reflecting surface is plane is called plane mirror. It's one side is silvered and reflection takes place from other side as shown in the figure.

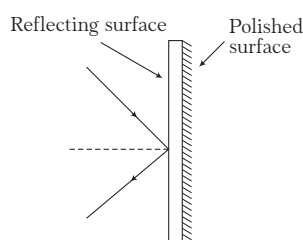


Fig. 9.4 Plane mirror

- (ii) **Spherical mirror** It is the part of a spherical transparent medium. One of the curved surfaces is polished and reflection takes place from other side. Spherical mirrors are of two types

- (a) **Concave mirror** In this mirror, outer curved surface is polished and reflection takes place from inner curved surface as shown in figure.

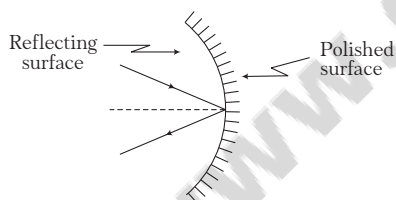


Fig. 9.5 Concave mirror

- (b) **Convex mirror** In this mirror, inner curved surface is polished and reflection takes place from outer curved surface as shown in figure.

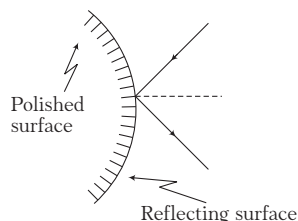


Fig. 9.6 Convex mirror

Reflection by a plane mirror

Characteristics of a plane mirror are given below

Properties of image formed by plane mirror

The image formed by a plane mirror is **virtual, erect**, of **same size** and **at the same distance from the mirror** as the distance of object from mirror. The ray diagram of the image of a point object and of an extended object is as shown below.

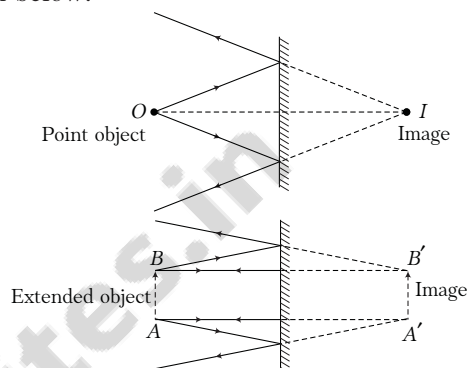


Fig. 9.7 Formation of images by a plane mirror

When a convergent beam is incident on the plane mirror, the object (O) is virtual as shown in figure. In this case image (I) formed by the plane mirror is real.

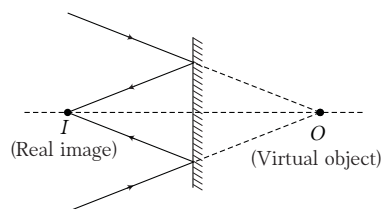


Fig. 9.8 Formation of image by plane mirror of a virtual object

Note To find the location of image of an object from an inclined plane mirror, you have to see the perpendicular distance of the object from the mirror.

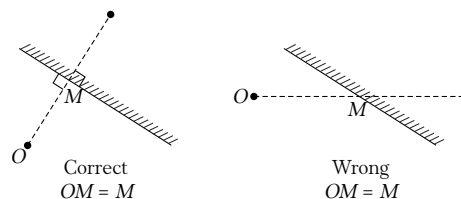


Fig. 9.9

Deviation produced by a plane mirror

The angle between reflected ray and the ray extended in the direction of incident ray, is called deviation produced by plane mirror.

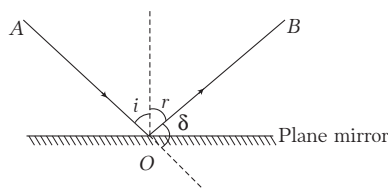


Fig. 9.10 Deviation produced by plane mirror due to single reflection

Deviation produced by plane mirror is given by

$$\begin{aligned}\delta &= 180^\circ - (i + r) \\ &= 180^\circ - 2i \\ &= 180^\circ - 2r \quad (\because i = r)\end{aligned}$$

The deviation produced by reflection at two mirrors inclined to each other at an angle θ is

$$\delta = 360^\circ - 2\theta$$

It is independent of the angle of incidence.

- (i) If two plane mirrors are inclined to each other at 90° , then the ray reflected at two mirrors becomes parallel to incident ray.

$$\text{As,} \quad \delta = 360^\circ - 2\theta$$

$$\text{Here,} \quad \theta = 90^\circ$$

$$\therefore \delta = 360^\circ - 2 \times 90^\circ = 180^\circ$$

i.e. the reflected ray is parallel to incident ray.

- (ii) If two plane mirrors are inclined to each other at 60° and a ray of light is incident on one mirror parallel to other, then after reflection from two mirrors, the reflected ray becomes parallel to the first mirror.

Rotation produced in reflected ray due to rotation of a plane mirror

- (i) For a given fixed incident ray, if the plane mirror is rotated through an angle θ in the plane of incidence, the reflected ray turns through an angle 2θ .

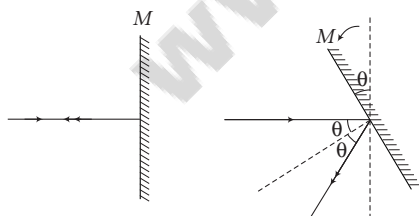


Fig. 9.11 Rotating plane mirror by angle θ

- (ii) If a plane mirror is rotated in its own plane, the incident ray and the reflected ray remain at same positions.

Field of view of an object for a plane mirror

The field of view is the region between the extreme reflected rays and depends on the location of the object in front of the mirror.

Two important results based on the field of view are given below

- (i) The minimum height of a plane mirror to see object's full height in it is $H/2$, where H is the height of object. But the mirror should be placed in a fixed position as shown in Fig. 9.12.

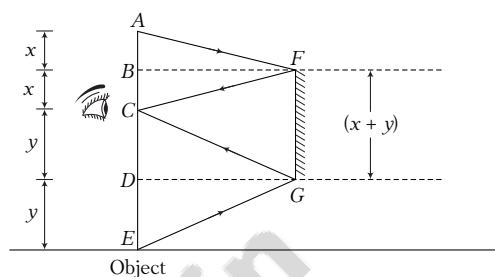


Fig. 9.12 Minimum height of plane mirror to see object's full height

$$\text{Height of the object } AE = 2(x + y) = H$$

$$\text{Height of the mirror} = (x + y) = \frac{H}{2}$$

Note The mirror can be placed anywhere between the centre line BF (of AC) and DG (of CE).

- (ii) Minimum height of the plane mirror fixed on the wall of a room in which an observer at the centre of the room can see the full image of the wall behind him, is one third the height of wall.

The minimum height of mirror in this case should be $H/3$, where H is the height of wall. The ray diagram in this case is drawn in Fig. 9.13.

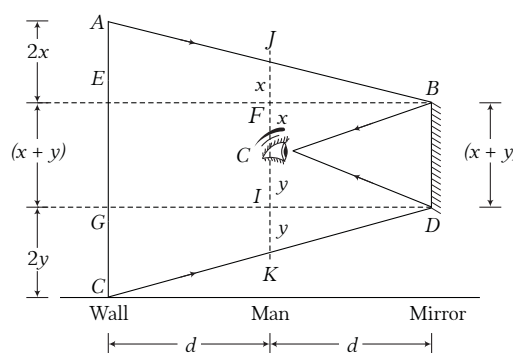


Fig. 9.13 Minimum height of plane mirror to see the wall of the room

Height of the wall is $3(x + y)$ while that of the mirror is $(x + y)$.

Special cases of image formation through a plane mirror

- (i) If the object is displaced by a distance a towards or away from the plane mirror, image will also be displaced by a distance a towards or away from the mirror.

- (ii) If an object approaches or recedes from the plane mirror with velocity v , then the image also moves in the same manner from the mirror with velocity v . But the velocity of the image with respect to object will be $2v$.
- (iii) If plane mirror moves a distance x towards or away from the object, the image will move a distance $2x$ towards or away from the mirror/object.
- (iv) If both plane mirror and object are moved by a distance x , each in opposite directions, the image will be displaced by a distance $3x$ in the direction of the displacement of the mirror.
- (v) If a luminous object is placed in front of a thick glass mirror, multiple images are formed due to multiple reflections. The second image formed by the first reflection by the polished surface is much brighter than the others. The intensity of the other images rapidly fade away.
- (vi) If an object is placed between two mirrors facing each other at an angle θ , then the number of images is given by $\frac{360^\circ}{\theta} = N$ and actual number of images are n , where

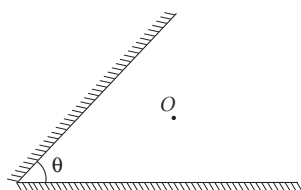


Fig. 9.14

- (a) $n = N - 1$, if N is even integer.
- (b) $n = N$, if N is odd integer and object is not on the bisector of mirrors.
- (c) $n = N - 1$, if N is odd integer and object is on the bisector of mirrors.
- (d) If $\frac{360^\circ}{\theta} = N$ is a fraction, the number of images will be equal to its lower boundary of integral part.

Note

- (i) If the mirrors are placed parallel to each other like shown below, then Number of images = ∞ and distance between n th images,

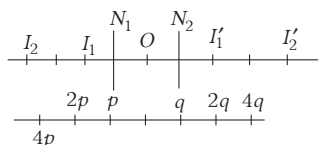
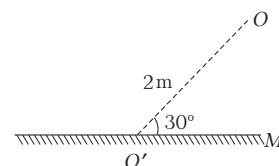


Fig. 9.15

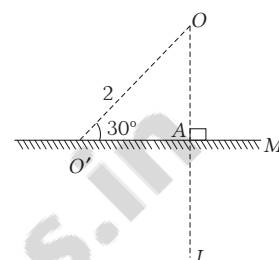
$$I_n I_n' = 2np + 2nq \quad \text{or} \quad I_n I_n' = 2n(p + q)$$

- (ii) The number of images formed by two mutually perpendicular ($\theta = 90^\circ$) mirrors will be 3. All these three images will lie on a circle with centre at C , the point of intersection of mirrors M_1 and M_2 and whose radius is equal to distance between C and the object O .

Example 9.2 A point object O is at an angle of 30° from the plane mirror M , as shown in figure. If $OO' = 2$ m, then find the location of image.



Sol. To find the location of image, we have to draw a perpendicular on the mirror M from point O as shown in figure.

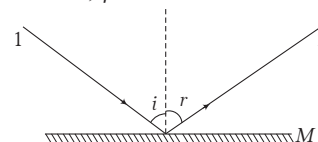


Now from the triangle $O'AO$, we have

$$\sin 30^\circ = \frac{OA}{2} \Rightarrow OA = 2 \times \sin 30^\circ = 1 \text{ m}$$

$\therefore$ Distance of the image from the mirror is $AI = OA = 1$ m.

Example 9.3 Consider a ray of light is incident on a plane mirror M . If deviation produced by the mirror in the incident light ray is 120° then, find i and r .



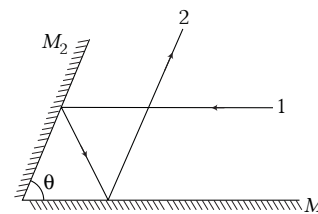
Sol. Deviation produced by the mirror is,

$$\delta = 180^\circ - 2i = 120^\circ \quad (\text{given})$$

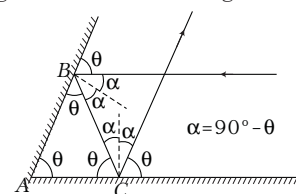
$$\Rightarrow 2i = 60^\circ \Rightarrow i = 30^\circ$$

$$\therefore r = i = 30^\circ$$

Example 9.4 Two plane mirrors M_1 and M_2 are inclined at angle θ as shown below. A ray of light 1, which is parallel to M_1 strikes M_2 and after two reflections, the ray 2 becomes parallel to M_2 . Find the angle θ .



Sol. Different angles are as shown in figure. In triangle ABC ,

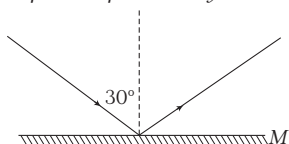


From $\triangle ABC$,

$$\theta + \theta + \theta = 180^\circ$$

$$\therefore \theta = 60^\circ$$

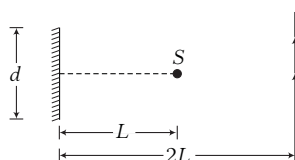
Example 9.5 A ray of light is incident on a plane mirror at an angle of 30° as shown in figure. Now the mirror is rotated through 30° in anti-clockwise direction. Find the angle and sense of rotation of the reflected ray.



Sol. Angle rotated by the plane mirror, $\theta = 30^\circ$, anti-clockwise.

$\therefore$ Angle rotated by reflected ray, $\theta_r = 2\theta = 60^\circ$, anti-clockwise.

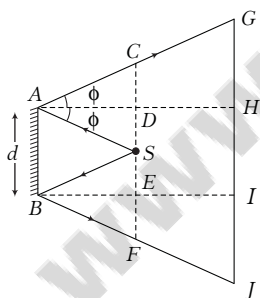
Example 9.6 A point source of light S , placed at a distance L in front of the centre of a mirror of width d , hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown below. Find the greatest distance over which he can see the image of the light source in the mirror.



Sol. The ray diagram will be as shown in figure given below.

$$HI = AB = d \Rightarrow DS = CD = d/2$$

Since, $AH = 2AD$

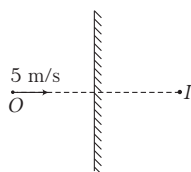


$$\therefore GH = 2CD = 2(d/2) = d$$

Similarly, $IJ = d$

$$\therefore GJ = GH + HI + IJ = d + d + d = 3d$$

Example 9.7 Consider a point object moving towards a stationary plane mirror with velocity 5 m/s . Find velocity of image w.r.t.



(i) the mirror

(ii) the object

Sol. The object is moving towards right with a velocity of $u = 5 \text{ m/s}$

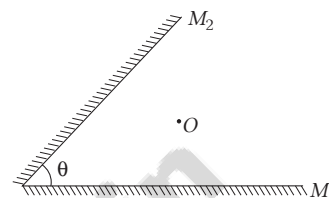
(i) Velocity of image w.r.t. the mirror,

$$\mathbf{v}_{IM} = -\mathbf{u} = 5 \text{ m/s, towards left.}$$

(ii) Velocity of image w.r.t. the object,

$$\mathbf{v}_{IO} = -2\mathbf{u} = 2 \times 5 \text{ m/s} = 10 \text{ m/s, towards left.}$$

Example 9.8 Consider two plane mirrors inclined at an angle θ as shown in figure. Find the number of images of object O formed by these mirrors when



(i) $\theta = 60^\circ$

(ii) $\theta = 72^\circ$ and the object is at 40° from M_1

(iii) $\theta = 72^\circ$ and the object is at 36° from M_1

(iv) $\theta = 80^\circ$

Sol. Two mirrors M_1 and M_2 are inclined at angle θ .

$$\text{Let, } N = \frac{360^\circ}{\theta}$$

(i) For $\theta = 60^\circ$, $N = \frac{360^\circ}{60^\circ} = 6 = \text{even integer}$

$\therefore$ Number of images formed

$$n = N - 1 = 6 - 1 = 5$$

(ii) For $\theta = 72^\circ$, $N = \frac{360^\circ}{72^\circ} = 5 = \text{odd integer}$ and the object is not on the bisector,

$\therefore$ Number of images formed, $n = N = 5$

(iii) For $\theta = 72^\circ$, $N = \frac{360^\circ}{72^\circ} = 5 = \text{odd integer}$ and the object is also at the bisector which is at 36° .

$\therefore$ Number of images formed

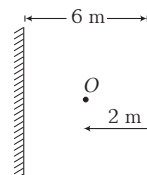
$$n = N - 1 = 5 - 1 = 4$$

(iv) For $\theta = 80^\circ$, $N = \frac{360^\circ}{80^\circ} = 4.5 = \text{a fraction}$

$\therefore$ Number of images formed,

$$n = \text{integral part} = 4$$

Example 9.9 Two plane mirrors are placed parallel to each other at a separation of 6 m as shown in figure. Find the (i) number of images of the object O and (ii) separation between 5th image.



Sol. Given, mirrors are placed parallel to each other. Therefore,

(i) Number of images formed $= \infty$

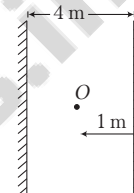
(ii) Distance between 5th images, $I_5 I_5' = 2 \times 5(x + y)$

$$\text{where, } x = 6 - 2 = 4 \text{ m} \Rightarrow y = 2 \text{ m}$$

$$\Rightarrow I_5 I_5' = 10(4 + 2) = 60 \text{ m}$$

CHECK POINT 9.1

- Light falls on a plane reflecting surface. For what angle of incidence is the reflected ray normal to the incident ray?
(a) 60° (b) 45°
(c) 90° (d) 30°
- Which of the following is correct for the image formed by a plane mirror?
(a) Always real
(b) Always virtual
(c) Virtual and laterally inverted
(d) Real and laterally inverted
- A plane mirror reflects a pencil of light to form a real image, then the pencil of light incident on the mirror is
(a) parallel (b) convergent
(c) divergent (d) None of these
- A ray of light is incident on a plane mirror at an angle of 30° . The deviation produced in the ray after reflection is
(a) 30° (b) 60° (c) 90° (d) 120°
- If the reflected ray is rotated by an angle of 40° in anti-clockwise direction, then the mirror was rotated by
(a) 20° in anti-clockwise direction
(b) 40° in anti-clockwise direction
(c) 20° in clockwise direction
(d) 40° in clockwise direction
- A man is 180 cm tall and his eyes are 10 cm below the top of his head. In order to see this entire height right from top to head, he uses a plane mirror kept at a distance of 1 m from him. The minimum length of the plane mirror is
(a) 180 cm (b) 90 cm (c) 85 cm (d) 170 cm
- An object is moving towards a stationary plane mirror with a speed of 2 m/s. Velocity of the image w.r.t. the object is
(a) 2 m/s (b) 4 m/s
(c) 1 m/s (d) None of these
- To get three images of a single object, one should have two plane mirrors at an angle of
(a) 30° (b) 60° (c) 90° (d) 150°
- An object (O) is placed between two parallel plane mirrors as shown in figure. Distance between 4th image is
(a) 16 m (b) 32 m (c) 8 m (d) 64 m

**SPHERICAL MIRRORS**

These mirrors are those types of mirrors whose reflecting surface is a part of a hollow sphere.

Spherical mirrors are of two types

(i) Concave mirror

A spherical mirror whose reflecting surface is towards the centre of the sphere is called concave mirror.

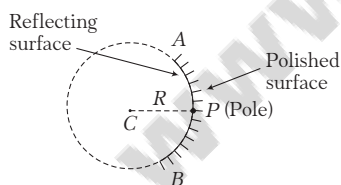


Fig. 9.16 Concave mirror as a part of hollow sphere

(ii) Convex Mirror

A spherical mirror whose reflecting surface is away from the centre of the sphere is called convex mirror.

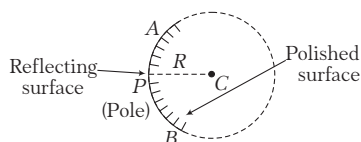


Fig. 9.17 Convex mirror as a part of hollow sphere

Terms related to spherical mirror

In case of these thin spherical mirrors important terms and their definitions are as follows

- Pole (P)** It is the mid-point of the mirror.
- Centre of curvature (C)** It is the centre of the sphere of which the mirror is a part.
- Radius of curvature (R)** Distance between pole and centre of curvature is called radius of curvature. In the given figure, the distance CP is radius of curvature.
- Principal axis** The line joining the pole P and centre of curvature C when produced is called principal axis.
- Aperture (AB)** Effective diameter of light reflecting area of the mirror is called aperture.
- Principal focus or focus** It is a point on the principal axis of the mirror at which the light rays coming parallel to principal axis actually meet after reflection (or appears to meet).

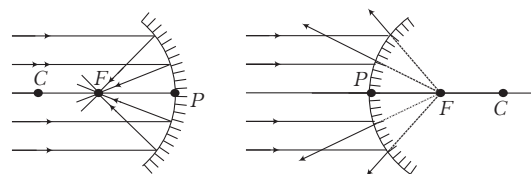


Fig. 9.18 Principal focus of (a) concave mirror (a converging mirror) (b) convex mirror (a diverging mirror)

Focus of concave mirror is real, while focus of convex mirror is virtual.

- (vii) **Focal length** The distance between pole and focus of a spherical mirror is called its focal length. It is represented by f . Focal length is independent of medium. *i.e.*, If a spherical mirror is placed in water or any other mediums, its focal length remains unchanged. Focal length of a mirror is equal to the half of radius of curvature of the mirror. *i.e.* $f = \frac{R}{2}$.

Note Focal length of a plane mirror is infinity and hence its power is zero.

Sign convention

To derive the relevant formulae for reflection by spherical mirrors (concave or convex), we must first adopt a sign convention for measuring distances.

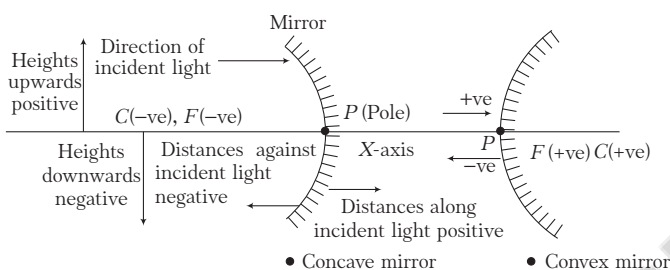


Fig. 9.19 Cartesian sign convention

According to the Cartesian sign convention,

- All the distances are measured from pole (P) of the mirrors.
- The principal axis of the mirror is taken as X -axis and the pole as origin.
- Distances measured in the direction of the incident light are taken as positive and opposite to the direction of incident light as negative.
- The heights measured upwards with respect to X -axis and normal to the principal axis of the mirror are taken as positive and the heights measured downwards are taken as negative.

Rules for image formation in spherical mirrors

In ray optics, to locate the image of an object, tracing of a ray as it reflects is very important. Following four types of rays are used for image formation.

Ray 1. A ray through the centre of curvature which strikes the mirror normally and is reflected back along the same path.

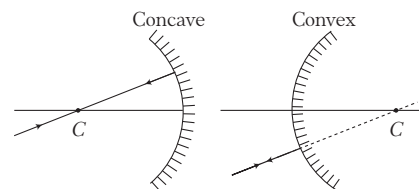


Fig. 9.20

Ray 2. A ray parallel to principal axis after reflection either actually passes through the principal focus F or appears to diverge from it.

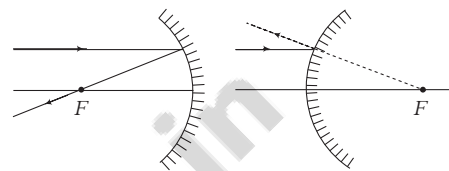


Fig. 9.21

Ray 3. A ray passing through the principal focus F or a ray which appears to converge at F is reflected parallel to the principal axis.

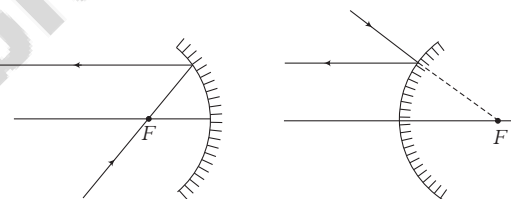


Fig. 9.22

Ray 4. A ray striking at pole P is reflected symmetrically back in the opposite side following the laws of reflection.

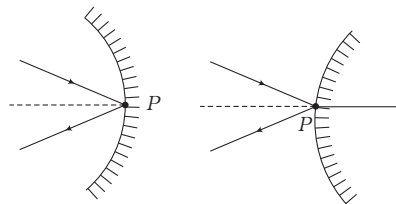


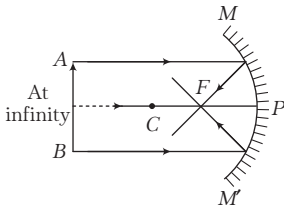
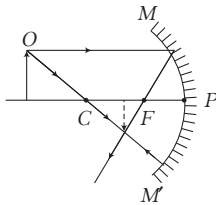
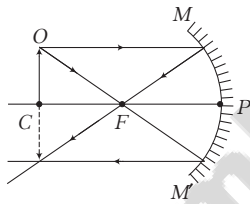
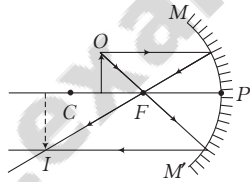
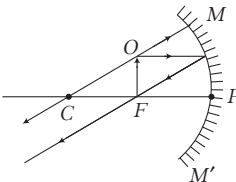
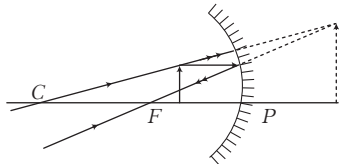
Fig. 9.23

Image formation by spherical mirrors

1. Image formation by concave mirror

In case of a concave mirror, the image is erect and virtual when the object is placed between F and P . In all other positions of object, the image is real and inverted as shown in the table.

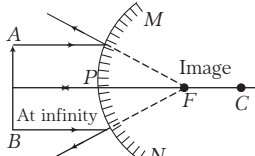
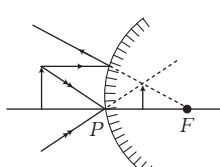
Table for image formation by concave mirror

S.No.	Position of Object	Ray Diagram	Properties of Image
(i)	At infinity		Real, inverted, very small at F
(ii)	Between infinity and C		Real, inverted, diminished between F and C
(iii)	At C		Real, inverted, equal in size at C
(iv)	Between F and C		Real, inverted and very large between C and infinity
(v)	At F		Real, inverted, very large at infinity
(vi)	Between F and P		Virtual, erect, large in size behind the mirror

2. Image formation by convex mirror

Image formed by convex mirror is always virtual, erect and diminished no matter where the object is placed. All the images formed by this mirror will be between pole and focus as shown in the table given below.

Table for image formation by convex mirror

S.No.	Position of Object	Ray Diagram	Properties of Image
(i)	At infinity		Virtual, erect, very small in size at F
(ii)	In front of mirror (at finite distance)		Virtual, erect, diminished between P and F

Mirror formula

The relation among object distance (u), image distance (v) and focal length (f) of a mirror (concave or convex) can be established as

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

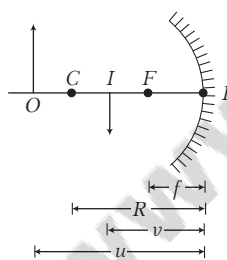


Fig. 9.24 Image and object distances in a concave mirror

Important informations related to mirror formula

(i) Newton's formula

In case of spherical mirrors, if object distance x_1 and image distance x_2 are measured from focus instead of pole, then $u = (f + x_1)$ and $v = (f + x_2)$. The mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ reduces to, } \frac{1}{f + x_2} + \frac{1}{f + x_1} = \frac{1}{f}$$

which on simplification gives,

$$x_1 x_2 = f^2$$

This formula is called **Newton's formula**.

(ii) v versus u graph for a plane mirror

The focal length and the radius of curvature are infinite for a plane mirror.

$$\therefore \text{ From mirror formula } \frac{1}{v} + \frac{1}{u} = 0 \Rightarrow v = -u$$

Thus, v versus u graph for a plane mirror is a straight line with slope equal to -1 as shown in figure.

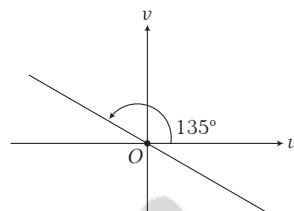


Fig. 9.25 v versus u graph of a plane mirror

(iii) v versus u graph for a spherical mirror

For a spherical mirror v versus u graph is a rectangular hyperbola as shown in figure.

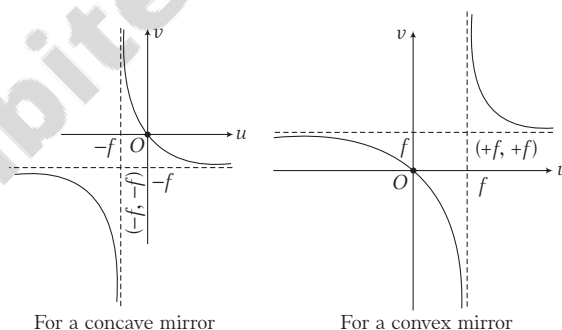


Fig. 9.26 v versus u graph for a spherical mirror

(iv) $\frac{1}{v}$ versus $\frac{1}{u}$ graph for a spherical mirror

For a spherical mirror $\frac{1}{v}$ versus $\frac{1}{u}$ graph is a straight line as shown in figure.

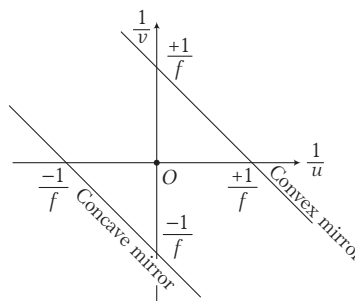


Fig. 9.27 $\frac{1}{v}$ versus $\frac{1}{u}$ graph for a spherical mirror

Note Coordinates of an object if given by (x_0, y_0) , then coordinates of image is

$$(x_1, y_1) = \left(\frac{fx_0}{x_0 - f}, \frac{fy_0}{f - x_0} \right)$$

Magnification

The ratio of height of image formed by the mirror to height of object is called magnification produced by the mirror.

The lateral, transverse or linear magnification (m)

When an object is placed perpendicular to principal axis then the lateral, transverse or linear magnification is given as,

$$m = \frac{\text{Height of image}}{\text{Height of object}}$$

Also,
$$m = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

Axial or longitudinal magnification (m_L)

When an object is placed along principal axis, then

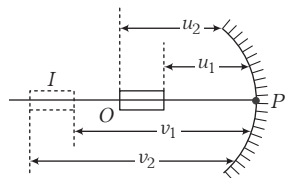


Fig. 9.28 An object placed in front of concave mirror

$$m_L = \frac{-\text{Length of image}}{\text{Length of object}} = -\frac{(v_2 - v_1)}{u_2 - u_1} = -\frac{\Delta v}{\Delta u}$$

If length of the object is small,

$$m_L = \frac{dv}{du}$$

from mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

By differentiation, we get

$$\frac{1}{v^2} \frac{dv}{du} - \frac{1}{u^2} = 0$$

or
$$\frac{dv}{du} = -\left(\frac{v^2}{u^2}\right) \Rightarrow m_L = -m^2$$

Here, du = object length and dv = image length

Also, when object is moving, then we can write

$$\frac{\text{Velocity of image } (v_I)}{\text{Velocity of object } (v_O)} = -\left(\frac{v}{u}\right)^2 = -\left(\frac{f}{u-f}\right)^2$$

On the basis of the above relation, following inferences can be given

- (a) When object moves from $-\infty$ to F , the real image moves from F to $-\infty$ and when the object is placed between $-\infty$ to C , then

$$|v_I| < |v_O|$$

But, when the object is placed between C to F , then

$$|v_I| < |v_O|$$

- (b) When the object moves from F to P , the virtual image moves from infinity (∞) to P , then

$$|v_I| > |v_O|$$

- (c) When object is either at C or P , then

$$|v_I| = |v_O|$$

Areal magnification (m_a)

It is given by ratio of area of image (A_I) and area of object (A_O)

$$\therefore m_a = \frac{A_I}{A_O} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

Few important points related to magnification of mirrors

- (i) For real extended object, if the image is erect and virtual, then for,
Concave mirror : $m > 1$,
Plane mirror : $m = 1$ and
Convex mirror : $m < 1$

- (ii) If the image is real, then the mirror is concave and for the object

Beyond C : $m < -1$

At C : $m = -1$ and Between C and F : $m > -1$

Example 9.10 An object is 30.0 cm from a spherical mirror, along the central axis. The absolute value of lateral magnification is $1/2$. The image produced is inverted. What is the focal length of the mirror?

Sol. Image is inverted, so it is real, u and v both are negative.

Magnification is $\frac{1}{2}$, therefore, $v = \frac{u}{2}$.

Given, $u = -30$ cm and $v = -15$ cm

Using the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

we have,
$$\frac{1}{f} = \frac{1}{-15} - \frac{1}{30} = -\frac{1}{10}$$

$\therefore$ Focal length $f = -10$ cm

Example 9.11 A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror?

Sol. Using the relation,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}, \text{ we get}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} - \left(-\frac{1}{12}\right)$$

$\therefore v = \frac{20}{3}$ cm = + 6.67 cm $\approx$ 6.7 cm

The positive sign shows that the image is formed behind the mirror.

Using the relation, magnification $m = -\frac{v}{u}$, we get

$$m = -\frac{(20/3)}{-12} = +\frac{5}{9}$$

Hence, the image is virtual, erect and diminished.

As the needle is moved farther from the mirror, the image moves towards the focus (upto the focus only) and gets smaller and smaller in size.

Example 9.12 A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image.

Sol. (a) Using the relation, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-18} - \frac{1}{(-27)}$$

$$\therefore v = -54 \text{ cm}$$

The negative sign shows that the image is formed in front of the mirror, i.e., on the side of the object itself. Thus, the screen must be placed at a distance of 54 cm in front of the mirror.

(b) Magnification, $m = -\frac{v}{u} = \frac{I}{O}$

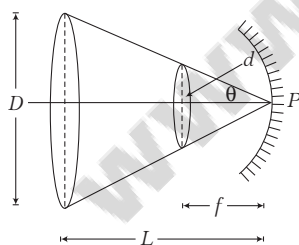
$$\text{we get, } \frac{I}{2.5} = -\frac{(-54)}{(-27)}$$

$$\text{or } I = (2.5) \times (-2) = -5 \text{ cm}$$

Thus, clearly the image is real, inverted and magnified.

Example 9.13 Determine the diameter of the image formed by a spherical concave mirror of focal length 8 m. The diameter of the moon is 3450 km and the distance between the earth and the moon is approx. 4×10^5 km.

Sol.



The image will be formed at focus because object is at large distance.

The angle subtended by diameter of moon at pole,

$$\tan \theta = \frac{D}{L}, \text{ since } \theta \text{ is small, } \tan \theta = \frac{D}{L} \text{ radian}$$

$$\text{By similar triangles, } \theta = \frac{d}{f} = \frac{D}{L}$$

Diameter of image of moon,

$$d = f\theta = f \frac{D}{L} = 8 \times \frac{3450 \times 10^3}{4 \times 10^8} = 0.069 \text{ m} = 6.9 \text{ cm}$$

Example 9.14 Find the distance of object from a concave mirror of focal length 10 cm, so that image size is four times the size of the object.

Sol. Concave mirror can form real as well as virtual image. Here nature of image is not given in the question. So, we will consider two possible cases.

Case I (When image is real) Real image is formed on the same side of the object, i.e., u , v and f all are negative. So let,

$$u = -x$$

$$\text{then, } v = -4x \text{ as } \left| \frac{v}{u} \right| = |m| = 4$$

$$\text{and } f = -10 \text{ cm}$$

$$\text{Substituting in, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\text{We have } \frac{1}{-4x} - \frac{1}{x} = \frac{1}{-10} \text{ or } \frac{5}{4x} = \frac{1}{10}$$

$$\therefore x = 12.5 \text{ cm}$$

Note $|x| > |f|$ and we know that in case of a concave mirror, image is real when object lies beyond F .

Case II (When image is virtual) Mirror image is virtual when it is formed behind the mirror, i.e., u and f are negative while v is positive. So let,

$$u = -y$$

$$\text{then, } v = +4y \text{ and } f = -10 \text{ cm}$$

$$\text{Substituting in, } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\text{we have, } \frac{1}{4y} - \frac{1}{y} = \frac{1}{-10}$$

$$\text{or } \frac{3}{4y} = \frac{1}{10}$$

$$\text{or } y = 7.5 \text{ cm}$$

Note Here $|y| < |f|$, as we know that image is virtual when the object lies between F and P .

Example 9.15 A concave mirror has a radius of curvature of 24 cm. How far is an object from the mirror, if an image is formed that is

- virtual and 3.0 times the size of the object,
- real and 3.0 times the size of the object and
- real and 1/3rd the size of the object?

Sol. Given, $R = -24 \text{ cm}$ (concave mirror)

$$\text{Hence, } f = \frac{R}{2} = -12 \text{ cm}$$

- Image is virtual and 3 times larger. Hence, u is negative and v is positive. Simultaneously, $|v| = 3|u|$. So let,

$$u = -x \text{ then, } v = +3x$$

$$\text{Substituting values in the mirror formula } \frac{1}{v} + \frac{1}{u} = \frac{1}{f}, \text{ we}$$

have

$$\frac{1}{3x} - \frac{1}{x} = \frac{1}{-12}$$

$$\therefore x = 8 \text{ cm}$$

Therefore, object distance is 8 cm.

- (ii) Image is real and three times larger. Hence, u and v both are negative and $|v| = 3|u|$. So let,

$$u = -x$$

$$\text{then, } v = -3x$$

Substituting values in mirror formula, we have

$$\frac{1}{-3x} - \frac{1}{x} = \frac{1}{-12}$$

$$\text{or } x = 16 \text{ cm}$$

$\therefore$ Object distance should be 16 cm.

- (iii) Image is real and $\frac{1}{3}$ rd the size of object. Hence, both u

and v are negative and $|v| = \frac{|u|}{3}$. So let,

$$u = -x$$

$$\text{then, } v = -\frac{x}{3}$$

Substituting values in the mirror formula, we have

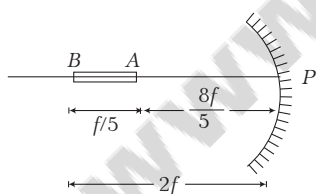
$$-\frac{3}{x} - \frac{1}{x} = \frac{1}{-12}$$

$$\text{or } x = 48 \text{ cm}$$

$\therefore$ Object distance should be 48 cm.

Example 9.16 A thin stick of length $f/5$ is placed along the principal axis of a concave mirror of focal length f such that its image which is real and elongated just touches the stick. What is the magnification?

Sol. One end of image coincides with one end of stick i.e., either A or B is centre of curvature.



Since image is elongated, i.e. point B is centre of curvature. Image of B is formed on B itself.

Image of A

$$u = -(2f - f/5) = \frac{-8f}{5}$$

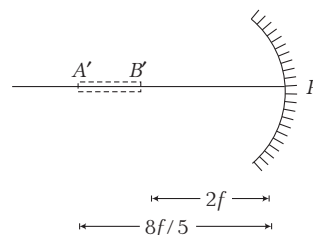
$$f = -f, v = ?$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} + \frac{1}{-8f/5} = \frac{1}{-f}$$

$$\frac{1}{v} = -\frac{1}{f} + \frac{5}{8f} = \frac{-8+5}{8f} = \frac{-3}{8f}$$

$$\Rightarrow v = -\frac{8f}{3}$$



$$\text{Image length, } A'B' = \frac{8f}{5} - 2f = -2f/5$$

$$\text{Longitudinal magnification, } m_L = \frac{-A'B'}{AB} = \frac{2f/5}{f/5} = 2$$

Uses of spherical mirrors

Uses of spherical mirrors (concave and convex mirrors) are given below

Concave mirrors

- It is used as reflectors in search light, cinema projectors and headlights of vehicles, etc.
- Concave mirror is also used as face looking or shaving mirror because it forms erect and magnified image.
- These mirrors are used in dish antennas.
- Concave mirrors are used by dentists and ENT specialists, in solar cookers, etc.

Convex mirrors

- It is used as a reflector in street lamps to diverge the light over a large area.
- Convex mirrors are used as rear view mirror (or driver's mirror) in vehicles, because it has a wider field of view than a plane mirror as shown in figure.

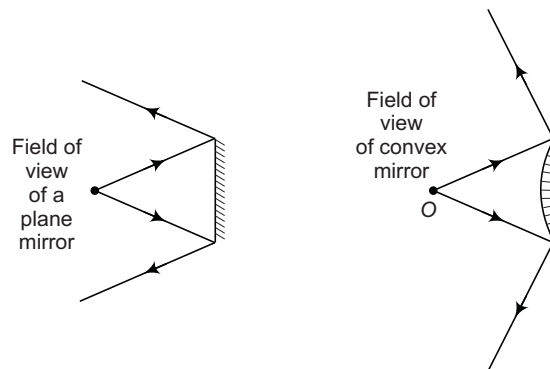


Fig. 9.29

CHECK POINT 9.2

1. A concave mirror of focal length f (in air) is immersed in water ($\mu = 4/3$). The focal length of the mirror in water will be
 (a) f (b) $\frac{4}{3}f$ (c) $\frac{3}{4}f$ (d) $\frac{7}{3}f$
2. An object is placed 40 cm from a concave mirror of focal length 20 cm. The image formed is
 (a) real, inverted and same in size
 (b) real, inverted and smaller in size
 (c) virtual, erect and larger in size
 (d) virtual, erect and smaller in size
3. A point object is placed at a distance of 30 cm from a convex mirror of focal length 30 cm. The image will form at
 (a) infinity (b) pole
 (c) focus (d) 15 cm behind the mirror
4. An object is placed at a distance of 30 cm from a concave mirror and its real image is formed at a distance of 30 cm from the mirror. The focal length of the mirror is
 (a) -15 cm (b) -45 cm
 (c) -30 cm (d) -20 cm
5. A convex mirror of focal length f forms an image which is $1/n$ times the object. The distance of the object from the mirror is
 (a) $(n-1)f$ (b) $\left[\frac{n-1}{n}\right]f$
 (c) $\left[\frac{n+1}{n}\right]f$ (d) $(n+1)f$
6. The focal length of a concave mirror is 50 cm. Where should be the object placed, so that its image is two times and inverted?
 (a) 75 cm (b) 60 cm
 (c) 125 cm (d) 50 cm
7. The image formed by a convex mirror of focal length 30 cm is a quarter of the size of the object. The distance of the object from the mirror is
 (a) 30 cm (b) 90 cm (c) 120 cm (d) 60 cm
8. A concave mirror gives an image three times as large as the object placed at a distance of 20 cm from it. For the image to be real, the focal length should be
 (a) 10 cm (b) 15 cm
 (c) 20 cm (d) 30 cm
9. An object of size 7.5 cm is placed in front of a convex mirror of radius of curvature 25 cm at a distance of 40 cm. The size of the image should be
 (a) 2.3 cm (b) 1.78 cm
 (c) 1 cm (d) 0.8 cm
10. A point object is placed at a distance of 10 cm and its real image is formed at a distance of 20 cm from a concave mirror. If the object is moved by 0.1 cm towards the mirror, the image will shift by about
 (a) 0.4 cm away from the mirror
 (b) 0.8 cm away from the mirror
 (c) 0.4 cm towards the mirror
 (d) 0.8 cm towards the mirror

REFRACTION OF LIGHT

When a light passes from one medium to another medium, it deviates from its original path. The phenomenon of change in path of light at the boundary of separation of two media, as it goes from one medium to another medium is called **refraction**.

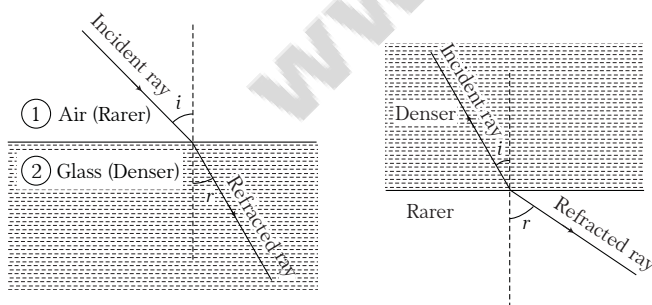


Fig.9.30 Refraction of light

- When a ray of light goes from a rarer medium to a denser medium, it bends towards the normal.
- When a ray of light goes from a denser medium to a rarer medium, it bends away from the normal.

Laws of refraction

There are two laws of refraction as follows

- (i) For two particular media, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant, i.e. $\frac{\sin i}{\sin r} = \text{constant}$... (i)

This is known as **Snell's law**.

- (ii) The incident ray, the normal and the refracted ray at the point of incidence all lie in the same plane.

Note The angle of deviation on refraction is $\delta = |i - r|$. If the incident ray is normal, refraction takes place without bending. In this case $i = r = 0$, i.e., incident ray passes undeviated.

Refractive index (μ or n)

The constant ratio $\frac{\sin i}{\sin r}$ is called the relative **refractive index**, for light passing from the first to the second medium.

Thus,
$$\mu = \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \dots (ii)$$

Absolute refractive index

When light travels from vacuum (or air) to any transparent medium, then refractive index of medium with respect to vacuum (or air) is called absolute refractive index *i.e.*,

$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v}$$

Relative refractive index

When light travels from medium (1) to medium (2), then refractive index of medium (2) with respect to medium (1) is called relative refractive index *i.e.*,

$${}_1\mu_2 = \frac{\mu_2}{\mu_1}$$

Refractive index is a dimensionless quantity and have no unit.

Dependence of refractive index

The value of refractive index depends on

- The nature of the two media.
- The temperature of the media.
- The wavelength of the light.
- It does not depend on the value of i (however refractive index = $\frac{\sin i}{\sin r}$ for a pair of media.
- According to Cauchy's formula,

$$\mu = A + \frac{B}{\lambda^2}$$

where, A and B are constants.

$\therefore$ As λ increases, μ decreases.

Important points about refractive index

- Refractive index is a property of the medium.
- A medium with higher value of μ is called denser medium and with the lower value of μ is called rarer medium.
- The minimum value of refractive index (absolute) is 1 for vacuum. For air, it is 1.003 but practically, we will use $\mu = 1$ for air.
- Maximum value of μ is 2.46 for diamond.

Special points about refraction

- We can write Snell's law as,

$$\mu \sin i = \text{constant} \quad \dots(i)$$

For two media, $\mu_1 \sin i = \mu_2 \sin r$

$$\text{or} \quad \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = {}_1\mu_2 \quad \dots(ii)$$

- From Eq. (i), we can see that $i > r$ if $\mu_2 > \mu_1$, *i.e.* if a ray of light passes from a rarer to a denser medium it bends towards normal and *vice-versa*.

$$(iii) \text{ Since, } {}_1\mu_2 = \frac{\mu_2}{\mu_1} \quad \text{and} \quad {}_2\mu_1 = \frac{\mu_1}{\mu_2}$$

$$\therefore {}_1\mu_2 = \frac{1}{{}_2\mu_1}$$

Similarly, for three mediums,

$${}_1\mu_2 = \frac{\mu_2}{\mu_1}, \quad {}_2\mu_3 = \frac{\mu_3}{\mu_2} \quad \text{and} \quad {}_3\mu_1 = \frac{\mu_1}{\mu_3}$$

$$\therefore {}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_1 = 1$$

For n mediums,

$${}_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_4 \times \dots \times (n-1)\mu_n = {}_1\mu_n$$

It is called reversibility of light. According to this principle, when a light ray, after suffering n number of reflections and refractions, will has its final path reversed, *i.e.* it travels back along its entire initial path.

- Snell's law can also be written as,

$${}_1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1} \quad \dots(iii)$$

Here, v_1 is the speed of light in medium 1 and v_2 in medium 2. Similarly, λ_1 and λ_2 are the corresponding wavelengths.

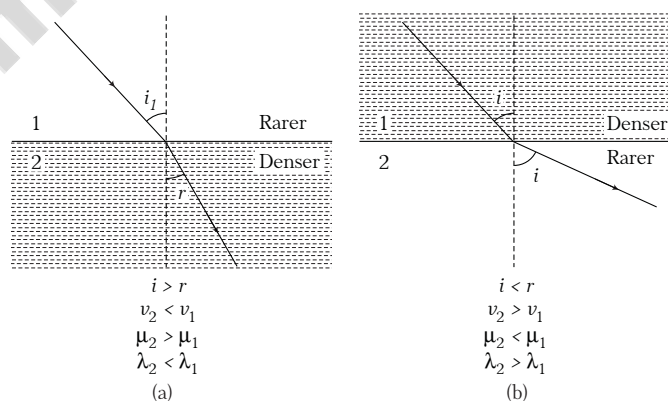


Fig. 9.31 Light ray travelling from (a) Rarer to denser medium and (b) denser to rarer medium

- As a ray of light moves from medium (1) to medium (2), its wavelength changes but its frequency remains constant.

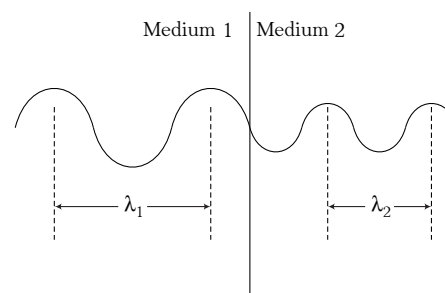


Fig. 9.32

- (vi) Experiments show that, if the boundaries of the media are **parallel**, then the emergent ray CD , is parallel to the incident ray, however it is laterally displaced.

We can also directly apply the Snell's law, in medium 1 and 5, i.e. $\mu_1 \sin i_1 = \mu_5 \sin i_5$

$$\Rightarrow i_1 = i_5 \text{ if } \mu_1 = \mu_5 \quad (\because \mu \sin i = \text{constant})$$

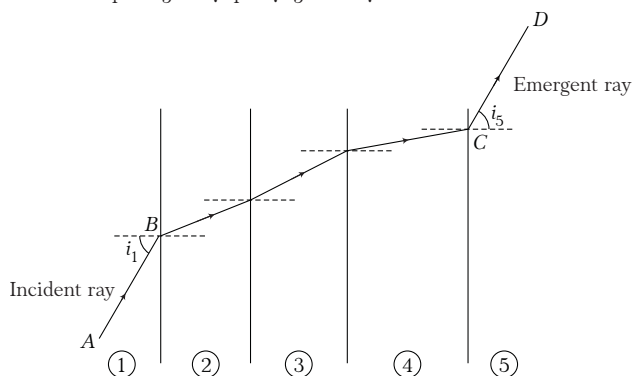


Fig. 9.33

If any of the boundary is not parallel we cannot use this law directly by jumping the intervening media.

Example 9.17 (i) Find the speed of light of wavelength $\lambda = 780 \text{ nm}$ (in air) in a medium of refractive index $\mu = 1.55$.

(ii) What is the wavelength of this light in the given medium?

Sol. (i) Speed of light, $v = \frac{c}{\mu} = \frac{3.0 \times 10^8}{1.55}$
 $= 1.94 \times 10^8 \text{ m/s}$

(ii) Wavelength of light in the given medium,

$$\lambda_{\text{medium}} = \frac{\lambda_{\text{air}}}{\mu} = \frac{780}{1.55} \approx 503 \text{ nm}$$

Example 9.18 Light of wavelength 300 nm in medium A enters into medium B through a plane surface. If frequency of light is $5 \times 10^{14} \text{ Hz}$ and $v_A/v_B = 4/5$, then find absolute refractive indices of media A and B .

Sol. Speed, $v_A = v\lambda_A = 5 \times 10^{14} \times 300 \times 10^{-9} = 1.5 \times 10^8 \text{ m/s}$

$$\text{Absolute refractive index, } \mu_A = \frac{c}{v_A} = \frac{3 \times 10^8}{1.5 \times 10^8} = 2$$

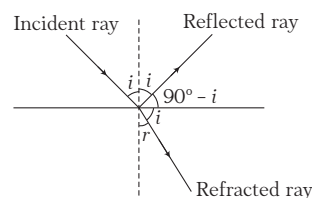
We have, $\mu \propto \frac{1}{v}$

$$\therefore \frac{\mu_A}{\mu_B} = \frac{v_B}{v_A} \Rightarrow \frac{2}{\mu_B} = \frac{5}{4}$$

$$\text{Refractive index, } \mu_B = \frac{2 \times 8}{5} = 3.2$$

Example 9.19 A ray of light falls on a glass plate of refractive index $\mu = 1.5$. What is the angle of incidence of the ray, if the angle between the reflected and refracted rays is 90° ?

Sol. In the figure, $r = 90^\circ - i$



From Snell's law,

$$1.5 = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin (90^\circ - i)} = \frac{\sin i}{\cos i} = \tan i$$

$$\therefore \text{Angle of incidence, } i = \tan^{-1}(1.5) = 56.3^\circ$$

Example 9.20 Refractive index of glass with respect to water is $(9/8)$. Refractive index of glass with respect to air is $(3/2)$. Find the refractive index of water with respect to air.

Sol. Given, ${}_w\mu_g = 9/8$ and ${}_a\mu_g = 3/2$

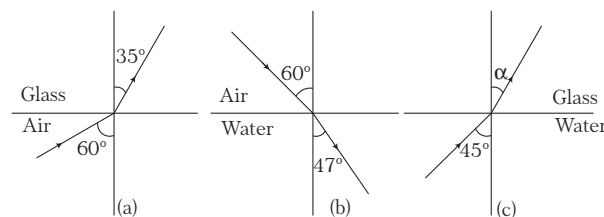
As, ${}_a\mu_g \times {}_g\mu_w \times {}_w\mu_a = 1$

$$\therefore \frac{1}{{}_w\mu_a} = {}_a\mu_w = {}_a\mu_g \times {}_g\mu_w = \frac{{}_a\mu_g}{{}_w\mu_g}$$

$\therefore$ Refractive index of water with respect to air,

$${}_a\mu_w = \frac{3/2}{9/8} = \frac{4}{3}$$

Example 9.21 Figures (a) and (b) show refraction of an incident ray in air at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface [Fig. (c)]



Sol. From Fig. (a),

$i = \text{angle of incidence} = 60^\circ$

$r = \text{angle of refraction} = 35^\circ$

$\therefore$ Refractive index of glass with respect to air,

$${}_a\mu_g = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 35^\circ} = \frac{0.8660}{0.5736} = 1.51$$

From Fig. (b), here, $i = 60^\circ$ and $r = 47^\circ$

$\therefore$ Refractive index of water with respect to air,

$${}_a\mu_w = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 47^\circ} = \frac{0.8660}{0.7314} = 1.18$$

$\therefore$ We have

$${}_a\mu_g = {}_a\mu_w \times {}_w\mu_g$$

or

$${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.51}{1.18} = 1.28$$

From Fig. (c), $i = \text{angle of incidence} = 45^\circ$

$\therefore$ Using the relation,

$${}_w\mu_g = \frac{\sin i}{\sin r}$$

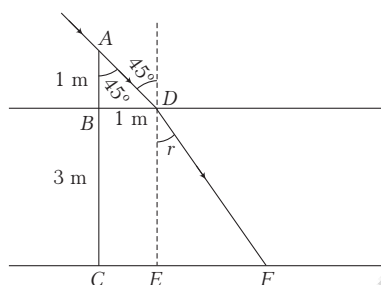
or $1.28 = \frac{\sin 45^\circ}{\sin r}$

or $\sin r = \frac{\sin 45^\circ}{1.28} = \frac{1/\sqrt{2}}{1.28}$
 $= \frac{0.707}{1.28} = 0.5525$
 $= \sin 33.53^\circ$

$\therefore$ Angle of refraction, $r = 33.53^\circ$

Example 9.22 A pile 4 m high driven into the bottom of a lake is 1 m above the water. Determine the length of the shadow of the pile on the bottom of the lake, if the sun rays make an angle of 45° with the water surface. The refractive index of water is $4/3$.

Sol. From Snell's law, $\frac{4}{3} = \frac{\sin 45^\circ}{\sin r}$



Solving this equation, we get, $r = 32^\circ$

Further, $EF = (DE) \tan r = (3) \tan 32^\circ = 1.88 \text{ m}$

$\therefore$ Total length of shadow, $L = CF$

or $L = (1 + 1.88) \text{ m} = 2.88 \text{ m}$

Image due to refraction at a plane surface

When a light ray passes from one medium to another medium, then it bends at the interface of the two media. Due to this bending or refraction of light, image is formed which creates an illusion of the shifting of the object position.

Following two types of shifts are possible after refraction from a plane surface

Type I The object (O) is in denser medium (glass, water, etc.) and is seen from rarer medium (air) normally through plane surface.

In this condition, due to refraction of light, object in the denser medium as seen from rarer medium seems to be raised above as compared to its actual depth below the boundary of denser medium.

In the figure, $\angle AOB$ will be r and $\angle AIB$ is i . For normal incidence, (i.e. small angles of i and r)

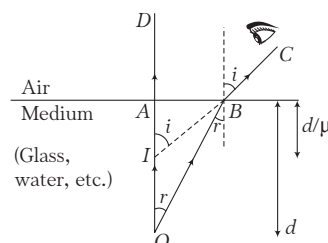


Fig. 9.34 Object is in denser medium

$$\sin i \approx \tan i = \frac{AB}{AI} \quad \dots(i)$$

and

$$\sin r \approx \tan r = \frac{AB}{AO} \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we have

$$\frac{\sin i}{\sin r} = \frac{AO}{AI}$$

or

$$\mu = \frac{AO}{AI} \quad \left(\because \frac{\sin i}{\sin r} = \mu \right)$$

$\therefore$

$$AI = \frac{AO}{\mu} = \frac{d}{\mu}$$

If point O is at a depth of d from a water surface, then the above result is also sometimes written as

$$AI = d_{\text{apparent}} = \frac{AO (= d_{\text{actual}})}{\mu} = \frac{d}{\mu}$$

$$\therefore \text{Shift} = OI = AO - AI = d \left(1 - \frac{1}{\mu} \right)$$

$$\text{Apparent shift} = d \left(1 - \frac{1}{\mu} \right)$$

where, d is actual depth of the object in the denser medium of refractive index μ .

Lateral magnification

If immiscible liquids of refractive indices μ_1 , μ_2 and μ_3 (with $\mu_3 > \mu_2 > \mu_1$) are filled in a vessel. Their depths are d_1 , d_2 and d_3 , respectively. Then, the apparent depth (for normal incidence) when seen from top of the first liquid will be

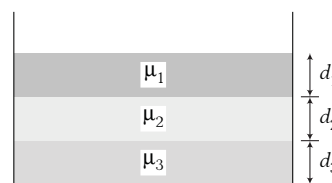


Fig. 9.35

$$d_{\text{app}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3}$$

Similarly for n liquids, $d_{\text{app}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \dots + \frac{d_n}{\mu_n}$

Also apparent shift of the bottom will be upwards which is given as

$$\text{Apparent shift} = d_1 \left(1 - \frac{1}{\mu_1}\right) + d_2 \left(1 - \frac{1}{\mu_2}\right) + \dots + d_n \left(1 - \frac{1}{\mu_n}\right)$$

Type II The object (O) is in rarer medium (air) and is seen from denser medium (glass, water, etc.), normally through plane surface.

In this condition, due to refraction of light, object in the rarer medium as seen from denser medium seems to be raised above as compared to its actual height above the boundary of rarer medium. As the ray of light passes from rarer to denser medium, it bends towards the normal and hence the object will appear farther (at I) as shown in figure.

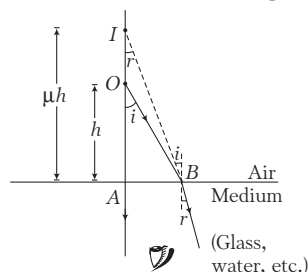


Fig. 9.36 Object is in rarer medium

Proceeding as previous Eqs. (i) and (ii), we get

$$\mu = \frac{\sin i}{\sin r} = \frac{\tan i}{\tan r} = \frac{AB}{AO} \times \left(\frac{AI}{AB}\right)$$

$$\Rightarrow AI = \mu (AO) = \mu(h)$$

$\therefore$ Apparent height (AI) = μ (actual height)

and shift = $OI = AI - AO = (\mu - 1)h$

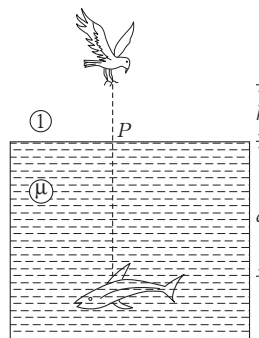
Note In all the above cases, the change in the value of x (depth or height) is μ times whether it is increasing or decreasing. All the relations can be derived for small angles of incidence as done in type I.

Example 9.23 A printed page is kept pressed by a glass cube ($\mu = 1.5$) of edge 9.0 cm. By what amount will the printed letters appear to be shifted when viewed from the top?

Sol. The thickness of the cube = $t = 9.0$ cm. The shift in the position of the printed letters is

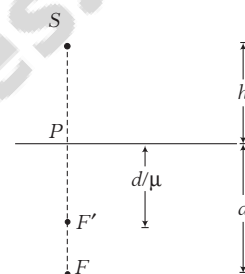
$$\Delta d = \left(1 - \frac{1}{\mu}\right)d = \left(1 - \frac{1}{1.5}\right) \times 9.0 \text{ cm} = 3.0 \text{ cm}$$

Example 9.24 Consider the situation as shown in the diagram.



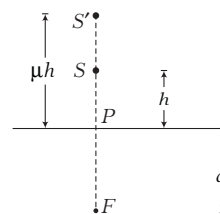
Find the distance between sparrow and fish as seen by (i) sparrow and (ii) fish.

Sol. (i) For sparrow, fish will appear nearer at distance d/μ from P ,



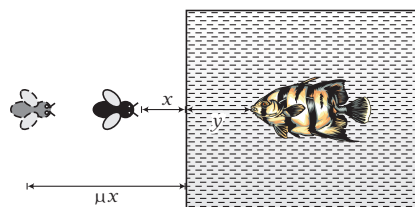
$$SF' = h + \frac{d}{\mu}$$

(ii) For fish, sparrow will appear farther at distance μh from P



$$FS' = d + \mu h$$

Example 9.25 A fish in an aquarium, approaches the left wall at a rate of 3 ms^{-1} , and observes a fly approaching it at 8 ms^{-1} . If the refractive index of water is $(4/3)$, then find the actual velocity of the fly.



Sol. For the fish, apparent distance of the fly from the wall of the aquarium is μx . If x is actual distance, then apparent velocity will be $= \frac{d(\mu x)}{dt}$

$$(v_{app})_{fly} = \mu v_{fly}$$

Now, the fish observes the velocity of the fly to be 8 ms^{-1} .

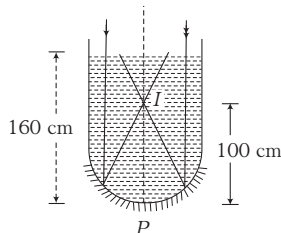
Therefore, apparent relative velocity $= 8 \text{ ms}^{-1}$

$$\Rightarrow v_{fish} + (v_{app})_{fly} = 8 \text{ ms}^{-1} \Rightarrow 3 + \mu v_{fly} = 8$$

$$\therefore \text{Velocity of fly } v_{fly} = 5 \times \frac{3}{4} = 3.75 \text{ ms}^{-1}$$

Example 9.26 A concave mirror of radius of curvature 2 m is placed at the bottom of a tank of water. The mirror forms an image of the sun when it is directly overhead. Calculate the distance of the images from the mirror for (i) 160 cm and (ii) 80 cm of water in the tank. (Take, $\mu = 4/3$ for water)

Sol. (i) When the depth of the water is 160 cm.

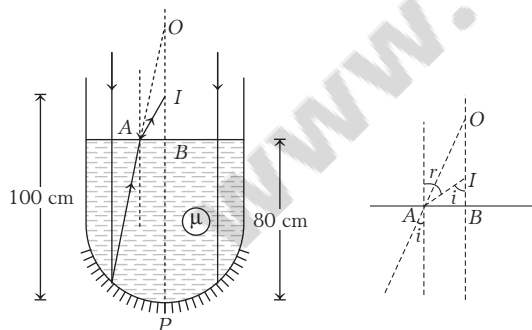


$$\text{Focal length of mirror, } f = \frac{R}{2} = \frac{200}{2} = 100 \text{ cm}$$

For parallel rays, image is formed at focus of mirror,

$$PI = 100 \text{ cm}$$

(ii) When depth of water is 80 cm



$$\mu \sin i = 1 \times \sin r$$

As angles i and r are small, $\sin i = \tan i$, $\sin r \approx \tan r$

$$\mu \tan i = \tan r$$

$$\frac{4}{3} \cdot \frac{AB}{OB} = \frac{AB}{BI} \Rightarrow \frac{4}{3} \cdot \frac{1}{20} = \frac{1}{BI}$$

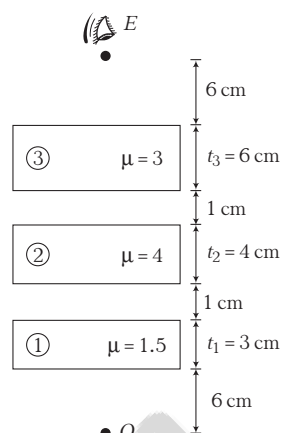
$$BI = 15 \text{ cm}$$

$$\Rightarrow PI = 80 + BI = 80 + 15 = 95 \text{ cm}$$

Alternatively

$$\mu(BI) = BO \Rightarrow \frac{4}{3} \times BI = 20 \Rightarrow BI = 15 \text{ cm}$$

Example 9.27 Consider the situation shown in figure.



Find distance of image of O from eye E .

Sol. Shift by (1); $t_1(1 - 1/\mu_1) = 3(1 - 2/3) = 1 \text{ cm}$

Shift by (2); $t_2(1 - 1/\mu_2) = 4(1 - 1/4) = 3 \text{ cm}$

Shift by (3); $t_3(1 - 1/\mu_3) = 6(1 - 1/3) = 4 \text{ cm}$

Total shift $= 1 + 3 + 4 = 8 \text{ cm}$

Image of O is formed at L .

$$OI = 8 \text{ cm} \Rightarrow EO = 6 + 6 + 1 + 4 + 1 + 3 + 6 = 27 \text{ cm}$$

$$EI = EO - OI = 27 - 8 = 19 \text{ cm}$$

Refraction through a glass slab

Normal shift

If a glass slab is placed in the path of a converging or diverging beam of light, then point of convergence or point of divergence appears to be shifted as shown in figure.

$$\text{Normal shift, } OO' = x = \left(1 - \frac{1}{\mu}\right)t$$

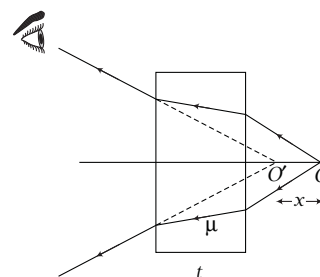


Fig. 9.37 Normal shift by a glass slab

Note Due to slab, if the $\mu_{slab} > \mu_{medium}$, then the shift is along the direction of incident ray and if, $\mu_{slab} < \mu_{medium}$, then the shift will be in opposite direction.

Lateral shift

When a light ray passes through a glass slab it is refracted twice at the two parallel faces and finally emerges out parallel to its incident direction i.e., the ray undergoes no deviation $\delta = 0$.

The angle of emergence (e) is equal to the angle of incidence (i). The lateral shift of the ray is the perpendicular distance between the incident and emergent ray and it is given by $BC = d = t \sec r \sin(i - r)$.

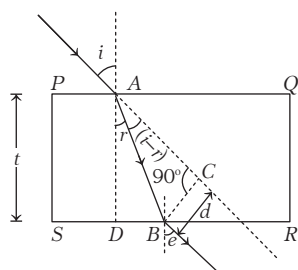


Fig. 9.38 Lateral shift by a glass slab

or
$$d = \left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right] t \sin i$$

For small angles of incidence, $d = ti \left(\frac{\mu - 1}{\mu} \right)$

Note Optical path is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in a medium. Time taken by light ray to pass through the other medium is $\frac{\mu x}{c}$, where x = geometrical path and μx optical path.

Example 9.28 A small pin fixed on a table top is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Sol. As we know, $d = \text{normal shift} = t - \frac{t}{\mu} = t \left(1 - \frac{1}{\mu} \right)$

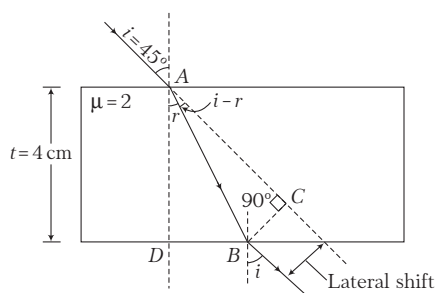
$$= 15 \left(1 - \frac{1}{1.5} \right) = 15 \left(1 - \frac{2}{3} \right) = 15 \times \frac{1}{3} = 5 \text{ cm}$$

i.e., the pin appears to be raised by 5 cm.

No, the answer does not depend upon the location of the slab for small angles of incidence.

Example 9.29 A light ray is incident at an angle of 45° with the normal on a 4 cm thick plate ($\mu = 2.0$). Find the shift in the path of the light as it emerges out from the plate.

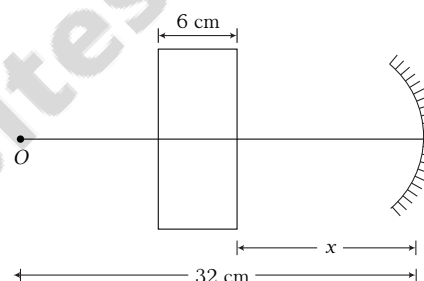
Sol. Consider the ray diagram as shown below



$$\begin{aligned} \text{Lateral shift, } BC &= \frac{t \sin(i - r)}{\cos r} = t \sin i \left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}} \right] \\ &= 4 \sin 45^\circ \left[1 - \frac{\cos 45^\circ}{\sqrt{(2)^2 - (\sin^2 45^\circ)}} \right] \\ &= \frac{4}{\sqrt{2}} \left[1 - \frac{1/\sqrt{2}}{\sqrt{4 - 1/2}} \right] \\ &= 2\sqrt{2} \left(1 - \frac{1}{\sqrt{7}} \right) = 1.76 \text{ cm} \end{aligned}$$

Example 9.30 A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between object and mirror. Find the position of final image when the distance x shown in figure is

- (i) 5 cm and (ii) 20 cm



Sol. As we know, the normal shift produced by a glass slab,

$$\Delta x = \left(1 - \frac{1}{\mu} \right) t = \left(1 - \frac{2}{3} \right) (6) = 2 \text{ cm}$$

i.e., for the mirror the object is placed at a distance $(32 - \Delta x) = 32 - 2 = 30 \text{ cm}$ from it.

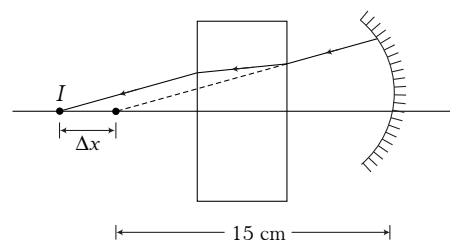
Applying mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

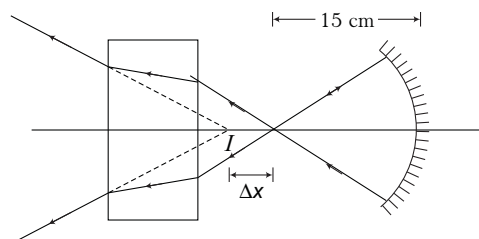
or
$$\frac{1}{v} - \frac{1}{30} = -\frac{1}{10}$$

or
$$v = -15 \text{ cm}$$

- (i) **When $x = 5 \text{ cm}$** The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance, $\Delta x = 2 \text{ cm}$. Hence, the final **real** image is formed at a distance $(15 + 2) = 17 \text{ cm}$ from the mirror.



- (ii) **When $x = 20$ cm** This time also the final image is at a distance 17 cm from the mirror but it is **virtual** as shown.



Critical angle and Total Internal Reflection (TIR)

Whenever a ray of light goes from a denser medium to a rarer medium, it bends away from the normal. As angle of incidence in denser medium increases, angle of refraction also increases in rarer medium.

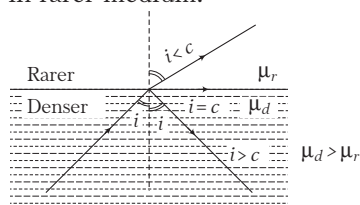


Fig. 9.39 Total internal reflection

The angle of incidence ($\angle i$) in denser medium for which the angle of refraction ($\angle r$) in rarer medium is 90° is called the critical angle ($\angle C$ or $\angle \theta_C$).

$$\Rightarrow \frac{\sin C}{\sin 90^\circ} = \frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}} = \frac{\mu_r}{\mu_d}$$

$$\Rightarrow \sin C = \frac{\mu_r}{\mu_d}$$

$$\Rightarrow C = \sin^{-1} \left(\frac{\mu_r}{\mu_d} \right)$$

Now, if the angle of incidence ($\angle i$) in the rarer medium is greater than the critical angle ($\angle C$), then the ray instead of suffering refraction is totally reflected back in the same (denser) medium.

This phenomenon is called total internal reflection.

For total internal reflection to take place following set of conditions must be obeyed.

- The ray must travel from denser medium to rarer medium.
- The angle of incidence ($\angle i$) must be greater than the critical angle ($\angle C$).

When the rarer medium is air, $\mu_r = 1$ and $\mu_d = \mu$

$$\therefore \text{Critical angle, } C = \sin^{-1} \left(\frac{1}{\mu} \right)$$

Applications of total internal reflection

TIR has following applications

(i) Totally reflecting prisms

Prisms are right-angled isosceles triangle which turn, the light ray by 90° or 180° . They are based on the phenomenon of total internal reflection. Refractive index of crown glass is $3/2$.

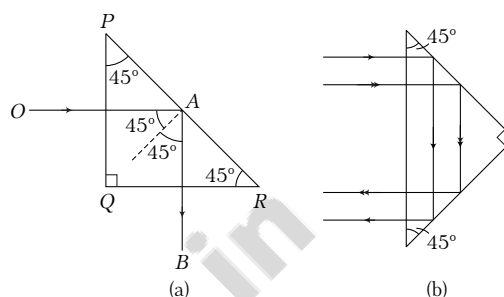


Fig. 9.40 Prism reflectors

Hence, $C = \sin^{-1} \left(\frac{1}{\mu} \right) = \sin^{-1} \left(\frac{2}{3} \right) \approx 42^\circ$

In totally reflecting glass prisms, angle of incidence is made $45^\circ (> C)$.

(ii) Optical fibres

It is a thin tube of transparent material that allows light to pass through, without being refracted into the air or another external medium. These fibres work on the principle of total internal reflection.

Optical fibres are fabricated with high quality composite glass/quartz fibres. The refractive index of core is higher than that of the cladding. When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflection along the length of the fibre and finally comes out from other end. Due to total internal reflection at each stage, no absorption of light takes place *i.e.*, there is no appreciable loss in the intensity of light.

Optical fibres are fabricated in such a way that light reflected at one side of the inner surface strikes the other surface at an angle larger than critical angle. Even, if fibre is bent, light can easily travel along the length.

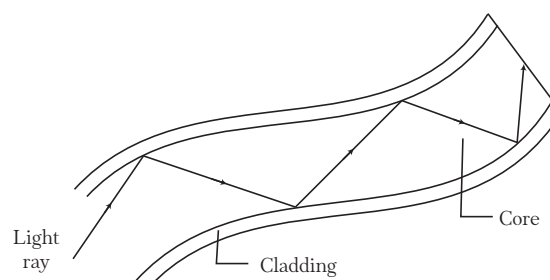


Fig. 9.41 Typical structure of optical fibre

Uses

- (i) In transmitting electrical and optical signals.
- (ii) In visual examination of internal organs like esophagus, stomach and intestine.
- (iii) In decorative lamps.

Note As, we have seen $\theta_C = \sin^{-1}\left(\frac{\mu_r}{\mu_d}\right)$

Suppose, we have two sets of media 1 and 2 and $\left(\frac{\mu_r}{\mu_d}\right)_1 < \left(\frac{\mu_r}{\mu_d}\right)_2$

then, $(\theta_C)_1 < (\theta_C)_2$

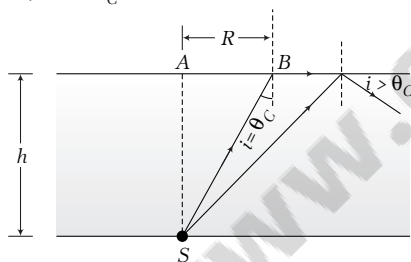
So, a ray of light has more chances to have TIR in case 1.

Some other examples of total internal reflection

- (i) The sparkling of diamond is due to total internal reflection inside it.
- (ii) Shining of air bubble in water is also due to total internal reflection. The critical angle of water-air interface is 48.75° .
- (iii) The phenomenon of mirage in deserts and looming in cold region is caused by total internal reflection.

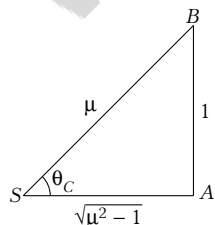
Example 9.31 An isotropic point source is placed at a depth h below the water surface. A floating opaque disc is placed on the surface of water, so that the bulb is not visible from the surface. What is the minimum radius of the disc? Take refractive index of water = μ .

Sol. As shown in figure, light from the source will not emerge out of water, if $i > \theta_C$.



Therefore, minimum radius R corresponds to, $i = \theta_C$

In $\triangle SAB$,

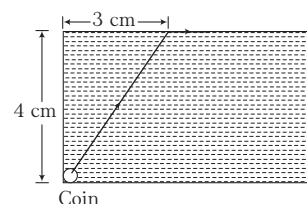


$$\sin \theta_C = \frac{1}{\mu}$$

Also, $\frac{R}{h} = \tan \theta_C$

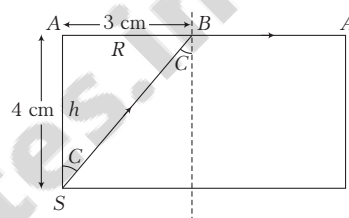
$$\therefore R = h \tan \theta_C \text{ or } R = \frac{h}{\sqrt{\mu^2 - 1}}$$

Example 9.32 A small coin is resting at the bottom of a beaker filled with a liquid. A ray of light from the coin travels upto the surface of the liquid and moves along its surface. Find the speed of light in the liquid.



Sol. As shown in figure, a light ray from the coin will not emerge out of liquid if, $i > C$.

Therefore, minimum radius R corresponds to, $i = C$

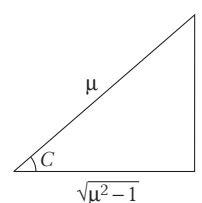


$$\text{In } \triangle SAB, \frac{R}{h} = \tan C$$

$$\Rightarrow R = h \tan C$$

$$\Rightarrow R = \frac{h}{\sqrt{\mu^2 - 1}}$$

Given, $R = 3 \text{ cm}$ and $h = 4 \text{ cm}$



$$\therefore \frac{3}{4} = \frac{1}{\sqrt{\mu^2 - 1}}$$

$$\Rightarrow \mu^2 = \frac{25}{9} \Rightarrow \mu = \frac{5}{3}$$

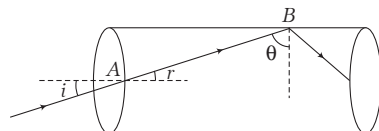
$$\text{But, } \mu = \frac{c}{v}$$

$$\Rightarrow \text{Speed of light in liquid, } v = \frac{c}{\mu} = \frac{3 \times 10^8}{5/3} = 1.8 \times 10^8 \text{ ms}^{-1}$$

Example 9.33 Light enters at an angle of incidence in a transparent rod of refractive index n . Find the range of n , so that the light once entered into it will not leave it through its lateral face.

Sol. Let a light ray enters at A and refracted beam is AB. At the lateral face, the angle of incidence is θ . For no refraction at this face, $\theta > C$.

$$\begin{aligned} \text{i.e.,} \quad & \sin \theta > \sin C \\ \text{but} \quad & \theta + r = 90^\circ \\ \Rightarrow \quad & \theta = 90^\circ - r \end{aligned}$$



$$\begin{aligned} \sin(90^\circ - r) &> \sin C \\ \text{or} \quad \cos r &> \sin C \end{aligned} \quad \dots(i)$$

By Snell's law, $\mu = \frac{\sin i}{\sin r}$

$$\Rightarrow \sin r = \frac{\sin i}{\mu}$$

$$\therefore \cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{\sin^2 i}{\mu^2}}$$

From Eq. (i), $\sqrt{1 - \frac{\sin^2 i}{\mu^2}} > \sin C$

$$\text{Also, } \sin C = \frac{1}{n} \Rightarrow 1 - \frac{\sin^2 i}{n^2} > \frac{1}{n^2}$$

$$\text{or} \quad n^2 > \sin^2 i + 1$$

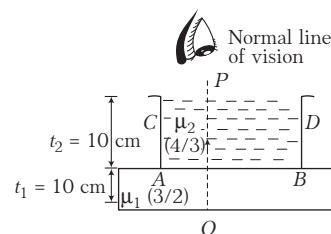
The maximum value of $\sin i$ is 1.
 $n^2 > 2$ or $n > \sqrt{2}$

Example 9.34 A rectangular glass block of thickness 10 cm and refractive index 1.5 is placed over a small coin. A beaker is filled with water of refractive index $4/3$ to a height of 10 cm and is placed over the small block.

- Find the apparent position of the object when it is viewed at normal incidence.
- If the eye is slowly moved away from the normal at a certain position, the image of coin is found to disappear due

to total internal reflection. At what surface does it happen and why?

Sol. Given,



$$\begin{aligned} \text{(i) Total shift, } OI &= t_1(1 - 1/\mu_1) + t_2(1 - 1/\mu_2) \\ &= 10\left(1 - \frac{1}{3/2}\right) + 10\left(1 - \frac{1}{4/3}\right) = \frac{10}{3} + \frac{10}{4} \end{aligned}$$

Distance of image from P,

$$\begin{aligned} PI &= PO - OI \\ &= (10 + 10) - \left(\frac{10}{3} + \frac{10}{4}\right) = 14.17 \text{ cm} \end{aligned}$$

- The image of coin disappears when the ray suffers TIR at either at AB or CD.

The critical angles for glass-water surface

$$\mu_1 \sin C_1 = \mu_2 \sin 90^\circ$$

$$\frac{3}{2} \sin C_1 = \frac{4}{3}$$

$$\sin C_1 = \frac{8}{9}$$

$$C_1 = \sin^{-1}(8/9)$$

The critical angles for water-air surface

$$\mu_2 \sin C_2 = 1 \times \sin 90^\circ$$

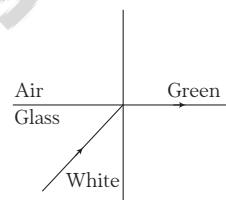
$$\frac{4}{3} \sin C_2 = 1$$

$$\sin C_2 = \frac{3}{4}$$

$C_2 < C_1$, TIR occurs earlier at the water-air surface as eye is moved away from the normal.

CHECK POINT 9.3

- A light wave has a frequency of 4×10^{14} Hz and a wavelength of 5×10^{-7} m in a medium. The refractive index of the medium is
 (a) 1.5 (b) 1.33 (c) 1.25 (d) 1.75
- μ_1 and μ_2 are the refractive indices of two mediums and v_1 and v_2 are the velocities of light in these two mediums respectively. Then, the relation connecting these quantities is
 (a) $v_1 = v_2$ (b) $\mu_2 v_1 = \mu_1 v_2$
 (c) $\mu_1^2 v_1 = \mu_2^2 v_2$ (d) $\mu_1 v_1 = \mu_2 v_2$
- Absolute refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$. The ratio of velocities of light in glass and water will be
 (a) $\frac{\sqrt{3}}{2}$ (b) 1.5 (c) 1.732 (d) 2
- The refractive index of a certain glass is 1.5 for light whose wavelength in vacuum is 6000 Å. The wavelength of this light when it passes through glass is
 (a) 4000 Å (b) 6000 Å
 (c) 9000 Å (d) 15000 Å
- When a ray of light falls on a given plate at an angle of incidence 60° , the reflected and refracted rays are found to be normal to each other. The refractive index of the material of the plate is
 (a) $\frac{\sqrt{3}}{2}$ (b) 1.5 (c) 1.732 (d) 2

6. The refractive indices of glass and water with respect to air are $3/2$ and $4/3$, respectively. The refractive index of glass with respect to water will be
 (a) $8/9$ (b) $9/8$
 (c) $7/6$ (d) None of these
7. If ${}_i\mu_j$ represents refractive index when a light ray goes from medium i to medium j , then the product ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3$ is equal to
 (a) ${}_3\mu_1$ (b) ${}_3\mu_2$ (c) $\frac{1}{{}_1\mu_4}$ (d) ${}_4\mu_2$
8. When light is refracted into a medium from vacuum
 (a) its wavelength and frequency both increase
 (b) its wavelength increases but frequency remains unchanged
 (c) its wavelength decreases but frequency remains unchanged
 (d) its wavelength and frequency both decrease
9. A spot is placed on the bottom of a slab made of transparent material of refractive index 1.5. The spot is viewed vertically from the top when it seems to be raised by 2 cm. Then, the height of the slab is
 (a) 10 cm (b) 8 cm (c) 6 cm (d) 4 cm
10. An air bubble inside a glass slab ($\mu = 1.5$) appears 6 cm when viewed from one side and 4 cm when viewed from the opposite side. The thickness of the slab is
 (a) 10 cm (b) 6.67 cm
 (c) 15 cm (d) None of these
11. An under water swimmer is at a depth of 12 m below the surface of water. A bird is at a height of 18 m from the surface of water, directly above his eyes. For the swimmer the bird appears to be at a distance from the surface of water equal to (Refractive index of water is $4/3$)
 (a) 24 m (b) 12 m (c) 18 m (d) 9 m
12. A vessel of depth $2d$ cm is half filled with a liquid of refractive index μ_1 and the upper half with a liquid of refractive index μ_2 . The apparent depth of the vessel when seen perpendicularly from above is
 (a) $d \left[\frac{\mu_1 \mu_2}{\mu_1 + \mu_2} \right]$ (b) $d \left[\frac{1}{\mu_1} + \frac{1}{\mu_2} \right]$
 (c) $2d \left[\frac{1}{\mu_1} + \frac{1}{\mu_2} \right]$ (d) $2d \left[\frac{1}{\mu_1 \mu_2} \right]$
13. Three immiscible transparent liquids with refractive indices $3/2$, $4/3$ and $6/5$ are arranged one on top of another. The depths of the liquids are 3 cm, 4 cm and 6 cm, respectively. The apparent depth of the vessel is
 (a) 10 cm (b) 9 cm
 (c) 8 cm (d) 7 cm
14. A glass slab is immersed in water. What will be the critical angle for a light ray at glass-water interface?
 [Take, ${}_a n_g = 1.50$, ${}_a n_w = 1.33$ and $\sin^{-1}(0.887) = 62.5^\circ$]
 (a) 48.8° (b) 72.8°
 (c) 62.5° (d) 64.5°
15. The wavelength of light in two liquids x and y is $3500 \text{ \AA}$ and $7000 \text{ \AA}$ respectively, then the critical angle of x relative to y will be
 (a) 60° (b) 45°
 (c) 30° (d) 15°
16. White light is incident on the interface of glass and air as shown in the figure. If green light is just totally internally reflected, then the emerging ray in air contains

 (a) yellow, orange, red
 (b) violet, indigo, blue
 (c) all colours
 (d) all colours except green
17. The critical angle of a prism is 30° . The velocity of light in the medium is
 (a) $1.5 \times 10^8 \text{ m/s}$
 (b) $3 \times 10^8 \text{ m/s}$
 (c) $4.5 \times 10^8 \text{ m/s}$
 (d) None of the above
18. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45° . The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value of n from the following.
 (a) 1.2 (b) $4/3$
 (c) 1.4 (d) 1.5

REFRACTION AT SPHERICAL SURFACES

When two transparent media are separated by a spherical surface, light incident on the surface from one side get refracted into the medium on the other side. Spherical surfaces are of two types as shown in figure.

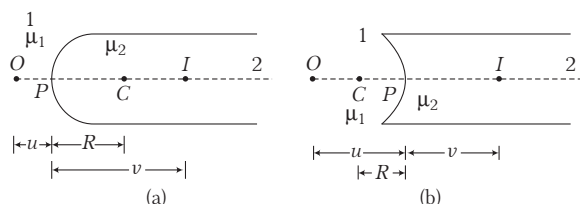


Fig. 9.42 Spherical surface (a) convex (b) concave

For both surfaces, refraction formula is given by

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

where μ_1 = refractive index of first medium,
 μ_2 = refractive index of second medium,
 R = radius of curvature of spherical surfaces,
 u = object distance
 and v = image distance.

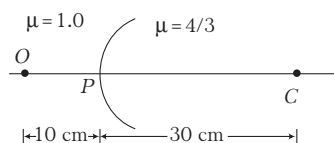
Transverse magnification, $m = \frac{h_I}{h_O} = \frac{\mu_1 v}{\mu_2 u}$

Note For a plane surface, $R = \infty$, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = 0$

Cartesian sign convention for spherical surfaces

- The principal axis of the spherical surface is taken as X-axis and the optical centre as origin. Here, the principal axis is the diameter extended.
- The direction of the incident light is taken as the positive direction of X-axis and opposite to it is taken as negative.
- The upward direction is taken as positive and the downward direction as negative.

Example 9.35 Locate the image of the point object O. The point C is centre of curvature of the spherical surface.



Sol. Here, $\mu_1 = 1$, $\mu_2 = 4/3$, $u = -10$ cm and $R = 30$ cm

Using the relation, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

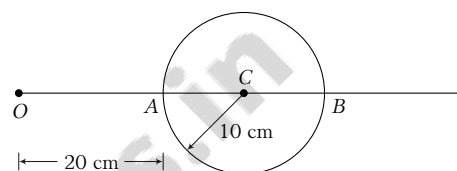
Substituting the values,

$$\Rightarrow \frac{4/3}{v} - \frac{1}{-10} = \frac{4/3 - 1}{30} \Rightarrow \frac{4}{3v} + \frac{1}{10} = \frac{1}{90}$$

$$\Rightarrow \frac{4}{3v} = \frac{1}{90} - \frac{1}{10} = \frac{-8}{90} \Rightarrow v = -15 \text{ cm}$$

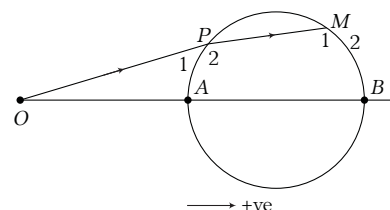
The image is formed 15 cm left of spherical surface and is virtual.

Example 9.36 A glass sphere of radius $R = 10$ cm is kept inside water. A point object O is placed at 20 cm from A as shown in figure. Find the position and nature of the image when seen from other side of the sphere. Also draw the ray diagram. Given, $\mu_g = 3/2$ and $\mu_w = 4/3$.



Sol. A ray of light from O gets refracted twice. The ray of light is travelling in a direction from left to right. Hence, the distances measured in this direction are taken positive.

Applying, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$, twice with proper signs.



We have, $\frac{3/2}{AI_1} - \frac{4/3}{-20} = \frac{3/2 - 4/3}{10}$

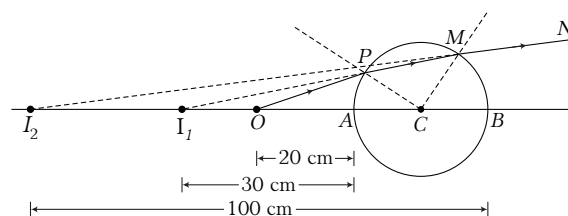
or $AI_1 = -30$ cm

Now, the first image I_1 , acts as an object for the second surface, where

$$BI_1 = u = -(30 + 20) = -50 \text{ cm}$$

$$\therefore \frac{4/3}{BI_2} - \frac{3/2}{-50} = \frac{4/3 - 3/2}{-10}$$

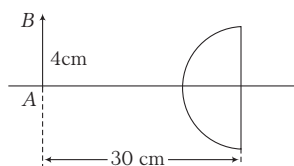
$\therefore BI_2 = -100$ cm, i.e. the final image I_2 is **virtual** and is formed at a distance 100 cm (towards left) from B. The ray diagram is as shown.



Following points should be noted while drawing the ray diagram.

- (i) At point P , the ray travels from rarer to a denser medium. Hence, it will bend towards normal PC . At point M , it travels from a denser to a rarer medium, hence, moves away from the normal MC .
- (ii) PM ray when extended backwards meets at I_1 and MN ray when extended meets at I_2 .

Example 9.37 A linear object of length 4 cm is placed at 30 cm from the plane surface of hemispherical glass of radius 10 cm. The hemispherical glass is surrounded by water. Find the final position and size of the image.



Sol. For 1st surface,

$$\mu_1 = \frac{4}{3}, \mu_2 = \frac{3}{2}, u = -20 \text{ cm and } R = +10 \text{ cm}$$

$$\text{Using, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$$

$$\Rightarrow \frac{(3/2)}{v} - \frac{(4/3)}{(-20)} = \frac{(3/2 - 4/3)}{10} \Rightarrow v = -30 \text{ cm}$$

$$\text{Using, } \frac{A'B'}{AB} = \frac{\mu_1 v}{\mu_2 u} \Rightarrow \frac{A'B'}{(4 \text{ cm})} = \frac{(4/3)(-30)}{(3/2)(-20)}$$

$$\Rightarrow A'B' = 5.3 \text{ cm}$$

$A'B'$ behaves as the object for plane surface,

$$\mu_1 = \frac{3}{2}, \mu_2 = \frac{4}{3} \text{ and } R = \infty, u' = -40$$

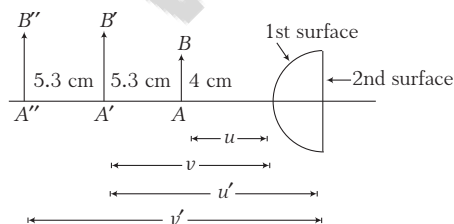
$$\Rightarrow \frac{\mu_2}{v'} = \frac{\mu_1}{u'} \Rightarrow \frac{(4/3)}{v'} = \frac{(3/2)}{(-40)}$$

Solving it, we will get, $v' = -35.5 \text{ cm}$

$$\text{Now, using, } \frac{A''B''}{A'B'} = \frac{(\mu_1 v')}{(\mu_2 u')}$$

$$\frac{A''B''}{(5.3)} = \frac{(3/2)(-35.5)}{(4/3)(-40)} \Rightarrow A''B'' \approx 5.3 \text{ cm}$$

The final images in all the above cases are shown in figure.



Lenses

Lens is a transparent medium (material) bounded by two surfaces at least one of which should be spherical. Refractive index of the medium (material) should also be different from the surroundings.

Lenses are of two types

- (i) **Convex or converging lens** In this lens, a transparent medium is bounded by two surfaces such that it is thicker in the middle.

It is of four types (as shown)

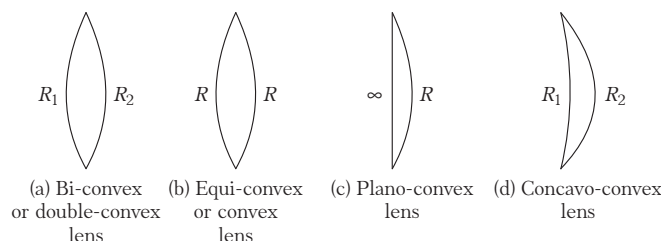


Fig. 9.43 Types of convex lens

- (ii) **Concave or diverging lens** In this lens, a transparent medium is bounded by two surfaces such that it is thinner in the middle. It is of four types (as shown)

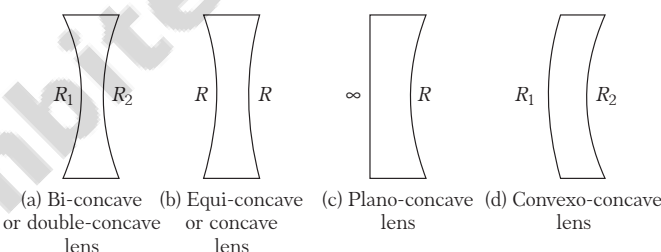


Fig. 9.44 Types of concave lens

Some definitions related to lenses

- (i) **Optical centre** The optical centre is a point within or outside the lens, at which incident rays refract without deviation in its path.

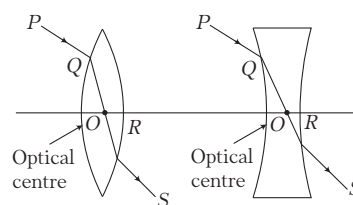


Fig. 9.45 Optical centre

- (ii) **Centre of curvature** The centres of the two imaginary spheres of which the lens is a part are called centres of curvature of the lens. A lens has two centres of curvature with respect to its two curved surfaces.
- (iii) **Radii of curvature** The radii of the two imaginary spheres of which the lens is a part are called radii of curvature of the lens. A lens has two radii of curvature. These may or may not be equal.

(iv) **Principal axis** The straight line passing through the optical centre and centre of curvature of lens is called principal axis of lens.

(v) **Principal focus** Lens has two principal focus

(a) **First principal focus** It is a point on the principal axis of lens, the rays starting from which (convex lens) or appear to converge at which (concave lens) become parallel to principal axis after refraction.

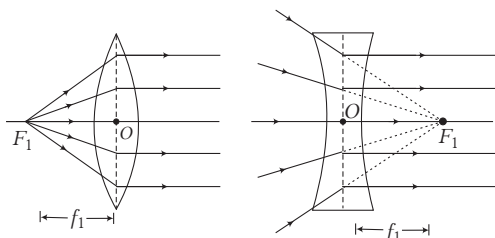


Fig. 9.46 First principal focus

(b) **Second principal focus** It is the point on the principal axis at which the rays coming parallel to the principal axis converge (convex lens) or appear to diverge (concave lens) after refraction from the lens.

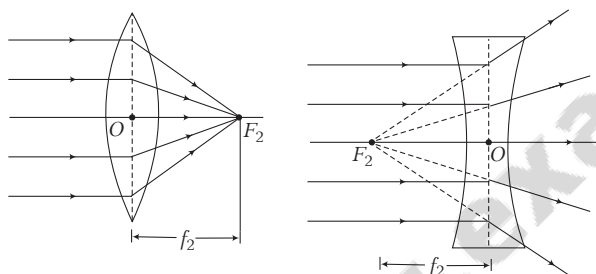


Fig. 9.47 Second principal focus

Both the focuses of convex lens are real, while that of concave lens are virtual.

(vi) **Focal length** The distance between focus and optical centre of lens is called focal length of lens.

(vii) **Aperture** The effective diameter of circular boundary of the lens is called aperture.

Note Intensity $\propto (\text{aperture})^2$

Sign convention for lenses

- Sign convention for lenses are same as for spherical surfaces.
- From the Figs. 9.46 and 9.47, we can see that f_1 is negative for a convex lens and positive for a concave lens. But f_2 is positive for convex lens and negative for concave lens.
- $|f_1| = |f_2|$, if the media on the two sides of a thin lens have same refractive index.

(iv) We are mainly concerned with the second focus f_2 .

Thus, wherever we write the focal length f , it means the second principal focal length. Thus, $f = f_2$ and hence, f is positive for a convex lens and negative for a concave lens.

Law for formation of images by lens

The position and nature of the image by lens, in any case can be obtained either from a ray diagram or by calculation.

Ray diagram

To construct the image of a small object perpendicular to the axis of a lens, two of the following three rays are drawn from the top of the object.

- A ray parallel to the principal axis after refraction passes through the principal focus or appears to diverge from it.

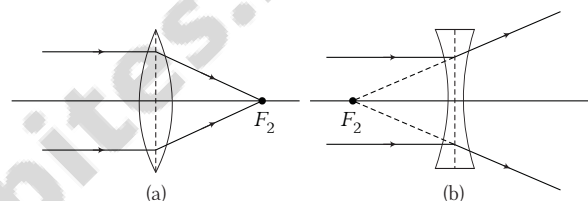


Fig. 9.48 Path of incident ray parallel to principal axis for (a) convex lens (b) concave lens

- A ray through the optical centre O passes undeviated because the middle of the lens acts like a thin parallel-sided slab.

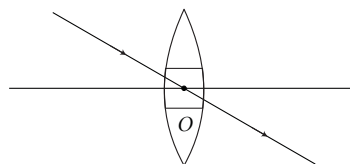


Fig. 9.49 Path of incident ray passing through centre for convex lens

- A ray passing through, the first focus F_1 becomes parallel to the principal axis after refraction.

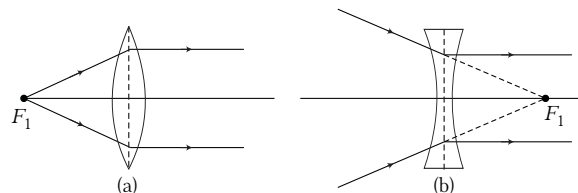


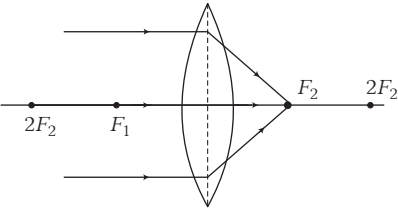
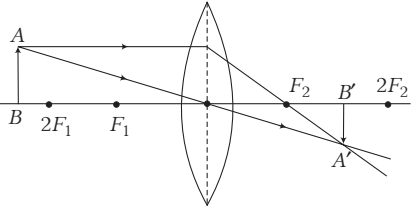
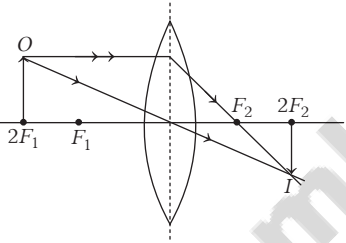
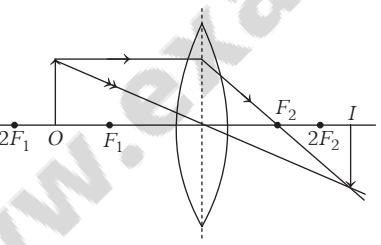
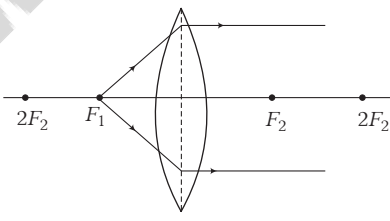
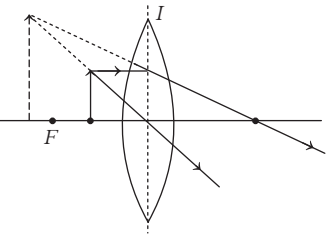
Fig. 9.50 Path of incident ray passing through the focus for (a) convex lens (b) concave lens

Image formation by lens

Convex lens

The image formed by convex lens depends on the position of object. Formation of image by this lens for different positions of object is shown in the table.

Formation of image by convex lens

S.No.	Position of object	Ray diagram	Position of image	Nature and size of image
(i)	At infinity		At the principal focus (F_2) or in the focal plane	Real, inverted and extremely diminished
(ii)	Beyond $2F_1$		Between F_2 and $2F_2$	Real, inverted and diminished
(iii)	At $2F_1$		At $2F_2$	Real, inverted and of same size as the object
(iv)	Between F_1 and $2F_1$		Beyond $2F_2$	Real, inverted and highly magnified
(v)	At F_1		At infinity	Real, inverted and extremely magnified
(vi)	Between F_1 and optical centre		On the same side as the object	Virtual, erect and magnified

Concave lens

The image formed by a concave lens is always virtual, erect and diminished. The image formation by this lens for different positions of object is shown in the table given below.

Image formation by concave lens

S.No.	Position of object	Ray diagram	Position of image	Nature and size of image
(i)	At infinity		At the focus	Virtual, erect and point sized
(ii)	Anywhere on the principal axis		Between optical centre and F2	Virtual, erect and diminished

Lens maker's formula and lens formula

Lens maker's formula

If R_1 and R_2 are the radii of curvature of first and second refracting surfaces of a thin lens of focal length f and refractive index μ (with respect to surrounding medium), the relation between f , μ , R_1 and R_2 is known as lens maker's formula and is given by,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Lens formula

The expression which shows the relation between u , v and f is called lens formula and given by,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

where, u is distance of object from the lens, v is distance of image from the lens and f is focal length of the lens.

Focal length of different lens

Focal lengths of different lenses are given below

(i) For **equi-biconvex lens**, $R_1 = +R$ and $R_2 = -R$

$$f = \frac{R}{2(\mu - 1)}$$

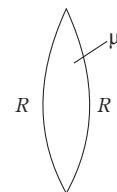


Fig. 9.51 Equi-biconvex lens

(ii) For **equi-biconcave lens**, $R_1 = -R$ and $R_2 = +R$

$$f = -\frac{R}{2(\mu - 1)}$$

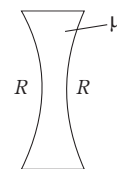


Fig. 9.52 Equi-biconcave lens

(iii) For **plano-convex lens** $R_1 = R$ and $R_2 = \infty$

$\therefore$

$$f = \frac{R}{\mu - 1}$$

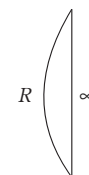


Fig. 9.53 Plano-convex lens

(iv) For plano-concave lens,

$$R_1 = \infty, R_2 = R$$

$\Rightarrow$

$$f = \frac{-R}{(\mu - 1)}$$

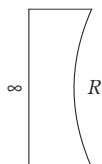


Fig. 9.54 Plano-concave lens

Note

(i) **Newton's formula**

In case of thin convex lens, if an object is placed at a distance x_1 from first focus and its image is formed at distance x_2 from the second focus, then $x_1 x_2 = f^2$.

(ii) If some portion of a lens is covered with black paper, then full image will be formed but the brightness will be reduced.

Lens immersed in a liquid

If a lens (made of glass) of refractive index μ_g is immersed in a liquid of refractive index μ_l , then its focal length f_l is given by

$$\frac{1}{f_l} = (\mu_l \mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

If f_a is the focal length of lens in air, then

$$\frac{1}{f_a} = (\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{f_l}{f_a} = \frac{(\mu_l \mu_g - 1)}{(\mu_g - 1)} = \frac{(\mu_l \mu_g - 1)}{\left(\frac{\mu_l \mu_g}{\mu_l} - 1 \right)}$$

There are three cases based on this

(i) If $\mu_g > \mu_l$, i.e. lens is immersed in a liquid whose refractive index is less than that of material of lens.

Then, f_l and f_a are of same sign and $f_l > f_a$ that is the nature of lens remains unchanged, but its focal length increases and hence power of lens decreases.

(ii) If $\mu_g = \mu_l$, i.e. lens is immersed in a liquid whose refractive index is same as that of material of lens.

Then, $f_l = \infty$. It means lens behaves as a plane glass plate and becomes invisible in the medium.

(iii) If $\mu_g < \mu_l$, i.e. lens is immersed in a liquid whose refractive index is greater than that of the material of lens.

Then, f_l and f_a have opposite signs and the nature of lens changes, i.e. a convex lens diverges the light rays and concave lens converges the light rays.

Focal length of lens if media are different

Consider a double convex lens of refractive index μ_2 . It is surrounded by media of refractive indices μ_1 and μ_3 as shown in figure.

(i) If rays are incident from left, then for refraction at surface 1.

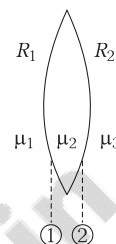


Fig. 9.55 Bi-convex lens with different surrounding media

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad \dots(i)$$

For refraction at surface (2),

$$\frac{\mu_3}{v'} - \frac{\mu_2}{v} = \frac{\mu_3 - \mu_2}{R_2} \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$\frac{\mu_3}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

If $u = \infty, v' = f$

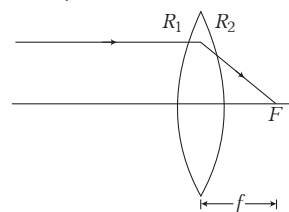


Fig. 9.56 Ray diagram when incident ray comes from left

$$\therefore \frac{\mu_3}{f} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

(ii) If rays are incident from right, replace μ_3 by μ_1 and R_2 by R_1 .

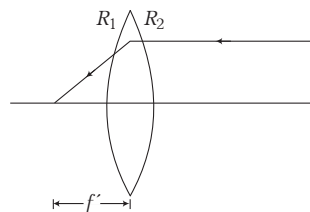
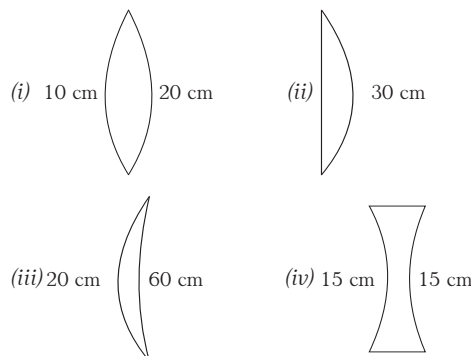


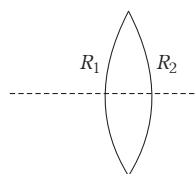
Fig. 9.57 Ray diagram when incident ray comes from right

$$\therefore \frac{\mu_1}{f'} = \frac{\mu_2 - \mu_3}{R_2} + \frac{\mu_1 - \mu_2}{R_1}$$

Example 9.38 Find focal lengths of lenses made of glass ($\mu = 3/2$) and place in air.



Sol. (i)



Given, $R_1 = 10$ cm, $R_2 = -20$ cm

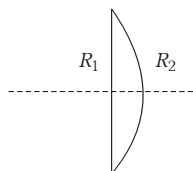
On applying Lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$= \left(\frac{3}{2} - 1 \right) \left(\frac{1}{10} - \frac{1}{(-20)} \right) = \left(\frac{1}{2} \right) \left(\frac{3}{20} \right)$$

$\therefore$ Focal length of lens, $f = 40/3$ cm

(ii)



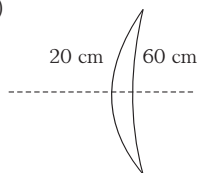
Given, $R_1 = \infty$, $R_2 = -30$ cm

From Lens maker's formula, we have

$$\frac{1}{f} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{\infty} - \frac{1}{-30} \right) = \frac{1}{60}$$

$\therefore$ Focal length, $f = 60$ cm

(iii)

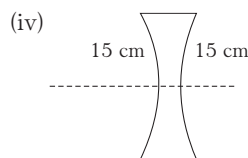


Given, $R_1 = 20$ cm, $R_2 = 60$ cm

From lens Maker's formula, we have

$$\frac{1}{f} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{20} - \frac{1}{60} \right) = \left(\frac{1}{2} \right) \left(\frac{2}{60} \right)$$

$\therefore$ Focal length, $f = 60$ cm



Given, $R_1 = -15$ cm, $R_2 = 15$ cm

From Lens maker's formula, we have

$$\frac{1}{f} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{-15} - \frac{1}{15} \right) = \left(\frac{1}{2} \right) \left(\frac{-2}{15} \right)$$

$\therefore$ Focal length, $f = -15$ cm

Example 9.39 Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required, if the focal length is to be 20 cm?

Sol. Using the relation,

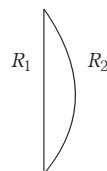
$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right), \text{ we get}$$

$$\frac{1}{20} = (1.55 - 1) \left[\frac{1}{R} - \left(\frac{1}{-R} \right) \right] = 0.55 \left[\frac{1}{R} + \frac{1}{R} \right] = 0.55 \times \frac{2}{R}$$

or $R = 1.10 \times 20 = 22$ cm

Example 9.40 Find the refractive index of the material of a plano-convex lens, if the radius of curvature of the convex surface is 20 cm and focal length of the lens is 60 cm.

Sol.



Given, $R_1 = \infty$, $R_2 = -20$ cm, $f = 60$ cm, $\mu = ?$

From Lens maker's formula, we have

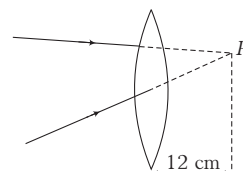
$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{60} = (\mu - 1) \left(\frac{1}{\infty} - \frac{1}{-20} \right) = \frac{\mu - 1}{20}$$

$\therefore$ Refractive index, $\mu = 1 + \frac{1}{3} = \frac{4}{3}$

Example 9.41 A beam of light converges to a point P. Now a lens is placed on the path of the convergent beam 12 cm apart from P. At what point does the beam converge if the lens is (i) a convex lens of focal length 20 cm, (ii) a concave lens of focal length 16 cm?

Sol. (i) For convex lens, $f = 20$ cm



Using the formula,

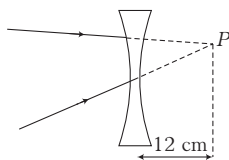
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}, \text{ we get}$$

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{12} + \frac{1}{20}$$

$$\therefore v = \frac{60}{8} = 7.5 \text{ cm}$$

i.e. the image is formed on the right of the lens and is real.

(ii) For concave lens, $f = -16 \text{ cm}$



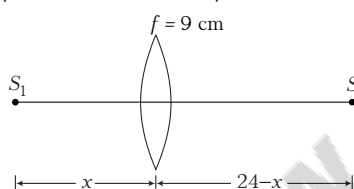
$$\therefore \text{ Using the relation, } \frac{1}{f} = \frac{1}{v} - \frac{1}{u},$$

$$\text{we get } \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-16} + \frac{1}{12}$$

$$\therefore v = +48 \text{ cm}$$

The image is formed at 48 cm to the right of the lens where the beam would converge and is real.

Example 9.42 The distance between two point sources of light is 24 cm. Find out where you would place a converging lens of focal length 9 cm, so that the images of both the sources are formed at the same point.



Sol. Using the relation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\text{For } S_1, \frac{1}{v_1} - \frac{1}{-x} = \frac{1}{9}$$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{9} - \frac{1}{x} \quad \dots(i)$$

$$\text{For } S_2, \frac{1}{v_2} - \frac{1}{-(24-x)} = \frac{1}{9}$$

$$\therefore \frac{1}{v_2} = \frac{1}{9} - \frac{1}{24-x} \quad \dots(ii)$$

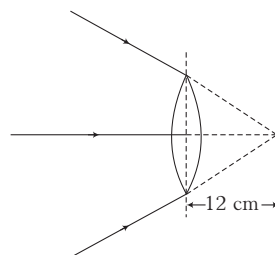
Since, sign convention for S_1 and S_2 is just opposite. Hence,

$$v_1 = -v_2 \quad \text{or} \quad \frac{1}{v_1} = -\frac{1}{v_2}$$

$$\therefore \frac{1}{9} - \frac{1}{x} = \frac{1}{24-x} - \frac{1}{9}$$

Solving this equation we get, $x = 6 \text{ cm}$. Therefore, the lens should be kept at a distance of 6 cm from either of the object.

Example 9.43 If the focal length of the lens is 20 cm, find the distance of the image from the lens in the following figure.



Sol. According to figure, the point on the right side of the lens at which rays converge will behave as virtual object for the lens.

$$\therefore u = +12 \text{ cm}, \quad f = 20 \text{ cm}$$

$$\text{By lens formula, } \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\therefore \frac{1}{20} = \frac{1}{v} - \frac{1}{12} \quad \text{or} \quad \frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{3+5}{60} = \frac{8}{60}$$

$$\text{or} \quad v = \frac{60}{8} = 7.5 \text{ cm}$$

So, image will be formed on same side of the virtual object at a distance of 7.5 cm from the lens.

Example 9.44 Focal length of a convex lens in air is 10 cm. Find its focal length in water. Given that $\mu_g = 3/2$ and $\mu_w = 4/3$.

Sol. Using the relations, $\frac{1}{f_{\text{air}}} = (\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$

$$\text{and } \frac{1}{f_{\text{water}}} = \left(\frac{\mu_g}{\mu_w} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{(\mu_g - 1)}{(\mu_g/\mu_w - 1)}$$

Substituting the values, we have

$$f_{\text{water}} = \frac{(3/2 - 1)}{\left(\frac{3/2}{4/3} - 1 \right)} f_{\text{air}} = 4 f_{\text{air}} = 4 \times 10 = 40 \text{ cm}$$

Note Students can remember the result, $f_{\text{water}} = 4 f_{\text{air}}$, if $\mu_g = 3/2$ and $\mu_w = 4/3$.

Magnification produced by a lens

The ratio of the size of the image to the size of object is called magnification.

Transverse magnification

The lateral, transverse or linear magnification (m) produced by a lens is defined by,

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{I}{O} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-u}{f}$$

Note Use sign convention while solving the problems.

Longitudinal magnification (m_L)

When an object is placed along the principal axis then

$$m_L = \frac{\text{Image length}}{\text{Object length}}$$

If length of the object is small,

$$m_L = \frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{u+f}\right)^2 = \left(\frac{f-u}{f}\right)^2$$

where, du = object length, dv = image length

Areal magnification (M_s)

$$m_s = \frac{A_I}{A_O} = m^2 = \left(\frac{f}{f+u}\right)^2$$

(where, A_I = Area of image and A_O = area of object)

Example 9.45 An object of size 3.0 cm is placed at 14 cm from concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Sol. Using the relation, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, we get

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-21} + \frac{1}{(-14)}$$

$$\therefore v = -8.4 \text{ cm}$$

$\therefore$ The image is virtual, erect and located at 8.4 cm from the lens on the same side as the object.

$$\text{Also, we know that } m = \frac{I}{O} = \frac{v}{u}$$

$$\therefore I = \frac{v}{u} \times O = \frac{-8.4}{-14} \times 3 = 1.8 \text{ cm}$$

i.e. the image is of diminished size.

If the object is moved away from the lens, the virtual image moves towards the focus of the lens (but never beyond focus).

Example 9.46 Find the distance of an object from a convex lens, if image is two times magnified. Focal length of the lens is 10 cm.

Sol. Convex lens forms both type of images real as well as virtual. Since, nature of the image is not mentioned in the question, we will have to consider both the cases.

When image is real Means v is positive and u is negative with $|v| = 2|u|$. Thus, if

$$u = -x \text{ then, } v = 2x \text{ and } f = 10 \text{ cm}$$

$$\text{Substituting in, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\text{we have, } \frac{1}{2x} + \frac{1}{x} = \frac{1}{10}$$

$$\text{or } \frac{3}{2x} = \frac{1}{10}$$

$$\therefore x = 15 \text{ cm}$$

This means object lies between F and $2F$.

When image is virtual Means v and u both are negative. So let,

$$u = -y \text{ then } v = -2y$$

$$\text{and } f = 10 \text{ cm}$$

$$\text{Substituting in, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\text{we have, } \frac{1}{-2y} + \frac{1}{y} = \frac{1}{10} \text{ or } \frac{1}{2y} = \frac{1}{10}$$

$$\therefore y = 5 \text{ cm}$$

This means object lies between F and P .

Displacement method to determine the focal length of a convex lens

The focal length of convex lens can be determined by displacement method. Consider an object and a screen placed at a distance D ($> 4f$) apart. Let a lens of focal length f be placed between the object and the screen.

For two different positions of lens, two images (I_1 and I_2) of an object are formed at the screen. Let the distance between the two positions of the lens is x , then the focal

length of the lens is given by $f = \frac{D^2 - x^2}{4D} = \frac{x}{m_1 - m_2}$

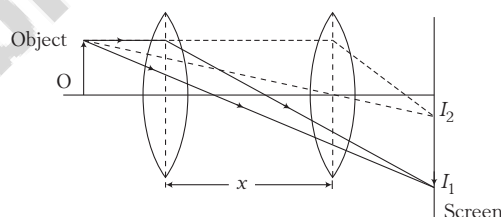


Fig. 9.58 Two convex lenses separated by distance x

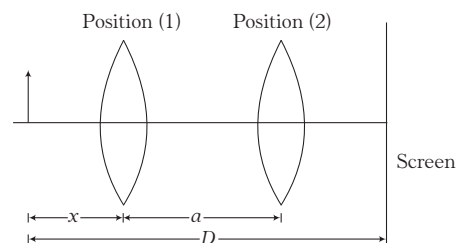
where $m_1 = \frac{I_1}{O}$, $m_2 = \frac{I_2}{O}$, $m_1 m_2 = 1$

Also, size of object $O = \sqrt{I_1 I_2}$

Example 9.47 An object is placed at a distance of 80 cm from a screen. Where should a convex lens of focal length 15 cm be placed so as to obtain a real image of the object?

Sol. Given, $D = 80$ cm and $f = 15$ cm

$D > 4f$, there will be two positions of lens for which real image can be obtain.



For lens in position (1), $u = -x$, $v = D - x = 80 - x$, $f = 15$ cm

$$\text{As, } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{80-x} - \frac{1}{-x} = \frac{1}{15}$$

$$\frac{80}{(80-x)x} = \frac{1}{15} \Rightarrow x^2 - 80x + 1200 = 0$$

$$x^2 - 60x - 20x + 1200 = 0$$

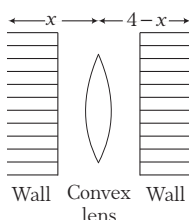
$$\Rightarrow x(x-60) - 20(x-60) = 0$$

$$\Rightarrow (x-20)(x-60) = 0$$

$$\Rightarrow x = 20 \text{ or } 60 \text{ cm}$$

Example 9.48 The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 4m away by means of a large convex lens. Find the maximum possible focal length of the lens required for this purpose.

Sol. Let a large convex lens is placed between two walls at a distance x from wall on which an electric bulb is fixed.



Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Putting, $u = x$ and $v = 4 - x$

$$\therefore \frac{1}{f} = \frac{1}{4-x} - \frac{1}{-x}$$

$$\text{or } \frac{1}{f} = \frac{x+4-x}{(4-x)(x)}$$

$$\text{or } \frac{1}{f} = \frac{4}{(4-x)(x)}$$

$$\text{or } f = \frac{(4-x)x}{4} \quad \dots(i)$$

$$\text{Now, magnification, } m = \frac{v}{u} = \frac{4-x}{x}$$

$$\text{or } 1 = \frac{4-x}{x}$$

$$\text{or } x = 4 - x \text{ or } 2x = 4 \text{ or } x = 2 \text{ m}$$

From Eq. (i), we get

$$f = \frac{(4-2)(2)}{4} = \frac{2 \times 2}{4} = 1 \text{ m}$$

Power of a lens

It is the ability of the lens to deviate the path of rays passing through it. If the lens converges the rays parallel to principal axis, its power is said positive and if it diverges the rays, its power is negative. It is given as

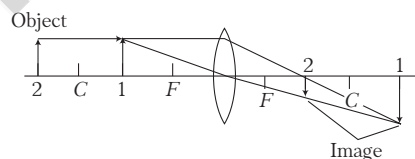
$$P (\text{in dioptre}) = \frac{1}{f (\text{in metre})}$$

Following table gives the sign of P and f for different type of lenses

Nature of lens	Focal length (f)	Power $P_L = \frac{1}{f}$	Converging /Diverging	Ray diagram
Convex lens	+ve	+ve	Converging	
Concave lens	-ve	-ve	Diverging	

Example 9.49 For a given lens, the magnification was found to be twice as large as when the object was 0.15 m distance from it as when the distance was 0.2 m. Find power of the lens.

Sol. Let the positions of objects are as shown 1 and 2 on left side and positions of images as shown 1 and 2 on right side in two different situations.



$$\text{It is given, } \left| \frac{v_1}{u_1} \right| = 2 \left| \frac{v_2}{u_2} \right|$$

$$\text{Here, } u_1 = -15 \text{ cm and } u_2 = -20 \text{ cm}$$

$$\therefore v_1 = 2v_2 \times \frac{u_1}{u_2} = 2v_2 \times \frac{15}{20} = \frac{3}{2} v_2$$

$$\text{Now, } \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\text{and } \frac{1}{f} = \frac{1}{v_2} - \frac{1}{u_2}$$

$$\text{So, } \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{v_2} - \frac{1}{u_2}$$

$$\Rightarrow \frac{2}{3v_2} + \frac{1}{15} = \frac{1}{v_2} + \frac{1}{20} \Rightarrow v_2 = 20 \text{ cm}$$

$$\frac{v_1}{u_1} = 2 \frac{v_2}{u_2} = 2 \times \frac{20}{20} = 2$$

$$\Rightarrow v_1 = 2u_1 = 2 \times 15 = 30 \text{ cm}$$

$$\therefore \frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{30} + \frac{1}{15} = \frac{3}{30}$$

$$\Rightarrow f = 10 \text{ cm} = 0.10 \text{ m}$$

$$\therefore \text{Power of the lens, } P = \frac{1}{f} = \frac{1}{0.1} = +10 \text{ D}$$

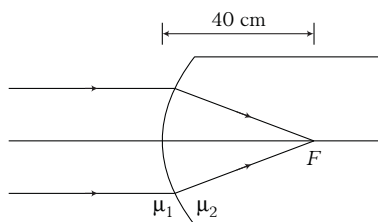
Example 9.50 A spherical convex surface separates object and image space of refractive index 1.0 and 4/3. If radius of curvature of the surface is 10 cm, find its power.

Sol. Let us see where does the parallel rays converge (or diverge) on the principal axis. Let us call it the focus and the corresponding length the focal length f .

$$\text{We have, } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{4/3}{f} - \frac{1.0}{\infty} = \frac{4/3 - 1.0}{+10}$$

$$\text{or focal length, } f = 40 \text{ cm} = 0.4 \text{ m}$$



Since, the rays are converging, its power should be positive. Hence,

$$P (\text{in dioptre}) = \frac{+1}{f (\text{in metre})} = \frac{1}{0.4}$$

$$\text{or } P = 2.5 \text{ D}$$

Example 9.51 A thin glass (refractive index 1.5) lens has optical power of -5 D in air. Calculate its optical power in a liquid medium with refractive index 1.6.

Sol. From Lens maker's formula we have,

$$\frac{1}{f_a} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

It is given that, $\mu = 1.5$

$$\therefore \frac{1}{f_a} = (1.5 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\text{and } \frac{1}{f_m} = \left(\frac{\mu_g - \mu_m}{\mu_m} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

$$\Rightarrow \frac{1}{f_m} = \left(\frac{1.5 - 1}{1.6} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

$$\text{Thus, } \frac{f_m}{f_a} = \frac{(1.5 - 1)}{\left(\frac{1.5 - 1}{1.6} \right)} = -8$$

$$\Rightarrow f_m = -8 \times f_a$$

$$= -8 \times \frac{-1}{5} = 1.6 \text{ m} \quad \left(\because f_a = \frac{1}{P} = -\frac{1}{5} \text{ m} \right)$$

$\therefore$ Power of the lens,

$$P_m = \frac{\mu}{f_m} = \frac{1.6}{1.6} = 1 \text{ D}$$

Combination of two or more thin lenses in contact

Combinations of lenses in contact are used in many optical instruments to improve their performance.

(i) **When two lenses are in contact** Combination will behave as a lens, which has lesser focal length.

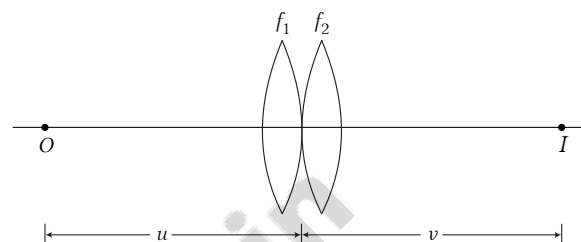


Fig. 9.59 Combination of two convex lenses with zero separation

In this situation, combined focal length (F) is given as

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Similarly, for more than two lenses in contact, the equivalent focal length is given by the formula,

$$\frac{1}{F} = \sum_{i=1}^n \frac{1}{f_i} = \frac{1}{f_1} + \frac{1}{f_2} + \dots$$

Note Here f_1, f_2 etc., are to be substituted with appropriate sign.

Similarly,

(a) Power of combination,

$$P = P_1 + P_2 + \dots = \sum_{i=1}^n P_i$$

(b) Magnification of combination,

$$M = m_1 \times m_2 \times \dots = \prod_{i=1}^n m_i$$

(ii) **Lenses separated by a distance**

If two lenses of focal lengths f_1 and f_2 are separated by a distance x , then its equivalent focal length

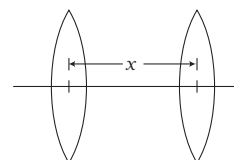


Fig. 9.60 Combination of two convex lenses with x separation

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$$

(a) Power of combination,

$$P = P_1 + P_2 - xP_1P_2$$

(b) Total magnification remains unchanged i.e.,

$$m = m_1 \times m_2$$

Note Combination of two convex lenses behaves as

- (i) convex lens, if $(f_1 + f_2) > d$
- (ii) glass plate, if $(f_1 + f_2) = d$
- (iii) concave lens, if $(f_1 + f_2) < d$

Example 9.52 A converging lens of focal length 5.0 cm is placed in contact with a diverging lens of focal length 10.0 cm. Find the combined focal length of the system.

Sol. Here, $f_1 = +5.0$ cm and $f_2 = -10.0$ cm

Therefore, the combined focal length F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{5.0} - \frac{1}{10.0} = +\frac{1}{10.0}$$

$$\therefore F = +10.0 \text{ cm}$$

i.e., the combination behaves as a converging lens of focal length 10.0 cm.

Example 9.53 Two thin equiconvex lenses each of focal length 0.2 m are placed coaxially with their optic centres 0.5 m apart. Then find the focal length of the combination.

Sol. Given, focal length of each lens, $f_1 = f_2 = 0.2$ m

Separation between the lenses, $d = 0.5$ m

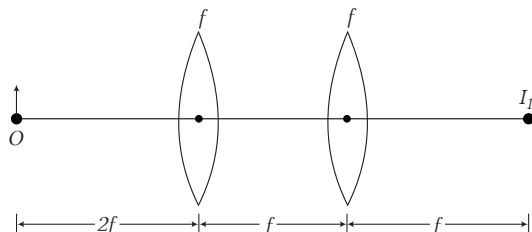
$\therefore$ Focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{0.2} + \frac{1}{0.2} - \frac{0.5}{(0.2)(0.2)} = -2.5$$

$$\Rightarrow F = -\frac{1}{2.5} = -0.4 \text{ m}$$

Example 9.54 Two thin converging lenses are placed on a common axis, so that the centre of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left-hand lens. Where will its image be? What is the lateral magnification? (The focal of each lens is f .)

Sol.



The image formed by first lens will be at a distance $2f$ with lateral magnification, $m_1 = -1$. For the second lens, this image will behave as a virtual object. Using the lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\text{we have, } \frac{1}{v} - \frac{1}{f} = \frac{1}{f}$$

$$\therefore v = \frac{f}{2}$$

$$\text{Magnification, } m_2 = \frac{v_2}{u_2} = \frac{(f/2)}{f} = \frac{1}{2} \quad \{\text{since, } v_2 = v \text{ and } u_2 = f\}$$

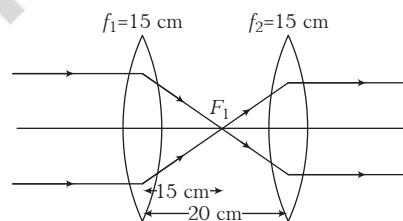
Therefore, final image is formed at a distance $f/2$ from the second lens with total lateral magnification,

$$m = m_1 \times m_2 = (-1) \times \left(\frac{1}{2}\right) = -\frac{1}{2}$$

Example 9.55 Two convex lenses, each of focal length 15 cm, are placed at a separation of 20 cm with their principal axes coinciding.

- (i) Show that a light beam coming parallel to the principal axis diverges as it comes out of the lens system.
- (ii) Find the location of the virtual image formed by the lens system of an object placed far away.
- (iii) Find the focal length of equivalent lens.

Sol. (i) The beam will diverge after coming out of the two convex lenses system because, the image formed by the first lens lies on its focus (F_1) as shown in the figure.



(ii) For the first convex lens,

$$u = \infty$$

$$\therefore \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} - \frac{1}{\infty} = \frac{1}{15}$$

$$\Rightarrow v_1 = 15 \text{ cm}$$

For the second convex lens,

$$u_2 = -(20 - 15) = -5 \text{ cm}$$

$$\therefore \frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2} \Rightarrow \frac{1}{v_2} - \frac{1}{-5} = \frac{1}{15}$$

$$\Rightarrow \frac{1}{v_2} = \frac{1}{15} - \frac{1}{5}$$

$$\Rightarrow v_2 = -7.5 \text{ cm}$$

Virtual image is formed by the lens system at distance of 7.5 cm left from the second lens.

(iii) Focal length of two equivalent lenses,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

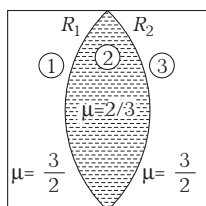
$$= \frac{1}{15} + \frac{1}{15} - \frac{20}{225} = -\frac{2}{45}$$

$$F = -22.5 \text{ cm}$$

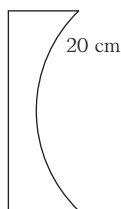
Note The sign of the focal length is negative, thus the lens system diverges a parallel beam incident on it.

Example 9.56 Two plano-concave lenses of glass of refractive index 1.5 have radii of curvature of 20 and 30 cm. They are placed in contact with curved surface towards each other and the space between them is filled with a liquid of refractive index $2/3$. Find the focal length of the system.

Sol.

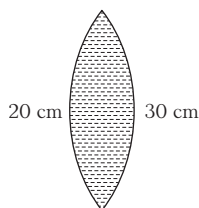


Lens 1



$$\frac{1}{f_1} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{\infty} - \frac{1}{20} \right) = -\frac{1}{40}$$

Lens 2



$$\begin{aligned} \frac{1}{f_2} &= \left(\frac{3}{2} - 1 \right) \left(\frac{1}{20} + \frac{1}{30} \right) \\ &= -\left(\frac{1}{3} \right) \left(\frac{5}{60} \right) = -\frac{1}{36} \end{aligned}$$

Lens 3



$$\frac{1}{f_3} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{-30} - \frac{1}{\infty} \right) = \frac{-1}{60}$$

$\therefore$

$$\begin{aligned} \frac{1}{F} &= \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{-1}{40} - \frac{1}{36} - \frac{1}{60} \\ &= \frac{-9 - 10 - 6}{360} = \frac{-25}{360} \end{aligned}$$

$$F = -14.4 \text{ cm}$$

Cutting of lens

- (i) If a symmetrical convex lens of focal length f is cut into two parts along its optical axis, then focal length of each part (a plano-convex lens) is $2f$. However, if

the two parts are joined as shown in figure, the focal length of combination is again f .

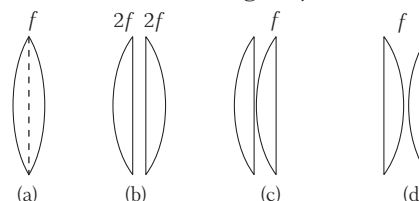


Fig. 9.61 Convex lens cut along its optical axis

- (ii) If a symmetrical convex lens of focal length f is cut into two parts along the principal axis, then focal length of each part remains unchanged to f . If these two parts are joined with curved ends on one side, then focal length of the combination is $f/2$. But on joining two parts in opposite sense the net focal length becomes ∞ (or net power = 0), i.e. it behaves like a plane glass.

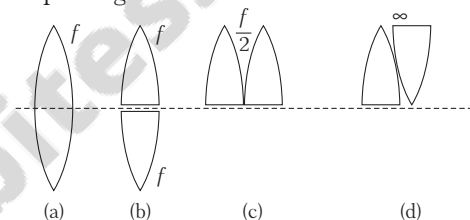


Fig. 9.62 Convex lens cut along its principal axis

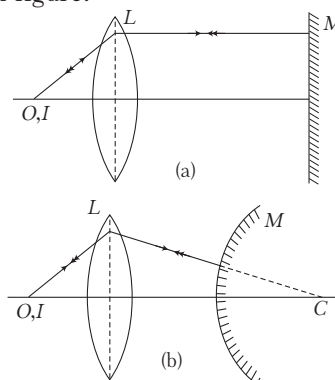
Combination of lenses and mirrors

- (i) When several lenses or mirrors are combined coaxially, image by first serves as an object for second and so on. The net magnification is given by

$$m = m_1 \times m_2 \times m_3 \times \dots$$

We should keep in mind that direction of incident rays is reversed after every reflection.

- (ii) In some cases, object and image are formed at same place after interacting the (lens + mirror) combination. For this, after refraction from lens, rays must retrace their path i.e., must be incident normally on the mirror after refraction from the lens. In other word, we can say that image by lens should be formed at centre of curvature (C) of mirror as shown in figure.



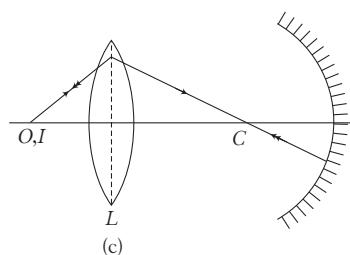
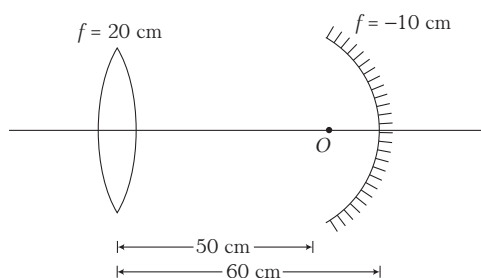


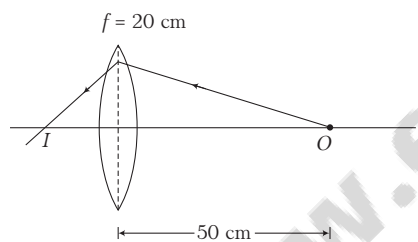
Fig. 9.63 Combination of lens and mirror

Example 9.57 A converging lens of focal length 20 cm and a converging mirror of focal length 10 cm are placed 60 cm apart with common principal axis. A point source is placed in between the lens and the mirror at a distance of 50 cm from the lens. Find the locations of the two images formed.

Sol. Consider the situation,



Directed by lens:



$$u = 50 \text{ cm}, f = -20 \text{ cm}, v = ?$$

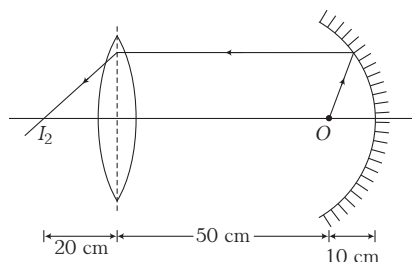
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{50} = -\frac{1}{20}$$

$$\frac{1}{v} = -\frac{1}{20} + \frac{1}{50} = -\frac{3}{100}$$

$$\Rightarrow v = \frac{100}{-3} \text{ cm} = -\frac{100}{3} \text{ cm}$$

By mirror and lens By mirror, image is formed at infinity because object is at focus of mirror.

By lens, image is formed at focus of lens.



Example 9.58 A converging lens and a diverging mirror are placed at a separation of 20 cm. The focal length of the lens is 30 cm and that of the mirror is 50 cm. Where should a point source be placed between the lens and the mirror, so that the light, after getting reflected by the mirror and then getting transmitted by the lens, comes out parallel to the principal axis?

Sol.

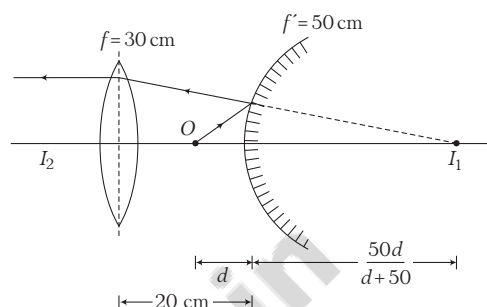


Image by mirror $u = -d, f' = 50 \text{ cm}, v = ?$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f'} \Rightarrow \frac{1}{v} + \frac{1}{-d} = \frac{1}{50}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{50} + \frac{1}{d} = \frac{d+50}{50d}$$

$$v = \frac{50d}{d+50}$$

Image of O is formed at I_1 .

Since, it is given that final beam is parallel to principal axis, for this, image by mirror should be formed at focus of the lens.

$$\Rightarrow 20 + \frac{50d}{d+50} = 30 \Rightarrow \frac{50d}{d+50} = 10$$

$$\Rightarrow 5d = d + 50 \Rightarrow 4d = 50$$

$$\Rightarrow d = \frac{25}{2} \text{ cm} = 12.5 \text{ cm}$$

Silvering of a lens

(i) Let a plano-convex lens has a curved surface of radius of curvature R and refractive index μ . If its plane surface is silvered, it behaves as a concave mirror of focal length f . Its focal length is $f = \frac{R}{2(\mu - 1)}$

(ii) If the curved surface of plano-convex lens is silvered, then it behaves as a concave mirror of focal length, $f = \frac{R}{2\mu}$

(iii) If one surface of a symmetrical double convex lens is silvered, then the lens behaves as a concave mirror of focal length, $f = \frac{R}{2(2\mu - 1)}$

(iv) If the plane surface of a plano-concave lens is silvered, then the lens behaves as a convex mirror of focal length, $f = \frac{R}{2(\mu - 1)}$

- (v) If the spherical surface of a plano-concave lens is silvered, then the lens behaves as a convex mirror of

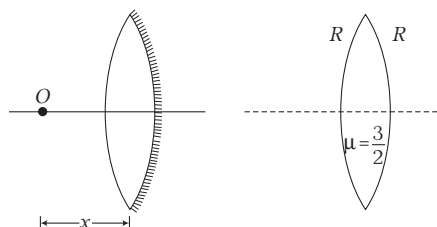
$$\text{focal length, } f = \frac{R}{2\mu}$$

- (vi) If one surface of a double concave lens is silvered, then the lens behaves as a convex mirror of

$$\text{focal length, } f = \frac{R}{2(2\mu - 1)}$$

Example 9.59 A biconvex thin lens is prepared from glass ($\mu = 1.5$), the two bounding surfaces having equal radii of 30 cm each. One of the surfaces is silvered from outside to make it reflecting. Where should an object be placed before this lens, so that the image is formed on the object itself?

Sol.



$$\text{We have, } \frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

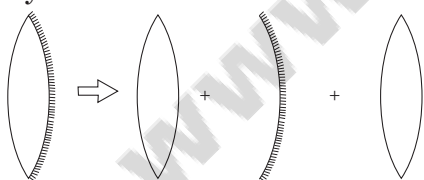
$$\frac{1}{f} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{30} - \frac{1}{-30} \right) = \left(\frac{1}{2} \right) \left(\frac{2}{30} \right)$$

$$\Rightarrow f = 30 \text{ cm}$$

The image by lens should be formed at centre of curvature of mirror.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-30} - \frac{1}{-x} = \frac{1}{30} \Rightarrow x = 15 \text{ cm}$$

Alternatively

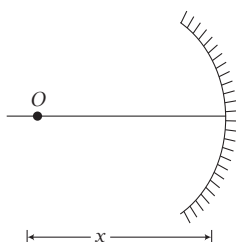


$$\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l} = \frac{2}{f_l} + \frac{1}{f_m} = \frac{2}{30} + \frac{1}{30/2} = \frac{4}{30}$$

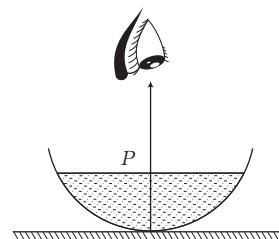
$$F = \frac{30}{4} \text{ cm}$$

Since, object and image coincide,

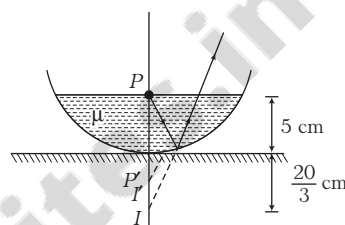
$$\text{Hence, } u = x = 2F = 15 \text{ cm}$$



Example 9.60 A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it upto a height of 5 cm. A small dust particle floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. (Take, $\mu_w = 2/3$)



Sol.



The dust particle is in water.

Image by concave mirror

$$u = -5 \text{ cm, } f = -20 \text{ cm, } v = ?$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{-5} = \frac{1}{-20}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{5} - \frac{1}{20} = \frac{3}{20}$$

$$\Rightarrow v = \frac{20}{3} \text{ cm}$$

The image of P is formed at I.

Image by refraction After reflection, direction of incident rays is reversed. I serves as an object

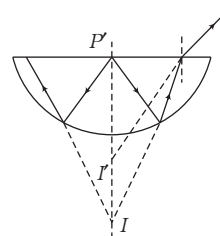
$$\mu_1 = \frac{2}{3}, u = - \left(5 + \frac{20}{3} \right) = - \frac{35}{3} \text{ cm}$$

$$\mu_2 = 1, v = ?$$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = 0 \Rightarrow \frac{1}{v} = \frac{2/3}{(-35/3)}$$

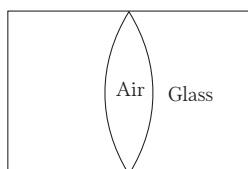
$$\Rightarrow v = - \frac{35}{2} = -17.5 \text{ cm}$$

Final image is formed at I'. $PI' = 17.5 \text{ cm}$

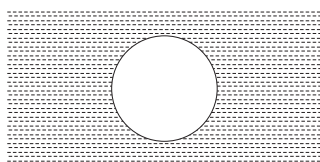


CHECK POINT 9.4

1. A convex lens has a focal length of 20 cm. It is used to form an image of an object placed 15 cm from lens. The image is
 - (a) virtual, inverted and enlarged
 - (b) real, inverted and diminished
 - (c) real, inverted and enlarged
 - (d) virtual, erect and enlarged
2. In the figure, an air lens of radii of curvature 10 cm ($R_1 = R_2 = 10$ cm) is cut in a cylinder of glass ($\mu = 2/3$). The focal length and the nature of the lens is

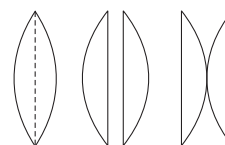


- (a) 15 cm, concave
 - (b) 15 cm, convex
 - (c) ∞ , neither concave nor convex
 - (d) 0, concave
3. An air bubble is contained inside water. It behaves as a



- (a) concave lens
 - (b) convex lens
 - (c) Neither convex nor concave
 - (d) Cannot say
4. A plano-convex lens is made of glass of refractive index 1.5. The radius of curvature of its convex surface is R . Its focal length is
 - (a) $R/2$
 - (b) R
 - (c) $2R$
 - (d) $1.5R$
 5. At what distance from a convex lens of focal length 30 cm an object should be placed, so that the size of image be $(1/4)$ th of the object?
 - (a) 30 cm
 - (b) 60 cm
 - (c) 15 cm
 - (d) 150 cm
 6. Distance of an object from a concave lens of focal length 20 cm is 40 cm. Then linear magnification of the image is
 - (a) 1
 - (b) < 1
 - (c) > 1
 - (d) zero
 7. In order to obtain a real image of magnification 2 using converging lens of focal length 20 cm, where should an object be placed?
 - (a) 50 cm
 - (b) 30 cm
 - (c) -50 cm
 - (d) -30 cm
 8. An object is placed at 10 cm from a lens and real image is formed with magnification of 0.5, then the lens is
 - (a) concave with focal length of $(10/3)$ cm
 - (b) convex with focal length of $(10/3)$ cm
 - (c) concave with focal length of 10 cm
 - (d) convex with focal length of 10 cm

9. The real image which is exactly equal to the size of an object is to be obtained on a screen with the help of a convex lens of focal length 15 cm. For this, what must be the distance between the object and screen?
 - (a) 15 cm
 - (b) 30 cm
 - (c) 45 cm
 - (d) 60 cm
10. A plano-convex lens of curvature of 30 cm and refractive index 1.5 produces a real image of an object kept 90 cm from it. What is the magnification?
 - (a) 4
 - (b) 0.5
 - (c) 1.5
 - (d) 2
11. The minimum distance between an object and its real image formed by a convex lens is
 - (a) $1.5f$
 - (b) $2f$
 - (c) $2.5f$
 - (d) $4f$
12. A convex lens of refractive index $3/2$ has a power of 2.5 D. If it is placed in a liquid of refractive index 2, the new power of the lens is
 - (a) 2.5 D
 - (b) -2.5 D
 - (c) 1.25 D
 - (d) -1.25 D
13. Two thin lenses, one of focal length +60 cm and the other of focal length -20 cm are put in contact. The combined focal length is
 - (a) +15 cm
 - (b) -15 cm
 - (c) +30 cm
 - (d) -30 cm
14. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm. The power of combination is
 - (a) -1.5 D
 - (b) -6.5 D
 - (c) +6.5 D
 - (d) +1.5 D
15. Two similar plano-convex lenses are combined together in three different ways as shown in the adjoining figure. The ratio of the focal lengths in three cases will be



- (a) 2 : 2 : 1
 - (b) 1 : 1 : 1
 - (c) 1 : 2 : 2
 - (d) 2 : 1 : 1
16. A biconvex lens has a focal length f . It is cut into two parts along a line perpendicular to principal axis. The focal length of each part will be
 - (a) $f/2$
 - (b) f
 - (c) $(3/2)f$
 - (d) $2f$
 17. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen, then
 - (a) half the image will disappear
 - (b) complete image will disappear
 - (c) intensity of image will increase
 - (d) intensity of image will decrease
 18. In a plano-convex lens the radius of curvature of the convex lens is 10 cm. If the plane side is polished, then the focal length will be (Refractive index = 1.5)
 - (a) 20.5 cm
 - (b) 10 cm
 - (c) 15.5 cm
 - (d) 5 cm

PRISM

It is a portion of a transparent medium generally made of crown or flint glass and bounded by three planes intersecting by three parallel straight lines AD , CF and BE as shown in figure.

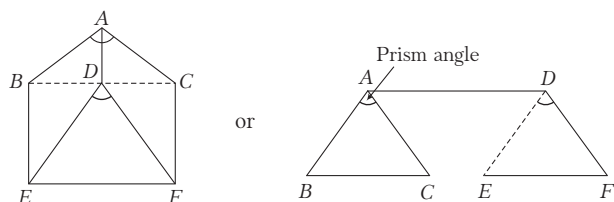


Fig. 9.64 Prism

$ABED$ and $ACFD$ are two refracting faces as shown in the figure. $\angle A$ is called angle of prism or refracting angle.

Refraction of light through a prism

The figure below shows the passage of light through a triangular prism ABC .

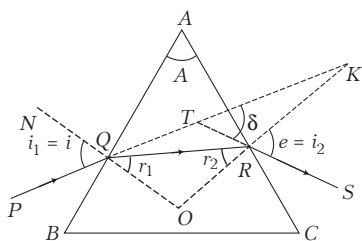


Fig. 9.65 Cross-sectional view of a prism

The angles of incidence and refraction at first face AB are i and r_1 , respectively.

The angle of refraction at the second face AC is r_2 and the angle of emergence is e .

The angle between the emergent ray RS and incident ray PQ is called angle of deviation (δ).

Here, angle of deviation is given by,

$$\delta = (i_1 + i_2) - A \quad \dots(i)$$

where, A is angle of prism and $\angle A = \angle r_1 + \angle r_2$, $\dots(ii)$

or $\delta = (i + e) - A$, where e is angle of emergence which is equal to i_2 .

If angle of incidence and refraction are small, then

$$\delta = (\mu - 1) A \quad \dots(iii)$$

This expression shows that for a given angle A all rays entering a small angled prism at small angles of incidence suffer the same deviation.

Minimum deviation

The angle of emergence of the ray from the second face equals the angle of incidence of the ray on the first face then deviation produced is minimum.

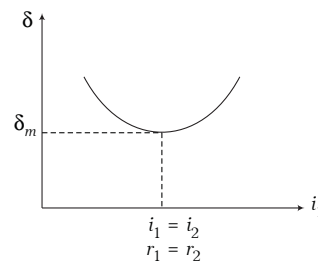


Fig. 9.66 i_1 - δ curve

$$\text{i.e.} \quad i_2 = i_1 = i \quad \dots(iv)$$

$$\text{and} \quad r_1 = r_2 = r \quad \dots(v)$$

From Eqs. (ii) and (v), we get

$$r = \frac{A}{2}$$

$$\text{Further at, } \delta = \delta_m = (i + i) - A \text{ or } i = \frac{A + \delta_m}{2}$$

$$\text{From Snell's law, } \mu = \frac{\sin i}{\sin r}$$

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} \quad \dots(vi)$$

or

Expression given as Eq. (vi) is also called **prism formula**.

For small angled prism, $\sin\left(\frac{A + \delta_m}{2}\right) \approx \frac{A + \delta_m}{2}$ and

$$\sin A/2 \approx A/2$$

$$\text{From Eq. (vi), } \mu = \frac{(A + \delta_m)}{A} \Rightarrow \delta_m = (\mu - 1)A$$

$$\Rightarrow \delta_m = (\mu - 1)A \quad \dots(vii)$$

Thus, Eq. (iii) is derived here.

Condition of no emergence

For no emergence of light, total internal reflections must take place at the second surface. For that, $r_2 > C$

$$\text{So,} \quad A > r_1 + C \quad (\because A = r_1 + r_2)$$

Maximum value of $r_1 = C$

So, $A \geq 2C$ for any angle of incidence.

If light ray is incident normally on first surface

$$\text{i.e.} \quad i = 0, \text{ it means } \angle r = 0^\circ$$

So in this case, condition of no emergence from second surface is $A > C$

$$\therefore \sin A > \sin C$$

$$\sin A > \frac{1}{\mu} \Rightarrow \mu > \operatorname{cosec} A$$

Example 9.61 A ray of light is incident at an angle of 60° on the face of a prism having refracting angle 30° . The ray emerging out of the prism makes an angle 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which, it emerges.

Sol. Given, $i_1 = 60^\circ$, $A = 30^\circ$ and $\delta = 30^\circ$.

From the relation, $\delta = (i_1 + i_2) - A$

we have, $i_2 = 0$

This means that the emergent ray is perpendicular to the face through which it emerges.

Example 9.62 The refracting angle of a glass prism is 30° . A ray is incident onto one of the faces perpendicular to it. Find the angle δ between the incident ray and the ray that leaves the prism. The refractive index of glass is $\mu = 1.5$.

Sol. Given, $A = 30^\circ$, $\mu = 1.5$ and $i_1 = 0^\circ$

Since, $i_1 = 0^\circ$, therefore, r_1 is also equal to 0° .

Further, since, $r_1 + r_2 = A$

$$\therefore r_2 = A = 30^\circ$$

$$\text{Using, } \mu = \frac{\sin i_2}{\sin r_2}$$

$$\text{we have, } 1.5 = \frac{\sin i_2}{\sin 30^\circ}$$

$$\text{or } \sin i_2 = 1.5 \sin 30^\circ = 1.5 \times \frac{1}{2} = 0.75$$

$$\therefore i_2 = \sin^{-1}(0.75) = 48.6^\circ$$

Now, the deviation, $\delta = (i_1 + i_2) - A = (0^\circ + 48.6^\circ) - 30^\circ$

$$\text{or } \delta = 18.6^\circ$$

Example 9.63 The angle of minimum deviation for a glass prism with $\mu = \sqrt{3}$ equals the refracting angle of the prism. What is the angle of the prism?

Sol. Given, $A = \delta_m$

$$\text{Using, } \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\text{we have, } \sqrt{3} = \frac{\sin\left(\frac{A + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\text{or } \sqrt{3} = \frac{\sin A}{\sin\left(\frac{A}{2}\right)} = \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{\sin\left(\frac{A}{2}\right)}$$

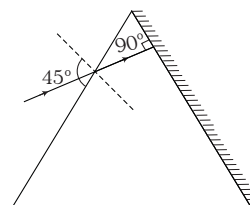
$$\therefore \cos \frac{A}{2} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{A}{2} = 30^\circ \text{ or } A = 60^\circ$$

$$\Rightarrow \delta_m = A = 60^\circ$$

Example 9.64 One face of a prism with a refracting angle of 30° is coated with silver. A ray incident on another face at an angle of 45° is refracted and reflected from the silver coated face and retraces its path. What is the refractive index of the prism?

Sol.



Given, $A = 30^\circ$, $i_1 = 45^\circ$ and $r_2 = 0$

Since, $r_1 + r_2 = A$

$$\therefore r_1 = A = 30^\circ$$

Now, refractive index of the prism,

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \sqrt{2}$$

Example 9.65 A prism is made of glass of unknown refractive index. A parallel beam of light is incident on a face of the prism. The angle of minimum deviation is measured to be 40° . What is the refractive index of the material of the prism? The refracting angle of the prism is 60° . If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

$$\begin{aligned} \text{Sol. Using the relation, } \mu &= \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60^\circ + 40^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)} \\ &= \frac{\sin 50^\circ}{\sin 30^\circ} = \frac{0.7660}{0.50} = 1.532 \approx 1.53 \end{aligned}$$

Let δ'_m be the new angle of minimum deviation of a parallel beam of light when the prism is placed in water of refractive index, ${}_a\mu_w = 1.33$ and ${}_a\mu_g = 1.532$

$$\therefore {}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.532}{1.33} = 1.152$$

$${}_w\mu_g = \frac{\sin\left(\frac{A + \delta'_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\text{or } 1.152 = \frac{\sin\left(\frac{60^\circ + \delta'_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$\text{or } \sin\left(30^\circ + \frac{\delta'_m}{2}\right) = 1.152 \times \frac{1}{2} = 0.5759 = \sin 35.17^\circ$$

$$\text{or } 30^\circ + \frac{\delta'_m}{2} = 35.17^\circ \text{ or } \frac{\delta'_m}{2} = 35.17^\circ - 30 = 5.17^\circ$$

$$\text{Angle of minimum deviation, } \delta'_m = 2 \times 5.17^\circ = 10.34^\circ = 10^\circ 20'$$

Example 9.66 At what angle should a ray of light be incident on the face of a prism of refracting angle 60° , so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Sol. Let C = critical angle for the material of the prism.

$$\therefore \sin C = \frac{1}{\mu} = \frac{1}{1.524} = 0.6562$$

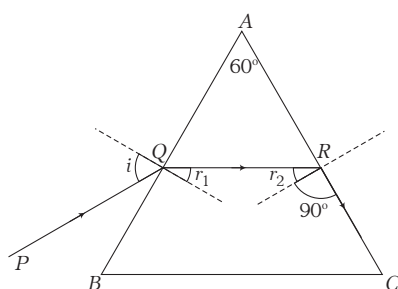
$$\text{or } C = \sin^{-1}(0.6562) = 41^\circ$$

We know that for a prism, $A = r_1 + r_2$

Here, $r_2 = C = 41^\circ$, $A = 60^\circ$

$$\therefore 60^\circ = r_1 + 41^\circ \text{ or } r_1 = 60^\circ - 41^\circ = 19^\circ$$

For the face AB , $r_1 = 19^\circ$, $\mu = 1.524$



$\therefore$ According to Snell's law,

$$\mu = \frac{\sin i}{\sin r_1}$$

$$\begin{aligned} \text{or } \sin i &= \mu \sin r_1 = 1.524 \sin 19^\circ \\ &= 1.524 \times 0.3256 = 0.4962 \\ i &= \sin^{-1}(0.4962) = 29.75^\circ \end{aligned}$$

$\therefore$ Angle of incidence, $i = 29.75^\circ \approx 30^\circ$

Dispersion of light by a prism

If a beam of white light (contains all colours), is sent through the prism, it is then separated into a spectrum of its constituents colours. The spreading of white light into its colour components is called **dispersion**.

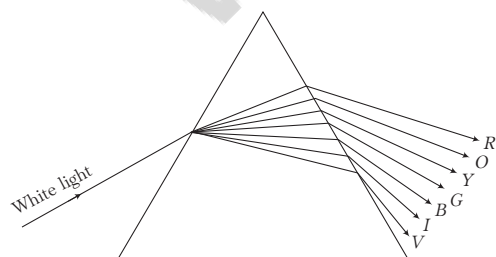


Fig. 9.67 Dispersion of white light

If δ_r , δ_y and δ_v are the deviations for red, yellow and violet components, then average deviation is measured by δ_y as yellow light falls in between red and violet. So, $\delta_v - \delta_r$ is called **angular dispersion**. The **dispersive power** of a material is defined as the ratio of angular

dispersion to the average deviation when a white beam of light is passed through it. It is denoted by ω .

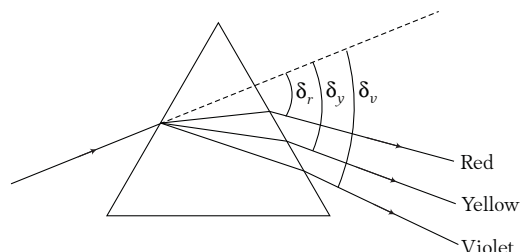


Fig. 9.68 Deviation of rays of different colours in a prism

As we know, $\delta = (\mu - 1) A$

This equation is valid, when A and i are small. Suppose, a beam of white light is passed through such a prism, the deviation of red, yellow and violet light are given by,

$$\delta_r = (\mu_r - 1)A,$$

$$\delta_y = (\mu_y - 1)A,$$

$$\text{and } \delta_v = (\mu_v - 1)A$$

The angular dispersion is, $\delta_v - \delta_r = (\mu_v - \mu_r) A$ and the average deviation is $\delta_y = (\mu_y - 1) A$.

Thus, the dispersive power of the medium is,

$$\omega = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\mu_v - \mu_r}{\mu_y - 1} \quad \dots(i)$$

Note

- Deviation corresponding to yellow coloured light is average or mean deviation, $\mu_y = \frac{\mu_v + \mu_r}{2}$
- A single prism produces both deviation and dispersion simultaneously.

Example 9.67 The refractive indices of flint glass for red and violet light are 1.613 and 1.632, respectively. Find the angular dispersion produced by the thin prism of flint glass having refracting angle 4° .

Sol. Angular dispersion,

$$\theta = (\mu_v - \mu_r)A = (1.632 - 1.613) \times 4^\circ = 0.076^\circ$$

Example 9.68 Evaluate the dispersive power of flint glass.

The refractive indices of flint glass for red, yellow and violet light are 1.613, 1.620 and 1.632, respectively.

Sol. Dispersive power is given by

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.632 - 1.613}{1.620 - 1} = 0.031$$

Combination of prisms

Figure shows two prisms of refracting angles A and A' and dispersive powers ω and ω' , respectively. They are placed in contact in such a way that the two refracting angles are reversed with respect to each other. A ray of light passes

through the combination as shown. The deviation produced by the two prisms are,

$$\delta_1 = (\mu - 1) A$$

and

$$\delta_2 = (\mu' - 1) A'$$

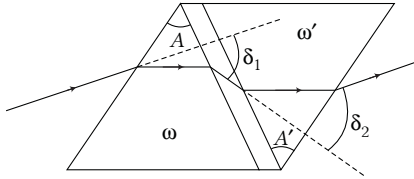


Fig. 9.69 Combination of two prisms

As the two deviations are opposite to each other, the net deviation is,

$$\delta = \delta_1 - \delta_2 = (\mu - 1)A - (\mu' - 1)A' \quad \dots(ii)$$

Using this equation, the average deviation produced by the combination, if white light is passed is,

$$\delta_y = (\mu_y - 1)A - (\mu'_y - 1)A' \quad \dots(iii)$$

and the net angular dispersion is,

$$\delta_v - \delta_r = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$

But, as $\mu_v - \mu_r = \omega(\mu_y - 1)$ from Eq. (i), we have

$$\delta_v - \delta_r = (\mu_y - 1)\omega A - (\mu'_y - 1)\omega' A' \quad \dots(iv)$$

Dispersion without average deviation

From Eq. (iii), $\delta_y = 0$, if

$$\frac{A}{A'} = \frac{\mu'_y - 1}{\mu_y - 1} \quad \dots(v)$$

This is the required condition of dispersion without average deviation. Using this in Eq. (iv), the net angular dispersion produced is :

$$\delta_v - \delta_r = (\mu_y - 1)A (\omega - \omega')$$

Average deviation without dispersion

From Eq. (iv), $\delta_v - \delta_r = 0$, if

$$\frac{A}{A'} = \frac{(\mu'_y - 1)\omega'}{\mu_y - 1} = \frac{\mu'_v - \mu'_r}{\mu_v - \mu_r} \quad \dots(vi)$$

This is the required condition of average deviation without dispersion. Using the above condition in Eq. (iii), the net

average deviation is, $\delta_y = (\mu_y - 1)A \left(1 - \frac{\omega}{\omega'}\right)$

Example 9.69 White light is passed through a prism of angle 4° . If the refractive indices for red and blue colours are 1.641 and 1.659 respectively, then calculate the angle of deviation between them, also calculate the dispersive power.

Sol. Angle of deviation,

$$\delta = (\mu_y - 1)A = \left(\frac{\mu_r + \mu_b}{2} - 1\right)4^\circ$$

$$= \left(\frac{1.641 + 1.659}{2} - 1\right) \times 4^\circ = (1.65 - 1) \times 4^\circ = 2.6^\circ$$

Dispersive power,

$$\omega = \frac{\mu_b - \mu_r}{\mu_y - 1} = \frac{1.659 - 1.641}{1.650 - 1} = 0.028$$

Example 9.70 We have to combine a crown glass prism of angle 4° with a flint glass prism in such a way that the mean ray passes undeviated, find (i) the angle of the flint glass prism needed and (ii) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.514, 1.517 and 1.523 respectively for crown glass and 1.613, 1.620 and 1.632, respectively for flint glass.

Sol. It is given that, for

Crown glass: $\mu_r = 1.514, \mu_y = 1.517, \mu_v = 1.523, A = 4^\circ$

Flint glass: $\mu'_r = 1.613, \mu'_y = 1.620, \mu'_v = 1.632, A' = ?$

(i) For no deviation $(\mu_y - 1)A = (\mu'_y - 1)A'$

$$\Rightarrow (1.517 - 1) \times 4 = (1.620 - 1)A'$$

$$\Rightarrow A' = 3.34^\circ$$

(ii) Also, angular dispersion,

$$\theta_0 = \theta_1 - \theta_2$$

$$= |(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'|$$

$$= |(1.523 - 1.514) \times 4 - (1.632 - 1.613) \times 3.34|$$

$$= 0.027^\circ$$

Example 9.71 It is given that the dispersive powers of crown and flint glasses are 0.06 and 0.10 respectively. Also the refractive indices for yellow light for these glasses are 1.517 and 1.621, respectively. We have to form an achromatic combination of prism of crown and flint glasses which can produce a deviation of 1° in the yellow ray. Evaluate the refracting angles of the two prisms needed.

Sol. Crown glass, $\omega = 0.06, \mu_y = 1.517$

Flint glass, $\omega' = 0.10, \mu'_y = 1.621$

For no dispersion,

$$(\mu_v - \mu_r)A = (\mu'_v - \mu'_r)A'$$

$$\Rightarrow \omega(\mu_y - 1)A = \omega'(\mu'_y - 1)A'$$

$$\left(\because \omega = \frac{\mu_v - \mu_r}{\mu_y - 1}, \omega' = \frac{\mu'_v - \mu'_r}{\mu'_y - 1} \right)$$

$$\Rightarrow 0.06(1.517 - 1)A = 0.10(1.621 - 1)A' \Rightarrow A = 2A'$$

$$\text{Deviation, } \delta_0 = |(\mu_y - 1)A - (\mu'_y - 1)A'|$$

$$1 = |(1.517 - 1)A - (1.621 - 1)A'|$$

$$= |0.517A - 0.621A'|$$

$$= |0.517 \times 2A' - 0.621A'|$$

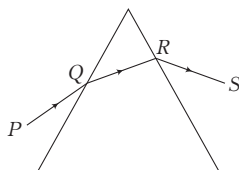
$$\Rightarrow 1 = 0.413A'$$

$$\Rightarrow A' = 2.42^\circ$$

or Angle of prism, $A = 2A' = 4.84^\circ$

CHECK POINT 9.5

1. A ray of light is incident on an equilateral glass prism placed on a horizontal table. For minimum deviation which of the following is true?



- (a) PQ is horizontal
 (b) QR is horizontal
 (c) RS is horizontal
 (d) Either PQ or RS is horizontal
2. A ray of light is incident at an angle of 60° on one face of a prism of angle 30° . The ray emerging out of the prism makes an angle of 30° with the incident ray. The emergent ray is
 (a) normal to the face through which it emerges
 (b) inclined at 30° to the face through which it emerges
 (c) inclined at 60° to the face through which it emerges
 (d) None of the above
3. When light rays are incident on a prism at an angle of 45° , the minimum deviation is obtained. If refractive index of the material of prism is $\sqrt{2}$, then the angle of prism will be
 (a) 30° (b) 75° (c) 90° (d) 60°

4. A ray of light passes through an equilateral glass prism in such a manner that the angle of incidence is equal to the angle of emergence and each of these angles is equal to $3/4$ of the angle of the prism. The angle of deviation is
 (a) 45° (b) 39° (c) 20° (d) 30°
5. In a thin prism of glass (refractive index 1.5), which of the following relations between the angle of minimum deviations δ_m and angle of prism r will be correct?
 (a) $\delta_m = r$ (b) $\delta_m = 1.5r$
 (c) $\delta_m = 2r$ (d) $\delta_m = r/2$
6. The angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of the prism. The angle of prism is (Given, $\cos 41^\circ = 0.75$)
 (a) 21° (b) 42° (c) 60° (d) 82°
7. Dispersive power depends upon
 (a) the angle of prism
 (b) material of prism
 (c) deviation produced by prism
 (d) height of the prism
8. A thin prism P_1 with angle 6° and made from glass of refractive index 1.54 is combined with another thin prism P_2 of refractive index 1.72 to produce dispersion without deviation. The angle of prism P_2 will be
 (a) $5^\circ 24'$ (b) $4^\circ 30'$
 (c) 6° (d) 8°

OPTICAL INSTRUMENTS

Instruments (e.g., microscope, telescope etc.) which are used to assist the eye in viewing an object are called optical instruments. We shall discuss these instruments in detail in coming sections followed by some basics of the eye.

The eye

It is the organ which gives us the sense of sight.

Few important points related to the eye are as follows

- The eye behaves as convex lens of refractive index 1.437.
- Real and inverted image of an object are obtained at retina, however brain sense it erect.
- Yellow spot is the most sensitive part of the eye, the image formed at yellow spot is brighter.
- Blind spot is not sensitive for light.
- Eye lens is fixed between ciliary muscles. The shape (curvature) of the eye can be changed by ciliary muscles.

Power of accommodation

It is the maximum variation in power of eye lens for focussing near or far objects clearly at retina. For a young adult with normal visions the power of accommodation is about 4 D. The eye loses its power of accommodation at old age.

Range of vision

The minimum distance for the clear vision of an object is called **least distance of distinct vision (D)**. For normal eye, this distance is generally taken to be 25 cm. The range of vision of eye is from 25 cm (near point) to ∞ (far point).

Persistence of vision

It is $1/10$ s i.e., if time interval, between two consecutive light pulses is lesser than 0.1s, eye cannot distinguish them separately.

Visual angle

The size of an object as sensed by us is related to the size of the image formed on the retina.

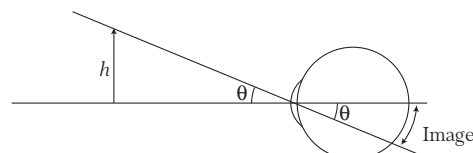


Fig. 9.70 Visual angle

The size of the image formed on the retina is roughly proportional to the angle subtended by the object on the eye.

This angle is known as the **visual angle** (θ). Optical instruments are used to increase this angle artificially in order to improve the clarity.

Note While testing your eye through reading chart, if doctor finds it to 6/12 it implies that you can read a letter from 6m which the normal eye can read from 12m. Thus, 6/6 means normal eye sight.

Magnifying power (M)

It is the factor by which the image on the retina can be enlarged by using the microscope or telescope.

Magnifying power of a microscope,

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle formed by the object}} \quad (\text{when kept at distance } D)$$

Magnifying power of a telescope,

$$M = \frac{\text{Visual angle formed by final image}}{\text{Visual angle subtended by the object}} \quad (\text{directly when seen from naked eye})$$

Microscope

It is an optical instrument used to see very small object.

Simple microscope

It consists of a single convex lens (converging lens) of small focal length. The image formed by this microscope is erect, virtual, enlarged and on same side of object.

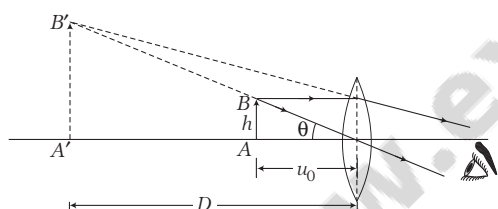


Fig. 9.71 Image formation in a simple microscope

- (i) When final image is formed at infinity.

$$\text{Magnifying power, } M_{\infty} = \frac{D}{f}$$

This is also called magnifying power for normal adjustment.

- (ii) When final image is formed at D (near point).

$$\text{Magnifying power, } M_D = 1 + \frac{D}{f}$$

Note

- $M_D > M_{\infty}$, i.e. when final image is formed at 25 cm, angular magnification is increased but eye is most strained. On the other hand, when final image is at infinity, angular magnification is slightly less but eye is relaxed. So, the choice is yours, whether you want to see bigger size with strained eye or smaller size with relaxed eye.
- M can be increased by decreasing f , but due to several other aberrations the image becomes too defective at large magnification with a simple microscope. Roughly speaking a magnification power upto 4 is trouble free.

Example 9.72 A 20 D lens is used as a magnifier. Where should the object be placed to obtain maximum angular magnification? (Given, $D = 25$ cm).

Sol. Focal length of the lens, $f = \frac{1}{20} \text{ m} = \frac{100}{20} \text{ cm} = 5 \text{ cm}$

Maximum angular magnification is obtained when final image is formed at D . Hence, by using lens formula,

$$\frac{1}{-25} - \frac{1}{-u_o} = \frac{1}{5} \Rightarrow u_o = 4.17 \text{ cm}$$

Compound microscope

Simple microscope has a limited maximum magnification. So, for larger magnification, one can use compound microscope. It consists of two converging lenses arranged coaxially. The one facing the object is called **objective** and the one close to eye is called **eye-piece**. The objective has a smaller aperture and smaller focal length than those of the eye-piece.

The separation between the objective and the eye-piece (called the length of the microscope L) can be varied by appropriate screws fixed on the panel of microscope.

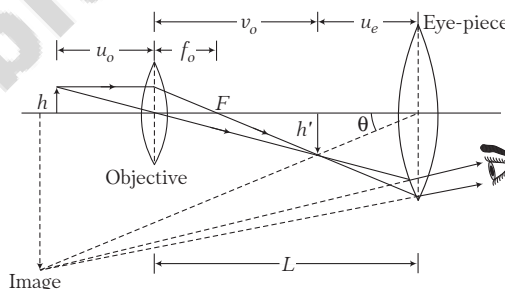


Fig. 9.72 Image formation in a compound microscope

The object is placed beyond first focus of objective, so that an inverted and real image (intermediate image) is formed by the objective. This intermediate image acts as an object for the eye-piece and lies between first focus and pole of eye-piece. The final magnified virtual image is formed by the eye-piece.

$\therefore$ Magnifying power of the compound microscope,

$$M = \frac{v_o}{u_o} \left(\frac{D}{u_e} \right)$$

or $M = M_o \times M_e$, where M_o is the magnifying power of objective and M_e is the magnifying power of eye-piece.

- (i) When the final image is formed at infinity,

$$\text{Magnifying power, } M_{\infty} = \frac{v_o}{u_o} \frac{D}{f_e}$$

and

$$L_{\infty} = v_o + f_e \left(\because M_{\infty} = \frac{(L_{\infty} - f_o - f_e)}{f_o f_e} \right)$$

(ii) When the final image (by eye-piece) is formed at D .

$$\text{Magnifying power, } M_D = \frac{v_o}{u_o} \left(1 + \frac{D}{f_e} \right)$$

and

$$L_D = v_o + |u_e| = v_o + \frac{Df_e}{D + f_e}$$

In general, object is placed very near to the principal focus of objective and f_o is also very small, then

$$M_D = -\frac{L}{f_o} \left(1 + \frac{D}{f_e} \right)$$

Example 9.73 The separation between the objective and the eye-piece of a compound microscope can be adjusted between 9.8 cm to 11.8 cm. If the focal lengths of the objective and the eye-piece are 1.0 cm and 6 cm respectively, find the range of the magnifying power, if the image is always needed at 24 cm from the eye.

Sol. For eye-piece, it is given that

$$f_e = 6 \text{ cm}, \quad v_e = -24 \text{ cm}$$

$$\therefore \frac{1}{-24} - \frac{1}{-u_e} = \frac{1}{6} \quad \text{or} \quad u_e = 4.8 \text{ cm}$$

When, $L = 9.8 \text{ cm}$, then $v_o = (9.8 - 4.8) \text{ cm} = 5.0 \text{ cm}$

$$\therefore \frac{1}{5.0} - \frac{1}{-u_o} = \frac{1}{1.0}$$

$$\text{or} \quad u_o = 1.25 \text{ cm}$$

$$\text{or Magnifying power, } |M| = \frac{v_o}{u_o} \cdot \frac{D}{u_e} = \left(\frac{5.0}{1.25} \right) \left(\frac{25.0}{4.8} \right) = 20.83$$

When, $L = 11.8 \text{ cm}$, then $v_o = (11.8 - 4.8) \text{ cm} = 7.0 \text{ cm}$

$$\therefore \frac{1}{7.0} - \frac{1}{-u_o} = \frac{1}{1.0} \quad \text{or} \quad u_o = 1.17 \text{ cm}$$

$$|M| = \frac{v_o}{u_o} \cdot \frac{D}{f_e} = \left(\frac{7.0}{1.17} \right) \left(\frac{25.0}{4.8} \right) = 31.16$$

Therefore, range of magnifying power is from 20.83 to 31.16.

Example 9.74 A compound microscope has a magnifying power of 100 when the image is formed at infinity. The objective has a focal length of 0.5 cm and the tube length is 6.5 cm. Find the focal length of the eye-piece.

Sol. When the final image is at infinity,

$$u_e = f_e = \text{tube length} - v_o$$

$$\therefore f_e = 6.5 - v_o \quad \dots(i)$$

$$\text{Since, } |M_\infty| = \frac{v_o}{u_o} \cdot \frac{D}{f_e}$$

$$\therefore 100 = \frac{v_o}{u_o} \cdot \frac{25}{f_e} \quad \text{or} \quad \frac{v_o}{u_o f_e} = 4 \quad \dots(ii)$$

For the objective,

$$\frac{1}{v_o} - \frac{1}{-u_o} = \frac{1}{f_o} = \frac{1}{0.5} \Rightarrow \frac{1}{v_o} + \frac{1}{u_o} = 2 \quad \dots(iii)$$

We have three unknowns v_o , u_o and f_e , solving these three equations, we get

$$f_e = 2 \text{ cm}$$

Telescopes

A microscope is used to view the objects placed closed to it. To look at distant objects such as a star, a planet or a distant tree etc., we use telescopes. There are three types of refracting telescopes in use

- Astronomical telescope,
- Terrestrial telescope and
- Galilean telescope.

Astronomical telescope

It is used to see the heavenly bodies. It consists of two converging lenses placed coaxially. The one facing the distant object is called the objective and has a large aperture and large focal length. The other is called the eye-piece, as the eye is placed closed to it.

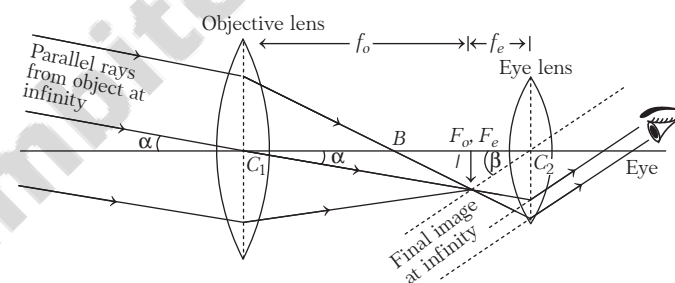


Fig. 9.73 Image formation in an astronomical telescope

The intermediate image formed by objective is real, inverted and small and final image formed by eye-piece is real, inverted and highly magnified.

Magnifying power

(i) When object is at infinity

$$\text{Magnifying power, } M = \frac{\beta}{\alpha} = \frac{f_o}{u_e} \quad \text{or} \quad M = \frac{f_o}{f_e}$$

where, α = angle subtended by object on objective and β = angle subtended by final image at eye-piece.

$$\therefore M_\infty = \frac{f_o}{f_e} \quad \text{and} \quad L_\infty = f_o + f_e \quad (\because u_e = f_e)$$

(ii) When the final image is at D

$$\text{Magnifying power, } M_D = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

and

$$L_D = f_o + \frac{Df_e}{D + f_e}$$

Terrestrial telescope

It consists of three converging lenses, objective, eye-piece lens and erecting lens. It is used to see far off objects on the earth and its final image is virtual, erect and smaller.

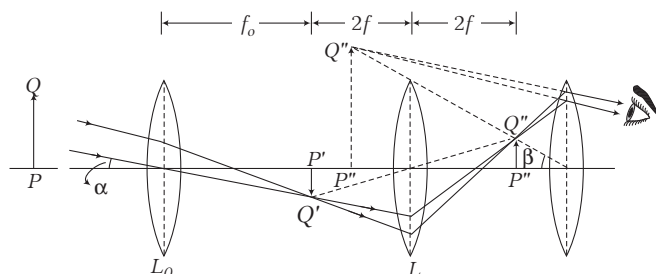


Fig. 9.74 Image formation in a terrestrial telescope

Magnifying power

(i) When image is formed at infinity

$$M_{\infty} = \frac{f_o}{f_e} \quad \text{and} \quad L_{\infty} = f_o + 4f + f_e$$

(ii) When image is formed at D

$$M_D = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$$

$$\text{and } L_D = f_o + 4f + \frac{Df_e}{D + f_e}$$

Galilean telescope

In this telescope, a convergent lens is used as the objective and a divergent lens as the eye-piece. The objective lens forms a real and inverted image but the divergent lens comes in between the objective and real image formed by

the objective. Real image formed by the objective is called intermediate image. This intermediate image acts as virtual object for eye-piece. Final image is erect and magnified as shown in figure. The intermediate image is formed at second focus of objective.

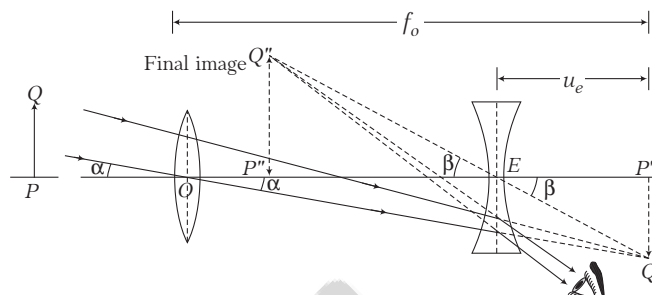


Fig. 9.75 Image formation in a Galilean telescope

Magnifying power

$$\text{Magnifying power, } M_{\infty} = \frac{f_o}{f_e}$$

$$\text{and } L_{\infty} = f_o - f_e = |f_o| - u_e$$

When the final image is formed at a distance D

$$\text{Thus, magnifying power, } M_D = \frac{f_o}{f_e} \left(1 - \frac{f_e}{D} \right)$$

$$\text{and } L_D = f_o - \frac{f_e D}{D - f_e}$$

Note

- (i) In all above formulae of M , we are considering only the magnitude of M .
- (ii) For telescopes, formulae have been derived when the object is at infinity. For the object at some finite distance different formulae will have to be derived.

Summary of formulae for optical instruments

Name of Optical Instruments	M	L	M_{∞}	M_D	L_{∞}	L_D
Simple microscope	$\frac{D}{u_o}$	—	$\frac{D}{f}$	$1 + \frac{D}{f}$	—	—
Compound microscope	$\frac{v_o}{u_o} \frac{D}{u_e}$	$v_o + u_e$	$\frac{v_o}{u_o} \frac{D}{f_e}$	$\frac{v_o}{u_o} \left(1 + \frac{D}{f_e} \right)$	$v_o + f_e$	$v_o + \frac{Df_e}{D + f_e}$
Astronomical telescope	$\frac{f_o}{u_e}$	$f_o + u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$	$f_o + f_e$	$f_o + \frac{Df_e}{D + f_e}$
Terrestrial telescope	f_o / u_e	$f_o + 4f + u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$	$f_o + 4f + f_e$	$f_o + 4f + \frac{Df_e}{D + f_e}$
Galilean telescope	$\frac{f_o}{u_e}$	$f_o - u_e$	$\frac{f_o}{f_e}$	$\frac{f_o}{f_e} \left(1 - \frac{f_e}{D} \right)$	$f_o - f_e$	$f_o - \frac{f_e D}{D - f_e}$

Reflecting telescope

This type of telescope is based upon the same principle except that the formation of images takes place by reflection instead of by refraction. It consists of concave mirror of large aperture and large focal length (objective). A plane mirror is placed between the concave mirror and its focus. A small convex lens works as eye-piece.

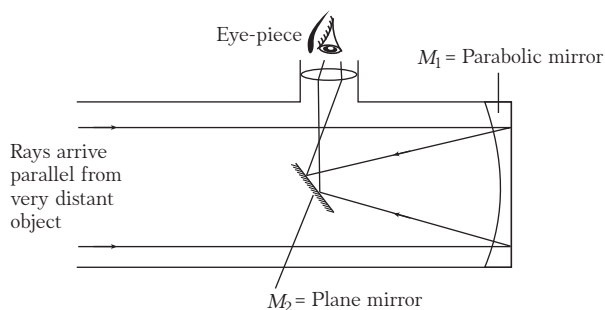


Fig. 9.76 Reflecting telescope

If f_o is focal length of the concave spherical mirror and f_e the focal length of the eye-piece, then magnifying power of the reflecting telescope is given by $m = \frac{f_o}{f_e}$

Example 9.75 A small telescope has an objective lens of focal length 140 cm and an eye-piece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distance objects when

- the telescope is in normal adjustment (i.e., when the final image is at infinity)?
- the final image is formed at the least distance of distinct vision 25 cm?

Sol. (i) When the telescope is in normal adjustment, the magnifying power is given by

$$M = \frac{f_o}{|f_e|} = + \frac{140}{5} = 28$$

- (ii) When the final image is formed at the least distance of distinct vision, then M is given by

$$M = \frac{f_o}{|f_e|} \left(1 + \frac{f_e}{D} \right) = \frac{140}{5} \left(1 + \frac{5.0}{25} \right) = 33.6$$

Example 9.76 A small telescope has an objective lens of focal length 144 cm and an eye-piece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eye-piece?

Sol. M = magnifying power of telescope

L = separation between the objective and the eyepiece

In normal adjustment (i.e., when the final image is formed at ∞),

$$M = \frac{f_o}{f_e} = - \frac{144}{6} = -24$$

$$\Rightarrow |M| = 24$$

$$\therefore L = f_o + f_e = 144 + 6 = 150 \text{ cm}$$

Example 9.77 The eye-piece of an astronomical telescope has a focal length of 10 cm. The telescope is focused for normal vision of distant objects when the tube length is 1.0 m. Find the focal length of the objective and the magnifying power of the telescope.

Sol. Given, $f_e = 10 \text{ cm}$, $L_\infty = 1.0 \text{ m} = 100 \text{ cm}$

$$\text{Since, } L_\infty = f_o + f_e$$

$$\therefore f_o = 90.0 \text{ cm}$$

$$\text{Further, magnification, } M_\infty = \frac{f_o}{f_e} = \frac{90.0}{10.0} = 9.0$$

Example 9.78 An astronomical telescope is to be designed to have a magnifying power of 50 in normal adjustment. If the length of the tube is 102 cm, find the powers of the objective and the eye-piece.

Sol. Given, $M_\infty = 50$

$$\therefore \frac{f_o}{f_e} = 50 \quad \dots(i)$$

$$\text{Further, } L_\infty = 102 \text{ cm}$$

$$\therefore f_o + f_e = 102 \text{ cm} \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we have

$$f_o = 100 \text{ cm} \quad \text{and} \quad f_e = 2 \text{ cm}$$

$$\therefore \text{Power of object, } P_o = \frac{1}{1.0} = 1 \text{ D}$$

$$\text{and power of eye-piece, } P_e = \frac{1}{0.02} = 50 \text{ D}$$

Example 9.79 A telescope has an objective of focal length 50 cm and an eye-piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focused for distinct vision on a scale 2 m away from the objective. Calculate (i) magnification produced and (ii) separation between objective and eyepiece.

Sol. Given, $f_o = 50 \text{ cm}$ and $f_e = 5 \text{ cm}$

$$\text{For objective, } \frac{1}{v_o} - \frac{1}{-200} = \frac{1}{50}$$

$$\therefore v_o = \frac{200}{3} \text{ cm} \Rightarrow m_o = \frac{v_o}{u_o} = \frac{(200/3)}{-200} = -\frac{1}{3}$$

$$\text{For eye-piece, } \frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$$

$$\therefore u_e = -\frac{25}{6} \text{ cm} \quad \text{and} \quad m_e = \frac{v_e}{u_e} = \frac{-25}{-(25/6)} = 6$$

$$(i) \text{ Magnification, } m = m_o \times m_e = -2$$

(ii) Separation between objective and eye-piece,

$$L = v_o + |u_e| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.83 \text{ cm}$$

Note Here, object is placed at finite distance from the objective. Hence, formulae derived for angular magnification M cannot be applied directly as they have been derived for the object to be at infinity. Here, it will be difficult to find angular magnification. So, only linear magnification can be obtained.

Example 9.80 A Galilean telescope is 27 cm long when focused to form an image at infinity. If the objective has a focal length of 30 cm, what is the focal length of the eyepiece?

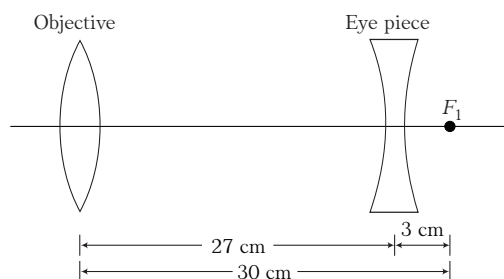
Sol. Given, $f_o = +30$ cm

Length of telescope is given, 27 cm.

Therefore, $u_e = +3$ cm

For the final image at infinity, the intermediate image should lie at first focus of eyepiece of the Galilean telescope.

$\therefore f_e = -3$ cm



Resolving power of a microscope and a telescope

Microscope

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope. It depends on the wavelength λ of the light, the refractive index μ of the medium between the object and the objective and the angle θ subtended by a radius of the objective on one of the objects.

Resolving power is reciprocal of resolution limit.

$$\text{Resolving power, } R_p = \frac{1}{\Delta d} = \frac{2\mu \sin \theta}{\lambda}$$

Resolving power (R_p) of microscope increases

- (i) on introducing a medium of higher refractive index (μ) between object and objective lens or
- (ii) by decreasing the wavelength of used light.

Telescope

The resolving power of a telescope is defined as the reciprocal of the angular separation between two distant objects which are just resolved by a telescope.

$$\text{It is given by, } R = \frac{1}{\Delta \theta} = \frac{a}{1.22 \lambda}$$

Here, a is the diameter of the objective. For increasing the resolving power of telescope, telescopes with larger objective aperture are used.

Example 9.81 Calculate the resolving power of a microscope with cone angle of light falling on the objective equal to 60° . Take, $\lambda = 600$ nm and μ for air = 1.

Sol. Here, $2\theta = 60^\circ \Rightarrow \theta = 30^\circ$, $\lambda = 600$ nm = 6×10^{-7} m, $\mu = 1$

$$RP = \frac{1}{d} = \frac{2\mu \sin \theta}{\lambda}$$

$$RP = \frac{2 \times 1 \times \sin 30^\circ}{600 \times 10^{-9}} = \frac{10}{6} \times 10^6 = 1.67 \times 10^6$$

Example 9.82 Assume that light of wavelength $6000 \text{ \AA}$ is coming from a star. What is the resolution of a telescope whose objective has diameter of 100 inch?

Sol. Diameter of objective of telescope,

$$D = 100 \text{ inch} = 100 \times 2.54 \text{ cm} = 254 \text{ cm}$$

$$\lambda = 6000 \text{ \AA} = 6000 \times 10^{-8} \text{ cm} = 6 \times 10^{-5} \text{ cm}$$

Limit of resolution of telescope,

$$d\theta = \frac{1.22 \lambda}{D} = \frac{1.22 \times 6 \times 10^{-5}}{254} = 2.9 \times 10^{-7} \text{ rad}$$

Example 9.83 The diameter of the pupil of human eye is about 2 mm. Human eye is most sensitive to the wavelength 555 nm. Find the limit of resolution of human eye.

Sol. Here, $D = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$, $\lambda = 555 \text{ nm} = 555 \times 10^{-9} \text{ m}$

$$\text{Limit of resolution, } d\theta = \frac{1.22 \lambda}{D} = \frac{1.22 \times 555 \times 10^{-9}}{2 \times 10^{-3}} = 3.39 \times 10^{-4} \text{ rad}$$

$$d\theta = 3.39 \times 10^{-4} \times \left(\frac{180}{\pi}\right)^\circ = 0.0194^\circ$$

$$= 0.0194 \times 60 \text{ min} = 1.2 \text{ min}$$

CHECK POINT 9.6

- The focal length of a normal eye lens is about
 - 1 mm
 - 2 cm
 - 25 cm
 - 1 m
- An object is placed at a distance u from a simple microscope of focal length f . The angular magnification obtained depends
 - on f but not on u
 - on u but not on f
 - on f as well as u
 - neither on f nor on u
- Magnifying power of a simple microscope is (when final image is formed at $D = 25$ cm from eye)
 - $\frac{D}{f}$
 - $1 + \frac{D}{f}$
 - $1 + \frac{f}{D}$
 - $1 - \frac{D}{f}$
- In a compound microscope, the intermediate image is
 - virtual, erect and magnified
 - real, erect and magnified
 - real, inverted and magnified
 - virtual, erect and reduced
- A compound microscope has two lenses. The magnifying power of one is 5 and the combined magnifying power is 100. The magnifying power of the other lens is
 - 10
 - 20
 - 50
 - 25
- The length of the compound microscope is 14 cm. The magnifying power for relaxed eye is 25. If the focal length of eye lens is 5 cm, then the object distance for objective lens will be
 - 1.8 cm
 - 1.5 cm
 - 2.1 cm
 - 2.4 cm
- If the focal length of objective and eye lens are 1.2 cm and 3 cm respectively and the object is put 1.25 cm away from the objective lens and the final image is formed at infinity. The magnifying power of the microscope is
 - 150
 - 200
 - 250
 - 400
- The focal length of objective and eye lens of a microscope are 4 cm and 8 cm, respectively. If the least distance of distinct vision is 24 cm and object distance is 4.5 cm from the objective lens, then the magnifying power of the microscope will be
 - 18
 - 32
 - 64
 - 20
- If the telescope is reversed, *i.e.* seen from the objective side, then
 - object will appear very small
 - object will appear very large
 - there will be no effect on the image formed by the telescope
 - image will be slightly greater than the earlier one
- The aperture of a telescope is made large, because to
 - increase the intensity of image
 - decrease the intensity of image
 - have greater magnification
 - have lesser resolution
- In an astronomical telescope, the focal length of the objective lens is 100 cm and of eyepiece is 2 cm. The magnifying power of the telescope for the normal eye is
 - 50
 - 10
 - 100
 - $\frac{1}{50}$
- The focal lengths of the objective and eye lenses of a telescope are respectively, 200 cm and 5 cm. The maximum magnifying power of the telescope will be
 - 40
 - 48
 - 60
 - 100
- The number of lenses in a terrestrial telescope is
 - two
 - three
 - four
 - six
- Magnifying power of a Galilean telescope is given by
 - $\frac{f_o}{f_e} \left(1 - \frac{f_e}{D} \right)$
 - $\frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$
 - $\frac{f_o}{f_e} \left(1 + \frac{2f_e}{D} \right)$
 - $\frac{f_o}{f_e} \left(1 - \frac{2f_e}{D} \right)$
- In Galilean telescope, the final image formed is
 - real, erect and enlarged
 - virtual, erect and enlarged
 - real, inverted and enlarged
 - virtual, inverted and enlarged
- Reflecting telescope consists of
 - convex mirror of large aperture
 - concave mirror of large aperture
 - concave lens of small aperture
 - None of the above
- Resolving power of a microscope is given by
 - $\frac{2\mu \sin\theta}{\lambda^2}$
 - $\frac{\mu \sin\theta}{\lambda}$
 - $\frac{2\mu \sin\theta}{\lambda}$
 - $\frac{2\mu \cos\theta}{\lambda}$
- The resolving power of telescope whose lens has a diameter of 1.22 m for a wavelength of $5000 \text{ \AA}$ is
 - 2×10^5
 - 2×10^6
 - 2×10^2
 - 2×10^4

DEFECTS OF VISION

The common defects of vision are as follows

Myopia or short-sightedness (near sightedness)

In this defect, distant objects are not clearly visible and eye can see only nearby object. The image of distant object is formed before the retina.

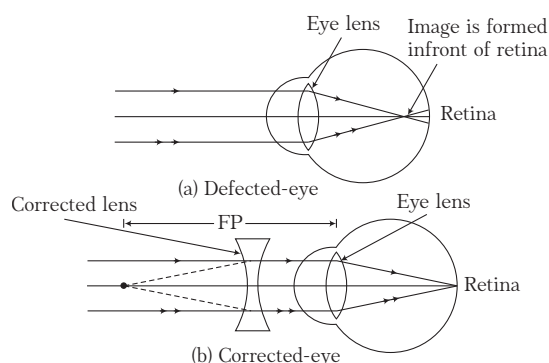


Fig. 9.77 Myopia defect in eye

In this defect focal length or radii of curvature of lens reduced or power of lens increases or distance between eye lens and retina increases.

The defect can be remedied by using a concave lens, placed between object and eye (as shown in figure), so that this lens forms the image of distant object at the far point (FP) of the person. Using lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{- (\text{FP})} - \frac{1}{- (\text{distance of object})} = \frac{1}{f}$$

If object is at infinity then, power of the lens,

$$P = \frac{1}{f} = \frac{1}{- (\text{FP})}$$

Farsightedness or longsightedness (hypermetropia or hyperopia)

In this defect, the near objects are not clearly visible and eye can see only distant object. The image of near object is formed behind the retina.

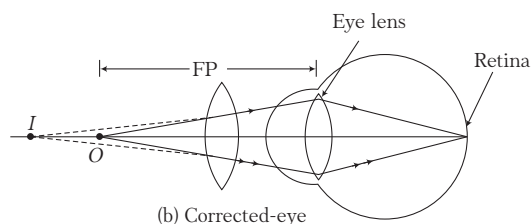
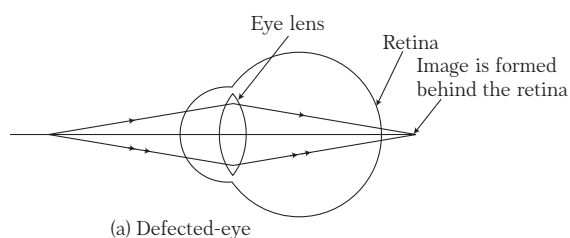


Fig. 9.78 Hypermetropia defect in eye

In this defect, focal length or radii of curvature of lens increases or power of lens decreases or distance between eye lens and retina decreases.

This defect can be remedied by using a convex lens placed between object and eye (as shown in figure,) so that this lens forms the image of near object at near point (NP) of the person.

Using lens formula, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow -\frac{1}{(\text{NP})} - \frac{1}{- (\text{distance of object})} = \frac{1}{f}$$

Power of the lens, $P = \frac{1}{f}$

Astigmatism

In this defect, eye cannot see objects in two orthogonal (perpendicular) directions clearly simultaneously or eye can not focus objects both in horizontal and vertical lens clearly.

This defect is remedied by using cylindrical lens. This defect arises due to imperfect shape (not perfectly spherical) of cornea or the eye lens or of both.

Presbyopia

In this, both near and far objects are not clearly visible.

This defect arises due to weakness of ciliary muscles and hardening of eye lens.

This is remedied either by using two separate lenses or by using single spectacle having bifocal lenses.

Example 9.84 A person who can see things most clearly at a distance of 10 cm, requires spectacles to enable to see clearly things at a distance of 30 cm. What should be the focal length of the spectacles?

Sol. Since, the person is not able to see distant objects beyond 10 cm. So, he should use concave lens in his spectacles. Concave lens will form the image of object distant 30 cm, at 10 cm, it means $u = -30$ cm, $v = -10$ cm.

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-10} - \frac{1}{-30} = -\frac{2}{30} \quad \text{or} \quad \frac{1}{f} = \frac{1}{15}$$

So, $f = -15$ cm

Minus sign signifies that lens used is concave.

Example 9.85 The power of a lens used by a short sighted person, is $-2D$. Find the maximum distance of an object which he can see without spectacles.

Sol. Power (P) = $\frac{1}{\text{Focal length (m)}}$

Given, $P = -2D$

$\therefore$ Focal length = $-\frac{1}{2} = 0.5\text{ m} \Rightarrow F = -50\text{ cm}$

Hence, maximum distance of an object, at which he can see without spectacles is 50 cm .

Example 9.86 If a person can see clearly at a distance of 100 cm , then find the power of lens used to see object at 40 cm .

Sol. By lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Given, $v = -100\text{ cm}$ and $u = -40\text{ cm}$

$$\therefore \frac{1}{f} = \frac{1}{-100} - \frac{1}{-40} \Rightarrow \frac{1}{f} = -\frac{1}{100} + \frac{1}{40}$$

$$\Rightarrow \frac{1}{f} = \frac{-1 + 2.5}{100} = \frac{1.5}{100}$$

Hence, power of lens is $P = \frac{100}{f(\text{cm})} = 100 \times \frac{1.5}{100} = 1.5D$

Example 9.87 A myopic person has been using a spectacles of power $1.0D$ for distant vision. During old age he also needs to use separate reading glass of power $+2.0D$. Explain what may have happened?

Sol. Here, $P = -1D$

$$\therefore f = \frac{1}{P} = \frac{1}{-1} \text{ m} = -100\text{ cm}$$

For normal vision, far point is at infinity

$$\therefore u = -\infty$$

Using lens formula, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$... (i)

$$\text{we get, } -\frac{1}{-\infty} + \frac{1}{v} = -\frac{1}{100}$$

$$\text{or } \frac{1}{v} = -\frac{1}{100}$$

$$\therefore v = -100\text{ cm}$$

Thus virtual image of the object at infinity is produced at 100 cm distance using spectacles.

To view objects at distance 25 cm to 100 cm , the person using ability of accommodation of eye which is partially lost in old age for which he needs another spectacles having power,

$$P = +2D$$

$$\therefore f = \frac{1}{P} = \frac{1}{2} = 0.5\text{ m} = 50\text{ cm} \quad \text{and} \quad v = 25\text{ cm}$$

Using Eq. (i), we get

$$-\frac{1}{u} + \frac{1}{25} = \frac{1}{50}$$

$$\text{or } u = 50\text{ cm}$$

$\therefore$ His near point shifts to 50 cm .

Defects of images

Actual image formed by an optical system is usually imperfect. The defects of images are called **aberrations**. The defect may be due to light or optical system. If the defect is due to light, it is called **chromatic aberration**, and if due to optical system, **monochromatic aberration**.

Chromatic aberration

The image of an object formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration. This defect arises due to the fact that focal length of a lens is different for different colours. For a lens,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

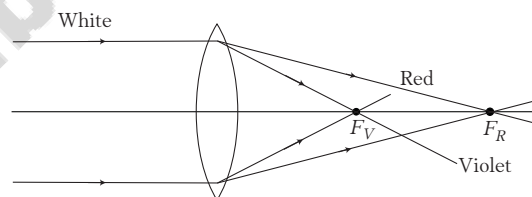


Fig. 9.79 Chromatic aberration

As, μ is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it.

The difference between f_R and f_V is a measure of longitudinal chromatic aberration (LCA). Thus,

$$\text{LCA} = f_R - f_V = \omega f_Y$$

where,

$$\omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}$$

Here,

$$\mu_Y = \frac{\mu_V + \mu_R}{2}$$

Condition of achromatism

To get achromatism, we use a pair of two lenses in contact. For two thin lenses in contact, we have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

On differentiating, we get

$$\therefore \frac{-dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration, if

$$dF = 0$$

$$\therefore \frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$$

$$\therefore \frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0$$

$$\therefore \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

This is the **condition of achromatism**. From the condition of achromatism, following conclusions can be drawn

(i) As ω_1 and ω_2 are positive quantities, f_1 and f_2 should have opposite signs, i.e., if one lens is convex, the other must be concave.

(ii) If $\omega_1 = \omega_2$, means both the lenses are of same material. Then,

$$\frac{1}{f_1} + \frac{1}{f_2} = 0 \quad \text{or} \quad \frac{1}{F} = 0 \quad \text{or} \quad F = \infty$$

Thus, the combination behaves as a plane glass plate. So, we can conclude that both the lenses should be of different materials or $\omega_1 \neq \omega_2$

(iii) Dispersive power of crown glass (ω_C) is less than that of flint glass (ω_F).

(iv) If we want the combination to behave as a convergent lens, then convex lens should have lesser focal length or its dispersive power should be more. Thus, convex lens should be made of flint glass and concave lens of crown.

Thus, combination is converging, if convex is made of flint glass and concave of crown. Similarly, for the combination to behave as diverging lens, convex is made of crown glass and concave of flint glass.

Monochromatic aberration

This is the defect in image due to optical system. Monochromatic aberration is of many types such as, spherical, coma, distortion, curvature and astigmatism. Here, we shall limit ourselves to spherical aberration only. Spherical aberration arises due to spherical nature of lens (or mirror).

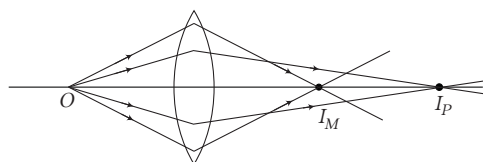


Fig. 9.80 Spherical aberration

The paraxial rays (close to optic axis) get focused at I_P and marginal rays (away from the optic axis) are focused at I_M . Thus, image of a point object O is not a point.

The inability of the lens to form a point image of an axial point object is called spherical aberration. Spherical aberration can never be eliminated but can be minimised by the following methods :

(i) **By using stops** By using stops either paraxial or marginal rays are cut-off.

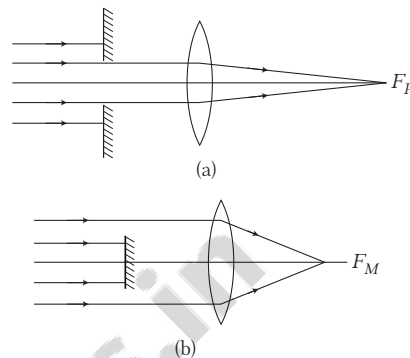


Fig. 9.81 Minimising spherical aberration using stops

(ii) **Using two thin lenses separated by a distance** Two thin lenses separated by a distance, $d = f_2 - f_1$, has the minimum spherical aberration.

(iii) **Using parabolic mirrors** If spherical mirror is replaced by parabolic mirror, spherical aberration is minimised.

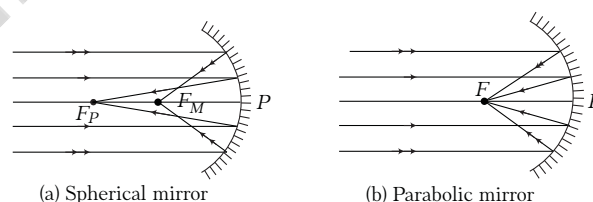


Fig. 9.82 Minimising spherical aberration using mirrors

(iv) **Using lens of large focal length** It has been found that spherical aberration varies inversely as the cube of the focal length. So, if f is large, spherical aberration will be reduced.

(v) **Using plano-convex lens** In case of plano-convex lens spherical aberration is minimised, if its curved surface faces the incident or emergent ray whichever is more parallel.

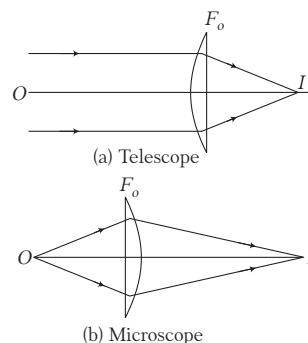


Fig. 9.83 Minimising spherical aberration using plano-convex lens

This is why in telescope, the curved surface faces the object while in microscope curved surface is towards the image.

- (vi) **Using crossed lens** For a single lens with object at infinity, spherical aberration is found to be minimum, when R_1 and R_2 have the following ratio,

$$\frac{R_1}{R_2} = \frac{2\mu^2 - \mu - 4}{\mu(2\mu + 1)}$$

A lens which satisfies this condition is called a crossed lens.

Some natural phenomena associated with sunlight

The spectrum of colours that we see around us all the time is possible only due to sunlight. The rainbow, blue colour of the sky, white clouds, the red view at sunrise and sunset, etc., are some of the natural phenomena.

The Rainbow

It is the combined effect of dispersion, refraction and internal reflection of sunlight by spherical water droplets of rain. An observer can see the rainbow only when his back is towards the sun.

Primary Rainbow

It is a result of three steps process shown in the figure.

- (i) Refraction with dispersion
- (ii) Internal reflection
- (iii) Refraction

In primary rainbow, red colour is on the top and violet on the bottom as shown in the figure.

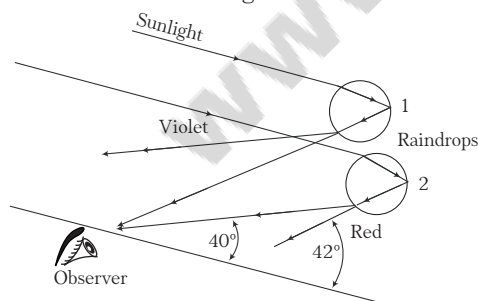


Fig. 9.84 Formation of primary rainbow

Secondary Rainbow

It is a result of four step process as shown in the figure.

- (i) Refraction with dispersion
- (ii) Internal reflection
- (iii) External reflection
- (iv) Refraction

It is found that red light emerges at an angle of 50° related to the incoming sunlight and violet light emerges at an angle of 53° .

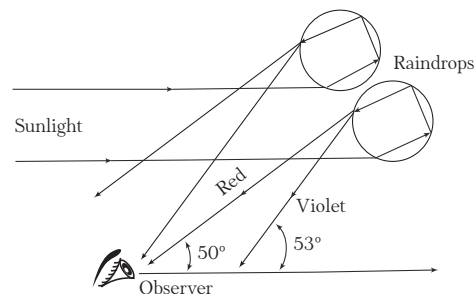


Fig. 9.85 Formation of secondary rainbow

Scattering of light

If the molecules of a medium after absorption of incoming radiations (light) emit them in all directions, this phenomenon is called scattering. If the wavelength of radiation remains unchanged in this process, then the scattering is called elastic otherwise, inelastic.

Rayleigh has shown, theoretically that in case of elastic scattering of light by molecules, the intensity of scattered light depends on both nature of molecules and wavelength of light. According to him,

$$\text{Intensity of scattered light} \propto \frac{1}{\lambda^4}$$

Raman effect is based on inelastic scattering. For this CV Raman was awarded the Nobel Prize in 1930.

Scattering helps us in understanding the following

- (i) **Why sky is blue** When white light from the sun enters the earth's atmosphere, scattering takes place. As scattering is proportional to $\frac{1}{\lambda^4}$, blue is scattered most. When we look at the sky we receive scattered light which is rich in blue and hence, the sky appears blue.

- (ii) **Why sun appears red in the morning and evening**

When sun is at the horizon, due to oblique incidence light reaches earth after traversing maximum path in the atmosphere and so suffers maximum scattering.

Now, as scattering $\propto \frac{1}{\lambda^4}$, shorter wavelengths are

scattered most leaving the longer one. As red light has longest wavelength in the visible region, it is scattered least. This is why sun appears red in the morning and evening. The same reason is why red light is used for danger signals.

CHECK POINT 9.7

- If a person is suffering from myopia, then it can be removed by
 - convex lens
 - concave lens
 - cylindrical lens
 - toric lens
- A short sighted person can see distinctly only those objects which lie between 10 cm and 100 cm from him. The power of the spectacle lens required to see a distant object is
 - + 0.5 D
 - 1.0 D
 - 10 D
 - + 4.0 D
- Which of the following statement(s) is/are correct for hypermetropia?
 - Near objects are not clearly visible.
 - Distant objects are not clearly visible.
 - Concave lens is used for remedy of hypermetropia.
 - None of the above
- Astigmatism (for a human eye) can be removed by using
 - concave lens
 - convex lens
 - cylindrical lens
 - prismatic lens
- The focal length of a converging lens is measured for violet, green and red colours as f_v , f_g and f_r , respectively. We can then infer that
 - $f_v = f_r$
 - $f_v < f_r$
 - $f_v > f_r$
 - $f_g > f_r$
- A combination is made of two lenses of focal lengths f_1 and f_2 and dispersive powers ω_1 and ω_2 , respectively. The combination will be achromatic, if
 - $\omega_1 = 2\omega_2$ and $f_1 = 2f_2$
 - $2\omega_1 = \omega_2$ and $f_1 = 2f_2$
 - $\omega_1 = 2\omega_2$ and $f_1 = -2f_2$
 - $2\omega_1 = \omega_2$ and $2f_1 = f_2$
- Rainbow is caused due to
 - Refraction
 - reflection
 - dispersion
 - All of these
- Phenomenon/Phenomena associated with scattering is/are
 - blue colour of the sky
 - appearance of reddish sun during sunset and sunrise
 - Both (a) and (b)
 - None of the above

Chapter Exercises

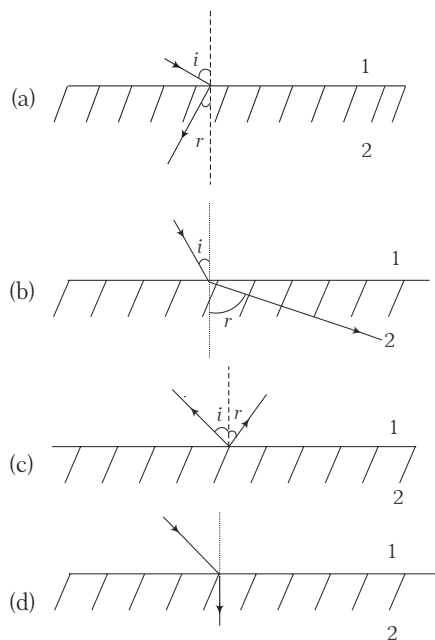
(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1 Myopia is due to
 - (a) elongation of eye ball
 - (b) irregular change in focal length
 - (c) shortening of eye ball
 - (d) older age
- 2 When the power of eye lens increases, the defect of vision is produced. The defect is known as
 - (a) shortsightedness
 - (b) longsightedness
 - (c) colourblindness
 - (d) None of the above
- 3 When white light is passed through a prism, the colour which shows maximum deviation is
 - (a) red
 - (b) violet
 - (c) yellow
 - (d) green
- 4 In human eye, the focusing is done by
 - (a) to and fro movement of eye lens
 - (b) to and fro movement of the retina
 - (c) change in the convexity of the lens surface
 - (d) change in the refractive index of the eye fluids
- 5 A man has height of 6 m. He observes his image of 2 m height and erect, then mirror used is
 - (a) concave
 - (b) convex
 - (c) plane
 - (d) None of these
- 6 The field of view is maximum for
 - (a) plane mirror
 - (b) concave mirror
 - (c) convex mirror
 - (d) cylindrical mirror
- 7 If there had been one eye of the man, then
 - (a) image of the object would have been inverted
 - (b) visible region would have decreased
 - (c) image would have not been seen three-dimensional
 - (d) Both (b) and (c)
- 8 What will be the colour of sky as seen from the earth, if there were no atmosphere?
 - (a) Black
 - (b) Blue
 - (c) White
 - (d) Red
- 9 The reason for shining of air bubble in water is
 - (a) diffraction of light
 - (b) dispersion of light
 - (c) scattering of light
 - (d) total internal reflection of light
- 10 The sky would appear red instead of blue, if
 - (a) atmospheric particles scatter blue light more than the red light
 - (b) atmospheric particles scatter all colours equally
 - (c) atmospheric particles scatter red light more than the blue light
 - (d) the sun was much hotter
- 11 A passenger in an aeroplane [NCERT Exemplar]
 - (a) should see a rainbow
 - (b) may see a primary and a secondary rainbow as concentric circles
 - (c) may see a primary and a secondary rainbow as concentric arcs
 - (d) should never see a secondary rainbow
- 12 The phenomena involved in the reflection of radiowaves by ionosphere is similar to [NCERT Exemplar]
 - (a) reflection of light by a plane mirror
 - (b) total internal reflection of light in air during a mirage
 - (c) dispersion of light by water molecules during the formation of a rainbow
 - (d) scattering of light by the particles of air
- 13 Which of the following quantities increases when wavelength of light is increased? Consider only the magnitudes.
 - (a) The power of a converging lens
 - (b) The focal length of a converging lens
 - (c) The power of a diverging lens
 - (d) None of the above
- 14 A transparent plastic bag filled with air forms a concave lens. Now, if this bag is completely immersed in water, then it behaves as
 - (a) divergent lens
 - (b) convergent lens
 - (c) equilateral prism
 - (d) rectangular slab
- 15 A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first colour to emerge is [NCERT Exemplar]
 - (a) blue
 - (b) green
 - (c) violet
 - (d) red
- 16 When light wave suffers reflection at the interface from air to glass, then the change in phase of the reflected wave is equal to
 - (a) zero
 - (b) $\frac{\pi}{2}$
 - (c) π
 - (d) 2π

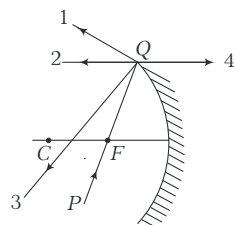
- 17** There are certain materials developed in laboratories which have a negative refractive index figure. A ray incident from air (medium 1) into such a medium (medium 2) shall follow a path given by

[NCERT Exemplar]



- 18** The minimum magnifying power of telescope is M . If the focal length of its eye lens is halved, the magnifying power will become
(a) $m/2$ (b) $2m$ (c) $3m$ (d) $4m$
- 19** A normal eye is not able to see objects closer than 25 cm because
(a) the focal length of the eye is 25 cm
(b) the distance of the retina from the eye lens is 25 cm
(c) the eye is not able to increase the focal length beyond a limit
(d) the eye is not able to decrease the focal length beyond a limit
- 20** A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is
(a) 2 cm (b) 4 cm (c) 6 cm (d) 12 cm
- 21** An object has an image thrice of its original size when kept at 8 cm and 16 cm from a convex lens. Focal length of the lens is
(a) less than 8 cm (b) 8 cm
(c) 16 cm (d) between 8 and 16 cm
- 22** A ray of light is incident on the surface of separation of a medium at an angle of 45° and is refracted in the medium at an angle of 30° . What will be the velocity of light in the medium?
(a) 1.96×10^8 m/s (b) 2.12×10^8 m/s
(c) 2.65×10^8 m/s (d) 1.24×10^8 m/s
- 23** An object approaches to a convergent of lens from the left of the lens with a uniform speed 5 m/s and stops at the focus. The image [NCERT Exemplar]
(a) moves away from the lens with a uniform speed of 5 m/s
(b) moves away from the lens with a uniform acceleration
(c) moves away from the lens with a non-uniform acceleration
(d) moves towards the lens with a non-uniform acceleration
- 24** If the aperture of a telescope is decreased, then resolving power will
(a) increase (b) decrease
(c) remain same (d) zero
- 25** A prism can have a maximum refracting angle of (C = critical angle for the material of the prism)
(a) 60° (b) C
(c) $2C$ (d) slightly less than 180°
- 26** When a lens of refractive index n_1 is placed in a liquid of refractive index n_2 , then the lens seems to be disappeared only, if
(a) $n_1 = n_2/2$ (b) $n_1 = 3n_2/2$
(c) $n_1 = n_2$ (d) $n_1 = 5n_2/2$
- 27** When sun light is scattered by minute particles of atmosphere, then the intensity of light scattered away is proportional to
(a) (wavelength of light) 4 (b) (frequency of light) 4
(c) (wavelength of light) 2 (d) (frequency of light) 2
- 28** Which amongst the following phenomena is not associated with the total internal reflection?
(a) The mirage formation
(b) Optical fibre communication
(c) The glittering of diamond
(d) Dispersion of light
- 29** Wavelength of light in vacuum is $5890 \text{ \AA}$, then its wavelength in glass ($\mu = 1.5$) will be
(a) $9372 \text{ \AA}$ (b) $7932 \text{ \AA}$
(c) $7548 \text{ \AA}$ (d) $3927 \text{ \AA}$
- 30** When a thin convex lens is put in contact with a thin concave lens of the same focal length f , then the resultant combination has focal length equal to
(a) $f/2$ (b) $2f$
(c) 0 (d) ∞
- 31** If refractive index of glass is 1.50 and of water is 1.33, then critical angle is
(a) $\sin^{-1}\left(\frac{8}{9}\right)$ (b) $\sin^{-1}\left(\frac{2}{3}\right)$
(c) $\cos^{-1}\left(\frac{8}{9}\right)$ (d) None of these

- 32** A plano-convex lens is made of refractive index of 1.6. The radius of curvature of the curved surface is 60 cm. The focal length of the lens is
(a) 400 cm (b) 200 cm
(c) 100 cm (d) 50 cm
- 33** A mark at the bottom of a liquid appears to rise by 0.1 m and the depth of the liquid is 1 m. The refractive index of the liquid is
(a) 1.33 (b) 9/10 (c) 10/9 (d) 1.5
- 34** Under minimum deviation condition in a prism, if a ray is incident at an angle of 30° , then the angle between the emergent ray and the second refracting surface of the prism is
(a) 0° (b) 30° (c) 45° (d) 60°
- 35** A beam of light composed of red and green rays is incident obliquely at a point on the face of a rectangular glass slab. When coming out on the opposite parallel face, then the red and green rays emerge from
(a) two points propagating in two different non-parallel directions
(b) two points propagating in two different parallel directions
(c) one point propagating in two different directions
(d) one point propagating in the same direction
- 36** If object is placed 15 cm from a concave mirror of focal length 10 cm, then the nature of image formed will be
(a) magnified and inverted
(b) magnified and erect
(c) small in size and inverted
(d) small in size and erect
- 37** If x_1 is the size of the magnified image and x_2 is the size of the diminished image in lens displacement method, then the size of the object is
(a) $\sqrt{x_1 x_2}$ (b) $x_1 x_2$ (c) $x_1^2 x_2$ (d) $x_1 x_2^2$
- 38** The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (figure). Which of the four rays correctly shows the direction of reflected ray?
[NCERT Exemplar]

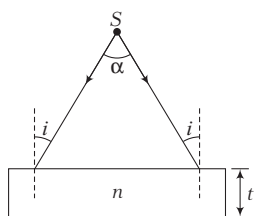


- (a) 1 (b) 2
(c) 3 (d) 4

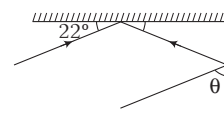
- 39** Two beams of red and violet colours made to pass separately through a prism ($A = 60^\circ$). In the minimum deviation position, the angle of refraction inside the prism will be
(a) greater for red colour
(b) equal but not 30° for both the colours
(c) greater for violet colour
(d) 30° for both the colours
- 40** An object is placed at a distance u from an equiconvex lens such that the distance between the object and its real image is minimum. The focal length of the lens is f . The value of u is
(a) ∞ (b) $1.5f$ (c) $2f$ (d) $4f$
- 41** A telescope has an objective of focal length 100 cm and an eyepiece of focal length 5 cm. What is the magnifying power of the telescope when it is in normal adjustment?
(a) 0.2 (b) 2.0 (c) 20 (d) 200
- 42** The time required for the light to pass through a glass slab (refractive index = 1.5) of thickness 4 mm is (Take, speed of light in free space, $c = 3 \times 10^8 \text{ ms}^{-1}$)
(a) 10^{-11} s (b) $2 \times 10^{-11} \text{ s}$
(c) $2 \times 10^{11} \text{ s}$ (d) $2 \times 10^{-5} \text{ s}$
- 43** Electromagnetic radiation of frequency n , wavelength λ , travelling with velocity v in air, enters a glass slab of refractive index μ . The frequency, wavelength and velocity of light in the glass slab will be respectively
(a) $\frac{n}{\mu}, \frac{\lambda}{\mu}, \frac{v}{\mu}$ (b) $n, \frac{\lambda}{\mu}, \frac{v}{\mu}$ (c) $n, \lambda, \frac{v}{\mu}$ (d) $\frac{n}{\mu}, \frac{\lambda}{\mu}, v$
- 44** We have combined a convex lens of focal length f_1 and concave lens of focal length f_2 and their combined focal length was F . The combination of these lenses will behave like a concave lens, if
(a) $f_1 > f_2$ (b) $f_1 < f_2$
(c) $f_1 = f_2$ (d) $f_1 \leq f_2$
- 45** Light travels in two media A and B with speeds $1.8 \times 10^8 \text{ ms}^{-1}$ and $2.4 \times 10^8 \text{ ms}^{-1}$, respectively. Then, the critical angle between them is
(a) $\sin^{-1}\left(\frac{2}{3}\right)$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$ (c) $\tan^{-1}\left(\frac{2}{3}\right)$ (d) $\sin^{-1}\left(\frac{3}{4}\right)$
- 46** The focal lengths of a thin convex lens for red and blue colours are 100.5 cm and 99.5 cm, respectively. The dispersive power of the lens is
(a) 0.01 (b) 0.02 (c) 1.005 (d) 0.995
- 47** Two mirrors are placed at right angle to each other. A man is standing between them combing his hair. How many images will he see?
(a) 2 (b) 3 (c) 1 (d) zero

- 48** Two mirrors are kept at 60° to each other and a body is placed at the middle. The total number of images formed are
(a) six (b) four (c) five (d) three
- 49** If the focal length of the eyepiece of a telescope is doubled, then its magnifying power (m) will be
(a) $2m$ (b) $3m$ (c) $m/2$ (d) $4m$
- 50** Angular resolving power of human eye is
(a) 3.4×10^3 rad (b) 3.4×10^2 rad
(c) 3.4×10^4 rad (d) 3.4×10^6 rad
- 51** If the red light is replaced by blue light illuminating the object in a microscope, then the resolving power of the microscope
(a) decreases (b) increases
(c) gets halved (d) remains unchanged
- 52** A telescope using light having wavelength $5000 \text{ \AA}$ and using lenses of focal lengths 2.5 cm and 30 cm . If the diameter of the aperture of the objective is 10 cm , then the resolving limit of telescope is
(a) 6.1×10^{-6} rad (b) 5.0×10^{-6} rad
(c) 8.3×10^{-4} rad (d) 7.3×10^{-3} rad
- 53** An astronomical telescope in normal adjustment receives light from a distance source S , the tube length is now decreased slightly, then
(a) no image will be formed
(b) a virtual image of S will be formed at a finite distance
(c) a large, real image of S will be formed behind the eye piece, far away from it
(d) a small, real image of S will be formed behind the eye-piece closes to it
- 54** An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of eyepiece are
(a) $f_o = 45 \text{ cm}$ and $f_e = -9 \text{ cm}$
(b) $f_o = -7.2 \text{ cm}$ and $f_e = 5 \text{ cm}$
(c) $f_o = 50 \text{ cm}$ and $f_e = 10 \text{ cm}$
(d) $f_o = 30 \text{ cm}$ and $f_e = 6 \text{ cm}$
- 55** The magnification produced by an astronomical telescope for normal adjustment is 10 and the length of the telescope is 1.1 m . The magnification, when the image is formed at least distance of distinct vision is
(a) 6 (b) 14 (c) 16 (d) 18
- 56** Where should a person stand straight from the pole of a convex mirror of focal length 2.0 m on its axis, so that the image formed become half of his original height?
(a) -2.60 m (b) -4.0 m (c) -0.5 m (d) -2.0 m
- 57** The radius of curvature of the curved surface of a plano-convex lens is 20 cm . If the refractive index of the material of the lens be 1.5, then it will [NCERT Exemplar]
(a) act as a convex lens only for the objects that lie on its curved side
(b) act as a concave lens for the objects that lie on its curved side
(c) act as a convex lens irrespective of the side on which the object lies
(d) act as a concave lens irrespective of side on which the object lies
- 58** Wavelength of given light waves in air and in a medium are $6000 \text{ \AA}$ and $3000 \text{ \AA}$, respectively. The critical angle is
(a) $\tan^{-1}\left(\frac{2}{3}\right)$ (b) $\tan^{-1}\left(\frac{3}{2}\right)$
(c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{2}{3}\right)$
- 59** A convex and a concave mirror of radii 10 cm each are placed facing each other and 15 cm apart. An object is placed exactly between them. If the reflection first takes place in concave and then in convex mirror, then the position of the final image will be
(a) 7 cm behind concave mirror
(b) at the pole of the concave mirror
(c) at the pole of the convex mirror
(d) 6.7 cm in front of concave mirror
- 60** The focal lengths of the lenses of an astronomical telescope are 50 cm and 5 cm . The length of the telescope when the image is formed at the least distance of distinct vision is
(a) 45 cm (b) 55 cm (c) $\frac{275}{6} \text{ cm}$ (d) $\frac{325}{6} \text{ cm}$
- 61** An object of 5 cm height is placed 1 m apart from a concave spherical mirror which has a radius of curvature of 20 cm . The size of the image is
(a) 0.11 cm (b) 0.5 cm
(c) 0.55 cm (d) 0.60 cm
- 62** Two thin lenses of focal lengths 20 cm and 25 cm are placed in contact. The effective power of the combination is
(a) 9 D (b) 2 D (c) 3 D (d) 7 D
- 63** The nearer point of hypermetropic eye is 40 cm . The lens to be used for its correction should have the power
(a) $+1.5 \text{ D}$ (b) -1.5 D (c) $+2.5 \text{ D}$ (d) $+0.5 \text{ D}$
- 64** In a plano-convex lens, the radius of curvature of convex surface is 10 cm and the focal length of the lens is 30 cm . The refractive index of the material of the lens will be
(a) 1.5 (b) 1.66 (c) 1.33 (d) 3

- 65** If the refractive index of a material of equilateral prism is $\sqrt{3}$, then angle of minimum deviation of the prism is
 (a) 30° (b) 45° (c) 60° (d) 75°
- 66** A ray of light is incident at 60° on one face of a prism of angle 30° and the emergent ray makes 30° with the incident ray. The refractive index of the prism is
 (a) 1.732 (b) 1.414 (c) 1.5 (d) 1.33
- 67** A diverging beam of light from a point source S having divergence angle α , falls symmetrically on a glass slab as shown in figure. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n , then the divergence angle of the emergent beam is



- (a) zero (b) α
 (c) $\sin^{-1} \frac{1}{n}$ (d) $2 \sin^{-1} \left(\frac{1}{n} \right)$
- 68** A convex lens forms an image of an object on a screen 30 cm from the lens. When the lens is moved 90 cm towards the object, then the image is again formed on the screen. Then, the focal length of the lens is
 (a) 13 cm (b) 24 cm
 (c) 33 cm (d) 40 cm
- 69** If tube length of astronomical telescope is 105 cm and magnifying power is 20 for normal setting, then the focal length of objective is
 (a) 100 cm (b) 10 cm
 (c) 20 cm (d) 25 cm
- 70** A double convex glass lens ($R_1 = R_2 = 10$ cm) having focal length equal to the focal length of a concave mirror. The radius of curvature of the concave mirror is ($\mu_g = 1.5$)
 (a) 10 cm (b) 20 cm
 (c) 40 cm (d) 15 cm
- 71** The radii of curvature of the two surfaces of a lens are 20 cm and 30 cm and the refractive index of the material of the lens is 1.5. If the lens is concave-convex, then the focal length of the lens is
 (a) 24 cm (b) 10 cm
 (c) 15 cm (d) 120 cm
- 72** An eye specialist prescribes spectacles having a combination of a convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination will be
 (a) +1.5 D (b) -1.5 D
 (c) +6.67 D (d) -6.67 D
- 73** The power of a biconvex lens is 10 D and the radius of curvature of each surface is 10 cm. Then, the refractive index of the material of the lens is
 (a) $\frac{3}{2}$ (b) $\frac{4}{3}$
 (c) $\frac{9}{8}$ (d) $\frac{5}{3}$
- 74** Two lenses are placed in contact with each other and the focal length of combination is 80 cm. If the focal length of one is 20 cm, then the power of the other will be
 (a) 1.66 D (b) 4.00 D
 (c) -1.00 D (d) -3.75 D
- 75** A ray of light falls on a denser-rarer boundary from denser side and the critical angle is 45° . The maximum deviation the ray can undergo is
 (a) 30° (b) 45°
 (c) 90° (d) 120°
- 76** A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, the lens has been used to form the image of an object. What should be the distance of the object from the lens in order to have a real image of the size of the object?
 (a) 20 cm (b) 30 cm
 (c) 60 cm (d) 80 cm
- 77** If light travels a distance x in t_1 seconds in air and $10x$ distance in t_2 seconds in a medium, the critical angle of the medium, will be
 (a) $\tan^{-1} \left(\frac{t_1}{t_2} \right)$ (b) $\sin^{-1} \left(\frac{t_1}{t_2} \right)$
 (c) $\sin^{-1} \left(\frac{10t_1}{t_2} \right)$ (d) $\tan^{-1} \left(\frac{10t_1}{t_2} \right)$
- 78** A ray of light is directed towards a corner reflector as shown in figure. The incident ray makes an angle of 22° with one of the mirrors. At what angle θ does the ray emerge?

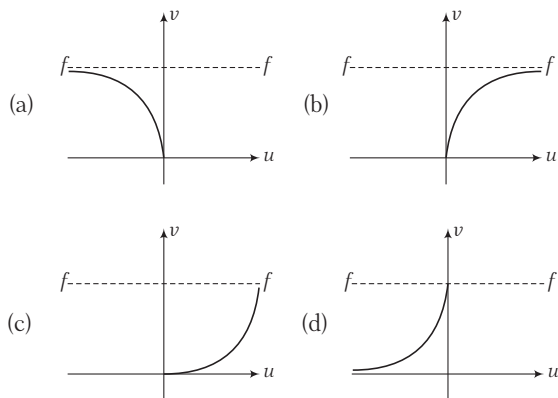


- (a) 22° (b) 68°
 (c) 44° (d) 46°

- 79** The focal lengths of the objective and eye lens of a microscope are 1 cm and 5 cm, respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is

(a) 30 cm (b) 25 cm
(c) 15 cm (d) 12 cm

- 80** The graph between u and v for a convex mirror is



- 81** A concave lens of focal length 20 cm placed in contact with a plane mirror acts as a

(a) convex mirror of focal length 10 cm
(b) concave mirror of focal length 40 cm
(c) concave mirror of focal length 60 cm
(d) concave mirror of focal length 10 cm

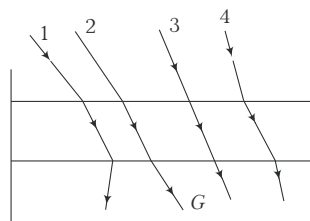
- 82** A diver at a depth of 12 m in water ($\mu = \frac{4}{3}$) sees the

sky at a cone of semivertical angle

(a) $\sin^{-1}\left(\frac{4}{3}\right)$ (b) $\tan^{-1}\left(\frac{4}{3}\right)$
(c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) 90°

- 83** The optical density of turpentine is higher than that of water while its mass density is lower. Figure shows, a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in figure, the path shown is correct?

[NCERT Exemplar]



(a) 1 (b) 2 (c) 3 (d) 4

- 84** A plano-convex lens ($f = 20$ cm) is silvered at plane surface. Now, focal length will be

(a) 20 cm (b) 40 cm (c) 30 cm (d) 10 cm

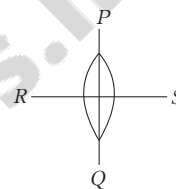
- 85** A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, then the angle of incidence is [NCERT Exemplar]

(a) 7.5° (b) 5° (c) 15° (d) 2.5°

- 86** For a divergent lens, the object is at m times its focal length, where m is linear magnification produced by the lens having the value

(a) m (b) $\frac{1}{m}$ (c) $m + 1$ (d) $\frac{1}{m + 1}$

- 87** The figure shows an equiconvex lens of focal length f . If the lens is cut along PQ , the focal length of each half will be



(a) $\frac{f}{2}$ (b) f (c) $2f$ (d) $4f$

- 88** The optical path of a monochromatic light is same whether, if it goes through 4.0 cm of glass or 4.5 cm of water. If the refractive index of glass is 1.53, then the refractive index of the water is

(a) 1.30 (b) 1.36
(c) 1.42 (d) 1.46

- 89** A ray of light travelling in transparent medium of refractive index μ falls on a surface separating the medium from air at an angle of incidence of 45° . For which of the following value of μ , the ray can undergo total internal reflection?

(a) $\mu = 1.33$ (b) $\mu = 1.41$ (c) $\mu = 1.50$ (d) $\mu = 1.25$

- 90** A ray of light, travelling in a medium of refractive index μ , is incident at an angle i on a composite transparent plate consisting of three plates of refractive indices μ_1 , μ_2 and μ_3 . The ray emerges from the composite plate into a medium of refractive index μ_4 , at angle x , then

(a) $\sin x = \sin i$ (b) $\sin x = \frac{\mu}{\mu_4} \sin i$
(c) $\sin x = \frac{\mu_4}{\mu} \sin i$ (d) $\sin x = \frac{\mu_1 \mu_3}{\mu_2 \mu_4} \frac{\mu}{\mu} \sin i$

- 91** What should be the angle between two plane mirrors, so that whatever be the angle of incidence, the incident ray and the reflected ray from the two mirrors be parallel to each other?

(a) 60° (b) 90°
(c) 120° (d) 45°

- 92** How much water should be filled in a container of height 21 cm, so that it appears half filled to the observer when viewed from the top of the container? (Take, $\mu = 4/3$)
 (a) 8 cm (b) 10.5 cm (c) 12 cm (d) 14 cm
- 93** When a plane mirror is placed horizontally on ground at a distance of 60 m from the base of a tower, then the top of the tower and its image in the mirror subtend an angle of 90° at the corner of the mirror nearer to the base of tower. The height of the tower is
 (a) 30 m (b) 60 m (c) 90 m (d) 120 m
- 94** A prism having refractive index 1.414 and refracting angle 30° has one of the refracting surfaces silvered. A beam of light incident on the other refracting surface will retrace its path, if the angle of incidence is
 (a) 0° (b) 30° (c) 60° (d) 45°
- 95** Two thin lenses have a combined power of +9 D. When they are separated by a distance of 20 cm, then their equivalent power becomes $+\frac{27}{5}$ D. Their individual powers (in dioptre) are
 (a) 1, 8 (b) 2, 7 (c) 3, 6 (d) 4, 5
- 96** A ray of light strikes a horizontal plane mirror at an angle of 45° . A second plane mirror is attached at an angle θ with it. If ray after reflection from second mirror runs parallel to the first mirror, then θ is
 (a) 45° (b) 60° (c) 67.5° (d) 135°
- 97** A convex lens of focal length 1.0 m and a concave lens of focal length 0.25 m are 0.75 m apart. A parallel beam of light is incident in the convex lens. The beam emerging after refraction from both lenses is
 (a) parallel to principal axis (b) convergent
 (c) divergent (d) None of these
- 98** For an angle of incidence θ on an equilateral prism of refractive index $\sqrt{3}$, the ray refracted is parallel to the base inside the prism. The value of θ is
 (a) 30° (b) 45° (c) 60° (d) 75°
- 99** A thin prism of angle 6° made up of glass of refractive index 1.5 is combined with another prism made up of glass of refractive index 1.75 to produce dispersion without deviation. The angle of second prism is
 (a) 7° (b) 9° (c) 4° (d) 5°
- 100** A double convex lens made of glass (refractive index $n = 1.5$) has both radii of curvature of magnitude 20 cm. Incident light rays parallel to the axis of the lens will converge at a distance L such that
 (a) $L = 20$ cm (b) $L = 10$ cm
 (c) $L = 40$ cm (d) $L = \frac{20}{3}$ cm
- 101** When sunlight is scattered by atmospheric atoms and molecules, the amount of scattering of light of wavelength 440 nm is A . The amount of scattering for the light of wavelength 660 nm is approximately
 (a) $\frac{4}{9}A$ (b) $2.25A$ (c) $1.5A$ (d) $\frac{A}{6}$
- 102** A prism, having refractive index $\sqrt{2}$ and refracting angle 30° , has one of the refracting surfaces polished. A beam of light incident on the other refracting surface will trace its path, if the angle of incidence is
 (a) 0° (b) 30°
 (c) 45° (d) 60°
- 103** A telescope has a objective of focal length 50 cm and an eyepiece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focused for distinct vision on a scale of 200 cm away. The separation between the objective and the eyepiece is
 (a) 75 cm (b) 60 cm
 (c) 71 cm (d) 74 cm
- 104** A glass prism has a refractive angle of 90° and a refractive index of 1.5. A ray is incident at an angle of 30° . The ray emerges from an adjacent face at an angle of
 (a) 60° (b) 30°
 (c) 45° (d) the ray does not emerge
- 105** An equilateral prism deviates a ray through 45° for the two angles of incidence differing by 20° . The angle of incidence is
 (a) 62.5° (b) 42.5°
 (c) Both (a) and (b) (d) None of these
- 106** The focal length of the objective of a terrestrial telescope is 80 cm and it is adjusted for parallel rays and its magnifying power is 20. If the focal length of erecting lens is 20 cm, then full length of telescope will be
 (a) 84 cm (b) 100 cm
 (c) 124 cm (d) 164 cm
- 107** The magnifying power of a microscope with an objective of 5 mm focal length is 400. The length of its tube is 20 cm. Then, the focal length of the eye-piece is
 (a) 200 cm (b) 160 cm
 (c) 2.5 cm (d) 0.1 cm
- 108** A thin plano-convex lens acts like a concave mirror of radius of curvature 20 cm when its plane surface is silvered. The radius of curvature of the curved surface, if index of refraction of its material is 1.5 will be
 (a) 40 cm (b) 30 cm
 (c) 10 cm (d) 20 cm

- 109** A convex lens has mean focal length of 20 cm. The dispersive power of the material of the lens is 0.02. The longitudinal chromatic aberration for an object at infinity is

(a) 10^3 (b) 0.80
(c) 0.40 (d) 0.20

- 110** A convex lens of focal length f is placed some, where in between an object and a screen. The distance between object and screen is x . If numerical value of magnification produced by lens is m , then focal length of lens is

(a) $\frac{mx}{(m+1)^2}$ (b) $\frac{mx}{(m-1)^2}$
(c) $\frac{(m+1)^2}{m}x$ (d) $\frac{(m-1)^2}{m}x$

- 111** A square of side 3 cm is located at a distance 25 cm from a concave mirror of focal length 10 cm. The centre of square is at the axis of the mirror and the plane is normal to axis of mirror. The area enclosed by the image of the square is

(a) 4 cm^2 (b) 6 cm^2
(c) 16 cm^2 (d) 36 cm^2

- 112** A plano-convex lens has a maximum thickness of 6 cm. When placed on a horizontal table with the curved surface in contact with the table surface, then the apparent depth of the bottom most point of the lens is found to be 4 cm. If the lens is inverted such that the plane face of the lens is in contact with the surface of the table, then the apparent depth of the centre of the plane face is found to be $\frac{17}{3}$ cm.

The radius of curvature of the lens is

(a) 68 cm (b) 75 cm
(c) 128 cm (d) 34 cm

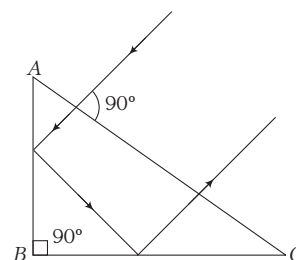
- 113** An object is 20 cm away from a concave mirror with focal length 15 cm. If the object moves with a speed of 5 m/s along the axis, then the speed of the image will be

(a) 45 m/s (b) 27 m/s
(c) 9 m/s (d) 10 m/s

- 114** The dispersive powers of glasses of lenses used in an achromatic pair are in the ratio 5 : 3. If the focal length of the concave lens is 15 cm, then the nature and focal length of the other lens would be

(a) convex, 9 cm (b) concave, 9 cm
(c) convex, 25 cm (d) concave, 25 cm

- 115** A ray falls on a prism ABC ($AB = BC$) and travels as shown in figure. The minimum refractive index of the prism material should be

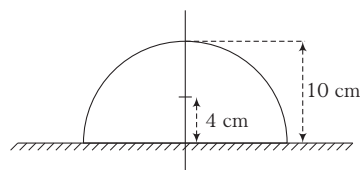


(a) $4/3$ (b) $\sqrt{2}$
(c) 1.5 (d) $\sqrt{3}$

- 116** If the distances of an object and its virtual image from the focus of a convex lens of focal length f are 1 cm each, then f is

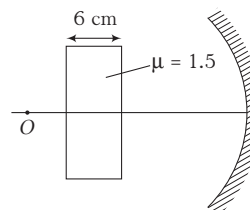
(a) 4 cm (b) $(\sqrt{2} + 1)$ cm
(c) $2\sqrt{2}$ cm (d) $(2 + \sqrt{2})$ cm

- 117** A hemispherical paper weight contains a small flower on its axis of symmetry at a distance of 4 cm from its flat surface. Where does the flower appear to an observer when he looks at it along the axis of symmetry from the top? (Take, refractive index of glass = 1.5)



(a) 15 cm from top (b) 20 cm from top
(c) 5 cm from top (d) 25 cm from top

- 118** A slab of glass, of thickness 6 cm and refractive index $\mu = 1.5$ is placed in front of a concave mirror as shown in the figure. If the radius of curvature of the mirror is 40 cm and the reflected image coincides with the object, then the distance of the object from the mirror is



(a) 30 cm (b) 22 cm (c) 42 cm (d) 38 cm

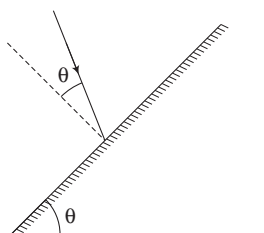
- 119** An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens, then the image formed is virtual. If the size of the image formed are equal, then the focal length of the lens will be

(a) 19 cm (b) 17 cm (c) 21 cm (d) 11 cm

- 120** A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror, what is the size of image?

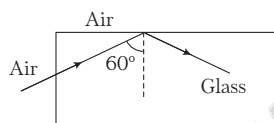
(a) $\left(\frac{f}{u-f}\right)b$ (b) $\left(\frac{f}{u-f}\right)^2 b$
 (c) $\left(\frac{f}{u-f}\right)b^2$ (d) $\left(\frac{f}{u-f}\right)$

- 121** A mirror is inclined at an angle of θ with the horizontal. If a ray of light is incident at an angle of incidence θ , then the reflected ray makes the following angle with horizontal



- (a) θ (b) 2θ
 (c) $\theta/2$ (d) None of these

- 122** A light ray going from air is incident (as shown in figure) at one end of a glass fibre (refractive index $\mu = 1.5$) making an incidence angle of 60° on the lateral surface,



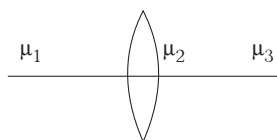
so that it undergoes a total internal reflection. How much time would it take to traverse the straight fibre of length 1 km?

- (a) $3.3 \mu\text{s}$ (b) $6.6 \mu\text{s}$ (c) $5.7 \mu\text{s}$ (d) $3.85 \mu\text{s}$

- 123** A thin convergent glass lens $\mu = 1.5$ has a power of + 5.0 D. When this lens is immersed in a liquid of refractive index μ_l it acts as diverging lens of focal length 100 cm. The value of μ_l should be

- (a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) 2

- 124** The figure shows an equiconvex lens. What should be the condition on the refractive indices, so that the lens becoming diverging?

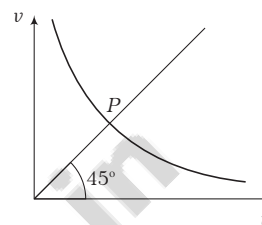


- (a) $2\mu_3 > \mu_1 - \mu_2$ (b) $2\mu_2 < \mu_1 + \mu_3$
 (c) $2\mu_2 > 2\mu_1 - \mu_3$ (d) None of these

- 125** A thin rod of length $f/3$ lies along the axis of a concave mirror of focal length f . One end of its magnified image touches one end of the rod, the length of the image is

- (a) f (b) $\frac{1}{2}f$ (c) $2f$ (d) $\frac{1}{4}f$

- 126** The graph shows part of variation of v with change in u for a concave mirror. Points plotted above the point P on the curve are for values of v



- (a) smaller than f
 (b) smaller than $2f$
 (c) larger than $2f$
 (d) larger than f but less than $2f$

- 127** When an object is at distance x and y from a lens, a real image and a virtual image is formed respectively having same magnification. The focal length of the lens is

- (a) $\frac{x+y}{2}$ (b) $x-y$ (c) $\sqrt{xy}$ (d) $x+y$

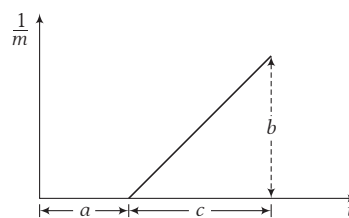
- 128** A symmetric double convex lens is cut in two equal parts by a plane perpendicular to the principal axis. If the power of the original lens is 4D, then the power of a cut lens will be

- (a) 2D (b) 3D (c) 4D (d) 5D

- 129** A ray incident at a point as an angle of incidence of 60° enters a glass sphere of refractive index $n = \sqrt{3}$ and is reflected and refracted further at the surface of the sphere. The angle between the reflected and refracted rays at this surface is

- (a) 50° (b) 60° (c) 90° (d) 40°

- 130** The graph shows, how the inverse of magnification $\frac{1}{m}$ produced by a convex thin lens, with variation in object distance u . What was the focal length of the lens used?



- (a) $\frac{b}{c}$ (b) $\frac{b}{ca}$ (c) $\frac{bc}{a}$ (d) $\frac{c}{b}$

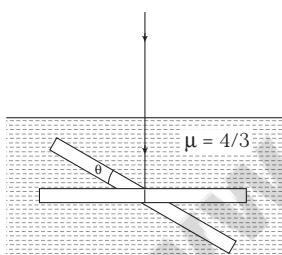
- 131** A convex lens of focal length 30 cm forms a real image three times larger than the object on a screen. Object and screen are moved until the image becomes twice the size of the object. If the shift of the object is 6 cm. The shift of screen is
(a) 28 cm (b) 14 cm (c) 18 cm (d) 16 cm

- 132** An object is placed at 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and $\mu = 1.5$ is then placed close to the mirror in the space between the object and the mirror. The position of final image formed is
(a) -3.94 cm (b) 4.3 cm
(c) -4.93 cm (d) 3.94 cm

- 133** An infinitely long rod lies along with the axis of a concave mirror of focal length f . The near end of the rod is at a distance $u > f$ from the mirror. Its image will have a length

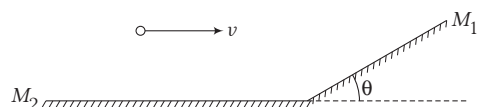
- (a) $\frac{f^2}{u-f}$ (b) $\frac{uf}{u-f}$
(c) $\frac{f^2}{u+f}$ (d) $\frac{uf}{u+f}$

- 134** A plane mirror is placed horizontally inside water ($\mu = 4/3$). A ray falls normally on it, then mirror is rotated through an angle θ , the minimum value of θ for which ray does not come out of the water surface is



- (a) $\pi/4$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$
(c) $\frac{1}{2} \sin^{-1}\left(\frac{3}{4}\right)$ (d) $2 \sin^{-1}\left(\frac{3}{4}\right)$

- 135** A point object is moving with a speed v before an arrangement of two mirrors as shown in figure.



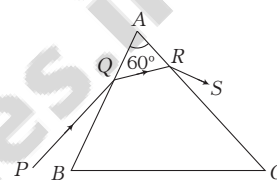
Find the velocity of image in mirror M_1 with respect to image in mirror M_2

- (a) $2v \sin(\theta)$ (b) $v \sin(\theta/2)$
(c) $2v \cos(\theta/2)$ (d) $v \cos(\theta/2)$

- 136** If a ray of light in a denser medium strikes a rarer medium at an angle of incidence i , the angles of reflection and refraction are respectively, r and r' . If the reflected and refraction rays are at right angles to each other, the critical angle for the given pair of media is

- (a) $\sin^{-1}(\tan r')$ (b) $\sin^{-1}(\tan r)$
(c) $\tan^{-1}(\sin i)$ (d) $\cot^{-1}(\tan i)$

- 137** A ray PQ incident on the refracting face BA is refracted in the prism BAC as shown in the figure and emerges from the other refracting face AC as RS , such that $AQ = AR$. If the angle of prism $A = 60^\circ$ and the refractive index of the material of prism is $\sqrt{3}$, then the angle of deviation of the ray is



- (a) 60° (b) 45°
(c) 30° (d) None of these

- 138** The XZ -plane separates two media A and B with refractive indices μ_1 and μ_2 , respectively. A ray of light travels from A and B . Its directions in the two media are given by the unit vectors $\mathbf{r}_A = a\hat{i} + b\hat{j}$ and $\mathbf{r}_B = \alpha\hat{i} + \beta\hat{j}$ respectively, where $\hat{i}$ and $\hat{j}$ are unit vectors in the x and y -directions. Then,

- (a) $\mu_1 a = \mu_2 \alpha$ (b) $\mu_1 \alpha = \mu_2 a$
(c) $\mu_1 b = \mu_2 \beta$ (d) None of these

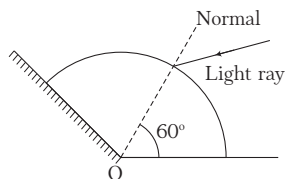
- 139** A ray of light is incident on surface of glass slab at an angle 45° . If the lateral shift produced per unit thickness is $\frac{1}{\sqrt{3}}$ m, the angle of refraction produced is

- (a) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) $\tan^{-1}\left(1 - \sqrt{\frac{2}{3}}\right)$
(c) $\sin^{-1}\left(1 - \sqrt{\frac{2}{3}}\right)$ (d) $\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{3}-1}}\right)$

- 140** In a lake, a fish rising vertically to the surface of water uniformly at the rate of 3 m/s, observes a bird diving vertically towards the water at the rate of 9 m/s. The actual velocity of the dive of the bird is (Take, refractive index of water = $4/3$)

- (a) 3.6 m/s (b) 4.5 m/s
(c) 6.0 m/s (d) 12.0 m/s

- 141** Consider the situation as shown in figure. The point O is the centre and the light ray forms an angle of 60° with the normal.



The normal makes an angle 60° with the horizontal and the mirror makes an angle 60° with the normal. The value of refractive index of that spherical portions, so that light ray retraces its path is

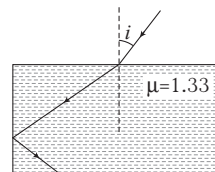
- (a) $\sqrt{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{3}{2}$ (d) $\sqrt{3}$

- 142** A concave mirror is placed at the bottom of an empty tank with face upwards and axis vertical. When sunlight falls normally on the mirror, then it is focused at distance of 32 cm from the mirror. If the tank filled with water ($\mu = \frac{4}{3}$) upto a height of

20 cm, then the sunlight will now get focused at

- (a) 16 cm above water level
(b) 9 cm above water level
(c) 24 cm below water level
(d) 9 cm below water level

- 143** Given a slab with index $\mu = 1.33$ and incident light striking the top horizontal face at angle i as shown in figure. The maximum value of i for which total internal reflection occurs is

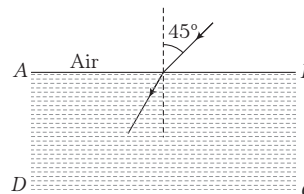


- (a) $\sin^{-1}(\sqrt{0.77})$ (b) $\cos^{-1}(\sqrt{0.77})$
(c) $\sin^{-1}(0.77)$ (d) $\sin^{-1}(0.38)$

- 144** The apparent depth of water in cylindrical water tank of diameter $2R$ cm is reducing at the rate of x cm/min when water is being drained out at a constant rate. The amount of water drained (in cc/min) is (n_1 = refractive index of air, n_2 = refractive index of water)

- (a) $\frac{x\pi R^2 n_1}{n_2}$ (b) $\frac{x\pi R^2 n_2}{n_1}$ (c) $\frac{2\pi R n_1}{n_2}$ (d) $\pi R^2 x$

- 145** In the figure shown, for an angle of incidence 45° , at the top surface, what is the minimum refractive index needed for total internal reflection at vertical face AD ?



- (a) $\frac{\sqrt{2} + 1}{2}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{\frac{1}{2}}$ (d) $\sqrt{2}$

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-6) These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

- 1 Assertion** The formula connecting u , v and f for a spherical mirror is valid only for mirrors sizes which are very small compared to their radii of curvature.

Reason Laws of reflection are strictly valid for plane surface, but not for large spherical.

- 2 Assertion** The images formed by total internal reflection are much brighter than those formed by mirrors or lenses.

Reason There is no loss of intensity in total internal reflection.

- 3 Assertion** The refractive index of diamond is $\sqrt{6}$ and that of liquid is $\sqrt{3}$. If the light travels from diamond to the liquid, it will totally reflected when the angle of incidence is 30°

Reason $\mu = \frac{1}{\sin C}$, where μ is the refractive index of diamond with respect to liquid.

- 4 Assertion** In front of a concave mirror, a point object is placed between focus and centre of curvature. If a glass slab is placed between object

and mirror, then image from mirror may become virtual.

Reason Glass slab always makes a virtual image of a real object.

- 5 Assertion** A diverging lens (in air) cannot be made more diverging whatever be the medium we choose to completely immerse the lens.

Reason The minimum refractive index of any medium is 1.

- 6 Assertion** A hollow lens behaves like a thin glass plate.

Reason Power of this lens becomes zero.

Statement based questions

- 1** I. Image formed by a convex mirror can never be real.
II. Convex mirror is diverging in nature.
Which amongst these statement(s) is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
- 2** I. Shining of diamond is due to the phenomenon of total internal reflection.
II. Refractive index of diamond is large, hence critical angle is small.
Which amongst these statement(s) is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II
- 3** I. If angle of incidence in case of a prism is gradually increased, then deviation produced by prism will first decrease, then increase.
II. At minimum deviation, $r_1 = \frac{A}{2}$.
Which amongst these statement(s) is/are correct?
(a) I only (b) II only
(c) Both I and II (d) Neither I nor II
- 4** Mark the correct statement.
(a) Our eyes can distinguish between real and virtual image.
(b) Virtual image can also be taken on screen.
(c) If the incident rays are converging at a point, then the object is real.
(d) None of the above

- 5** You are given four sources of light each one providing a light of a single colour-red, blue, green and yellow. Suppose the angle of refraction for a beam of yellow light corresponding to a particular angle of incidence at the interface of two media is 90° . Which of the following statements is correct, if the source of yellow light is replaced with that of other lights without changing the angle of incidence?

[NCERT Exemplar]

- (a) The beam of red light would undergo total internal reflection.
- (b) The beam of red light would bend towards normal while it gets refracted through the second medium.
- (c) The beam of blue light would undergo total internal reflection.
- (d) The beam of green light would bend away from the normal as it gets refracted through the second medium.

- 6** When a ray of light is incident normally on one refracting surface of an equilateral prism (refractive index of the material of the prism = 1.5), then choose the correct statement from the following.
(a) Emerging ray is deviated by 30° .
(b) Emerging ray is deviated by 45° .
(c) Emerging ray just grazes the second refracting surface.
(d) The ray undergoes total internal reflection at the second refracting surface.

- 7** A car is moving with at a constant speed of 60 kmh^{-1} on a straight road. Looking at the rear view mirror, the driver finds that the car following him is at a distance of 100 m and is approaching with a speed of 5 kmh^{-1} . In order to keep track of the car in the rear, the driver begins to glance alternatively at the rear and side mirror of his car after every 2 s till the other car overtakes. If the two cars were maintaining their speeds, which of the following statement(s) is/are correct? [NCERT Exemplar]
(a) The speed of the car in the rear is 65 kmh^{-1} .
(b) In the side mirror, the car in the rear would appear to approach with a speed of 5 kmh^{-1} to the driver of the leading car.
(c) In the rear view mirror, the speed of the approaching car would appear to decrease as the distance between the cars decreases.
(d) In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases.

Match the columns

- 1** Match Column I (Phenomenon) with Column II (Principle) and mark the correct option from the codes given below.

Column I (Phenomenon)	Column II (Principle)
(A) Blue colour of a sky	(p) Total internal reflection
(B) Glittering of diamond	(q) Dispersion of light
(C) Formation of rainbow	(r) Scattering of light
(D) In the evening when the sun goes down below the horizon, it continues to remain visible for sometime	(s) Refraction of light

Codes

A	B	C	D	A	B	C	D
(a) p	s	r	q	(b) p	q	r	s
(c) r	p	q	s	(d) s	p	r	q

- 2 Focal length of a biconvex lens is f . Regarding the focal lengths, match the Column I with Column II and mark the correct option from the codes given below.

Column I		Column II	
(A) 	(p)	f	
(B) 	(q)	$2f$	
(C) 	(r)	$\frac{f}{2}$	
(D) 	(s)	Zero	

Codes

A	B	C	D	A	B	C	D
(a) q	r	s	p	(b) q	p	r	q
(c) p	q	r	s	(d) s	p	r	q

- 3 A thin convex lens has focal length f_1 and a concave lens has focal length f_2 . They are kept in contact. Now, match the Column I with Column II and mark the correct option from the codes given below.

Column I		Column II	
(A) If $ f_1 = f_2 $	(p)	Power of system = 0	
(B) If $ f_1 > f_2 $	(q)	Power of system = ∞	
(C) If $ f_1 < f_2 $	(r)	Power of system is positive	
	(s)	Power of system is negative	

Codes

A	B	C	A	B	C
(a) p	s	r	(b) r	q	p
(c) p	r	q	(d) q	r	p

(C) Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

- 1 If the critical angle for total internal reflection from a medium to vacuum is 45° , then velocity of light in the medium is [NEET 2020]
 (a) 1.5×10^8 m/s (b) $\frac{3}{\sqrt{2}} \times 10^8$ m/s
 (c) $\sqrt{2} \times 10^8$ m/s (d) 3×10^8 m/s
- 2 A microscope is focused on an ink mark on the top of a table. If we place a glass slab of 3 cm thick on it, how should the microscope be moved to focus the ink spot again? The refractive index of glass is 1.5. [NEET 2020]
 (a) 2 cm upwards (b) 2 cm downwards
 (c) 1 cm upwards (d) 1 cm downwards
- 3 A plano-convex lens of unknown material and unknown focal length is given. With the help of a spherometer, we can measure the [NEET 2020]
 (a) focal length of the lens
 (b) radius of curvature of the curved surface
 (c) aperture of the lens
 (d) refractive index of the material
- 4 A ray is incident at an angle of incidence i on one surface of a small angle prism (with angle of prism A) and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ , then the angle of incidence is nearly equal to [NEET 2020]
 (a) $\frac{2A}{\mu}$ (b) μA (c) $\frac{\mu A}{2}$ (d) $\frac{A}{2\mu}$
- 5 An object is placed on the principal axis of a concave mirror at a distance of $1.5f$ (f is the focal length). The image will be at [NEET 2020]
 (a) $-3f$ (b) $1.5f$ (c) $-1.5f$ (d) $3f$
- 6 The power of a biconvex lens is 10 D and the radius of curvature of each surface is 10 cm. Then, the refractive index of the material of the lens is [NEET 2020]
 (a) $\frac{4}{3}$ (b) $\frac{9}{8}$ (c) $\frac{5}{3}$ (d) $\frac{3}{2}$
- 7 Pick the incorrect statement in the context with rainbow. [NEET 2019]
 (a) The order of colours is reversed in the secondary rainbow.
 (b) An observer can see a rainbow when his front is towards the sun.
 (c) Rainbow is a combined effect of dispersion, refraction and reflection of sunlight.
 (d) When the light rays undergo two internal reflections in a water drop, a secondary rainbow is formed.
- 8 Two similar thin equi-convex lenses, of focal length f each, are kept co-axially in contact with each other such that the focal length of the combination is F_1 . When the space between the two lenses is filled with glycerine (which has the same refractive index $\mu = 1.5$, as that of glass), then the equivalent focal length is F_2 . The ratio of $F_1 : F_2$ will be [NEET 2019]
 (a) 1 : 2 (b) 2 : 3
 (c) 3 : 4 (d) 2 : 1

- 9 Which colour of the light has the longest wavelength? [NEET 2019]

(a) Blue (b) Green
(c) Violet (d) Red

- 10 An equi-convex lens has power P . It is cut into two symmetrical halves by a plane containing the principal axis. The power of one part will be [NEET (Odisha) 2019]

(a) 0 (b) $\frac{P}{2}$ (c) $\frac{P}{4}$ (d) P

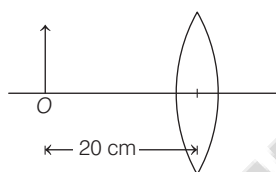
- 11 A double convex lens has focal length 25 cm. The radius of curvature of one of the surfaces is doubled of the other. Find the radii, if the refractive index of the material of the lens is 1.5. [NEET (Odisha) 2019]

(a) 100 cm and 50 cm
(b) 25 cm and 50 cm
(c) 18.75 cm and 37.5 cm
(d) 50 cm and 100 cm

- 12 In total internal reflection, when the angle of incidence is equal to the critical angle for the pair of media in contact, what will be angle of refraction? [NEET 2019]

(a) 0° (b) Equal to angle of incidence
(c) 90° (d) 180°

- 13 Calculate the focal length of the given lens, if the magnification is -0.5 . [AIIMS 2019]

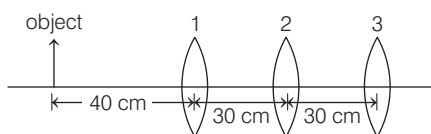


(a) 6.66 cm (b) 5.44 cm
(c) 3.88 cm (d) 1.38 cm

- 14 If focal length of objective and eye lenses are 10 cm and 10 mm respectively and tube length is 11 cm, then angular magnification of telescope is [AIIMS 2019]

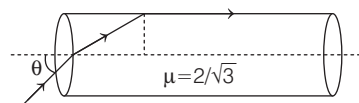
(a) 10 (b) 5
(c) 100 (d) 50

- 15 Find the position of final image from first lens. Given, focal length of each lens is 10 cm. [JIPMER 2019]



(a) 40 cm (b) 50 cm
(c) 45 cm (d) 55 cm

- 16 Find the value of θ in the given diagram. [JIPMER 2019]



(a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

- 17 If velocity of light in air is c , what will be the velocity of light in a medium of refractive index 1.4? [JIPMER 2019]

(a) 2.1×10^8 m/s (b) 1.4×10^8 m/s
(c) 2.8×10^8 m/s (d) None of these

- 18 If frequency (ν) of light is 5×10^{16} Hz and speed of light in air is 3×10^8 m/s. Find the ratio of wavelength of light in a medium of refractive index 2 to air. [JIPMER 2019]

(a) $\frac{1}{2}$ (b) 3 (c) $\frac{3}{2}$ (d) 2

- 19 When a light ray enters from oil to glass on oil-glass interface, then velocity of light changes by a factor of [take, $\mu_{\text{oil}} = 2$ and $\mu_{\text{glass}} = 3/2$] [JIPMER 2019]

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) 3 (d) 1

- 20 An astronomical refracting telescope will have large angular magnification and high angular resolution when it has an objective lens of [NEET 2018]

(a) large focal length and large diameter
(b) large focal length and small diameter
(c) small focal length and large diameter
(d) small focal length and small diameter

- 21 An object is placed at a distance of 40 cm from a concave mirror of focal length 15 cm. If the object is displaced through a distance of 20 cm towards the mirror, the displacement of the image will be [NEET 2018]

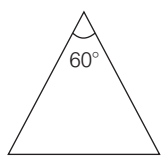
(a) 30 cm towards the mirror
(b) 36 cm away from the mirror
(c) 30 cm away from the mirror
(d) 36 cm towards the mirror

- 22 **Assertion** Angle of deviation depends on the angle of prism.

Reason For thin prism, $\delta = (\mu - 1)A$

where, δ = angle of deviation, μ = refractive index and A = angle of prism. [NEET 2018]

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.
- 23** A thin prism having refracting angle 10° is made of glass of refractive index 1.42. This prism is combined with another thin prism of glass of refractive index 1.7. This combination produces dispersion without deviation. The refracting angle of second prism should be [NEET 2018]
 (a) 4° (b) 6° (c) 8° (d) 10°
- 24** A lens of refractive index μ is put in a liquid of refractive index μ' . If the focal length of lens in air f , then its focal length in liquid will be [NEET 2018]
 (a) $\frac{-f\mu'(\mu-1)}{(\mu'-\mu)}$ (b) $\frac{-f(\mu'-\mu)}{\mu'(\mu-1)}$
 (c) $-\frac{\mu'(\mu-1)}{f(\mu'-\mu)}$ (d) $\frac{f\mu'\mu}{(\mu-\mu')}$
- 25** Two identical glass ($\mu_g = 3/2$) equi-convex lenses of focal length f each are kept in contact. The space between the two lenses is filled with water ($\mu_w = 4/3$). The focal length of the combination is [NEET 2018, 16]
 (a) $f/3$ (b) f (c) $\frac{4f}{3}$ (d) $\frac{3f}{4}$
- 26** The refractive index of glass is 1.5. The speed of light in glass is [AIIMS 2018]
 (a) 3×10^3 m/s (b) 2×10^8 m/s
 (c) 1×10^8 m/s (d) 4×10^8 m/s
- 27** The refractive index of the material of a prism is $\sqrt{2}$ and the angle of the prism is 30° . One of the two refracting surfaces of the prism is made a mirror inwards by silver coating. A beam of monochromatic light entering the prism from the other face will retrace its path (after reflection from the silvered surface) if its angle of incidence on the prism is [JIPMER 2018]
 (a) 30° (b) 45° (c) 60° (d) zero
- 28** If minimum deviation = 30° , then speed of light in shown prism will be



[JIPMER 2018]

- (a) $\frac{3}{\sqrt{2}} \times 10^8$ m/s (b) $\frac{1}{\sqrt{2}} \times 10^8$ m/s
 (c) $\frac{2}{\sqrt{3}} \times 10^8$ m/s (d) $\frac{2}{3} \times 10^8$ m/s

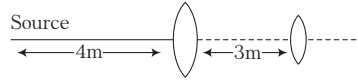
- 29** A thin prism P_1 of angle 4° and refractive index 1.54 is combined with another prism P_2 of refractive index 1.72 produce dispersion without deviation, the angle of P_2 is [NEET 2017]
 (a) 4° (b) 5.33 (c) 2.6° (d) 3°
- 30** A thin symmetrical double convex lens of refractive index $\mu_2 = 1.5$ is placed between a medium of refractive index $\mu_1 = 1.4$ to the left and another medium of refractive index $\mu_3 = 1.6$ to the right. Then, the system behaves as [JIPMER 2017]
 (a) a convex lens (b) a concave lens
 (c) a glass plate (d) a convexo concave lens
- 31** An air bubble in a glass slab with refractive index 1.5 (near normal incidence) is 5 cm deep when viewed from one surface and 3 cm deep when viewed from the opposite face. The thickness (in cm) of the slab is [NEET 2016]
 (a) 8 (b) 10 (c) 12 (d) 16
- 32** A person can see clearly objects only when they lie between 50 cm and 400 cm from his eyes. In order to increase the maximum distance of distinct vision to infinity, the type and power of the correcting lens, the person has to use, will be [NEET 2016]
 (a) convex, + 2.25 D (b) concave, - 0.25 D
 (c) concave, - 0.2 D (d) convex, + 0.15 D
- 33** A astronomical telescope has objective and eyepiece of focal lengths 40 cm and 4 cm, respectively. To view an object 200 cm away from the objective, the lenses must be separated by a distance [NEET 2016]
 (a) 46.0 cm (b) 50.0 cm (c) 54.0 cm (d) 37.3 cm
- 34** Match the Column I with Column II and mark the correct option from the codes given below. [where, m is the magnification produced by the mirror.] [NEET 2016]

Column I		Column II	
(A)	$m = -2$	(p)	Convex mirror
(B)	$m = -1/2$	(q)	Concave mirror
(C)	$m = +2$	(r)	Real image
(D)	$m = +1/2$	(s)	Virtual image

Codes

- A B C D A B C D
 (a) p,r p,s p,q r,s (b) p,s q,r q,s q,r
 (c) r,s q,s q,r p,d (d) q,r q,r q,s p,s

- 35** The angle of incidence for a ray of light at a refracting surface of a prism is 45° . The angle of prism is 60° . If the ray suffers minimum deviation through the prism, then the angle of minimum deviation and refractive index of the material of the prism respectively, are [NEET 2016]

- (a) $30^\circ, \sqrt{2}$ (b) $45^\circ, \sqrt{2}$
 (c) $30^\circ, \frac{1}{\sqrt{2}}$ (d) $45^\circ, \frac{1}{\sqrt{2}}$
- 36** Two identical thin plano-convex glass lenses (refractive index 1.5) each having radius of curvature of 20 cm are placed with their convex surfaces in contact at the centre. The intervening space is filled with oil of refractive index 1.7. The focal length of the combination is [CBSE AIPMT 2015]
 (a) -20 cm (b) -25 cm (c) -50 cm (d) 50 cm
- 37** The refracting angle of a prism is A and refractive index of the material of the prism is $\cot(A/2)$. The angle of minimum deviation is [CBSE AIPMT 2015]
 (a) $180^\circ - 3A$ (b) $180^\circ - 2A$
 (c) $90^\circ - A$ (d) $180^\circ + 2A$
- 38** The near point and far point of a person are 40 cm and 250 cm, respectively. Determine the power of the lens he/she should use while reading a book kept at distance 25 cm from the eye. [AIIMS 2015]
 (a) 2.5 D (b) 5 D
 (c) 1.5 D (d) 3.5 D
- 39** An object is seen through a simple microscope of focal length 12 cm. What will be the angular magnification produced, if the image is formed at the near point of the eye which is 25 cm away from it?
 (a) 6.08 (b) 3.08 [UK PMT 2015]
 (c) 9.03 (d) 5.09
- 40** Angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of prism of given prism. Then, the angle of prism is (Take, $\sin 48^\circ 36' = 0.75$) [Guj. CET 2015]
 (a) 80° (b) $41^\circ 24'$ (c) 60° (d) $82^\circ 48'$
- 41** A ray of light passes from a medium A having refractive index 1.6 to the medium B having refractive index 1.5. The value of critical angle of medium A is [Guj. CET 2015]
 (a) $\sin^{-1} \sqrt{\frac{16}{15}}$ (b) $\sin^{-1} \left(\frac{16}{15} \right)$ (c) $\sin^{-1} \left(\frac{1}{2} \right)$ (d) $\sin^{-1} \left(\frac{15}{16} \right)$
- 42.** The focal lengths of a converging lens are f_v and f_r for violet and red lights, respectively. Which of the following is correct? [CG PMT 2015]
 (a) $f_v < f_r$
 (b) $f_v = f_r$
 (c) $f_v > f_r$
 (d) It depends on the average refractive index
- 43** Aperture of human eye is 0.2 cm. The minimum magnifying power of a visual telescope, whose objective has diameter 100 cm, is [CG PMT 2015]
 (a) 500 (b) 0.002 (c) 0.02 (d) 100
- 44** An object is located 4 m from the first of two thin converging lenses of focal lengths 2 m and 1 m, respectively. The lenses are separated by 3 m. The final image formed by the second lens is located from the source at a distance of [WB JEE 2015]
- 
- (a) 8 m (b) 5.5 m
 (c) 6 m (d) 6.5 m
- 45** If $\mu_V = 1.5230$ and $\mu_R = 1.5145$, then dispersive power of a crown glass is [UP CPMT 2015]
 (a) 0.0164 (b) 0.00701
 (c) 0.0132 (d) 0.0320
- 46** Dispersion of light is caused due to [Manipal 2015]
 (a) intensity of light (b) density of medium
 (c) wavelength (d) None of these
- 47** Calculate the focal length of a reading glass of a person, if the distance of distinct vision is 75 cm.
 (a) 75.2 cm (b) 25.6 cm [KCET 2015]
 (c) 100.4 cm (d) 37.5 cm
- 48** A person wants a real image of his own 3 times enlarged. Where should he stand in front of a concave mirror of radius of curvature of 30 cm? [KCET 2015]
 (a) 90 cm (b) 10 cm
 (c) 20 cm (d) 30 cm
- 49** The magnifying power of a convex lens of focal length 10 cm, when the image is formed at the near point is [Kerala CEE 2015]
 (a) 6 (b) 5.5
 (c) 4 (d) 3.5
 (e) 2
- 50** The velocity of image when object and mirror both are moving towards each other with velocities 4 ms^{-1} and 5 ms^{-1} respectively, is [UP CPMT 2015]
 (a) -14 ms^{-1} (b) 15 ms^{-1}
 (c) -9 ms^{-1} (d) 14 ms^{-1}
- 51** The limiting angle of incidence for an optical ray that can be transmitted by an equilateral prism of refractive index $\mu = \sqrt{\frac{7}{3}}$ is given by (angles can be assumed to be small, so that sine of the angle is angle itself) [CG PMT 2015]
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$
 (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$

- 52** A ray enters a glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence of 60° , a ray is reflected and refracted at the farther surface of the sphere. The angle between the reflected and refracted rays at this surface is [UP CPMT 2015]
 (a) 50° (b) 60°
 (c) 90° (d) 40°
- 53** To measure the roughness of the surface of a material, which of the following microscope is preferred for better result output? [Manipal 2015]
 (a) Compound microscope (b) Electron microscope
 (c) Atomic force microscope (d) None of the above
- 54** The angle of a prism is A and one of its refracting surfaces is silvered. Light rays falling at an angle of incidence $2A$ on the first surface returns back through the same path after suffering reflection at the silvered surface. The refractive index μ of the prism is [CBSE AIPMT 2014]
 (a) $2 \sin A$ (b) $2 \cos A$
 (c) $\frac{1}{2} \cos A$ (d) $\tan A$
- 55** If the focal length of objective lens is increased, then magnifying power of [CBSE AIPMT 2014]
 (a) microscope will increase but that of telescope decrease
 (b) microscope and telescope both will increase
 (c) microscope and telescope both will decrease
 (d) microscope will decrease but that of telescope will increase
- 56** An object placed at 20 cm in front of a concave mirror produces three times magnified real image. What is the focal length of the concave mirror? [KCET 2014]
 (a) 15 cm (b) 6.6 cm
 (c) 10 cm (d) 7.5 cm
- 57** In vacuum, to travel distance d , light takes time t and in medium to travel distance $5d$, it takes time T . The critical angle of the medium is [MHT CET 2014]
 (a) $\sin^{-1}\left(\frac{5T}{t}\right)$ (b) $\sin^{-1}\left(\frac{5t}{3T}\right)$
 (c) $\sin^{-1}\left(\frac{5t}{T}\right)$ (d) $\sin^{-1}\left(\frac{3t}{5T}\right)$
- 58** The equiconvex lens has focal length f . If it is cut perpendicular to the principal axis passing through optical centre, then focal length of each half is [MHT CET 2014]
 (a) $\frac{f}{2}$ (b) f (c) $\frac{3f}{2}$ (d) $2f$
- 59** Two lenses of power 15 D and -3 D are placed in contact. The focal length of the combinations is [Kerala CEE 2014]
 (a) 10 cm (b) 15 cm (c) 12 cm (d) 18 cm
 (e) 8.33 cm
- 60** A luminous object is separated from a screen by distance d . A convex lens is placed between the object and the screen such that it forms a distinct image on the screen. The maximum possible focal length of this convex lens is [WB JEE 2014]
 (a) $4d$ (b) $2d$ (c) $\frac{d}{2}$ (d) $\frac{d}{4}$
- 61** An object is placed 30 cm away from a convex lens of focal length 10 cm and a sharp image is formed on a screen. Now, a concave lens is placed in contact with the convex lens. The screen now has to be moved by 45 cm to get a sharp image again. The magnitude of focal length of the concave lens is (in cm) [WB JEE 2014]
 (a) 72 (b) 60 (c) 36 (d) 20
- 62** A thin glass (refractive index 1.5) lens has optical power of -5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be [UK PMT 2014]
 (a) 1 D (b) -1 D (c) 25 D (d) -25 D
- 63** A concave lens of focal length f forms an image which is $1/3$ times the size of the object. Then, the distance of object from the lens is [EAMCET 2014]
 (a) $2f$ (b) f (c) $\frac{2}{3}f$ (d) $\frac{3}{2}f$
- 64** A focal length of a lens is 10 cm. What is power of a lens in dioptre? [KCET 2014]
 (a) 0.1 D (b) 10 D (c) 15 D (d) 1 D
- 65** Astigmatism is corrected by using [Kerala CEE 2014]
 (a) cylindrical lens (b) plano-convex lens
 (c) plano-concave lens (d) convex lens
 (e) concave lens
- 66** The intermediate image formed by the objective of a compound microscope is [WB JEE 2014]
 (a) real, inverted and magnified
 (b) real, erect and magnified
 (c) virtual, erect and magnified
 (d) virtual, inverted and magnified
- 67** An astronomical telescope arranged for normal adjustment has a magnification of 6. If the length of the telescope is 35 cm, then the focal lengths of objective and eyepiece respectively, are [EAMCET 2014]
 (a) 30 cm, 5 cm (b) 5 cm, 30 cm
 (c) 40 cm, 5 cm (d) 30 cm, 6 cm
- 68** A microscope is having objective of focal length 1 cm and eyepiece of focal length 6 cm. If tube length is 30 cm and image is formed at the least distance of distinct vision, what is the magnification produced by the microscope? (Take, $D = 25$ cm) [KCET 2014]
 (a) 6 (b) 150 (c) 25 (d) 125

- 69** Diameter of the objective of a telescope is 200 cm. What is the resolving power of a telescope? (Take, wavelength of light = 5000 Å) [KCET 2014]
 (a) 6.56×10^6 (b) 3.28×10^5
 (c) 1×10^6 (d) 3.28×10^6
- 70** A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If lenses are made of different materials of refractive indices μ_1 and μ_2 and R is the radius of curvature of the curved surface of the lenses, then the focal length of the combination is [NEET 2013]
 (a) $\frac{R}{2(\mu_1 + \mu_2)}$ (b) $\frac{R}{2(\mu_1 - \mu_2)}$
 (c) $\frac{R}{(\mu_1 - \mu_2)}$ (d) $\frac{2R}{(\mu_2 - \mu_1)}$
- 71** For a normal eye, the cornea of eye provides a converging power of 40 D and the least converging power of the eye lens behind the cornea is 20 D. Using this information, the distance between the retina and the cornea-eye lens can be estimated to be [NEET 2013]
 (a) 5 cm (b) 2.5 cm
 (c) 1.67 cm (d) 1.5 cm
- 72** An object is 8 cm high. It is desired to form a real image 4 cm high at 60 cm from the mirror. The type of mirror needed with the focal length is [J&K CET 2013]
 (a) convex mirror with focal length $f = 40$ cm
 (b) convex mirror with focal length $f = 20$ cm
 (c) concave mirror with focal length $f = -40$ cm
 (d) concave mirror with focal length $f = -20$ cm
- 73** When an object is placed 40 cm from a diverging lens, its virtual image is formed 20 cm from the lens. The focal length and power of lens are [J&K CET 2013]
 (a) $f = -20$ cm, $P = -5$ D
 (b) $f = -40$ cm, $P = -5$ D
 (c) $f = -40$ cm, $P = -2.5$ D
 (d) $f = -20$ cm, $P = -2.5$ D
- 74** A magnifying glass of focal length 5 cm is used to view an object by a person whose smallest distance of distinct vision is 25 cm. If he holds the glass close to eye, then the magnification is [J&K CET 2013]
 (a) 5 (b) 6
 (c) 2.5 (d) 3
- 75** A person has a minimum distance of distinct vision as 50 cm. The power of lenses required to read a book at a distance of 25 cm is [J&K CET 2013]
 (a) 3 D (b) 1 D
 (c) 2 D (d) 4 D
- 76** A small angled prism of refractive index 1.4 is combined with another small angled prism of refractive index 1.6 to produce dispersion without deviation. If the angle of first prism is 6° , then the angle of the second prism is [EAMCET 2013]
 (a) 8° (b) 6°
 (c) 4° (d) 2°
- 77** The magnifying power of the astronomical telescope for normal adjustment is 50. The focal length of the eyepiece is 2 cm. The required length of the telescope for normal adjustment is [EAMCET 2013]
 (a) 102 cm (b) 100 cm
 (c) 98 cm (d) 25 cm
- 78** The speed of light in media M_1 and M_2 are $1.5 \times 10^8 \text{ ms}^{-1}$ and $2 \times 10^8 \text{ ms}^{-1}$, respectively. A ray travels from medium M_1 to the medium M_2 with an angle of incidence θ . The ray suffers total internal reflection. Then, the value of the angle of incidence θ is [CET 2013]
 (a) $> \sin^{-1}\left(\frac{3}{4}\right)$ (b) $< \sin^{-1}\left(\frac{3}{4}\right)$
 (c) $= \sin^{-1}\left(\frac{3}{4}\right)$ (d) $\leq \sin^{-1}\left(\frac{3}{4}\right)$
- 79** Radii of curvature of a converging lens are in the ratio 1 : 2. Its focal length is 6 cm and refractive index is 1.5. Then, its radii of curvature are ..., respectively. [Kerala CET 2013]
 (a) 9 cm and 18 cm (b) 6 cm and 12 cm
 (c) 3 cm and 6 cm (d) 4.5 cm and 9 cm
- 80** The sunlight reaches us as white and not as its components because [KCET 2013]
 (a) air medium is dispersive
 (b) air medium is non-dispersive
 (c) air medium scatter the sunlight
 (d) air medium absorbs the sunlight
 (e) speed of light depends on wavelength in vacuum
- 81** The radius of curvature of the convex face of a plano-convex lens is 12 cm and the refractive index of the material of the lens is 1.5. Then, the focal length of the lens is [MP PMT 2013]
 (a) 6 cm (b) 12 cm
 (c) 18 cm (d) 24 cm
- 82** Two thin lenses of focal lengths f_1 and f_2 are placed in contact with each other, then the equivalent focal length of the combination will be [MP PMT 2013]
 (a) $f_1 + f_2$ (b) $\frac{1}{f_1 + f_2}$
 (c) $\frac{f_1 f_2}{f_1 + f_2}$ (d) $\frac{f_1 + f_2}{f_1 f_2}$

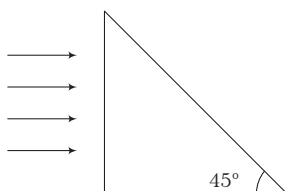
- 83** The mirrors are inclined at an angle of 50° . The number of images formed for an object placed in between the mirrors is [MP PMT 2013]

(a) 5 (b) 6
(c) 7 (d) 8

- 84** If c is the velocity of light in free space, then the time taken by light to travel a distance x in a medium refractive index μ is [MP PMT 2013]

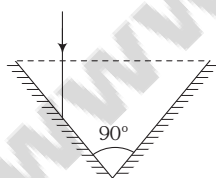
(a) $\frac{x}{c}$ (b) $\frac{\mu x}{c}$
(c) $\frac{x}{\mu c}$ (d) $\frac{c}{\mu x}$

- 85** A beam of light consisting of red, green and blue colours is incident on a right angled prism as shown in figure. The refractive indices of the material of the prism for above red, green and blue wavelengths are 1.39, 1.44 and 1.47, respectively. The prism will [UPCPMT 2013]



- (a) separate part of the blue colour from the red and green
(b) separate part of the red colour from the green and blue colours
(c) separate all the three colours from one another
(d) None of the above

- 86** A vessel consists of two plane mirrors at right angles as shown in figure. The vessel is filled with water. The total deviation in incident ray is [UP CPMT 2013]



- (a) 0° (b) 60°
(c) 90° (d) 180°

- 87** For having large magnification power of a compound microscope, [MPPMT 2013]

- (a) length of the microscope tube must be small
(b) focal lengths of objective lens and eyepiece should be large
(c) focal lengths of objective lens and eyepiece should be small
(d) focal length of eyepiece must be smaller than the focal length of objective lens

- 88** A ray of light is incident at an angle of incidence i , on one face of a prism of angle A (assumed to be small) and emerges normally from the opposite face.

If the refractive index of the prism is μ , then the angle of incidence i , is nearly equal to

[CBSE AIPMT 2012]

- (a) μA (b) $\frac{\mu A}{2}$
(c) A/μ (d) $A/2\mu$

- 89** A concave mirror of focal length f_1 is placed at a distance d from a convex lens of focal length f_2 . A beam of light coming from infinity and falling on this convex lens concave mirror combination returns to infinity. The distance d must be equal [CBSE AIPMT 2012]

- (a) $f_1 + f_2$ (b) $-f_1 + f_2$
(c) $2f_1 + f_2$ (d) $-2f_1 + f_2$

- 90** A thin convex lens of refractive index 1.5 has 20 cm focal length in air. If the lens is completely immersed in a liquid of refractive index 1.6, then its focal length will be [AIIMS 2012]

- (a) - 160 cm (b) - 100 cm
(c) + 10 cm (d) + 100 cm

- 91 Assertion** The resolving power of a telescope is more, if the diameter of the objective lens is more.

Reason Objective lens of large diameter collects more light. [AIIMS 2012]

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion
(b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion
(c) If Assertion is true but Reason is false
(d) If both Assertion and Reason are false

- 92** A thin equiconvex lens of refractive index $3/2$ and radius of curvature 30 cm is put in water (refractive index = $4/3$), its focal length is [UP CPMT 2012]

- (a) 0.15 m (b) 0.30 m
(c) 0.45 m (d) 1.20 m

- 93** If the focal length of a concave mirror is 50 cm, then where should the object be placed, so that its image is two times and inverted [BCECE (Mains) 2012]

- (a) 55 cm (b) - 75 cm
(c) 65 cm (d) 85 cm

- 94** The diameter of the eye ball of a normal eye is about 2.5 cm. The power of the eye lens varies from [BCECE (Mains) 2012]

- (a) 9 D to 8 D (b) 40 D to 32 D
(c) 44 D to 40 D (d) None of these

- 95** Two thin lenses when placed in contact, then the power of combination is +10 D. If they are kept 0.25 m apart, then the power reduces to +6 D. The focal lengths of the lenses (in m) will be

- (a) 0.5 and 0.125 [BCECE (Mains) 2012]
(b) 0.5 and 0.75
(c) 0.125 and 0.75
(d) Both have the same focal length 1.25

96 If the image formed by a convex mirror of focal length 30 cm is a quarter of the size of the object, then the distance of the object from the mirror will be [BHU 2012]

- (a) 30 cm (b) 60 cm
(c) 90 cm (d) 120 cm

97 In a compound microscope, the focal length of the objective is 2.5 cm and of eye lens is 5 cm. If an object is placed at 3.75 cm before the objective and the image is formed at the least distance of distinct vision, then the distance between two lenses will be [BHU 2012]

- (a) 11.67 cm (b) 12 cm
(c) 12.75 cm (d) 13 cm

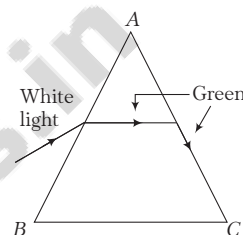
98 A short linear object of length L lies on the axis of a spherical mirror of focal length f at a distance u from the mirror. Its image has an axial length L' equal to [AMU 2012]

- (a) $L \left[\frac{f}{u-f} \right]^{1/2}$ (b) $L \left[\frac{u+f}{f} \right]^{1/2}$
(c) $L \left[\frac{u-f}{f} \right]^2$ (d) $L \left[\frac{f}{u-f} \right]^2$

99 An object is placed at a distance of 10 cm from a co-axial combination of two lenses A and B in contact. The combination forms a real image three times the size of the object. If lens B is concave with a focal length of 30 cm, then the nature and focal length of lens A is [AMU 2012]

- (a) convex, 12 cm (b) concave, 12 cm
(c) convex, 6 cm (d) convex, 18 cm

100 White light is incident on face AB of a glass prism. The path of the green component is shown in the figure. If the green light is just totally internally reflected at face AC as shown, then the light emerging from face AC will contain [AMU 2012]



- (a) yellow, orange and red colours
(b) violet, indigo and blue colours
(c) All colours
(d) All colours except green

ANSWERS

CHECK POINT 9.1

1. (b) 2. (c) 3. (b) 4. (d) 5. (a) 6. (b) 7. (b) 8. (c) 9. (b)

CHECK POINT 9.2

1. (a) 2. (a) 3. (d) 4. (a) 5. (a) 6. (a) 7. (b) 8. (b) 9. (b) 10. (a)

CHECK POINT 9.3

1. (a) 2. (d) 3. (c) 4. (a) 5. (c) 6. (b) 7. (c) 8. (c) 9. (c) 10. (c)
11. (a) 12. (b) 13. (a) 14. (c) 15. (c) 16. (a) 17. (a) 18. (c)

CHECK POINT 9.4

1. (d) 2. (a) 3. (a) 4. (c) 5. (d) 6. (b) 7. (d) 8. (b) 9. (b) 10. (d)
11. (d) 12. (d) 13. (d) 14. (a) 15. (b) 16. (d) 17. (d) 18. (b)

CHECK POINT 9.5

1. (b) 2. (a) 3. (d) 4. (d) 5. (a) 6. (d) 7. (b) 8. (b)

CHECK POINT 9.6

1. (b) 2. (a) 3. (b) 4. (c) 5. (b) 6. (a) 7. (b) 8. (b) 9. (a) 10. (a)
11. (a) 12. (b) 13. (b) 14. (a) 15. (b) 16. (b) 17. (c) 18. (b)

CHECK POINT 9.7

1. (b) 2. (b) 3. (a) 4. (c) 5. (b) 6. (c) 7. (d) 8. (c)

(A) Taking it together

1. (a)	2. (a)	3. (b)	4. (c)	5. (b)	6. (c)	7. (d)	8. (a)	9. (d)	10. (c)
11. (b)	12. (b)	13. (b)	14. (b)	15. (d)	16. (c)	17. (a)	18. (b)	19. (d)	20. (c)
21. (d)	22. (b)	23. (c)	24. (b)	25. (c)	26. (c)	27. (b)	28. (d)	29. (d)	30. (d)
31. (a)	32. (c)	33. (c)	34. (d)	35. (b)	36. (a)	37. (a)	38. (b)	39. (d)	40. (c)
41. (c)	42. (b)	43. (b)	44. (a)	45. (d)	46. (a)	47. (b)	48. (c)	49. (c)	50. (a)
51. (b)	52. (a)	53. (b)	54. (d)	55. (b)	56. (d)	57. (c)	58. (c)	59. (c)	60. (d)
61. (c)	62. (a)	63. (c)	64. (c)	65. (c)	66. (a)	67. (b)	68. (b)	69. (a)	70. (b)
71. (d)	72. (b)	73. (a)	74. (d)	75. (c)	76. (a)	77. (c)	78. (b)	79. (c)	80. (a)
81. (a)	82. (c)	83. (b)	84. (d)	85. (a)	86. (d)	87. (c)	88. (b)	89. (b)	90. (b)
91. (b)	92. (d)	93. (b)	94. (d)	95. (c)	96. (c)	97. (a)	98. (c)	99. (c)	100. (a)
101. (d)	102. (c)	103. (c)	104. (d)	105. (c)	106. (d)	107. (c)	108. (c)	109. (c)	110. (a)
111. (a)	112. (d)	113. (a)	114. (a)	115. (b)	116. (b)	117. (c)	118. (c)	119. (d)	120. (b)
121. (d)	122. (d)	123. (c)	124. (b)	125. (b)	126. (c)	127. (a)	128. (a)	129. (c)	130. (d)
131. (a)	132. (c)	133. (a)	134. (c)	135. (a)	136. (b)	137. (a)	138. (a)	139. (b)	140. (b)
141. (d)	142. (b)	143. (a)	144. (b)	145. (b)					

B. Medical entrance special format questions

• Assertion and reason

1. (c) 2. (a) 3. (d) 4. (c) 5. (a) 6. (a)

• Statement based questions

1. (b) 2. (c) 3. (c) 4. (d) 5. (c) 6. (d) 7. (d)

• Match the columns

1. (c) 2. (b) 3. (a)

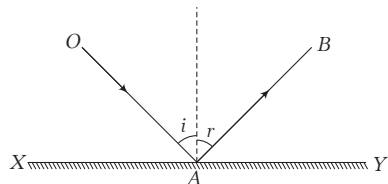
(C) Medical entrances' gallery

1. (b)	2. (c)	3. (b)	4. (b)	5. (a)	6. (d)	7. (b)	8. (a)	9. (d)	10. (d)
11. (c)	12. (c)	13. (a)	14. (a)	15. (b)	16. (b)	17. (a)	18. (a)	19. (a)	20. (a)
21. (b)	22. (a)	23. (b)	24. (a)	25. (d)	26. (b)	27. (b)	28. (a)	29. (d)	30. (c)
31. (c)	32. (b)	33. (c)	34. (d)	35. (a)	36. (c)	37. (b)	38. (c)	39. (b)	40. (d)
41. (d)	42. (a)	43. (a)	44. (b)	45. (a)	46. (c)	47. (d)	48. (c)	49. (d)	50. (a)
51. (b)	52. (c)	53. (c)	54. (b)	55. (d)	56. (a)	57. (c)	58. (d)	59. (e)	60. (d)
61. (d)	62. (a)	63. (a)	64. (b)	65. (a)	66. (a)	67. (a)	68. (b)	69. (d)	70. (c)
71. (c)	72. (a)	73. (c)	74. (b)	75. (c)	76. (c)	77. (a)	78. (a)	79. (d)	80. (b)
81. (d)	82. (c)	83. (b)	84. (b)	85. (b)	86. (d)	87. (d)	88. (a)	89. (c)	90. (a)
91. (a)	92. (d)	93. (b)	94. (c)	95. (a)	96. (c)	97. (a)	98. (d)	99. (c)	100. (d)

Hints & Explanations

• CHECK POINT 9.1

1 (b)



Here, XY is a plane reflecting surface,
 OA and AB are the incident and reflected rays, respectively

i = angle of incidence
 r = angle of reflection

As, $AB \perp OA$,

$\therefore i + r = 90^\circ$

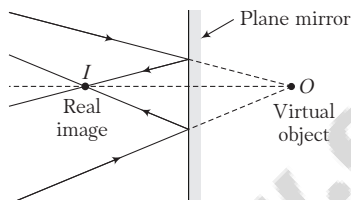
Also, according to laws of reflection,

$r = i$

$\therefore i + i = 90^\circ \Rightarrow 2i = 90^\circ$

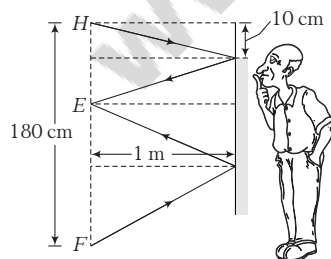
$i = \frac{90^\circ}{2} = 45^\circ$

3 (b) When convergent beam is incident on a plane mirror, then mirror forms real image, for a virtual object.



4 (d) Deviation by a plane mirror,
 $\delta = 180^\circ - 2\theta = 180^\circ - 60^\circ = 120^\circ$

6 (b) According to ray diagram shown in figure



Length of mirror = $\frac{1}{2}(10 + 170) = 90$ cm

8 (c) As number of images, $n = 3$

So, $n = \frac{360}{\theta} - 1$

$3 = \frac{360}{\theta} - 1$

$\therefore \theta = 90^\circ$

9 (b) Distance between 4th images

$$I_u I_u' = 2n(x + y) = 2 \times 4 [3 + 1] = 32 \text{ m}$$

• CHECK POINT 9.2

1 (a) On immersing a mirror in water, focal length of the mirror remains unchanged.

2 (a) Since, the object is at the centre of curvature of the mirror. So, image formed is real, inverted and same in size.

3 (d) Given, $u = -30$ cm and $f = 30$ cm

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} + \frac{1}{30} = \frac{1}{15}$$

$\therefore$ Distance from the mirror, $v = +15$ cm

The positive sign shows that the image is formed behind the mirror.

4 (a) Given, $u = -30$ cm

$v = -30$ cm

Using relation, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{-30} + \frac{1}{-30} = \frac{1}{f}$$

$\therefore$ Focal length of the mirror, $f = -15$ cm

5 (a) Magnification, $m = +\frac{1}{n} = -\frac{v}{u}$

$\therefore v = -\frac{u}{n}$

From mirror formula, $\frac{1}{f} = \frac{1}{\left(-\frac{u}{n}\right)} + \frac{1}{u}$

$\therefore$ The distance of the object from the mirror, $u = -(n-1)f$

6 (a) In case of concave mirror, $m = -2$

Also, $m = \frac{f}{f-u}$

$$\Rightarrow -2 = \frac{-50}{-50-u}$$

$\therefore$ Distance of the object from the mirror, $u = -75$ cm

7 (b) Magnification, $m = \frac{f}{f-u}$

$$\Rightarrow \left(+\frac{1}{4}\right) = \frac{+30}{+30-u}$$

$\Rightarrow$ The distance of the object from the mirror, $u = -90$ cm

8 (b) Magnification,

$$m = \frac{f}{f-u} \Rightarrow -3 = \frac{f}{f-(-20)}$$

$\Rightarrow$ The focal length of mirror, $f = -15$ cm

- 9 (b) Magnification,

$$m = \frac{I}{O} = \frac{f}{f - u}$$

$$\Rightarrow \frac{I}{7.5} = \frac{(R/2)}{(R/2) - u} = \frac{(25/2)}{(25/2) - (-40)}$$

$\therefore$ The size of the image, $I = 1.78$ cm

- 10 (c) From the relation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

We have, $\frac{-dv}{v^2} - \frac{du}{u^2} = 0$

$$\therefore dv = \frac{v^2}{u^2} (-du)$$

$$= -\left(\frac{20}{10}\right)^2 (0.1) = -0.4 \text{ cm away from the mirror}$$

CHECK POINT 9.3

- 1 (a) Refractive index of the medium,

$$\mu = \frac{c}{v} = \frac{c}{n\lambda} = \frac{3 \times 10^8}{4 \times 10^{14} \times 5 \times 10^{-7}} = 1.5$$

- 3 (c) As, $\mu = \frac{c}{v} \Rightarrow v \propto \frac{1}{\mu}$

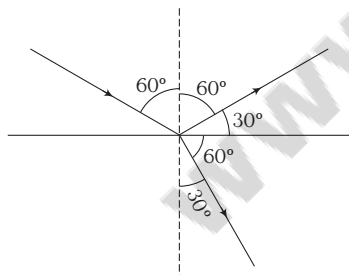
$$\therefore \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$$

$\Rightarrow$ Ratio of velocities of light

$$\frac{v_g}{v_w} = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$$

- 4 (a) Wavelength of light, $\lambda' = \frac{\lambda}{\mu} = \frac{6000}{1.5} = 4000 \text{ \AA}$

- 5 (c)



Refractive index of the material,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 30^\circ} = \sqrt{3} = 1.732$$

- 6 (b) Refractive index of glass with respect to water,

$${}_w\mu_g = \frac{a\mu_g}{a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

- 7 (c) ${}_2\mu_1 \times {}_3\mu_2 \times {}_4\mu_3 = \frac{\mu_1}{\mu_2} \times \frac{\mu_2}{\mu_3} \times \frac{\mu_3}{\mu_4} = \frac{\mu_1}{\mu_4} = {}_4\mu_1 = \frac{1}{\mu_4}$

- 9 (c) We have, $d_{\text{app}} = \frac{d}{\mu}$

or $(d - 2) = \frac{d}{1.5}$

$$\therefore d = 6 \text{ cm}$$

- 10 (c) $d_{\text{actual}} = (d_{\text{app}})\mu$

From one side, $d_1 = 6 \times 1.5 = 9$

From other side, $d_2 = 4 \times 1.5 = 6$

$$\therefore d = d_1 + d_2 = 15 \text{ cm}$$

- 11 (a) Distance from the surface of water,

$$h' = \mu h = \frac{4}{3} \times 18 = 24 \text{ m}$$

- 12 (b) Apparent depth, $h' = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} = \frac{d}{\mu_1} + \frac{d}{\mu_2} = d \left(\frac{1}{\mu_1} + \frac{1}{\mu_2} \right)$

- 13 (a) As, $d_{\text{app}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3} = \frac{3}{3/2} + \frac{4}{4/3} + \frac{6}{6/5}$
- $$= 2 + 3 + 5 = 10 \text{ cm}$$

- 14 (c) The refractive index of glass relative to water is given by

$${}_w\mu_g = \frac{a\mu_g}{a\mu_w} = \frac{1.50}{1.33}$$

If C be the critical angle at glass water interface, then

$$\sin C = \frac{1}{{}_w\mu_g} = \frac{1.33}{1.50} = 0.8867$$

$$C = \sin^{-1}(0.8867) = 62.5^\circ$$

- 15 (c) Critical angle,

$$\sin C = \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow \sin C = \frac{3500}{7000} = \frac{1}{2}$$

$$\therefore C = 30^\circ$$

- 16 (a) Critical angle,

$$\sin C = \frac{1}{\mu}$$

or $C = \sin^{-1}\left(\frac{1}{\mu}\right)$

As μ decreases with increase in λ . Also, yellow, orange and red have higher wavelength than green, so μ will be less for these rays. Thus, critical angle for these rays will be high, hence if green is just totally internally reflected, then yellow, orange and red rays will emerge out.

- 17 (a) We have, $\sin \theta_C = \frac{1}{\mu}$

$$\therefore \mu = \frac{1}{\sin 30^\circ} = 2$$

The velocity of light, $v = \frac{c}{\mu} = \frac{3.0 \times 10^8}{2}$

$$= 1.5 \times 10^8 \text{ m/s}$$

- 18 (c) We have, $\sin i > \sin \theta_C$

$$\therefore \sin 45^\circ > \frac{1}{n} \text{ or } \frac{1}{\sqrt{2}} > \frac{1}{n}$$

or $n > \sqrt{2} \text{ or } n > 1.414$

• CHECK POINT 9.4

- 1 (d) When object lies between focus and pole of convex lens, image formed is virtual, erect and enlarged in size.

- 2 (a) Focal length of air lens in glass is given as

$$\frac{1}{f} = (\mu_a - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$= \left(\frac{2}{3} - 1 \right) \left[\frac{1}{10} - \left(-\frac{1}{10} \right) \right] \Rightarrow f = -15 \text{ cm}$$

So, nature of lens is concave type.

- 4 (c) Focal length of plano-convex lens,

$$f = \frac{R}{\mu - 1} = \frac{R}{(1.5 - 1)} = 2R$$

- 5 (d) Image is real, because virtual image is enlarged.

$$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{x/4} + \frac{1}{x} = \frac{1}{30} \quad (\because u = -x)$$

$$\frac{5}{x} = \frac{1}{30}$$

$$\therefore x = 150 \text{ cm}$$

- 6 (b) From a concave lens, image is always smaller in size.

- 7 (d) For real image from converging lens,

$$m = -2$$

$$\therefore \text{Magnification, } m = \frac{f}{f + u} \Rightarrow -2 = \frac{f}{u + f} = \frac{20}{u + 20}$$

$$\Rightarrow u = -30 \text{ cm}$$

- 8 (b) We have $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{+5} - \frac{1}{-10} = \frac{1}{f} \left(|v| = \frac{|u|}{2} \right)$

$$\therefore \text{Focal length, } f = \frac{10}{3} \text{ cm}$$

Positive sign implies that the lens is convex.

- 9 (b) Keep the object at a distance $2f$ from the lens.

- 10 (d) We have, $\frac{1}{v} - \frac{1}{-90} = \frac{1}{f}$ and $(1.5 - 1) \left(\frac{1}{30} \right) = \frac{1}{f}$

$$\therefore v = 180 \text{ cm}$$

$$\text{or } |m| = \left| \frac{v}{u} \right| = \frac{180}{90} = 2.0$$

- 12 (d) We have, $P = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$

$$2.5 = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

$$P' = \left(\frac{3/2}{2} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$\therefore P' = -1.25 \text{ D}$$

- 13 (d) Focal length of combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{60} + \frac{1}{(-20)} \Rightarrow F = -30 \text{ cm}$$

- 14 (a) Focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{F} = \frac{1}{(+40)} + \frac{1}{(-25)}$$

$$\Rightarrow F = -\frac{200}{3} \text{ cm}$$

$$\text{As, Power, } P = \frac{100}{F} = \frac{100}{(-200/3)} = -1.5 \text{ D}$$

- 15 (b) In each case, there is a combination of two plano-convex lenses placed close to each other. Focal length of combination

$\frac{1}{F} = \frac{1}{2f} + \frac{1}{2f}$ is same in all cases, so ratio of focal length in three cases is 1 : 1 : 1.

- 18 (b) Focal length, $F = \frac{R}{2(\mu - 1)}$

$$\Rightarrow F = \frac{10}{2(1.5 - 1)} = 10 \text{ cm}$$

• CHECK POINT 9.5

- 1 (b) In minimum deviation position, refracted ray inside an isosceles prism is parallel to the base of the prism.

- 2 (a) Deviation of ray in prism is given by

$$\delta = i + e - A$$

$$\text{So, } e = \delta + A - i = 30^\circ + 30^\circ - 60^\circ = 0^\circ$$

So, emergent ray will make an angle of 90° with the face through which it emerges.

- 3 (d) Refractive index of prism, $\mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$

$$\text{As, } i = 45^\circ = \frac{A + \delta_m}{2}$$

$$\text{So, } \frac{\sin 45^\circ}{\sin (A/2)} = \sqrt{2}$$

$$\Rightarrow \frac{1}{2} = \sin \frac{A}{2} \Rightarrow A = 60^\circ$$

- 4 (d) Given, $i = e = \frac{3}{4} A = \frac{3}{4} \times 60^\circ = 45^\circ$

In the position of minimum deviation,

$$2i = A + \delta_m$$

$$\text{or } \delta_m = 2i - A$$

$$= 2 \times 45^\circ - 60 = 30^\circ$$

- 5 (a) Angle of minimum deviation in prism,

$$\delta_m = (\mu - 1)A = (\mu - 1)(2r) \quad (\because r = A/2)$$

$$= (1.5 - 1)(2r) = 0.5 \times 2r = r$$

- 6 (d) Refractive index of prism, $\mu = \frac{\sin \left(\frac{A + \delta_m}{2} \right)}{\sin \left(\frac{A}{2} \right)}$

Substituting $A = \delta_m$ and $\mu = 1.5$, we get

$$\Rightarrow 1.5 = \frac{\sin A}{\sin\left(\frac{A}{2}\right)}$$

$$\frac{1.5}{2} = \cos \frac{A}{2}$$

$$0.75 = \cos \frac{A}{2}$$

We get,

$$A = 82^\circ$$

- 8 (b) To produce dispersion without deviation,

$$\frac{A'}{A} = \frac{(\mu_y - 1)}{(\mu'_y - 1)} \Rightarrow \frac{A'}{6^\circ} = \frac{(1.54 - 1)}{(1.72 - 1)}$$

$$\Rightarrow A' = 4.5^\circ = 4^\circ 30'$$

• CHECK POINT 9.6

- 3 (b) When final image is formed at $D = 25$ cm from eye.

In this situation, $v = -D$

From lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\text{We have } \frac{1}{-D} - \frac{1}{(-u)} = \frac{1}{f}$$

$$\text{i.e. } \frac{D}{u} = 1 + \frac{D}{f}$$

$$\text{So, magnifying power} = \frac{D}{u} = \left(1 + \frac{D}{f}\right)$$

- 5 (b) Magnifying power, $m = m_o \times m_e$

$$100 = 5 \times m_e$$

$$m_e = 20$$

- 6 (a) Length of compound microscope,

$$L_\infty = v_o + f_e$$

$$\Rightarrow 14 = v_o + 5$$

$$\therefore v_o = 9 \text{ cm}$$

Magnifying power of microscope for relaxed eye,

$$M = \frac{v_o}{u_o} \cdot \frac{D}{f_e} \text{ or } 25 = \frac{9}{u_o} \cdot \frac{25}{5}$$

$$\text{or distance of object for objective, } u_o = \frac{9}{5} = 1.8 \text{ cm}$$

- 7 (b) When final image is formed at infinity, then magnifying power,

$$M_\infty = -\frac{v_o}{u_o} \times \frac{D}{f_e}$$

$$\text{From } \frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o},$$

$$\frac{1}{+1.2} = \frac{1}{v_o} - \frac{1}{(-1.25)}$$

$$\Rightarrow v_o = 30 \text{ cm}$$

$$\therefore |M_\infty| = \frac{30}{1.25} \times \frac{25}{3} = 200$$

- 8 (b) For objective lens,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

$$\text{or } \frac{1}{4} = \frac{1}{v_o} - \frac{1}{(-4.5)} \Rightarrow |v_o| = 36 \text{ cm}$$

$$\therefore \text{Magnifying power, } |M| = \frac{v_o}{u_o} \left(1 + \frac{D}{f_e}\right) = \frac{36}{4.5} \left(1 + \frac{24}{8}\right) = 32$$

- 9 (a) If the telescope is reversed i.e., seen from the objective side, then object will appear very small, because in this case magnification becomes reciprocal of initial magnification.

- 10 (a) In a telescope, large aperture of objective helps in improving the brightness (image) by gathering more light from distant object.

- 11 (a) Magnification of astronomical telescope for normal eye,

$$m = -\frac{f_o}{f_e} = -\frac{100}{2} = -50$$

- 12 (b) Magnifying power of astronomical telescope,

$$m = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{D}\right) = -\frac{200}{5} \left(1 + \frac{5}{25}\right) = -48$$

- 13 (b) In terrestrial telescope, there are three lenses, objective, eye-piece and third is erecting lens, it is placed at $2f$ from intermediate image.

- 15 (b) If in an astronomical telescope, the convergent eyepiece is replaced by a divergent lens which is placed in such a way that rays from objective are directed towards its focus, final image will be erect, enlarged and virtual and hence, it is called Galilean telescope.

- 18 (b) Resolving power of telescope

$$= \frac{d}{1.22\lambda} = \frac{1.22}{1.22 \times 5000 \times 10^{-10}} = 2 \times 10^6$$

• CHECK POINT 9.7

- 2 (b) Focal length, $f = -d = -100 \text{ cm} = -1 \text{ m}$

$$\therefore P = \frac{1}{f} = \frac{1}{-1} = -1 \text{ D}$$

- 3 (a) In case of hypermetropia, eye can see distant objects clearly but cannot see nearby objects clearly.

- 6 (c) Condition of achromatic combination is satisfied, if

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

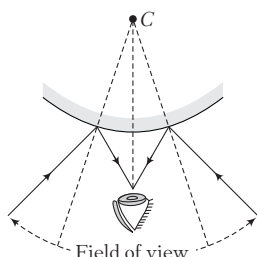
This condition is satisfied for values in given in option (c) only.

(A) Taking it together

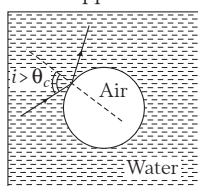
- 1 (a) In myopia, the focal length or radii of curvature of lens reduces or power of lens increases or distance between eye lens and retina increases.

- 2 (a) In shortsightedness, the focal length of eye lens decreases and so the power of eye lens increases.

- 6 (c) The field of view in convex mirror is more as compared to plane or concave mirror. This is why it is used as rear view mirror in vehicles.



- 7 (d) If there had been one eye of the man, then visible region decreases, so the depth of image will not be seen.
- 8 (a) Sky appears blue due to scattering of light. In absence of atmosphere, no scattering will occur. Therefore sky will be seen black.
- 9 (d) An air bubble in water shines due to total internal reflection at its outer surface. Here, light is propagating from denser medium (water) to rarer medium (air) and if $i > \theta_c$, then total internal reflection takes place at the surface of the bubble. Hence, air bubble appears to be shining in water.



- 12 (b) The phenomenon involved in the reflection of radiowaves by ionosphere is similar to total internal reflection of light in air during a mirage when angle of incidence is greater than critical angle.
- 13 (b) With increase in wavelength, refractive index gets decreased. Therefore, focal length of lens will be increased.
- 15 (d) Since $v \propto \lambda$, the light of red colour is of highest wavelength and therefore of highest speed. So, after travelling through the slab, the red colour emerge first.
- 16 (c) On reflection from a rarer to denser medium, a phase difference of π occurs.
- 17 (a) The materials of negative refractive index are those in which incident ray from air (medium 1) bend differently as compare to medium of positive refractive index.
- 18 (b) $|m| = \frac{f_o}{f_e}$
- When f_e is halved, so $m' = \frac{2 \times f_o}{f_e} = 2m$
- 20 (c) All rays coming from object at centre are normal. So, no bending will take place. Hence, virtual image of point object will form at the centre of glass sphere. So, the distance of the virtual image from the surface of sphere is 6 cm.
- 21 (d) Magnification, $m = \pm 3$

We have, $m = \frac{f}{f + u}$

For virtual image, $3 = \frac{f}{f - 8}$... (i)

For real image, $-3 = \frac{f}{f - 16}$... (ii)

Solving Eqs. (i) and (ii), we get
 $f = 12 \text{ cm}$

22 (b) As, $\frac{c}{v} = \frac{\sin i}{\sin r}$

$\Rightarrow v = \frac{c \times \sin r}{\sin i} = \frac{3 \times 10^8 \times \sin 30^\circ}{\sin 45^\circ} = 2.12 \times 10^8 \text{ m/s}$

24 (b) The resolving power of telescope = $\frac{a}{1.22 \lambda}$

where, a = aperture of telescope.

$\therefore$ Resolving power $\propto$ Aperture of telescope

If aperture of telescope is decreased, then the resolving power will decrease.

- 25 (c) For $A > 2C$, the ray of light does not emerge from the opposite face of the prism.

- 26 (c) If refractive index of lens n_1 is equal to refractive index of liquid n_2 , then lens behaves as a plane glass plate and becomes invisible in the medium.

- 27 (b) Molecules of a medium after absorbing incoming light radiations, emit them in all directions. This phenomenon is called scattering. According to scientist Rayleigh, the intensity of scattered light $\propto \frac{1}{\lambda^4} \propto \nu^4$

where, ν is the frequency.

- 29 (d) Given, wavelength in vacuum = $5890 \text{ \AA}$

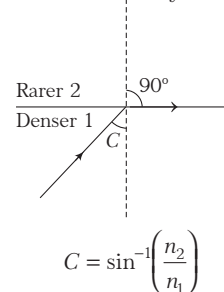
In a medium of refractive index, the wavelength of light decreases to $\frac{\lambda}{\mu} = \frac{5890}{1.5} \approx 3927 \text{ \AA}$.

- 30 (d) Resultant focal length of combination,

$$F_{\text{comb}} = \frac{f_1 f_2}{f_1 + f_2} = \frac{f(-f)}{f + (-f)} = \infty$$

- 31 (a) When a ray of light passes from a denser medium to a rarer medium, then it bends away from the normal at the interface of the two media.

The angle of incidence is measured with respect to the normal at the refractive boundary. It is given by



where, C is critical angle, n_2 is the refractive index of rarer medium and n_1 of the denser medium.

Given, $n_2 = 1.33, n_1 = 1.50$

$$C = \sin^{-1}\left(\frac{1.33}{1.50}\right)$$

$$\Rightarrow C = \sin^{-1}\left(\frac{8}{9}\right)$$

32 (c) Here, $\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$

$$\frac{1}{f} = (1.6 - 1) \left[\frac{1}{60} - \frac{1}{\infty} \right] = \frac{1}{100} \Rightarrow f = 100 \text{ cm}$$

33 (c) Real depth = 1 m

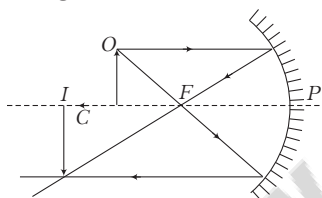
Apparent depth = $1 - 0.1 = 0.9 \text{ m}$

$$\text{Refractive index, } \mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{0.9} = \frac{10}{9}$$

34 (d) Under minimum deviation, $i = e = 30^\circ$, so angle between emergent ray and second refracting surface is $90^\circ - 30^\circ = 60^\circ$.

35 (b) In any medium other than air or vacuum, the velocities of different colours of light are different. Therefore, both red and green colours are refracted at different angles of refractions. Hence, after emerging from glass slab through opposite parallel face, they appear at two different points and move in two different parallel directions.

36 (a) When object is placed between centre of curvature and focus (i.e., $f < u < 2f$), then image will be between $2f$ and ∞ , real, inverted, large in size and $m > -1$



37 (a) The size of the object in accordance with the Fresnel's displacement law is given by $\sqrt{x_1 x_2}$.

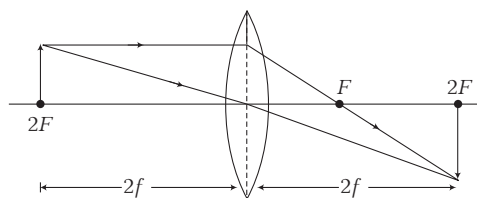
38 (b) The PQ ray of light passes through focus F and incident on the concave mirror, after reflection the ray should become parallel to the principal axis and shown by ray 2 in the figure.

39 (d) Here, angle of prism, $A = 60^\circ$

For minimum deviation, $A = 2r$

$$\text{or } r = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ \text{ for both colours.}$$

40 (c) For real image, object should be placed at distance $2f$ for minimum distance between object and its real image.



41 (c) The magnifying power of the telescope in normal adjustment,

$$m = \frac{f_o}{f_e} = \frac{100}{5} = 20$$

42 (b) We have, $n_a v_a = n_g v_g$

$$\frac{n_g}{n_a} = \frac{v_a}{v_g}$$

$$\frac{3}{2} = \frac{3 \times 10^8}{v_g}$$

$$v_g = 2 \times 10^8$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$t = \frac{4 \times 10^{-3}}{2 \times 10^8}$$

$$t = 2 \times 10^{-11} \text{ s}$$

43 (b) In going from one medium to another, frequency remains same but wavelength and velocity decrease with the increase in refractive index i.e., wavelength becomes $\lambda' = \lambda / \mu$ and velocity becomes,

$$c = \frac{v}{\mu}$$

44 (a) For combination of one convex and one concave lens

$$\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2}$$

$$\Rightarrow F = \frac{f_1 f_2}{f_2 - f_1}$$

If $f_1 > f_2$, then F will be negative.

45 (d) We have,

$$C = \sin^{-1}\left(\frac{v_1}{v_2}\right) = \sin^{-1}\left(\frac{1.8 \times 10^8}{2.4 \times 10^8}\right) = \sin^{-1}\left(\frac{3}{4}\right)$$

46 (a) We have, $f_y = \sqrt{f_R f_b}$

$$f_y = \sqrt{100.5 \times 99.5} = 99.99$$

$$\text{Also, } \omega = \frac{f_R - f_b}{f_y} = \frac{100.5 - 99.5}{99.99}$$

$$\text{Dispersive power, } \omega = \frac{1}{99.99} = 0.01$$

47 (b) The number of images formed depends upon the angle between the mirrors. If two mirrors make an angle θ with each other, then the number of images formed is

$$n = \frac{360^\circ}{\theta} - 1$$

When mirrors are kept mutually perpendicular to each other, then $\theta = 90^\circ$.

$$n = \frac{360}{90} - 1 = 3$$

48 (c) As, $n = \frac{360^\circ}{60^\circ} = 6$

Since, n is an even integer, so total number of images will be $(n - 1)$ or 5.

49 (c) The magnifying power (m) of a telescope, $m = -\frac{f_o}{f_e}$. Now, if the focal length of the eyepiece (f_e) is doubled, then the new magnification would be equal to $m' = -\frac{f_o}{2f_e} = \frac{m}{2}$.

50 (a) The minimum angular separation between two objects, so that they are just resolved is called resolving limit. For eye, it is $\theta' = \left[\frac{1}{60} \right]^\circ$.

Reciprocal of resolving limit is called resolving power,

$$\begin{aligned} (\text{RP}) &= \frac{1}{\left(\frac{1}{60} \right)^\circ} = \frac{1}{\frac{1}{60} \times \frac{\pi}{180}} \\ &= \frac{10800}{\pi} = 3436 \times 10^3 \text{ rad} \end{aligned}$$

51 (b) Resolving power of microscope $= \frac{2\mu \sin \theta}{\lambda}$

Since, $\lambda_R > \lambda_B$, hence resolving power increases.

52 (a) Resolving limit of telescope

$$= \frac{1.22\lambda}{a} = \frac{1.22 \times (5000 \times 10^{-10})}{0.1} = 6.1 \times 10^{-6} \text{ rad}$$

53 (b) In an astronomical telescope, $L = f_o + f_e$

When tube length is decreased, then the image formed by the objective lens will lie between principal focus and optical centre of eye lens instead of lying at the focus of eye lens. Therefore, a virtual image will be formed at a finite distance from the eye lens.

54 (d) In this case, $|m| = \frac{f_o}{f_e} = 5$... (i)

and length of telescope $= f_o + f_e = 36$... (ii)

On solving Eqs. (i) and (ii), we get

$$f_e = 6 \text{ cm}, f_o = 30 \text{ cm}$$

55 (b) Magnification, $|M| = \frac{f_o}{f_e} = 10$ or $f_o = 10 f_e$

Given,

$$\begin{aligned} f_e + f_o &= 1.1 \text{ m} \\ f_e + 10 f_e &= 1.1 \times 100 \text{ cm} \\ 11 f_e &= 110 \\ f_e &= 10 \end{aligned}$$

Magnification at least distance of distinct vision,

$$\begin{aligned} M_D &= \frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right) = 10 \left(1 + \frac{10}{25} \right) \\ &= 10 \left(\frac{35}{25} \right) = 14 \end{aligned}$$

56 (d) Given, $m = +\frac{1}{2}$, $f = 2 \text{ m} = 200 \text{ cm}$, $u = ?$

$$\therefore \text{Magnification, } m = \frac{f}{(f - u)}$$

$$\Rightarrow +\frac{1}{2} = \frac{200}{(200 - u)}$$

$$\Rightarrow 200 - u = 400$$

$$\Rightarrow u = -200 \text{ cm}$$

$$\text{or } u = -2 \text{ m}$$

57 (c) Here, $R = 20 \text{ cm}$, $\mu = 1.5$, on substituting the values in

$$f = \frac{R}{\mu - 1} = \frac{20}{1.5 - 1} = 40 \text{ cm of converging nature as } f > 0.$$

Therefore, lens act as a convex lens irrespective of the side on which the object lies.

58 (c) We have, $\frac{1}{\sin C} = \frac{\lambda_1}{\lambda_2}$

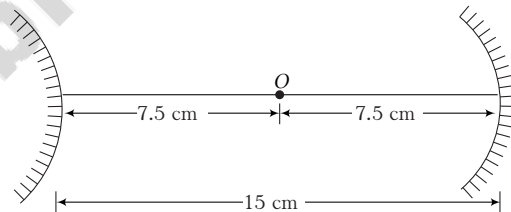
$$\Rightarrow \frac{1}{\sin C} = \frac{6000}{3000}$$

$$\Rightarrow \frac{1}{\sin C} = \frac{6}{3} = 2$$

$$\Rightarrow \sin C = \frac{1}{2}$$

$$\Rightarrow \text{Critical angle between two medium, } C = \sin^{-1} \left(\frac{1}{2} \right)$$

59 (c) For concave mirror,



$$\text{Using the relation, } \frac{1}{v} - \frac{1}{u} = -\frac{1}{f}$$

$$\therefore v = -15 \text{ cm}$$

Therefore position of final Image will be at the pole of the convex mirror.

60 (d) Length of the telescope when final image is formed at least distance of distinct vision,

$$\begin{aligned} L &= f_o + u_e = f_o + \frac{f_e D}{f_e + D} \\ &= 50 + \frac{5 \times 25}{(5 + 25)} = \frac{325}{6} \text{ cm} \end{aligned}$$

61 (c) We have, $\frac{I}{O} = \frac{f}{(f - u)}$

$$\Rightarrow \frac{I}{5} = \frac{-10}{-10 - (-100)}$$

$$\Rightarrow I = -0.55$$

Size of the image, $I = 0.55 \text{ cm}$

62 (a) We have, the effective power given by

$$P_{\text{eff}} = P_1 + P_2$$

$$\text{Also, we know } P = \frac{1}{f}$$

$$\text{Hence, } P_{\text{eff}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.20} + \frac{1}{0.25} = 5 + 4 = 9 \text{ D}$$

- 63** (c) Hypermetropia can be removed by using a convex lens.

Focal length of used lens, $f = +d = +40$ cm
 $= +(\text{defected near point})$

$$\therefore \text{Power of lens} = \frac{100}{f(\text{cm})} = \frac{100}{40} = 2.5 \text{ D}$$

- 64** (c) We have, $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$$\frac{1}{30} = (\mu - 1) \left(\frac{1}{10} \right)$$

The refractive index of the material of the lens,

$$\mu = \frac{4}{3} = 1.33$$

- 65** (c) Refractive index of the material of prism,

$$\mu = \frac{\sin \left(\frac{A + \delta_m}{2} \right)}{\sin \left(\frac{A}{2} \right)}$$

$$\Rightarrow \sqrt{3} = \frac{\sin \left(\frac{60^\circ + \delta_m}{2} \right)}{\sin \left(\frac{60^\circ}{2} \right)}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \sin \left(30^\circ + \frac{\delta_m}{2} \right)$$

$\therefore$ Minimum deviation of the prism, $\delta_m = 60^\circ$

- 66** (a) As, $A + \delta = i + e$... (i)

$$\Rightarrow 30^\circ + 30^\circ = 60^\circ + e$$

Angle of emergence, $e = 60^\circ - 60^\circ = 0$

$$r_1 = A = 30^\circ$$

$$\therefore \text{Refractive index, } \mu = \frac{\sin i}{\sin r} = \frac{\sin 60^\circ}{\sin 30^\circ} = \sqrt{3} = 1.732$$

- 67** (b) Since, rays after passing through the glass slab just suffer lateral displacement. So, angle between emergent ray is also α .

- 68** (b) We have, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\therefore \frac{1}{30} - \frac{1}{(-u)} = \frac{1}{f} \text{ or } \frac{1}{30} + \frac{1}{u} = \frac{1}{f} \quad \dots (i)$$

$$\text{Similarly, } \frac{1}{120} + \frac{1}{u-90} = \frac{1}{f} \quad \dots (ii)$$

On solving Eqs. (i) and (ii), we get

The focal length of the lens, $f = 24$ cm

- 69** (a) Given, $f_o + f_e = 105$

$$f_o + \frac{f_o}{20} = 105$$

$$\Rightarrow f_o = 100 \text{ cm}$$

- 70** (b) We have, $\frac{1}{f} = (1.5 - 1) \left(\frac{1}{10} + \frac{1}{10} \right)$

$\therefore$ Focal length, $f = 10$ cm

Radius of curvature, $R = 2f = 20$ cm

- 71** (d) The focal length of the lens,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{20} - \frac{1}{30} \right)$$

$$\Rightarrow \frac{1}{f} = 0.5 \left(\frac{30 - 20}{600} \right)$$

$$\text{or } \frac{1}{f} = \frac{1}{2} \times \frac{10}{600} = \frac{1}{120}$$

or focal length, $f = 120$ cm

- 72** (b) Power of the lens = $\frac{1}{\text{Focal length}}$

Focal length of combination of convex and concave lenses is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

where, f_1 and f_2 be the focal lengths of convex and concave lenses, respectively.

$$\text{Now, } \frac{1}{F} = \frac{1}{0.4} + \frac{1}{(-0.25)}$$

$$\frac{1}{F} = -1.5$$

$$\text{Power, } P = -1.5 \text{ D} \quad \left(\because P = \frac{1}{F} \right)$$

- 73** (a) Power of a biconvex lens is 10 D
 and radius of curvature of each surface = 10 cm.

$$\Rightarrow P = \frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\Rightarrow 10 = (\mu - 1) \left[\frac{100}{10} - \left(-\frac{100}{10} \right) \right]$$

$$\Rightarrow 10 = (\mu - 1) [20]$$

$$\Rightarrow (\mu - 1) = \frac{1}{2}$$

$$\Rightarrow \text{Refractive index of the material, } \mu = \frac{3}{2}$$

- 74** (d) Focal length of the combination,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \frac{1}{80} = \frac{1}{20} + \frac{1}{f_2} \text{ or } f_2 = -\frac{80}{3} \text{ cm}$$

$$\therefore \text{Power of second lens, } P' = \frac{100}{f_2} = \frac{100}{(-80/3)} = -3.75 \text{ D}$$

- 75** (c) The maximum deviation can be, when it is reflected back from boundary.

$$i = \theta_c = 45^\circ$$

$$\therefore \text{Deviation, } \delta = 180^\circ - 2i = 90^\circ$$

- 76** (a) When plano-convex lens is silvered at curved surface, then it becomes concave mirror of focal length,

$$f = \frac{R}{2\mu} = \frac{30}{2 \times 1.5} = 10 \text{ cm}$$

If we have to find real image of same size of the object, then object must be placed at centre of curvature $u = 2f = 20 \text{ cm}$.

77 (c) From the formula, $\sin C = \frac{1}{{}_1\mu_2} \Rightarrow \sin C = {}_2\mu_1$

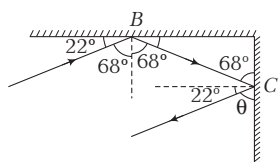
$$\therefore \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} \Rightarrow \sin C = \frac{10x/t_2}{x/t_1}$$

$$\sin C = \frac{10t_1}{t_2}$$

$$C = \sin^{-1}\left(\frac{10t_1}{t_2}\right)$$

78 (b) From geometry of the figure,

$$\text{Angle, } \theta = 90^\circ - 22^\circ = 68^\circ$$

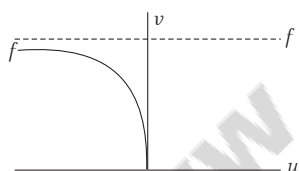


79 (c) Magnifying power, $m_\infty = \frac{(L_\infty - f_o - f_e) \cdot D}{f_o f_e}$

$$\Rightarrow 45 = \frac{(L_\infty - 1 - 5) \times 25}{1 \times 5}$$

$$\Rightarrow \text{The length of the tube, } L_\infty = 15 \text{ cm}$$

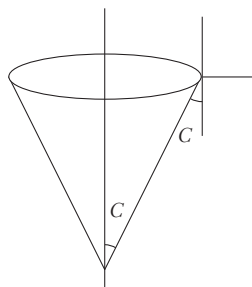
80 (a) u changes from 0 to $-\infty$, then v will change from 0 to $+f$.



81 (a) Power, $P = 2P_L + P_M$ or $-\frac{1}{F} = 2\left(-\frac{1}{20}\right) + 0$ or $F = 10 \text{ cm}$

Therefore, the combination behaves as a convex mirror of focal length 10 cm.

82 (c) We can represent this diagrammatically as



$$\text{Thus, we see that } C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{3}{4}\right)$$

83 (b) Here, light ray initially goes from (optically) rarer medium (air) to optically denser medium (turpentine), so it bends towards the normal i.e., $i > r$ whereas when it goes from optically denser medium (turpentine) to rarer medium (water), then it bends away the normal i.e., $i < r$. This has been correctly depicted by ray 2.

84 (d) The focal length of the plano-convex lens is 20 cm with the plane surface being silvered.

$$\text{So, the net power of the lens is given by } P = \frac{1}{f} \times 2 = \frac{2}{f}$$

$$\text{Thus, the net focal length, } \frac{f}{2} = \frac{20}{2} = 10 \text{ cm}$$

85 (a) Since, deviation, $\delta = (\mu - 1)A = (1.5 - 1) \times 5^\circ = 2.5^\circ$

By geometry, angle of refraction by first surface is 5° .

But $\delta = \theta - r$, so we have $2.5^\circ = \theta - 5^\circ$

On solving, $\theta = 7.5^\circ$

86 (d) Given, $u = mf$

$$\therefore \frac{1}{v} - \frac{1}{(-mf)} = -\frac{1}{f} \Rightarrow \frac{1}{v} = -\frac{1}{f} \left(1 + \frac{1}{m}\right)$$

$$\Rightarrow \frac{1}{v} = \frac{m+1}{-mf} \Rightarrow -\frac{v}{u} = \frac{1}{1+m}$$

87 (c) According to lens Maker's formula,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{(-R)} \right)$$

$$\Rightarrow \frac{1}{f} = (\mu - 1) \left(\frac{2}{R} \right) \quad \dots(i)$$

$$\text{On cutting, } \frac{1}{f'} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right)$$

$$\frac{1}{f'} = (\mu - 1) \left(\frac{1}{R} \right) \quad \dots(ii)$$

From Eqs. (i) and (ii), we get $f' = 2f$.

88 (b) As optical paths are equal, hence

$$n_g x_g = n_w x_w$$

$$n_w = n_g \frac{x_g}{x_w}$$

Refractive index of the water,

$$n_w = 1.53 \times \frac{4.0}{4.5} = 1.36$$

89 (b) For total internal reflection, $i > C$

$$\Rightarrow \sin i > \sin C \Rightarrow \sin 45^\circ > \frac{1}{\mu}$$

$$\Rightarrow \mu > \sqrt{2} \Rightarrow \mu > 1.414$$

90 (b) From Snell's law,

$$\mu \sin i = \text{constant}$$

$$\mu_4 \sin x = \mu \sin i$$

$$\therefore \sin x = \frac{\mu}{\mu_4} \sin i$$

- 91 (b) As, incident ray and reflected ray are parallel to each other it means, deviation, $\delta = 180^\circ$

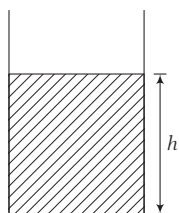
$$\therefore \delta = 360^\circ - 2\theta$$

$$\therefore 180^\circ = 360^\circ - 2\theta$$

Angle between two plane mirrors,

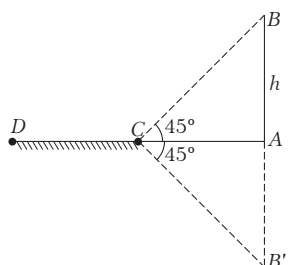
$$\theta = 90^\circ$$

- 92 (d) According to the question, $\frac{h}{\mu} = \frac{21}{2} = 10.5$



$$\therefore \text{Height, } h = 10.5 \times \frac{4}{3} \Rightarrow h = 14 \text{ cm}$$

- 93 (b) Given, $AC = 60 \text{ m}$ and $\angle BCB' = 90^\circ$



From the figure,

$$\therefore \text{Angle, } \angle BCA = \frac{\angle BCB'}{2} = 45^\circ$$

$$\text{We have, } \tan 45^\circ = \frac{h}{AC}$$

$$\Rightarrow AC = h \Rightarrow h = 60 \text{ m}$$

- 94 (d) At second surface, there is no refraction, so

$$r_2 = 0$$

$$\therefore r_1 = A = 30^\circ$$

From Snell's law,

$$n = \frac{\sin i_1}{\sin r_1}$$

$$\Rightarrow n = \frac{\sin i_1}{\sin 30^\circ}$$

$$\Rightarrow \sin i_1 = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin i_1 = \frac{1}{\sqrt{2}} \Rightarrow i_1 = 45^\circ$$

- 95 (c) Given, $P_1 + P_2 = 9$

...(i)

$$\text{We have, } P = P_1 + P_2 - dP_1P_2$$

$$\Rightarrow \frac{27}{5} = 9 - \frac{20}{100} \times P_1P_2$$

$$\text{Again } (P_2 - P_1)^2 = (P_1 + P_2)^2 - 4P_1P_2$$

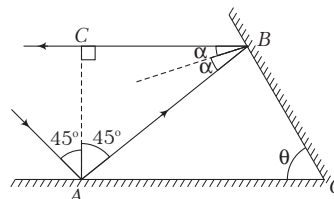
$$\Rightarrow P_2 - P_1 = 3$$

...(ii)

On solving Eqs. (i) and (ii), we get

$$P_1 = 3 \text{ and } P_2 = 6$$

- 96 (c) From the figure in $\triangle ABC$, $45^\circ + 90^\circ + 2\alpha = 180^\circ$



or

$$\alpha = 22.5^\circ$$

$$\angle OBA = 90^\circ - \alpha$$

$$\text{In } \triangle OAB, \theta + 45^\circ + 90^\circ - \alpha = 180^\circ$$

$$\Rightarrow \theta = 67.5^\circ$$

$$\begin{aligned} 97 \text{ (a) Power of system} &= \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \\ &= \frac{1}{1} + \left(\frac{1}{-0.25} \right) - \frac{0.75}{(1)(-0.25)} \\ &= 1 - 4 + 3 = -3 + 3 = 0 \end{aligned}$$

Since, power of the system is zero, therefore the incident parallel beam of light will remain parallel after emerging from the system.

- 98 (c) Angle of incidence, $i = \theta$

Angle of equilateral prism,

$$A = 60^\circ$$

$$\mu = \sqrt{3}$$

In the case of minimum deviation, the refracted ray inside the prism is parallel to the base.

$$\text{Hence, } r = \frac{A}{2} = \frac{60}{2} = 30^\circ$$

$$\therefore \mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \mu = \frac{\sin \theta}{\sin r}$$

$$\Rightarrow \sqrt{3} = \frac{\sin \theta}{\sin 30^\circ}$$

$$\Rightarrow \sin \theta = \sqrt{3} \sin 30^\circ$$

$$\Rightarrow \sin \theta = \sqrt{3} \times \frac{1}{2}$$

$$\Rightarrow \sin \theta = \sin 60^\circ$$

$$\Rightarrow \theta = 60^\circ$$

- 99 (c) For dispersion without deviation,

$$\frac{A}{A'} = \frac{(\mu' - 1)}{(\mu - 1)}$$

$$\frac{6^\circ}{A'} = \frac{0.75}{0.50}$$

$$A' = \frac{0.50 \times 6^\circ}{0.75}$$

$$A' = 4^\circ$$

- 100 (a) Here, $n = 1.5$, as per sign convention followed

$$R_1 = +20 \text{ cm and } R_2 = -20 \text{ cm}$$

$$\therefore \frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$= (1.5-1) \left[\frac{1}{(+20)} - \frac{1}{(-20)} \right] = 0.5 \times \frac{2}{20} = \frac{1}{20}$$

$$\Rightarrow f = +20 \text{ cm}$$

Incident rays travelling parallel to the axis of lens will converge at its second principal focus, hence

$$L = +20 \text{ cm}$$

- 101 (d) By Rayleigh's law of scattering,

$$I \propto \frac{1}{\lambda^4} \text{ or } \frac{I_2}{I_1} = \left(\frac{\lambda_1}{\lambda_2} \right)^4$$

$$\frac{I_2}{A} = \left(\frac{440}{660} \right)^4 \text{ or } I_2 \cong \frac{A}{6}$$

- 102 (c) Light should fall normally on the silvered face, i.e.

$$r_2 = 0$$

$$\therefore r_1 = A = 30^\circ$$

Now, refractive index, $\mu = \frac{\sin i_1}{\sin r_1}$ or $\sqrt{2} = \frac{\sin i_1}{\sin 30^\circ}$

Angle of incidence, $i_1 = 45^\circ$

- 103 (c) For objective, $\frac{1}{v_o} - \frac{1}{-200} = \frac{1}{50}$

$$\therefore v_o = \frac{200}{3} \text{ cm}$$

$$\text{For eyepiece, } \frac{1}{-25} - \frac{1}{(-u_e)} = \frac{1}{5}$$

$$\therefore u_e = \frac{25}{6}$$

Therefore, separation between objective and eye-piece,

$$L = v_o + u_e = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 71 \text{ cm}$$

- 104 (d) Critical angle, $\theta_C = \sin^{-1} \left(\frac{1}{1.5} \right) = 41.8^\circ$

Incident angle, $i_1 = 30^\circ$

$$\therefore \sin r_1 = \frac{\sin i_1}{\mu} = \frac{\sin(30^\circ)}{1.5}$$

$$r_1 = 19.5^\circ$$

or angle of emergence, $r_2 = A - r_1 = 90^\circ - 19.5^\circ = 70.5^\circ$

Since, $r_2 > \theta_C$

$\therefore$ The ray will not emerge from the prism.

- 105 (c) We have, $\delta = i_1 + i_2 - A$

$$45^\circ = (i_1 + i_2) - 60^\circ$$

$$\therefore i_1 + i_2 = 105^\circ \quad \dots(i)$$

$$\text{and } i_1 - i_2 = 20^\circ \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$i_1 = 62.5^\circ \text{ and } i_2 = 42.5^\circ$$

- 106 (d) For terrestrial telescope, magnification,

$$M = \frac{f_o}{f_e} = \frac{80}{f_e} = 20$$

$$\Rightarrow f_e = 4 \text{ cm}$$

Hence, length of terrestrial telescope

$$= f_o + f_e + 4f$$

$$= 80 + 4 + 4 \times 20$$

$$= 164 \text{ cm}$$

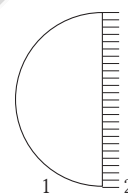
- 107 (c) We assume that, final image is formed at infinity and

$$M_\infty = \frac{(L_\infty - f_o - f_e) \cdot D}{f_o f_e} = \frac{LD}{f_o f_e}$$

$$\text{or } 400 = \frac{20 \times 25}{0.5 \times f_e}$$

$$\therefore f_e = 2.5 \text{ cm}$$

- 108 (c) We have, $\frac{1}{f} = \frac{2\mu}{R_2} - \frac{2(\mu-1)}{R_1}$



$$\frac{1}{-10} = \frac{2 \times 1.5}{\infty} - \frac{2(1.5-1)}{R}$$

$$\therefore R = 10 \text{ cm}$$

- 109 (c) The separation between the images formed by extreme wavelengths of visible range is called the longitudinal chromatic aberration, given by

$$f_1 - f_2 = \omega \times f$$

where, ω is dispersive power.

Given, $\omega = 0.02$, $f = 20 \text{ cm}$

$$\therefore f_1 - f_2 = 0.02 \times 20 = 0.40$$

- 110 (a) Here, $x = u + v \quad \dots(i)$

$$\text{Magnification, } m = \frac{f}{f-u} = \frac{f-v}{f}$$

Image is real, magnification is negative

$$-m = \frac{f}{f-u}$$

$$\Rightarrow u = \frac{(m+1)f}{m}$$

$$\text{From } -m = \frac{f-v}{f}$$

$$\Rightarrow v = (m+1)f$$

Putting the value of v in Eq. (i), we get

$$x = \frac{(m+1)}{m} f + (m+1)f$$

$$\Rightarrow f = \frac{mx}{(m+1)^2}$$

- 111 (a) Linear magnification,

$$m = \frac{I}{O} = \frac{f}{u-f} = \frac{-10}{-25+10}$$

$$m = \frac{10}{15} = \frac{2}{3}$$

$$\text{Areal magnification} = m^2 = \frac{4}{9}$$

$$\begin{aligned} \text{Area of image} &= \frac{4}{9} \times \text{Area of object} \\ &= \frac{4}{9} \times (3)^2 = 4 \text{ cm}^2 \end{aligned}$$

- 112 (d) As,
- $n = \frac{\text{Real depth}}{\text{Apparent depth}}$

$$\Rightarrow n = \frac{6}{4} = \frac{3}{2}$$

$$\text{Also, } \frac{n_1}{u} - \frac{n_2}{v} = \frac{n_1 - n_2}{R}$$

$$\Rightarrow \frac{1.5}{6} - \frac{4}{17} = \frac{1.5 - 1}{R}$$

$$\text{Radius of curvature, } R = 34 \text{ cm}$$

- 113 (a) We have,
- $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{v} - \frac{1}{20} = \frac{1}{-15}$$

$$v = -60 \text{ m}$$

$$\text{Image speed} = \left(\frac{v^2}{u^2} \right) (\text{Object speed}) = \left(\frac{60}{20} \right)^2 (5) = 45 \text{ m/s}$$

- 114 (a) From condition of achromatism,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$$

$$\Rightarrow \frac{5}{3} = \frac{-(-15)}{f_2}$$

$$\text{Focal length, } f_2 = 9 \text{ cm (convex)}$$

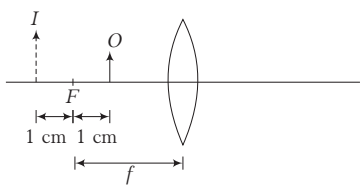
- 115 (b) Angle
- i
- at both faces will be
- 45°
- . For TIR to take place,

$$i > \theta_C \text{ or } \sin i > \sin \theta_C$$

$$\therefore \frac{1}{\sqrt{2}} > \frac{1}{\mu} \text{ or } \mu > \sqrt{2}$$

- 116 (b) As, per question,

$$\therefore \frac{1}{-(f+1)} - \frac{1}{-(f-1)} = \frac{1}{f}$$



$$\text{On solving, we get } f = (\sqrt{2} + 1) \text{ cm}$$

- 117 (c) We have,
- $\frac{\mu_2}{v} = \frac{\mu_1}{u} - \frac{\mu_2 - \mu_1}{R}$

$$\frac{1}{v} = \frac{1.5}{-6} - \frac{1 - 1.5}{-10}$$

$$\therefore v = -5 \text{ cm}$$

- 118 (c) Shift from the slab,
- $S = \left(1 - \frac{1}{\mu} \right) t = 2 \text{ cm}$

Let x be the distance between object and mirror, then

$$x - 2 = 40 \text{ or } x = 42 \text{ cm}$$

- 119 (d) Lens should be convex. Images should be magnified, if object at 16 cm length should lie between
- f
- and
- $2f$
- . Because corresponding to 6 cm (
- $< f$
-), length image should be virtual.

From the given options, if $f = 11 \text{ cm}$.

Then, 6 cm, $< f$ and $11 \text{ cm} < 16 \text{ cm} < 22 \text{ cm}$ or $f < 16 \text{ cm} < 2f$

- 120 (b) Using mirror formula,
- $\frac{1}{v} + \frac{1}{(-u)} = \frac{1}{-f}$

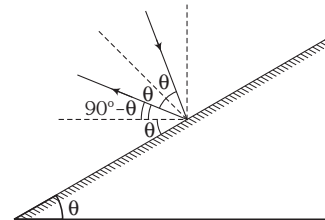
$$\text{Multiplying with } u, \frac{u}{v} = \frac{-u}{f} + 1 = \frac{f-u}{f}$$

$$\therefore \text{Magnification, } m = -\frac{v}{u} = \left(\frac{f}{f-u} \right)$$

$$\text{Now, } |dv| = m^2 |du|$$

$$\text{Size of image} = \left(\frac{f}{f-u} \right)^2 \cdot b = \left(\frac{f}{u-f} \right)^2 \cdot b$$

- 121 (d) We have,
- $90^\circ - \theta + 2\theta = 90^\circ$



$$\therefore \theta = 0^\circ$$

Angle of reflected ray with horizontal $= 90^\circ - \theta = 90^\circ$

- 122 (d) When total internal reflection just takes place from lateral surface,
- $i = C$
- , i.e.
- $60^\circ = C$

$$\Rightarrow \sin 60^\circ = \sin C = \frac{1}{\mu} \Rightarrow \mu = \frac{2}{\sqrt{3}}$$

Time taken by light to traverse some distance in a medium,

$$t = \frac{\mu x}{c} = \frac{\frac{2}{\sqrt{3}} \times 10^3}{3 \times 10^8} = 3.85 \mu\text{s}$$

- 123 (c) When the lens is in air, we have

$$P_a = \frac{1}{f_a} = \frac{\mu_g - \mu_a}{\mu_a} \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

When the lens is in liquid, we have

$$P_l = \frac{1}{f_l} = \frac{\mu_g - \mu_l}{\mu_l} \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\text{Given, } P_a = 5, P_l = -1, \mu_a = 1 \text{ and } \mu_g = 1.5$$

$$\text{On solving, we get } \mu_l = \frac{5}{3}$$

124 (b) We have, $\frac{\mu_2 - \mu_1}{v_1} = \frac{\mu_2 - \mu_1}{R}$... (i)

and $\frac{\mu_3 - \mu_2}{f} = \frac{\mu_3 - \mu_2}{-R}$... (ii)

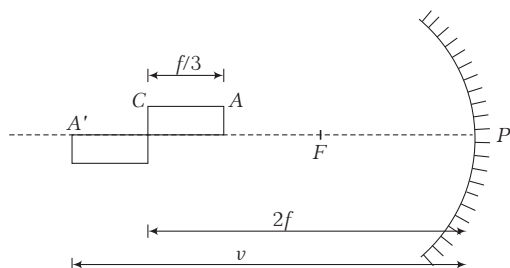
From Eqs. (i) and (ii), we get

$$\frac{\mu_3}{f} = \frac{\mu_2 - \mu_1 - \mu_3 + \mu_2}{R} = \frac{2\mu_2 - (\mu_1 + \mu_3)}{R}$$

Lens will become diverging, iff f is negative

i.e. $2\mu_2 < (\mu_1 + \mu_3)$

125 (b) For end A of the rod, $u = 2f - \frac{f}{3} = \frac{5f}{3}$

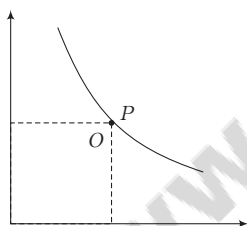


By using, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-f} = \frac{1}{v} + \frac{1}{(-5f/3)}$

$\therefore v = -\frac{5}{2}f$

So, length of image = $\frac{5}{2}f - 2f = \frac{f}{2}$

126 (c) At P, $u = v$ which is possible only when $u = 2f$



If we take another point Q just above P, then $v > 2f$.

127 (a) For real image,

$$u = -x$$

$$v = +mx$$

$\therefore \frac{1}{mx} - \frac{1}{-x} = \frac{1}{f}$... (i)

For virtual image, $u = -y$

Then, $v = -my$

$\therefore \frac{1}{-my} - \frac{1}{(-y)} = \frac{1}{f}$... (ii)

On solving Eqs. (i) and (ii), we get

$$f = \frac{x+y}{2}$$

128 (a) Biconvex lens is cut perpendicularly to the principal axis, it will become a plano-convex lens.

Focal length of biconvex lens,

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (n-1) \frac{2}{R} \quad (\because R_1 = R, R_2 = -R)$$

$$\Rightarrow f = \frac{R}{2(n-1)} \quad \dots (i)$$

For plano-convex lens,

$$\frac{1}{f_1} = (n-1) \left(\frac{1}{R} - \frac{1}{\infty} \right)$$

$$f_1 = \frac{R}{(n-1)} \quad \dots (ii)$$

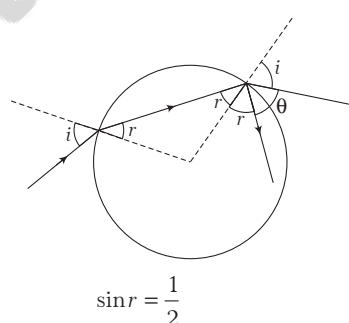
On comparing Eqs. (i) and (ii), we see the focal length become double.

As power of lens, $P \propto \frac{1}{\text{focal length}}$.

Hence, power will become half.

New power = $\frac{4}{2} = 2 \text{ D}$

129 (c) We have, refractive index, $\mu = \frac{\sin i}{\sin r} \Rightarrow \sqrt{3} = \frac{\sin 60^\circ}{\sin r}$



$$\sin r = \frac{1}{2}$$

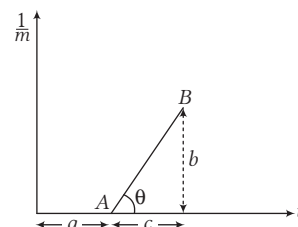
Refracted angle, $r = 30^\circ$

The angle between the reflected and refracted rays,

$$\theta = 180^\circ - i - r = 90^\circ$$

130 (d) Equation of straight line AB,

$$\frac{1}{m} = \tan \theta u - \frac{ba}{c}$$

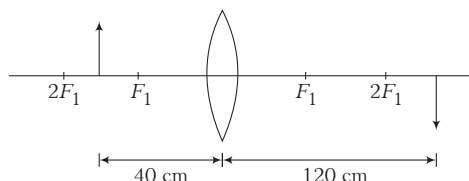


From, $\frac{1}{v} - \frac{1}{-u} = \frac{1}{+f}$

$$v = \frac{uf}{u-f} \quad \text{or} \quad \frac{v}{u} = m = \frac{f}{u-f} \Rightarrow \frac{1}{m} = \left(\frac{1}{f} \right) u - 1$$

Comparing this with equation of straight line, $1/f$ is the slope of line, which is b/c . So, the focal length of the lens used is $\frac{c}{b}$.

- 131 (a)** We have, $\frac{1}{3x} - \frac{1}{(-x)} = \frac{1}{+30}$
 $\therefore x = 40 \text{ cm}$ and $3x = 120 \text{ cm}$



To decrease the magnification, object should be moved towards $2F_1$.

Hence, image will move towards $2F_2$. Let displacement is y , then

$$2(40 + y) = (120 - y) \quad \left(\because m = \frac{v}{u} \right)$$

$$\therefore y = 28 \text{ cm}$$

- 132 (c)** Due to glass slab, increase in path
 $= (\mu - 1)t = (1.5 - 1)3 \text{ cm}$
 $= 1.5 \text{ cm}$

$$\therefore u' = -(21 + 1.5) \text{ cm}$$

$$\text{or } u' = -22.5 \text{ cm}$$

$$\text{Now, } \frac{1}{-22.5} + \frac{1}{v} = \frac{1}{-5}$$

$$\text{or } \frac{1}{v} = \frac{1}{22.5} - \frac{1}{5} \text{ or } \frac{1}{v} = \frac{5 - 22.5}{22.5 \times 5}$$

$$\text{or } v = \frac{22.5 \times 5}{17.5} \text{ or } v = -6.43 \text{ cm}$$

The position of the final image $= -(6.43 - 1.5) \text{ cm} = -4.93 \text{ cm}$

- 133 (a)** From the relation,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ or } \frac{1}{v} - \frac{1}{u} = \frac{1}{-f}$$

$$\frac{1}{v} = \frac{1}{u} - \frac{1}{f} \text{ or } v = \left(\frac{uf}{f - u} \right)$$

Since, $u > f$, v is negative

$$\text{or } |v| = \left(\frac{uf}{u - f} \right) > f$$

The end which is at infinity will have its image at focus.

$\therefore$ Length of image,

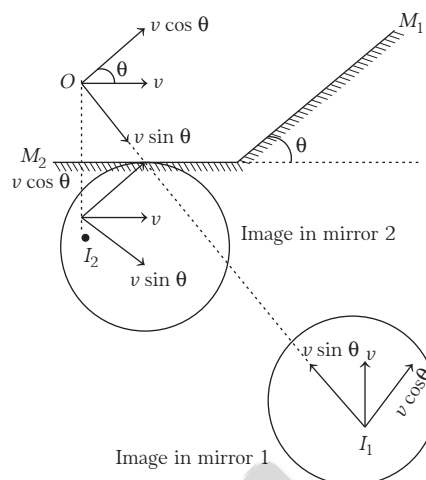
$$L = |v| - f = \frac{f^2}{u - f}$$

- 134 (c)** The reflected ray will rotate by angle 2θ . For TIR to take place at water-air boundary,

$$\sin 2\theta > \sin \theta_c \text{ or } \sin 2\theta > \frac{1}{\mu}$$

$$\therefore \theta > \frac{1}{2} \sin^{-1} \left(\frac{3}{4} \right)$$

- 135 (a)** Velocity of image, $v_r = \sqrt{v^2 + v^2 - 2v \cdot v \cdot \cos 2\theta}$
 $= 2v \sin(\theta)$



- 136 (b)** We know that, $\frac{1}{\mu} = \frac{\sin i}{\sin r'}$... (i)

$$\text{But } r + r' = 90^\circ \text{ or } r' = 90^\circ - r$$

$$\sin r' = \sin(90^\circ - r)$$

$$= \cos r$$

$$\text{or } \sin r' = \cos i \quad (\text{since, } i = r) \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\therefore \frac{1}{\mu} = \frac{\sin i}{\cos i} \text{ or } \frac{1}{\mu} = \tan i$$

$$\text{or } \sin i_c = \tan i = \tan r$$

The critical angle for pair of media is,

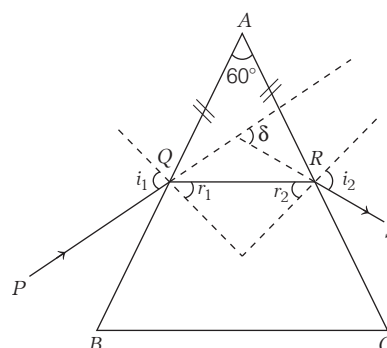
$$i_c = \sin^{-1}(\tan r)$$

- 137 (a)** $AQ = AR$

$$\Rightarrow \angle AQR = \angle ARQ = 60^\circ$$

$$\therefore r_1 = r_2 = 30^\circ$$

$$\sin i_1 = \mu \sin r_1$$



$$\Rightarrow \sin i_1 = \sqrt{3} \sin 30^\circ$$

$$= \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

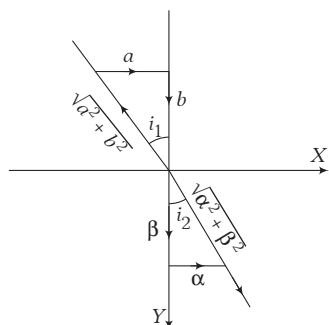
$$\Rightarrow i_1 = 60^\circ$$

$$\text{Similarly, } i_2 = 60^\circ$$

$$\text{In prism, deviation, } \delta = i_1 + i_2 - A$$

$$= 60^\circ + 60^\circ - 60^\circ = 60^\circ$$

- 138 (a) From the Snell's law, $\mu_1 \sin i_1 = \mu_2 \sin i_2$



$$\therefore \frac{\mu_1 a}{\sqrt{a^2 + b^2}} = \frac{\mu_2 \alpha}{\sqrt{\alpha^2 + \beta^2}} \left(\because \sin \theta = \frac{\text{Height}}{\text{Hypotenuse}} \right)$$

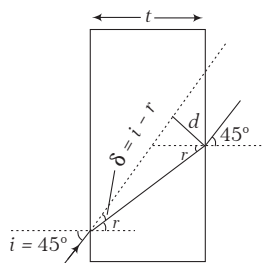
$$\left(\because r_A = a\hat{i} + b\hat{j} \text{ and } r_B = \alpha\hat{i} + \beta\hat{j} \text{ are unit vectors} \right)$$

$$\sqrt{a^2 + b^2} = \sqrt{\alpha^2 + \beta^2} = 1$$

$$\therefore \mu_1 a = \mu_2 \alpha$$

- 139 (b) Here, angle of incidence, $i = 45^\circ$

$$\frac{\text{Lateral shift (d)}}{\text{Thickness of glass slab (t)}} = \frac{1}{\sqrt{3}}$$



$$\text{Lateral shift, } d = \frac{t \sin \delta}{\cos r} = \frac{t \sin (i - r)}{\cos r} \Rightarrow \frac{d}{t} = \frac{\sin (i - r)}{\cos r}$$

$$\text{or } \frac{d}{t} = \frac{\sin i \cos r - \cos i \sin r}{\cos r}$$

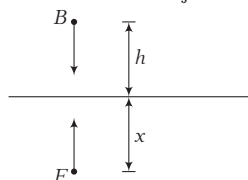
$$\text{or } \frac{d}{t} = \frac{\sin 45^\circ \cos r - \cos 45^\circ \sin r}{\cos r} = \frac{\cos r - \sin r}{\sqrt{2} \cos r}$$

$$\text{or } \frac{d}{t} = \frac{1}{\sqrt{2}} (1 - \tan r) \text{ or } \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{2}} (1 - \tan r)$$

$$\text{or } \tan r = 1 - \frac{\sqrt{2}}{\sqrt{3}}$$

$$\text{or Angle of refraction, } r = \tan^{-1} \left(1 - \frac{\sqrt{2}}{\sqrt{3}} \right)$$

- 140 (b) Fish is observer and bird is object.



Apparent distance between F and B at some instant will be

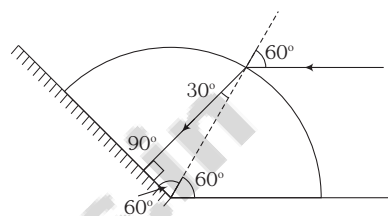
$$y = (x + \mu h)$$

$$\left(-\frac{dy}{dt} \right) = \left(-\frac{dx}{dt} \right) + (\mu) \left(-\frac{dh}{dt} \right)$$

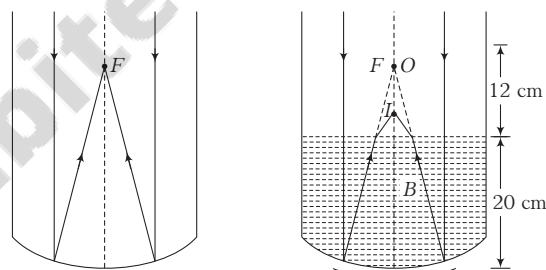
$$9 = 3 + \frac{4}{3} \left(-\frac{dh}{dt} \right)$$

$$\therefore -\frac{dh}{dt} = 4.5 \text{ m/s}$$

141 (d) Refractive index, $\mu = \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$



- 142 (b) In first case, when sun is at infinity, i.e.



$$\therefore \frac{1}{f} = \frac{1}{-32} + \frac{1}{(-\infty)}$$

$$\therefore f = -32 \text{ cm}$$

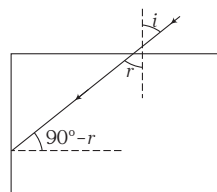
When water is filled in that tank upto a height of 20 cm, the image formed by the mirror will act as virtual object O for water surface.

$$\therefore BI = BO \times \frac{3}{4} = 12 \times \frac{3}{4} = 9 \text{ cm}$$

- 143 (a) TIR to take place,

$$90^\circ - r > \theta_c$$

$$\Rightarrow r < 90^\circ - \theta_c$$



From the relation,

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin i = \mu \sin r$$

Since, for TIR to take place, $r < 90^\circ - \theta_C$

$$\therefore i = \sin^{-1} \{\mu \sin r\}$$

$$i \leq \sin^{-1} [n \sin (90^\circ - \theta_C)]$$

or $i \leq \sin^{-1} \{\mu \cos \theta_C\}$

or $i \leq \sin^{-1} \left\{ \mu \sqrt{1 - \frac{1}{\mu^2}} \right\} \quad \left(\because \sin \theta_C = \frac{1}{\mu_C} \right)$

or $i \leq \sin^{-1} \left\{ 1.33 \sqrt{1 - \frac{1}{(1.33)^2}} \right\}$

or $i \leq \sin^{-1} \{\sqrt{0.77}\}$

Note For simplifying the calculation, take $1.33 = \frac{4}{3}$

144 (b) As, $y_a = \left(\frac{n_1}{n_2} \right) y$

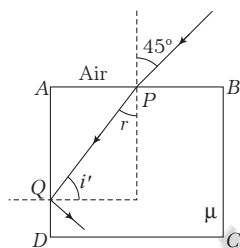
Here, y = actual depth and y_a = apparent depth.

$$\therefore y = \left(\frac{n_2}{n_1} \right) \cdot y_a \Rightarrow \left(-\frac{dy}{dt} \right) = \frac{n_2}{n_1} \left(-\frac{dy_a}{dt} \right) = \frac{n_2 x}{n_1}$$

The amount of water drained in cc/min

$$\therefore \frac{dv}{dt} = A \cdot \left(-\frac{dy}{dt} \right) = \frac{\pi R^2 n_2 x}{n_1}$$

145 (b) At point P,



Refractive index, $\mu = \frac{\sin 45^\circ}{\sin r} \Rightarrow \sin r = \frac{1}{\mu\sqrt{2}}$... (i)

At point Q, for total internal reflection, $\sin i' = \frac{1}{\mu}$

From figure, $i' = 90^\circ - r$

$$\therefore \sin(90^\circ - r) = \frac{1}{\mu}$$

$$\Rightarrow \cos r = \frac{1}{\mu} \quad \dots (ii)$$

Now, $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{1}{2\mu^2}} = \sqrt{\frac{2\mu^2 - 1}{2\mu^2}} \quad \dots (iii)$

From Eqs. (ii) and (iii), we get

$$\frac{1}{\mu} = \sqrt{\frac{2\mu^2 - 1}{2\mu^2}}$$

Refractive index, $\mu = \sqrt{\frac{3}{2}}$

This is the minimum value of refractive index for total internal reflection at vertical face AD.

(B) Medical entrance special format questions

• Assertion and reason

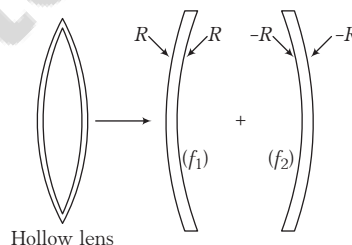
2 (a) In total internal reflection, 100% of incident light is reflected back into the same medium and there is no loss of intensity, while in reflection from mirrors and refraction from lenses, there is always some loss of intensity. Therefore, images formed by total internal reflection are much brighter than those formed by mirrors or lenses.

3 (d) For total internal reflection, the angle of incidence should be greater than the critical angle. As critical angle is 35° . Therefore, total internal reflection is not possible.

4 (c) Glass slab will shift the object towards mirror. If it comes between pole and focus, the image will become virtual.

5 (a) When a diverging lens is immersed in a liquid, then its diverging power will either decrease or it will become converging.

6 (a) As, $\frac{1}{f_1} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{R} \right) = 0$



$$\therefore P_1 = 0$$

$$\frac{1}{f_2} = (\mu - 1) \left(\frac{1}{-R} - \frac{1}{(-R)} \right) = 0$$

$$\therefore P_2 = 0$$

• Statement based questions

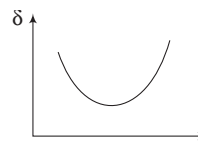
1 (b) For virtual object (u = positive),

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

If $u < f$, then $v = -ve$, i.e. image may be real.

3 (c) For minimum deviation,



and

$$r_1 = r_2$$

$\therefore$

$$A = r_1 + r_2$$

$\therefore$

$$r_1 = r_2 = \frac{A}{2}$$

- 4 (d) Only real image can be taken on screen. Object is real only when incident rays are diverging from it. Our eyes cannot distinguish between real and virtual image.
- 5 (c) According to VIBGYOR, among all given sources of light, the blue light have smallest wavelength. According to Cauchy relationship, smaller the wavelength higher the refractive index and consequently smaller the critical angle. So, corresponding to blue colour, the critical angle is least which facilitates total internal reflection for the beam of blue light. The beam of green light would also undergo total internal reflection.
- 6 (d) Here, $i_1 = 0 = r_1$
Therefore, $r_2 = A = 60^\circ$
 $\theta_C = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ$
 $\Rightarrow r_2 > \theta_C$
- 7 (d) The speed of the image of the car would appear to increase as the distance between the cars decreases.

• Match the columns

- 3 (a) If $|f_1| = |f_2|$, power of the system is zero.

$$\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{-f_2} = \frac{1}{f_1} - \frac{1}{f_2} \text{ or } P = P_1 - P_2$$

If $|f_1| > |f_2|$, power of system is negative

If $|f_1| < |f_2|$, power of system is positive.

Hence, A $\rightarrow$ p, B $\rightarrow$ s, C $\rightarrow$ r.

(C) Medical entrances' gallery

- 1 (b) Critical angle, $i_c = 45^\circ$

We know that,

$$\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 45^\circ} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$$

$$\Rightarrow \mu = \sqrt{2}$$

$$\Rightarrow \frac{\text{Velocity of light in air}}{\text{Velocity of light in medium}} = \sqrt{2}$$

$$\Rightarrow \frac{3 \times 10^8}{v_m} = \sqrt{2} \text{ or } v_m = \frac{3}{\sqrt{2}} \times 10^8 \text{ m/s}$$

- 2 (c) Given, $n = 1.5$, $h_o = 3 \text{ cm}$

$$\text{Refractive index, } n = \frac{h_o}{h_i}$$

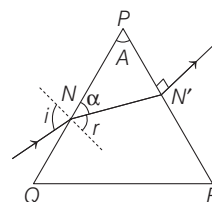
$$\Rightarrow 1.5 = \frac{3}{h_i}$$

$$\Rightarrow h_i = \frac{3}{1.5} = 2 \text{ cm}$$

Hence, microscope is moved $3 - 2 = 1 \text{ cm}$ upwards.

- 3 (b) Spherometer is used to measure the radius of curvature of an item such as a lens and curved mirrors that are spherical in shape.

- 4 (b) According to the question, the ray diagram can be shown as



From figure in $\triangle PNN'$,

$$A + \alpha + 90^\circ = 180^\circ$$

$$\Rightarrow \alpha = 90^\circ - A \quad \dots(i)$$

$$\text{Also, } r + \alpha = 90^\circ$$

$$\Rightarrow r = 90^\circ - \alpha = 90^\circ - (90^\circ - A) = A \quad [\because \text{from Eq. (i)}]$$

$$\Rightarrow r = A$$

From Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

$$\Rightarrow \sin i = \mu \sin A \quad (\text{here, } \mu_1 = 1 \text{ and } \mu_2 = \mu)$$

For small angle, $\sin \theta \approx \theta \Rightarrow i = \mu A$

- 5 (a) Given, object distance, $u = -1.5f$

By mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} + \frac{1}{-1.5f} = \frac{1}{-f}$$

$$\Rightarrow \frac{1}{v} = -\frac{1}{f} + \frac{1}{1.5f}$$

$$= \frac{1}{f} \left[-1 + \frac{1}{1.5} \right] = \frac{1}{f} \left[-1 + \frac{2}{3} \right]$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} \left(-\frac{1}{3} \right)$$

$$\text{or } v = -3f$$

- 6 (d) Power of biconvex lens,

$$P = 10 \text{ D}$$

$$\therefore f = \frac{1}{P} = \frac{1}{10} = 0.1 \text{ m} = 10 \text{ cm}$$

$$\text{Given, } R_1 = R_2 = 10 \text{ cm}$$

By lens Maker's formula,

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{10} = (\mu - 1) \left(\frac{1}{10} - \frac{1}{-10} \right)$$

$$\Rightarrow \frac{1}{10} = (\mu - 1) \left(\frac{2}{10} \right)$$

$$\Rightarrow 1 = (\mu - 1) \times 2$$

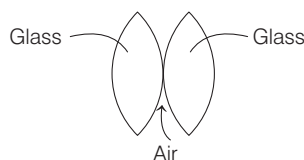
$$\Rightarrow \mu - 1 = \frac{1}{2} \text{ or } \mu = 1 + \frac{1}{2} = \frac{3}{2}$$

- 7 (b) The necessary conditions for the rainbow to take place are as follows

- Rainbow should take place at the opposite part of the sky as compared to the part where sun is shining.
 - The observer must stand with his back towards the sun.
- ∴ Statement in option (b) is incorrect.

However, rest statements regarding the rainbow are correct.

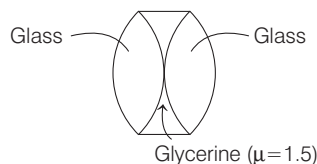
- 8 (a) **Case I** When two equi-convex lenses of focal lengths f_1 and f_2 respectively, are kept co-axially in contact, then the equivalent focal length of combination is



$$\frac{1}{F_1} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} + \frac{1}{f} \quad [\because \text{Here, } f_1 = f_2]$$

$$= \frac{2}{f} \Rightarrow F_1 = \frac{f}{2} \quad \dots (i)$$

Case II When glycerine of same refractive index as that of the glass is filled in the space between two lenses, then the combination will now comprises of three lenses; first bi-convex, second bi-concave and third is bi-convex. So, the focal length of the combination now is given as



$$\frac{1}{F_2} = \frac{1}{f} + \frac{1}{(-f)} + \frac{1}{f} = \frac{1}{f}$$

$$\Rightarrow F_2 = f \quad \dots (ii)$$

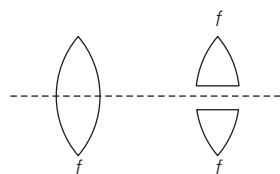
From Eqs. (i) and (ii), we get

$$F_1 : F_2 = 1 : 2$$

- 9 (d) Different colours of white light have different wavelengths. The descending order of the wavelength of the component of white light is

$$\lambda_{\text{red}} > \lambda_{\text{green}} > \lambda_{\text{blue}} > \lambda_{\text{violet}}$$

- 10 (d) When the lens is cut along its principal axis, the focal length of the two halves will remain same because the radius of curvature of both the surfaces are still same. So, the power also remains same as $P = \frac{1}{f}$



- 11 (c) Using the lens Maker's formula,

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Given, $f = 25 \text{ cm}$

$$\mu_2 = 1.5, \mu_1 = 1 (\text{for air})$$

let $R_1 = R$ and $R_2 = -2R$

$$\Rightarrow \frac{1}{25} = \left(\frac{1.5 - 1}{1} \right) \left[\frac{1}{R} - \left(-\frac{1}{2R} \right) \right]$$

$$\frac{1}{25 \times 0.5} = \frac{1}{R} + \frac{1}{2R}$$

$$\Rightarrow R = \frac{3}{2} \times 25 \times 0.5$$

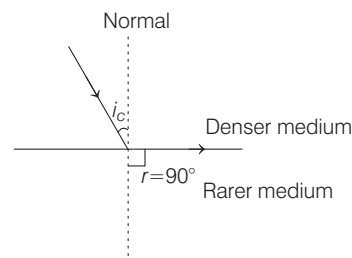
$$= 18.75 \text{ cm}$$

$$\therefore R_1 = 18.75 \text{ cm}$$

$$R_2 = 2 \times 18.75$$

$$= 37.5 \text{ cm}$$

- 12 (c) The total internal reflection is the phenomenon of reflecting back of light in the denser medium when it travels from denser to rarer medium, such that the angle of incidence is greater than the critical angle. While the critical angle for a pair of given media in contact is the angle of incidence in denser medium for which the angle of refraction in rarer medium is 90° .



- 13 (a) From the given figure in the question, we get

Distance of object, $u = -20 \text{ cm}$

Magnification, $m = -0.5$

Magnification of lens in terms of u and f is given by

$$m = \frac{f}{u + f}$$

$$\Rightarrow -0.5 = \frac{f}{-20 + f}$$

$$\Rightarrow 10 - 0.5f = f$$

$$\Rightarrow 15f = 10$$

$$f = \frac{10}{15}$$

or $f = 6.66 \text{ cm}$

- 14 (a) Given, focal length of objective lens,

$$f_o = 10 \text{ cm}$$

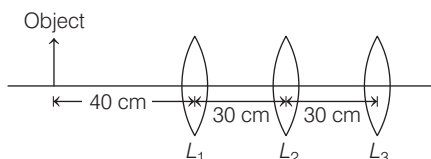
Focal length of eye lens, $f_e = 10 \text{ mm} = 1 \text{ cm}$

$$\begin{aligned}\text{Total length} &= f_o + f_e = (10 + 1) \text{ cm} \\ &= 11 \text{ cm}\end{aligned}$$

This is the case when the final image is formed at infinity, therefore angular magnification,

$$|m| = \left| \frac{f_o}{f_e} \right| = \frac{10}{1} = 10$$

15 (b)



(i) For refraction from 1st lens L_1 ,

$$\begin{aligned}\frac{1}{f} &= \frac{1}{v} - \frac{1}{u} \\ \Rightarrow \frac{1}{v} &= \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{40} = \frac{3}{40} \text{ cm} \\ \Rightarrow v &= \frac{40}{3} \text{ cm}\end{aligned}$$

This image acts as an object for 2nd lens.

$$\therefore \text{Object distance for 2nd lens } L_2 = 30 - \frac{40}{3} = \frac{50}{3} \text{ cm}$$

(ii) Now for refraction from L_2 ,

$$\begin{aligned}\frac{1}{v} - \frac{1}{u} &= \frac{1}{f} \\ \Rightarrow \frac{1}{v} &= \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{3}{50} = \frac{2}{50} \\ \Rightarrow v &= 25 \text{ cm}\end{aligned}$$

This image acts as an object for 3rd lens L_3 .

$$\begin{aligned}\therefore \text{Object distance for } L_3, \\ u &= 30 - 25 = 5 \text{ cm}\end{aligned}$$

(iii) Now for refraction from L_3 ,

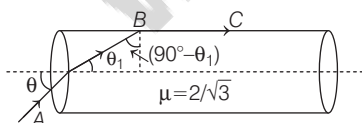
$$\Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{5}$$

$$\text{or } v = -10 \text{ cm}$$

(location of final image = 10 cm to left of L_3)

$$\therefore \text{Distance from } L_1 = 30 + (30 - 10) = 50 \text{ cm}$$

16 (b)



Since, BC grazes along surface.

$$\Rightarrow \text{Angle } (90^\circ - \theta_1) = \text{Critical angle } C$$

$$\therefore \sin C = \frac{1}{\mu}$$

$$\sin(90^\circ - \theta_1) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos \theta_1 = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta_1 = 30^\circ$$

Now, using Snells' law on 1st interface, $\mu_1 \sin \theta = \mu_2 \sin \theta_1$

$$\Rightarrow (1) \sin \theta = \frac{2}{\sqrt{3}} \sin 30^\circ = \frac{2}{\sqrt{3}} \cdot \frac{1}{2}$$

$$\Rightarrow \sin \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

17 (a) Refractive index of medium = $\frac{\text{Speed of light in vacuum}}{\text{Speed in medium}}$

$$\Rightarrow \text{Speed of light in medium} = \frac{\text{Speed of light in vacuum}}{\text{Refractive index}}$$

$$\begin{aligned}\Rightarrow v &= \frac{c}{n} = \frac{3 \times 10^8}{1.4} \\ &= 2.14 \times 10^8 \text{ m/s}\end{aligned}$$

18 (a) As, $v = \nu \lambda$, where ν is same for a source of light.

$$\Rightarrow v \propto \lambda \Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$\text{But } v_1 = \frac{c}{n_1} \text{ and } v_2 \text{ (in air)} = \frac{c}{n_2} = c \quad (\because n_2 = 1)$$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{c/n_1}{c} = \frac{1}{n_1} = \frac{1}{2} \quad (\because \text{given, } n_2 = 2)$$

19 (a) Given, $\mu_{\text{oil}} = 2$ and $\mu_{\text{glass}} = 3/2$

Velocity of light in oil is given by

$$v_{\text{oil}} = \frac{\text{Velocity of light } (c)}{\mu_{\text{oil}}} = \frac{c}{\mu_{\text{oil}}}$$

$$\text{Similarly, } v_{\text{glass}} = \frac{c}{\mu_{\text{glass}}}$$

$$\therefore \frac{v_{\text{glass}}}{v_{\text{oil}}} = \frac{c/\mu_{\text{glass}}}{c/\mu_{\text{oil}}} = \frac{\mu_{\text{oil}}}{\mu_{\text{glass}}} = \frac{2}{3/2} = \frac{4}{3}$$

20 (a) Angular magnification of an astronomical refracting telescope is given as

$$M = \frac{f_o}{f_e}$$

where, f_o and f_e are the focal length of objective and eyepiece, respectively.

From the given relation, it is clear that for large magnification either f_o has to be large or f_e has to be small.

Angular resolution of an astronomical refracting telescope is given as

$$R = \frac{a}{1.22\lambda}$$

where, a is the diameter of the objective.

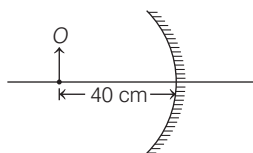
Thus, to have large resolution, the diameter of the objective should be large.

Hence, for the given condition in the question to be satisfied, objective lens should have large focal length (f_o) and large diameter (a).

- 21 (b) Case I** When the object distance,

$$u_1 = -40 \text{ cm}$$

Focal length of mirror, $f = -15 \text{ cm}$



Using the mirror formula, we get

$$\frac{1}{f} = \frac{1}{v_1} + \frac{1}{u_1}$$

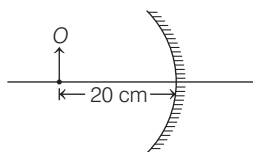
Substituting the given values, we get

$$-\frac{1}{15} = \frac{1}{v_1} + \left(\frac{-1}{40}\right)$$

$$\Rightarrow \frac{1}{v_1} = \frac{1}{40} - \frac{1}{15} = \frac{3-8}{120} = \frac{-5}{120}$$

$$\Rightarrow v_1 = \frac{-120}{5} = -24 \text{ cm}$$

Case II When the object distance, $u_2 = -20 \text{ cm}$



Using the mirror formula, we get

$$\frac{1}{f} = \frac{1}{v_2} + \frac{1}{u_2}$$

Substituting the given values, we get

$$-\frac{1}{15} = \frac{1}{v_2} + \left(\frac{-1}{20}\right)$$

$$\Rightarrow \frac{1}{v_2} = \frac{1}{20} - \frac{1}{15} = \frac{3-4}{60} = \frac{-1}{60}$$

$$\Rightarrow v_2 = -60 \text{ cm}$$

$\therefore$ The displacement of the image

$$= v_2 - v_1 = -60 - (-24) = -60 + 24 = -36 \text{ cm}$$

or $= 36 \text{ cm}$, away from the mirror

- 22 (a)** The relation between angle of deviation δ for a thin prism of angle of prism A and refractive index μ of material of prism is given by $\delta = (\mu - 1)A$. Thus, angle of deviation depends on the angle of prism.

- 23 (b)** For dispersion without deviation, net deviation produced by the combination of prisms must be zero.

Let prism angle of the first and second prisms are A_1 and A_2 , respectively. Similarly, their refractive indices are μ_1 and μ_2 .

Condition for dispersion without deviation is

$$\delta_1 - \delta_2 = 0 \Rightarrow (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 = 0$$

$$\Rightarrow A_2 = \left(\frac{\mu_1 - 1}{\mu_2 - 1}\right)A_1 = \left(\frac{1.42 - 1}{1.7 - 1}\right)(10^\circ)$$

$$\text{or } A_2 = 6^\circ$$

- 24 (a)** According to the question,

Here, μ = refractive index of lens,

μ' = refractive index of liquid
and f = focal length of lens in air.

We have, by lens Maker's formula

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

When lens is dipped in liquid,

$$\frac{1}{f'} = \left(\frac{\mu_g}{\mu_l} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$= \left(\frac{\mu_g - \mu_l}{\mu_l} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\therefore \frac{f'}{f} = \frac{(\mu - 1)\mu'}{(\mu - \mu')} = \frac{-(\mu - 1)\mu'}{(\mu' - \mu)} \quad (\because \mu_l = \mu' \text{ and } \mu_g = \mu)$$

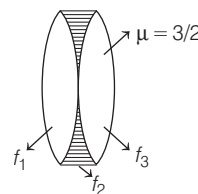
$$\text{or } f' = \frac{-f\mu'(\mu - 1)}{(\mu' - \mu)}$$

- 25 (d)** Consider the situation shown in figure.

Let radius of curvature of lens surfaces is R .

The combination is equivalent to three lenses in contact,

$$\therefore \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{2}{f_1} + \frac{1}{f_2} \quad (\because f_1 = f_3)$$



$$\text{Now, } \frac{1}{f_1} = \frac{1}{f_3} = (\mu - 1) \left(\frac{2}{R} \right) = \frac{1}{f} \quad \dots(i)$$

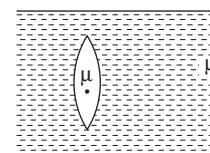
$$\frac{1}{f_2} = (\mu_w - 1) \left(-\frac{2}{R} \right) = \left(\frac{4}{3} - 1 \right) \left(\frac{-2}{R} \right)$$

$$= \left(\frac{1}{3} \right) \left(-\frac{2}{R} \right) = \left(\frac{-2}{3} \right) \left(\frac{1}{2f(\mu - 1)} \right) \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \frac{1}{f_2} = \left(-\frac{1}{3} \right) \left(\frac{1}{\frac{3}{2} - 1} \right) \left(\frac{1}{f} \right) = -\frac{2}{3} \times \frac{1}{f}$$

$$\therefore \frac{1}{f_{eq}} = \frac{2}{f} - \frac{2}{3} \times \frac{1}{f} = \frac{6-2}{3f} = \frac{4}{3f}$$

$$\Rightarrow f_{eq} = \frac{3f}{4}$$



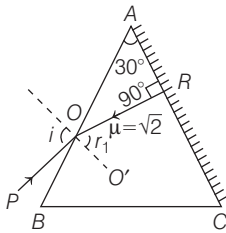
- 26 (b) From Snell's law of refraction, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$

Given, $v_1 = 3 \times 10^8$ m/s

$$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = 1.5$$

$$v_2 = \frac{v_1}{1.5} = \frac{3 \times 10^8}{1.5} \text{ m/s} = \frac{3 \times 10^8}{3/2} = 2 \times 10^8 \text{ m/s}$$

- 27 (b) According to the question, the figure of mentioned prism is given as (since, there is no refraction at the face AC)



Given, refractive index of the material of prism, $\mu = \sqrt{2}$

and angle of prism, $A = 30^\circ$.

If the ray OR has to retrace its path after reflection (as per the given condition), then the ray has to fall normally on the surface AC.

This means $\angle ARO = \angle ORC = 90^\circ$

In $\triangle AOR$,

$$\angle AOR + \angle ARO + \angle OAR = 180^\circ$$

$$\Rightarrow \angle AOR + 90^\circ + 30^\circ = 180^\circ$$

$$\Rightarrow \angle AOR = 180^\circ - 120^\circ = 60^\circ$$

As we know, $\angle AOR + \angle r_1 = 90^\circ$

$$\Rightarrow \angle r_1 = 90^\circ - 60^\circ = 30^\circ$$

[from Eq. (i)]

Applying Snell's law at the face AB, we get

$$\mu = \frac{\sin i}{\sin r}$$

Substituting the given values, we get

$$\begin{aligned} \sqrt{2} &= \frac{\sin i}{\sin 30^\circ} \\ \Rightarrow \sin i &= \sin 30^\circ \times \sqrt{2} \\ &= \frac{1}{2} \times \sqrt{2} = \frac{1}{\sqrt{2}} \quad \left(\because \sin 30^\circ = \frac{1}{2} \right) \end{aligned}$$

$$\text{or} \quad i = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ \quad \left(\because \sin 45^\circ = \frac{1}{\sqrt{2}} \right)$$

Thus, the angle of incidence of the ray on the prism is 45° .

- 28 (a) Given, $A = 60^\circ$ and $\delta_m = 30^\circ$

$$\text{According to prism formula, } \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\text{or} \quad \mu = \frac{\sin\left(\frac{60^\circ + 30^\circ}{2}\right)}{\sin\frac{60^\circ}{2}} = \frac{\sin 45^\circ}{\sin 30^\circ} \quad \text{or} \quad \mu = \sqrt{2}$$

$$\therefore \mu = \frac{\text{Speed of light in air (c)}}{\text{Speed of light in prism (v)}}$$

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times 10^8 \text{ m/s}$$

- 29 (d) Given, angle of prism, $A_1 = 4^\circ$

$$A_2 = ?, \quad n_1 = 1.54, \quad n_2 = 1.72$$

Angle of deviation, $\delta = (n - 1)A$

Dispersion produced by both the prism will be equal,

$$\text{i.e. } (n_1 - 1)A_1 = (n_2 - 1)A_2$$

$$\begin{aligned} \Rightarrow A_2 &= \frac{(n_1 - 1)A_1}{(n_2 - 1)} \\ &= \frac{(1.54 - 1) \times 4}{(1.72 - 1)} = 3^\circ \end{aligned}$$

Hence, the angle of prism P_2 is 3° .

- 30 (c) For first refracting surface,

For parallel beam of light,

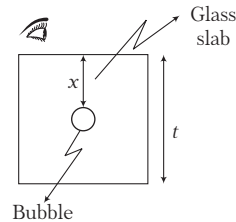
$$\begin{aligned} -\frac{\mu_1}{u} + \frac{\mu_2}{v} &= \frac{\mu_2 - \mu_1}{R} \\ \Rightarrow -\frac{1.4}{(-\infty)} + \frac{1.5}{v} &= \frac{1.5 - 1.4}{R} \\ v &= 15R \end{aligned}$$

For second refracting surface,

$$\begin{aligned} \frac{\mu_2}{-15R} + \frac{\mu_3}{v'} &= \frac{\mu_3 - \mu_2}{R} \\ \Rightarrow \frac{1.5}{15R} + \frac{1.6}{v'} &= \frac{1.6 - 1.5}{R} \\ \Rightarrow v' &= \infty \end{aligned}$$

Hence, the combination behaves as glass plate.

- 31 (c) Let thickness of the given slab is t . According to the question, when viewed from both the surfaces



$$\Rightarrow \frac{x}{\mu} + \frac{t - x}{\mu} = 3 + 5 \Rightarrow \frac{t}{\mu} = 8 \text{ cm}$$

$$\Rightarrow \text{Thickness of the slab, } t = 8 \times \mu = 8 \times \frac{3}{2} = 12 \text{ cm}$$

- 32 (b) Given, image distance, $v = 400 \text{ cm} = 4 \text{ m} \Rightarrow u = \infty$

Using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-4} - \frac{1}{\infty} = \frac{1}{f} \Rightarrow f = -4 \text{ m}$$

Now, power of the required lens,

$$P = \frac{1}{f} = \frac{1}{-4} = -0.25 \text{ D}$$

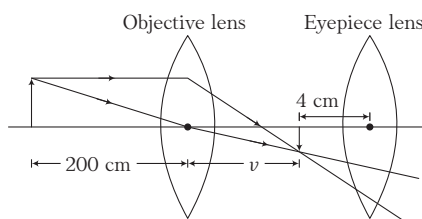
Thus, the person require a concave lens of power -0.25 D .

33 (c) According to question,

Focal length of objective lens, $f_o = +40$ cm

Focal length of eyepiece lens, $f_e = 4$ cm

Object distance for objective lens, $u_o = -200$ cm



Applying lens formula for objective lens,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-200)} = \frac{1}{40}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{40} - \frac{1}{200} = \frac{5-1}{200} = \frac{4}{200}$$

$$\Rightarrow v = 50 \text{ cm}$$

Image will be form at first focus of eyepiece lens.

So, for normal adjustment distance between objective and eyepiece lens (length of tube) will be

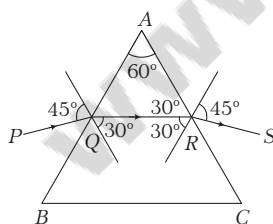
$$v + f_e \Rightarrow 50 + 4 \Rightarrow 54 \text{ cm}$$

34 (d) A concave mirror forms real and virtual images, whose magnification can be negative or positive depending upon the position of the object.

If object is placed between focus and pole, then the image obtained will be virtual, enlarged and its magnification will be positive. In all other cases, concave mirror forms real images whose magnification will be negative.

A convex mirror always forms a virtual and diminished image whose magnification will always be positive.

35 (a) Consider a ray of light PQ incident at the surface AB and moves along RS , after passing through the prism ABC .



It is given that, the incident ray suffers minimum deviation. Therefore, the ray inside the prism must be parallel to the base BC of the prism.

From the geometry of the prism and the ray diagram, it is clear that,

Angle of incidence, $i = 45^\circ$

Angle of refraction, $r = r' = 30^\circ$

Angle of emergence, $e = 45^\circ$

A = angle of prism = 60°

Therefore, minimum deviation suffered by the ray is

$$\delta_{\min} = i + e - (r + r') = 90^\circ - 60^\circ = 30^\circ$$

Refractive index of the material,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

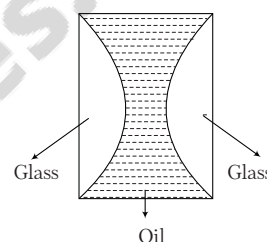
$$\begin{aligned} \therefore \mu &= \frac{\sin\left(\frac{60^\circ + 30^\circ}{2}\right)}{\sin\frac{60^\circ}{2}} = \frac{\sin 45^\circ}{\sin 30^\circ} \\ &= \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2} \end{aligned}$$

36 (c) Given, $\mu_g = 1.5$,

$\mu_{\text{oil}} = 1.7$ and $R = 20$ cm

From lens Maker's formula for the plano-convex lens,

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$



Here, $R_1 = R$ and for plane surface, $R_2 = \infty$

$$\therefore \frac{1}{f_{\text{lens}}} = (1.5 - 1) \left(\frac{1}{R} - 0 \right) \Rightarrow \frac{1}{f_{\text{lens}}} = \frac{0.5}{R}$$

When the intervening medium is filled with oil, then focal length of the concave lens formed by the oil

$$\frac{1}{f_{\text{concave}}} = (1.7 - 1) \left(-\frac{1}{R} - \frac{1}{R} \right) = -0.7 \times \frac{2}{R} = \frac{-1.4}{R}$$

Here, we have two concave surfaces.

$$\begin{aligned} \text{So, } \frac{1}{f_{\text{eq}}} &= 2 \times \frac{1}{f_{\text{lens}}} + \frac{1}{f_{\text{concave}}} \\ &= 2 \times \frac{0.5}{R} + \left(\frac{-1.4}{R} \right) = \frac{1}{R} - \frac{1.4}{R} = \frac{-0.4}{R} \end{aligned}$$

Focal length of the combination,

$$\therefore f_{\text{eq}} = -\frac{R}{0.4} = -\frac{20}{0.4} = -50 \text{ cm}$$

$$\text{37 (b) Refractive index, } \mu = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow \cot\frac{A}{2} = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A + D_m}{2}\right)$$

$$\Rightarrow \frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{D_m}{2}$$

$$\Rightarrow D_m = \pi - 2A$$

Angle of minimum deviation, $D_m = 180^\circ - 2A$

- 38** (c) If the object is placed at a distance 25 cm from the corrected lens, then it should produce the virtual image at 40 cm.

Thus, $u = -25$ cm and $v = -40$ cm

The lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{-1}{40} + \frac{1}{25}$

$$\Rightarrow f = \frac{200}{3} \text{ cm} = \frac{2}{3} \text{ m}$$

$$\therefore \text{Power of the lens, } P = \frac{1}{f} = \frac{1}{2/3} = \frac{3}{2} = +1.5 \text{ D}$$

- 39** (b) Given, focal length of simple microscope, $f = 12$ cm

Near point, $v = 25$ cm

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow -\frac{1}{25} - \frac{1}{u} = \frac{1}{12}$

$$\Rightarrow \frac{1}{u} = -\frac{1}{12} - \frac{1}{25}$$

$$\Rightarrow \frac{1}{u} = -\frac{25 - 12}{300} = -\frac{37}{300}$$

$$\therefore u = -\frac{300}{37}$$

$$\text{Now, magnification, } |m| = \left| \frac{v}{u} \right| = \frac{25}{\frac{300}{37}} = \frac{25 \times 37}{300}$$

$$\Rightarrow m = 3.08$$

- 40** (d) Given, $\mu = 1.5$

$$\text{As refractive index of a prism, } \mu = \frac{\sin \frac{A + \delta}{2}}{\sin \frac{A}{2}}$$

where, δ = angle of minimum deviation
and A = refracting angle of prism.

As per question, $\delta = A$, then

$$\mu = \frac{\sin \frac{A + A}{2}}{\sin \frac{A}{2}} = \frac{\sin A}{\sin \frac{A}{2}} = \frac{2 \sin \frac{A}{2} \cos \frac{A}{2}}{\sin \frac{A}{2}} = 2 \cos \frac{A}{2}$$

$$\Rightarrow \frac{1.5}{2} = \cos \frac{A}{2} = \sin(90^\circ - \frac{A}{2})$$

$$\Rightarrow 90^\circ - \frac{A}{2} = \sin^{-1}(0.75) = 48^\circ 36'$$

$$\Rightarrow 90^\circ - 48^\circ 36' = \frac{A}{2} \Rightarrow 41^\circ 24' = \frac{A}{2}$$

$$\therefore A = 82^\circ 48'$$

- 41** (d) As the law of refraction, we have

$$\frac{\sin i_c}{\sin 90^\circ} = \frac{1.5}{1.6}$$

$$\therefore \sin i_c = \frac{15}{16}$$

where, i_c = critical angle of medium.

$$\text{or } i_c = \sin^{-1}\left(\frac{15}{16}\right)$$

- 42** (a) Let wavelengths of red and violet lights are λ_r and λ_v , respectively.

As we know that,

$$\lambda_r > \lambda_v$$

$$\Rightarrow \mu_v > \mu_r \quad \left(\because \mu \propto \frac{1}{\lambda} \right)$$

$$\Rightarrow f_r > f_v \quad \left(\because f \propto \frac{1}{\mu} \right)$$

- 43** (a) Smallest angle that can be resolved by telescope is given by

$$d\theta = \frac{1.22 \lambda}{D}$$

Smallest angle that can be resolved by human eye is given by

$$d\theta' = \frac{1.22 \lambda}{D_e}$$

where, D_e is the aperture of human eye, therefore the telescope must have a minimum magnifying power which must be such that, if an object subtend an angle $d\theta$ at its objective, then the final image produced must subtend an angle $d\theta'$ at the human eye.

$\therefore$ Minimum magnifying power is given by

$$M = \frac{d\theta'}{d\theta} = \frac{D}{D_e}$$

$$\Rightarrow M = \frac{100 \text{ cm}}{0.2 \text{ cm}} = 500$$

[$\because D = 100$ cm and $D_e = 0.2$ cm (given)]

- 44** (b) From the lens formula (for first lens),

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{v_1} - \frac{1}{(-4)}$$

$$\Rightarrow \frac{1}{2} - \frac{1}{4} = \frac{1}{v_1} = \frac{3}{4} \quad \dots(i)$$

$$\Rightarrow v_1 = \frac{4}{3}, \quad u_2 = 3 - \frac{4}{3} = 5/3$$

This image will be treated as the source for second lens, then again from lens formula, we have

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2} \Rightarrow \frac{1}{1} = \frac{1}{v_2} + 5/3 \quad (\text{by Eq. (i)})$$

$$\Rightarrow 1 - 5/3 = \frac{1}{v_2}$$

$$\Rightarrow v_2 = -3/2$$

This is the final image distance from 2nd lens, so the overall distance of image from the primary source (or object).

Then, $d = 4 + 3 - 1.5 = 5.5$ m

- 45 (a) Here, $\mu_V = 1.5230$ and $\mu_R = 1.5145$

Therefore, mean refractive index,

$$\mu = \frac{1.5230 + 1.5145}{2} = 1.5187$$

Now, dispersive power ω is given by

$$\omega = \frac{\mu_V - \mu_R}{(\mu - 1)} = \frac{1.5230 - 1.5145}{(1.5187 - 1)} = 0.0164$$

- 46 (c) The splitting of white light into its constituent colour is called dispersion of light. In this phenomenon, different colours contained in white light emerge from the glass prism in different directions due to their different wavelengths.

- 47 (d) Given, $v = -75$ cm

$$u = -25$$
 cm

We know that, lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{-1}{75} + \frac{1}{25}$$

$$\frac{1}{f} = \frac{1}{25} - \frac{1}{75}$$

$$\frac{1}{f} = \frac{3-1}{75}$$

$$\frac{1}{f} = \frac{2}{75}$$

Focal length of a glass, $f = \frac{75}{2} = 37.5$ cm

- 48 (c) Given, $m = -3$ and $R = 30$ cm

$$f = -\frac{R}{2} = -15$$
 cm

We know that, $m = -\frac{v}{u} \Rightarrow -3 = \frac{-v}{u}$

$$v = 3u$$

Also,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{-15} = \frac{1}{3u} + \frac{1}{u} \Rightarrow \frac{-1}{15} = \frac{1+3}{3u}$$

$$-3u = 4 \times 15$$

$$\Rightarrow u = \frac{-4 \times 15}{3}$$

Initial position of a person, $u = -20$ cm

- 49 (d) Given, $f = 10$ cm, $D = 25$ cm

We know that,

$$m = 1 + \frac{D}{f} = 1 + \frac{25}{10}$$

$$m = 1 + 2.5 = 3.5$$

- 50 (a) v_{OM} = velocity of object with respect to mirror

$$= -v_{IM}$$

(Here, v_{IM} = velocity of image with respect to mirror)

$$\Rightarrow v_O - v_M = -(v_I - v_M)$$

$$4\text{ms}^{-1} - (-5\text{ms}^{-1}) = -v_I + (-5\text{ms}^{-1})$$

$$v_I = -14\text{ms}^{-1}$$

- 51 (b) Given, refractive index of prism, $\mu = \sqrt{7/3}$

and angle of prism, $A = 60^\circ$

$$\text{Using, } \mu = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\frac{A}{2}}$$

Since, angles can be assumed to be small.

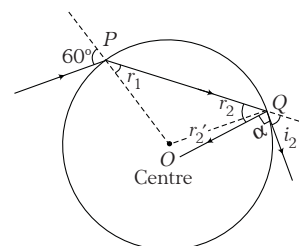
$$\text{Hence, } \sin\left(\frac{A + D_m}{2}\right) \approx \frac{A + D_m}{2} \text{ and } \sin\frac{A}{2} \approx \frac{A}{2}$$

$$\Rightarrow \mu = \frac{\frac{A + D_m}{2}}{\frac{A}{2}}$$

$$\text{So, } \sqrt{\frac{7}{3}} = \frac{(60^\circ + D_m)/2}{60^\circ/2} \text{ or } 1.53 = \frac{60^\circ + D_m}{2} \times \frac{1}{30^\circ}$$

$$\therefore D_m \approx 30^\circ = \frac{\pi}{6}$$

- 52 (c) The situation can be represented as



For the refraction at point P,

$$\frac{\sin 60^\circ}{\sin r_1} = \sqrt{3} \Rightarrow r_1 = 30^\circ$$

Since, $r_2 = r_1 \Rightarrow r_2 = 30^\circ$

For refraction at point Q, $\frac{\sin r_2}{\sin i_2} = \frac{1}{\sqrt{3}}$

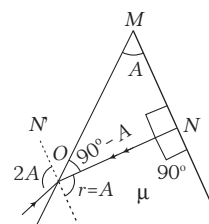
When $r_2 = 30^\circ$, then $i_2 = 60^\circ$

$$\Rightarrow r_2 = r'_2 = 30^\circ$$

$$\therefore \alpha = 180^\circ - (r'_2 + i_2) = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$$

- 53 (c) Atomic force microscope is used for measuring the roughness of a material as it has a much better optical diffraction limit.

- 54 (b) For the condition giving in question, ray diagram will be



From the ray diagram, $\angle MON = 90^\circ - A$

So, $\angle r = 90^\circ - (90^\circ - A)$
 $\Rightarrow \angle r = A$
 By Snell's law, $\frac{\sin i}{\sin r} = \mu \Rightarrow \frac{\sin(2A)}{\sin(A)} = \mu$
 $\frac{2 \sin A \cos A}{\sin A} = \mu \Rightarrow \mu = 2 \cos A$

55 (d) For microscope, $m = \frac{L}{f_o} \frac{D}{f_e} \Rightarrow m \propto \frac{1}{f_o}$

For telescope, $m = \frac{f_o}{f_e} \Rightarrow m \propto f_o$

The magnifying power of microscope will decrease but the magnifying power telescope will increase, if f_o is increased.

56 (a) We know that, linear magnification, $m = \frac{f}{f-u}$

Given, object displaced, $u = -20$ cm

$m = -3$ ($\because$ all real images are inverted)

So, $-3 = \frac{f}{f - (-20)}$

$\Rightarrow -3 = \frac{f}{f+20} \Rightarrow -3f - 60 = f$

$4f = -60 \Rightarrow f = -\frac{60}{4} = -15$ cm

Since, mirror is concave, $f = -15$ cm $\Rightarrow |f| = 15$ cm

57 (c) In vacuum, $c = d/t$

In medium, $v = \frac{5d}{T}$

As refractive index, $\mu = \frac{c}{v} = \frac{d/t}{5d/T} = \frac{T}{5t}$

Also, $\sin C = \frac{1}{\mu}$ (where, C is critical angle)

$\therefore C = \sin^{-1} \left[\frac{5t}{T} \right]$

58 (d) When an equiconvex lens is cut parallel to principal axis, then focal length remains the same and when the lens is cut perpendicular to principal axis, then its focal length becomes twice the original.

59 (e) Given, $P_1 = 15$ D and $P_2 = -3$ D

$\therefore P = P_1 + P_2 = 15$ D $- 3$ D $= 12$ D

$\therefore P = \frac{1}{f(\text{m})} \Rightarrow P = \frac{100}{f(\text{cm})} \Rightarrow 12 = \frac{100}{f}$

Focal length of the combination, $f = \frac{100}{12} = 8.33$ cm

60 (d) Focal length of a convex lens by displacement method

is given as $f = \frac{a^2 - b^2}{4a}$

where, a = distance between the image and object

and b = distance between two positions of lens.

Here, $a = d$ and to form a distinct image $b = 0$, so the maximum possible focal length is $d/4$.

61 (d) For the first condition, $f_1 = 10$ cm, $u = -30$ cm

then $\frac{1}{f_1} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{10} = \frac{1}{v} + \frac{1}{30}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} \Rightarrow v = \frac{30}{2} = 15$ cm

For the second condition, when concave lens is placed,

$v' = (15 + 45) = 60$ cm $\Rightarrow \frac{1}{F} = \frac{1}{v'} - \frac{1}{u}$

(where, F = focal length of combination)

$\therefore \frac{1}{F} = \frac{1}{60} + \frac{1}{30} \Rightarrow F = \frac{60}{3} = 20$ cm

The magnitude of focal length of concave lens,

$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow \frac{1}{20} = \frac{1}{10} + \frac{1}{f_2}$

$\Rightarrow f_2 = -20$ cm (negative sign for concave lens)

62 (a) As we know that, from lens Maker's formula,

$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

As, $\frac{1}{f}$ = power of lens (P)

where, f = focal length,

μ_2 and μ_1 = refractive index of 2nd and 1st medium

R_1 and R_2 = radius of curvature of 1st and 2nd curved surface of lens.

So, for 1st case, $P_1 = \left(\frac{1.5}{1} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$... (i)

As, $\mu_2 = 1.5$, $\mu_1 = 1$ (air)

where, P_1 = power in initial condition.

Now, for 2nd case, $P_2 = \left(\frac{1.5}{1.6} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$... (ii)

where, P_2 = power in 2nd condition.

On dividing Eqs. (i) and (ii), we get

$\frac{P_1}{P_2} = \frac{1.5 - 1}{\frac{1.5}{1.6} - 1} = \frac{0.5 \times 1.6}{-0.1}$

$\Rightarrow \frac{-5}{P_2} = \frac{-0.8}{0.1}$ (as, $P_1 = -5$ D)

$\Rightarrow P_2 = \frac{+5 \times 0.1}{0.8} = \frac{5}{8} = 0.625$ D

So, option (a) is close to the calculated value as $0.625 \approx 1$ D.

63 (a) Given, $f = -f \Rightarrow v = -\frac{1}{3}u$ (for concave lens)

According to the lens formula,

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \frac{1}{f} = \frac{1}{-(1/3)u} + \frac{1}{u} = \frac{-3+1}{u}$

$\Rightarrow \frac{1}{f} = -\frac{2}{u} \Rightarrow u = 2f$

- 64 (b) Given, $f = 10$ cm and power, $P = ?$

The power of lens, $P = \frac{1}{\text{Focal length (m)}}$

$$P = \frac{1}{f(\text{m})} \Rightarrow P = \frac{100}{f(\text{cm})} = \frac{100}{10} = 10 \text{ D}$$

- 65 (a) In this defect, eye cannot see perpendicular directions clearly, i.e. horizontal and vertical lines simultaneously. It is due to imperfect spherical nature of eye lens.

This defect can be removed by using cylindrical lenses. (Toric lenses).

- 67 (a) In normal adjustment, the object and the final image both are at infinity and the separation between the objective and the eyepiece is $f_o + f_e$.

$$\text{Thus, } f_o + f_e = 35 \text{ cm} \quad \dots(i)$$

Also, the magnifying power of the telescope in normal

$$\text{adjustment is } |m| = \frac{f_o}{f_e} = 6 \quad (\because \text{given})$$

$$f_o = 6f_e \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$f_o = 30 \text{ cm and } f_e = 5 \text{ cm}$$

- 68 (b) Given, length of tube, $L = 30$ cm, focal length of objective lens, $f_o = 1$ cm, focal length of eyepiece, $f_e = 6$ cm and $D = 25$ cm.

$$\text{For compound microscope, } m = \frac{L}{f_o} \left(1 + \frac{D}{f_e} \right)$$

$$m = \frac{30}{1} \left(1 + \frac{25}{6} \right) = 30 \times \frac{(6+25)}{6} \\ = 5 \times 31 = 155 \approx 150$$

- 69 (d) Given, wavelength of light,

$$\lambda = 5000 \text{ \AA} = 5000 \times 10^{-10} \text{ m} = 5 \times 10^{-7} \text{ m}$$

Diameter of telescope, $D = 200$ cm = 2 m

$$\text{Resolving power of a telescope} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

where, $d\theta$ = limit of resolution,

λ = wavelength of light used

and D = diameter of aperture of objective.

$$\text{So, resolving power} = \frac{2}{1.22 \times 5 \times 10^{-7}} = \frac{2 \times 10^7}{6.1} \\ = 3.278 \times 10^6 = 3.28 \times 10^6$$

- 70 (c) Focal length of the combination

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \dots(i)$$

$$\text{We have, } \frac{1}{f_1} = (\mu_1 - 1) \left(\frac{1}{\infty} - \frac{1}{-R} \right) = \frac{(\mu_1 - 1)}{R}$$

$$\text{and } \frac{1}{f_2} = (\mu_2 - 1) \left(\frac{1}{-R} - \frac{1}{\infty} \right) = -\frac{(\mu_2 - 1)}{R}$$

Putting these values of $\frac{1}{f_1}$ and $\frac{1}{f_2}$ in Eq. (i), we get

$$\frac{1}{f} = \frac{(\mu_1 - 1)}{R} - \frac{(\mu_2 - 1)}{R} \\ = \frac{[\mu_1 - 1 - \mu_2 + 1]}{R} = \frac{\mu_1 - \mu_2}{R} \\ f = \frac{R}{\mu_1 - \mu_2}$$

- 71 (c) Given, power (P_1) = 40D and power (P_2) = 20D

We have, $P_{eq} = P_1 + P_2 = 40 + 20 = 60 \text{ D}$

$$\text{So, } \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow f_{eq} = \frac{100}{P_{eq}} = \frac{100}{60} = 1.67 \text{ cm}$$

- 73 (c) We have, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \frac{1}{f} = \frac{1}{-20} - \left(-\frac{1}{40} \right) = \frac{-2+1}{40} = -\frac{1}{40}$$

$$\text{or } f = -40 \text{ cm}$$

$$\text{Power of the lens, } P = -\frac{1}{0.40} = -2.5 \text{ D}$$

- 74 (b) Given, $f = 5$ cm, $D = 25$ cm

$$\therefore \text{Magnification, } m = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$$

- 76 (c) We know about the thin prism, $\delta_m = (\mu - 1)A$

In given condition, $A_1(\mu_1 - 1) = A_2(\mu_2 - 1)$

Given, $\mu_1 = 1.4$, $\mu_2 = 1.6$ and $A_1 = 6^\circ$

Now, $6(1.4 - 1) = A_2(1.6 - 1)$

$$A_2 = \frac{6 \times 0.4}{0.6} \Rightarrow A_2 = \frac{2.4}{0.6}$$

$$A_2 = 4^\circ$$

Hence, the angle of the second prism is 4° .

- 78 (a) For total internal reflection, the angle of incidence of light in denser medium must be greater than the critical angle for the pair of media in contact, i.e. $i > C$.

$$\text{We have, } \frac{1}{\sin C} = \frac{v_2}{v_1}$$

$$\text{Given, } v_1 = 1.5 \times 10^8 \text{ ms}^{-1}$$

$$v_2 = 2 \times 10^8 \text{ ms}^{-1}$$

$$\text{Here, } C = \sin^{-1} \left(\frac{v_1}{v_2} \right)$$

For total internal reflection, $\theta > C$

$$\therefore \theta > \sin^{-1} \left(\frac{v_1}{v_2} \right)$$

$$\Rightarrow \theta > \sin^{-1} \left(\frac{1.5 \times 10^8}{2 \times 10^8} \right)$$

$$\Rightarrow \theta > \sin^{-1} \left(\frac{3}{4} \right)$$

- 79 (d) Given, $\frac{R_1}{R_2} = \frac{1}{2}$, $f = 6$ cm and $\mu = 1.5$.

Let, $R_1 = R$ and $R_2 = 2R$

We know that, the focal length of converging lens is given

by using relation, $\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} + \frac{1}{R_2} \right]$

Substituting all the values, we get

$$R = 4.5 \text{ cm}$$

$$\therefore R_1 = 4.5 \text{ cm}$$

$$\text{and } R_2 = 2 \times 4.5 = 9 \text{ cm}$$

- 81 (d) We know that, for plano-convex lens, $\frac{1}{f} = (\mu - 1) \frac{1}{R}$

Given, $R = 12$ cm and $\mu = 1.5$

$$\text{Here, } f = \frac{R}{\mu - 1} = \frac{12}{1.5 - 1} = \frac{12}{0.5} = 24 \text{ cm}$$

- 82 (c) Focal length of combination of two thin lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{f_1 + f_2}{f_1 f_2} \Rightarrow f = \frac{f_1 f_2}{f_1 + f_2}$$

- 83 (b) We know that, the relation for number of images formed for an object placed between the mirror.

$$n = \frac{360^\circ}{\theta} - 1$$

Given, $\theta = 50^\circ$

$$\text{Now, number of images, } n = \frac{360^\circ}{50^\circ} - 1$$

$$= 7.2 - 1 = 6.2 \approx 6$$

- 84 (b) Velocity of light in a medium, $v = \frac{c}{\mu}$

we know that, time taken, $t = \frac{x}{v}$

$$\Rightarrow t = \frac{x}{c/\mu} \Rightarrow t = \frac{\mu x}{c}$$

- 85 (b) The critical angles $\left[C = \sin^{-1} \frac{1}{\mu} \right]$ for red ($\mu = 1.39$), green

($\mu = 1.44$) and blue ($\mu = 1.47$) lights are 46° , 44° and 43° , respectively.

All colours will strike the hypotenuse face at 45° . Hence, green and blue rays will be totally reflected while red rays will pass through the prism.

- 86 (d) Angle between the two plane mirrors, $\theta = 90^\circ$

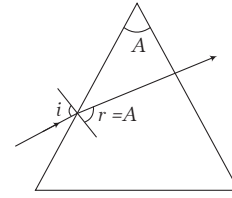
As, reflection is independent of medium.

$\therefore$ Deviation produced by the combination of two plane mirrors is $\delta = 2\pi - 2\theta = 2\pi - 2(\pi/2) = \pi = 180^\circ$.

- 87 (d) In order to increase the magnifying power of a compound microscope, the object should have large focal length or eyepiece should have small focal length.

- 88 (a) Refractive index,

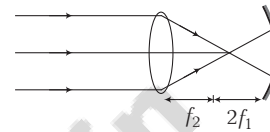
$$\mu = \frac{\sin i}{\sin r} \Rightarrow \mu = \frac{\sin i}{\sin A}$$



In case of small angle, $\mu = \frac{i}{A}$

Angle of incidence, $i = \mu A$

- 89 (c) $d = 2f_1 + f_2$



- 90 (a) From lens formula,

$$\frac{1}{f} = (\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = (1.5 - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(i)$$

$$\text{Also, } \mu_g = \frac{\mu_g}{\mu_l} = \frac{1.5}{1.6}$$

$$\therefore \frac{1}{f'} = (\mu_g - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f'} = \left(\frac{1.5}{1.6} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f}{f'} = \frac{\left(\frac{1.5}{1.6} - 1 \right)}{(1.5 - 1)} = -\frac{1}{16 \times 0.5}$$

$$f' = -16 \times 0.5 \times f$$

$$= 16 \times 0.5 \times 20$$

$$\Rightarrow f' = -160 \text{ cm}$$

- 91 (a) Resolving power of telescope $= \frac{a}{1.22} \lambda$

where, a is the diameter of objective lens and λ is the wavelength of light used. It is obvious that on increasing the diameter of objective lens, the amount of light collected by it increases. So, the image formed is more bright. Thus, resolving power of telescope increases.

- 92 (d) $\therefore \frac{1}{f} = \left[\frac{\mu_1}{\mu_2} - 1 \right] \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$

Given, $R_1 = 0.30$ m,

$$R_2 = -0.30 \text{ m}, \mu_1 = 3/2, \mu_2 = 4/3$$

$$\Rightarrow \frac{1}{f} = \left[\frac{3/2}{4/3} - 1 \right] \left[\frac{1}{0.30} - \left(-\frac{1}{0.30} \right) \right]$$

By solving the above equation, we get, $f = 1.20$ m

- 93 (b) $\therefore$ Magnification, $m = \frac{f}{f - u}$

Here, $m = -2$ and $f = -50$ cm

$$\therefore -2 = \frac{-50}{-50 - u}$$

$$\Rightarrow 100 + 2u = -50$$

$$\text{or } u = \frac{-150}{2} = -75 \text{ cm}$$

94 (c) An eye see distant objects with full relaxation.

$$\text{So, } \frac{1}{2.5 \times 10^{-2}} - \frac{1}{-\infty} = \frac{1}{f}$$

$$\text{or } P = \frac{1}{f} = \frac{1}{2.5 \times 10^{-2}} = 40 \text{ D}$$

An eye see an object at 25 cm with strain

$$\frac{1}{2.5 \times 10^{-2}} - \frac{1}{25 \times 10^{-2}} = \frac{1}{f}$$

$$\therefore P = \frac{1}{f} = 40 + 4 = 44 \text{ D}$$

The power of the eye lens varies from 44 D to 40 D.

95 (a) When lenses are in contact, then power,

$$P = \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$10 = \frac{1}{f_1} + \frac{1}{f_2} \quad \dots(i)$$

When they are placed a distance d apart

$$P' = \frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$6 = \frac{1}{f_1} + \frac{1}{f_2} - \frac{0.25}{f_1 f_2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$f_1 f_2 = \frac{1}{16} \quad \dots(iii)$$

From Eqs. (i) and (iii), we get

$$f_1 + f_2 = \frac{5}{8} \quad \dots(iv)$$

Also, $(f_1 - f_2)^2 = (f_1 + f_2)^2 - 4 f_1 f_2$

$$= \left(\frac{5}{8}\right)^2 - 4 \times \frac{1}{16} = \frac{9}{64}$$

$$f_1 - f_2 = \frac{3}{8} \quad \dots(v)$$

On solving Eqs. (iv) and (v), we get

$$f_1 = 0.5 \text{ m and } f_2 = 0.125 \text{ m}$$

96 (c) Given, $f = +30 \text{ cm}$ and $m = \frac{1}{4}$

$$\text{Magnification, } m = \frac{f}{f - u}$$

$$\therefore \frac{30}{30 - u} = \frac{1}{4}$$

$$30 - u = 120$$

$$\Rightarrow u = -90 \text{ cm}$$

$$\Rightarrow |u| = 90 \text{ cm}$$

97 (a) For objective lens,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$$

Using proper sign convention,

$$\frac{1}{+2.5} = \frac{1}{v_o} - \frac{1}{(-3.75)}$$

$$\Rightarrow v_o = 7.5 \text{ cm}$$

$$\text{For eye lens, } \frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\frac{1}{+5} = \frac{1}{(-25)} - \frac{1}{u_e}$$

$$\Rightarrow u_e = -4.16 \text{ cm or } |u_e| = 4.16 \text{ cm}$$

$\therefore$ Distance between the two lens = $7.5 + 4.16 = 11.67 \text{ cm}$

98 (d) From mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \dots(i)$$

On differentiating Eq. (i), we obtain

$$0 = \frac{1}{v^2} dv - \frac{1}{v^2} du$$

$$dv = -\left(\frac{v}{u}\right)^2 du \quad \dots(ii)$$

Also, from Eq. (i), we get

$$\frac{v}{u} = \frac{f}{u - f} \quad \dots(iii)$$

From Eqs. (ii) and (iii), we get

$$dv = -\left(\frac{f}{u - f}\right)^2 L$$

Therefore, size of image is $\left(\frac{f}{u - f}\right)^2 L$.

99 (c) $\therefore$ Magnification, $m = \frac{v}{-u} = -3$

$$v = 3u = 30 \text{ cm}$$

Focal length of co-axial combination, $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u}$

$$-\frac{1}{30} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30}$$

$$\frac{1}{f_2} = \frac{4}{30} + \frac{1}{30}$$

$$\frac{1}{f_2} = \frac{4+1}{30} = \frac{5}{30}$$

$$f_2 = \frac{30}{5} = 6 \text{ cm}$$

Hence, lens A is convex lens.

100 (d) White light is incident on face AB of a glass prism, the green light is just totally internally reflected at face AC, thus the light emerging from face AC will contain all colours except green.

Wave Optics

The ray optics uses the geometry of straight lines to explain the macroscopic phenomena like rectilinear propagation of light, reflection of light and refraction of light, etc. However, the microscopic phenomena like interference, diffraction and polarisation could not be accounted by ray optics. To explain these phenomena concept of waves was introduced. The new branch of physics based on the wave concept of light was called **wave optics**. According to wave theory of light, light is a form of energy which travels through a medium in the form of transverse wave motion. In this chapter, we shall study the various phenomena related to wave nature of light.

NATURE OF LIGHT

Various theories about the nature of light have been proposed from time-to-time. The main two theories are as follows

1. Newton's corpuscular theory of light

In 1675 AD Newton proposed this theory. According to him,

- (i) Light consists of tiny particles called **corpuscles** which are emitted by a luminous object.
- (ii) These corpuscles travels with speed of light in all directions.
- (iii) The corpuscles carry energy and momentum with them. When they strike retina of the eye, they produce sensation of vision.
- (iv) The corpuscles of different colours are of different sizes. Red coloured corpuscles are larger than blue coloured corpuscles.
- (v) The corpuscular theory could explain the reflection, refraction and rectilinear propagation of light but could not explain interference, diffraction and polarisation of light.
- (vi) According to this theory, speed of light in denser medium is more than speed of light in a rarer medium, which is incorrect. Therefore, the Newton's corpuscular theory is wrong.

2. Huygens' wave theory of light

In 1678, a Dutch scientist, Christian Huygens' propounded the wave theory of light. According to him,

- (i) Light travels in the form of waves.
- (ii) These waves travel in all the directions with the velocity of light.

Inside

1 Nature of light

- Wavefront
- Huygens' principle of secondary wavelets
- Principle of superposition of waves
- Interference of light wave
- Necessary conditions for interference of light

2 Young's double slit experiment

- Intensity of the fringes
- Lloyd's mirror

3 Diffraction of light

- Fraunhofer diffraction of light due to a single narrow slit
- Width of central maxima
- Fresnel's distance

4 Polarisation of light

- Malus's law
- Polarisation of transverse mechanical waves

- (iii) The waves of light of different colours have different wavelengths.
- (iv) Initially, the light waves were assumed to be longitudinal. But later on while explaining the phenomenon of polarisation, the light waves were considered to be transverse.
- (v) The whole universe with all matter and space is filled with a luminiferous medium called **ether**, of very low density and very high elasticity.
- (vi) Huygen's theory could explain reflection, refraction, interference, diffraction, polarisation but could not explain photoelectric effect and Compton's effect.
- (vii) Wave theory introduced the concept of wavefront.

Wavefront

A wavefront is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations. A wavelet is the point of disturbance due to propagation of light. A line perpendicular to a wavefront is called a **ray**.

Types of wavefront

Depending on the shape of source of light, wavefronts can be of three types which are given below

(i) Spherical wavefront

When light is emerging from a point source wavefronts are spherical and concentric with point source at their centre as shown in figure by spheres 1, 2, 3 and 4. Rays are radial as shown by arrows.

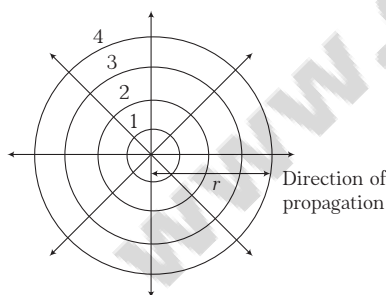


Fig. 10.1 Spherical wavefront of a point source

In spherical wavefront, amplitude of light waves, $A \propto \frac{1}{r}$

and intensity of light waves, $I \propto \frac{1}{r^2}$.

(ii) Cylindrical wavefront

When the source of light is linear, *e.g.* a straight line source as shown in figure. All the points are equidistant from the source.

Therefore wavefronts 1, 2 and 3 are cylindrical and co-axial with the source as their common axis.

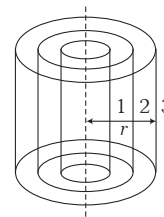


Fig. 10.2 Cylindrical wavefront of a linear source

In this case, intensity of light waves, $I \propto \frac{1}{r}$

and amplitude of light waves, $A \propto \frac{1}{\sqrt{r}}$.

(iii) Plane wavefront

When light sources are emitting parallel rays, or when the light is coming from a very far-off source wavefronts will be planes as shown in figure.

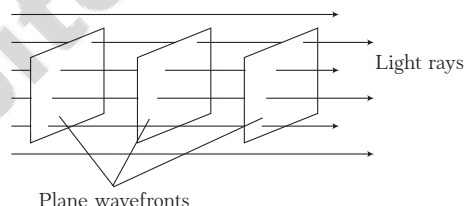


Fig. 10.3 Plane wavefront of a distant source

Note wavefronts parallel to xy -plane can be represented by $z = \text{constant}$. Similarly, wavefronts parallel to yz -plane and zx -plane are $x = \text{constant}$ and $y = \text{constant}$, respectively.

In this case, intensity of light waves, $I \propto r^0$

and amplitude of light waves, $A \propto r^0$.

Shapes of wavefronts of reflected and refracted waves

We have discussed the various types of wavefront. Keeping in mind those shapes, now we are going to see the shapes of wavefronts associated with reflected as well as refracted waves.

- (i) Shapes of wavefronts before and after reflection from a plane and curved mirrors are as shown below.

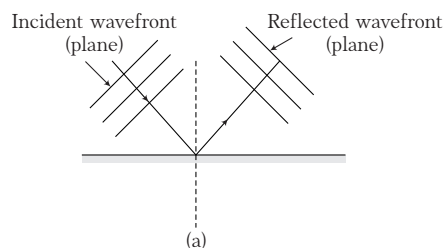


Fig. 10.4 Reflection from a plane mirror

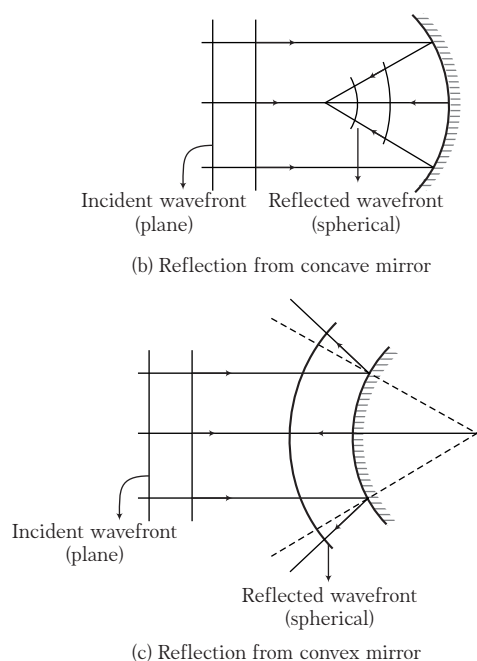


Fig. 10.5 Reflection from curved mirrors

- (ii) Shapes of wavefronts before and after refraction from a plane and curved surfaces are as shown below.

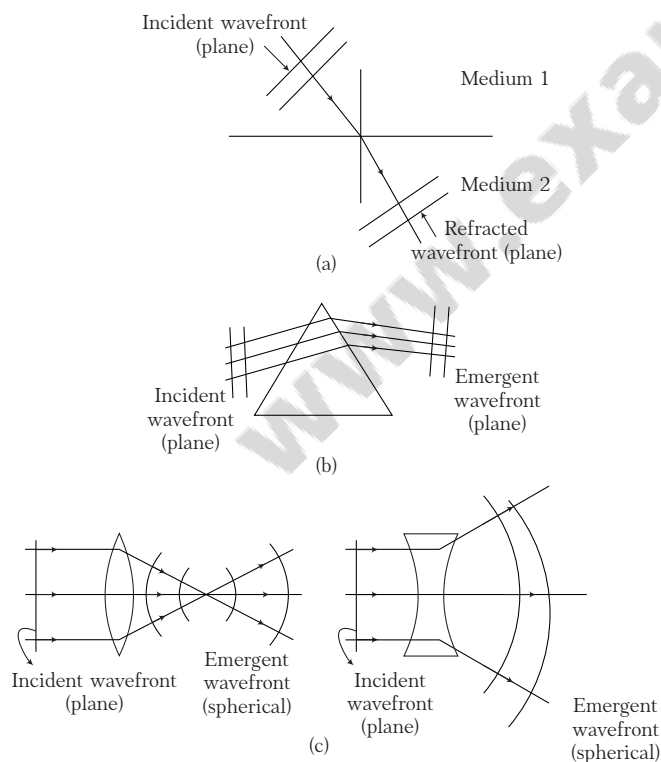
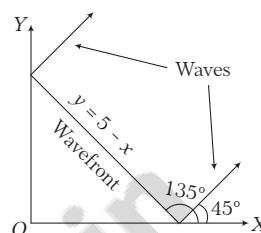


Fig. 10.6 (a) Refraction from a plane surface
(b) Refraction from a prism
(c) Refraction through lenses

Example 10.1 A wavefront is represented by the plane $y = 5 - x$. Find the direction of propagation of wave.

Sol. Given, wavefront, $y = 5 - x$

This wavefront can be represented by a line as shown in the figure. This wavefront makes an angle of 135° with X-axis. Therefore, the wave must be perpendicular to the wavefront and it will make an angle of 45° with X-axis as shown in figure.



Huygens' principle of secondary wavelets

In 1678, in order to explain the propagation of wave in a medium, Huygens' propounded a principle known as **Huygens' principle of secondary wavelets** by which at any instant, we can geometrically obtain the position of wavefront. He made following three assumptions

- Every point on a given wavefront (called **primary wavefront**) can be regarded as fresh source of new disturbance and sends out its own spherical wavelets called **secondary wavelets**.
- The secondary wavelets spread in all directions with the velocity of wave (*i.e.* velocity of light).
- A surface touching these secondary wavelets, tangentially in the forward direction at any instant (Δt) gives the position and shape of the new wavefront at the instant. This is called **secondary wavefront**.

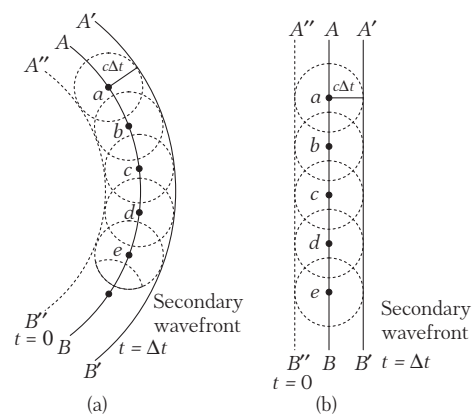


Fig. 10.7 Formation of secondary wavefront using Huygens' principle

In Fig. (a), AB is the section of the given spherical wavefront and $A'B'$ is the new wavefront in the forward direction.

In Fig. (b), AB is the section of the given plane wavefront and $A'B'$ is the new wavefront in the forward direction.

Note Huygens' argued that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction. Hence, the backward secondary wavelet is absent.

Merits

On the basis of Huygens' principle many optical phenomena like interference, diffraction, reflection (or laws of reflection) as well as refraction (or Snell's law) could be explained.

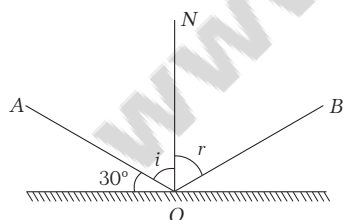
Demerits

Following are demerits of Huygens' principle

- This principle could not explain, why wavefronts of secondary wavelets are formed in forward direction and not in backward direction?
- This theory assumes light waves as longitudinal waves, therefore could not explain polarisation of light.
- Michelson-Morley does not assume the ether particle as a medium in his experiment.
- This principle could not explain electromagnetic nature of light.

Example 10.2 A plane wavefront incident on a reflecting surface at an angle of 30° with horizontal. Find the angle of reflected wavefront with horizontal.

Sol. Consider a plane wavefront OA incident at an angle of 30° with horizontal, ON is normal to the reflecting surface, $\angle i$ is angle of incidence and $\angle r$ is angle of reflection.



Now, from figure, we have

$$\angle i + 30^\circ = 90^\circ$$

$$\Rightarrow \angle i = 60^\circ = \angle r$$

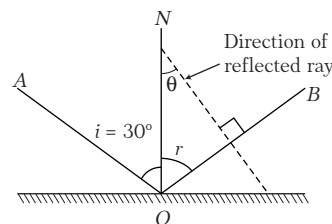
($\because \angle i = \angle r$, according to law of reflection)

$$\text{Angle with horizontal} = 90^\circ - \angle r = 30^\circ$$

Example 10.3 A plane wavefront incident on a reflecting surface at an angle of 30° with vertical. Find the angle (acute) of reflected wavefront with vertical.

Sol. Consider a plane wavefront OA incident at an angle of 30° with vertical, ON is normal to the reflecting surface, $\angle i$ is angle of incidence and $\angle r$ is angle of reflection.

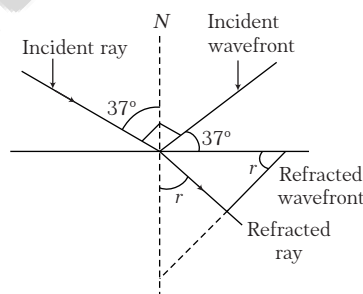
According to law of reflection, $\angle i = \angle r = 30^\circ$



Reflected ray must be perpendicular to the reflected wavefront (OB) as shown in the figure. Therefore, acute angle made by reflected ray with the vertical is $\theta = 90^\circ - r$
 $= 90^\circ - 30^\circ = 60^\circ$

Example 10.4 A plane wavefront is incident from air ($\mu = 1$) at an angle of 37° with a horizontal boundary of a refractive medium from air of refractive index $\mu = \frac{3}{2}$. Find the angle of refracted wavefront with the horizontal boundary.

Sol. It has been given that incident wavefront makes 37° with horizontal. Hence, incident ray makes 37° with normal as the ray is perpendicular to the wavefront.



$$\text{Now, by Snell's law, } \frac{\sin 37^\circ}{\sin r} = \frac{3}{2}$$

$$\Rightarrow \sin r = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$

$\Rightarrow r = \sin^{-1}\left(\frac{2}{5}\right)$, which is same as angle of refracted wavefront with horizontal.

Example 10.5 A plane wavefront of yellow light with wavelength $0.5 \mu\text{m}$ in air suffers refraction in a medium in which velocity of light is $2 \times 10^8 \text{ ms}^{-1}$. Find the wavelength of the light associated with a wavefront in the medium.

Sol. For wavelength and speed of light associated with a wavefront,

$$\text{we have } \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\text{Here, } \lambda_1 = 0.5 \mu\text{m}, v_1 = 3 \times 10^8 \text{ ms}^{-1} \text{ and } v_2 = 2 \times 10^8 \text{ ms}^{-1}$$

$$\therefore \frac{0.5}{\lambda_2} = \frac{3 \times 10^8}{2 \times 10^8}$$

Hence, the wavelength of the light associated with a wavefront in the medium is $\lambda_2 = 0.33 \mu\text{m}$.

Principle of superposition of waves

According to superposition principle, at a particular point in the medium, the resultant displacement y produced by a number of waves is the vector sum of the displacements produced by each of the waves ($y_1, y_2, \dots$).

$$\text{i.e. } y = y_1 + y_2 + y_3 + y_4 + \dots$$

Clearly, each wave contributes as, if the other wave is not present. The superposition principle, which was stated first for mechanical waves is equally applicable to the electromagnetic (light) waves.

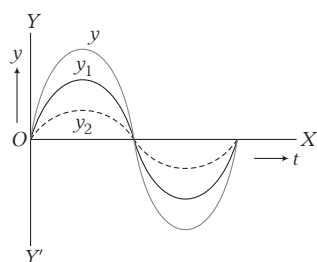


Fig. 10.8 Superposition of waves

Interference of light wave

When two light waves of exactly equal frequency having constant phase difference with respect to time travelled in same direction and superimpose (overlap) with each other, then intensity of resultant wave does not remain uniform in space.

This phenomenon of formation of maximum intensity at some points and minimum intensity at some other points by the two identical light waves travelling in same direction is called the **interference of light**.

Types of interference

Depending on phase difference in superimposing waves, interference is divided into two categories as follows

(i) Constructive interference

When the two waves meet in same phase, i.e. the intensity of light is maximum, is called the constructive interference.

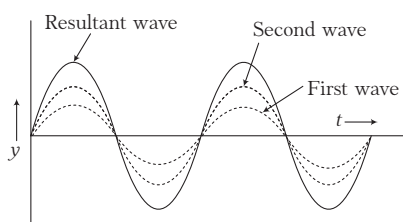


Fig. 10.9 Constructive interference

(ii) Destructive interference

When the two waves meet in opposite phase, i.e. the intensity of light is minimum is called the destructive interference.

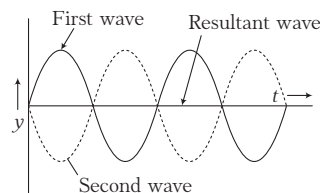


Fig. 10.10 Destructive interference

Note During interference of light redistribution of energy (intensity) takes place but total energy remains conserved.

Expression for resultant intensity in interference of two waves

Let the waves from two sources of light S_1 and S_2 , interfere at the point P as shown in figure.

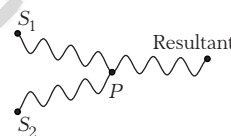


Fig. 10.11 Interference of two waves

$$y_1 = a \sin \omega t \quad \text{and} \quad y_2 = b \sin(\omega t + \phi)$$

where, a and b are the respective amplitudes of the two waves and ϕ is the constant phase angle by which second wave leads the first wave.

Applying superposition principle, the magnitude of the resultant displacement of the waves is

$$y = y_1 + y_2 \quad \text{or} \quad y = a \sin \omega t + b \sin(\omega t + \phi)$$

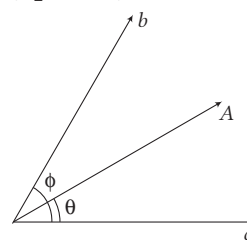


Fig. 10.12 Resultant of amplitudes a and b

Therefore, we get

$$y = A \sin(\omega t + \theta)$$

where, A is the resultant amplitude and θ is the resultant phase difference as shown in figure.

$$\therefore A = \sqrt{a^2 + b^2 + 2ab \cos \phi}$$

$$\text{and} \quad \tan \theta = \frac{b \sin \phi}{a + b \cos \phi}$$

As, intensity is directly proportional to the square of the amplitude of the wave, i.e. $I \propto a^2$.

So, for two different cases,

$$\therefore I_1 = ka^2, I_2 = kb^2$$

$$\text{and } I = kA^2 = k(a^2 + b^2 + 2ab \cos \phi)$$

$$\therefore \boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi}$$

Note Relation between path difference, phase difference and time difference,

$$\frac{2\pi}{\text{Phase diff.}} = \frac{\lambda}{\text{Path diff.}} = \frac{T}{\text{Time diff.}}$$

For constructive interference

Intensity, I should be maximum, for which
 $\cos \phi = \text{maximum} = +1$

$$\therefore \text{Phase difference, } \phi = 0, 2\pi, 4\pi, \dots$$

$$\text{i.e. } \phi = 2n\pi$$

where, $n = 1, 2, 3, \dots$

If Δx be the path difference between the two waves, then

$$\Delta x = \left(\frac{\lambda}{2\pi}\right)\phi = \left(\frac{\lambda}{2\pi}\right)(2n\pi) = n\lambda$$

$$\therefore \text{Path difference, } \Delta x = n\lambda$$

where, $n = 1, 2, 3, \dots$

Hence, condition for constructive interference at a point is that, the phase difference between the two waves reaching the point should be zero or an even integral multiple of π . Equivalent path difference between the two waves reaching the point should be zero or an even integral multiple of wavelength.

$$\begin{aligned} \therefore \text{Maximum intensity, } I_{\max} &= I_1 + I_2 + 2\sqrt{I_1 I_2} \\ &= (\sqrt{I_1} + \sqrt{I_2})^2 = k(a + b)^2 \end{aligned}$$

For destructive interference

Intensity, I should be minimum, for which
 $\cos \phi = \text{minimum} = -1$

$$\therefore \text{Phase difference, } \phi = \pi, 3\pi, 5\pi, \dots$$

$$\text{i.e. } \phi = (2n - 1)\pi$$

where, $n = 1, 2, 3, \dots$

The corresponding path difference between the two waves is

$$\Delta x = \left(\frac{\lambda}{2\pi}\right)\phi = \left(\frac{\lambda}{2\pi}\right)(2n - 1)\pi = (2n - 1)\frac{\lambda}{2}$$

$$\therefore \text{Path difference, } \Delta x = (2n - 1)\frac{\lambda}{2}, \text{ where } n = 1, 2, 3, \dots$$

Hence, condition for destructive interference at a point is that, the phase difference between the two waves reaching the point should be an odd integral multiple of π or path difference between the two waves reaching the point should be an odd integral multiple of half the wavelength.

$$\begin{aligned} \therefore \text{Minimum intensity, } I_{\min} &= I_1 + I_2 - 2\sqrt{I_1 I_2} \\ &= (\sqrt{I_1} - \sqrt{I_2})^2 = k(a - b)^2 \end{aligned}$$

Comparison of intensities of maxima and minima

As, we know that,

$$I_{\max} \propto (a + b)^2 \text{ and } I_{\min} \propto (a - b)^2$$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(a + b)^2}{(a - b)^2} = \left(\frac{\frac{a}{b} + 1}{\frac{a}{b} - 1}\right)^2 = \frac{(r + 1)^2}{(r - 1)^2}$$

where, $r = \frac{a}{b}$ (ratio of amplitudes).

Also, we can write as

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{a + b}{a - b}\right)^2$$

Example 10.6 Two sources of intensity I and $4I$ are used in an interference experiment. Find the intensity at a point, where the waves from two sources superimpose with a phase difference of (i) zero, (ii) $\pi/2$, (iii) π and (iv) ratio of maximum and minimum intensities.

Sol. Given, $I_1 = I$ and $I_2 = 4I$

$$\text{Intensity, } I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

(i) At $\phi = 0$,

$$\text{Intensity, } I_R = I + 4I + 2\sqrt{I \cdot 4I} \cos 0^\circ = 9I = I_{\max}$$

(ii) At $\phi = \frac{\pi}{2}$,

$$\text{Intensity, } I_R = I + 4I + 2\sqrt{I \cdot 4I} \cos(\pi/2) = 5I$$

(iii) At $\phi = \pi$,

$$\text{Intensity, } I_R = I + 4I + 2\sqrt{I \cdot 4I} \cos(\pi) = I = I_{\min}$$

(iv) Ratio of maximum and minimum intensities,

$$\frac{I_{\max}}{I_{\min}} = \frac{9I}{I} = 9:1$$

Example 10.7 Two sources with intensity I_0 and $4I_0$ respectively, interfere at a point in a medium. Find the ratio of (i) maximum and minimum intensities, (ii) amplitudes.

Sol. (i) Maximum intensity, $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$$\text{Here, } I_1 = I_0 \text{ and } I_2 = 4I_0$$

$$\therefore I_{\max} = (\sqrt{I_0} + \sqrt{4I_0})^2 = 9I_0$$

$$\text{and minimum intensity, } I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 = I_0$$

$$\Rightarrow \frac{I_{\max}}{I_{\min}} = \frac{9I_0}{I_0} = 9:1$$

$$\text{(ii) Ratio of intensities, } \frac{I_{\max}}{I_{\min}} = \left(\frac{r + 1}{r - 1}\right)^2$$

where, r = ratio of amplitudes.

$$\Rightarrow \frac{9}{1} = \left(\frac{r + 1}{r - 1}\right)^2 \Rightarrow 3 = \frac{r + 1}{r - 1}$$

$$\Rightarrow 3r - 3 = r + 1 \Rightarrow 2r = 4 \Rightarrow r = 2:1$$

Coherent and incoherent sources

Light sources can be coherent or incoherent which are discussed below.

Coherent sources

Two sources of light which continuously emit light waves of same frequency with a zero or constant phase difference between them are called coherent sources.

Incoherent sources

Two sources of light which do not emit light waves with a constant phase difference are called incoherent sources.

Need of coherent sources for the production of interference pattern

When two monochromatic waves of intensity I_1, I_2 and phase difference ϕ meet at a point, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

Here the term $2\sqrt{I_1 I_2} \cos \phi$ is called **interference term**.

There are two possibilities.

- (i) If $\cos \phi$ remains constant with time, then the total intensity at any point will be constant. The intensity will be maximum $(\sqrt{I_1} + \sqrt{I_2})^2$ at points, where $\cos \phi$ is 1 and minimum $(\sqrt{I_1} - \sqrt{I_2})^2$ at points, where $\cos \phi = -1$. The sources in this case are coherent.
- (ii) If $\cos \phi$ varies continuously with time assuming both positive and negative values, then the average value of $\cos \phi$ will be zero over time interval of measurement. The interference term averages to zero. There will be same intensity $I = I_1 + I_2$ at every point. The two sources in this case are incoherent.

Note In practice, coherent sources are produced either by dividing the wavefront or by dividing the amplitude of the incoming waves.

Example 10.8 Two incoherent sources emitting lights of intensities I_0 and $3I_0$ interfere in a medium. Calculate the resultant intensity at any point.

Sol. Since, the two sources are incoherent, so no sustained interference pattern is obtained.

We observe an average intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

Now, $\cos \phi = 0$ in one complete cycle.

$$\therefore I = I_1 + I_2 = 4I_0$$

Requirements to find out coherent sources

- (i) Coherent sources of light should be obtained from a single source by some device.

Two coherent sources can be obtained either by

- (a) the source and its virtual image (Lloyd's mirror).
- (b) the two virtual images of the same source (Fresnel's biprism).
- (c) two real images of the same source (Young's double slit).
- (ii) The two sources should give monochromatic light.
- (iii) The path difference between light waves from two sources should be small.
- (iv) Coherent sources are produced by
 - (a) division of wavefront (Young's double slit experiment).
 - (b) division of amplitude (partial reflection or refraction in thin films).

Necessary conditions for interference of light

In order to obtain a sustained (permanent or stable) and observable interference pattern, the following conditions must be fulfilled

- (i) **Sources must be coherent** In order to produce a stable interference pattern, the individual waves must maintain a constant phase relationship with one another, i.e. the two interfering sources must emit waves having a constant phase difference between them. If the phase difference between two sources does not remain constant, then the places of maxima and minima shift.

In case of mechanical waves, it is possible to keep a constant phase relationship between two different sources. But in case of light, two different light sources cannot be coherent.

- (ii) **Same frequency or wavelength** Phase relationship between two waves can be kept constant only when their frequencies are same. Thus we can say that, two coherent sources must have the same frequency.
- (iii) **Equality of amplitudes** The amplitudes of two interfering waves should be equal or approximately equal. Otherwise the difference between the intensities of maxima and minima will be too small and the contrast will be poor. Maximum contrast is, however obtained when $A_1 = A_2$, because then minimum intensity will be zero.

CHECK POINT 10.1

- According to corpuscular theory of light, the different colours of light are due to
 - different electromagnetic waves
 - different force of attraction among the corpuscles
 - different sizes of the corpuscles
 - None of the above
- Two identical light sources S_1 and S_2 emit light of same wavelength λ . These light rays will exhibit interference, if
 - their phase differences remain constant
 - their phases are distributed randomly
 - their light intensities remain constant
 - their light intensities change randomly
- When interference of light takes place
 - energy is created in the region of maximum intensity
 - energy is destroyed in the region of maximum intensity
 - conservation of energy holds good and energy is redistributed
 - conservation of energy does not hold good
- If two waves represented by $y_1 = 4 \sin \omega t$ and $y_2 = 3 \sin \left(\omega t + \frac{\pi}{3} \right)$ interfere at a point, then the amplitude of the resulting wave will be about
 - 7 m
 - 6 m
 - 5 m
 - 3.5 m
- Two coherent sources of intensities I_1 and I_2 produce an interference pattern. The maximum intensity in the interference pattern will be
 - $I_1 + I_2$
 - $I_1^2 + I_2^2$
 - $(I_1 + I_2)^2$
 - $(\sqrt{I_1} + \sqrt{I_2})^2$
- What is the path difference in case of destructive interference?
 - $n\lambda$
 - $\frac{n(\lambda + 1)}{2}$
 - $\frac{(n + 1)\lambda}{2}$
 - $\frac{(2n + 1)\lambda}{2}$
- If the amplitude ratio of two sources producing interference is 3 : 5, then the ratio of intensities at maxima and minima is
 - 25 : 16
 - 5 : 3
 - 16 : 1
 - 25 : 9
- Two sources of light of the same frequency produce intensities I and $4I$ at a point P when used individually. If they are used together such that the light from them reach P with a phase difference of $2\pi/3$, then the intensity at P will be
 - $2I$
 - $3I$
 - $4I$
 - $5I$
- Two incoherent sources of intensities I and $4I$ superpose, then the resultant intensity is
 - $5I$
 - $9I$
 - $3I$
 - I
- Which of the following is not an essential condition for interference?
 - The two interfering waves must propagate in almost the same direction.
 - The waves must have the same period and wavelength.
 - The amplitudes of the two waves must be equal.
 - The two interfering beams of light must originate from the same source.

YOUNG'S DOUBLE SLIT EXPERIMENT (YDSE)

One of the first to demonstrate the interference of light was Thomas Young in 1801.

In his experimental arrangement (as shown in Fig. 10.13), monochromatic light (single wavelength) from a narrow vertical slit S falls on two other narrow slits S_1 and S_2 which are very close together and equidistant from S . S_1 and S_2 act as two coherent sources (both being derived from S).

The emerging beams spread into the region beyond the slits. Superposition occurs in the shaded area, where the beams overlap. Alternate bright and dark equally spaced vertical bands (interference fringes) can be observed on a screen placed at some distance from the slits.

If either of S_1 or S_2 is covered, then the fringes disappear.

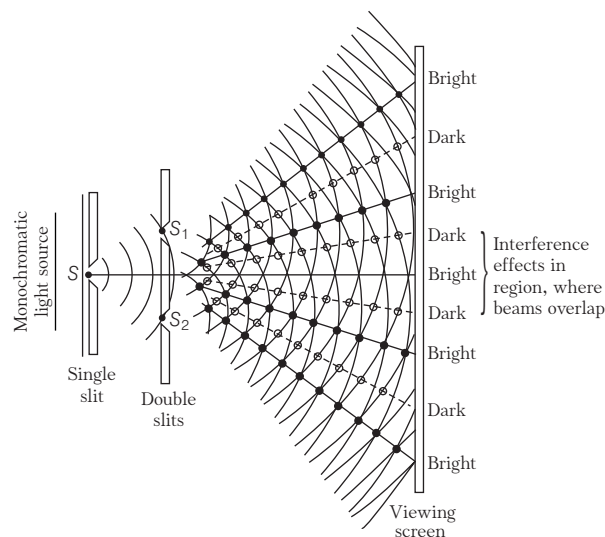


Fig. 10.13 Young's experimental arrangement to produce interference pattern

Path difference

Figure shows the light waves from S_1 and S_2 meeting at an arbitrary point P on the screen.

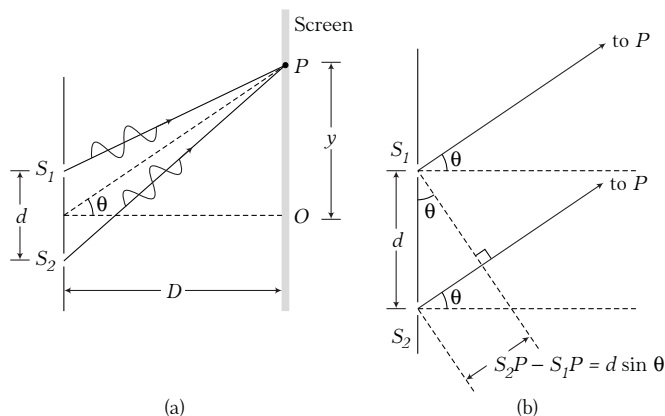


Fig. 10.14 (a) To reach P , the light waves from S_1 and S_2 must travel different distances. (b) The path difference between the two rays is $d \sin \theta$.

Since $D \gg d$, the two light rays are approximately parallel with a path difference,

$$\Delta x = S_2P - S_1P \approx d \sin \theta$$

In addition to our assumption that $D \gg d$, we assume that $d \gg \lambda$. These can be valid assumptions because in practice D is often of the order of 1 m, d is a fraction of a millimetre and λ is a fraction of a micrometre for visible light. Under these conditions θ is small, thus we can use

the approximation, $\sin \theta \approx \tan \theta = \frac{y}{D}$

$$\therefore \text{Path difference, } \Delta x = \frac{y}{D} d \quad \dots(i)$$

This is the path difference between two waves meeting at a point on the screen. Due to this path difference, some points on the screen are bright and some points are dark. Now, we will discuss the position of these bright and dark fringes and fringe width under following points

Position of bright fringe

For maximum intensity at P ,

$$\Delta x = n\lambda$$

$$\text{or} \quad d \sin \theta = n\lambda \quad (\text{where, } n = 0, \pm 1, \pm 2, \dots) \quad \dots(ii)$$

The bright fringe corresponding to the integer n is called the **n th order (or just n th) bright fringe**. The bright fringe for $n = 0$, is known as the central fringe and its centre (point O) is called the **central (or zero order) maximum**. Higher order bright fringes are situated symmetrically about the central fringe. It is useful to obtain expression for the position of the bright fringe measured vertical from O .

From Eqs. (i) and (ii), we get

Position of bright fringe,

$$y_{\text{bright}} = \frac{n\lambda}{d} D \quad \dots(iii)$$

Example 10.9 In a YDSE, green light of wavelength 500 nm is used. Where will be the second bright fringe formed for a set up in which separation between slits is 4 mm and the screen is placed 1 m from the slits?

Sol. Position of second bright fringe,

$$\begin{aligned} y_2 &= \frac{2\lambda D}{d} \\ &= \frac{2 \times 500 \times 10^{-9} \times 1}{4 \times 10^{-3}} \\ &= 0.25 \text{ mm} \end{aligned}$$

Note While reporting our answer we should report $y_2 = \pm 0.25$ mm as fringe will be formed both, above and below central bright fringe.

Example 10.10 Bichromatic light is used in YDSE having wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 700$ nm. Find minimum order of λ_1 which overlaps with λ_2 .

Sol. Let n_1 bright fringe of λ_1 overlaps with n_2 bright fringe of λ_2 .

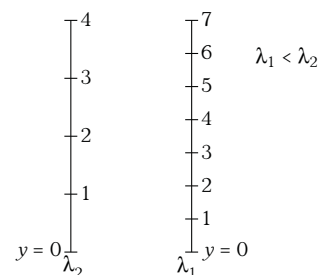
Then, their position will be same, i.e.

$$\frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d}$$

or

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1}$$

$$= \frac{700}{400} = \frac{7}{4}$$



The ratio $\frac{n_1}{n_2} = \frac{7}{4}$ implies that 7th bright fringe of λ_1 will

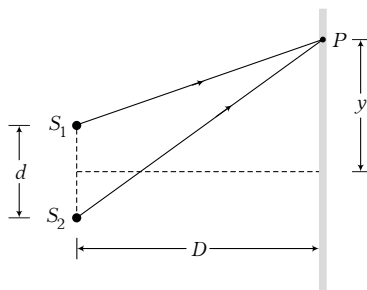
overlap with 4th bright fringe of λ_2 .

Similarly, 14th bright fringe of λ_1 will overlap with 8th bright fringe of λ_2 and so on.

So, the minimum order of λ_1 which overlaps with λ_2 is 7.

Example 10.11 Young's double slit experiment is carried out using microwaves of wavelength $\lambda = 3$ cm. Distance between the slits is $d = 5$ cm and the distance between the plane of slits and the screen is $D = 100$ cm. Find the number of maxims and their positions on the screen.

Sol. The maximum path difference that can be produced = distance between the sources or 5 cm



Thus, in this case, we can have only three maximas, one central maxima and two on its either side for a path difference of $\lambda = 3$ cm.

For maximum intensity at P,

$$S_2P - S_1P = \lambda$$

$$\text{or } \sqrt{(y + d/2)^2 + D^2} - \sqrt{(y - d/2)^2 + D^2} = \lambda$$

Here, $d = 5$ cm, $D = 100$ cm

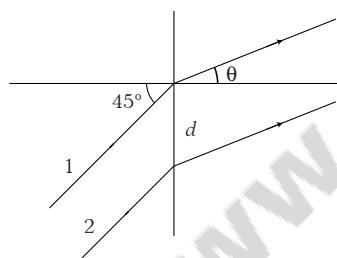
and $\lambda = 3$ cm

$\therefore y = \pm 75$ cm

Thus, the three maximas will be at

$$y = 0 \text{ and } y = \pm 75 \text{ cm}$$

Example 10.12 Distance between the slits, in YDSE, shown in figure is $d = 20\lambda$, where λ is the wavelength of light used. Find the angle θ , where



- (i) central maxima (where path difference is zero) is obtained.
 (ii) third order maxima is obtained.

Sol. Ray 1 has a longer path than that of ray 2 by a distance $d \sin 45^\circ$, before reaching the slits. Afterwards ray 2 has a path longer than ray 1 by a distance $d \sin \theta$. The net path difference is therefore, $d \sin \theta - d \sin 45^\circ$.

- (i) Central maximum is obtained, where net path difference is zero,

$$\text{or } d \sin \theta - d \sin 45^\circ = 0 \text{ or } \theta = 45^\circ$$

- (ii) Third order maxima is obtained, where net path difference is 3λ ,

$$\text{or } d \sin \theta - d \sin 45^\circ = 3\lambda$$

$$\therefore \sin \theta = \sin 45^\circ + \frac{3\lambda}{d}$$

Putting $d = 20\lambda$,

$$\therefore \sin \theta = \sin 45^\circ + \frac{3\lambda}{20\lambda} \text{ or } \theta \approx 59^\circ$$

Position of dark fringe

In previous point, we obtained the position of bright fringe. In the similar way, we can obtain position of dark fringe as follows

For minimum intensity at P,

$$\Delta x = (2n - 1) \frac{\lambda}{2}$$

$$\text{or } d \sin \theta = (2n - 1) \frac{\lambda}{2} \text{ (where, } n = \pm 1, \pm 2, \dots) \dots \text{(iv)}$$

The first minima ($n = \pm 1$) are adjacent to the central maximum on either side. We will obtain expression for position of dark fringe measured vertically from O.

From Eqs. (i) and (iv), we get

Position of dark fringe,

$$y_{\text{dark}} = \frac{(2n - 1) \lambda}{2d} D \dots \text{(v)}$$

Example 10.13 In YDSE, the two slits are separated by 0.1 mm and they are 0.5 m from the screen. The wavelength of light used is 5000 Å. Find the distance between 7th maxima and 11th minima on the screen.

Sol. Given, $d = 0.1$ mm = 10^{-4} m, $D = 0.5$ m

$$\text{and } \lambda = 5000 \text{ Å} = 5.0 \times 10^{-7} \text{ m}$$

The distance between 7th maxima and 11th minima is

$$\Delta y = (y_{11})_{\text{dark}} - (y_7)_{\text{bright}}$$

$$= \frac{(2 \times 11 - 1) \lambda D}{2d} - \frac{7\lambda D}{d}$$

$$\text{or } \Delta y = \frac{7\lambda D}{2d} = \frac{7 \times 5.0 \times 10^{-7} \times 0.5}{2 \times 10^{-4}} = 8.75 \times 10^{-3} \text{ m}$$

$$= 8.75 \text{ mm}$$

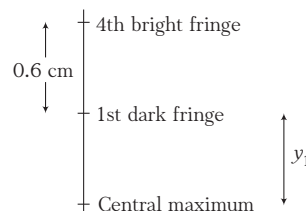
Example 10.14 In YDSE, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the first dark fringe and fourth bright fringe is obtained to be 0.6 cm. Determine the wavelength of the light used in the experiment.

Sol. Let us first identify the parameters given in the question, $d = 0.28$ mm, $D = 1.4$ m.

Also, separation between first dark fringe and fourth bright fringe is 0.6 cm.

Now, position of 1st dark fringe from central maximum be

$$y_1 = \frac{\lambda D}{2d}$$



Position of 4th bright fringe will be $y_2 = \frac{4\lambda D}{d}$
 Separation between them is equal to $\frac{4\lambda D}{d} - \frac{\lambda D}{2d} = 0.6 \text{ cm}$

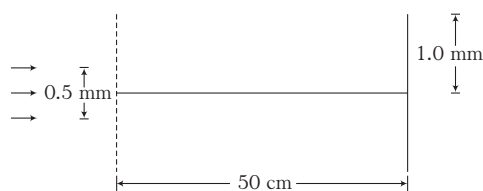
$$\Rightarrow \frac{7\lambda D}{2d} = 0.6 \text{ cm}$$

Wavelength of the light,

$$\lambda = \frac{0.6 \times 10^{-2} \times 2 \times 0.28 \times 10^{-3}}{7 \times 1.4} \\ = 3.428 \times 10^{-7} \text{ m}$$

Example 10.15 White coherent light (400 nm – 700 nm) is sent through the slits of a YDSE. There is a hole in the screen at a point 1.0 mm away (along the width of the fringes) from the central line. (Given, $d = 0.05 \text{ mm}$ and $D = 50 \text{ cm}$)

- (i) Which wavelength will be absent in the light coming from the hole?
 (ii) Which wavelength will have a strong intensity?



Sol. Given, $d = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$, $D = 50 \text{ cm} = 0.5 \text{ m}$
 and $y = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

- (i) For absent wavelength, use condition of minima,

$$y = (2n + 1) \frac{D\lambda}{2d} \\ \Rightarrow 10^{-3} = \frac{(2n + 1)(0.5)\lambda}{2 \times 0.5 \times 10^{-3}}$$

$$\text{Wavelength, } \lambda = \frac{2 \times 10^{-6}}{(2n + 1)} = \frac{2000}{2n + 1} \times 10^{-9} \text{ m} \\ = \frac{2000}{2n + 1} \text{ nm}$$

For $n = 0$, $\lambda = 2000 \text{ nm}$

For $n = 1$, $\lambda = \frac{2000}{3} = 667 \text{ nm}$

For $n = 2$, $\lambda = \frac{2000}{5} = 400 \text{ nm}$

In the range (400 – 700) nm, absent wavelengths are 400 nm, 667 nm.

- (ii) For wavelengths of strong intensity, use condition of maxima,

$$y = n \frac{D\lambda}{d} \Rightarrow 10^{-3} = \frac{n(0.5)(\lambda)}{0.5 \times 10^{-3}}$$

$$\text{Wavelength, } \lambda = \frac{10^{-6}}{n} = \frac{1000}{n} \times 10^{-9} \text{ m} = \frac{1000}{n} \text{ nm}$$

For $n = 1$, $\lambda = 1000 \text{ nm}$

For $n = 2$, $\lambda = 500 \text{ nm}$

For $n = 3$, $\lambda = 333 \text{ nm}$

In the range (400 – 700) nm, wavelength having strong intensity is 500 nm.

Fringe width

Distance between two adjacent bright (or dark) fringes is called the **fringe width**. It is denoted by β .

Thus,

$$\beta = \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d} = \frac{\lambda D}{d} \quad \text{or} \quad \boxed{\beta = \frac{\lambda D}{d}} \quad \dots(\text{vi})$$

Alternatively, we can show that distance between two successive dark fringes is also $\frac{\lambda D}{d}$.

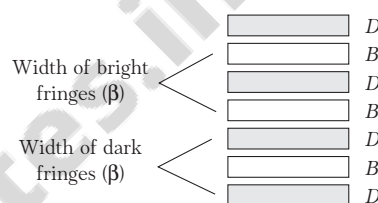


Fig. 10.15 Width of bright and dark fringes

Example 10.16 In YDSE, two slits are placed 2 mm from each other. Interference pattern is observed on a screen placed 2 m from the plane of slits. What is the fringe width for a light of wavelength 400 nm?

Sol. Given, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$, $D = 2 \text{ m}$
 and $\lambda = 400 \text{ nm} = 400 \times 10^{-9} \text{ m}$

$$\therefore \text{Fringe width, } \beta = \frac{\lambda D}{d} = \frac{400 \times 10^{-9} \times 2}{2 \times 10^{-3}} = 400 \times 10^{-6} \text{ m}$$

Example 10.17 In Young's double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from the plane of slits. If the screen is moved by 5 cm towards the slits, then the change in fringe width is $30 \mu\text{m}$. If the distance between the slits is 1 mm, then calculate wavelength of the light used.

Sol. Fringe width, $\beta = \frac{D\lambda}{d} \quad \dots(\text{i})$

When D decreases, then fringe width decreases,

$$\beta - 30 \times 10^{-6} = \frac{(D - 5 \times 10^{-2})\lambda}{d} \quad \dots(\text{ii})$$

Subtracting Eq. (ii) from Eq. (i), we get

$$30 \times 10^{-6} = \frac{5 \times 10^{-2}\lambda}{d} = \frac{5 \times 10^{-2}\lambda}{1 \times 10^{-3}}$$

Wavelength of the light,

$$\lambda = \frac{30 \times 10^{-6} \times 10^{-3}}{5 \times 10^{-2}} \\ = 6 \times 10^{-7} = 600 \times 10^{-9} \text{ m} \\ = 600 \text{ nm}$$

Important points related with fringe width

As, we know that, fringe width (β) is the distance between two successive maxima or minima. It is given by

$$\beta = \frac{\lambda D}{d} \quad \text{or} \quad \boxed{\beta \propto \lambda}$$

Two conclusions can be drawn from this relation

- (i) If Young's double slit experiment apparatus is immersed in a liquid of refractive index μ , then wavelength of light and hence fringe width decreases μ times.
- (ii) If white light is used in place of a monochromatic light, then coloured fringes are obtained on the screen with red fringes of larger size than that of violet because $\lambda_{\text{red}} > \lambda_{\text{violet}}$.

But note that centre is still white because there, path difference is zero for all colours. Hence, all the wavelengths interfere constructively. At other places, light will interfere destructively for those wavelengths for whom path difference is $\lambda/2, 3\lambda/2, \dots$, etc., and they will interfere constructively for the wavelengths for whom path difference is $\lambda, 2\lambda, \dots$, etc.

Note Shape of fringes on the screen is hyperbolic. But, if the screen is placed at very large distance from slits, then the hyperbola nearly looks straight line in shape.

Example 10.18 Fringe width in a particular YDSE is measured to be β . What will be the fringe width, if wavelength of the light is doubled, separation between the slits is halved and separation between the screen and slits is tripled?

Sol. We know that, $\beta = \frac{\lambda D}{d}$

Now, the changed values are $\lambda' = 2\lambda$, $d' = \frac{d}{2}$ and $D' = 3D$

The new fringe width will be

$$\beta' = \frac{\lambda' D'}{d'} = \frac{(2\lambda)(3D)}{\left(\frac{d}{2}\right)} = 12 \frac{\lambda D}{d} = 12\beta$$

The fringe width increases to 12 times of the original.

Example 10.19 Whose fringe width will be larger, the one for red light or the one for yellow light, all other fringes be the same?

Sol. We know that, fringe width, $\beta = \frac{\lambda D}{d}$

Now, for same value of D and d , $\beta \propto \lambda$.

Now, $\lambda_{\text{red}} > \lambda_{\text{yellow}}$

Hence, $\beta_{\text{red}} > \beta_{\text{yellow}}$

Hence, fringe width will be greater when red coloured light is used.

Angular width of fringes

Let angular position of n th bright fringe is θ_n and because of its small value $\tan \theta_n \approx \theta_n$

$$\therefore \theta_n = \frac{Y_n}{D} = \frac{nD\lambda/d}{D} = \frac{n\lambda}{d}$$

Similarly, if angular position of $(n+1)$ th bright fringe is

$$\theta_{n+1}, \text{ then } \theta_{n+1} = \frac{Y_{n+1}}{D} = \frac{(n+1)D\lambda/d}{D} = \frac{(n+1)\lambda}{d}$$

$\therefore$ Angular width of a fringe,

$$\theta = \theta_{n+1} - \theta_n = \frac{(n+1)\lambda}{d} - \frac{n\lambda}{d} = \frac{\lambda}{d}$$

$$\text{Also, } \beta = \frac{D}{d} \lambda$$

$$\therefore \boxed{\theta = \frac{\lambda}{d} = \frac{\beta}{D}}$$

It is independent of n , i.e. angular width of all fringes are same.

Example 10.20 Two slits in YDSE are placed 1 mm from each other. Interference pattern is observed on a screen placed 1 m from the plane of slits. What is the angular fringe width for a light of wavelength 400 nm.

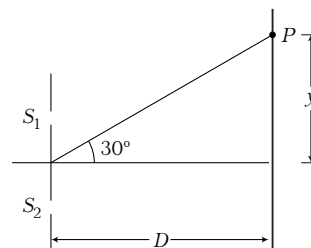
Sol. Given, $d = 1 \text{ mm} = 10^{-3} \text{ m}$

$$D = 1 \text{ m}, \lambda = 400 \text{ nm} = 400 \times 10^{-9} \text{ m}$$

$$\text{Angular fringe width, } \theta = \frac{\lambda}{d} = \frac{400 \times 10^{-9}}{10^{-3}} = 400 \times 10^{-6} \text{ rad}$$

Example 10.21 Two slits are separated by 0.32 mm. A beam of 500 nm strikes the slits producing an interference pattern. Determine the number of maxima observed in the angular range $-30^\circ < \theta < 30^\circ$.

Sol. Fringe width, $\beta = \frac{\lambda D}{d}$ and $y = \frac{D}{\sqrt{3}}$



Therefore, number of fringe width in a distance y ,

$$n = \frac{y}{\beta} = \frac{d}{\sqrt{3}\lambda} = \frac{0.32 \times 10^{-3}}{(\sqrt{3})(500 \times 10^{-9})} = 369.5$$

Therefore, total number of maxima obtained in the angular range $-30^\circ < \theta < 30^\circ$ (including the central one) is

$$N = 2n + 1 = 2 \times 369.5 + 1 = 739 + 1 = 740$$

Example 10.22 In Young's double slit experiment, separation between slits is 1 mm, distance of screen from slits is 2 m. If wavelength of incident light is 500 nm. Determine

- fringe width
- angular fringe width
- distance between 4th bright fringe and 3rd dark fringe.
- if whole arrangement is immersed in water ($\mu_w = 4/3$), new angular fringe width.

Sol. Given, $d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$ $D = 2 \text{ m}$,

$$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$$

- (i) Fringe width,

$$\beta = \frac{D\lambda}{d} = \frac{2 \times 500 \times 10^{-9}}{1 \times 10^{-3}} = 10^{-3} \text{ m} = 1 \text{ mm}$$

- (ii) Angular fringe width,

$$\theta = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{1 \times 10^{-3}} = 5 \times 10^{-4} \text{ rad}$$

- (iii) Distance of 4th bright fringe from centre,

$$y_4 = \frac{nD\lambda}{d} = \frac{4D\lambda}{d}$$

Distance of 3rd dark fringe from centre,

$$y'_3 = \frac{(2n-1)D\lambda}{2d} = \frac{(2 \times 3 - 1)D\lambda}{2d} = \frac{5D\lambda}{2d}$$

Distance between 4th bright fringe and 3rd dark fringe is

$$\begin{aligned} y_4 - y'_3 &= \frac{4D\lambda}{d} - \frac{5D\lambda}{2d} = \frac{3D\lambda}{2d} \\ &= \frac{3}{2}\beta = \frac{3}{2} \times 1 = \frac{3}{2} \text{ mm} \end{aligned}$$

- (iv) New angular fringe width,

$$\theta_w = \frac{\theta}{\mu_w} = \frac{5 \times 10^{-4}}{4/3} = \frac{15}{4} \times 10^{-4} \text{ rad}$$

Intensity of the fringes

In Young's double slit experiment, we are basically interested in finding the resultant intensity at a general point P at a distance y from point O on the screen or at an angle θ from C as shown below. Rather we can say we will find intensity I as a function of y or θ .

Since, $d \ll D$, we can assume that intensity at P due to independent sources S_1 and S_2 are almost equal.

or $I_1 \approx I_2 = I_0$ (say)

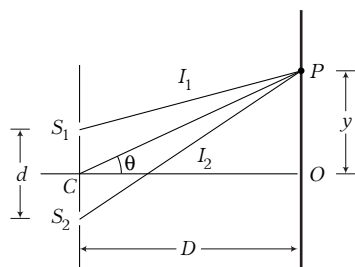


Fig. 10.16 Intensity in YDSE

For the two coherent sources S_1 and S_2 , the resultant intensity at point P is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

Putting $I_1 = I_2 = I_0$,

$$I = I_0 + I_0 + 2\sqrt{I_0 \times I_0} \cos \phi$$

Simplifying the above expression, we get

$$I = 4I_0 \cos^2 \frac{\phi}{2}$$

Maximum intensity

From above expression, we can see that intensity is maximum at points, where

$$\cos \frac{\phi}{2} = \pm 1 \quad \text{or} \quad \frac{\phi}{2} = n\pi, \text{ where } n = 0, \pm 1, \pm 2, \dots$$

$$\text{or} \quad \phi = 2n\pi \quad \text{or} \quad \frac{2\pi}{\lambda} \Delta x = 2n\pi$$

$$\Rightarrow \Delta x = n\lambda$$

We know that this path difference is for maxima. Thus, intensity of bright points are maximum and given by

$$I_{\max} = 4I_0$$

Therefore, we can write $I = I_{\max} \cos^2 \frac{\phi}{2}$

Minimum intensity

Minimum intensity on the screen is found at points, where

$$\cos \frac{\phi}{2} = 0 \quad \text{or} \quad \frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi$$

(where, $n = \pm 1, \pm 2, \pm 3, \dots$)

$$\text{or} \quad \phi = (2n - 1)\pi$$

$$\text{or} \quad \frac{2\pi}{\lambda} \Delta x = (2n - 1)\pi \Rightarrow \Delta x = (2n - 1) \frac{\lambda}{2}$$

We know that this path difference corresponds to minima. Thus, intensity of dark points are minimum and given by

$$I_{\min} = 0$$

Distribution of intensity

The distribution of intensity in Young's double slit experiment is shown below

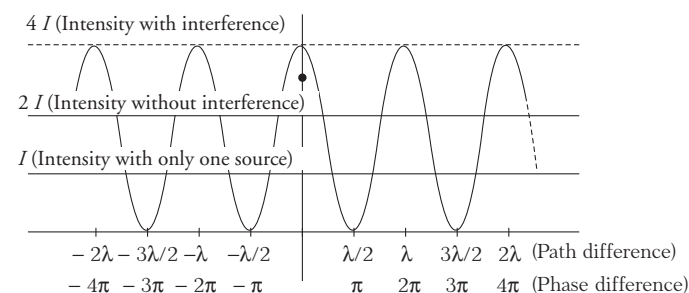


Fig. 10.17 Distribution of intensity in YDSE

Example 10.23 Two coherent sources each emitting light of intensity I_0 interfere, in a medium at a point, where phase difference between them is $\frac{2\pi}{3}$. Then, find the resultant intensity at that point.

Sol. Given, phase difference, $\phi = \frac{2\pi}{3}$

and resultant intensity, $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right) = 4I_0 \cos^2(\pi/3) = I_0$

Example 10.24 In a Young's double slit set up using monochromatic light of wavelength λ , the intensity of light at a point, where path difference is 2λ is found to be I_0 . What will be the intensity at a point where path difference is $\lambda/3$?

Sol. We know that, path difference is related to the phase difference according to the relation, $\phi = \frac{2\pi x}{\lambda}$

Now, phase difference, where path difference ($\lambda/3$) is equal to

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

Also, at the point, where path difference = 2λ

Phase difference will be $\frac{2\pi}{\lambda} \times 2\lambda = 4\pi$

Let individual source intensity be I .

$$\text{Then, } I_0 = 4I \cos^2 \frac{4\pi}{2} = 4I$$

$$\text{Hence, } I = \frac{I_0}{4}$$

Now, at the point, where phase difference is $\phi = \frac{2\pi}{3}$

$$I' = 4I \cos^2 \frac{2\pi}{3 \times 2}$$

$$I' = 4I \cos^2\left(\frac{\pi}{3}\right) = I = \frac{I_0}{4}$$

Example 10.25 Maximum intensity in YDSE is I_0 . Find the intensity at a point on the screen, where

(i) the phase difference between the two interfering

beams is $\frac{\pi}{3}$.

(ii) the path difference between them is $\frac{\lambda}{4}$.

Sol. (i) We know that, intensity,

$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

Here, $I_{\max}$ is I_0 (i.e. intensity due to independent sources is $I_0/4$).

Therefore, at phase difference, $\phi = \frac{\pi}{3}$ or $\frac{\phi}{2} = \frac{\pi}{6}$

$$\therefore \text{Intensity, } I = I_0 \cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4} I_0$$

(ii) Phase difference corresponding to the given path difference, $\Delta x = \frac{\lambda}{4}$ is

$$\phi = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) \quad \left(\because \phi = \frac{2\pi}{\lambda} \Delta x\right)$$

$$\text{or } \frac{\phi}{2} = \frac{\pi}{4}$$

$$\therefore I = I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

Example 10.26 In a Young's double slit set up the wavelength of light used is 546 nm. The distance of screen from slits is 1 m. The slit separation is 0.3 mm.

(i) Compare the intensity at a point P distant 10 mm from the central fringe, where the intensity is I_0 .

(ii) Find the number of bright fringes between P and the central fringe.

Sol. Given, $\lambda = 546 \text{ nm} = 5.46 \times 10^{-7} \text{ m}$,

$$D = 1 \text{ m and } d = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$$

(i) At a distance, $y = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$ from the central fringe, the path difference will be

$$\Delta x = \frac{y \cdot d}{D} = \frac{(10 \times 10^{-3})(0.3 \times 10^{-3})}{1} = 3.0 \times 10^{-6} \text{ m}$$

The corresponding phase difference between the two beams will be

$$\begin{aligned} \phi &= \frac{2\pi}{\lambda} \cdot \Delta x \\ &= \left(\frac{2\pi}{5.46 \times 10^{-7}}\right)(3.0 \times 10^{-6}) \text{ rad} \\ &= 34.5 \times \frac{180^\circ}{\pi} \\ &\approx 1977^\circ \end{aligned}$$

$$\therefore \frac{\phi}{2} = 988^\circ$$

$$\therefore I = I_0 \cos^2 \frac{\phi}{2} = I_0 \cos^2(988^\circ) = 3.0 \times 10^{-4} I_0$$

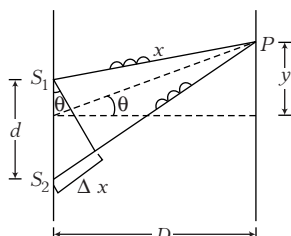
$$\begin{aligned} \text{(ii) Fringe width, } \beta &= \frac{\lambda D}{d} = \frac{(5.46 \times 10^{-7})(1)}{0.3 \times 10^{-3}} \text{ m} \\ &= 1.82 \text{ mm} \end{aligned}$$

$$\text{Since, } \frac{y}{\beta} = \frac{10}{1.82} = 5.49$$

Therefore, number of bright fringes between P and central fringe will be 5 (excluding the central fringe).

Example 10.27 Consider an interference arrangement similar to YDSE. Slits S_1 and S_2 are illuminated with coherent microwave sources each of frequency 2 MHz. The sources are synchronised to have zero phase difference. The slits are separated by distance $d = 75 \text{ m}$. The intensity I_θ is measured as a function of θ , where θ is defined as shown in figure.

If I_0 is maximum intensity, then calculate I_θ for (i) $\theta = 0^\circ$,
(ii) $\theta = \pi/6$ and (iii) $\theta = \pi/2$.



Sol. Given, $\nu = 2 \text{ MHz} = 2 \times 10^6 \text{ Hz}$, $d = 75 \text{ m}$

As, $c = \nu \lambda$

$$\Rightarrow \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{2 \times 10^6} = 150 \text{ m}$$

Path difference, $\Delta x = d \sin \theta$

$$\text{Phase difference, } \theta = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{\lambda} d \sin \theta = \frac{2\pi}{150} \times 75 \sin \theta = \pi \sin \theta$$

$$\text{Intensity at point } P, I = I_0 \cos^2(\theta/2) = I_0 \cos^2\left(\frac{\pi \sin \theta}{2}\right)$$

(i) For $\theta = 0^\circ$,

$$I = I_0$$

(ii) For $\theta = \frac{\pi}{6} = 30^\circ$,

$$I = I_0 \cos^2\left(\frac{\pi \sin 30^\circ}{2}\right) = I_0 \cos^2(\pi/4) = \frac{I_0}{2}$$

(iii) For $\theta = \frac{\pi}{2} = 90^\circ$,

$$I = 0$$

Insertion of transparent slab in YDSE : fringe shift

If a thin transparent plate (or slab) of thickness t and refractive index μ is placed in the path of one of interfering waves in YDSE, then fringe pattern shifts. First we will discuss the optical path, path difference due to the slab and then shifting in pattern as follows

- (i) **Optical path of a slab** If light travels a distance t in a slab (medium) of refractive index μ in a given time t_0 , then in this same time, it travels a distance μt in vacuum.

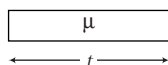


Fig. 10.18 Optical path of a slab

This distance μt is called **optical path** of the medium in air corresponding to distance t in the slab (medium).

- (ii) **Path difference produced by a slab** Consider two light waves 1 and 2 moving in air parallel to each

other. If a slab of refractive index μ and thickness t is inserted between the path of one of the rays, then a path difference produced among them is

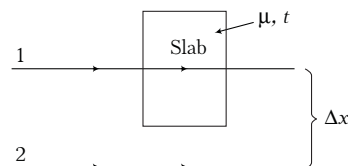


Fig. 10.19 Path difference between rays 1 and 2

$$\Delta x = (\mu - 1) t$$

- (iii) **Shifting of fringes** Suppose a glass slab of thickness t and refractive index μ is inserted onto the path of the ray emanating from source S_1 , then the whole fringe pattern shifts upwards by a distance $\frac{(\mu - 1) t D}{d}$. This can be calculated as follows

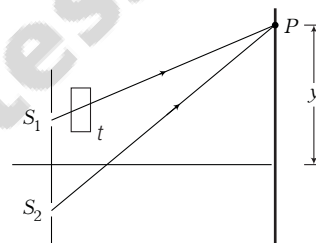


Fig. 10.20 Shift of fringes due to a slab

The net path difference between the two waves reaching at point P is

$$\Delta x = \Delta x_1 - \Delta x_2 \quad \text{or} \quad \Delta x = \frac{yd}{D} - (\mu - 1) t$$

For n th maxima on upper side, $\Delta x = n\lambda$

$$\text{or} \quad \frac{yd}{D} - (\mu - 1) t = n\lambda \Rightarrow y = \frac{n\lambda D}{d} + \frac{(\mu - 1) t D}{d}$$

Earlier it was $\frac{n\lambda D}{d}$

$$\therefore \text{Shift} = \frac{(\mu - 1) t D}{d}$$

Following three points are important with regard to above equation

- Shift is independent of n , (the order of the fringe) i.e. shift of zero order maximum = shift of 7th order maximum or shift of 5th order maximum = shift of 9th order minimum and so on.
- Shift is independent of λ , i.e. if white light is used, then shift of red colour fringe = shift of violet colour fringe.

$$(c) \text{ Number of fringes shifted} = \frac{\text{Shift}}{\text{Fringe width}}$$

$$= \frac{(\mu - 1) t D / d}{\lambda D / d} = \frac{(\mu - 1) t}{\lambda}$$

These numbers are inversely proportional to λ . This is because shift is same for all colours but fringe width of the colour having smaller value of λ is small, so more number of fringes will shift for this colour.

Example 10.28 Light travels in a mica sheet of refractive index 1.4 and length 10 cm. Find the optical path equivalent to length of the sheet.

Sol. Given, length of the sheet, $t = 10$ cm

Refractive index, $\mu = 1.4$

$\therefore$ Optical path $= \mu t = (1.4) \times (10) = 14$ cm

Example 10.29 Interference fringes are produced by a double slit arrangement and a piece of plane parallel glass of refractive index 1.5 is interposed in one of the interfering beam. If the fringes are displaced through 30 fringe widths for light of wavelength 6×10^{-5} cm, then find the thickness of the plate.

Sol. Path difference due to the glass slab, $\Delta x = (\mu - 1)t$... (i)

Thirty fringes are displaced due to the slab.

Hence, $\Delta x = 30\lambda$... (ii)

From Eqs. (i) and (ii), we get

$$(\mu - 1)t = 30\lambda$$

$$\therefore t = \frac{30\lambda}{\mu - 1} = \frac{30 \times 6 \times 10^{-5}}{1.5 - 1} = 3.6 \times 10^{-3} \text{ cm}$$

Example 10.30 In YDSE using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.5 and thickness 2 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the light.

Sol. Shift $= \frac{(\mu - 1)tD}{d}$

$$\text{New fringe width} = \frac{2D\lambda}{d} \Rightarrow \frac{(\mu - 1)tD}{d} = \frac{2D\lambda}{d}$$

Therefore, wavelength of light,

$$\lambda = \frac{(\mu - 1)t}{2} = \frac{(1.5 - 1) \times 2 \times 10^{-6}}{2} \text{ m} = 0.5 \times 10^{-6} \text{ m}$$

Example 10.31 In YDSE, find the thickness of a glass slab ($\mu = 1.5$) which should be placed before the upper slit S_1 , so that the central maximum now lies at a point, where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is $5000 \text{ \AA}$.

Sol. According to the question, shift $= 5 \times$ (fringe width)

$$\Rightarrow \frac{(\mu - 1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \text{Thickness of a glass slab, } t = \frac{5\lambda}{\mu - 1} = \frac{25000}{1.5 - 1} = 50000 \text{ \AA}$$

Example 10.32 In a double slit pattern ($\lambda = 6000 \text{ \AA}$), the first order and tenth order maxima fall at 12.50 mm and 14.75 mm from a particular reference point. If λ is changed to $5500 \text{ \AA}$, then find the position of zero order and tenth order fringes, other arrangements remaining the same.

Sol. Distance between 10 fringes is

$$10\beta - 1\beta = 14.75 \text{ mm} - 12.50 \text{ mm}$$

$$\Rightarrow 9\beta = 2.25 \text{ mm}$$

$$\therefore \text{Fringe width, } \beta = 0.25 \text{ mm}$$

As, fringe width $\propto \lambda$

When the wavelength is changed from $6000 \text{ \AA}$ to $5500 \text{ \AA}$, then the new fringe width will become,

$$\beta' = \left(\frac{5500}{6000} \right) \beta = \left(\frac{5500}{6000} \right) (0.25)$$

$$\therefore \beta' = 0.23 \text{ mm}$$

The position of central (or zero order) maxima will remain unchanged. Earlier it was at a position,

$$y_0 = y_1 - \beta = 12.50 - 0.25 = 12.25 \text{ mm}$$

The new position of tenth order maxima will be

$$y_{10} = y_0 + 10\beta' = 12.25 + (10)(0.23) = 14.55 \text{ mm}$$

Example 10.33 A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 800 nm .

(i) Calculate the fringe width.

(ii) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minima on the axis.

Sol. Given, $\mu_m = 1.33 = \frac{4}{3}$, $d = 1 \text{ mm}$, $D = 1.33 = \frac{4}{3} \text{ m}$

and $\lambda = 800 \text{ nm}$

$$\text{Wavelength, } \lambda_m = \frac{\lambda}{\mu_m} = \frac{800}{4/3} = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$$

$$\begin{aligned} \text{(i) Fringe width, } \beta &= \frac{D\lambda_m}{d} = \frac{\frac{4}{3} \times 600 \times 10^{-9}}{10^{-3}} \\ &= 800 \times 10^{-6} \text{ m} \end{aligned}$$

(ii) Let thickness of plate be t .

$$\mu_p = 1.53$$

$$\therefore \text{Refractive index, } {}_m\mu_p = \frac{\mu_p}{\mu_m} = \frac{1.53}{4/3}$$

$$\text{Shift due to plate} = \frac{({}_m\mu_p - 1)tD}{d} \Rightarrow \frac{({}_m\mu_p - 1)tD}{d} = \frac{D\lambda_m}{2d}$$

$$\begin{aligned} \text{Thickness of sheet, } t &= \frac{\lambda_m}{2({}_m\mu_p - 1)} = \frac{600 \times 10^{-9}}{2\left(\frac{1.53}{4/3} - 1\right)} \\ &= 2033 \times 10^{-9} \text{ m} \end{aligned}$$

Example 10.34 A monochromatic light of $\lambda = 500 \text{ nm}$ is incident on two identical slits separated by a distance of $5 \times 10^{-4} \text{ m}$. The interference pattern is seen on a screen placed at a distance of 1 m from the plane of slits. A thin glass plate of thickness $2.5 \times 10^{-6} \text{ m}$ and refractive index ($\mu = 1.5$) is placed between one of the slits and the screen. Find the intensity at the centre of the screen, if the intensity is I_0 in the absence of the plate. Also, find the lateral shift of the central maxima and number of fringes crossed through centre.

Sol. Given, $\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$, $d = 5 \times 10^{-4} \text{ m}$, $D = 1 \text{ m}$,

$$t = 2.5 \times 10^{-6} \text{ m} \text{ and } \mu = 1.5$$

Path difference due to plate,

$$\Delta x = (\mu - 1)t = (1.5 - 1) \times 2.5 \times 10^{-6} = 1.25 \times 10^{-6} \text{ m}$$

$$\text{Phase difference, } \phi = \frac{2\pi}{\lambda} \Delta x$$

$$= \frac{2\pi}{500 \times 10^{-9}} \times 1.25 \times 10^{-6} = 5\pi$$

$$\text{Intensity, } I = I_0 \cos^2(\phi / 2) = I_0 \cos^2(2.5\pi) = 0$$

Lateral shift,

$$y_0 = \frac{(\mu - 1)tD}{d} = \frac{1.25 \times 10^{-6} \times 1}{5 \times 10^{-4}} = 0.25 \times 10^{-2} \text{ m} = 2.5 \text{ mm}$$

$$\text{Number of fringes crossing the centre} = \frac{y_0}{\beta}$$

$$= \frac{(\mu - 1)tD}{\frac{d\lambda}{D}} = \frac{(\mu - 1)t}{\lambda} = \frac{1.25 \times 10^{-6}}{500 \times 10^{-9}} = 2.5$$

(iv) **Path difference due to insertion of two slabs in YDSE** In the problems of YDSE, our first task is to find path difference. Let us take a typical case. In the figure shown, path of ray 1 is more deviated than path of ray 2 by a distance,

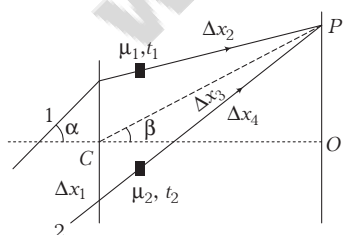


Fig. 10.21 Insertion of two slabs in a YDSE

$$\Delta x_1 = d \sin \alpha \text{ and } \Delta x_2 = (\mu_1 - 1)t_1$$

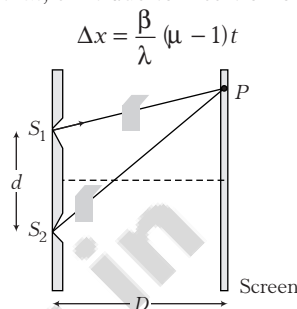
and path of ray 2 is greater than path of ray 1 by a distance, $\Delta x_3 = d \sin \beta$ and $\Delta x_4 = (\mu_2 - 1)t_2$

Therefore, net path difference is

$$\Delta x = (\Delta x_1 + \Delta x_2) - (\Delta x_3 + \Delta x_4)$$

Example 10.35 In a double slit arrangement, fringes are produced using light of wavelength $4800 \text{ \AA}$. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7 . By doing so, the central bright shifts to original 4th bright fringe from centre. Find the thickness of the glass plate.

Sol. As we know that, shift due to insertion of a slab is



$$\Delta x = \frac{\beta}{\lambda} (\mu - 1)t$$

$$\text{Shift due to one plate, } \Delta x_1 = \frac{\beta}{\lambda} (\mu_1 - 1)t$$

$$\text{Shift due to another path, } \Delta x_2 = \frac{\beta}{\lambda} (\mu_2 - 1)t$$

$$\text{Net shift, } \Delta x = \Delta x_2 - \Delta x_1 = \frac{\beta}{\lambda} (\mu_2 - \mu_1)t \quad \dots(i)$$

$$\text{Here, it is given that, } \Delta x = 4\beta \quad \dots(ii)$$

$$\text{Hence, } 4\beta = \frac{\beta}{\lambda} (\mu_2 - \mu_1)t$$

$$\Rightarrow t = \frac{4\lambda}{(\mu_2 - \mu_1)} = \frac{4 \times 4800 \times 10^{-10}}{(1.7 - 1.4)} = 6.4 \times 10^{-6} \text{ m} = 6.4 \mu\text{m}$$

Interference in thin films

Interference effects are commonly observed in thin films, such as thin layers of oil on water or the thin surface of a soap bubble. The various colours are observed when white light is incident on such films as the result of the interference of waves reflected from the two surfaces of the film. Consider a film of uniform thickness t and index of refraction μ , as shown in figure.

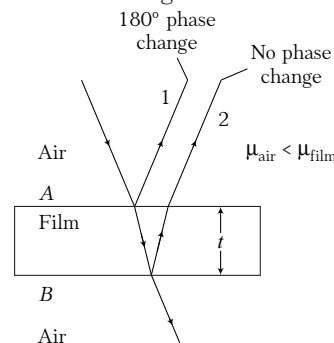


Fig. 10.22 Interference in light reflected from a thin film is due to a combination of rays reflected from the upper and lower surfaces of the film

Let us assume that the light rays travelling in air are nearly normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, we first note the following facts

- (i) The wavelength of light in a medium whose refractive index μ is

$$\lambda_{\mu} = \frac{\lambda}{\mu} \quad \dots(i)$$

where, λ is the wavelength of light in vacuum (or air).

- (ii) If a wave is reflected from a denser medium, then it undergoes a phase change of 180° . Let us apply these rules to the film shown in figure. The path difference between the two rays (ray 1 and ray 2) is $2t$, while the phase difference between them is 180° .

Hence, condition of constructive interference will be

$$2t = (2n - 1) \frac{\lambda_{\mu}}{2} \quad (\text{where, } n = 1, 2, 3, \dots)$$

or $2\mu t = \left(n - \frac{1}{2}\right)\lambda$ [from Eq. (i)]

Similarly, condition of destructive interference will be

$$2\mu t = n\lambda \quad (\text{where, } n = 0, 1, 2, \dots)$$

Example 10.36 Calculate the minimum thickness of a soap bubble film ($\mu = 1.33$) that results in constructive interference in the reflected light, if the film is illuminated with light whose wavelength in free space is $\lambda = 600 \text{ nm}$.

Sol. For constructive interference in case of soap bubble film,

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \quad (\text{where, } n = 1, 2, 3, \dots)$$

For minimum thickness t , $n = 1$

$$\Rightarrow 2\mu t = \lambda/2$$

Thus, the minimum thickness of a soap bubble film is

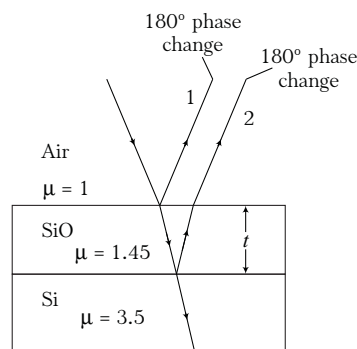
$$t = \frac{\lambda}{4\mu} = \frac{600}{4 \times 1.33} = 112.78 \text{ nm}$$

Example 10.37 In solar cells, silicon Si ($\mu = 3.5$) is coated with a thin film of silicon monoxide SiO ($\mu = 1.45$) to minimise reflective losses from the surface. Determine the minimum thickness of SiO that produces the least reflection at a wavelength of 550 nm , near the centre of the visible spectrum.

Sol. The reflected light is minimum when ray 1 and ray 2 (shown in figure) meet the condition of destructive interference.

Both rays undergo a 180° phase change upon reflection. The net change in phase due to reflection is therefore zero and the

condition for a minimum reflection requires a path difference of $\lambda_{\mu}/2$.



For minimum thickness t , $n = 1$

$$\Rightarrow 2t = \frac{\lambda}{2\mu}$$

Thus, the minimum thickness of silicon monoxide is

$$t = \frac{\lambda}{4\mu} = \frac{550}{4(1.45)} = 94.8 \text{ nm}$$

Note When we observe this film fringes through refracted light waves, then condition for maxima is $2t = n\lambda_{\mu}$ and for maxima, $2t = (2n - 1) \frac{\lambda_{\mu}}{2}$ (where, $n = 1, 2, 3, \dots$).

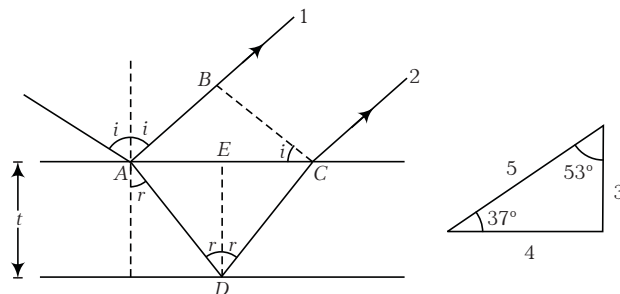
Example 10.38 A parallel beam of white light falls on a thin film whose refractive index is equal to $4/3$. The angle of incidence $i = 53^\circ$. What must be the minimum film thickness, if the reflected light is to be coloured yellow (λ of yellow = $0.6 \mu\text{m}$) most intensively? (Given, $\tan 53^\circ = 4/3$)

Sol. From the relation, $\mu = \frac{\sin i}{\sin r} \Rightarrow \frac{4}{3} = \frac{\sin 53^\circ}{\sin r}$

We have, $r \approx 37^\circ$

Interference has to be taken place between rays 1 and 2.

They have already a phase difference of π , because ray 1 is reflected from a denser medium.



They have a path difference of $\Delta x = 2\mu(AD) - AB$

$$= 2\mu \left(\frac{t}{\cos r} \right) - 2t(\tan r)(\sin i)$$

$$= 2 \left(\frac{4}{3} \right) \left(\frac{5}{4} \right) t - 2t \left(\frac{3}{4} \right) \left(\frac{4}{5} \right) = \frac{146}{45} t$$

For the two waves to interfere constructively,

$$\Delta x = (2n - 1) \frac{\lambda}{2} \quad (\text{where, } n = 1, 2, 3, \dots)$$

or
$$\frac{146}{45} t = (2n - 1) \frac{\lambda}{2}$$

For minimum thickness t , $n = 1$

or
$$\frac{146}{45} t = \frac{\lambda}{2} \quad \text{or} \quad t = \frac{45}{146} \times \frac{\lambda}{2} = \frac{45 \times 0.6}{146 \times 2} \mu\text{m} = 0.09 \mu\text{m}$$

Lloyd's mirror

A plane glass plate (acting as a mirror) is illuminated at almost grazing incidence by a light from a slit S_1 . A virtual image S_2 of S_1 is formed close to S_1 by reflection and these two act as coherent sources. The expression giving the fringe width is the same as for the double slit but the fringe system differs in one important respect.

In Lloyd's mirror, if the point P , for example, is such that the path difference $S_2P - S_1P$ is a whole number of wavelengths, the fringe at point P is dark not bright. This is due to 180° phase change which occurs when light is reflected from a denser medium.

This is equivalent to adding an extra half wavelength to the path of the reflected wave. At grazing incidence a fringe is formed at O , where the geometrical path difference between the direct and reflected waves is zero and it follows that it will be dark rather than bright.

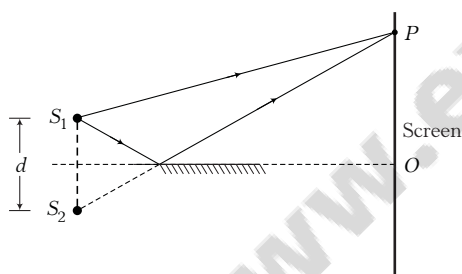


Fig. 10.23 Reflection of light from Lloyd's mirror

Thus, whenever there exists a phase difference of π between the two interfering beams of light, conditions of maximas and minimas are interchanged.

i.e. for minimum intensity, $\Delta x = n\lambda$

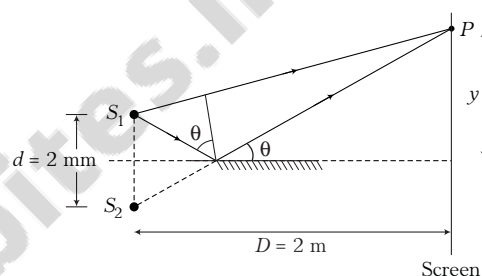
and for maximum intensity, $\Delta x = (2n - 1)\lambda/2$

(where, $n = 1, 2, 3, \dots$)

Example 10.39 Light source of wavelength 400 nm used in a Lloyd's mirror, lies 1 mm above the plane mirror. Find the distance of first minima above the mirror. Distance of the screen from the plane of the sources is 2 m.

Sol. Consider the formation of minimum at point P as shown in figure. Path difference between the rays reaching at point P is

$$\Delta x = \frac{d}{2} \sin \theta = \lambda \quad (\text{for first minimum})$$



$$\Rightarrow d \times \tan \theta \approx 2\lambda \quad (\because \sin \theta \approx \tan \theta)$$

$$\Rightarrow d \times \frac{y}{D} = 2\lambda$$

$$\Rightarrow y = \frac{D}{d} (2\lambda)$$

$\therefore$ Distance of first minima above the mirror,

$$y = \frac{2}{2 \times 10^{-3}} \times 2 \times 400 \times 10^{-9} = 0.8 \text{ mm}$$

CHECK POINT 10.2

- In Young's double slit experiment, an interference pattern is obtained for $\lambda = 6000 \text{ \AA}$, coming from two coherent sources S_1 and S_2 . At certain point P on the screen third dark fringe is formed. Then, the path difference $S_1P - S_2P$ (in micron) is
 - 0.75
 - 1.5
 - 3.0
 - 4.5
- In Young's double slit experiment, when two light waves form third minimum intensity, they have
 - phase difference of 3π
 - phase difference of $\frac{5\pi}{2}$
 - path difference of 3λ
 - path difference of $\frac{5\lambda}{2}$
- Two slits are separated by a distance of 0.5 mm and illuminated with light of $\lambda = 6000 \text{ \AA}$. If the screen is placed 2.5 m from the slits. The distance of the third bright image from the centre will be
 - 1.5 mm
 - 3 mm
 - 6 mm
 - 9 mm
- Fringe width decreases with increase in
 - λ
 - D
 - d
 - None of these
- In Young's double slit experiment, the fringe width will remain same, if (D = distance between screen and plane of slits, d = separation between two slits and λ = wavelength of light used)
 - Both λ and D are doubled
 - Both d and D are doubled
 - D is doubled but d is halved
 - λ is doubled but d is halved
- Interference was observed in interference chamber when air is present. Now, the chamber is evacuated and if the same light is used, then for the same arrangement
 - no interference pattern will be obtained
 - exactly same interference pattern will be obtained with better contrast
 - the fringe width is slightly decreased
 - the fringe width is slightly increased
- Monochromatic green light of wavelength $5 \times 10^{-7} \text{ m}$ illuminates a pair of slits 1 mm apart. The separation of bright lines on the interference pattern formed on a screen 2 m away is
 - 0.25 mm
 - 0.1 mm
 - 1.0 mm
 - 0.01 mm
- In double slit experiment, for light of which colour, the fringe width will be minimum?
 - Violet
 - Red
 - Green
 - Yellow
- In Young's double slit experiment, green light ($\lambda = 5461 \text{ \AA}$) is used and 60 fringes were seen in the field view. Now, sodium light is used ($\lambda = 5890 \text{ \AA}$), then number of fringes observed are
 - 40
 - 60
 - 50
 - 55
- In Young's double slit experiment, the ratio of intensities of bright and dark fringes is 9. This means that
 - the intensities of individual sources are 5 and 4 units respectively
 - the intensities of individual sources are 4 and 1 units respectively
 - the ratio of their amplitudes is 3
 - the ratio of their amplitudes is 4
- What happens to the fringe pattern, if in the path of one of the slits a glass plate which absorbs 50% energy is interposed?
 - The bright fringes become bright and dark fringes become darker
 - No fringes are observed
 - The fringe width decreases
 - None of the above
- In Young's double slit experiment, instead of taking slits of equal widths, one slit is made twice as wide as the other. Then, in the interference pattern
 - the intensities of both the maxima and the minima increases
 - the intensity of maxima increases and the minima has zero intensity
 - the intensity of maxima decreases and that of the minima increases
 - the intensity of maxima decreases and the minima has zero intensity
- A thin mica sheet of thickness $2 \times 10^{-6} \text{ m}$ and refractive index ($\mu = 1.5$) is introduced in the path of the light from upper slit. The wavelength of the wave used is $5000 \text{ \AA}$. The central bright maximum will shift
 - two fringes upward
 - two fringes downward
 - ten fringes upward
 - None of these
- In Young's double slit experiment, the aperture screen distance is 2 m. The fringe width is 1 mm. Light of 600 nm is used. If a thin plate of glass ($\mu = 1.5$) of thickness 0.06 mm is placed over one of the slits, then there will be a lateral displacement of the fringes by
 - zero
 - 5 cm
 - 10 cm
 - 15 cm
- Interference fringes were produced in Young's double slit experiment using light of wavelength $5000 \text{ \AA}$. When a film of thickness $2.5 \times 10^{-3} \text{ cm}$ was placed in front of one of the slits, then the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is
 - 1.25
 - 1.35
 - 1.4
 - 1.5
- What is the minimum thickness of a thin film ($\mu = 1.2$) that results in constructive interference in the reflected light? If the film is illuminated with light whose wavelength in free space is $\lambda = 500 \text{ nm}$?
 - 104 nm
 - 200 nm
 - 300 nm
 - 400 nm

DIFFRACTION OF LIGHT

The phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength is called **diffraction**. Other waves, such as sound waves and water waves, also have this property of spreading when passing through apertures or by sharp edges.

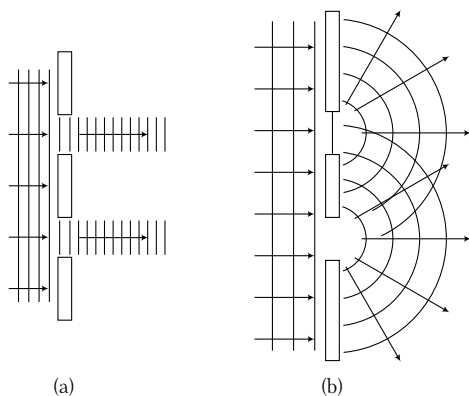


Fig. 10.24 (a) If light waves did not spread out after passing through the slits, no interference would occur. (b) The light waves from the two slits overlap as they spread out, filling the shadowed regions with light and producing interference fringes.

Diffraction pattern

In general, diffraction occurs when waves pass through small openings, around obstacles, or past sharp edges, as shown in figure. When an opaque object is placed between a point source of light and a screen, no sharp boundary exists on the screen between a shadowed region and an illuminated region. The illuminated region above the shadow of the object contains alternating light and dark fringes. Such a display is called a diffraction pattern.

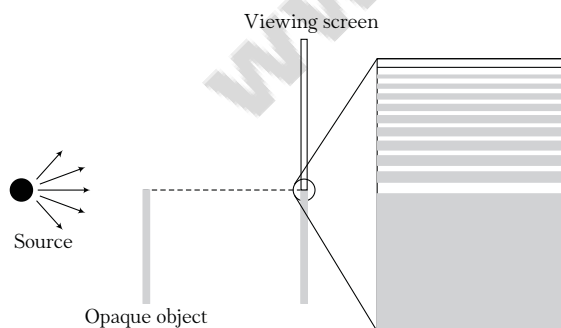


Fig. 10.25 Light from a small source passes by the edge of an opaque object. In reality, light bends around the top edge of the object and enters this region. Because of these effects, a diffraction pattern consisting of bright and dark fringes appears in the region above the edge of the object.

Types of diffraction of light

The phenomenon of diffraction is divided mainly in the following two classes

(i) Fresnel diffraction

If source and screen are at finite distance from the obstacle or aperture, then diffraction observed is known as Fresnel diffraction (as shown in the figure).

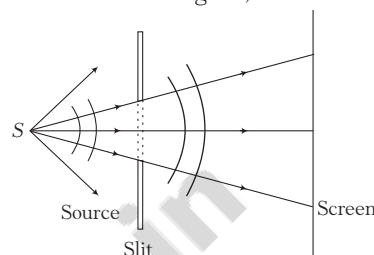


Fig. 10.26 Fresnel diffraction through a slit

(ii) Fraunhofer diffraction

When all the rays passing through a narrow slit are approximately parallel to one another. This can be achieved experimentally either by placing the screen far from the opening used to create the diffraction or by using a converging lens to focus the rays once they pass through the opening, as shown in figure.

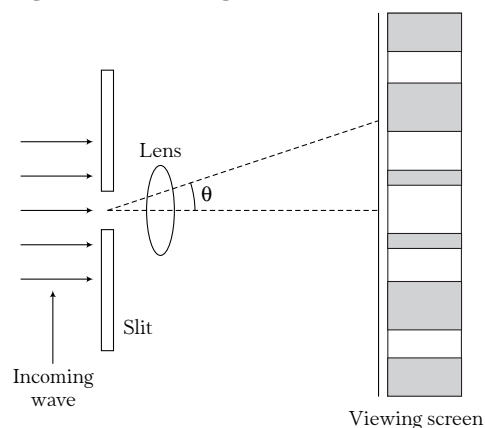


Fig. 10.27 Fraunhofer diffraction pattern of a single slit. The pattern consists of a central bright fringe flanked by much weaker maxima alternating with dark fringes.

The diffraction produced in such an arrangement is called Fraunhofer diffraction.

Fraunhofer Diffraction of light due to a single narrow slit

Suppose a plane wavefront is incident on a slit AB (of width a). Each and every part of the exposed part of the plane wavefront (i.e. every part of the slit) acts as a source of secondary wavelets spreading in all directions.

The diffraction is obtained on a screen placed at a large distance. (In practice, this condition is achieved by placing the screen at the focal plane of a converging lens placed just after the slit).

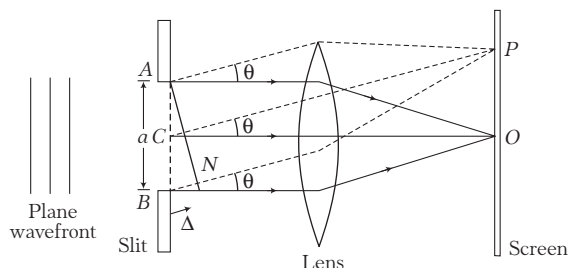


Fig. 10.28 A plane wavefront is incident on the slit AB, which forms a diffraction pattern on the screen

The diffraction pattern consists of a central bright fringe (central maxima) surrounded by dark and bright lines (called **secondary minima** and **maxima**).

At point O on the screen, the central maxima is obtained. The wavelets originating from points A and B meet in the same phase at this point, hence at point O, intensity is maximum.

Secondary minima For obtaining n th secondary minima at point P on the screen, path difference between the diffracted waves $a \sin \theta = \pm n\lambda$, where $n = 1, 2, 3, \dots$

(i) Angular position of n th secondary minima,

$$\sin \theta \approx \theta = \frac{n\lambda}{a}$$

(ii) Distance of n th secondary minima from central maxima,

$$y_n = D \cdot \theta \Rightarrow \frac{n\lambda D}{a}$$

$$y_n = \frac{n\lambda f}{a}$$

where, D = distance between slit and screen,
 $f \approx D$ = focal length of converging lens.

Secondary maxima For obtaining n th secondary maxima at point P on the screen, path difference $= a \sin \theta = (2n + 1) \frac{\lambda}{2}$, where $n = 1, 2, 3, \dots$

(i) Angular position of n th secondary maxima,

$$\sin \theta \approx \theta = \frac{(2n + 1)\lambda}{2a}$$

(ii) Distance of n th secondary maxima from central maxima,

$$y_n = D \cdot \theta = \frac{(2n + 1)\lambda D}{2a}$$

$$y_n = \frac{(2n + 1)\lambda f}{2a}$$

Example 10.40 A screen is placed 50 cm from a single slit, which is illuminated with 6000 Å light. If distance between first and third minima in the diffraction pattern is 3 mm. What is the width of the slit?

Sol. Given, $D = 50$ cm and $\lambda = 6000 \text{ Å} = 600 \text{ nm}$

Distance between first and third minima,

$$y_3 - y_1 = \frac{3D\lambda}{a} - \frac{D\lambda}{a} = \frac{2D\lambda}{a}$$

$$\Rightarrow 3 \times 10^{-3} = \frac{2 \times 0.5 \times 600 \times 10^{-9}}{a}$$

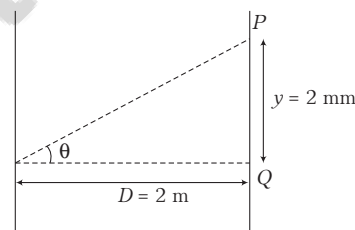
Therefore, width of the slit,

$$a = 200 \times 10^{-6} \text{ m} = 0.2 \text{ mm}$$

Example 10.41 A beam of light of wavelength 400 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 2 m away from the slit. It is observed that 2nd order minima occurs at a distance of 2 mm from the position of central maxima. Find the width of the slit.

Sol. Given, $\lambda = 400 \text{ nm} = 400 \times 10^{-9} \text{ m}$,

$$D = 2 \text{ m}, y = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$



Let point P is the point of second order minima.

$$\text{Now, } \sin \theta \approx \tan \theta \approx \theta = \frac{2 \times 10^{-3}}{2} = 10^{-3}$$

Let the slit separation be a .

$$\text{Now, for second order minima, } \theta = \frac{2\lambda}{a} = 10^{-3}$$

$$\Rightarrow \frac{2 \times 400 \times 10^{-9}}{a} = 10^{-3}$$

Thus, we obtain $a = 0.8 \text{ mm}$.

Example 10.42 A beam of light of wavelength 600 nm from a distant source falls on a single slit 2 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. What is the distance between the first dark fringe on either side of central bright fringe?

Sol. Given, $\lambda = 600 \text{ nm}$, $a = 2 \times 10^{-3} \text{ m}$ and $D = 2 \text{ m}$

$$\text{Distance of first dark fringe from centre, } y_1 = \frac{D\lambda}{a}$$

Distance between the first dark fringe on either side of the central bright fringe,

$$2y_1 = \frac{2D\lambda}{a} = \frac{2 \times 2 \times 600 \times 10^{-9}}{2 \times 10^{-3}} = 1200 \times 10^{-6} \text{ m}$$

$$= 1.2 \times 10^{-3} \text{ m} = 1.2 \text{ mm}$$

Example 10.43 A slit of width d is illuminated by red light of wavelength $6500\text{\AA}$. For what value of d will the first maximum fall at angle of diffraction of 30° ?

Sol. For first secondary maximum of the diffraction pattern,

$$d \sin \theta = \frac{3\lambda}{2}$$

$$\therefore d = \frac{3\lambda}{2 \sin \theta} = \frac{3 \times 6500 \times 10^{-10}}{2 \times \sin 30^\circ}$$

$$= 1.95 \times 10^{-6} \text{ m}$$

Example 10.44 A beam of light of wavelength 600 nm from a distant source falls on a single slit 1.0 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. What is the distance between the second bright fringe on either side of the central bright fringe?

Sol. For the diffraction at a single slit, the position of maxima is given by

$$a \sin \theta = (2n + 1) \frac{\lambda}{2}$$

For small value of θ , $\sin \theta \approx \theta = \frac{y}{D}$

For second bright fringe ($n = 2$),

$$\therefore a \times \frac{y}{D} = \frac{5\lambda}{2} \text{ or } y = \frac{5\lambda}{2a} D$$

Substituting the values, we have

$$y = \frac{5 \times 2 \times 6 \times 10^{-7}}{2 \times 1 \times 10^{-3}} = 3.0 \times 10^{-3} \text{ m} = 3.0 \text{ mm}$$

Distance between second maxima on either side of central maxima $= 2y = 6.0 \text{ mm}$.

Example 10.45 In a single slit diffraction experiment first minima for $\lambda_1 = 660 \text{ nm}$ coincides with first maxima for wavelength λ_2 . Calculate the value of λ_2 .

Sol. Position of minima in diffraction pattern is given by

$$a \sin \theta = n\lambda$$

For first minima of λ_1 , we have

$$a \sin \theta_1 = (1) \lambda_1 \text{ or } \sin \theta_1 = \frac{\lambda_1}{a} \quad \dots(i)$$

The first maxima approximately lies between first and second minima. For wavelength λ_2 , its position will be

$$a \sin \theta_2 = \frac{3}{2} \lambda_2$$

$$\therefore \sin \theta_2 = \frac{3\lambda_2}{2a} \quad \dots(ii)$$

The two will coincide, if

$$\theta_1 = \theta_2 \text{ or } \sin \theta_1 = \sin \theta_2$$

$$\therefore \frac{\lambda_1}{a} = \frac{3\lambda_2}{2a}$$

$$\text{or } \lambda_2 = \frac{2}{3} \lambda_1 = \frac{2}{3} \times 660 \text{ nm}$$

$$= 440 \text{ nm}$$

Example 10.46 A lens of focal length 1 m forms Fraunhofer diffraction pattern of a single slit of width 0.04 cm in its focal plane. The incident light contains two wavelength λ_1 and λ_2 . It is found that the fourth minimum corresponding to λ_1 and the fifth minimum corresponding to λ_2 occur at the same point 0.5 cm from the central maximum. Compute λ_1 and λ_2 .

Sol. The angular position of n th minima,

$$a \sin \theta = n\lambda$$

$$\text{For } \lambda_1, a \sin \theta_1 = 4\lambda_1 \quad \dots(i)$$

$$\text{For } \lambda_2, a \sin \theta_2 = 5\lambda_2 \quad \dots(ii)$$

$$\text{We have, } \theta_1 = \theta_2 = \theta$$

$$\therefore a \sin \theta = 4\lambda_1 = 5\lambda_2 \quad \dots(iii)$$

$$y = f\theta \Rightarrow \theta = y/f \quad [\because D = f]$$

Separation, $y = 0.5 \text{ cm} = 5 \times 10^{-3} \text{ m}$, focal length, $f = 1 \text{ m}$

$$\theta = \frac{y}{f} = \frac{5 \times 10^{-3}}{1} = 0.005 \text{ rad}$$

$$\text{Wavelength, } \lambda_1 = \frac{a \sin \theta}{4} = \frac{a\theta}{4} \quad (\text{for small angle, } \theta \approx \sin \theta)$$

$$= \frac{0.04 \times 10^{-2} \times 0.005}{4}$$

$$= 5 \times 10^{-7} \text{ m} = 500 \text{ nm}$$

$$\text{Wavelength, } \lambda_2 = \frac{4\lambda_1}{5} = \frac{4 \times 500}{5} = 400 \text{ nm}$$

Width of central maxima

Diffraction pattern can be graphically shown as below

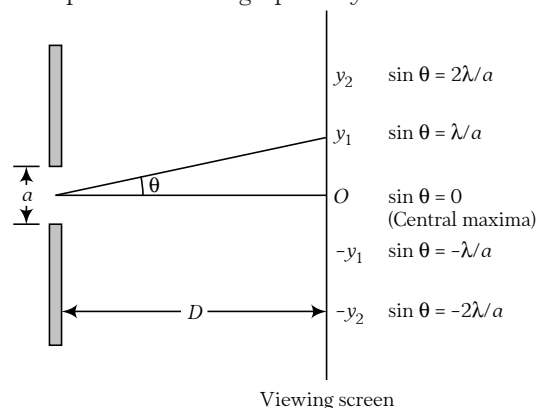


Fig. 10.29 Fraunhofer diffraction pattern from a single slit of width a . The positions of two minima on each side of the central maximum are labelled.

The point O corresponds to the position of central maxima and the position $-3\lambda, -2\lambda, -\lambda, \lambda, 2\lambda, 3\lambda, \dots$ are secondary minima. The above conditions for diffraction maxima and minima are exactly reverse of mathematical conditions for interference maxima and minima. The width of central maximum is the distance between first secondary minimum on either side of the central bright fringe at point O .

For first secondary minimum,

$$a \sin \theta = \lambda \quad \text{or} \quad \sin \theta = \frac{\lambda}{a} \quad \dots(i)$$

$$\text{If } \theta \text{ is small, } \sin \theta \approx \theta = \frac{y}{D} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{y}{D} = \frac{\lambda}{a}$$

$$\text{or} \quad y = \frac{D\lambda}{a}$$

$$\text{Width of central maximum} = 2y = \frac{2D\lambda}{a}$$

As the slit width a increases, width of central maximum decreases.

$$\text{Angular width of central maxima, } 2\theta = \frac{2\lambda}{a}.$$

Example 10.47 Light of wavelength 580 nm is incident on a slit having a width of 0.300 mm. The viewing screen is 2.00 m from the slit. Find the position of the first dark fringe and the width of the central bright fringe.

Sol. The two dark fringes that flank the central bright fringe correspond to $n = \pm 1$, hence we find that

$$\sin \theta = \pm \frac{\lambda}{a} = \pm \frac{5.80 \times 10^{-7} \text{ m}}{0.300 \times 10^{-3} \text{ m}} = \pm 1.93 \times 10^{-3}$$

Therefore, the position of the first minima measured from the central axis are given by

$$y_1 \approx D \sin \theta = \pm D \frac{\lambda}{a} = \pm 2 \times 1.93 \times 10^{-3} = \pm 3.86 \times 10^{-3} \text{ m}$$

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is equal to

$$2 |y_1| = 7.72 \times 10^{-3} \text{ m} = 7.72 \text{ mm}$$

Note that this value is much greater than the width of the slit. However, as the slit width is increased, the diffraction pattern narrows, corresponding to smaller values of θ . In fact, for large values of a , the various maxima and minima are so closely spaced that only a large central bright area resembling the geometric image of the slit is observed.

Example 10.48 Angular width of central maximum in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, then the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find the refractive index of the liquid.

Sol. Angular width of central maximum = $\frac{2\lambda}{a}$

$$\Rightarrow \theta_1 = \frac{2\lambda_1}{a} \quad \dots(i)$$

$$\theta_2 = \theta_1 \times 0.7 = \frac{2\lambda_2}{a} \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{1}{0.7} = \frac{\lambda_1}{\lambda_2} = \frac{600}{\lambda_2} \quad (\because \lambda_1 = 60000 \text{ Å} = 600 \text{ nm})$$

Wavelength, $\lambda_2 = 420 \text{ nm}$

When immersed in liquid, $\lambda_2 = \lambda_1/\mu$

Using Eqs. (i) and (ii), we get

$$\Rightarrow \frac{2\lambda_1/\mu}{a} = \frac{2\lambda_1}{a} \times 0.7$$

$$\Rightarrow \frac{1}{0.7} = \mu$$

$$\text{Refractive index of the liquid, } \mu = \frac{10}{7} = 1.42$$

Intensity of single slit diffraction pattern

We can use phasors to determine the light intensity distribution for a single slit diffraction pattern. The proof is beyond our syllabus, we are just writing here the intensity at angle θ .

$$I = I_0 \left[\frac{\sin \beta/2}{\beta/2} \right]^2$$

$$\text{Here, } \frac{\beta}{2} = \frac{\pi a \sin \theta}{\lambda}$$

$$\text{or} \quad I = I_0 \left[\frac{\sin (\pi a \sin \theta)/\lambda}{(\pi a \sin \theta)/\lambda} \right]^2 \quad (\text{at angle } \theta)$$

Here, I_0 is the intensity at $\theta = 0^\circ$ (the central maximum).

From this result we see that minima occurs when,

$$\frac{\pi a \sin \theta}{\lambda} = m\pi \quad \text{or} \quad \sin \theta = m \frac{\lambda}{a}$$

where, $m = \pm 1, \pm 2, \dots$ (not zero)

Note The $\sin \theta = 0$, corresponds to central maximum, while

$$\frac{\pi a \sin \theta}{\lambda} = \pi \text{ corresponds to first minima.}$$

Intensity distribution curve

The equation of minima gives the values of θ for which the diffraction pattern has zero light intensity, i.e. when a dark fringe is formed.

However, it tells us nothing about the variation in light intensity along the screen. The general features of the intensity distribution are shown in figure. A broad central bright fringe is observed; this fringe is flanked by much weaker bright fringes alternating with dark fringes. The various dark fringes occur at the values of θ . Each bright fringe peak lies approximately half-way between its bordering dark fringe minima.

Note that the central bright maximum is twice as wide as the secondary maxima.

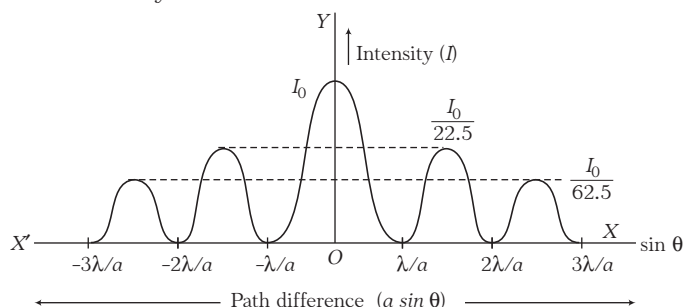
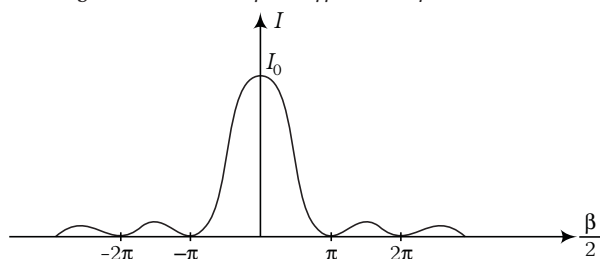


Fig. 10.30 Intensity distribution curve on the screen in diffraction pattern with $\sin \theta$

Example 10.49 Find the ratio of the intensities of the secondary maxima to the intensity of the central maximum for the single-slit Fraunhofer diffraction pattern.



Sol. To a good approximation, the secondary maxima lie mid-way between the zero points. From figure, we see that this corresponds to $\beta/2$ values of $3\pi/2, 5\pi/2, 7\pi/2, \dots$

$$\therefore \frac{I_1}{I_0} = \left[\frac{\sin(3\pi/2)}{(3\pi/2)} \right]^2 = \frac{1}{9\pi^2/4} = 0.045$$

$$\frac{I_2}{I_0} = \left[\frac{\sin(5\pi/2)}{5\pi/2} \right]^2 = \frac{1}{25\pi^2/4} = 0.016$$

i.e. the first secondary maxima (the ones adjacent to the central maximum) have an intensity of 4.5% that of the central maximum and the next secondary maxima have an intensity of 1.6% that of the central maximum.

Fresnel's distance

When a slit or hole of size a is illuminated by a parallel beam, then it is diffracted into an angle of $\approx \frac{\lambda}{a}$.

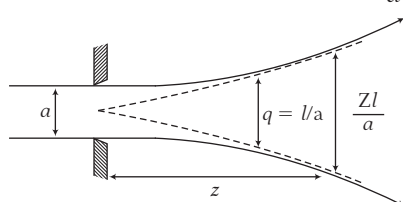


Fig. 10.31 Diffraction of a parallel beam

In travelling a distance Z , size of beam is $Z\lambda/a$.

So, taking
$$\frac{Z\lambda}{a} \geq a$$

or
$$Z \geq \frac{a^2}{\lambda}$$

Now, distance Z_F is called Fresnel's distance,

$$Z_F = a^2 / \lambda$$

Spreading due to diffraction is comfortable upto distance $Z_F/2$ and for distance much greater than Z_F , spreading due to diffraction is prominent.

So, image formation can be explained by ray optics for distance less than Z_F .

Example 10.50 For what distance is ray optics a good approximation when a plane light wave is incident on a circular aperture of width 2 mm having wavelength 600 nm?

Sol. Ray optics is a good approximation upto Fresnel's distance only.

$$Z_F = \frac{a^2}{\lambda} = \frac{(2 \times 10^{-3})^2}{600 \times 10^{-9}} = 6.7 \text{ m}$$

Hence, the ray optics hold good upto distance of 6.7 m.

Example 10.51 Estimate how large can be the aperture opening to work with laws of ray optics using a monochromatic light of wavelength 450 nm to a distance of around 20 m.

Sol. Now, here we are given Fresnel's distance equal to 20 m, $\lambda = 450 \text{ nm}$, we have to estimate value of a .

$$\text{Using } Z_F = \frac{a^2}{\lambda}, a = \sqrt{Z_F \lambda} = \sqrt{450 \times 20 \times 10^{-9}}$$

$$= 3 \times 10^{-3} \text{ m or } 3 \text{ mm}$$

Difference between interference and diffraction

Interference	Diffraction
It is due to superposition of waves from two coherent sources.	It is due to superposition of wavelets from same wavefronts.
All bright fringes are of same intensity.	Intensity decreases with the increase in the order of maxima.
The dark fringes are almost black.	Dark fringes are not perfectly black.
Fringes are of same width $\frac{\lambda D}{d}$.	The central maximum is of double the width of the secondary maxima. So, width is not equal.
The number of bands is large.	The number of bands is small.
Bands are equally spaced.	Bands are unequally spaced.

Diffraction grating

It consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, then the direction of principal maxima is given by

$$d \sin \theta = n\lambda$$

where, $n = 1, 2, 3, \dots$ is the order of principal maxima and d is the distance between two consecutive slits and is called **grating element**.

Note Diffraction grating is based on combined phenomena of interference and diffraction. It is the results of interference in diffracted waves.

Example 10.52 A diffraction grating has 1.26×10^4 rulings uniformly spaced over width, $w = 25.4$ mm. It is illuminated at normal incidence by blue light of wavelength 450 nm. At what angles to the central axis do the second order maxima occur.

Sol. The grating spacing d is

$$d = \frac{w}{N} = \frac{25.4 \times 10^{-3}}{1.26 \times 10^4} \\ = 2.016 \times 10^{-6} \text{ m} = 2016 \text{ nm}$$

The second order maxima corresponds to $n = 2$.

For $\lambda = 450$ nm, we have

$$d \sin \theta = 2\lambda$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{2\lambda}{d} \right)$$

$$\Rightarrow \theta = \sin^{-1} \left(\frac{2 \times 450 \times 10^{-9}}{2.016 \times 10^{-6}} \right)$$

$$\therefore \text{Angle, } \theta \approx 27^\circ$$

Resolving power of the diffraction grating

The diffraction grating is most useful for measuring wavelengths accurately. Like the prism, the diffraction grating can be used to disperse a spectrum into its wavelength components. Of the two devices, the grating is the more precise, if one wants to distinguish two closely spaced wavelengths.

For two nearly equal wavelengths λ_1 and λ_2 between which a diffraction grating can just barely distinguish, the resolving power R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda}$$

where, $\lambda = (\lambda_1 + \lambda_2)/2$ and $\Delta\lambda = \lambda_2 - \lambda_1$.

CHECK POINT 10.3

- To observe diffraction, the size of an obstacle
 - should be of the same order as wavelength
 - should be much larger than the wavelength
 - has no relation to wavelength
 - should be exactly $(\lambda/2)$
- Yellow light is used in a single slit diffraction experiment with slit width of 0.6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal
 - that the central maximum is narrower
 - more number of fringes
 - less number of fringes
 - no diffraction pattern
- In Fresnel's class of diffraction, the
 - obstacle screen distance is finite
 - the diffracted wavefront is considered as spherical
 - no convex lens is used to focus the diffraction fringes on the screen
 - All of the above
- A slit of width d is illuminated by white light. For red light ($\lambda = 6500 \text{ \AA}$), the first minima is obtained at $\theta = 30^\circ$. Then, the value of d will be

(a) 3250 $\text{\AA}$	(b) $13 \times 10^{-4} \text{ mm}$
(c) 1.24 microns	(d) $26 \times 10^{-4} \text{ cm}$
- Direction of the first secondary maxima in the Fraunhofer diffraction pattern at a single slit is given by (a is the width of the slit)

(a) $a \sin \theta = \frac{\lambda}{2}$	(b) $a \cos \theta = \frac{3\lambda}{2}$
(c) $a \sin \theta = \lambda$	(d) $a \sin \theta = \frac{3\lambda}{2}$
- A light wave is incident normally over a single slit of width $24 \times 10^{-5} \text{ cm}$. The angular position of second dark fringe from the central maxima is 30° . What is the wavelength of light?

(a) 6000 $\text{\AA}$	(b) 5000 $\text{\AA}$	(c) 3000 $\text{\AA}$	(d) 1500 $\text{\AA}$
-----------------------	-----------------------	-----------------------	-----------------------
- The main difference in the phenomenon of interference and diffraction is that
 - diffraction is due to interaction of light from the same wavefront, whereas the interference is the interaction of two waves from isolated sources
 - diffraction is due to interaction of light from the same wavefront, whereas the interference is the interaction of two waves derived from the same source
 - diffraction is due to interaction of waves derived from the same source, whereas the interference is the bending of light from the same wavefront
 - diffraction is caused by the reflected waves from a source, whereas interference is caused due to refraction of waves from a source
- The X-ray cannot be diffracted by means of an ordinary grating due to

(a) large wavelength	(b) high speed
(c) short wavelength	(d) All of these

POLARISATION OF LIGHT

Light is an electromagnetic wave in which electric and magnetic field vectors vary sinusoidally perpendicular to each other as well as perpendicular to the direction of propagation of wave of light.

“The phenomenon of limiting the vibration of electric field vector in one direction in a plane perpendicular to the direction of propagation of light wave is called polarisation of light”.

The plane perpendicular to the plane of oscillation is called **plane of polarisation**.

Note In ordinary light (sun light, bulb light, etc.), the electric field vectors are distributed in all directions is called **unpolarised light**.

Polaroids

These are thin film of ultramicroscopic crystals of herapathite or iodoquinine sulfate with their optic axis parallel to each other and it is used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal.

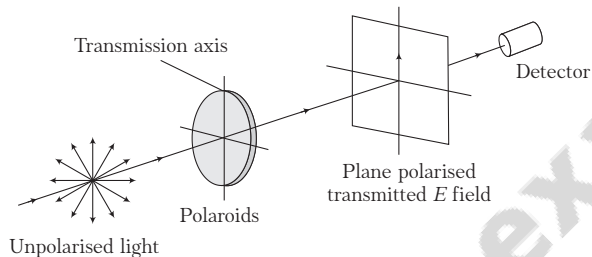


Fig. 10.32 Unpolarised light passes through polaroids

- (i) Polaroids allow the light oscillations parallel to the transmission axis, to pass through them.
- (ii) The crystal or polaroid on which unpolarised light is incident is called **polariser**. Crystal or polaroid on which polarised light is incident is called **analyser**.

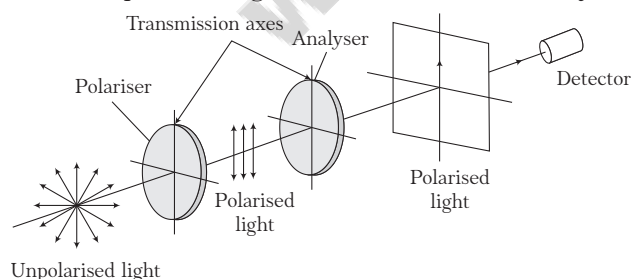


Fig. 10.33 Transmission axis of the polariser and analyser are parallel to each other, so whole of the polarised light passes through analyser

- (iii) If the transmission axis of analyser is perpendicular to that of the polariser, then no light passes through the analyser.

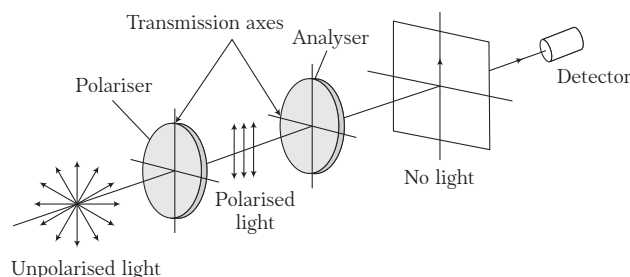


Fig. 10.34 Transmission axis of the analyser is perpendicular to the polariser, hence no light passes through the analyser

Uses of polaroids

Polaroids are used

- (i) in sun glasses.
- (ii) to prepare filters.
- (iii) for laboratory purpose.
- (iv) in head light of automobiles.
- (v) in three-dimensional motion pictures.
- (vi) to improve colour contrast in old paintings.

Malus's law

This law states that “the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser”.

If I_0 be the intensity of the polarised light incident on the analyser and θ be the angle between the transmission axes of the polariser and the analyser, the intensity of the light transmitted through the analyser is given by $I = I_0 \cos^2 \theta$.

Note If an unpolarised light is converted into plane polarised light (say by passing it through a polaroid), then its intensity becomes half.

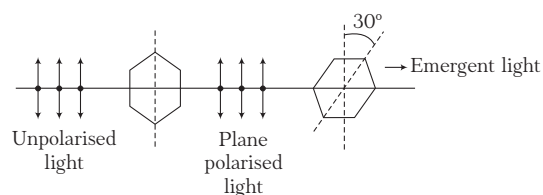
$$\text{i.e. } I_0 = \frac{I}{2}$$

$$\text{So, } I = \frac{I_0}{2} \cos^2 \theta$$

where, I_i = intensity of unpolarised light.

Example 10.53 Two polaroids are oriented with their planes perpendicular to incident light and transmission axis making an angle of 30° with each other. What fraction of incident unpolarised light is transmitted?

Sol.



Suppose intensity of unpolarised light = I'

∴ Intensity of polarised light

$$= \frac{I'}{2}$$

Intensity of emergent light is $I = \frac{I'}{2} \cos^2 30^\circ = \frac{I'}{2} \times \frac{3}{4} = \frac{3I'}{8}$

∴ Fraction of incident unpolarised light transmitted,

$$\frac{I}{I'} = \frac{3}{8}$$

or $\frac{I}{I'} \times 100 = \frac{300}{8} = 37.5\%$

Polarisation of transverse mechanical waves

If a transverse mechanical wave is passed through a narrow slit, so that the plane of vibration of the wave is parallel to the slit, then the wave passes through the slit with its vibrations being unaffected and the wave is said to be plane polarised.

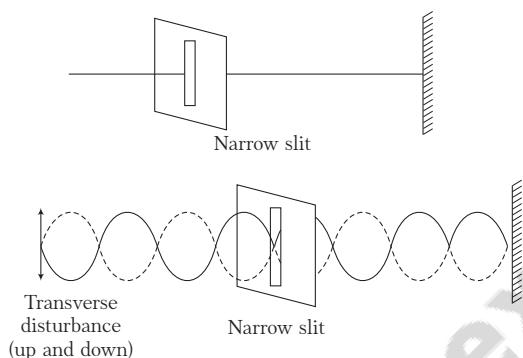


Fig. 10.35 Polarisation of transverse wave

Polarisation by reflection

When a beam of unpolarised light is reflected from a transparent medium (refractive index $= \mu$), then the reflected light is completely plane polarised at a certain angle of incidence (called the **angle of polarisation** θ_p).

In this case, reflected and refracted waves are perpendicular to each other.

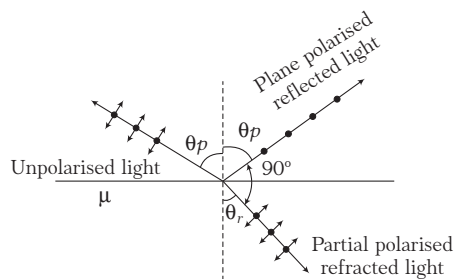


Fig. 10.36 Polarisation of light by reflection

By Snell's law, $\frac{\sin \theta_p}{\sin \theta_r} = \mu$

Now, since reflected ray is perpendicular to refracted ray

$$90^\circ + \theta_p + \theta_r = 180^\circ$$

$$\Rightarrow \theta_p + \theta_r = 90^\circ$$

$$\Rightarrow \theta_r = 90^\circ - \theta_p$$

$$\text{Now, } \frac{\sin \theta_p}{\sin \theta_r} = \frac{\sin \theta_p}{\sin(90^\circ - \theta_p)} = \mu \Rightarrow \tan \theta_p = \mu$$

It is called **Brewster's law**.

Example 10.54 When light of particular wavelength falls on a plane surface at an angle of incidence 60° , then the reflected light becomes completely plane polarised. Find the refractive index of surface material and the angle of refraction through it.

Sol. Using Brewster's law, $\mu = \tan \theta_p = \tan 60^\circ = \sqrt{3}$

At this condition, $\theta_p + \theta_r = 90^\circ$

$$\therefore \theta_r = 90^\circ - 60^\circ$$

Angle of refraction, $\theta_r = 30^\circ$

Polarisation of light by scattering

When a beam of white light passes through a medium consisting of small particles of size of the order of the wavelength of light (dust, smoke, air molecules), then the light seen in a direction perpendicular to the incident beam appears bluish. This phenomenon is known as **scattering of light** and the particles causing scattering are called **scatterers**. The scattering of light is, in fact, absorption and re-radiation of light by the scatterers.

The scattering of light can be demonstrated by a simple experiment. A few drops of dilute sulphuric acid is added to a dilute solution of hypo (sodium thiosulphate) prepared in a glass tank.

A precipitate of fine sulphur particles is formed, the particles grow in size with time. Now, a beam of light from a bright source is sent through the tank. The light scattered by the sulphur particles in a direction at right angles to the incident light appears bluish as shown in figure below.

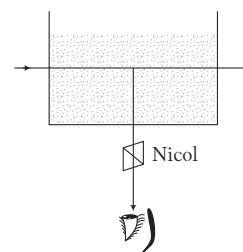


Fig. 10.37 Polarisation of light by sulphur particles

When the scattered bluish light is seen through an analyser (e.g. a rotating Nicol) a variation in intensity with minimum intensity zero is found. This shows that the light scattered in a direction perpendicular to the incident light is plane polarised.

CHECK POINT 10.4

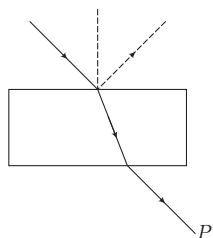
- Which of the following cannot be polarised?
 (a) Radio waves (b) Ultraviolet rays
 (c) Infrared rays (d) Ultrasonic waves
- Light waves can be polarised as they are
 (a) transverse
 (b) of high frequency
 (c) longitudinal
 (d) reflected
- An optically active compound
 (a) rotates the plane polarised light
 (b) changes the direction of polarised light
 (c) does not allow plane polarised light to pass through
 (d) None of the above
- A polaroid is placed at 45° to an incoming light of intensity I_0 . Now, the intensity of light passing through polaroid after polarisation would be
 (a) I_0 (b) $I_0 / 2$
 (c) $I_0 / 4$ (d) zero
- When the angle of incidence on a material is 60° , then the reflected light is completely polarised. The velocity (in ms^{-1}) of the refracted ray inside the material is
 (a) 3×10^8 (b) $\left(\frac{3}{\sqrt{2}}\right) \times 10^8$
 (c) $\sqrt{3} \times 10^8$ (d) 0.5×10^8
- The angle of polarisation for any medium is 60° , what will be critical angle for this
 (a) $\sin^{-1} \sqrt{3}$ (b) $\tan^{-1} \sqrt{3}$
 (c) $\cos^{-1} \sqrt{3}$ (d) $\sin^{-1} \left(\frac{1}{\sqrt{3}}\right)$
- A light has amplitude A and angle between analyser and polariser is 60° . Light reflected by analyser has amplitude
 (a) $A\sqrt{2}$ (b) $A/\sqrt{2}$
 (c) $\sqrt{3}A/2$ (d) $A/2$
- Light is incident on a glass surface at polarising angle of 57.5° . Then, the angle between the incident ray and the refracted ray is
 (a) 57.5° (b) 115°
 (c) 65° (d) 205°
- When unpolarised light beam is incident from air onto glass ($n = 1.5$) at the polarising angle,
 (a) reflected beam is polarised 100 per cent
 (b) reflected and refracted beams are partially polarised
 (c) the reason for option (a) is that almost all the light is reflected
 (d) All of the above
- The Brewster angle for the glass-air interface is 54.74° . If a ray of light going from air to glass strikes at an angle of incidence 45° , then the angle of refraction is (take, $\tan 54.74^\circ = \sqrt{2}$)
 (a) 60° (b) 30°
 (c) 25° (d) 54.74°

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1 In a Young's double slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case, [NCERT Exemplar]
 - (a) there shall be alternate interference patterns of red and blue
 - (b) there shall be an interference pattern for red distinct from that for blue
 - (c) there shall be no interference fringes
 - (d) there shall be an interference pattern for red mixing with one for blue
- 2 In Young's double slit experiment, the fringe width is β . If entire arrangement is placed in a liquid of refractive index n , then the fringe width becomes
 - (a) $\frac{\beta}{n+1}$
 - (b) $n\beta$
 - (c) $\frac{\beta}{n}$
 - (d) $\frac{\beta}{n-1}$
- 3 Consider the sunlight incident on a slit of width $10^4 \text{ \AA}$. The image seen through the slit shall [NCERT Exemplar]
 - (a) be a fine sharp slit white in colour at the centre
 - (b) a bright slit white at the centre diffusing to zero intensities at the edges
 - (c) a bright slit white at the centre diffusing to regions of different colours
 - (d) only be a diffused slit white in colour
- 4 In Young's double slit experiment, the angular width of a fringe formed on a distant screen is 1° , the wavelength of light used is $6000 \text{ \AA}$. The spacing between the slits is
 - (a) $6 \times 10^{-7} \text{ m}$
 - (b) $6.8 \times 10^{-5} \text{ m}$
 - (c) $3.4 \times 10^{-5} \text{ m}$
 - (d) $3.4 \times 10^{-4} \text{ m}$
- 5 Consider a light beam incident from air to the glass slab at Brewster's angle as shown in figure. A polaroid is placed in the path of the emergent ray at point P and rotated about an axis passing through the centre and perpendicular to the plane of the polaroid. [NCERT Exemplar]
 - (a) For a particular orientation, there shall be darkness as observed through the polaroid
 - (b) The intensity of light as seen through the polaroid shall be independent of the rotation
 - (c) The intensity of light as seen through the polaroid shall go through a minimum but not zero for two orientations of the polaroid
 - (d) The intensity of light as seen through the polaroid shall go through a minimum for four orientations of the polaroid
- 6 Two waves of same frequency and same amplitude from two monochromatic sources are allowed to superpose at a certain point. If in one case, the phase difference is 0 and in other case it is $\pi/2$, then the ratio of the intensities in the two cases will be
 - (a) 1 : 1
 - (b) 2 : 1
 - (c) 4 : 1
 - (d) None of the above
- 7 What is the minimum thickness of a soap film needed for constructive interference in reflected light, if the light incident on the film is 750 nm ? Assume that the index for the film is $n = 1.33$.
 - (a) 282 nm
 - (b) 70.5 nm
 - (c) 141 nm
 - (d) 387 nm
- 8 Monochromatic light from a narrow slit illuminates two parallel narrow slits producing an interference pattern on a screen. The separation between the two slits is now doubled and the distance between the screen and the slits is reduced to half. The fringe width
 - (a) is doubled
 - (b) becomes four times
 - (c) becomes one-fourth
 - (d) remains the same
- 9 Two slits separated by a distance of 1 mm , are illuminated with red light of wavelength $6.5 \times 10^{-7} \text{ m}$. The interference fringes are observed on a screen placed 1 m from the slits. The distance between the third dark fringe and the fifth bright fringe is equal to
 - (a) 0.65 mm
 - (b) 1.63 mm
 - (c) 3.25 mm
 - (d) 4.88 mm



- 10** In Young's double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen, when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by

(a) 12 (b) 18
(c) 24 (d) 30

- 11** A parallel beam of monochromatic light of wavelength $5000 \text{ \AA}$ is incident normally on a single narrow slit of width 0.001 mm . The light is focused by a convex lens on a screen placed on the focal plane. The first minima will be formed for the angle of diffraction equal to

(a) 0° (b) 15°
(c) 30° (d) 60°

- 12** In Young's double slit experiment, the 10th maxima of wavelength λ_1 is at a distance of y_1 from the central maximum. When the wavelength of the source is changed to λ_2 , 5th maxima is at a distance of y_2 from its central maximum. The ratio $\left(\frac{y_1}{y_2}\right)$ is

(a) $\frac{2\lambda_1}{\lambda_2}$ (b) $\frac{2\lambda_2}{\lambda_1}$
(c) $\frac{\lambda_1}{2\lambda_2}$ (d) $\frac{\lambda_2}{2\lambda_1}$

- 13** It is found that when waves of same intensity from two coherent sources superpose at a certain point, then the resultant intensity is equal to the intensity of one wave only. This means that the phase difference between the two waves at that point is

(a) zero (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) π

- 14** Young's double slit experiment is performed in a liquid. The 10th bright fringe in liquid lies on screen where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately

(a) 1.8 (b) 1.54
(c) 1.67 (d) 1.2

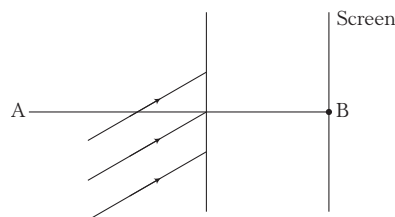
- 15** In a Young's double slit experiment using red and blue lights of wavelengths 600 nm and 480 nm respectively, the value of n for which the n th red fringe coincides with $(n+1)$ th blue fringe is

(a) 5 (b) 4
(c) 3 (d) 2

- 16** In Young's double slit experiment, how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringes, if $\lambda = 2000 \text{ \AA}$ and $d = 7000 \text{ \AA}$?

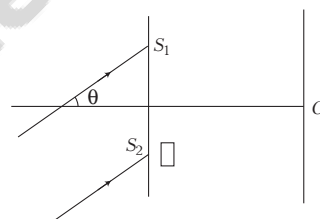
(a) 12 (b) 7
(c) 18 (d) 4

- 17** A beam of light parallel to central line AB is incident on the plane of slits. The number of minima obtained on the large screen is n_1 . Now, if the beam is tilted by some angle ($\neq 90^\circ$) as shown in figure, then the number of minima obtained is n_2 . Then,



(a) $n_1 = n_2$ (b) $n_1 > n_2$
(c) $n_2 > n_1$ (d) n_2 will be zero

- 18** A monochromatic beam of light falls on YDSE apparatus at some angle (say θ) as shown in the figure. A thin sheet of glass is inserted in front of the lower slit S_2 . The central bright fringe (path difference = 0) will be obtained



(a) at O
(b) above O
(c) below O
(d) anywhere depending on angle θ , thickness of plate t and refractive index of glass μ

- 19** Unpolarised light of intensity 32 Wm^{-2} passes through three polarisers such that the transmission axis of the last polariser is crossed with the first. If the intensity of the emerging light is 3 Wm^{-2} . What is the angle between the transmission axes of the first two polarisers?

(a) 60° (b) 45° (c) 90° (d) 120°

- 20** In a Young's double slit experiment, one of the slits is covered with a transparent sheet of thickness $3.6 \times 10^{-3} \text{ cm}$ due to which position of central fringe shifts to a position originally occupied by 30th bright fringe. The refractive index of the sheet, if $\lambda = 6000 \text{ \AA}$ is

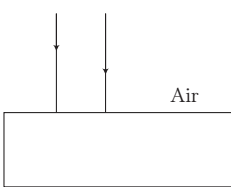
(a) 1.5 (b) 1.2 (c) 1.4 (d) 1.6

- 21** In Young's double slit experiment, the y -coordinates of central maximum and 10th maxima are 2 cm and 5 cm, respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5. The corresponding y -coordinates will be

(a) 2 cm, 7.5 cm (b) 3 cm, 6 cm
(c) 2 cm, 4 cm (d) $\frac{4}{3}$ cm, $\frac{10}{3}$ cm

- 22** In Young's double slit experiment, wavelength $\lambda = 5000 \text{ \AA}$, the distance between the slits is 0.2 mm and the screen is at 200 cm from the slits. The central maximum is at $x = 0$. The third maxima (taking the central maximum as zeroth maxima) will be at x equal to
 (a) 1.67 cm (b) 1.5 cm (c) 0.5 cm (d) 5.0 cm

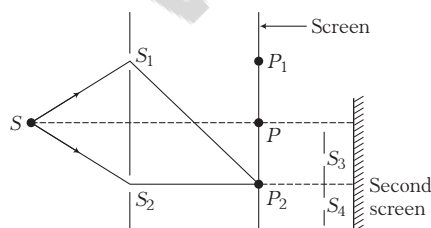
- 23** A parallel beam of light of intensity I is incident on a glass plate. 25% of light is reflected in any reflection by upper surface and 50% of light is reflected by any reflection from lower surface. Rest is refracted. The ratio of maximum to minimum intensity in interference region of reflected rays is



(a) $\left(\frac{\frac{1}{2} + \sqrt{\frac{3}{8}}}{\frac{1}{2} - \sqrt{\frac{3}{8}}} \right)^2$ (b) $\left(\frac{\frac{1}{4} + \sqrt{\frac{3}{8}}}{\frac{1}{2} - \sqrt{\frac{3}{8}}} \right)^2$
 (c) $\frac{5}{8}$ (d) None of these

- 24** In the Young's double slit experiment, the intensities at two points P_1 and P_2 on the screen are respectively I_1 and I_2 . If P_1 is located at the centre of a bright fringe and P_2 is located at a distance equal to a quarter of fringe width from P_1 , then $\frac{I_1}{I_2}$ is
 (a) 2 (b) 3 (c) 4 (d) None of these

- 25** Figure shows a standard two slit arrangement with slits S_1, S_2 . P_1, P_2 are the two minima points on either side of P (figure).



At point P_2 on the screen, there is a hole and behind P_2 is a second 2-slit arrangement with slits S_3, S_4 and a second screen behind them. [NCERT Exemplar]

- (a) There would be no interference pattern on the second screen but it would be lighted
 (b) The second screen would be totally dark

- (c) There would be a single bright point on the second screen
 (d) There would be a regular two slit pattern on the second screen

- 26** In Young's double slit experiment, the two slits acts as coherent sources of equal amplitude A and wavelength λ . In another experiment with the same set up the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is
 (a) 4 : 1 (b) 1 : 1 (c) 2 : 1 (d) 1 : 4

- 27** In the ideal double slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is
 (a) 2λ (b) $\frac{2\lambda}{3}$ (c) $\frac{\lambda}{3}$ (d) λ

- 28** In the standard Young's double slit experiment, the intensity on the screen at a point distant 1.25 fringe widths from the central maximum is (assuming slits to be identical)
 (a) $\frac{1}{2} I_{\max}$ (b) $\frac{1}{4} I_{\max}$ (c) $\frac{1}{3} I_{\max}$ (d) $I_{\max}$

- 29** In a Young's double slit experiment, D equals the distance of screen and d is the separation between the slits. The distance of the nearest point to the central maximum where the intensity is same as that due to a single slit, is equal to
 (a) $\frac{D\lambda}{d}$ (b) $\frac{D\lambda}{2d}$ (c) $\frac{D\lambda}{3d}$ (d) $\frac{2D\lambda}{d}$

- 30** The intensity of each of the two slits in Young's double slit experiment is I_0 . Calculate the minimum separation between the two points on the screen where intensities are $2I_0$ and I_0 . (Given, fringe width = α)
 (a) $\frac{\alpha}{4}$ (b) $\frac{\alpha}{3}$ (c) $\frac{\alpha}{12}$ (d) None of these

- 31** One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4, and the other by a thin glass plate of the refractive index 1.7. The point on the screen where the central maximum fall before the glass plate was inserted, is now occupied by what had been the fifth bright fringe was seen before. Assume the plate have the same thickness t and wavelength of light 480 nm . Then, the value of t is
 (a) $2.4 \text{ }\mu\text{m}$ (b) $4.8 \text{ }\mu\text{m}$
 (c) $8 \text{ }\mu\text{m}$ (d) $16 \text{ }\mu\text{m}$

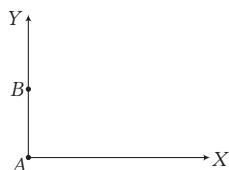
- 32** An interference is observed due to two coherent sources separated by a distance 5λ along Y -axis, where λ is the wavelength of light. A detector D is moved along the positive X -axis. The number of point on the X -axis excluding the points $x = 0$ and $x = \infty$ at which resultant intensity will be maximum, are

(a) 4 (b) 5
(c) ∞ (d) zero

- 33** In a Young's double slit experiment, using unequal slit widths, the intensity at a point midway between a bright and dark fringes is $4I$. If one slit is covered by an opaque film, intensity at that point becomes $2I$. If the other is covered instead, then the intensity at that point is

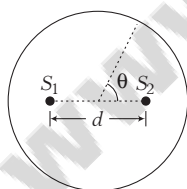
(a) $2I$ (b) $5I$
(c) $(5 + 2\sqrt{2})I$ (d) $(5 + 4\sqrt{2})I$

- 34** Two coherent light sources A and B are at a distance 3λ from each other (λ = wavelength). The distance from A on the X -axis at which the interference is constructive, is



(a) 3λ (b) 8λ (c) $5\lambda/4$ (d) 8.75λ

- 35** Two coherent sources separated by distance d are radiating in phase having wavelength λ . A detector moves in a big circle around the two sources in the plane of the two sources. The angular position of $n = 4$ interference maxima is given as

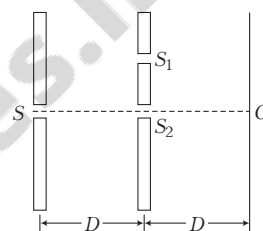


(a) $\sin^{-1} \frac{n\lambda}{d}$ (b) $\cos^{-1} \frac{4\lambda}{d}$ (c) $\tan^{-1} \frac{d}{4\lambda}$ (d) $\cos^{-1} \frac{\lambda}{4d}$

- 36** Consider a ray of light incident from air onto a slab of glass (refractive index n) of width d , at an angle θ . The phase difference between the ray reflected by the top surface of the glass and the bottom surface is

(a) $\frac{4\pi d}{\lambda} \left(1 - \frac{1}{n^2} \sin^2 \theta\right)^{-1/2} + \pi$ [NCERT Exemplar]
(b) $\frac{4\pi d}{\lambda} \left(1 - \frac{1}{n^2} \sin^2 \theta\right)^{1/2}$
(c) $\frac{4\pi d}{\lambda} \left(1 - \frac{1}{n^2} \sin^2 \theta\right)^{1/2} + \frac{\pi}{2}$
(d) $\frac{4\pi d}{\lambda} \left(1 - \frac{1}{n^2} \sin^2 \theta\right)^{1/2} + 2\pi$

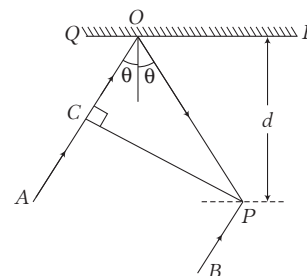
- 37** Two ideal slits S_1 and S_2 are at a distance d apart and illuminated by light of wavelength λ passing through an ideal source slit S placed on the line through S_2 as shown below. The distance between the plane of slits and the source slit is D .



A screen is held at a distance D from the plane of the slits. The minimum value of d for which there is darkness at O is

(a) $\sqrt{\frac{3\lambda D}{2}}$ (b) $\sqrt{\lambda D}$ (c) $\sqrt{\frac{\lambda D}{2}}$ (d) $\sqrt{3\lambda D}$

- 38** In the diagram below, CP represents a wavefront and AO and BP , the corresponding incident rays. Find the condition of θ for constructive interference at point P between the rays BP and reflected ray OP



(a) $\cos \theta = 3\lambda/2d$ (b) $\cos \theta = \lambda/4d$
(c) $\sec \theta - \cos \theta = \lambda/d$ (d) $\sec \theta - \cos \theta = 4\lambda/d$

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
- (b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
- (c) If Assertion is true but Reason is false.
- (d) If Assertion is false but Reason is true.

1 Assertion Two coherent sources transmit waves of equal intensity I_0 . Resultant intensity at a point where path difference is $\frac{\lambda}{3}$ is also I_0 .

Reason In interference resultant intensity at any point is the average intensity of two individual intensities.

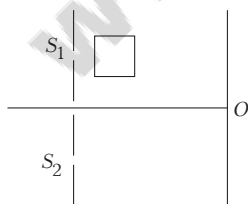
2 Assertion In Young's double slit experiment, ratio $\frac{I_{\max}}{I_{\min}}$ is infinite.

Reason If width of any one of the slits is slightly increased, then this ratio will decrease.

3 Assertion If a glass slab is placed in front of one of the slits, then fringe width will decrease.

Reason Glass slab will produce an additional path difference.

4 Assertion In the YDSE set up shown in figure, a glass slab is inserted in front of the slit S_1 as shown in figure. Then, zeroth order maxima (where path difference = 0) will lie above point O .



Reason Glass slab will produce an extra path difference in the path of the ray passing through S_1 .

5 Assertion The unpolarised light and polarised light can be distinguished from each other by using polaroid.

Reason A polaroid is capable of producing plane polarised beam of light.

Statement based questions

1 Which one of the following statement is incorrect?

- (a) If the wave is longitudinal, then it must be a mechanical wave.
- (b) If the wave is mechanical, then it may or may not be transverse wave.
- (c) Mechanical waves cannot propagate in vacuum.
- (d) Polarisation help us to distinguish between sound wave and light wave.

2 Two lights of wavelength $\lambda_1 = 4500 \text{ \AA}$ and $\lambda_2 = 6000 \text{ \AA}$ are sent through a Young's double slit apparatus simultaneously, then which of the following statement(s) is/are correct?

- (a) No interference pattern will be formed.
- (b) The third order bright fringe of λ_1 will coincide with the fourth order bright fringe of λ_2 .
- (c) The third order bright fringe of λ_2 will coincide with the fourth order bright fringe of λ_1 .
- (d) The fringes of wavelength λ_1 will be wider than the fringes of wavelength λ_2 .

3 Consider the following statements in case of Young's double slit experiment.

- I. A slit S is necessary, if we use an ordinary extended source of light.
 - II. A slit S is not needed, if we use an ordinary but well collimated beam of light.
 - III. A slit S is not needed, if we use a spatially coherent source of light.
- (a) I, II and III (b) Both I and II
(c) Both II and III (d) Both I and III

Match the columns

1 In Young's double slit experiment, match the following two columns.

Column I	Column II
(A) When width of one slit is slightly increased	(p) maximum intensity will increase
(B) When one slit is closed	(q) maximum intensity will decrease
(C) When both the sources are made incoherent	(r) maximum intensity will remain same
(D) When a glass slab is inserted in front of one of the slits	(s) fringe pattern will disappear

Note Assume absorption from glass slab to be negligible.

Codes

- | | | | | | | | |
|---------|---|-----|-----|---------|-----|-----|---|
| A | B | C | D | A | B | C | D |
| (a) q,s | r | p,q | s | (b) p | q,s | q,s | r |
| (c) p | q | r,s | r,s | (d) p,s | p, | q,r | q |

2. In normal YDSE experiment, match the following two columns.

Column I	Column II
(A) When apparatus is immersed in a liquid	(p) fringe width will increase
(B) When wavelength of light used is increased	(q) fringe width will decrease

Column I	Column II
(C) When distance between slits and screen (D) is increased	(r) fringe width will remain constant
(D) When distance between two slits (d) is increased	(s) fringe pattern will disappear

Codes

A	B	C	D	A	B	C	D
(a) q	p	p	q	(b) p	r	s	q
(c) q	r	p	r	(d) p	s	r	p

(C) Medical entrances' gallery

Collection of questions asked in NEET & various medical entrance exams

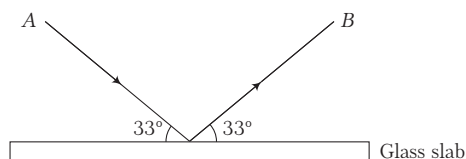
- 1 In Young's double slit experiment, if the separation between coherent sources is halved and the distance of the screen from the coherent sources is doubled, then the fringe width becomes [NEET 2020]
(a) half (b) four times (c) one-fourth (d) double
- 2 The Brewster's angle i_b for an interface should be
(a) $30^\circ < i_b < 45^\circ$ (b) $45^\circ < i_b < 90^\circ$ [NEET 2020]
(c) $i_b = 90^\circ$ (d) $0^\circ < i_b < 30^\circ$
- 3 Two coherent sources of light interfere and produce fringe pattern on a screen. For central maximum, the phase difference between the two waves will be [NEET 2020]
(a) zero (b) π (c) $3\pi/2$ (d) $\pi/2$
- 4 In a Young's double slit experiment, when light of wavelength 400 nm was used, the angular width of the first minima formed on a screen placed 1 m away, was found to be 0.2° . What will be the angular width of the first minima, if the entire experimental apparatus is immersed in water? (Take, for water $\mu = 4/3$) [NEET 2019]
(a) 0.15° (b) 0.051° (c) 0.1° (d) 0.266°
- 5 In a Young's double slit experiment, if there is no initial phase difference between the light from the two slits, a point on the screen corresponding to the 5th minima has path difference [NEET (Odisha) 2019]
(a) $5\frac{\lambda}{2}$ (b) $10\frac{\lambda}{2}$ (c) $9\frac{\lambda}{2}$ (d) $11\frac{\lambda}{2}$
- 6 Angular width of the central maximum in the fraunhofer diffraction for $\lambda = 6000 \text{ \AA}$ is θ_0 . When the same slit is illuminated by another monochromatic light, the angular width decreases by 30%. The wavelength of this light is [NEET (Odisha) 2019]
(a) $1800 \text{ \AA}$ (b) $4200 \text{ \AA}$ (c) $6000 \text{ \AA}$ (d) $420 \text{ \AA}$
- 7 Distance of 5th dark fringe from centre is 4 mm. If $D = 2 \text{ m}$, $\lambda = 600 \text{ nm}$, then distance between slits is [AIIMS 2019]
(a) 1.35 mm (b) 2.00 mm
(c) 3.25 mm (d) 10.35 mm
- 8 A light of wavelength 500 nm is incident on a Young's double slit experiment. The distance between slit and screen is $D = 1.8 \text{ m}$ and distance between slits is $d = 0.4 \text{ mm}$. If screen moves with a speed of 4 ms^{-1} , then with what speed first maxima will move? [AIIMS 2019]
(a) 5 mms^{-1} (b) 4 mms^{-1} (c) 3 mms^{-1} (d) 2 mms^{-1}
- 9 **Assertion** Distance between position of bright and dark fringes remain same in Young's double slit experiment.
Reason Fringe width, $\beta = \frac{\lambda D}{d}$ [AIIMS 2019]
(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Both Assertion and Reason are incorrect.
- 10 **Assertion** Incoming light reflected by earth is partially polarised.
Reason Atmospheric particle polarise the light. [AIIMS 2019]
(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Both Assertion and Reason are incorrect.
- 11 Huygens' principle does not use [JIPMER 2019]
(a) reflection (b) refraction
(c) diffraction (d) point of spectra origin
- 12 In a Young's double slit experiment, the separation d between the slits is 2 mm, the wavelength λ of the light used is $5896 \text{ \AA}$ and distance D between the screen and slit is 100 cm. It is found that, the angular width of the fringe is 0.20° . To increase the

- fringe angular width to 0.21° (with same λ and D), the separation between the slits needs to be changed to [NEET 2018]
- (a) 2.1 mm (b) 1.9 mm
(c) 1.8 mm (d) 1.7 mm
- 13** Unpolarised light is incident from air on a plane surface of a material of refractive index μ . At a particular angle of incidence i , it is found that the reflected and refracted rays are perpendicular to each other. Which one of the following option is correct for this situation? [NEET 2018]
- (a) $i = \sin^{-1}\left(\frac{1}{\mu}\right)$
(b) Reflected light is polarised with its electric vector perpendicular to the plane of incidence.
(c) Reflected light is polarised with its electric vector parallel to the plane of incidence.
(d) $i = \tan^{-1}\left(\frac{1}{\mu}\right)$
- 14** Young's double slit experiment is first performed in air and then in a medium other than air. It is found that 8th bright fringe in the medium lies, where 5th dark fringe lies in air. The refractive index of the medium is nearly [NEET 2017]
- (a) 1.25 (b) 1.59 (c) 1.69 (d) 1.78
- 15** Two polaroids P_1 and P_2 are placed with their axis perpendicular to each other. Unpolarised light I_0 is incident on P_1 . A third polaroid P_3 is kept in between P_1 and P_2 such that its axis makes an angle 45° with that of P_1 . The intensity of transmitted light through P_2 is [NEET 2017]
- (a) $\frac{I_0}{2}$ (b) $\frac{I_0}{4}$ (c) $\frac{I_0}{8}$ (d) $\frac{I_0}{16}$
- 16** The Young's double slit experiment is performed with blue and green light of wavelengths $4360 \text{ \AA}$ and $5460 \text{ \AA}$, respectively. If x is the distance of 4th maxima from the central one, then [AIIMS 2017]
- (a) $x_{\text{blue}} = x_{\text{green}}$ (b) $x_{\text{blue}} > x_{\text{green}}$
(c) $x_{\text{blue}} < x_{\text{green}}$ (d) $x_{\text{blue}} / x_{\text{green}}$
- 17** White light is used to illuminate two slits in Young's double slit experiment. The separation between the slits is b and the screen is at a distance $d(>> b)$ from the slits. At a point on the screen directly in front of one of the slits, which wavelengths are missing? [JIPMER 2017]
- (a) $\frac{b}{d}, \frac{b}{3d}, \frac{b}{5d}$ (b) $\frac{b^2}{2d}, \frac{b^2}{4d}, \frac{b^2}{6d}$
(c) $\frac{b^2}{d}, \frac{b^2}{3d}, \frac{b^2}{5d}$ (d) $\frac{b}{2d}, \frac{b}{4d}, \frac{b}{6d}$
- 18** The maximum numbers of possible interference maxima for slit separation equal to twice the wavelength in Young's double slit experiment, is [JIPMER 2017]
- (a) infinite (b) five (c) three (d) zero
- 19** The interference pattern is obtained with two coherent light sources of intensity ratio n . In the interference pattern, the ratio $\frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$ will be [NEET 2016]
- (a) $\frac{\sqrt{n}}{n+1}$ (b) $\frac{2\sqrt{n}}{n+1}$
(c) $\frac{\sqrt{n}}{(n+1)^2}$ (d) $\frac{2\sqrt{n}}{(n+1)^2}$
- 20** A linear aperture whose width is 0.02 cm is placed immediately in front of a lens of focal length 60 cm . The aperture is illuminated normally by a parallel beam of wavelength $5 \times 10^{-5} \text{ cm}$. The distance of the first dark band of the diffraction pattern from the centre of the screen is [NEET 2016]
- (a) 0.10 cm (b) 0.25 cm
(c) 0.20 cm (d) 0.15 cm
- 21** The intensity at the maximum in a Young's double slit experiment is I_0 . Distance between two slits is $d = 5\lambda$, where λ is the wavelength of light used in the experiment. What will be the intensity in front of one of the slits on the screen placed at a distance, $D = 10 d$? [NEET 2016]
- (a) $\frac{I_0}{4}$ (b) $\frac{3}{4} I_0$ (c) $\frac{I_0}{2}$ (d) I_0
- 22** In a diffraction pattern due to a single slit of width a , the first minima is observed at an angle 30° when light of wavelength $5000 \text{ \AA}$ is incident on the slit. The first secondary maxima is observed at an angle of [NEET 2016]
- (a) $\sin^{-1}\left(\frac{2}{3}\right)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$
(c) $\sin^{-1}\left(\frac{3}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{4}\right)$
- 23** For a parallel beam of monochromatic light of wavelength λ diffraction is produced by a single slit whose width a is of the order of the wavelength of the light. If D is the distance of the screen from the slit, the width of the central maximum will be [CBSE AIPMT 2015]
- (a) $\frac{2D\lambda}{a}$ (b) $\frac{D\lambda}{a}$
(c) $\frac{Da}{\lambda}$ (d) $\frac{2Da}{\lambda}$

- 24** In a Young's double slit experiment, the two slits are 1 mm apart and the screen is placed 1 m away. A monochromatic light of wavelength 500 nm is used. What will be the width of each slit for obtaining 10th maxima of double slit within the central maximum of single slit pattern? [CBSE AIPMT 2015]
(a) 0.2 mm (b) 0.1 mm (c) 0.5 mm (d) 0.02 mm
- 25** Light of wavelength λ is incident on slit of width d . The resulting diffraction pattern is observed on a screen placed at distance D . The linear width of central maximum is equal to width of the slit, then $D = \dots$. [Guj. CET 2015]
(a) $\frac{2\lambda^2}{d}$ (b) $\frac{d^2}{2\lambda}$ (c) $\frac{d}{\lambda}$ (d) $\frac{2\lambda}{d}$
- 26** A beam of light contains two wavelengths 6500 Å and 5200 Å. They form interference fringes in Young's double slit experiment. What is the least distance (approx) from central maximum, where the bright fringes due to both wavelength coincide? The distance between the slits is 2 mm and distance between the screen and slit is 120 cm. [CG PMT 2015]
(a) 0.16 cm (b) 0.32 cm (c) 0.48 cm (d) 1.92 cm
- 27** Find the right condition for Fraunhofer diffraction due to a single slit. [WB JEE 2015]
(a) Source is at infinite distance and the incident beam has converged at the slit.
(b) Source is near to the slit and the incident beam is parallel.
(c) Source is at infinity and the incident beam is parallel.
(d) Source is near to the slit and the incident beam has converged at the slit.
- 28** The parallel beams of monochromatic light of wavelength 4.5×10^{-7} m passes through a long slit with width 0.2×10^{-3} m. The angular divergence in which most of the light is [UP CPMT 2015]
(a) 4.5×10^{-3} rad (b) 4.5π rad
(c) 2.25×10^{-3} rad (d) 9×10^{-3} rad
- 29** The condition for obtaining secondary maxima in the diffraction pattern due to single slit is [Manipal 2015]
(a) $a \sin \theta = \frac{n\lambda}{2}$ (b) $a \sin \theta = (2n - 1)\lambda/2$
(c) $a \sin \theta = n\lambda$ (d) $a \sin \theta = (2n - 1)\lambda$
- 30** In Young's double slit experiment, when wavelength used is 6000 Å and the screen is 40 cm from the slits, the fringes are 0.012 cm wide. What is the distance between the slits? [Manipal 2015]
(a) 0.24 cm (b) 0.2 cm (c) 2.4 cm (d) 0.024 cm
- 31** The polarising angle of glass is 57° . A ray of light which is incident at this angle will have an angle of refraction as [KCET 2015]
(a) 43° (b) 25° (c) 38° (d) 33°
- 32** To observe diffraction, the size of the obstacle [KCET 2015]
(a) should be much larger than the wavelength
(b) has no relation to wavelength
(c) should be of the order of wavelength
(d) should be $\lambda/2$, where λ is the wavelength
- 33** A plane wave of wavelength 6250 Å is incident normally on a slit of width 2×10^{-2} cm. The width of the principal maximum of diffraction pattern on a screen at a distance of 50 cm will be [EAMCET 2015]
(a) 312.5×10^{-3} cm
(b) 312.5×10^{-4} cm
(c) 312 cm
(d) 312.5×10^{-5} cm
- 34** A beam of light of wavelength 590 nm is focussed by a converging lens of diameter 10 cm at a distance of 20 cm from it. Find the diameter of the disc image formed [UK PMT 2015]
(a) 5.76×10^{-4} cm (b) 7.51×10^{-4} cm
(c) 8.72×10^{-4} cm (d) 9.80×10^{-4} cm
- 35** A plane polarised light is incident normally on a tourmaline plate. Its $\mathbf{E}$ vector make an angle of 60° with the optic axis of the plate. Find the percentage difference between initial and final intensities. [Guj. CET 2015]
(a) 50% (b) 25%
(c) 75% (d) 90%
- 36** In Young's double slit experiment, the locus of the point P lying in a plane with a constant path difference between two interfering waves is [Kerala CEE 2015]
(a) a hyperbola (b) a straight line
(c) an ellipse (d) a parabola
(e) a circle
- 37** Wavefront is the locus of all points, where the particles of the medium vibrate with the same [KCET 2014]
(a) phase (b) amplitude
(c) frequency (d) period
- 38** In the Young's double slit experiment, the intensity of light at a point on the screen (where the path difference is λ) is K , (λ being the wavelength of light used). The intensity at a point where the path difference is $\lambda/4$, will be [J&K CET 2014]
(a) K (b) $K/4$
(c) $K/2$ (d) zero
- 39** Two coherent monochromatic beams of intensities I and $4I$ respectively, are superposed. Then, the maximum and minimum intensities in the resulting pattern are [WB JEE 2014]
(a) $5I$ and $3I$ (b) $9I$ and $3I$
(c) $4I$ and I (d) $9I$ and I

- 40** Colours appear on a thin soap film and soap bubbles due to the phenomenon of [UK PMT 2014]
 (a) interference (b) scattering
 (c) diffraction (d) dispersion
- 41** In Young's double slit experiment, the ratio of maximum and minimum intensities in the fringe system is 9 : 1. The ratio of amplitudes of coherent sources is [UK PMT 2014]
 (a) 9 : 1 (b) 3 : 1
 (c) 2 : 1 (d) 1 : 1
- 42** In Young's double slit interference experiment, using two coherent waves of different amplitudes, the intensities ratio between bright and dark fringes is 3. Then, the value of the wave amplitudes ratio that arrive there is [EAMCET 2014]
 (a) $\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ (b) $\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$
 (c) $\sqrt{3}:1$ (d) $1:\sqrt{3}$
- 43** The angle of incidence at which reflected light is totally polarised for refraction from air to glass (refractive index μ) is [UK PMT 2014]
 (a) $\sin^{-1}\mu$ (b) $\sin^{-1}\left(\frac{1}{\mu}\right)$
 (c) $\tan^{-1}\left(\frac{1}{\mu}\right)$ (d) $\tan^{-1}(\mu)$
- 44** A polarised light of intensity I_0 is passed through another polariser whose pass axis makes an angle of 60° with the pass axis of the former. What is the intensity of emergent polarised light from second polariser? [KCET 2014]
 (a) $I = I_0$ (b) $I = I_0 / 6$
 (c) $I = I_0 / 5$ (d) $I = I_0 / 4$
- 45** A fringe width of a certain interference pattern is $\beta = 0.002$ cm. What is the distance of 5th dark fringe from centre? [MHT CET 2014]
 (a) 1×10^{-2} cm (b) 11×10^{-2} cm
 (c) 1.1×10^{-2} cm (d) 3.28×10^6 cm
- 46** In Young's double slit experiment, the slits are 2 mm apart and are illuminated by photons of two wavelengths $\lambda_1 = 12000 \text{ \AA}$ and $\lambda_2 = 10000 \text{ \AA}$. At what minimum distance from the common central bright fringe on the screen 2 m from the slit will a bright fringe from one interference pattern coincide with a bright fringe from the other? [NEET 2013]
 (a) 8 mm (b) 6 mm
 (c) 4 mm (d) 3 mm
- 47** A parallel beam of fast moving electrons is incident normally on a narrow slit. A fluorescent screen is placed at a large distance from the slit. If the speed of the electrons is increased, then which of the following statement is correct? [NEET 2013]
 (a) Diffraction pattern is not observed on the screen in the case of electrons.
 (b) The angular width of the central maximum of the diffraction pattern will increase.
 (c) The angular width of the central maximum will decrease.
 (d) The angular width of the central maximum will be unaffected.
- 48** If two slits in Young's experiment are 0.4 mm apart and fringe width on a screen 200 cm away is 2 mm, the wavelength of light illuminating the slits is [J&K CET 2013]
 (a) 500 nm (b) 600 nm (c) 400 nm (d) 300 nm
- 49** Light of wavelength λ from a point source falls on a small circular obstacle of diameter d . Dark and bright circular rings around a central bright spot are formed on a screen beyond the obstacle. The distance between the screen and obstacle is D . Then, the condition for the formation of rings is [EAMCET 2013]
 (a) $\sqrt{\lambda} \approx \frac{d}{4D}$ (b) $\lambda \approx \frac{d^2}{4D}$ (c) $d \approx \frac{\lambda^2}{D}$ (d) $\lambda \approx \frac{D}{4}$
- 50** Which of the following phenomena support the wave theory of light?
 1. Scattering 2. Interference
 3. Diffraction
 4. Velocity of light in a denser medium is less than the velocity of light in the rarer medium. [Karnataka CET 2013]
 (a) 1, 2 and 3 (b) 1, 2 and 4
 (c) 2, 3 and 4 (d) 1, 3 and 4
- 51** λ_1 and λ_2 are used to illuminate the slits. β_1 and β_2 are the corresponding fringe widths. The wavelength λ_1 can produce photoelectric effect when incident on a metal. But the wavelength λ_2 cannot produce photoelectric effect. The correct relation between β_1 and β_2 is [Karnataka CET 2013]
 (a) $\beta_1 < \beta_2$ (b) $\beta_1 = \beta_2$
 (c) $\beta_1 > \beta_2$ (d) $\beta_1 \geq \beta_2$
- 52** When we close one slit in the Young's double slit experiment, then [Kerala CET 2013]
 (a) the bandwidth increased
 (b) the bandwidth is decreased
 (c) the bandwidth remains unchanged
 (d) the interference pattern is shifted
 (e) the diffraction pattern is observed

- 53** A beam of light is incident on a glass slab ($\mu = 1.54$) in a direction as shown in the figure. The reflected light is analysed by a polaroid prism. On rotating the polaroid (take, $\tan 57^\circ = 1.54$) [Kerala CET 2013]



- (a) the intensity remains unchanged
 (b) the intensity is reduced to zero and remains at zero
 (c) the intensity gradually reduced to zero and then again increases
 (d) the intensity increases continuously
 (e) the intensity increases initially and remains constant afterwards.
- 54** In Young's double slit experiment, the phase difference between the two waves reaching at the location of the third dark fringe is [MP PMT 2013]
- (a) π (b) $\frac{3\pi}{2}$
 (c) 5π (d) 3π

Directions (Q. Nos. 55-57) These questions consist of two statements each linked as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
 (b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
 (c) If Assertion is true but Reason is false.
 (d) If both Assertion and Reason are false.
- 55 Assertion** To observe diffraction of light the size of obstacle/aperture should be of the order of 10^{-7} m.
Reason 10^{-7} m is the order of wavelength of visible light. [AIIMS 2012]
- 56 Assertion** The pattern and position of fringes always remain same even after the introduction of transparent medium in a path of one of the slit.
Reason The central fringe is bright or dark depends upon the initial phase difference between the two coherence sources. [AIIMS 2012]
- 57 Assertion** Corpuscular theory fails in explaining the velocities of light in air and water.
Reason According to corpuscular theory, light should travel faster in denser medium than in rarer medium. [AIIMS 2012]

- 58** In Young's double slit experiment, the fringe width is 1×10^{-4} m. If the distance between the slit and screen is doubled and the distance between two slits is reduced to half and the wavelength is changed from 6.4×10^{-7} m to 4.0×10^{-7} m, then the value of new fringe width will be [BCECE (Mains) 2012]
- (a) 2.5×10^{-4} m (b) 2.0×10^{-4} m
 (c) 1×10^{-4} m (d) 0.5×10^{-4} m

- 59 Assertion** The clouds in sky generally appear to be whitish.
Reason Diffraction due to clouds is efficient in equal measure at all wavelengths. [AIIMS 2011]
- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Both Assertion and Reason are correct.

- 60** In a Young's double slit interference experiment, the fringe width obtained with a light of wavelength $5900 \text{ \AA}$ was 1.2 mm for parallel narrow slits placed 2 mm apart. In this arrangement, if the slit separation is increased by one-and-half times the previous value, then the fringe width is [EAMCET 2011]
- (a) 0.9 mm (b) 0.8 mm
 (c) 1.8 mm (d) 1.6 mm

- 61** In Young's double slit experiment, the angular width of the fringes is 0.20° for the sodium light ($\lambda = 5890 \text{ \AA}$). In order to increase the angular width of the fringes by 10%, the necessary change in wavelength is [Manipal 2011]
- (a) zero (b) increased by $6479 \text{ \AA}$
 (c) decreased by $589 \text{ \AA}$ (d) increased by $589 \text{ \AA}$

- 62** In Young's double slit experiment, the intensity of light coming from one of the slits is doubled the intensity from the other slit. The ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed is [Punjab PMET, Manipal 2011]
- (a) 34 (b) 40
 (c) 25 (d) 38

- 63** In Young's double slit experiment, if d , D and λ represents, the distance between the slits, the distance of the screen from the slits and wavelength of light used respectively, then the bandwidth is inversely proportional to [Kerala CEE 2011]
- (a) λ (b) d (c) D (d) λ^2
 (e) D^2

- 64** A Young's double slit set up for interference is shifted from air to within water, then the fringe width

[J&K CET 2011]

- (a) becomes infinite (b) decreases
(c) increases (d) remain unchanged

- 65** In Young's double slit experiment, the two slits are separated by 1 mm and the screen is placed 1 m away. The fringe separation for blue light of wavelength 500 nm is

[J&K CET 2011]

- (a) 10 mm (b) 0.5 mm
(c) 20 mm (d) 15 mm

- 66** In the case of light waves from two coherent sources S_1 and S_2 , there will be constructive interference at an arbitrary point P , if the path difference, $S_1P - S_2P$ is

[J&K CET 2011]

- (a) $\left(n + \frac{1}{2}\right)\lambda$ (b) $n\lambda$
(c) $\left(n - \frac{1}{2}\right)\lambda$ (d) $\frac{\lambda}{2}$

- 67** In Young's double slit experiment, fringes of width β are produced on a screen kept at a distance of 1 m from the slit. When the screen is moved away by 5×10^{-2} m, fringe width changes by 3×10^{-5} m. The separation between the slits is 1×10^{-3} m. The wavelength of the light used is ... nm.

[KCET 2011]

- (a) 400 (b) 500 (c) 600 (d) 700

- 68** In a single slit diffraction of light of wavelengths by a slit of width e , the size of the central maximum on a screen at a distance b is

[MP PMT 2011]

- (a) $2b\lambda + e$ (b) $\frac{2b\lambda}{e}$
(c) $\frac{2b\lambda}{e} + e$ (d) $\frac{2b\lambda}{e} - e$

- 69** A grating which would be most suitable for constructing a spectrometer for the visible and ultraviolet region, should have

[Haryana PMT, CG PMT 2011]

- (a) 100 lines cm^{-1} (b) 1000 lines cm^{-1}
(c) 10000 lines cm^{-1} (d) 1000000 lines cm^{-1}

- 70** In a single slit diffraction experiment, the width of the slit is made double its original width. Then, the central maximum on the diffraction pattern will become

[BCECE, J&K CET 2011]

- (a) narrower and fainter
(b) narrower and brighter
(c) broader and fainter
(d) broader and brighter

- 71** The slit width, when a light of wavelength 6500 Å is incident on a slit, if first minima for red light is at 30° , is

[DUMET 2011]

- (a) 1×10^{-6} m (b) 5.2×10^{-6} m
(c) 1.3×10^{-6} m (d) 2.6×10^{-6} m

- 72** The critical angle of a certain medium is $\sin^{-1}\left(\frac{3}{5}\right)$.

The polarising angle of the medium is [KCET 2011]

- (a) $\sin^{-1}\left(\frac{4}{5}\right)$ (b) $\tan^{-1}\left(\frac{5}{3}\right)$
(c) $\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

ANSWERS

CHECK POINT 10.1

1. (c)	2. (a)	3. (c)	4. (b)	5. (d)	6. (d)	7. (c)	8. (b)	9. (a)	10. (a)
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CHECK POINT 10.2

1. (b)	2. (d)	3. (d)	4. (c)	5. (b)	6. (d)	7. (c)	8. (a)	9. (d)	10. (b)
11. (d)	12. (a)	13. (a)	14. (b)	15. (c)	16. (a)				

CHECK POINT 10.3

1. (a)	2. (d)	3. (d)	4. (b)	5. (b)	6. (d)	7. (a)	8. (b)	9. (c)	
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CHECK POINT 10.4

1. (d)	2. (a)	3. (a)	4. (b)	5. (c)	6. (d)	7. (d)	8. (d)	9. (a)	10. (b)
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(A) Taking it together

1. (c)	2. (c)	3. (a)	4. (c)	5. (c)	6. (b)	7. (c)	8. (c)	9. (b)	10. (b)
11. (c)	12. (a)	13. (c)	14. (a)	15. (b)	16. (b)	17. (a)	18. (a)	19. (a)	20. (a)
21. (c)	22. (b)	23. (d)	24. (a)	25. (d)	26. (c)	27. (a)	28. (a)	29. (c)	30. (c)
31. (c)	32. (a)	33. (a)	34. (c)	35. (b)	36. (a)	37. (c)	38. (b)		

(B) Medical entrance special format questions

• Assertion and reason

1. (c) 2. (b) 3. (d) 4. (a) 5. (a)

• Statement based questions

1. (d) 2. (c) 3. (d)

• Match the columns

1. (b) 2. (a)

(C) Medical entrances' gallery

1. (b)	2. (b)	3. (a)	4. (a)	5. (c)	6. (b)	7. (a)	8. (a)	9. (a)	10. (b)
11. (d)	12. (b)	13. (b)	14. (d)	15. (c)	16. (c)	17. (c)	18. (b)	19. (b)	20. (d)
21. (c)	22. (c)	23. (a)	24. (a)	25. (b)	26. (a)	27. (c)	28. (a)	29. (b)	30. (b)
31. (d)	32. (c)	33. (a)	34. (a)	35. (c)	36. (a)	37. (a)	38. (c)	39. (d)	40. (a)
41. (c)	42. (c)	43. (d)	44. (d)	45. (c)	46. (b)	47. (c)	48. (c)	49. (b)	50. (c)
51. (a)	52. (e)	53. (c)	54. (c)	55. (a)	56. (b)	57. (a)	58. (a)	59. (a)	60. (b)
61. (d)	62. (a)	63. (b)	64. (b)	65. (b)	66. (b)	67. (c)	68. (c)	69. (d)	70. (a)
71. (c)	72. (b)								

Hints & Explanations

• CHECK POINT 10.1

1 (c) According to corpuscular theory of light, the different colours of light are due to different sizes of the corpuscles.

2 (a) For phenomenon of interference, it is necessary that there should be a constant phase difference between two light rays.

4 (b) Here, phase difference, $\phi = \pi / 3$

So, amplitude of resulting wave,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$$

$$= \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \cos \pi/3} \approx 6 \text{ m}$$

5 (d) Resultant intensity for two interfering waves is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi.$$

For maximum, $\cos \phi = \text{maximum} = 1$

$$\therefore I_{\max} = I_1 + I_2 + 2\sqrt{I_1I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$

6 (d) For destructive interference,

$$\cos \phi = \text{minimum} = -1$$

$$i.e. \quad \phi = \pi, 3\pi, 5\pi$$

$$\text{or} \quad \phi = (2n+1)\pi$$

$$\text{where,} \quad n = 0, 1, 2, 3, \dots$$

$$\text{or} \quad \Delta x = \frac{\lambda}{2\pi} \times \phi = \frac{(2n+1)\lambda}{2}$$

7 (c) In interference,

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

$$\text{Given,} \quad \frac{A_1}{A_2} = \frac{3}{5}$$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(3+5)^2}{(3-5)^2} = \frac{16}{1}$$

8 (b) $I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$

$$= I + 4I + 2\sqrt{I \times 4I} \times \cos \frac{2\pi}{3}$$

$$= 5I + 4I \times \left(-\frac{1}{2}\right) = 5I - 2I = 3I$$

9 (a) $I = I_1 + I_2 = I + 4I = 5I$

• CHECK POINT 10.2

1 (b) For dark fringe at point P,

$$S_1P - S_2P = \frac{5\lambda}{2}$$

$$= 5 \times \frac{6000}{2} = 15000 \text{ \AA}$$

$$= 1.5 \text{ micron}$$

2 (d) For minima, path difference should be an odd multiple of half wavelength.

$$i.e. \quad \Delta x = (2n-1) \frac{\lambda}{2}$$

$$\text{For third minima } (n=3), \quad \Delta x = (2 \times 3 - 1) \frac{\lambda}{2} = \frac{5\lambda}{2}$$

3 (d) Distance of n th bright fringe from the centre, $y_n = \frac{nD\lambda}{d}$

$$\text{So,} \quad y_3 = \frac{3 \times 6000 \times 10^{-10} \times 2.5}{0.5 \times 10^{-3}}$$

$$= 9 \times 10^{-3} \text{ m} = 9 \text{ mm}$$

$$4 (c) \beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \frac{1}{d}$$

Hence, fringe width decreases with increase in d .

5 (b) Fringe width, $\beta = \frac{D\lambda}{d}$

$$\text{If both } d \text{ and } D \text{ are doubled, then } \beta' = \frac{(2D)\lambda}{2d} = \frac{D\lambda}{d} = \beta$$

6 (d) Refractive index is slightly decreased, so fringe width is slightly increased.

7 (c) Separation of bright lines or fringe width

$$\beta = \frac{D\lambda}{d} = \frac{5 \times 10^{-7} \times 2}{10^{-3}} = 10^{-3} \text{ m} = 1.0 \text{ mm}$$

8 (a) Fringe width in double slit experiment is given by

$$\beta = \frac{D\lambda}{d}$$

It implies that $\beta \propto \lambda$, so in given options λ_{violet} = minimum.

9 (d) We have,

$$n_1\lambda_1 = n_2\lambda_2$$

$$\text{Number of fringes, } n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{60 \times 5461 \times 10^{-10}}{5890 \times 10^{-10}} = 55$$

10 (b) Given, $\frac{I_{\text{bright}}}{I_{\text{dark}}} = \frac{I_{\max}}{I_{\min}} = 9$

$$\text{or} \quad \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = 9 \quad \text{or} \quad \frac{a_1 + a_2}{a_1 - a_2} = 3$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{3+1}{3-1} \Rightarrow \frac{a_1}{a_2} = 2$$

$$\text{So,} \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{4}{1}$$

11 (d) In normal Young's double slit apparatus,

$$I_1 = I_2 = I_0$$

$$\therefore I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 = 4I_0$$

$$\text{and} \quad I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 = 0$$

When one of the slit is covered by a glass plate,

$$I_1 < I_0, I_2 = I_0$$

$$\therefore I_{\max} < 4I_0 \text{ and } I_{\min} > 0$$

- 12 (a)** In interference between waves of equal amplitudes a , the minimum intensity is zero and the maximum intensity is proportional to $4a^2$ (because slit width $\propto$ amplitude). For waves of unequal amplitudes a and A ($A > a$), the minimum intensity is non-zero and the maximum intensity is proportional to $(a + A)^2$, which is greater than $4a^2$. So, the intensities of both the maxima and the minima increases.

- 13 (a)** On introducing mica sheet, fringe shift $= \frac{\beta}{\lambda} (\mu - 1)t$

where, t is the thickness of sheet.

$$\text{So, shift} = \frac{\beta}{(5000 \times 10^{-10})} \times (1.5 - 1) \times (2 \times 10^{-6}) = 2\beta$$

So, central bright maximum will shift two fringes upward.

- 14 (b)** When thin plate of glass is introduced, then lateral displacement of fringes

$$= \frac{\beta}{\lambda} (\mu - 1)t$$

$$= \frac{1 \times 10^{-3}}{600 \times 10^{-9}} (1.5 - 1) \times 0.06 \times 10^{-3} = \frac{1}{20} \text{ m} = 5 \text{ cm}$$

- 15 (c)** Shift $= \frac{(\mu - 1)tD}{d}$ or $20 \left(\frac{\lambda D}{d} \right) = \frac{(\mu - 1)tD}{d}$

$$\therefore \mu = 1 + \frac{20\lambda}{t} = 1 + \frac{20(5.0 \times 10^{-7})}{2.5 \times 10^{-5}} = 1.4$$

- 16 (a)** For constructive interference in case of thin film,

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda \quad (\text{where, } n = 1, 2, 3, \dots)$$

For minimum thickness t , $n = 1$ or $2\mu t = \lambda / 2$

The minimum thickness of a thin film,

$$t = \frac{\lambda}{4\mu} = \frac{500}{4 \times 1.2} = 104.17 \approx 104 \text{ nm}$$

• CHECK POINT 10.3

- 3 (d)** In Fresnel's class of diffraction, the source of light and the screen on which the diffraction pattern is observed are at finite distance from diffracting obstacle. In this no lenses are used and the incident wavefront is either spherical or cylindrical.

- 4 (b)** For first minima, $d \sin \theta = n\lambda = (1)\lambda$

$$\Rightarrow d = \frac{\lambda}{\sin \theta} = \frac{6.5 \times 10^{-7}}{\sin 30^\circ} = 13 \times 10^{-4} \text{ mm}$$

- 5 (b)** Angular fringe width, $\theta = \frac{\lambda}{d} = \frac{6328 \times 10^{-10}}{2 \times 10^{-4}} \times \frac{180}{\pi} = 0.18^\circ$

- 6 (d)** In single slit, for n th secondary maxima path difference

$$a \sin \theta = (2n + 1) \frac{\lambda}{2}$$

For first secondary maximum, $n = 1$

$$\therefore a \sin \theta = \frac{3\lambda}{2}$$

- 7 (a)** For n th dark fringe, in single slit

$$a \sin \theta = n\lambda$$

For second dark fringe, $a \sin \theta = 2\lambda$

$$\text{So, } 24 \times 10^{-5} \times 10^{-2} \times \sin 30^\circ = 2\lambda$$

$$\therefore \lambda = 6 \times 10^{-7} \text{ m} = 6000 \text{ \AA}$$

• CHECK POINT 10.4

- 1 (d)** Ultrasonic waves cannot be polarised because these are longitudinal waves.

- 2 (a)** Only transverse waves can be polarised.

- 3 (a)** When the plane polarised light passes through an optically active compound, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle.

- 4 (b)** $I = I_0 \cos^2 \theta = I_0 \cos^2 45^\circ = \frac{I_0}{2}$

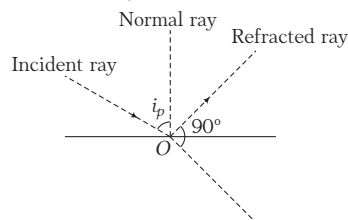
- 5 (c)** From Brewster's law, $\mu = \tan i_p \Rightarrow \frac{c}{v} = \tan 60^\circ = \sqrt{3}$
 $\Rightarrow v = \frac{c}{\sqrt{3}} = \frac{3 \times 10^8}{\sqrt{3}} = \sqrt{3} \times 10^8 \text{ ms}^{-1}$

- 6 (d)** By using $\mu = \tan \theta_p \Rightarrow \mu = \tan 60^\circ = \sqrt{3}$

$$\text{Also, } C = \sin^{-1} \left(\frac{1}{\mu} \right) \Rightarrow C = \sin^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

- 7 (d)** The amplitude will be $A \cos 60^\circ = A/2$.

- 8 (d)** Required angle $= 2i_p + 90^\circ = 2 \times 57.5^\circ + 90^\circ = 205^\circ$



- 9 (a)** According to Brewster's law, when a beam of ordinary light (*i.e.* unpolarised) is reflected from a transparent medium (like glass), the reflected light is completely plane polarised at certain angle of incidence called the angle of polarisation.

- 10 (b)** From Brewster's law, $\mu = \tan i_p = \tan 54.74^\circ = \sqrt{2}$

$$\Rightarrow \sqrt{2} = \frac{\sin 45^\circ}{\sin r} \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30^\circ$$

(A) Taking it together

- 1 (c)** For the interference pattern to be formed on the screen, the sources should be coherent and emits lights of same frequency and wavelength.

In a Young's double slit experiment, when one of the holes is covered by a red filter and another by a blue filter. In this case, due to filtration only red and blue lights are present. In YDSE, monochromatic light is used for the formation of fringes on the screen. Hence, in this case there shall be no interference fringes.

2 (c) We have, $\beta = \frac{\lambda D}{d}$

$\therefore$ The arrangement is placed in a liquid of refractive index n .

$\therefore \lambda' = \frac{\lambda}{n}$

Therefore, fringe width, $\beta' = \frac{\lambda D}{dn}$

or $\beta' = \frac{\beta}{n}$

3 (a) Given, width of the slit $= 10^4 \text{ \AA}$
 $= 10^4 \times 10^{-10} \text{ m}$
 $= 10^{-6} \text{ m} = 1 \text{ }\mu\text{m}$

Wavelength of (visible) sunlight varies from $4000 \text{ \AA}$ to $7000 \text{ \AA}$.

As the width of slit is comparable to that of wavelength, hence diffraction occurs with maxima at centre. So, at the centre all colours appear, i.e. mixing of colours form white colour at the centre.

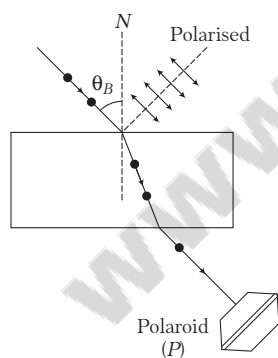
4 (c) θ (in radian) $= \frac{\lambda}{d}$

$\therefore d = \frac{\lambda}{\theta} = \frac{6 \times 10^{-7}}{\pi/180^\circ} = 3.4 \times 10^{-5} \text{ m}$

5 (c) Consider the diagram in which the light beam incident from air to the glass slab at Brewster's angle (i_p). The incident ray is unpolarised and is represented by dot ($\cdot$).

The reflected light is plane polarised represented by arrows.

As the emergent ray is unpolarised, hence intensity is minimum but cannot be zero when passes through polaroid.



6 (b) When phase difference is 0, intensity is maximum, i.e. $I_{\max}$.

When $\phi = \frac{\pi}{2}$ or $\frac{\phi}{2} = \frac{\pi}{4}$, the intensity will be

$$I = I_{\max} \cos^2 \frac{\phi}{2} = \frac{I_{\max}}{2}$$

$\therefore$ The desired ratio is 2 : 1.

7 (c) For constructive interference in case of soap film,

$$2\mu t = \left(n - \frac{1}{2}\right)\lambda$$

For minimum thickness t , $n = 1$

$$\Rightarrow 2\mu t = \frac{\lambda}{2}$$

$$\therefore t = \frac{\lambda}{4\mu} = \frac{750}{4 \times 1.33} = 141 \text{ nm}$$

8 (c) Fringe width, $\beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \frac{D}{d}$

$$\frac{\beta_2}{\beta_1} = \frac{D_2}{D_1} \times \frac{d_1}{d_2} = \frac{\frac{1}{2}D_1}{D_1} \times \frac{d_1}{D_1} = \frac{1}{4} \Rightarrow \beta_2 = \frac{\beta_1}{4}$$

9 (b) $\Delta y = \frac{5\lambda D}{d} - \frac{5\lambda D}{2d} = \frac{5\lambda D}{2d}$
 $= \frac{5 \times 6.5 \times 10^{-7} \times 1}{2 \times 10^{-3}} \text{ m} = 1.63 \text{ mm}$

10. (b) $n_1\lambda_1 = n_2\lambda_2 \Rightarrow n_2 = \frac{n_1\lambda_1}{\lambda_2} = 12 \times \frac{600}{400} = 18$

11. (c) In single narrow slit, for first minima,

$$d \sin \theta = \lambda$$

$$\Rightarrow \sin \theta = \frac{\lambda}{d}$$

$$\therefore \theta = \sin^{-1} \left(\frac{5000 \times 10^{-10}}{0.001 \times 10^{-3}} \right) = 30^\circ$$

12 (a) We have, $y_1 = \frac{10\lambda_1 D}{d}$, $y_2 = \frac{5\lambda_2 D}{d}$

$$\therefore \frac{y_1}{y_2} = \frac{2\lambda_1}{\lambda_2}$$

13 (c) Intensity, $I = 4I_0 \cos^2 \frac{\phi}{2}$

$$\therefore I = I_0 \Rightarrow \cos^2 \frac{\phi}{2} = \frac{1}{4}$$

$$\therefore \phi = \frac{2\pi}{3}$$

14 (a) We have, $\frac{2(6)-1}{2}\beta = 10\beta'$

$$\therefore \frac{\beta}{\beta'} = \frac{10}{5.5} = 1.8 = \mu$$

15 (b) Let n th red fringe coincides with m th blue fringe, then

$$\frac{n\lambda_R D}{d} = \frac{m\lambda_B D}{d}$$

$$\therefore \frac{n}{m} = \frac{\lambda_B}{\lambda_R} = \frac{480}{600} = \frac{4}{5}$$

i.e. 4th red fringe coincides with 5th blue fringe. Next 8th red fringe will coincide with 10th blue fringe and so on. Thus, $n = 4$

and $m = n + 1 = 5$

16 (b) For maximum (brightness) intensity on the screen, path difference should be integer multiple of wavelength, i.e.

$$d \sin \theta = n\lambda$$

$$\sin \theta = \frac{n\lambda}{d} = \frac{n(2000)}{7000} = \frac{n}{3.5}$$

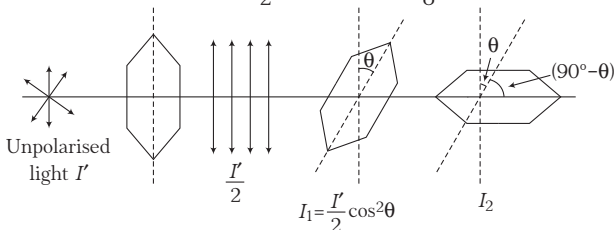
As, $(\sin \theta)_{\max} = 1 \Rightarrow n = 3.5$

So, number of maxims = $0, \pm 1, \pm 2, \pm 3$

Thus, only seven maxims can be obtained on both sides of the screen.

17 (a) Fringe size will not change, i.e. $n_1 = n_2$.

19 (a) $I_2 = I_1 \cos^2(90^\circ - \theta) = \frac{I'}{2} \cos^2 \theta \sin^2 \theta = \frac{I'}{8} (\sin 2\theta)^2$



$$3 = \frac{32}{8} (\sin 2\theta)^2$$

$$\sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ \text{ or } \sin 120^\circ$$

$$\theta = 30^\circ \text{ or } 60^\circ$$

20 (a) Number of fringes shifted, $N = \frac{(\mu - 1)t}{\lambda}$

$\therefore$ Refractive index of the sheet,

$$\mu = \frac{N\lambda}{t} + 1 = \frac{30 \times 6000 \times 10^{-10}}{3.6 \times 10^{-5}} + 1 = 1.5$$

21 (c) Fringe width, $\beta = \frac{D\lambda}{d}$

$$\text{i.e. } \beta \propto \lambda$$

So, wavelength λ and hence fringe width β decreases 1.5 or $\frac{3}{2}$

times when immersed in liquid. The distance between central maximum and 10th maxima is 3 cm in vacuum. When immersed in liquid it will reduce to 2 cm. Position of central maximum will not change while 10th maxima will be obtained at $y = 4$ cm.

22 (b) Position of third maxima,

$$x_3 = \frac{3\lambda D}{d} = \frac{3(5 \times 10^{-5})(200)}{0.02} = 1.5 \text{ cm}$$

23 (d) $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{32}$

$$\therefore \frac{I_1}{I_2} = \frac{8}{9}$$

$$\text{or } \sqrt{\frac{I_1}{I_2}} = \frac{2\sqrt{2}}{3}$$

$$\therefore \frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1} \right)^2 = \left(\frac{2\sqrt{2} + 3}{2\sqrt{2} - 3} \right)^2$$

24 (a) According to the question,

$$y = \frac{\omega}{4} = \frac{\lambda D}{4d} \Rightarrow \Delta x = \frac{yd}{D} = \frac{\lambda}{4}$$

$$\text{Phase difference, } \phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{\pi}{2}$$

$$\therefore \frac{\phi}{2} = \frac{\pi}{4}$$

$$\text{Now, intensity, } I_2 = I_1 \cos^2 \frac{\phi}{2}$$

$$\text{or } \frac{I_1}{I_2} = \frac{1}{\cos^2 \phi/2} = 2$$

25 (d) According to question, there is a hole at point P_2 . From Huygens' principle, wave will propagate from the sources S_1 and S_2 . Each point on the screen will act as secondary sources of wavelets.

Now, there is a hole at point P_2 (minima). The hole will act as a source of fresh light for the slits S_3 and S_4 .

Therefore, there will be a regular two slit pattern on the second screen.

26 (c) Intensity in the first case, $I_1 = (\sqrt{I_0} + \sqrt{I_0})^2 = 4I_0$

where, I_0 = intensity due to each source.

Intensity in second case, $I_2 = I_0 + I_0 = 2I_0$

$$\therefore \frac{I_1}{I_2} = \frac{4I_0}{2I_0} = \frac{2}{1}$$

27 (a) Shift = $\frac{(\mu - 1)tD}{d} = \omega = \frac{\lambda D}{d}$

$$\therefore t = 2\lambda$$

(as $\mu = 1.5$)

28 (a) Path difference, $\Delta x = \frac{yd}{D} = \left(\frac{1.25\lambda D}{d} \right) \left(\frac{d}{D} \right)$

$$= 1.25\lambda = \frac{5}{4}\lambda$$

$$\text{Phase difference, } \phi = \frac{2\pi}{\lambda} \cdot \Delta x$$

$$= \frac{2\pi}{\lambda} \times \frac{5}{4}\lambda = \frac{5}{2}\pi$$

$$\therefore \frac{\phi}{2} = \frac{5}{4}\pi = 225^\circ$$

$$\therefore \text{Intensity, } I = I_{\max} \cos^2 \frac{\phi}{2} = \frac{1}{2} I_{\max}$$

29 (c) Intensity, $I = 4I_0 \cos^2 \frac{\phi}{2}$

$$\therefore I = I_0$$

$$\therefore \phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{\lambda} \left(\frac{yd}{D} \right)$$

$$\therefore y = \frac{\lambda D}{3d}$$

30 (c) $2I_0 = 4I_0 \cos^2 \frac{\phi}{2}$

$$\phi = \frac{\pi}{2} = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{\lambda} \left(\frac{yd}{D} \right)$$

$$\therefore y_1 = \frac{\lambda D}{4d}$$

Further, $I_0 = 4I_0 \cos^2 \frac{\phi}{2}$

$$\Rightarrow \phi = \frac{2\pi}{3} = \frac{2\pi}{\lambda} \Delta x = \frac{2\pi}{\lambda} \left(\frac{y_2 d}{D} \right)$$

$$\therefore y_2 = \frac{\lambda D}{3d}$$

$$\text{Minimum separation, } \Delta y = y_2 - y_1 = \frac{1}{12} \left(\frac{\lambda D}{d} \right) = \frac{\alpha}{12}$$

31 (c) Five fringes have to be shifted.

$$\therefore 5 = \frac{(\mu_1 - 1)t - (\mu_2 - 1)t}{\lambda} = \frac{(\mu_1 - \mu_2)t}{\lambda}$$

$$\therefore \text{Thickness, } t = \frac{5\lambda}{(\mu_1 - \mu_2)} = \frac{5 \times 480 \times 10^{-9}}{1.7 - 1.4} = 8 \times 10^{-6} \text{ m or } 8 \mu\text{m}$$

32 (a) At $x = 0$, $\Delta x = 5\lambda$

which corresponds to a maximum.

At $x = \infty$, $\Delta x = 0$, this also corresponds to a maximum.

$\therefore$ Other possible maxima between $x = 0$ and $x = \infty$ are $\Delta x = \lambda, 2\lambda, 3\lambda, 4\lambda$.

33 (a) Location of the point, $y = \frac{\omega}{4} = \frac{\lambda D}{4d}$

$$\Rightarrow \Delta x = \frac{y d}{D} = \frac{\lambda}{4}$$

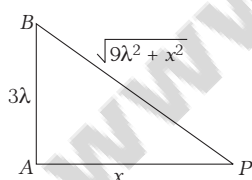
$$\Rightarrow \phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{\pi}{2}$$

$$\therefore I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

$$\therefore 4I = 2I + I_2 + 0$$

$$\Rightarrow I_2 = 2I$$

34 (c) Say P be the point on X -axis, where constructive interference is obtained.



$$BP - AP = 2\lambda \text{ or } \lambda$$

$$\therefore \sqrt{9\lambda^2 + x^2} - x = 2\lambda$$

$$\therefore 9\lambda^2 + x^2 = x^2 + 4\lambda^2 + 4x\lambda$$

$$\Rightarrow x = \frac{5\lambda}{4}$$

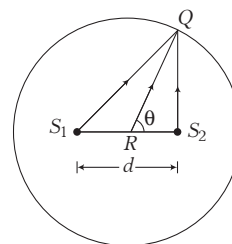
$$\text{Further, } \sqrt{9\lambda^2 + x^2} - x = \lambda$$

$$\therefore 9\lambda^2 + x^2 = x^2 + \lambda^2 + 2x\lambda$$

$$\therefore x = 4\lambda$$

Hence, the distance from A on the X -axis at which the interference is constructive is $5\lambda/4$.

35 (b) Path difference at a point Q on the circle is



$$\Delta y = d \cos \theta \quad \dots(i)$$

For maxima at Q , path difference should be integer multiple of wavelength

$$\Delta y = n\lambda \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$n\lambda = d \cos \theta$$

$$\therefore \theta = \cos^{-1} \left(\frac{n\lambda}{d} \right)$$

For $n = 4$

$$\text{Angular position, } \theta = \cos^{-1} \left(\frac{4\lambda}{d} \right)$$

36 (a) Consider the diagram, the ray P is incident at an angle θ and gets reflected in the direction P' and refracted in the direction P'' . Due to reflection from the glass medium, there is a phase change of π .

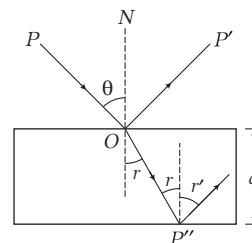
Time taken to travel along OP''

$$\Delta t = \frac{OP''}{v} = \frac{d / \cos r}{c / n} = \frac{nd}{c \cos r}$$

$$\text{From Snell's law, } n = \frac{\sin \theta}{\sin r} \Rightarrow \sin r = \frac{\sin \theta}{n}$$

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{\sin^2 \theta}{n^2}}$$

$$\therefore \Delta t = \frac{nd}{c \left(1 - \frac{\sin^2 \theta}{n^2} \right)^{1/2}} = \frac{nd}{c} \left(1 - \frac{\sin^2 \theta}{n^2} \right)^{-1/2}$$



$$\text{Phase difference} = \Delta \phi = \frac{2\pi}{T} \times \Delta t = \frac{2\pi nd}{\lambda} \left(1 - \frac{\sin^2 \theta}{n^2} \right)^{-1/2}$$

$$\text{So, net phase difference} = \Delta \phi + \pi = \frac{2\pi nd}{\lambda} \left(1 - \frac{1}{n^2} \sin^2 \theta \right)^{-1/2} + \pi$$

which is close to option (a).

- 37 (c) Path difference between the waves reaching at O ,

$$\Delta = \Delta_1 + \Delta_2$$

where, Δ_1 = initial path difference and Δ_2 = path difference between the waves after emerging from slits.

$$\text{Now, } \Delta_1 = SS_1 - SS_2 = \sqrt{(D^2 + d^2)} - D$$

$$\text{and } \Delta_2 = S_1O - S_2O = \sqrt{(D^2 + d^2)} - D$$

$$\begin{aligned} \therefore \Delta &= 2 \{ \sqrt{(D^2 + d^2)} - D \} = 2 \{ (D^2 + d^2)^{1/2} - D \} \\ &= 2 \left\{ \left(D + \frac{d^2}{2D} \right) - D \right\} \text{ (from binomial expansion)} \\ &= \frac{d^2}{D} \end{aligned}$$

For obtaining dark at O , Δ must be equals to $(2n-1) \frac{\lambda}{2}$.

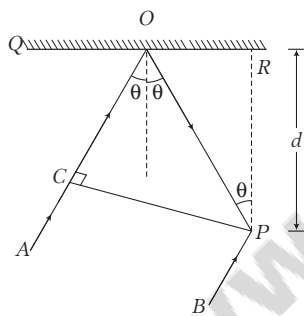
$$\text{i.e. } \frac{d^2}{D} = (2n-1) \frac{\lambda}{2}$$

$$\therefore d^2 = \frac{(2n-1) \lambda D}{2} \quad \text{or} \quad d = \sqrt{\frac{(2n-1) \lambda D}{2}}$$

For minimum distance ($n=1$),

$$\text{So, } d = \sqrt{\frac{\lambda D}{2}}$$

- 38 (b) Path difference between the two rays is given by



$$\Delta = CO + PO$$

$$\therefore PR = d$$

$$\text{So, } PO = d \sec \theta$$

$$\text{and } CO = PO \cos 2\theta = d \sec \theta \cos 2\theta$$

$$\text{So, } \Delta = (d \sec \theta + d \sec \theta \cos 2\theta)$$

Phase difference between two rays is $\phi = \pi$ (as one ray is reflected one and another is direct).

Now, for constructive interference, path difference should be even multiple of half-wavelength.

$$\text{i.e. } \Delta = \lambda / 2, 3\lambda / 2, \dots$$

$$\text{So, } d \sec \theta + d \sec \theta \cos 2\theta = \frac{\lambda}{2}$$

$$\text{or } d \sec \theta (1 + \cos 2\theta) = \frac{\lambda}{2}$$

$$\text{or } d \sec \theta (2 \cos^2 \theta) = \frac{\lambda}{2}$$

$$\therefore \cos \theta = \lambda / 4d$$

(B) Medical entrance special format questions

• Assertion and reason

$$1 \text{ (c) } \phi = \frac{2\pi}{\lambda} \cdot \Delta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

$$\text{and } I = 4I_0 \cos^2 \left(\frac{\phi}{2} \right) = 4I_0 \times \frac{1}{4} = I_0$$

$$2 \text{ (b) } I_{\max} > 4I_0 \text{ and } I_{\min} = 0$$

$$\therefore \frac{I_{\max}}{I_{\min}} = \text{infinite}$$

If width of one slit is slightly increased, $I_{\min} > 0$. Therefore, this ratio will be less than infinite.

- 3 (d) Fringe width does not change by the introduction of glass slab.

- 4 (a) Whole fringe pattern shifts towards that side, where slab is inserted.

- 5 (a) When a polaroid is rotated in the path of unpolarised light, the intensity of light transmitted from polaroid remain undiminished (because unpolarised light contains waves vibrating in all possible planes with equal probability).

However, when the polaroid is rotated in path of plane polarised light, its intensity will vary from maximum (when the vibrations of the plane polarised light are parallel to the axis of the polaroid) to minimum (when the direction of the vibrations becomes perpendicular to the axis of the crystal). Thus, using polaroid we can easily identify that whether the light is polarised or not.

• Statement based questions

- 1 (d) Polarisation help us to distinguish between sound wave and light wave.

- 2 (c) Let n th bright fringe of λ_1 coincide with m th bright fringe of λ_2 .

$$\begin{aligned} \therefore n\lambda_1 &= m\lambda_2 \\ \Rightarrow \frac{n}{m} &= \frac{\lambda_2}{\lambda_1} = \frac{6000}{4000} = \frac{4}{3} \end{aligned}$$

Hence, 3rd order bright fringe of λ_2 will coincide with 4th order bright fringe of λ_1 .

- 3 (d) In case of Young's double slit experiment, a slit S is necessary, if we use an ordinary extended source of light while it is not needed, if we use a spatially coherent source of light.

• Match the columns

- 1 (b)

$$\text{A. } I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 \Rightarrow I_1 = I_0 \text{ and } I_2 > I_0$$

$$\therefore I_{\max} > 4I_0$$

- B. When one slit is closed, fringe pattern will disappear.

$$I_{\max} = I_{\min} = I_0$$

- C. When both slits are made incoherent, again fringe pattern will disappear.

$$I_{\max} = I_{\min} = I_1 + I_2 = 2I_0$$

- D. Glass slab will only shift the fringe pattern.

- 2 (a) Fringe width, $\beta = \frac{\lambda D}{d}$

In a liquid, λ decreases μ times, therefore β also decrease μ times.

When wavelength λ and distance between slits and screen D increases, β increases.

When distance between two slits d increases, β decreases.

(C) Medical entrances' gallery

- 1 (b) Fringe width, $\beta = \frac{\lambda D}{d}$

$$\therefore \frac{\beta_2}{\beta_1} = \frac{\frac{\lambda_2 D_2}{d_2}}{\frac{\lambda_1 D_1}{d_1}} \quad [\text{As, } \lambda_1 = \lambda_2, D_2 = 2D_1 \text{ and } d_2 = \frac{d_1}{2}]$$

$$\therefore \frac{\beta_2}{\beta_1} = \frac{2D_1}{\left(\frac{d_1}{2}\right)} \times \frac{d_1}{D_1} = 4$$

$$\Rightarrow \beta_2 = 4\beta_1$$

i.e. Fringe width becomes four times.

- 2 (b) The Brewster's angle i_b for an interface should lie between 45° to 90° , i.e. $45^\circ < i_b < 90^\circ$.

- 3 (a) For central maximum, path difference is zero for both coherent sources in interference.

$$\begin{aligned} \text{Hence, phase difference} &= \frac{2\pi}{\lambda} \times \text{path difference} \\ &= \frac{2\pi}{\lambda} \times 0 = 0 \end{aligned}$$

- 4 (a) The angular width in YDSE is given by

$$\theta = \frac{\beta}{D}$$

where, β is the separation between two fringes and D is the distance between the slits and screen. If YDSE apparatus is immersed in a liquid of refractive index μ , then the wavelength of light and hence the angular width decreases μ times.

$$\text{i.e. } \theta' = \frac{\beta}{\mu D} = \frac{\theta}{\mu}$$

Here, μ (for water) = $4/3$ and $\theta = 0.2^\circ$

$$\Rightarrow \theta' = \frac{0.2^\circ}{4/3} = 0.15^\circ$$

- 5 (c) In a Young's double slit experiment, the path difference for n th minima is given by, $\Delta y = (2n - 1) \frac{\lambda}{2}$

For 5th minima, $n = 5$

$$\therefore \Delta y = [2(5) - 1] \frac{\lambda}{2} = \frac{9\lambda}{2}$$

- 6 (b) The angular width of central maximum is given by

$$2\theta = \frac{2\lambda}{a} \quad \dots(i)$$

where, λ = wavelength of light used

and a = width of the slit.

Angular width, $2\theta = \theta_0$ (given)

$$\text{So, } \theta_0 = \frac{2\lambda_1}{a} \quad \dots(ii) \text{ [from Eq. (i)]}$$

For another light of wavelength λ_2 (says), the angular width decreases by 30%, so

$$2\theta = \left(\frac{100 - 30}{100} \right) \theta_0$$

$$2\theta = \frac{70}{100} \theta_0 = 0.7\theta_0$$

$$\therefore \text{Angular width, } 0.7\theta_0 = \frac{2\lambda_2}{a} \quad \dots(iii)$$

As, slit width is constant, so on dividing Eq. (ii) by Eq. (iii), we get

$$\begin{aligned} \Rightarrow \frac{\theta_0}{0.7\theta_0} &= \frac{\lambda_1}{\lambda_2} \\ \lambda_2 &= \lambda_1 \times 0.7 \\ &= 6000 \times 0.7 \\ &= 4200 \text{ \AA} \end{aligned}$$

- 7 (a) Given, distance between source and observer,

$$D = 2 \text{ m}$$

$$\begin{aligned} \text{Distance of 5th dark fringe from centre} \\ &= 4 \times 10^{-3} \text{ m} \end{aligned}$$

$$\text{Wavelength, } \lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$$

$$\begin{aligned} \text{Distance of 5th dark fringe from centre} \\ &= \beta + \beta + \beta + \beta + \frac{\beta}{2} = \frac{9\beta}{2} \end{aligned}$$

where, β = fringe width.

$$\text{According to question, } \frac{9\beta}{2} = 4 \times 10^{-3}$$

$$\frac{9}{2} \times \frac{D\lambda}{d} = 4 \times 10^{-3}$$

$$\frac{9}{2} \times \frac{2 \times 6 \times 10^{-7}}{d} = 4 \times 10^{-3}$$

$$\begin{aligned} \Rightarrow d &= 1.35 \times 10^{-3} \text{ m} \\ &= 1.35 \text{ mm} \end{aligned}$$

- 8 (a) Given, wavelength of light,
 $\lambda = 500 \text{ nm} = 5 \times 10^{-7} \text{ m}$

$$\begin{aligned} \text{Distance between slit and screen,} \\ D &= 1.8 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Distance between two slits,} \\ d &= 0.4 \text{ mm} = 4 \times 10^{-4} \text{ m} \end{aligned}$$

$$\text{Velocity of screen} = \frac{dD}{dt} = 4 \text{ ms}^{-1}$$

$$\text{Speed of first maxima} = \frac{d\beta}{dt} = ?$$

In Young's double slit experiment, fringe width,

$$\beta = \frac{D\lambda}{d}$$

Differentiating with respect to t , we get

$$\begin{aligned}\frac{d\beta}{dt} &= \frac{\lambda}{d} \cdot \frac{dD}{dt} = \frac{5 \times 10^{-7}}{4 \times 10^{-4}} \times 4 \\ &= 5 \times 10^{-3} \text{ ms}^{-1} \\ &= 5 \text{ mm s}^{-1}\end{aligned}$$

- 9 (a) Distance between position of bright and dark fringes is called fringe width in Young's double slit experiment, which is given by $\beta = \frac{D\lambda}{d}$.

where, β = fringe width,

D = distance between source and screen,

d = distance between two slits

and λ = wavelength of monochromatic light wave.

Therefore, in Young's double slit experiment, fringes are of equal width, i.e. width of bright and dark fringes are same.

Hence, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

- 10 (b) When unpolarised light coming from sun, enters the atmosphere of earth, then it scatters due to presence of atmospheric particles. The scattered light seen in a direction perpendicular to the direction of incidence is found to be plane polarised.

When the light from the sun incident on reflecting surface of earth at an angle of polarisation, then reflecting light is polarised. Little amount of light is incident on polarising direction, hence reflected light is partially polarised.

Hence, Assertion and Reason both are correct but Reason is not the correct explanation of Assertion.

- 11 (d) Huygens' principle use the phenomenon of light as reflection, refraction and diffraction but does not use point of spectra origin.

- 12 (b) In a Young's double slit experiment, angular width of a fringe is given as

$$\theta = \frac{\lambda}{d}$$

where, λ is the wavelength of the light source and d is the distance between the two slits.

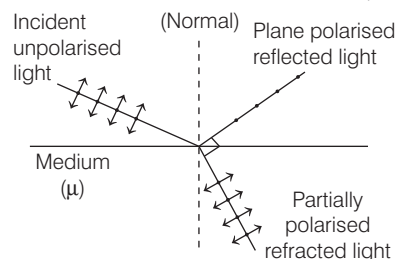
$$\Rightarrow \theta \propto \frac{1}{d} \text{ or } \frac{\theta_1}{\theta_2} = \frac{d_2}{d_1} \quad \dots(i)$$

Given, $\theta_1 = 0.20^\circ$, $\theta_2 = 0.21^\circ$,
 $d_1 = 2 \text{ mm}$

Substituting the given values in Eq. (i), we get

$$\begin{aligned}\Rightarrow \frac{0.20^\circ}{0.21^\circ} &= \frac{d_2}{2} \\ \Rightarrow d_2 &= 2 \times \frac{0.20^\circ}{0.21^\circ} \\ &= \frac{0.40}{0.21} = 1.9 \text{ mm}\end{aligned}$$

- 13 (b) The figure shown below represents the course of path an unpolarised light follows, when it is incident from air on plane surface of material of refractive index μ .



If reflected and refracted rays are perpendicular to each other, then the reflected light is completely plane polarised at a certain angle of incidence. This means, the reflected light has electric vector perpendicular to the plane of incidence.

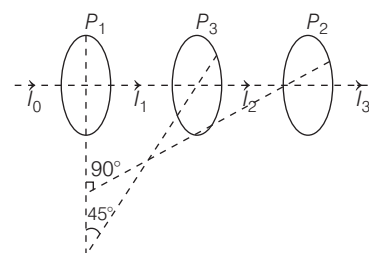
- 14 (d) According to question,
5th dark fringe in air = 8th bright fringe in the medium

$$\begin{aligned}(2n-1) \frac{D\lambda}{2d} &= \frac{nD\lambda}{\mu d} \\ (2 \times 5 - 1) \frac{\lambda D}{2d} &= 8 \frac{\lambda D}{\mu d} \\ 9 \frac{\lambda D}{2d} &= 8 \frac{\lambda D}{\mu d} \\ \Rightarrow \frac{9}{2} &= \frac{8}{\mu} \Rightarrow \mu = \frac{8 \times 2}{9}\end{aligned}$$

$\therefore$ Refractive index of the medium,

$$\mu = \frac{16}{9} = 1.7777 \approx 1.78$$

- 15 (c) According to the question,



Given, $\theta = 45^\circ$

From the above diagram, intensity transmitted through P_3 ,

$$I_2 = \frac{I_0}{2} \cos^2 45^\circ$$

$$\Rightarrow I_2 = \frac{I_0}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow I_2 = \frac{I_0}{4}$$

Similarly, intensity transmitted through P_2 ,

$$I_3 = \frac{I_0}{4} \cos^2 45^\circ \Rightarrow I_3 = \frac{I_0}{4} \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow I_3 = \frac{I_0}{4} \times \frac{1}{2} \Rightarrow I_3 = \frac{I_0}{8}$$

- 16 (c) In Young's double slit experiment, position of bright fringes on the screen are given by

$$x = \frac{nD\lambda}{d}$$

Hence, distance of n th maxima will be $x = n\lambda \frac{D}{d} \propto \lambda$

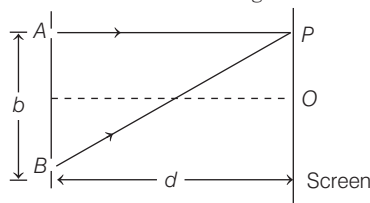
$$\lambda_{\text{blue}} = 4360 \text{ \AA}, \lambda_{\text{green}} = 5480 \text{ \AA}$$

where, x = distance of 4th maxima from the central one.

As, $\lambda_{\text{blue}} < \lambda_{\text{green}}$

$\therefore x_{\text{blue}} < x_{\text{green}}$

- 17 (c) The situation is shown in the figure



The path difference between the waves reaching the point P is

$$\begin{aligned} \Delta p &= BP - AP \\ &= (d^2 + b^2)^{1/2} - d \\ &= d \left(1 + \frac{b^2}{d^2} \right)^{1/2} - d \\ &\approx d \left[1 + \frac{1}{2} \cdot \frac{b^2}{d^2} \right] - d = \frac{b^2}{2d} \end{aligned}$$

For a dark band, $\Delta p = \frac{b^2}{2d} = (2n-1) \frac{\lambda}{2}$

where, $n = 1, 2, 3$ or $\lambda = \frac{1}{(2n-1)} \cdot \frac{b^2}{d}$

Hence, the missing wavelengths are $\frac{b^2}{d}, \frac{b^2}{3d}, \frac{b^2}{5d}, \dots$

- 18 (b) The condition of interference maxima is

$$d \sin \theta = n\lambda, \quad \sin \theta = \frac{n\lambda}{d}$$

$$\text{Given, } d = 2\lambda \text{ and } \sin \theta = \frac{n\lambda}{2\lambda} = \frac{n}{2}$$

The magnitude of $\sin \theta$ lies between 0 and 1.

When $n = 0$, $\sin \theta = 0 \Rightarrow \theta = 0^\circ$

When $n = 1$, $\sin \theta = 1/2 \Rightarrow \theta = 30^\circ$

When $n = 2$, $\sin \theta = 1 \Rightarrow \theta = 90^\circ$

Thus, there is central maximum at $\theta = 0^\circ$, on either side of it maxima lie at $\theta = 30^\circ$ and $\theta = 90^\circ$, so maximum number of possible interference maxima is five.

- 19 (b) It is given that, $\frac{I_2}{I_1} = n \Rightarrow I_2 = nI_1$

Maximum intensity of interference,

$$I_{\text{max}} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$\begin{aligned} \therefore I_{\text{max}} &= (\sqrt{I_1} + \sqrt{nI_1})^2 \\ &= (1 + \sqrt{n})^2 I_1 \end{aligned}$$

$$= (1 + n + 2\sqrt{n})I_1$$

Minimum intensity of interference,

$$I_{\text{min}} = (I_1 - I_2)^2$$

$$\therefore I_{\text{min}} = (\sqrt{I_1} - \sqrt{nI_1})^2$$

$$= (1 - \sqrt{n})^2 I_1$$

$$= (1 + n - 2\sqrt{n})I_1$$

$$\therefore \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{2\sqrt{n} - (-2\sqrt{n})}{2(1+n)} = \frac{2\sqrt{n}}{n+1}$$

- 20 (d) For the distance of the first dark band of the diffraction pattern from the centre of the screen is given by position of first minima.

$$i.e. \quad y = \frac{\lambda D}{a}$$

where, λ = wavelength of parallel beam,

D = focal length and a = width of linear aperture.

Here, $a = 0.02 \text{ cm} = 2 \times 10^{-4} \text{ m}$,

$$\lambda = 5 \times 10^{-5} \text{ cm} = 5 \times 10^{-7} \text{ m},$$

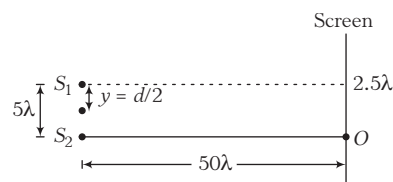
$$D = 60 \text{ cm} = 0.6 \text{ m}$$

$$\Rightarrow y = \frac{(5 \times 10^{-7})(0.6)}{2 \times 10^{-4}}$$

$$= 15 \times 10^{-4} \text{ m}$$

$$\Rightarrow y = 0.15 \text{ cm}$$

- 21 (c)



In the above figure, S_1 and S_2 are the two different slits.

Given, distance between slits S_1 and S_2 , $d = 5\lambda$

Distance between screen and slits, $D = 10d = 50\lambda$

According to the question, the intensity at maximum in Young's double slit experiment is I_0 .

$$\Rightarrow I_{\text{max}} = I_0$$

$$\therefore \text{Path difference} = \frac{dy}{D} = \frac{d \times (d/2)}{10d} = \frac{d}{20} = \frac{\lambda}{4} \quad [\because d = 5\lambda]$$

A path difference of λ corresponds to phase difference 2π .

So, for path difference $\lambda/4$, phase difference

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \pi/2 = 90^\circ$$

$$\text{As we know, } I = I_0 \cos^2 \frac{\phi}{2} \Rightarrow I = I_0 \cos^2 \frac{90^\circ}{2}$$

$$\Rightarrow I = I_0 \times \left(\frac{1}{\sqrt{2}} \right)^2 \Rightarrow I = \frac{I_0}{2}$$

- 22 (c) As the first minima is observed at an angle of 30° in a diffraction pattern due to a single slit of width a .

$$i.e. \quad n = 1, \theta = 30^\circ$$

$\therefore$ According to Bragg's law of diffraction,

$$\begin{aligned} a \sin \theta &= n\lambda \\ \Rightarrow a \sin 30^\circ &= (1) \lambda & (n=1) \\ \Rightarrow a &= 2\lambda & \dots(i) \left(\because \sin 30^\circ = \frac{1}{2} \right) \end{aligned}$$

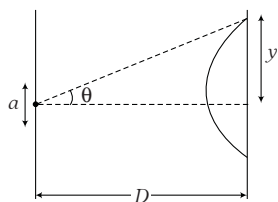
For first secondary maxima,

$$\Rightarrow a \sin \theta_1 = \frac{3\lambda}{2} \Rightarrow \sin \theta_1 = \frac{3\lambda}{2a} \quad \dots(ii)$$

Substitute value of a from Eq. (i) to Eq. (ii), we get

$$\sin \theta_1 = \frac{3\lambda}{4\lambda} \Rightarrow \sin \theta_1 = \frac{3}{4} \Rightarrow \theta_1 = \sin^{-1} \left(\frac{3}{4} \right)$$

- 23 (a)** For the condition of maxima, $\sin \theta = \lambda/a$



From the geometry,

$$\sin \theta \approx \theta = \frac{y}{D} \quad (\text{for small angle})$$

$$\text{So, } \frac{y}{D} = \frac{\lambda}{a} \Rightarrow y = \frac{\lambda D}{a}$$

$$\text{Hence, width of central maximum} = 2y = \frac{2\lambda D}{a}$$

- 24 (a)** Given, $d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$, $D = 1 \text{ m}$

$$\lambda = 500 \text{ nm} = 5 \times 10^{-7} \text{ m}$$

As, width of central maximum = width of 10th maxima

$$\therefore \frac{2D\lambda}{a} = 10 \left(\frac{\lambda D}{d} \right)$$

$$\Rightarrow a = \frac{d}{5} = \frac{10^{-3}}{5} = 0.2 \times 10^{-3} \text{ m}$$

$$a = 0.2 \text{ mm}$$

- 25 (b)** In diffraction, fringe width, $\beta = \frac{\lambda D}{d}$

$$\text{When } \frac{d}{2} = \beta, \text{ then } \frac{d}{2} = \frac{\lambda D}{d} \Rightarrow D = \frac{d^2}{2\lambda}$$

- 26 (a)** To find the point of coincidence of bright fringes, we can equate the distance of bright fringes from the central maximum, made by both the wavelengths of light.

Given, distance between the screen and slit,

$$D = 120 \text{ cm} = 120 \times 10^{-2} \text{ m}$$

Slit width, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$$\lambda_1 = 6500 \text{ \AA} \text{ and } \lambda_2 = 5200 \text{ \AA}$$

Let n th fringe due to $\lambda_2 = 5200 \text{ \AA}$ coincide with $(n-1)$ th bright fringe due to $\lambda_1 = 6500 \text{ \AA}$.

$$\therefore \frac{Dn\lambda_2}{d} = \frac{D(n-1)\lambda_1}{d}$$

$$\text{or } n \times 5200 = (n-1) 6500$$

$$\text{or } 4n = 5n - 5$$

$$\Rightarrow n = 5$$

$$\begin{aligned} \text{So, the least distance required, } y_n &= \frac{n\lambda_2 D}{d} \\ &= \frac{5 \times 5200 \times 10^{-10} \times 120 \times 10^{-2}}{2 \times 10^{-3}} \\ &= 0.16 \times 10^{-2} \text{ m} = 0.16 \text{ cm} \end{aligned}$$

- 27 (c)** The source is at infinity and incident beam is parallel.

- 28 (a)** Most of the light is diffracted between the two first order minima. These minima occur at angles given by $b \sin \theta = \pm \lambda$.

$$\Rightarrow \sin \theta = \pm \frac{\lambda}{b} = \pm \frac{4.5 \times 10^{-7}}{0.2 \times 10^{-3}} = \pm 2.25 \times 10^{-3}$$

$$\Rightarrow \theta \approx 2.25 \times 10^{-3} \text{ rad} \quad (\text{angle is very small})$$

The angular divergence, $2\theta = 4.5 \times 10^{-3} \text{ rad}$

- 29 (b)** $a \sin \theta = (2n-1)\lambda/2$ is the condition for obtaining secondary maxima in the diffraction pattern due to single slit.

- 30 (b)** Fringe width, $\beta = \frac{D\lambda}{d}$

$$\Rightarrow d = \frac{D\lambda}{\beta} = \frac{6000 \times 10^{-10} \times 40 \times 10^{-2}}{0.012 \times 10^{-2}} = 0.2 \text{ cm}$$

- 31 (d)** Given, $\theta_p = 57^\circ$

According to Brewster's law, we have

$$\theta_p + r = 90^\circ \Rightarrow r = 90^\circ - \theta_p = 90^\circ - 57^\circ$$

Angle of refraction, $r = 33^\circ$

- 32 (c)** To observe diffraction, the size of the obstacle should be of the order of wavelength, i.e. $\lambda \approx d$.

- 33 (a)** According to the question,

$$\lambda = 6250 \text{ \AA} = 6250 \times 10^{-10} \times 10^2 \text{ cm} = 6250 \times 10^{-8} \text{ cm}$$

Width of the slit, $a = 2 \times 10^{-2} \text{ cm}$

The angular width of central maximum, $\beta = 2\theta = 2 \frac{\lambda}{a}$

Thus, linear width is

$$\begin{aligned} L &= \beta \times D = \frac{2\lambda}{a} \times D = \frac{2 \times 6250 \times 10^{-8}}{2 \times 10^{-2}} \times 50 \\ &= 6250 \times 10^{-6} \times 50 = 3125 \times 10^{-4} \\ &= 312.5 \times 10^{-3} \text{ cm} \end{aligned}$$

- 34 (a)** Given, wavelength, $\lambda = 590 \text{ nm} = 590 \times 10^{-9} \text{ m}$

$$\text{Diameter of lens} = 10 \times 10^{-2} \text{ m}$$

$$\therefore \text{Radius of lens} = 5 \times 10^{-2} \text{ m}$$

$$\text{Focal length of lens} = 20 \times 10^{-2} \text{ m}$$

Now, the diameter of image disc

$$\begin{aligned} D &= 2R = 2 \left(1.22 \frac{\lambda D}{b} \right) = \left(\frac{1.22 \times 590 \times 10^{-9} \times 20 \times 10^{-2}}{5 \times 10^{-2}} \right) \times 2 \\ &\approx 5.76 \times 10^{-4} \text{ cm} \end{aligned}$$

- 35 (c)** According to law of Malus, when a beam of completely plane polarised light is incident on an analyser, the resultant intensity of light I transmitted from an analyser varies directly as the square of cosine angle between plane of analyser and polariser.

Let initial intensity is I and final intensity is I' , then

$$I' = I \cos^2 60^\circ = I \left(\frac{1}{2} \right)^2 = \frac{I}{4}$$

Now, % difference between initial and final intensities,

$$\begin{aligned} &= \frac{I - (I/4)}{I} \times 100 = \frac{3I}{4} \times \frac{1}{I} \times 100 \\ &= \frac{3}{4} \times 100 = 75\% \end{aligned}$$

- 36 (a)** In Young's double slit experiment, the locus of the point P lying in a plane with a constant path difference between the two interfering waves is a hyperbola.

- 37 (a)** On the wavefront, all the points are in the same phase.

- 38 (c)** For net intensity, $I' = 4I_0 \cos^2 \frac{\phi}{2}$

Case I $K = 4I_0 \cos^2 \left(\frac{2\pi}{2} \right) \quad \left(\because \phi = \frac{2\pi}{\lambda} \times \lambda \right)$

$$K = 4I_0 \cos^2 (\pi) \Rightarrow K = 4I_0 \quad \dots(i)$$

Case II $K' = 4I_0 \cos^2 \left(\frac{\pi/2}{2} \right) \quad \left(\because \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} \right)$

$$K' = 4I_0 \cos^2 (\pi/4) \Rightarrow K' = 2I_0 \quad \dots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$K' = K/2$$

- 39 (d)** We know that,

The maximum intensity, $I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2 \quad \dots(i)$

The minimum intensity, $I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2 \quad \dots(ii)$

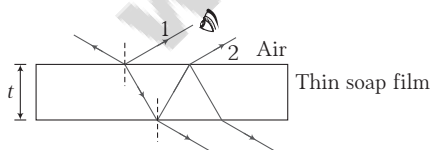
So, the ratio of the maximum and minimum intensities is

$$\frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \frac{(\sqrt{4I} + \sqrt{I})^2}{(\sqrt{4I} - \sqrt{I})^2}$$

$$\Rightarrow \frac{I_{\max}}{I_{\min}} = \left(\frac{3\sqrt{I}}{\sqrt{I}} \right)^2 = \frac{9}{1} = 9:1$$

Hence, the maximum and minimum intensities in the resulting pattern are $9I$ and I .

- 40 (a)** The diagram below shows a thin soap film and soap bubbles.



The rays 1 and 2 produce the phenomenon of interference when they meet. The path difference between the rays are proportional to thickness of film, i.e. path difference $(\Delta x) \propto t$. As, thickness t varies, so Δx varies. Maximas and minimas appear for different wavelengths in incident white light. When white light is used, the path difference (phase difference) will satisfy the condition of maxima for certain wavelengths and condition of minima for other wavelengths. This gives coloured appearance of the film.

41 (c) $\frac{I_{\max}}{I_{\min}} = \left(\frac{a_{\max}}{a_{\min}} \right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2} \right)^2$

According to given question, $\frac{I_{\max}}{I_{\min}} = \frac{9}{1}$

$$\Rightarrow 9 = \left(\frac{a_1/a_2 + 1}{a_1/a_2 - 1} \right)^2 \Rightarrow (3)^2 = \left(\frac{a_1/a_2 + 1}{a_1/a_2 - 1} \right)^2$$

$$\Rightarrow 3 = \frac{a_1/a_2 + 1}{a_1/a_2 - 1} \Rightarrow 3 \left(\frac{a_1}{a_2} \right) - 3 = \frac{a_1}{a_2} + 1$$

$$\Rightarrow 2 \left(\frac{a_1}{a_2} \right) = 4 \Rightarrow \frac{a_1}{a_2} = \frac{4}{2} = \frac{2}{1} \Rightarrow a_1 : a_2 = 2 : 1$$

- 42 (c)** According to given question,

$$I_1 : I_2 = 3 : 1, \quad a_1 : a_2 = ?$$

The intensity of light due to an illuminated slit is directly proportional to the width of the slit, if w_1 and w_2 be the width of two slits and I_1 and I_2 are the intensities of light on the screen due to the respective slits, then

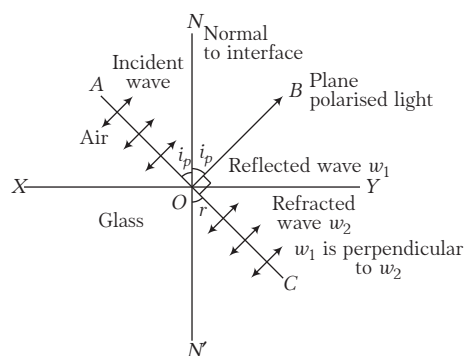
$$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \left(\frac{a_1}{a_2} \right)^2$$

where, a_1 and a_2 are the amplitudes of the corresponding light wave.

Here, $\frac{w_1}{w_2} = \frac{3}{1}$

$$\Rightarrow \left(\frac{a_1}{a_2} \right)^2 = \frac{3}{1} \quad \text{or} \quad a_1 : a_2 = \sqrt{3} : 1$$

- 43 (d)** Suppose, i_p is the Brewster's angle, when unpolarised light is incident at an polarising angle, the reflected component along OB and refracted component along OC in the following figure.



Now, $\angle BOY + \angle YOC = 90^\circ$

$$(90^\circ - i_p) + (90^\circ - r) = 90^\circ$$

Here, r is the angle of refraction.

$$\Rightarrow 90^\circ - i_p = r$$

According to Snell's law, $\mu = \frac{\sin i}{\sin r}$

When $i = i_p$, $r = 90^\circ - i_p$, then

$$\mu = \frac{\sin i_p}{\sin (90^\circ - i_p)} = \frac{\sin i_p}{\cos i_p} = \tan i_p$$

$$\therefore i_p = \tan^{-1}(\mu)$$

where, i_p is polarised angle.

44 (d) By Malus's law, $I = I_0 \cos^2 \theta$

(where, I = intensity of emergent polarised light, I_0 = intensity passed through polariser)

Given, $\theta = 60^\circ$, $I = ?$

$$\Rightarrow I = I_0 \times \cos^2 60^\circ = I_0 \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

45 (c) The distance of dark fringe from centre is given by

$$x_n = (2n + 1) \frac{\lambda D}{2d}$$

As, $n = 5$ for 5th dark fringe.

$$\text{So, } x_5 = \frac{11 \lambda D}{2d} = \frac{11}{2} \beta \quad \left(\because \frac{\lambda D}{d} = \beta \right)$$

$$= 11/2 \times 0.002 = 0.011 \text{ cm} = 1.1 \times 10^{-2} \text{ cm}$$

46 (b) Given, $\lambda_1 = 12000 \text{ \AA}$ and $\lambda_2 = 10000 \text{ \AA}$, $D = 2 \text{ m}$

and $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$$\text{We have, } \frac{\lambda_1}{\lambda_2} = \frac{12000}{10000} = \frac{6}{5} = \frac{n_2}{n_1}$$

$$\text{As, } x = \frac{n_1 \lambda_1 D}{d} = \frac{5 \times 12000 \times 10^{-10} \times 2}{2 \times 10^{-3}}$$

$$= 5 \times 1.2 \times 10^4 \times 10^{-10} \times 10^3 = 6 \text{ mm}$$

47 (c) We know, $\lambda = \frac{h}{mv}$ and $\theta = \frac{2\lambda}{d} \Rightarrow \theta \propto \frac{1}{v}$

48 (c) Fringe width, $\beta = \frac{D\lambda}{d}$

Given, $d = 0.4 \text{ mm}$, $D = 200 \text{ cm} = 2000 \text{ mm}$

$$\beta = 2 \text{ mm}$$

$$\therefore 2 = \frac{2000 \times \lambda}{0.4} \Rightarrow \lambda = 400 \text{ nm}$$

49 (b) Here, the condition for the formation of rings is $\lambda \approx \frac{d^2}{4D}$.

50 (c) Wave theory of light explains the phenomenon of interference, diffraction and velocity of light in a denser medium or rarer medium, but this theory fails to explain the scattering of light.

51 (a) Fringe width $\beta \propto \lambda$. As given that λ_1 produces photoelectric effect, so $\lambda_1 < \lambda_2$.

Hence, $\beta_1 < \beta_2$

52 (e) When we close one slit in the Young's double slit experiment, then the single slit diffraction pattern is observed.

53 (c) Given, $\mu = 1.54$

We know that, $\theta = \tan^{-1}(\mu) \Rightarrow \theta = \tan^{-1}(1.54) \Rightarrow \theta = 57^\circ$

On rotating the polaroid, the reflected light is fully polarised. Hence, the intensity gradually reduced to zero and then again increases.

54 (c) $\Delta\phi = (2n - 1)\pi$

For $n = 3$, $\Delta\phi = 5\pi$

55 (a) For diffraction to occur, the size of an obstacle/aperture is comparable to the wavelength of light wave.

56 (b) If a transparent medium of thickness t and refractive index μ is introduced in the path of one of the slits, then effective path in air is increased by an amount $(\mu - 1)t$ due to introduction of plate. Therefore, the zeroth fringe shifts to a new position where the two optical paths are equal. In such case fringe width remains unchanged.

57 (a) According to Newton's corpuscular theory of light, the light should travel faster in denser medium than in rarer medium. It is contrary to present theory of light which explains that light travels faster in air (rarer) than in water (denser).

58 (a) Fringe width, $\beta = \frac{D\lambda}{d}$

According to question, $\frac{\beta_2}{\beta_1} = \frac{\lambda_2 D_2 d_1}{\lambda_1 D_1 d_2}$

Here, $D_2 = 2D_1$ and $d_2 = \frac{d_1}{2}$

$$\Rightarrow \frac{\beta_2}{\beta_1} = \frac{\lambda_2 (2D_1) \times d_1}{\lambda_1 (D_1) \times \left(\frac{d_1}{2}\right)} = 4 \frac{\lambda_2}{\lambda_1} = \frac{4 \times 4 \times 10^{-7}}{6.4 \times 10^{-7}}$$

$$\Rightarrow \beta_2 = \frac{4 \times 4 \times 10^{-7} \times 1 \times 10^{-4}}{6.4 \times 10^{-7}} = 2.5 \times 10^{-4} \text{ m}$$

59 (a) As, the water droplets in cloud is responsible for diffraction of light and is efficient in equal measure at all wavelength and hence cloud look whitish.

60 (b) By Young's double slit interference experiment, $\beta = \frac{\lambda D}{d}$

Given, $\beta_1 = 1.2 \text{ mm}$, $\frac{d_2}{d_1} = 1 \frac{1}{2} = 1.5$

$$\text{So, } \beta \propto \frac{1}{d} \Rightarrow \frac{\beta_1}{\beta_2} = \frac{1/d_1}{1/d_2}$$

$$\Rightarrow \frac{\beta_1}{\beta_2} = \frac{d_2}{d_1} = 1.5 \Rightarrow \frac{1.2}{\beta_2} = 1.5$$

$$\Rightarrow \beta_2 = \frac{1.2}{1.5} = \frac{4}{5} = 0.8 \text{ mm}$$

61 (d) Let λ be the wavelength of monochromatic light incident on slit S , then the angular distance between two consecutive fringes, i.e. the angular fringe width is

$$\theta = \lambda/d \quad \dots(i)$$

where, d = distance between coherent sources.

$$\text{Given, } \frac{\Delta\theta}{\theta} = \frac{10}{100}$$

So, from Eq. (i),

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\theta}{\theta} = \frac{10}{100} = 0.1$$

$$\Rightarrow \Delta\lambda = 0.1\lambda = 0.1 \times 5890 \text{ \AA} = 589 \text{ \AA} \text{ (increases)}$$

$$62 \text{ (a)} \quad \frac{I_{\max}}{I_{\min}} = \frac{(\sqrt{2I} + \sqrt{I})^2}{(\sqrt{2I} - \sqrt{I})^2} = \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)^2 \quad (\text{given})$$

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \left[\frac{(\sqrt{2} + 1)(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} \right]^2$$

$$= \left(\frac{2 + 1 + 2\sqrt{2}}{2 - 1} \right)^2 = \frac{9 + 8 + 12\sqrt{2}}{1} = 34$$

- 63 (b) The bandwidth is inversely proportional to the distance between the slits d .

$$\text{As, } \beta = \frac{\lambda D}{d} \Rightarrow \beta \propto \frac{1}{d}$$

- 64 (b) When a Young's double slit set up for interference is shifted from air to within water, then the fringe width decreases.

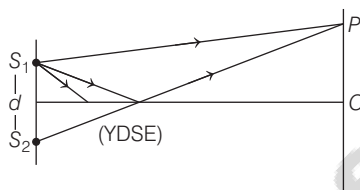
- 65 (b) Given, $d = 1\text{ mm} = 1 \times 10^{-3}\text{ m}$, $\lambda = 500\text{ nm} = 500 \times 10^{-9}\text{ m}$ and $D = 1\text{ m}$

$$\text{Fringe separation, } \beta = \frac{\lambda D}{d} = \frac{500 \times 10^{-9} \times 1}{1 \times 10^{-3}} = 500 \times 10^{-6}$$

$$= 0.5 \times 10^{-3}\text{ m} = 0.5\text{ mm}$$

- 66 (b) The path difference $S_1P - S_2P$ is a whole number of wavelength, for the interference at an arbitrary point P .

$$\text{So, } S_1P - S_2P = n\lambda \quad (\text{for maximum intensity})$$



- 67 (c) In Young's double slit experiment, we have

$$\beta = \frac{\lambda D}{d}$$

$$\Delta\beta = \frac{\lambda \Delta D}{d}$$

$$\Rightarrow \quad \lambda = \frac{\Delta\beta d}{\Delta D}$$

$$\lambda = \frac{3 \times 10^{-5} \times 1 \times 10^{-3}}{5 \times 10^{-2}} = 6 \times 10^{-7}\text{ m}$$

$$\text{or } \lambda = 600\text{ nm}$$

- 68 (c) The direction in which the first minima occurs is θ (say), then $e \sin \theta = \lambda$

$$\text{or } e\theta = \lambda \text{ or } \theta = \frac{\lambda}{e} \quad (\because \theta \approx \sin \theta, \text{ when } \theta \text{ is small})$$

$$\text{Width of the central maximum} = 2b\theta + e = \frac{2b\lambda}{e} + e$$

- 69 (d) Grating with 10^6 lines/cm is suitable for constructing spectrometer for visible and ultraviolet region because this corresponds to the wavelength, equivalent to the wavelengths of visible and ultraviolet region.

- 70 (a) Width of central maximum,

$$\beta = \frac{2D\lambda}{d}$$

$$\text{As, we increase } d \text{ to } 2d, \beta \text{ becomes } \frac{\beta}{2}.$$

So, it becomes narrower and fainter.

- 71 (c) When rays of monochromatic light of wavelength λ are incident on a diffraction grating in which slit separation is d , then for angle of diffraction θ , the following relation holds true.

$$d \sin \theta = n\lambda$$

where, n is called the spectrum order.

$$\text{Given, } n = 1, \lambda = 6500 \text{ \AA} = 6500 \times 10^{-10}\text{ m}$$

$$\text{and } \sin 30^\circ = \frac{1}{2}$$

$$\Rightarrow \quad d = \frac{n\lambda}{\sin 30^\circ} = \frac{6500 \times 10^{-10}}{(1/2)}$$

$$\Rightarrow \quad d = 1.3 \times 10^{-6}\text{ m}$$

- 72 (b) For polarising angle, $\mu = \tan i_p$

$$\text{and } \mu = \frac{1}{\sin C}$$

$$\tan i_p = \frac{1}{\sin C}$$

where, C = critical angle.

$$\therefore \cot i_p = \sin \left(\sin^{-1} \left(\frac{3}{5} \right) \right)$$

$$\Rightarrow \cot i_p = \frac{3}{5} \text{ or } \tan i = \frac{5}{3}$$

$$\text{or } i_p = \tan^{-1} \left(\frac{5}{3} \right)$$

Dual Nature of Radiation and Matter

We know that, light have the properties of electromagnetic waves. Various phenomena like interference, diffraction, polarisation, etc., demonstrated by light were explained by considering the light as waves. But some other phenomena like photoelectric effect, Compton effect, etc., could not be explained by the wave theory of light. To explain photoelectric effect, Einstein used Planck's quantum theory. According to which light consists photons or quanta of light that travel in a straight line with the speed of light. Thus, the particle nature of light was established.

Hence, it was concluded that light is of dual nature, as some phenomena were explained by wave theory of light and some by particle nature of light.

Similar to light, matter also has dual nature, *e.g.* electron is a matter particle as it contains mass and charge but fast moving electrons, *i.e.* cathode rays show the properties of waves. Thus, matter also exhibits dual nature.

In this chapter, we will study the dual nature of radiation and matter followed by electron emission, photoelectric effect and matter waves.

Electron emission

In metals, the electrons in the outer shells (valence electrons) are loosely bound to the atoms, hence they are free to move easily within the metal surface but cannot leave their own surface. Such electrons are called **free electrons**.

These free electrons can be emitted or ejected from the metals, if they have sufficient energy to overcome the attractive pull of metal surface. The phenomenon of emission of these electrons from the surface of a metal is called electron emission.

The minimum required energy for the electron emission from the metal surface can be supplied to the free electrons by any of the following physical processes

- (i) **Thermionic emission** The energy to the free electrons can be given by heating the metal. This process of emission of electrons is known as thermionic emission. The electrons, so emitted, are known as **thermions**.
- (ii) **Field emission** When a conductor is put under strong electric field ($\approx 10^8$ V/m), the free electrons on it experience an electric force in the opposite direction of field. Beyond a certain limit, electrons start coming out of the metal surface. Emission of electrons from a metal surface by this method is called the field emission.

Note Field emission is also known as **cold emission** or **cold cathode emission**. One of the example of cold emission is spark plug.

Inside

- 1 **Photoelectric effect**
 - Experimental study of photoelectric effect
 - Laws of photoelectric emission
- 2 **Einstein's photoelectric equation**
- 3 **Planck's quantum theory (Particle nature of light : the photon)**
- 4 **Photocell**
- 5 **Compton effect**
- 6 **Dual nature of radiation**
 - Matter waves : de-Broglie waves
- 7 **Davisson and Germer experiment**
- 8 **Electron microscope**

- (iii) **Secondary emission** Emission of electrons from a metal surface by the bombardment of high speed electrons or other particles is known as secondary emission. The electrons so emitted are called secondary electrons.
- (iv) **Photoelectric emission** It is the phenomenon of emission of electrons from the surface of metal when light radiations of suitable frequency fall on it. Here, the energy to free electrons is supplied by light photons.

Work function

To pull out electrons from the surface of the metal, a certain minimum amount of energy is required. This minimum energy required by the electrons is called the work function of the metal. It is denoted by ϕ_0 or W .

It is measured in eV (electron-volt). 1 eV is defined as the energy gained by an electron when it has been accelerated by a potential difference of one volt (1 V), so that

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

Amongst metals, cesium is regarded as the best photosensitive material as it has lowest work function ($\phi_0 = 2.14 \text{ eV}$), whereas work function for platinum is highest ($\phi_0 = 5.65 \text{ eV}$).

Note Work function depends on the properties of the metal and the nature of its surface. It decreases with increase in temperature.

PHOTOELECTRIC EFFECT

The phenomenon of emission of electrons from the surface of metal, when radiations of suitable frequency fall on it, is known as photoelectric effect.

The emitted electrons (as shown in Fig. 11.1) are called **photoelectrons** and the current, so produced is called **photoelectric current**.

Alkali metals like lithium, sodium, potassium, cesium, etc., show photoelectric effect with visible light, whereas the metals like zinc, cadmium, magnesium etc., are sensitive only to ultraviolet light.

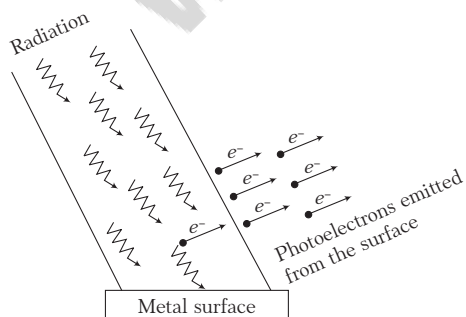


Fig. 11.1 Emission of photoelectrons from the metal surface

The phenomenon of photoelectric effect was discovered by Heinrich Hertz in 1887, while detailed investigation on photoelectric emission was done by Wilhelm Hallwachs and Philipp Lenard.

Experimental study of photoelectric effect

An extensive study of photoelectric effect was made by Lenard and RA Millikan. Fig. 11.2 shows the experimental arrangement for the study of photoelectric effect.

It consists of an evacuated glass or quartz tube which encloses a photosensitive plate C (called **emitter**) and a metal plate A (called **collector**). A transparent quartz window is sealed onto the glass tube which permits ultraviolet radiation to pass through it and fall on photosensitive plate C . The electrons are emitted by the plate C and are collected by the plate A . When the collector plate A is positive with respect to the emitter plate C , the electrons are attracted towards it and hence photoelectric current is constituted.

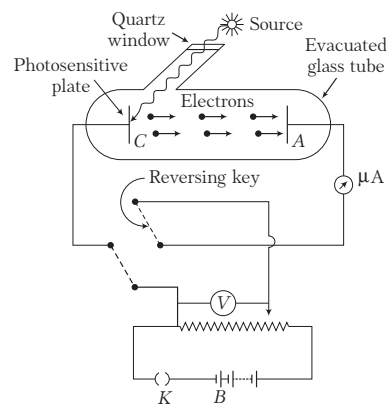


Fig. 11.2 Experimental arrangement for the study of photoelectric effect

The experimental arrangement given above is used to study the variation of photoelectric current with intensity of radiation, frequency of radiation and the potential difference between the plates A and C .

These variations are described below

Effect of intensity of light on photocurrent

For a fixed frequency of incident radiation and accelerating potential, the photoelectric current increases linearly with increase in intensity of incident light as shown in graph.

As, the photoelectric current is directly proportional to the number of photoelectrons emitted per second.

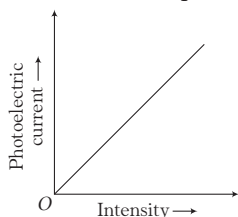


Fig. 11.3 Variation of photoelectric current with intensity of light

So for constant frequency of incident radiation, the number of photoelectrons emitted per second is directly proportional to the intensity of the incident radiations. *i.e.* Intensity $\propto$ Photoelectric current

$$\propto \text{number of photoelectrons emitted/second} \propto \frac{1}{(\text{distance})^2}$$

Effect of potential on photoelectric current

For a fixed frequency and intensity of incident light, photoelectric current increases with increase in potential applied to the collector, as shown in the graph below.

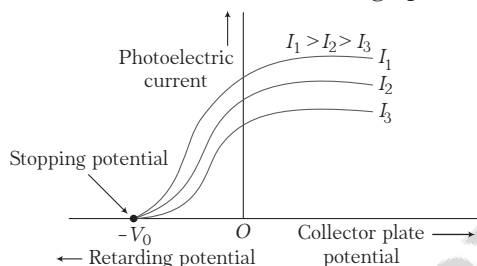


Fig. 11.4 Variation of photoelectric current with collector plate potential for different intensities but constant frequency of incident radiation

From the above graph, we observe that

- At some stage, for a certain positive potential of plate A, all the emitted electrons are collected by the plate A, and the photoelectric current becomes maximum or saturates. If we further increase the accelerating potential of plate A, the photocurrent does not increase. This maximum value of photoelectric current is called **saturation current**.

Note Saturation current corresponds to the case when all the photoelectrons emitted by the plate C reaches plate A.

- When the potential is decreased, the current decreases but does not become zero at zero potential. This shows that even in the absence of accelerating potential, few photoelectrons manage to reach the plate A on their own due to their kinetic energy.
- For a particular frequency of incident radiation, the minimum negative (retarding) potential V_0 given to plate A for which the photoelectric current stops or becomes zero is called **cut-off** or **stopping potential**.

In this condition, no electron (even the most energetic electron) reaches to plate A, *i.e.* the stopping potential is sufficient to repel even the most energetic photoelectron with maximum kinetic energy $K_{\max}$.

Hence, maximum kinetic energy of emitted electrons is given as

$$K_{\max} = eV_0 = \frac{1}{2}mv_{\max}^2$$

where, m is the mass of photoelectron and $v_{\max}$ is the maximum velocity of emitted photoelectron.

Note For the radiation of a given frequency and material of plate C, the value of stopping potential V_0 is independent of the intensity of the incident radiation. It means that, the maximum kinetic energy of emitted photoelectron depends on the light source and the emitter plate material but is independent of intensity of incident radiation.

Example 11.1 The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Sol. Maximum kinetic energy of photoelectrons $K_{\max} = eV_0$

where, $e = 1.6 \times 10^{-19}$ C, $V_0 = 1.5$ V

$$\therefore K_{\max} = 1.6 \times 10^{-19} \times 1.5 = 2.4 \times 10^{-19} \text{ J}$$

Example 11.2 In an experimental study of photoelectric effect, the stopping potential for a metallic surface is 5.68 V. What will be the maximum velocity of the electrons?

Sol. Stopping potential, $V_0 = 5.68$ V, $m_e = 9.1 \times 10^{-31}$ kg

$$\text{Maximum KE, } K_{\max} = \frac{1}{2}mv_{\max}^2 = eV_0$$

$$\Rightarrow v_{\max} = \sqrt{\frac{2eV_0}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 5.68}{9.1 \times 10^{-31}}} = 1.414 \times 10^6 \text{ ms}^{-1}$$

Effect of frequency of incident radiation on stopping potential

Let us take radiations of different frequencies but of same intensity. For each radiation, the variation of photoelectric current against the potential difference between the plates is as shown in the graph below

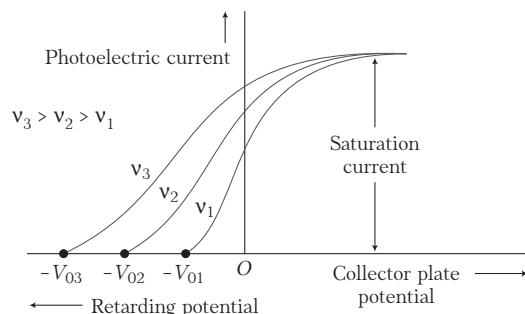
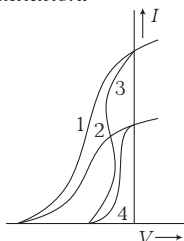


Fig. 11.5 Variation of photoelectric current with collector plate potential for different frequencies but same intensity of incident radiation

From the above graph, we observe that

- (i) The value of stopping potential is different for radiation of different frequencies but value of saturation current (for given intensity) remains same for all incident radiations as intensity remains constant.
- (ii) The value of stopping potential is more negative for radiation of higher incident frequency. That means, the energy of the emitted electrons depends on the frequency of incident radiations. Greater the frequency of incident radiation, greater is the maximum kinetic energy of photoelectrons, consequently greater retarding potential or stopping potential is required to stop them completely.
- (iii) The value of saturation current depends on the intensity of incident radiation but is independent of the frequency of the incident radiation.

Example 11.3 The given graph shows the variation of photoelectric current I versus applied voltage V for two different photosensitive materials and for two different intensities of the incident radiations. Identify the pairs of curves that correspond to different materials but same intensity of incident radiation.



Sol. Curves 1 and 2 correspond to similar materials, while curves 3 and 4 represent different materials, since the value of stopping potential for the pair of curves (1 and 2) and (3 and 4) are the same. For given frequency of the incident radiation, the stopping potential is independent of its intensity. So, the pairs of curves (1 and 3) and (2 and 4) correspond to different materials but same intensity of incident radiation.

Stopping potential versus frequency graph

If we plot a graph between stopping potential and the frequency of the incident radiation for two different metals A and B, we get the graph as shown in the figure.

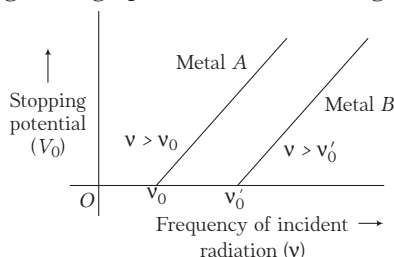


Fig. 11.6 Variation of stopping potential with frequency of incident light for a given photosensitive material

From the above graph, we observe that

- (i) The stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.
- (ii) There exists a certain minimum cut-off frequency ν_0 for which the stopping potential is zero. This minimum frequency is called **threshold frequency**.

It can be defined as,

“the minimum frequency of light which can emit photoelectrons from a material is called **threshold frequency** or **cut-off frequency** of that material”. It is a characteristic property of material.

For a frequency lower than cut-off frequency, no photoelectric emission is possible even if the intensity is large.

If frequency of incident radiation is more than the threshold frequency, the photoelectric emission starts instantaneously without any apparent time lag ($\sim 10^{-9}$ s or less), even when the incident radiation has low intensity.

Laws of photoelectric emission

On the basis of the experimental results on photoelectric effect, Lenard and Millikan gave the following laws of photoelectric emission

- (i) For a given photosensitive material and frequency of incident radiation, the photoelectric current or number of photoelectrons ejected per second is directly proportional to the intensity of the incident light.
- (ii) For a given material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation, whereas the stopping potential is independent of its intensity.
- (iii) For a given material, there exists a certain minimum frequency of the incident radiation below which no emissions of photoelectrons take place. This frequency is called threshold frequency.

Above the threshold frequency, the maximum kinetic energy of the emitted photoelectrons or equivalent stopping potential is independent of the intensity of the incident light but depends only upon the frequency (or wavelength) of the incident light.

- (iv) The photoelectric emission is an instantaneous process. The time lag between the incidence of radiations and emission of photoelectrons is very small, less than 10^{-9} s, even when incident radiation is made exceedingly dim.

Failure of wave theory in explaining photoelectric effect

The wave theory of light given by Huygens' could not explain the photoelectric emission due to the following main reasons

- (i) According to the wave picture of light, the free electrons at the surface of the metal absorb the radiant energy continuously.
The greater the intensity of radiation, the greater should be the energy absorbed by each electron. The maximum kinetic energy of the photoelectrons on the surface is then expected to increase with increase in intensity.
But according to experimental facts, the maximum kinetic energy of ejected photoelectrons is independent of intensity of incident radiation.
- (ii) According to wave theory of light, no matter what the frequency of radiation is, a sufficiently intense beam of radiation should be able to impart enough energy to the electrons, so that they exceed the minimum energy needed to escape from metal surface.
A threshold frequency, therefore should not exist which contradicts the experimental fact that no photoelectric emission takes place below that threshold frequency, no matter whatsoever may be its intensity.
- (iii) According to the wave theory of light, the absorption of energy by electron takes place continuously over the entire wavefront of the radiation. Since a large number of electrons absorb energy, the energy absorbed per electron per unit time turns out to be small.
Hence, it will take hours or more for a single electron to come out of the metal which contradicts the experimental fact that photoelectron emission is instantaneous ($\sim 10^{-9}$ s).

EINSTEIN'S PHOTOELECTRIC EQUATION

In 1905, Albert Einstein explained the various laws of photoelectric emission on the basis of Planck's quantum theory. According to Planck's quantum theory, light radiations consist of tiny packets of energy called **quanta**. One quantum of light radiation is called a **photon** which travels with the speed of light.

The energy of a photon is given by

$$E = h\nu$$

where, h is Planck's constant and ν is the frequency of light radiation.

When a photon of energy $h\nu$ falls on a metal surface, the energy of the photon is absorbed by the electron and is used in following two ways

- (i) A part of energy is utilised to overcome the surface barrier and come out of the metal surface. This part of energy is called **work function**.

It is expressed as $\phi_0 = h\nu_0$.

- (ii) The remaining part of the energy is used in giving a velocity v to the emitted photoelectron. This is equal to the maximum kinetic energy of the photoelectrons $\left(\frac{1}{2}mv_{\max}^2\right)$, where m is the mass of the photoelectron.

This can be seen in the figure given below

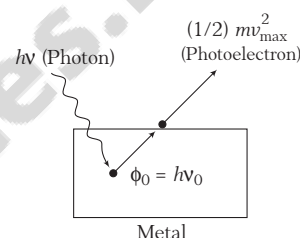


Fig. 11.7 Emission of photoelectron by a metal when a photon is absorbed by it

According to the law of conservation of energy,

$$h\nu = \phi_0 + \frac{1}{2}mv_{\max}^2 = h\nu_0 + \frac{1}{2}mv_{\max}^2$$

$$\therefore \frac{1}{2}mv_{\max}^2 = K_{\max} = h(\nu - \nu_0) = h\nu - \phi_0$$

or $K_{\max} = h\nu - \phi_0$

This equation is called **Einstein's photoelectric equation**.

Relation between stopping potential (V_0), frequency (ν) and wavelength (λ) of incident light

Maximum kinetic energy is given by

$$K_{\max} = h(\nu - \nu_0)$$

If stopping potential is V_0 , then

$$K_{\max} = eV_0$$

$$\therefore eV_0 = h(\nu - \nu_0) \quad \dots(i)$$

If λ = wavelength of the incident radiation,

λ_0 = threshold wavelength for the metal surface

and c = velocity of light.

Then, $\nu = \frac{c}{\lambda}$ and $\nu_0 = \frac{c}{\lambda_0}$

Putting these values in Eq. (i), we get

$$eV_0 = h \left(\frac{c}{\lambda} - \frac{c}{\lambda_0} \right) \Rightarrow eV_0 = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \dots (ii)$$

Slope of the graph between V_0 and ν_0

Consider the equation

$$eV_0 = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \Rightarrow V_0 = \frac{h}{e} \left(\frac{c}{\lambda} - \frac{c}{\lambda_0} \right)$$

or
$$V_0 = \frac{h}{e} \nu - \frac{h\nu_0}{e} \dots (iii)$$

Comparing Eq. (iii) with the straight line equation, i.e. $y = mx + c$, it is clear that graph between V_0 and ν_0 is a straight line. The slope of this line is given by $\tan \theta = \frac{h}{e}$ and intercept on V_0 -axis is given by $\frac{h\nu_0}{e}$, as shown in the graph,

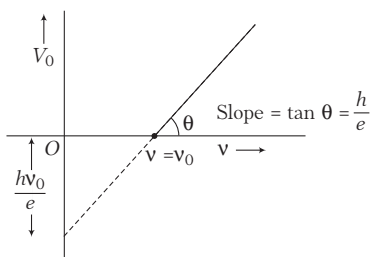


Fig. 11.8 Graph of V_0 versus ν

In terms of wavelength, we can write

$$V_0 = \frac{hc}{\lambda e} - \frac{hc}{\lambda_0 e} \dots (iv)$$

From the Eq. (iv), it is clear that graph between V_0 and $\frac{1}{\lambda}$ is also a straight line. The slope of this line is given by $\tan \theta = \frac{hc}{e}$ and intercept on V_0 -axis is given by $\frac{hc}{\lambda_0 e}$, as shown in the graph.

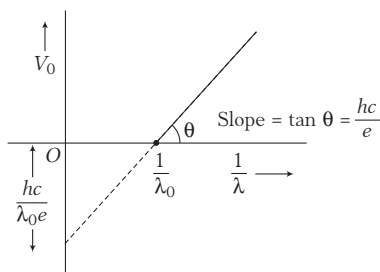


Fig. 11.9 Graph of V_0 versus $1/\lambda$

Explanation of photoelectric effect from Einstein's equation

Einstein explained all the observations of photoelectric effect using Einstein's photoelectric equation.

These are as follow

- (i) From photoelectric equation it is clear that, $K_{\max}$ is independent of intensity of radiation but depends linearly on ν .
- (ii) Photoelectric emission is possible only, if $h\nu > \phi_0$.
or $h\nu > h\nu_0$ ($\because \phi_0 = h\nu_0$)
 $\Rightarrow \nu > \nu_0$

where ν_0 is threshold frequency. If $\nu < \nu_0$, then kinetic energy of electron comes out to be negative, which is not possible, so no photoelectric emission is possible below ν_0 , even if the incident radiation of high intensity and falls on the surface for long duration.

- (iii) Intensity of radiation (I) is proportional to the number of energy quanta per unit area per unit time (n). Higher the value of n , higher the number of electrons emitted (for $\nu > \nu_0$).

That is the reason (for $\nu > \nu_0$),

$$\text{Photocurrent} \propto \text{Intensity}$$

- (iv) The absorption of photon by electron is instantaneous process, hence emission of electrons is instantaneous.

Note Practically only one electron is ejected by 1 million photons.

$$\text{Quantum efficiency} = \frac{\text{Number of electrons ejected / sec}}{\text{Number of photons incident / sec}}$$

$$\Rightarrow \eta = \frac{n'}{n} \times 100\%$$

Example 11.4 The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Sol. Here, W = Work function of certain metal = 4.2 eV

$$= (4.2 \times 1.6 \times 10^{-19}) \text{ J} = 6.72 \times 10^{-19} \text{ J} \quad (\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

If E be the energy of the photon of incident light, then

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} \text{ J} = 6.01 \times 10^{-19} \text{ J}$$

The photoelectric emission is possible only if $E = h\nu > \phi$, where ϕ is work function of metal. Here $E < \phi$, so this metal will not give photoelectric emission for incident radiation of wavelength 330 nm.

Example 11.5 The threshold frequency for a certain metal is $3.3 \times 10^{14} \text{ Hz}$. If light of frequency $8.2 \times 10^{14} \text{ Hz}$ is incident on the metal, predict the cut-off voltage for the photoelectric emission.

Sol. Using the relation, $h\nu - h\nu_0 = eV_0$

$$\text{We get } V_0 = h \left(\frac{\nu - \nu_0}{e} \right)$$

$$\text{Here, } \nu_0 = 3.3 \times 10^{14} \text{ Hz, } \nu = 8.2 \times 10^{14} \text{ Hz}$$

$$= 6.62 \times 10^{-34} \times \left(\frac{8.2 \times 10^{14} - 3.3 \times 10^{14}}{1.6 \times 10^{-19}} \right) = 2.03 \text{ V}$$

Example 11.6 Light of frequency $7.21 \times 10^{14} \text{ Hz}$ is incident on a metal surface. Electrons with a maximum speed of $6.0 \times 10^5 \text{ m/s}$ are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Sol. Using Einstein's photoelectric equation, $h\nu - h\nu_0 = \frac{1}{2} m v_{\max}^2$,

$$\text{We get } h\nu_0 = h\nu - \frac{1}{2} m v_{\max}^2$$

$$\therefore \nu_0 = \nu - \frac{m v_{\max}^2}{2h}$$

$$\text{Here, } \nu = 7.21 \times 10^{14} \text{ Hz}$$

$$v_{\max} = 6.0 \times 10^5 \text{ m/s}$$

$$\text{and } m = \text{mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$$\therefore \text{Threshold frequency, } \nu_0$$

$$= 7.21 \times 10^{14} - \frac{9.1 \times 10^{-31} \times (6.0 \times 10^5)^2}{2 \times 6.62 \times 10^{-34}}$$

$$= 7.21 \times 10^{14} - 2.47 \times 10^{14} = 4.74 \times 10^{14} \text{ Hz}$$

Example 11.7 The photoelectric work function of potassium is 2.3 eV . If light having a wavelength of $2800 \text{ \AA}$ falls on potassium, find

(a) the kinetic energy in electron volts of the most energetic electrons ejected

(b) and the stopping potential in volts.

Sol. Given, $W = 2.3 \text{ eV}$ and $\lambda = 2800 \text{ \AA}$

$$\therefore E \text{ (in eV)} = \frac{hc}{\lambda \text{ (in \AA)}} = \frac{12375}{\lambda \text{ (in \AA)}} = \frac{12375}{2800} = 4.4 \text{ eV}$$

$$(a) \text{ As } K_{\max} = E - W = (4.4 - 2.3) \text{ eV} = 2.1 \text{ eV}$$

$$(b) \text{ Using the relation, } K_{\max} = eV_0$$

$$\therefore 2.1 \text{ eV} = eV_0 \text{ or } V_0 = 2.1 \text{ V}$$

Example 11.8 Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V . Find the work function of the material from which the cathode is made.

Sol. Energy of incident photon,

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{488 \times 10^{-9}} = 4.08 \times 10^{-19} \text{ J}$$

Using formula $eV_0 = E - W$, we get

$$W = E - eV_0$$

$$\text{where, } E = h\nu$$

and $W = \text{Work function of the material}$

$$\text{Here, } E = 4.08 \times 10^{-19} \text{ J}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\text{and } V_0 = 0.38 \text{ V}$$

$$\therefore W = 4.08 \times 10^{-19} \text{ J} - (1.6 \times 10^{-19} \times 0.38)$$

$$= (4.08 - 0.608) \times 10^{-19} \text{ J} = 3.47 \times 10^{-19} \text{ J}$$

$$= \frac{3.47 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 2.17 \text{ eV}$$

Example 11.9 The work function of cesium is 2.14 eV .

Calculate

(i) the threshold frequency for cesium and

(ii) the wavelength of the incident light, if the photocurrent is brought to zero by a stopping potential of 0.60 V .

$$\text{Given, } h = 6.63 \times 10^{-34} \text{ J-s.}$$

Sol. Here, $V_0 = 0.60 \text{ V}$

$$\phi_0 = 2.14 \text{ eV} = 2.14 \times 1.6 \times 10^{-19} \text{ J}$$

$$(i) \text{ Threshold frequency, } \nu_0 = \frac{\phi_0}{h} \quad (\because \phi_0 = h\nu_0)$$

$$= \frac{2.14 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 5.16 \times 10^{14} \text{ Hz}$$

$$(ii) \text{ As, } eV_0 = \frac{hc}{\lambda} - \phi_0 \text{ or } \lambda = \frac{hc}{(eV_0 + \phi_0)}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{(1.6 \times 10^{-19} \times 0.6 + 2.14 \times 1.6 \times 10^{-19})} \approx 454 \times 10^{-9} \text{ m} = 454 \text{ nm}$$

Example 11.10. The work function of cesium metal is 2.14 eV . When light of frequency $6 \times 10^{14} \text{ Hz}$ is incident on the metal surface, photoemission of electrons occurs. What is the

(a) maximum kinetic energy of the emitted electrons,

(b) stopping potential

(c) and maximum speed of the emitted photoelectrons?

Sol. (a) Using Einstein's photoelectric equation,

$$\begin{aligned} E_{\max} &= h\nu - \phi_0 \\ &= 6.62 \times 6 \times 10^{-20} - 2.14 \times 1.6 \times 10^{-19} \\ &= 39.72 \times 10^{-20} - 34.24 \times 10^{-20} \text{ J} \\ &= 5.48 \times 10^{-20} \text{ J} \\ &= \frac{0.548 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \quad (\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}) \\ &= 0.342 \text{ eV} \end{aligned}$$

$$(b) \text{ As, } eV_0 = E_{\max}$$

$$\text{or } V_0 = \frac{E_{\max}}{e} = \frac{0.342 \times 1.6 \times 10^{-19} \text{ J}}{1.6 \times 10^{-19} \text{ C}}$$

$$= 0.342 \text{ JC}^{-1} = 0.342 \text{ V}$$

(c) As we know that, $\frac{1}{2}mv_{\max}^2 = E_{\max}$

$$\therefore v_{\max} = \sqrt{\frac{2E_{\max}}{m}} = \sqrt{\frac{2 \times 5.48 \times 10^{-20}}{9.1 \times 10^{-31}}} = 3.45 \times 10^5 \text{ ms}^{-1}$$

Example 11.11 When light of wavelength $4000 \text{ \AA}$ is incident on a barium emitter, emitted photoelectrons in a transverse magnetic field strength $B = 5.2 \times 10^{-6} \text{ T}$ moves in a circular path. Find the radius of the circle. (Take, work function of barium is 2.5 eV and mass of electron is $9.1 \times 10^{-31} \text{ kg}$, $e = 1.6 \times 10^{-19} \text{ C}$ and $h = 6.6 \times 10^{-34} \text{ J-s}$)

Sol. Using the relation, $K_{\max} = E - \phi_0$

$$\begin{aligned} \frac{1}{2}mv_{\max}^2 &= \frac{hc}{\lambda} - \phi_0 \Rightarrow v_{\max}^2 = \frac{2}{m} \left[\frac{hc}{\lambda} - \phi_0 \right] \\ &= \frac{2}{9.1 \times 10^{-31}} \left[\frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{4 \times 10^{-7}} - 2.5 \times 1.6 \times 10^{-19} \right] \\ &= 20.88 \times 10^{10} \end{aligned}$$

$$\therefore v_{\max} = 4.57 \times 10^5 \text{ m/s}$$

Radius of circular path described by emitted photoelectrons in a magnetic field is given by

$$\begin{aligned} Bev_{\max} &= \frac{mv_{\max}^2}{r} \\ \Rightarrow r &= \frac{mv_{\max}}{Be} = \frac{(9.1 \times 10^{-31}) \times (4.57 \times 10^5)}{5.2 \times 10^{-6} \times 1.6 \times 10^{-19}} = \frac{1}{2} \text{ m} \end{aligned}$$

Example 11.12 Light of wavelength $2475 \text{ \AA}$ is incident on barium. Photoelectrons emitted describe a circle of radius 100 cm by a magnetic field of flux density $\frac{1}{\sqrt{17}} \times 10^{-5} \text{ T}$.

Find the work function of the barium. (Take, $\frac{e}{m} = 1.7 \times 10^{11}$)

Sol. Radius of circular path described by a charged particle in a magnetic field B is given by

$$r = \frac{\sqrt{2mK}}{qB} \Rightarrow K = \frac{q^2 B^2 r^2}{2m} = \left(\frac{e}{m} \right) \frac{e B^2 r^2}{2}$$

where, K = kinetic energy of a particle,

q = charge on a particle

and r = radius of circular path.

$$\begin{aligned} \text{So, } K &= \frac{1}{2} \times 1.7 \times 10^{11} \times 1.6 \times 10^{-19} \times \left(\frac{1}{\sqrt{17}} \times 10^{-5} \right)^2 \times (1)^2 \\ &= 8 \times 10^{-20} \text{ J} = 0.5 \text{ eV} \end{aligned}$$

Using the relation, $E = \phi_0 + K_{\max}$

$$\Rightarrow \phi_0 = E - K_{\max}$$

$$\text{where, } E = h\nu = \frac{hc}{\lambda} = \frac{12375}{\lambda (\text{in \AA})}$$

$$= \left(\frac{12375}{2475} \right) \text{ eV} - 0.5 \text{ eV} = 4.5 \text{ eV}$$

Example 11.13 In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be $4.12 \times 10^{-15} \text{ Vs}$. Calculate the value of Planck's constant.

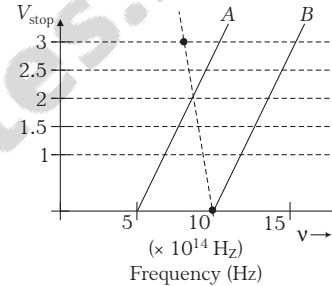
Sol. We know, $eV_0 = h\nu - W$

$$\therefore V_0 = \left(\frac{h}{e} \right) \nu - \frac{W}{e}$$

$$\text{Given, Slope} = 4.12 \times 10^{-15} \text{ Vs} = \frac{h}{e}$$

$$\begin{aligned} \therefore h &= 4.12 \times 10^{-15} \times e \\ &= 4.12 \times 10^{-15} \times 1.6 \times 10^{-19} \text{ Js}^{-1} \\ &= 6.592 \times 10^{-34} \text{ Js}^{-1} \end{aligned}$$

Example 11.14 A student performs an experiment on photoelectric effect, using two materials A and B. A plot of V_{stop} versus ν is given in the figure.



(i) Which material A or B has a higher work function?

(ii) Given the electric charge of an electron $= 1.6 \times 10^{-19} \text{ C}$, find the value of h obtained from the experiment for both A and B. Comment on whether it is consistent with the Einstein's theory.

Sol. (i) Given, threshold frequency of A is given by

$$\nu_{OA} = 5 \times 10^{14} \text{ Hz and for B, } \nu_{OB} = 10 \times 10^{14} \text{ Hz}$$

We know that, work function, $\phi = h\nu_0$ or $\phi_0 \propto \nu_0$

$$\text{So, } \frac{\phi_{OA}}{\phi_{OB}} = \frac{5 \times 10^{14}}{10 \times 10^{14}} < 1 \Rightarrow \phi_{OA} < \phi_{OB}$$

Thus, work function of B is higher than A.

$$(ii) \text{ For metal A, slope} = \frac{h}{e} = \frac{2}{(10 - 5) \times 10^{14}}$$

$$\begin{aligned} \text{or } h &= \frac{2 \times e}{5 \times 10^{14}} = \frac{2 \times 1.6 \times 10^{-19}}{5 \times 10^{14}} \\ &= 6.4 \times 10^{-34} \text{ Js} \end{aligned}$$

$$\text{For metal B, slope} = \frac{h}{e} = \frac{2.5}{(15 - 10) \times 10^{14}}$$

$$\begin{aligned} \text{or } h &= \frac{2.5 \times e}{5 \times 10^{14}} = \frac{2.5 \times 1.6 \times 10^{-19}}{5 \times 10^{14}} \\ &= 8 \times 10^{-34} \text{ Js} \end{aligned}$$

Since the value of h from experiment for metals A and B is different. Hence, experiment is not consistent with the Einstein's theory.

CHECK POINT 11.1

- Which of the following statement(s) is correct?
 - Hertz discovered the phenomenon of photoelectric emission
 - Lenard made detailed study of photoelectric effect.
 - Both (a) and (b)
 - None of the above
- When intensity of light increases, then photoelectric current
 - increases
 - decreases
 - remains same
 - None of the above
- The number of photoelectrons emitted per second from a metal surface increases when
 - the energy of incident photons increases
 - the frequency of incident light increases
 - the wavelength of the incident light increases
 - the intensity of the incident light increases
- When stopping potential is applied in an experiment on photoelectric effect, no photocurrent is observed. This means that
 - the emission of photoelectrons is stopped
 - the photoelectrons are emitted but are reabsorbed by the emitter metal
 - the photoelectrons are accumulated near the collector plate
 - the photoelectrons are dispersed from the sides of the apparatus.
- If the work function of potassium is 2 eV, then its photoelectric threshold wavelength is
 - 310 nm
 - 620 nm
 - 6200 nm
 - 3100 nm
- Maximum kinetic energy of emitted electron from the surface of a metal is K . When frequency of incident light is doubled, then maximum kinetic energy of emitted electrons is K' then
 - $K' > 2K$
 - $K' < 2K$
 - $K' = 2K$
 - Data is insufficient
- If the frequency of light in a photoelectric experiment is doubled, then the stopping potential will
 - be doubled
 - be halved
 - become more than double
 - become less than double
- When frequency increases, slope of stopping potential *versus* frequency graph
 - increases
 - decreases
 - remains same
 - slope is always zero
- The work function of aluminium is 4.2 eV. If two photons, each of energy 3.5 eV strike on an electron of aluminium, then emission of electrons will be
 - possible
 - not possible
 - data is incomplete
 - depend upon the area of the surface
- Photoelectric effect supports quantum nature of light because
 - the energy of released electron is discrete
 - the maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity
 - even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately
 - All of the above

PLANCK'S QUANTUM THEORY

(PARTICLE NATURE OF LIGHT : THE PHOTON)

In 1900, Max Planck proposed his quantum theory of radiation. According to this theory, the energy of an electromagnetic wave is not continuously distributed over the wavefront, instead it travels in the form of discrete packets or bundles of energy called **quanta**. One quantum of light radiation is called a **photon**, which travels with the speed of light. Photoelectric effect thus gave evidence that light consists of packets of energy.

Energy of a photon is given by

$$E = h\nu = \frac{hc}{\lambda}$$

where, h ($\approx 6.63 \times 10^{-34}$ Js) is the Planck's constant, ν is the frequency of radiation or photon, c is the speed of light and λ is the wavelength of photon. The energy E is expressed in electron-volt (eV).

The momentum of photon is given by

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \text{ kg}\cdot\text{ms}^{-1}$$

Characteristic properties of photons

Different characteristic properties of photons are given below

- In interaction of radiation with matter, radiation behaves as if it is made up of particles called **photons**.
- A photon in vacuum travels with a speed of light c (i.e. $3 \times 10^8 \text{ ms}^{-1}$) in straight line.
- According to the theory of relativity, the mass m of a particle moving with velocity v , comparable with the velocity of light c is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow m_0 = m \sqrt{1 - \frac{v^2}{c^2}}$$

where, m_0 is the mass of the particle at rest.

As, a photon moves with the speed of light, i.e. $v = c$, hence $m_0 = 0$. So, rest mass of photon is zero, i.e. the photon cannot exist at rest.

(iv) The inertial mass of a photon is given by

$$m = \frac{E}{c^2} = \frac{h\nu}{c\lambda} = \frac{h\nu}{c^2}$$

(v) Irrespective of the intensity of radiation, all the photons of a particular frequency ν or wavelength λ have the same energy $E \left(= h\nu = \frac{hc}{\lambda} \right)$ and momentum

$$p \left(= \frac{h\nu}{c} = \frac{h}{\lambda} \right).$$

(vi) Energy of a photon depends upon frequency of the photon, so the energy of the photon does not change when photon travels from one medium to another.

(vii) As, velocity of a photon is different in different media, so wavelength of the photon also changes in different media.

(viii) Photons are not deflected by electric and magnetic fields. This shows that photons are electrically neutral.

(ix) Total energy and total momentum are conserved in photon-particle collision. However, the number of photons may not be conserved in collision. As, the photon may be absorbed or a new photon may be created during a collision.

Note Single electron absorbs energy of a single photon.

Example 11.15 What is the energy (in joule) associated with a photon of wavelength $4000 \text{ \AA}$?

Sol. Energy of a photon, $E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^{-7}} = 4.95 \times 10^{-19} \text{ J}$

Example 11.16 The momentum of photon of electromagnetic radiation is $3.3 \times 10^{-29} \text{ kg-ms}^{-1}$. Find out the frequency and wavelength of the wave associated with it.

Sol. Given, $h = 6.63 \times 10^{-34} \text{ Js}$, $c = 3 \times 10^8 \text{ ms}^{-1}$

$$p = 3.3 \times 10^{-29} \text{ kg-ms}^{-1}$$

$$(i) \text{ Momentum, } p = \frac{h\nu}{c} \Rightarrow \nu = \frac{pc}{h} = \frac{3.3 \times 10^{-29} \times 3 \times 10^8}{6.63 \times 10^{-34}} = 1.5 \times 10^{13} \text{ Hz}$$

$$(ii) \quad \lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{1.5 \times 10^{13}} = 2 \times 10^{-5} \text{ m}$$

Example 11.17 The wavelength of a photon is $1.4 \text{ \AA}$. It collides with an electron. The energy of the scattered electron is $4.26 \times 10^{-16} \text{ J}$. Find the wavelength of photon after collision. (Take, $h = 6.63 \times 10^{-34} \text{ Js}$)

Sol. Let initial wavelength of photon is λ_1 and final wavelength of photon is λ_2 .

Since, total energy and momentum are conserved in photon-electron collision.

$$\Rightarrow 4.26 \times 10^{-16} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$\frac{hc}{\lambda_2} = \frac{hc}{\lambda_1} - 4.26 \times 10^{-16}$$

$$\begin{aligned} \Rightarrow \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda_2} &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.4 \times 10^{-10}} - 4.26 \times 10^{-16} \\ &= 14.20 \times 10^{-16} - 4.26 \times 10^{-16} \\ \lambda_2 &\approx 2 \times 10^{-10} \text{ m} \\ &= 2.0 \text{ \AA} \end{aligned}$$

Intensity of photons

A moving photon carries momentum with it. Therefore, pressure and intensity are also associated with photons.

Consider an area A through which light (photons) emitted by a source of total energy E and intensity I is passing normally as shown in the figure.

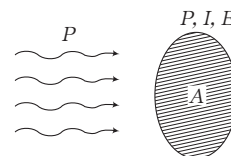


Fig. 11.10 Photon of energy E passing normally through an area A

If power of the source is P , then we can write

$$\text{Total energy of photons, } E = N h \nu = N \left(\frac{hc}{\lambda} \right)$$

$$\therefore \text{ Power of source, } P = \frac{E}{t} = \frac{N h \nu}{t} = n h \nu = \frac{n h c}{\lambda} \left(\because n = \frac{N}{t} \right)$$

Also, we can write

$$n = P/E = \frac{P}{h\nu} = \frac{P\lambda}{hc}$$

As, intensity or energy flux is defined as the energy crossing per unit area normally per second.

$$\Rightarrow \text{ Intensity, } I = \frac{P}{A} = \frac{E}{tA} = \frac{n h \nu}{A} = n \frac{h c}{A \lambda}$$

where, A = area normal to incident light,

N = number of photons

and n = number of photons/s.

Note At a distance r from a point source of power P , intensity is given

$$by I = \frac{P}{4\pi r^2} \Rightarrow I \propto \frac{1}{r^2}.$$

$$For a line source, I \propto \frac{P}{2\pi r l}$$

$$\Rightarrow I \propto \frac{1}{r}$$

Example 11.18 The eye can detect 5×10^4 photons per square metre per second of green light ($\lambda = 5000 \text{ \AA}$) while the ear can detect $10^{-13} \text{ (W/m}^2\text{)}$. Find the factor by which the eye is more sensitive as a power detector than the ear.

Sol. As, $E = \frac{hc}{\lambda(\text{\AA})}$

$$\Rightarrow E = \frac{12375}{5000} = 2.475 \text{ eV} = 4 \times 10^{-19} \text{ J}$$

$$I_{\text{Eye}} = (\text{Photon flux}) \times (\text{Energy of photon}) \\ = (5 \times 10^4) \times (4 \times 10^{-19}) = 2 \times 10^{-14} \text{ W/m}^2$$

$$\therefore \frac{S_{\text{Eye}}}{S_{\text{Ear}}} = \frac{I_{\text{Ear}}}{I_{\text{Eye}}} = \frac{10^{-13}}{2 \times 10^{-14}} = 5$$

As, lesser the intensity, more sensitive is the detector

$\therefore$ Eye is 5 times more sensitive than the ear.

Example 11.19 The intensity of sunlight reaching the surface of the earth is $1.388 \times 10^3 \text{ W/m}^2$. How many photons (nearly) per square metre are incident on the earth per second? Assume that, the photons in the sunlight have an average wavelength of 550 nm .

Sol. Let, n = number of photons incident on earth per sec

$$\text{per m}^2 \text{ and } E' = \text{energy of each photon} = \frac{hc}{\lambda}.$$

If E be the energy of all photons reaching the surface of earth per sec per unit area, then

$$E = nE' = \frac{nhc}{\lambda}$$

Intensity = Energy of photons reaching the earth per second per unit area

$$\text{Given, } I = 1.388 \times 10^3 \text{ W/m}^2 \\ = 1.388 \times 10^3 \text{ Js}^{-1} \text{ m}^{-2}$$

$$\therefore \frac{nhc}{\lambda} = I$$

$$\text{or } n = \frac{I}{\left(\frac{hc}{\lambda}\right)} = \frac{1.388 \times 10^3}{hc} \times \lambda \\ = \frac{1.388 \times 10^3 \times 550 \times 10^{-9}}{6.62 \times 10^{-34} \times 3 \times 10^8} \\ = 3.84 \times 10^{21}$$

Example 11.20 A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm . (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

Sol. (a) Using the formula, $E = \frac{hc}{\lambda}$, we get

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{589 \times 10^{-9}} \\ = 3.37 \times 10^{-19} \text{ J} \\ = \frac{3.37 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.1 \text{ eV}$$

(b) Let n = rate at which the photons are delivered to the sphere.

$$n = \frac{\text{Energy radiated per second}}{\text{Energy of each photon}} = \frac{P}{E} \\ = \frac{100}{3.37 \times 10^{-19}} = 2.96 \times 10^{20} \text{ photons/s}$$

Example 11.21 (a) Find the momentum of a photon of light of wavelength 660 nm .

(b) Calculate the number of photons emitted per second by a 66 W source of monochromatic light of wavelength 660 nm .

(c) How many photons per second does a one watt bulb emit, if its efficiency is 20% and the wavelength of light emitted is 400 nm ?

(d) Monochromatic light of wavelength 200 nm is incident normally on a surface of area 2 cm^2 . If the intensity of light is 100 W/m^2 , calculate the rate at which photons strike the surface.

Sol. (a) Momentum of a photon, $p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{660 \times 10^{-9}} \\ = 1.00 \times 10^{-27} \text{ kg-m/s}$

(b) Power of the source, $P = \frac{nhc}{\lambda}$

$$\therefore n = \frac{P\lambda}{hc} = \frac{66 \times 660 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 2.2 \times 10^{20}$$

(c) Efficiency = $\frac{\text{Output power}}{\text{Input power}}$

$$\Rightarrow \frac{20}{100} = \frac{P}{1}$$

$$\Rightarrow P = \frac{1}{5} \text{ W}$$

$\therefore$ Number of photons emitted,

$$n = \frac{P\lambda}{hc} = \frac{1}{5} \times \frac{400 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} \\ = 0.04 \times 10^{19} = 4 \times 10^{17}$$

(d) Rate at which photons strike the surface

$$n = \frac{P\lambda}{hc} = \frac{IA\lambda}{hc} = \frac{100 \times 2 \times 10^{-4} \times 200 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 2 \times 10^{16} \text{ per second}$$

Example 11.22 A small plate of metal (work function = 1.17 eV) is placed at a distance of 2 m from a monochromatic light source of wavelength 4.8×10^{-7} m and power 1.0 W. The light falls normally on the plate. Find the number of photons striking the metal plate per square meter per second. If a constant uniform magnetic field of strength 10^{-4} T is applied parallel to the metal surface, find the radius of the largest circular path followed by the emitted photoelectrons.

Sol. Intensity of light at distance r from point source,

$$I = \frac{P}{4\pi r^2} = n \frac{hc}{\lambda}$$

where, n is the number of photons striking the metal plate per square meter per second.

$$n = \frac{I\lambda}{hc} = \frac{P\lambda}{4\pi r^2 hc} = \frac{1 \times 4.8 \times 10^{-7}}{4\pi(2)^2 \times 6.6 \times 10^{-34} \times 3 \times 10^8} = 4.8 \times 10^{16}$$

$$E = \frac{1242}{\lambda(\text{nm})} \text{ eV} = \frac{1242}{480} = 2.59 \text{ eV}$$

According to Einstein's photoelectric equation,

$$E = \phi + K_{\max} \Rightarrow 2.59 = 1.17 + K_{\max} \Rightarrow K_{\max} = 1.42 \text{ eV}$$

When uniform magnetic field is applied, then

$$\frac{mv^2}{R} = qvB \Rightarrow \frac{mv}{R} = qB \Rightarrow \frac{m}{R} \left(\frac{2K_{\max}}{m} \right)^{1/2} = qB$$

$$\text{or } R = \frac{\sqrt{2mK_{\max}}}{eB} \quad \left(\because K_{\max} = \frac{1}{2}mv^2 \right)$$

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.42 \times 1.6 \times 10^{-19}}}{1.6 \times 10^{-19} \times 10^{-4}} = 4 \times 10^{-2} \text{ m}$$

Force and pressure exerted by incident light

When light (photons) is incident on a surface, then it exerts a force (or pressure) on the surface. Let light of intensity I is incident normally on a surface of area A as shown in the figure.

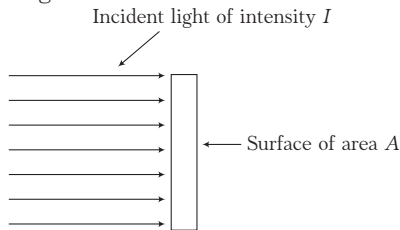


Fig. 11.11 Light is incident normally on a surface

We will now calculate the expression for force and pressure being exerted on perfectly reflecting and perfectly absorbing surfaces.

Force and pressure exerted on perfectly reflecting surface

Suppose the light with N photons are incident on a perfectly reflecting surface in time t as shown in the figure

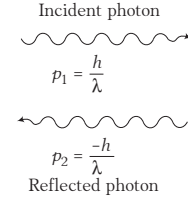


Fig. 11.12 Incident photon on perfectly reflecting surface

Momentum of photons before striking the surface, $p_1 = \frac{Nh}{\lambda}$

Momentum of photons after striking the surface, $p_2 = \frac{-Nh}{\lambda}$

Change in momentum of photons,

$$= p_2 - p_1 = -\frac{Nh}{\lambda} - \frac{Nh}{\lambda} = -\frac{2Nh}{\lambda}$$

But change in momentum of surface, $\Delta p = \frac{2Nh}{\lambda}$

Therefore, force on the surface

$$F = \frac{\Delta p}{\Delta t} = \frac{2Nh}{\lambda t} = n \left(\frac{2h}{\lambda} \right) \quad \left(\because n = \frac{N}{t} \right)$$

As, power $P = n \frac{hc}{\lambda}$

$$\therefore n = \frac{P\lambda}{hc} \Rightarrow F = \left(\frac{2h}{\lambda} \right) \times \left(\frac{P\lambda}{hc} \right) = \frac{2P}{c}$$

$$\Rightarrow \text{Pressure on the surface} = \frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c} \quad \left(\because I = \frac{P}{A} \right)$$

where, c is speed of light.

Special case

When a beam of light is incident at an angle ϕ on perfectly reflecting surface as shown in the figure, then

$$F = \frac{2P}{c} \cos \phi = n \left(\frac{2h}{\lambda} \right) \cos \phi$$

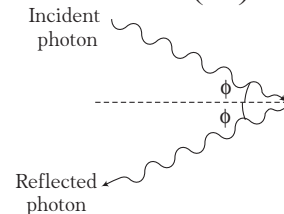


Fig. 11.13 Incident photon making angle ϕ on perfectly reflecting surface

$$\therefore \text{Pressure} = \frac{F}{A} = \frac{2P}{cA} \cos \phi = \frac{2I \cos \phi}{c}$$

Force and pressure on perfectly absorbing surface

When N photons are incident on a perfectly absorbing surface, in time t as shown in the figure below

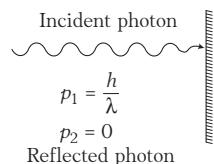


Fig. 11.14 Incident photon on perfectly absorbing surface

Force exerted on the surface,

$$F = \frac{\Delta p}{\Delta t} = \frac{\frac{Nh}{\lambda} - 0}{t} = \frac{nh}{\lambda} \quad \left(\because n = \frac{N}{t} \right)$$

Power, $P = \frac{nhc}{\lambda} \Rightarrow n = \frac{P\lambda}{hc}$

$$\therefore F = \left(\frac{P\lambda}{hc} \right) \left(\frac{h}{\lambda} \right) = \frac{P}{c}$$

$$\Rightarrow \text{Pressure on the surface} = \frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$$

Example 11.23 (a) A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly reflecting surface. The power through any cross section of the beam is 5 W. Find

(i) the number of photons reflected per second by the surface and

(ii) the force exerted by the light beam on the surface.

(b) A parallel beam of monochromatic light of wavelength 500 nm is incident on a totally reflecting plane mirror. The angle of incidence is 30° with the plane of the mirror and the number of photons striking the mirror per second is 2.0×10^{19} . Compute the force exerted by the light beam on the mirror.

(c) Radiation of wavelength 200 nm, propagating in the form of a parallel beam, fall normally on a plane metallic surface. The intensity of the beam is 10 W/m^2 and its cross-sectional area is 2.0 mm^2 . Find the pressure exerted by the radiation on the metallic surface, if the radiation is completely absorbed.

(d) A beam of white light is incident normally on a plane surface absorbing 60% of the light and reflecting the rest. If the incident beam carries 5 W of power, calculate the force exerted by it on the surface.

Sol. (a) (i) Power, $P = n \frac{hc}{\lambda}$

$\therefore$ Number of photons,

$$n = \frac{P\lambda}{hc} = \frac{5 \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 1.26 \times 10^{19}$$

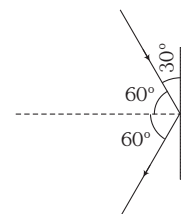
(ii) Force on the surface

$$F = \frac{2P}{c} = \frac{10}{3 \times 10^8} = 3.3 \times 10^{-8} \text{ N}$$

(b) Consider the situation shown in the figure given below.

Given that, angle of the incident ray with the normal,

$$\phi = 90^\circ - 30^\circ = 60^\circ$$



Force exerted by the light beam on the mirror,

$$\begin{aligned} F &= \frac{2P \cos \phi}{c} = \frac{2P \cos 60^\circ}{c} \\ &= \frac{2 \times n \frac{hc}{\lambda} \cos 60^\circ}{c} \\ &= \frac{2 \times 10^{19} \times 6.6 \times 10^{-34}}{500 \times 10^{-9}} \\ &= 2.64 \times 10^{-8} \text{ N} \end{aligned}$$

(c) Pressure exerted by the radiation,

$$p = \frac{I}{c} = \frac{10}{3 \times 10^8} = 3.3 \times 10^{-8} \text{ N/m}^2$$

(d) As we know that, $F = 2P/c$, if light is completely reflected and $F = P/c$, if light is completely absorbed.

Here, in this problem, 60% light is absorbed and 40% is reflected.

$$\begin{aligned} F &= \frac{60}{100} \times \frac{P}{c} + \frac{40}{100} \times \frac{2P}{c} = 1.4 \frac{P}{c} \\ &= \frac{1.4 \times 5}{3 \times 10^8} = 2.33 \times 10^{-8} \text{ N} \end{aligned}$$

PHOTOCELL

It is a device which converts light energy into electrical energy. It is also called as **electric eye**.

It works on the principle of photoelectric emission.

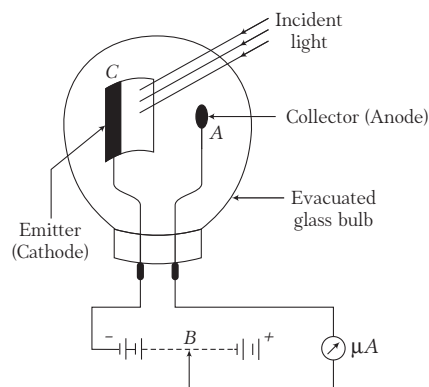


Fig. 11.15 Photocell

Construction

A photocell consists of a semi-cylindrical photosensitive metal plate *C* (emitter) and a wire loop *A* (collector) supported in an evacuated glass or quartz bulb as shown in Fig. 11.15. When light of suitable wavelength falls on the emitter *C*, photoelectrons are emitted. These photoelectrons are drawn to collector by electric field between cathode and anode constituting photocurrent.

Applications of photocell

Some applications of photocell are as below

- Used in television camera for telecasting scenes and in photo telegraphy.
- Used in counting devices.
- Used in burglar alarm and fire alarm.
- To measure the temperature of stars.
- Automatic switching of streetlights.
- Used as photoelectric sensors.
- Used for the determination of Planck's constant.

COMPTON EFFECT

The scattering of a photon by an electron is called Compton effect. The scattered photon will have less energy (more wavelength) than that of incident photon (less wavelength). The energy lost by the photon is taken by electron as kinetic energy. Hence, energy and momentum remains conserved. The schematic representation of Compton scattering is shown in the figure

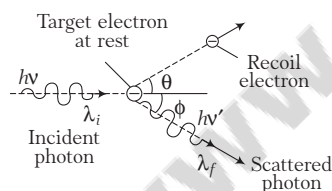


Fig. 11.16 Compton scattering

The change in wavelength due to Compton effect is called **Compton shift**.

$$\therefore \text{Compton shift, } \lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c} (1 - \cos\phi)$$

Here, m_0 is the rest mass of the electron.

(a) If $\phi = 0^\circ$, $\Delta\lambda = 0$

(b) If $\phi = 90^\circ$, $\Delta\lambda = \frac{h}{m_0 c} = 0.24 \text{ nm}$

(c) If $\phi = 180^\circ$, $\Delta\lambda = \frac{2h}{m_0 c} = 0.48 \text{ nm}$

(called **Compton wavelength**)

Compton shift depends only on the scattering angle. It does not depend on the incident wavelength or on the scattering material.

Note In Compton effect, only a part of energy of incident photon is absorbed by the electron but in photoelectric effect, the energy of incident photons is completely absorbed by the electron.

Example 11.24 X-rays of wavelength 10 pm are scattered from a target. (a) Find the wavelength of the X-rays scattered through 45° . (b) Find the maximum wavelength present in the scattered X-rays. (c) Find the maximum kinetic energy of the recoil electrons.

Sol. When X-rays are scattered from a target, the phenomenon is known as Compton scattering.

(a) The change in wavelength is given by

$$\Delta\lambda = \lambda' - \lambda = \lambda_c (1 - \cos\phi)$$

where, $\lambda_c = \text{Compton wavelength} = \frac{h}{m_0 c}$

For electron, $\lambda_c = 2.426 \times 10^{-12} \text{ m} = 2.426 \text{ pm}$

$\lambda' = \text{Wavelength of the X-rays scattered through angle } 45^\circ$

$$\therefore \lambda' = \lambda + \lambda_c (1 - \cos 45^\circ)$$

$$= 10.0 \text{ pm} + 0.293 \lambda_c = 10.7 \text{ pm}$$

(b) $\lambda' - \lambda = \Delta\lambda$ is maximum, when $(1 - \cos\phi) = 2$

$$\Rightarrow \lambda' = \lambda + 2\lambda_c = 10.0 \text{ pm} + 4.9 \text{ pm} = 14.9 \text{ pm}$$

(c) The maximum recoil kinetic energy is equal to the difference between the energy of the incident and scattered photon,

$$\text{so } KE_{\max} = h(\nu - \nu') = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda'} \right)$$

where, $\lambda' = \text{maximum wavelength present in the scattered X-rays.}$

$$\begin{aligned} \therefore KE_{\max} &= \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{10^{-12} \text{ m/pm}} \left(\frac{1}{10.0 \text{ pm}} - \frac{1}{14.9 \text{ pm}} \right) \\ &= 6.54 \times 10^{-15} \text{ J} \\ &= 40.8 \text{ keV} \end{aligned}$$

CHECK POINT 11.2

- The rest mass of photon is
 - zero
 - ∞
 - between 0 and ∞
 - equal to that of an electron
- The momentum of the photon of wavelength 5000 Å will be
 - $1.3 \times 10^{-27} \text{ kg-ms}^{-1}$
 - $1.3 \times 10^{-28} \text{ kg-ms}^{-1}$
 - $4 \times 10^{-29} \text{ kg-ms}^{-1}$
 - $4 \times 10^{-18} \text{ kg-ms}^{-1}$
- The momentum of a photon is $2 \times 10^{-16} \text{ g-cms}^{-1}$. Its energy is
 - $0.61 \times 10^{-26} \text{ erg}$
 - $2.0 \times 10^{-26} \text{ erg}$
 - $6 \times 10^{-8} \text{ erg}$
 - $6 \times 10^{-8} \text{ erg}$
- Amongst the following relation, which of one is correct?
 - $E^2 = p^2 c^2$
 - $E^2 = p^2 c$
 - $E^2 = pc^2$
 - $E^2 = p^2/c^2$
- Two photons having
 - equal wavelengths have equal linear momenta
 - equal energies have equal linear momenta
 - equal frequencies have equal linear momenta
 - All of the above
- The number of photons of wavelength 540 nm emitted per second by an electric bulb of power 100 W is (Take, $h = 6 \times 10^{-34} \text{ J-s}$)
 - 100
 - 1000
 - 3×10^{20}
 - 3×10^{18}
- Light of intensity 10^4 W/m^2 is falling on a perfectly reflecting surface. Pressure on the surface is
 - $6.67 \times 10^{-5} \text{ N/m}^2$
 - $4.5 \times 10^{-4} \text{ N/m}^2$
 - $3 \times 10^{-3} \text{ N/m}^2$
 - $4.28 \times 10^{-5} \text{ N/m}^2$
- Light of intensity I_0 , exerts a pressure of $4.5 \times 10^{-5} \text{ N/m}^2$ on a perfectly absorbing surface. Value of I_0 is
 - $1.4 \times 10^2 \text{ W/m}^2$
 - $1.4 \times 10^4 \text{ W/m}^2$
 - $1.4 \times 10^{-4} \text{ W/m}$
 - 1.4 W/m^2

DUAL NATURE OF RADIATION

With the experimental verification, it was proved that light has dual nature, in some phenomena it behaves like wave and in some phenomena it behaves like a particle depending upon the dimensions of object with which it interacts.

In 1924, the French physicist Louis de-Broglie put forward the bold hypothesis that moving particles of matter should also display wave like properties under suitable condition, *i.e.* the particles like electrons, protons, neutrons, etc., can have particle as well as wave nature.

Matter waves : de-Broglie waves

According to de-Broglie, a wave is associated with moving material particle which controls the particle in every respect. The wave associated with moving particle is called matter waves or de-Broglie wave and it propagates in the form of wave packets with group velocity. Wavelength associated with de-Broglie wave is called **de-Broglie wavelength** and is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

where, m , v and p are the mass, velocity and momentum of particle respectively and h is Planck's constant.

According to Planck's quantum theory, the energy of the photon is given by

$$E = h\nu = \frac{hc}{\lambda} \quad \dots(i)$$

According to Einstein's theory, the energy of the photon is given by

$$E = mc^2 \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\lambda = \frac{h}{mc} = \frac{h}{p}$$

where, $p = mc$ is momentum of a photon.

Calculation of de-Broglie wavelength of an accelerating electron

Let an electron of charge e having mass m be accelerated from rest through a potential difference V , then gain in kinetic energy of electron is given by

$$K = \frac{1}{2} m_e v^2$$

Work done on electron = eV

$$\therefore \frac{1}{2} m_e v^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m_e}}$$

Momentum is given by $p = m_e v = \sqrt{2eV m_e}$

The wavelength associated with moving electron is given by

$$\lambda = \frac{h}{\sqrt{2eV m_e}}$$

Substituting the value of h , e and m_e in above equation,

we get
$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \text{ V}}}$$

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

Similarly, de-Broglie wavelength for other charged

particles can be given by $\lambda = \frac{h}{p}$

where, $p = \sqrt{2m(KE)} = \sqrt{2mqV}$

(i) **For proton** $\lambda = \frac{h}{\sqrt{2m_p q}}$

On putting the values of h , m_p and q , we get

$$\lambda = \frac{0.286}{\sqrt{V}} \text{ \AA}$$

(ii) **For deuteron** $\lambda = \frac{0.202}{\sqrt{V}} \text{ \AA}$

(iii) **For α -particle** $\lambda = \frac{0.101}{\sqrt{V}} \text{ \AA}$

de-Broglie wavelength associated with uncharged particles

For neutron, de-Broglie wavelength is given as

$$\lambda_{\text{Neutron}} = \frac{0.286 \times 10^{-10}}{\sqrt{E \text{ (in eV)}}} \text{ m} = \frac{0.286}{\sqrt{E \text{ (in eV)}}} \text{ \AA}$$

Energy of thermal neutrons or gas molecules at ordinary temperature is given by

$$E = \frac{3}{2} kT \Rightarrow \lambda = \frac{h}{\sqrt{3mkT}}$$

where, T = absolute temperature

and k = Boltzmann's constant

$$= 1.38 \times 10^{-23} \text{ J/K.}$$

Also,
$$\lambda_{\text{Thermal neutrons}} = \frac{2517}{\sqrt{T}} \text{ \AA}$$

Characteristics of matter waves

Characteristics of matter waves are given below

- Matter waves are not electromagnetic waves in nature.
- These waves are non-mechanical waves, *i.e.* they can travel in vacuum.
- Matter waves are independent of charge, *i.e.* they are associated with every moving particle (whether charged or uncharged).

(iv) Observation of matter waves is possible only when the de-Broglie wavelength is of the order of size of the particle (*i.e.* the waves are diffracted).

(v) The phase velocity of the matter waves can be greater than the speed of the light.

(vi) The number of de-Broglie waves associated with n th orbital electron is n .

Example 11.25 A particle of mass 1 mg has the same wavelength as an electron moving with a velocity of $3 \times 10^6 \text{ ms}^{-1}$. What is the velocity of the particle?

Sol. de-Broglie wavelength,

$$\lambda = \frac{h}{mv}$$

As both particle and electron are having same wavelength, therefore their momentum will be equal.

$$\begin{aligned} m_p v_p &= m_e v_e \\ \Rightarrow v_p &= \frac{m_e v_e}{m_p} = \frac{9.1 \times 10^{-31} \times 3 \times 10^6}{10^{-6}} \\ \Rightarrow v_p &= 2.7 \times 10^{-18} \text{ ms}^{-1} \end{aligned}$$

Example 11.26 When the momentum of a proton is changed by an amount p_0 , the corresponding change in the de-Broglie wavelength is found to be 0.25%. Find the original momentum of the photon.

Sol. As $\lambda \propto \frac{1}{p} \Rightarrow \frac{\Delta p}{p} = -\frac{\Delta \lambda}{\lambda}$

$$\Rightarrow \frac{p_0}{p} = \frac{0.25}{100} \Rightarrow p = 400p_0$$

Example 11.27 The kinetic energy of an electron is 5 eV . Calculate the de-Broglie wavelength associated with it (Take, $h = 6.6 \times 10^{-34} \text{ Js}$ and $m_e = 9.1 \times 10^{-31} \text{ kg}$)

Sol. de-Broglie wavelength associated with the electron,

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2m(KE)}} \\ &= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 5 \times 1.6 \times 10^{-19}}} \\ &= 5.469 \times 10^{-10} \text{ m} = 5.47 \text{ \AA} \end{aligned}$$

Example 11.28 Find out the energy that should be added to an electron in terms of initial energy to reduce its de-Broglie wavelengths from 10^{-10} m to $0.5 \times 10^{-10} \text{ m}$.

Sol. de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{\sqrt{E}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}} \Rightarrow \frac{10^{-10}}{0.5 \times 10^{-10}} = \sqrt{\frac{E_2}{E_1}}$$

$$\Rightarrow E_2 = 4E_1$$

Hence, added energy $= E_2 - E_1 = 3E_1$

Example 11.29 What is the

- (a) momentum,
 (b) speed and
 (c) de-Broglie wavelength of an electron with kinetic energy of 120 eV?

Sol. (a) Using the relation, $E = \frac{p^2}{2m}$,

$$\begin{aligned}\text{We get } p &= \sqrt{2mE} \\ &= \sqrt{2 \times 9.1 \times 10^{-31} \times 120 \times 1.6 \times 10^{-19}} \\ &= 5.91 \times 10^{-24} \text{ kg-ms}^{-1}\end{aligned}$$

$$(b) \text{ Speed, } v = \frac{p}{m} = \frac{5.91 \times 10^{-24}}{9.1 \times 10^{-31}} = 6.5 \times 10^6 \text{ ms}^{-1}$$

(c) de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{6.62 \times 10^{-34}}{5.91 \times 10^{-24}} = 1.12 \times 10^{-10} \text{ m}$$

Example 11.30 What is the de-Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that, the molecule is moving with the root-mean-square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Sol. Let v = rms speed of N_2 molecule at 300 K

$$\text{or } v = \sqrt{\frac{3kT}{m}}$$

where, k = Boltzmann constant = $1.38 \times 10^{-23} \text{ JK}^{-1}$

$$\begin{aligned}\text{or } v &= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{28.0152 \times 1.67 \times 10^{-27}}} \\ &= 5.15 \times 10^2 \text{ ms}^{-1}\end{aligned}$$

Using the formula, $\lambda = \frac{h}{mv}$, we get

$$\begin{aligned}\lambda &= \frac{6.62 \times 10^{-34}}{28.0152 \times 1.67 \times 10^{-27} \times 5.15 \times 10^2} \\ &= 2.75 \times 10^{-11} \text{ m}\end{aligned}$$

Example 11.31 The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

- (a) an electron and
 (b) a neutron, would have the same de-Broglie wavelength?

Sol. (a) Let E_1 = KE of electron

Using the relation, $\lambda = \frac{h}{\sqrt{2m_e E}}$, we get

$$\begin{aligned}\lambda^2 &= \frac{h^2}{2m_e E} \\ \therefore E_1 &= \frac{h^2}{2m_e \lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (589 \times 10^{-9})^2} \\ &= 6.94 \times 10^{-25} \text{ J} \\ &= \frac{6.94 \times 10^{-25}}{1.6 \times 10^{-19}} \text{ eV} = 4.34 \times 10^{-6} \text{ eV}\end{aligned}$$

(b) E_2 = KE of neutron

$$\begin{aligned}\therefore E_2 &= \frac{h^2}{2m_n \lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (589 \times 10^{-9})^2} \\ &= 3.782 \times 10^{-28} \text{ J} \\ &= \frac{3.782 \times 10^{-28}}{1.6 \times 10^{-19}} \text{ eV} = 2.36 \times 10^{-9} \text{ eV}\end{aligned}$$

Example 11.32

- (a) For which value of kinetic energy of a neutron will the associated de-Broglie wavelength be $1.40 \times 10^{-10} \text{ m}$?
 (b) Also find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $\frac{3}{2} kT$ at 300 K.

Sol. (a) Using the formula $\lambda = \frac{h}{\sqrt{2mE}}$, we get

$$\begin{aligned}\lambda^2 &= \frac{h^2}{2mE} \\ \text{or } E &= \frac{h^2}{2m\lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (1.40 \times 10^{-10})^2} \\ &= 6.69 \times 10^{-21} \text{ J} \\ &= \frac{6.69 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} \\ &= 4.184 \times 10^{-2} \text{ eV}\end{aligned}$$

(b) Here, T = absolute temperature = 300 K

k = Boltzmann constant

$$= 1.38 \times 10^{-23} \text{ JK}^{-1}$$

E = kinetic energy of neutron

$$\begin{aligned}&= \frac{3}{2} kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 \\ &= 6.21 \times 10^{-21} \text{ J}\end{aligned}$$

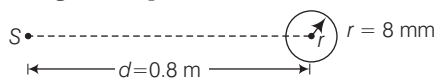
Using the relation, $\lambda = \frac{h}{\sqrt{2mE}}$, we get

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2 \times 1.67 \times 10^{-27} \times 6.21 \times 10^{-21}}} \\ &= 1.455 \times 10^{-10} \text{ m} = 1.455 \text{ \AA}\end{aligned}$$

Example 11.33 In a photoelectric effect set up, a point source of light of power $3.2 \times 10^{-3} \text{ W}$ emits mono-energetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius 8 mm. The efficiency of photoelectron emission is one for every 10^5 incident photons. Assume that, the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission.

- (a) Find the ratio of the wavelength of incident light to the de-Broglie wavelength of the fastest photoelectron emitted.
 (b) It is observed that, the photoelectron emission stops at a certain time t after the light source is switched ON. Why?
 (c) Evaluate the time t .

Sol. According to the question,



(a) Using the relation, $E = \phi + K_{\max} \Rightarrow 5 = 3 + K_{\max}$

$$K_{\max} = 2 \text{ eV}$$

de-Broglie wavelength is given as

$$\lambda_d = \frac{h}{p} = \frac{h}{\sqrt{2mK_{\max}}} \quad \dots(i)$$

and incident wavelength is given as

$$\lambda = \frac{hc}{E} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii),

$$\frac{\lambda}{\lambda_d} = \frac{c\sqrt{2mK_{\max}}}{E}$$

Substituting the values,

$$\begin{aligned} &= \frac{3 \times 10^8 \sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-19}}}{5 \times 1.6 \times 10^{-19}} \\ &= \frac{3 \times 10^8 \times 2}{5} \sqrt{\frac{9.1 \times 10^{-31}}{1.6 \times 10^{-19}}} \\ &= 1.2 \times 10^8 \times \frac{1}{4} \times 10^{-6} \sqrt{91} = 286.18 \end{aligned}$$

- (b) When electrons are ejected from sphere, it becomes positively charged and thus potential increases. When potential of sphere becomes equal to stopping potential, emission of electrons stopped.

- (c) Let charge on sphere be q when its potential equals stopping potential, $V = V_s$

As we know, $\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r} = V_s$

Substituting the values, we get

$$\Rightarrow \frac{9 \times 10^9 q}{8 \times 10^{-3}} = 2$$

$$\Rightarrow q = \frac{16}{9} \times 10^{-12} \text{ C} \quad \dots(iii)$$

Also, charge is given by $q = n_0 et$... (iv)

Comparing Eqs. (iii) and (iv), we get

$$\begin{aligned} \frac{16}{9} \times 10^{-12} &= 10^5 \times 1.6 \times 10^{-19} t \\ \Rightarrow t &= \frac{10}{9} \times 100 = \frac{1000}{9} \approx 111 \text{ s} \end{aligned}$$

DAVISSON AND GERMER EXPERIMENT

The wave nature of the material particles as predicted by de-Broglie, was confirmed by CI Davisson and LH Germer (1927) in United States and independently by GP Thomson (1928) in Scotland.

Experimental arrangement used by Davisson and Germer is as shown in the figure

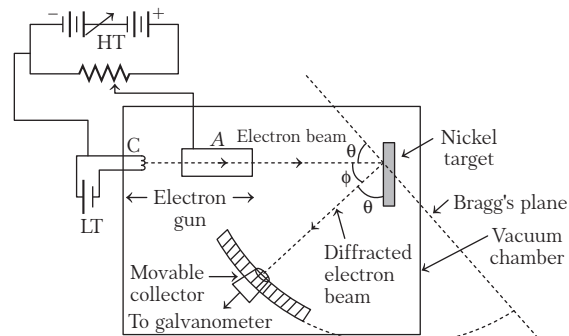


Fig. 11.17 Davisson-Germer electron diffraction arrangement

Electrons from hot tungsten cathode (C) are accelerated by a potential difference V between the cathode and anode (A).

A narrow hole in the anode renders the electrons into a fine beam of electrons and allows it to strike the nickel crystal.

The electrons are scattered in all directions by the atoms in the crystal. The intensity of the electron beam scattered in a given direction is found with the help of a detector.

By rotating the detector, the intensity of the scattered beam can be measured for different values of ϕ , the angle between incident and the scattered direction of electron beam.

Davisson and Germer found that, the intensity of scattered beam of electrons was not the same but different at different angles of scattering.

Intensity of scattered beam of electrons was found to be maximum when angle of scattering is 50° and accelerating potential is 54 V. Variation of intensity of scattered beam at 54 V is as shown below

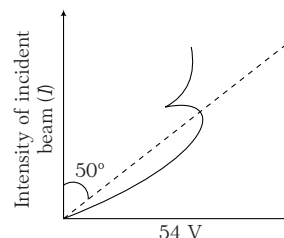


Fig. 11.18

Here, from the experimental set up

$$\theta + \phi + \theta = 180^\circ,$$

i.e.

$$\theta = 65^\circ$$

$$(\because \phi = 50^\circ)$$

For Ni crystal, lattice spacing $d = 0.91 \text{ \AA}$

For first principle maxima, $n = 1$

According to Bragg's equation for diffraction, $2d \sin \theta = n\lambda$ gives

$$\lambda = 1.65 \text{ \AA}$$

According to de-Broglie hypothesis,

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA} = \frac{12.27}{\sqrt{54}} = 0.167 \text{ nm}$$

$\therefore$ de-Broglie wavelength of moving electron at $V = 54 \text{ V}$ is $1.67 \text{ \AA}$ which is in close agreement with experimentally obtained value, i.e. $1.65 \text{ \AA}$.

This proves the existence of de-Broglie waves for slow moving electrons.

Example 11.34 In Davisson and Germer experiment, electron beam of wavelength $1.5 \text{ \AA}$ is incident on a crystal of lattice spacing $3 \text{ \AA}$. Find the incident angle for first maxima.

Sol. It is given that, $d = 3 \text{ \AA}$, $\lambda = 1.5 \text{ \AA}$ and $n = 1$

For the crystal, we can write, $2d \sin \theta = n\lambda$

$$\Rightarrow 2 \times 3 \times \sin \theta = 1 \times 1.5$$

$$\Rightarrow \sin \theta = \frac{1}{4}$$

$$\Rightarrow \theta = 14.4^\circ$$

$\therefore$ Angle of incidence of electron beams, $\theta = 14.4^\circ$

Example 11.35 In Davisson and Germer experiment, second maxima is obtained for an electron beam of wavelength $2.5 \text{ \AA}$ and incident at an angle of 30° . Evaluate the lattice spacing of the crystal.

Sol. It is given that, wavelength of electron beam, $\lambda = 2.5 \text{ \AA}$

Angle of incidence, $\theta = 30^\circ$

$n = 2$ (for second maxima), $d = ?$

As we know that, $2d \sin \theta = n\lambda$

$$\Rightarrow 2d \sin 30^\circ = 2 \times 2.5 \text{ \AA}$$

$$\Rightarrow 2 \times d \times \frac{1}{2} = 2 \times \frac{5}{2}$$

$$\Rightarrow d = 5 \text{ \AA}$$

$\therefore$ Lattice spacing, $d = 5 \text{ \AA}$.

ELECTRON MICROSCOPE

It is an important application of de-Broglie wave and used to study very minute objects like viruses, microbes, etc. Like light radiations, electron beams behave as waves but with much smaller wavelength. Just like any other microscope, its magnifying power is also inversely related to the wavelength of radiation used, i.e. electron beam. Here, the electrons are focused with the help of electric and magnetic lenses. Also, the viewing screen used should be fluorescent, so that image produced could be visible.

An electron microscope can have a very high magnification of $\approx 10^5$.

Example 11.36 An electron microscope uses electrons accelerated by a voltage of 50 kV . Determine the de-Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Sol. Here, $V = 50 \text{ kV} = 5 \times 10^4 \text{ V}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$

KE of an electron,

$$K = 50 \text{ eV} = (1.6 \times 10^{-19} \times 5 \times 10^4) \text{ J} = 8 \times 10^{-15} \text{ J}$$

$\therefore$ de-Broglie wavelength of electrons,

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2mK}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}} \text{ m} \\ &= \frac{6.63 \times 10^{-11}}{12.07} = 5.5 \times 10^{-12} \text{ m} \end{aligned}$$

Wavelength of yellow light, $\lambda_y = 5.9 \times 10^{-7} \text{ m}$

Resolving power of a microscope $\propto 1/\lambda$

$$\begin{aligned} \therefore \frac{\text{Resolving power of electron microscope}}{\text{Resolving power of optical microscope}} &= \frac{\lambda_y}{\lambda} \\ &= \frac{5.9 \times 10^{-7}}{5.5 \times 10^{-12}} \approx 10^5 \end{aligned}$$

Thus, the resolving power of an electron microscope is about 10^5 times greater than that of an optical microscope.

CHECK POINT 11.3

1. The wavelength of the matter wave is independent of which amongst the given associated variable/physical quantity?
(a) Mass (b) Velocity
(c) Momentum (d) Charge
2. If the velocity of an electron is doubled, its de-Broglie frequency will
(a) be halved (b) remain same
(c) be doubled (d) become four times
3. Proton and α -particles have the same de-Broglie wavelength. Which quantity will remain same for both of them?
(a) Charge (b) Energy
(c) Speed (d) Momentum
4. A photon, an electron and a very small unknown particle have the same wavelength. The one with the most energy is
(a) photon
(b) electron
(c) unknown particle
(d) depends upon the wavelength and the properties of the particle
5. If λ_p and λ_e denote the de-Broglie wavelength of proton and electron after they are accelerated from rest through the same potential difference, then
(a) $\lambda_e = \lambda_p$ (b) $\lambda_e < \lambda_p$
(c) $\lambda_e > \lambda_p$ (d) $\lambda_e = \lambda_p/2$
6. When the kinetic energy of an electron is increased, the wavelength of the associated wave will
(a) increase
(b) decrease
(c) not depend on the kinetic energy
(d) None of the above
7. The de-Broglie wavelength associated with an electron, accelerated through a potential difference of 100 V is
(a) 0.12 nm (b) 0.15 nm
(c) 0.20 nm (d) 0.25 nm
8. The de-Broglie wavelength of a neutron at 27°C is λ . What will be its wavelength at 927°C?
(a) $\frac{\lambda}{2}$ (b) $\frac{\lambda}{3}$
(c) $\frac{\lambda}{4}$ (d) $\frac{\lambda}{9}$
9. Which amongst the following property is exhibited by matter waves?
(a) These waves are longitudinal waves.
(b) These waves are electromagnetic waves in nature.
(c) These waves always travel with the speed of light.
(d) They can show diffraction.
10. In Davisson and Germer experiment, intensity of scattered beam of electron was found to be maximum when angle of scattering is
(a) 50° (b) 45°
(c) 30° (d) 90°

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1 The maximum kinetic energy of the photoelectrons varies
 - (a) inversely with the intensity and is independent of the frequency of the incident radiation
 - (b) inversely with the frequency and is independent of the intensity of the incident radiation
 - (c) linearly with the frequency and the intensity of the incident radiation
 - (d) linearly with the frequency and is independent of the intensity of the incident radiation
- 2 Let p and E denote the linear momentum and energy of a photon. If the wavelength is decreased, then
 - (a) Both p and E increase
 - (b) p increases and E decreases
 - (c) p decreases and E increases
 - (d) Both p and E decrease
- 3 If the particles listed below all have the same kinetic energy, which one would possess the shortest de-Broglie wavelength?
 - (a) Deuteron
 - (b) α -particle
 - (c) Proton
 - (d) Electron
- 4 An electron and proton are accelerated through the same potential difference. The ratio of their de-Broglie wavelength will be
 - (a) $(m_p/m_e)^{1/2}$
 - (b) m_e/m_p
 - (c) m_p/m_e
 - (d) 1
- 5 The energy of incident photons corresponding to maximum wavelength of visible light is
 - (a) 3.2 eV
 - (b) 7 eV
 - (c) 1.76 eV
 - (d) 1 eV
- 6 The momentum of a photon is $6.6 \times 10^{-29} \text{ kg-ms}^{-1}$. The frequency of the radiation (in Hz) is
 - (a) 3×10^{13}
 - (b) 3×10^8
 - (c) 3×10^{15}
 - (d) 3×10^6
- 7 The frequency of a photon having energy 100 eV is (Take, $h = 6.62 \times 10^{-34} \text{ Js}$; $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$)
 - (a) $2.417 \times 10^{-16} \text{ Hz}$
 - (b) $2.417 \times 10^{16} \text{ Hz}$
 - (c) $2.417 \times 10^{17} \text{ Hz}$
 - (d) $10.54 \times 10^{17} \text{ Hz}$
- 8 According to Einstein's photoelectric equation, the graph of KE of the photoelectron emitted from the metal *versus* the frequency of the incident radiation gives a straight line graph, whose slope
 - (a) depends on the intensity of the incident radiation
 - (b) depends on the nature of the metal and also on the intensity of incident radiation
 - (c) is same for all metals and independent of the intensity of the incident radiation
 - (d) depends on the nature of the metal
- 9 Sodium and copper have work functions 2.3 eV and 4.5 eV respectively, then the ratio of their threshold wavelengths is nearest to
 - (a) 1 : 2
 - (b) 2 : 1
 - (c) 1 : 4
 - (d) 4 : 1
- 10 Light of frequency $4\nu_0$ is incident on the metal of threshold frequency ν_0 . The maximum kinetic energy of the emitted photoelectron is
 - (a) $3h\nu_0$
 - (b) $2h\nu_0$
 - (c) $\frac{3}{2}h\nu_0$
 - (d) $\frac{1}{2}h\nu_0$
- 11 If the de-Broglie wavelengths for a proton and for α -particle are equal, then their velocities will be in the ratio
 - (a) 4 : 1
 - (b) 2 : 1
 - (c) 1 : 2
 - (d) 1 : 4
- 12 The de-Broglie wavelength of an electron having 80 eV of energy is nearly (Take $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, Mass of electron = $9.1 \times 10^{-31} \text{ kg}$ and Planck's constant = $6.6 \times 10^{-34} \text{ Js}$)
 - (a) $140 \text{ \AA}$
 - (b) $0.14 \text{ \AA}$
 - (c) $14 \text{ \AA}$
 - (d) $1.4 \text{ \AA}$
- 13 If the energy of the photon is increased by a factor of 4, then its momentum
 - (a) does not change
 - (b) decreases by a factor of 4
 - (c) increases by a factor of 4
 - (d) increases by a factor of 2

- 14** The work function for Al, K and Pt is 4.28 eV, 2.30 eV and 5.65 eV respectively. Their respective threshold frequencies would be
 (a) $Pt > Al > K$ (b) $Al > Pt > K$
 (c) $K > Al > Pt$ (d) $Al > K > Pt$
- 15** The de-Broglie wavelength of neutrons in thermal equilibrium is
 (a) $\frac{30.8}{\sqrt{T}} \text{ \AA}$ (b) $\frac{3.08}{\sqrt{T}} \text{ \AA}$ (c) $\frac{0.308}{\sqrt{T}} \text{ \AA}$ (d) $\frac{0.0308}{\sqrt{T}} \text{ \AA}$
- 16** Light of wavelength 3500 Å is incident on two metals A and B. A of work function 4.2 eV and B of work function 1.19 eV. The photoelectron will be emitted by
 (a) metal A
 (b) metal B
 (c) Both A and B
 (d) Neither metal A nor metal B
- 17** The de-Broglie wavelength of an electron in a metal at 27°C is
 (a) $3.5 \times 10^{-9} \text{ m}$ (b) $4.2 \times 10^{-9} \text{ m}$
 (c) $5.6 \times 10^{-9} \text{ m}$ (d) $6.2 \times 10^{-9} \text{ m}$
- 18** The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly [NCERT Exemplar]
 (a) 1.2 nm (b) $1.2 \times 10^{-3} \text{ nm}$
 (c) $1.2 \times 10^{-6} \text{ nm}$ (d) $1.2 \times 10 \text{ nm}$
- 19** An electron of mass m_e and a proton of mass m_p are moving with the same speed. The ratio of their de-Broglie wavelengths λ_e / λ_p is
 (a) 1 (b) 1836
 (c) $\frac{1}{1836}$ (d) 918
- 20** Photon and electron are given energy (10^{-2} J), then wavelengths associated with photon and electron are λ_{ph} and λ_{el} respectively, which are correctly related as
 (a) $\lambda_{ph} > \lambda_{el}$ (b) $\lambda_{ph} < \lambda_{el}$ (c) $\lambda_{ph} = \lambda_{el}$ (d) $\frac{\lambda_{el}}{\lambda_{ph}} = c$
- 21** A radio transmitter radiates 1 kW power at a wavelength 198.6 m. How many photons does it emit per second?
 (a) 10^{10} (b) 10^{20} (c) 10^{30} (d) 10^{40}
- 22** The retarding potential necessary to stop the emission of photoelectron, when a target material of work function 1.24 eV is irradiated with light of wavelength $4.36 \times 10^{-7} \text{ m}$ is
 (a) 0.36 V (b) 1.60 V (c) 2.84 V (d) 4.08 V
- 23** The surface of some material is radiated, by waves of $\lambda_1 = 3.5 \times 10^{-7} \text{ m}$ and $\lambda_2 = 5.4 \times 10^{-7} \text{ m}$, respectively. The ratio of the stopping potential in two cases is 2 : 1. The work function of the material is
 (a) 0.86 eV (b) 1.345 eV
 (c) 1.05 eV (d) 2.20 eV
- 24** The masses of two particles having same kinetic energies are in the ratio 1 : 2, then their de-Broglie wavelengths are in the ratio
 (a) 2 : 1 (b) 1 : 2
 (c) $\sqrt{2} : 1$ (d) $\sqrt{3} : 1$
- 25** Let n_r and n_b be respectively the number of photons emitted by red bulb and blue bulb of equal power in given time, then
 (a) $n_r = n_b$ (b) $n_r < n_b$
 (c) $n_r > n_b$ (d) None of these
- 26** When light of wavelength 300 nm falls on a photoelectric emitter, photoelectrons are liberated. For another emitter, light of wavelength 600 nm is sufficient for liberating photoelectrons. The ratio of the work function of the two emitters is
 (a) 1 : 2 (b) 2 : 1
 (c) 4 : 1 (d) 1 : 4
- 27** X-rays of wavelength 10.0 pm are scattered from a target. The wavelength of the X-rays scattered through 30° is
 (a) 10.32 pm (b) 12 pm
 (c) 10 nm (d) 10 Å
- 28** Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then, [NCERT Exemplar]
 (a) no electrons will be emitted as only photons can emit electrons
 (b) electrons can be emitted but all with an energy, E_0
 (c) electrons can be emitted with any energy, with a maximum of $E_0 - \phi$ (ϕ is the work function)
 (d) electrons can be emitted with any energy, with a maximum of E_0
- 29** A particle of mass M at rest decays into two particles of masses m_1 and m_2 , having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles, λ_1 / λ_2 is
 (a) m_1 / m_2 (b) m_2 / m_1
 (c) 1 : 1 (d) $\sqrt{m_2} / \sqrt{m_1}$
- 30** The de-Broglie wavelength associated with an electron moving with a velocity 0.5 cms^{-1} and rest mass $= 9.1 \times 10^{-31} \text{ kg}$ is
 (a) $4.2 \times 10^{-12} \text{ m}$ (b) $3.6 \times 10^{-12} \text{ m}$
 (c) $5.0 \times 10^{-12} \text{ m}$ (d) $4.2 \times 10^{-10} \text{ m}$

- 31** The work functions of metals A and B are in the ratio $1 : 2$. If light of frequencies f and $2f$ are incident on metal surfaces of A and B respectively, the ratio of the maximum kinetic energies of photoelectron emitted is (f is greater than threshold frequency of A , $2f$ is greater than threshold frequency of B)

(a) $1 : 1$ (b) $1 : 2$ (c) $1 : 3$ (d) $1 : 4$

- 32** A proton accelerated through a potential difference of 100 V , has de-Broglie wavelength λ_0 . The de-Broglie wavelength of an α -particle, accelerated through 800 V is

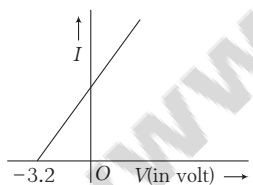
(a) $\frac{\lambda_0}{\sqrt{2}}$ (b) $\frac{\lambda_0}{2}$ (c) $\frac{\lambda_0}{4}$ (d) $\frac{\lambda_0}{8}$

- 33** An electron and a photon possess the same de-Broglie wavelength. If E_e and E_p respectively are the energies of electron and photon and v and c are

their respective velocities, then $\frac{E_e}{E_p}$ is equal to

(a) $\frac{v}{c}$ (b) $\frac{v}{2c}$ (c) $\frac{v}{3c}$ (d) $\frac{v}{4c}$

- 34** In a photoelectric experiment, the relation between applied potential difference between cathode and anode (V) and the photoelectric current (I) was found to be as shown in graph below. If Planck's constant $h = 6.6 \times 10^{-34}\text{ Js}$ and work function $\phi_0 = 4\text{ eV}$, the frequency of incident radiation would be nearly (in s^{-1})



(a) 0.436×10^{18} (b) 1.436×10^{17}
(c) 1.939×10^{14} (d) 0.775×10^{16}

- 35** Maximum KE of a photoelectron is E when the wavelength of incident light is λ . If energy becomes four times when wavelength is reduced to one-third, then work function of the metal is

(a) $\frac{3hc}{\lambda}$ (b) $\frac{hc}{3\lambda}$ (c) $\frac{hc}{\lambda}$ (d) $\frac{hc}{2\lambda}$

- 36** In a photoelectric experiment, the stopping potential V_s is plotted against the frequency ν of incident light. The resulting curve is a straight line which makes an angle θ with the ν -axis. The $\tan \theta$ will be equal to (ϕ = work function of surface)

(a) h/e (b) e/h
(c) $-\phi/e$ (d) eh/ϕ

- 37** The work function of a metal is $1.6 \times 10^{-19}\text{ J}$. When the metal surface is illuminated by the light of wavelength $6400\text{ \AA}$, then the maximum kinetic energy of emitted photoelectrons will be (Planck's constant $h = 6.4 \times 10^{-34}\text{ Js}$)

(a) $14 \times 10^{-19}\text{ J}$ (b) $2.8 \times 10^{-19}\text{ J}$
(c) $1.4 \times 10^{-19}\text{ J}$ (d) $1.4 \times 10^{-19}\text{ eV}$

- 38** The work function for tungsten and sodium are 4.5 eV and 2.3 eV , respectively. If the threshold wavelength λ for sodium is $5600\text{ \AA}$, then the value of λ for tungsten is

(a) $5893\text{ \AA}$ (b) $10683\text{ \AA}$
(c) $2862\text{ \AA}$ (d) $528\text{ \AA}$

- 39** In a photoelectric experiment for $4000\text{ \AA}$ incident radiation, the potential difference to stop the ejection is 2 V . If the incident light is changed to $3000\text{ \AA}$, then the potential required to stop the ejection of electrons will be

(a) 2 V (b) less than 2 V
(c) zero (d) greater than 2 V

- 40** An electron of mass m when accelerated through a potential difference V has de-Broglie wavelength λ . The de-Broglie wavelength associated with a proton of mass M accelerated through the same potential difference will be

(a) $\lambda \frac{m}{M}$ (b) $\lambda \sqrt{\frac{m}{M}}$
(c) $\lambda \frac{M}{m}$ (d) $\lambda \sqrt{\frac{M}{m}}$

- 41** A photocell is illuminated by a point source of light, which is placed at a distance d from the cell. If the distance becomes $d/2$, then number of electrons emitted per second will

(a) remain the same (b) become four times
(c) become two times (d) become one-fourth

- 42** The threshold frequency of the metal of the cathode in a photoelectric cell is $1 \times 10^{15}\text{ Hz}$. When a certain beam of light is incident on the cathode, it is found that a stopping potential 4.144 V is required to reduce the current to zero. The frequency of the incident radiation is

(a) $2.5 \times 10^{15}\text{ Hz}$ (b) $2 \times 10^{15}\text{ Hz}$
(c) $4.144 \times 10^{15}\text{ Hz}$ (d) $3 \times 10^{16}\text{ Hz}$

- 43** If maximum velocity with which an electron can be emitted from a photocell is $4 \times 10^8\text{ cm/s}$, the stopping potential is (Take, mass of electron $= 9 \times 10^{-31}\text{ kg}$)

(a) 30 V (b) 45 V
(c) 59 V (d) 49 V

- 44 An electron is moving with an initial velocity $\mathbf{v} = v_0 \hat{\mathbf{i}}$ and is in a magnetic field $\mathbf{B} = B_0 \hat{\mathbf{j}}$, then its de-Broglie wavelength will be [NCERT Exemplar]

(a) remains constant
(b) increase with time
(c) decrease with time
(d) increase and decrease periodically

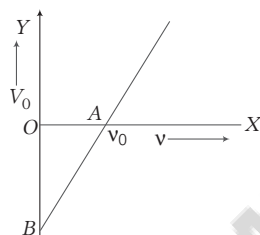
- 45 Threshold wavelength for a metal is $5200 \text{ \AA}$. The photoelectron will be ejected, if it is irradiated by light from

(a) 50 W infrared lamp (b) 1 W infrared lamp
(c) 50 W ultraviolet lamp (d) 0.5 W infrared lamp

- 46 When a monochromatic point source of light is at a distance r from a photoelectric cell, the cut-off voltage is V and the saturation current is I . If the same source is placed at a distance $3r$ away from the photoelectric cell, then

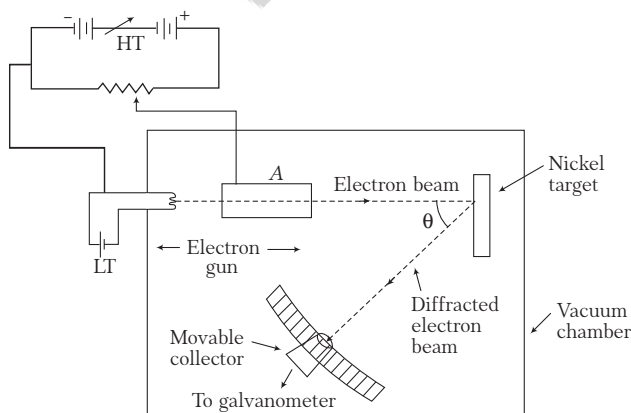
(a) the saturation current will not change
(b) the saturation current will change to $I/9$
(c) the stopping potential will not change
(d) the stopping potential will increase to $3V$

- 47 In an experiment on photoelectric effect, the frequency ν of the incident light is plotted against the stopping potential V_0 . The work function of the photoelectric surface is given by (e is electronic charge)



(a) $OB \times e$ in eV (b) OB in volt
(c) OA in eV (d) The slope of the line AB

- 48 Consider the figure given below. Suppose the voltage applied to A is increased. The diffracted beam will have the maxima at a value of θ that [NCERT Exemplar]



(a) will be larger than the earlier value
(b) will be the same as the earlier value
(c) will be less than the earlier value
(d) will depend on the target

- 49 Two electrons are moving with the same speed v . One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field. Then after sometime, if the de-Broglie wavelengths of the two are λ_1 and λ_2 , then

(a) $\lambda_1 = \lambda_2$ (b) $\lambda_1 > \lambda_2$
(c) $\lambda_1 < \lambda_2$ (d) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

- 50 The maximum wavelength of radiation that can produces photoelectric effect in a certain metal is 200 nm . The maximum kinetic energy acquired by electron due to radiation of wavelength 100 nm will be

(a) 12.4 eV (b) 6.2 eV
(c) 100 eV (d) 200 eV

- 51 A particle is dropped from a height H . The de-Broglie wavelength of the particle as a function of height is proportional to [NCERT Exemplar]

(a) H (b) $H^{1/2}$
(c) H^0 (d) $H^{-1/2}$

- 52 The photoelectric threshold wavelength for silver is λ_0 . The energy of the electron ejected from the surface of silver by an incident wavelength λ ($\lambda < \lambda_0$) will be [NCERT Exemplar]

(a) $hc(\lambda_0 - \lambda)$ (b) $\frac{hc}{\lambda_0 - \lambda}$
(c) $\frac{h}{c} \left(\frac{\lambda_0 - \lambda}{\lambda \lambda_0} \right)$ (d) $hc \left(\frac{\lambda_0 - \lambda}{\lambda \lambda_0} \right)$

- 53 If the momentum of an electron is changed by Δp , then the de-Broglie wavelength associated with it changes by 0.50% . The initial momentum of electron will be

(a) $\frac{\Delta p}{200}$ (b) $\frac{\Delta p}{199}$ (c) $199 \Delta p$ (d) $400 \Delta p$

- 54 When ultraviolet light of wavelength 100 nm is incident upon silver plate, a potential of 7.7 V is required to stop the photoelectrons from reaching the collector plate. How much potential will be required to stop the photoelectrons when light of wavelength 200 nm is incident upon silver?

(a) 1.5 V (b) 3.85 V (c) 2.35 V (d) 15.4 V

- 55 When 1 cm thick surface is illuminated with light of wavelength λ , the stopping potential is V . When the same surface is illuminated by light of wavelength 2λ , the stopping potential is $V/3$. Threshold wavelength for metallic surface is

(a) $4\lambda/3$ (b) 4λ (c) 6λ (d) $8\lambda/3$

- 56** If 5% of the energy supplied to a bulb is irradiated as visible light, how many quanta are emitted per second by a 100 W lamp? (Assume, wavelength of visible light as 5.6×10^{-5} cm)
- (a) 1.4×10^{19} (b) 3×10^3
(c) 1.4×10^{-19} (d) 3×10^4
- 57** Minimum light intensity that can be perceived by normal human eye is about 10^{-10} Wbm $^{-2}$. What is the minimum number of photons of wavelength 660 nm that must enter the pupil in 1 s for one to see the object? (Area of cross-section of the pupil is 10^{-4} m 2)
- (a) 3.3×10^2 (b) 3.3×10^3
(c) 3.3×10^4 (d) 3.3×10^5
- 58** A proton, a neutron, an electron and an α -particle have same energy, then their de-Broglie wavelengths compare as [NCERT Exemplar]
- (a) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (b) $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$
(c) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ (d) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$
- 59** A photon of energy E ejects a photoelectron from a metal surface whose work function is W_0 . If electron having maximum kinetic energy enters into a uniform magnetic field of induction B in a direction perpendicular to the field and describes a circular path of radius r , then the radius r is given by (in the usual notation)
- (a) $\frac{\sqrt{2m(E - W_0)}}{eB}$ (b) $\frac{\sqrt{2m(E - W_0)} eB}{eB}$
(c) $\frac{\sqrt{2m(E - W_0)}}{mB}$ (d) $\frac{2m\sqrt{W_0 - E}}{eB}$
- 60** Radiations of two photon having energies twice and five times the work function of metal are incident successively on the metal surface. The ratio of the maximum velocity of the photoelectrons emitted in the two cases will be
- (a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4
- 61** Work function of nickel is 5.01 eV. When ultraviolet radiation of wavelength 2000 Å is incident on the surface of nickel, electrons are emitted. What will be the maximum velocity of emitted electrons?
- (a) 3×10^8 ms $^{-1}$ (b) 6.46×10^5 ms $^{-1}$
(c) 10.36×10^5 ms $^{-1}$ (d) 8.54×10^6 ms $^{-1}$
- 62** When radiation of wavelength λ is incident on a metallic surface, the stopping potential is 4.8 V. If the same surface is illuminated with radiation of double the wavelength, then the stopping potential becomes 1.6 V. Then the threshold wavelength for the surface is
- (a) 2λ (b) 4λ (c) 6λ (d) 8λ
- 63** We wish to see inside an atom. Assuming the atom to have a diameter of 100 pm [1 picometer (pm) = 10^{-12} m], this means that one must be able to resolve a width of, say 10 pm. If an electron microscope is used, the minimum electron energy required is about
- (a) 1.5 keV (b) 15 keV (c) 150 keV (d) 1.5 MeV
- 64** A 1100 W light bulb is placed at the centre of a spherical chamber of radius 20 cm. Assume that, 60% the energy supplied to the bulb is converted into light and that the surface of the chamber is perfectly absorbing. The force and pressure exerted by the light on the surface of chamber is
- (a) 3.6×10^{-6} N, 4.4×10^{-6} N/m 2
(b) 7.3×10^{-6} N, 8.8×10^{-6} N/m 2
(c) 3.6×10^{-6} N, 8.8×10^{-6} N/m 2
(d) 7.3×10^{-6} N, 4.4×10^{-6} N/m 2
- 65** A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 1.0×10^{19} . The value of the force exerted by it on the surface is
- (a) 10^{-6} N (b) 10^{-7} N (c) 10^{-8} N (d) 10^{-9} N
- 66** In a photoemissive cell with exciting wavelength λ , the fastest electron has speed v . If the exciting wavelength is changed to $\frac{\lambda}{4}$, the speed of the fastest emitted electron will be
- (a) $2v$ (b) $\frac{v}{2}$ (c) $< 2v$ (d) $> 2v$
- 67** Light of wavelength λ , strikes a photoelectric surface and electrons are ejected with an energy E . If E is to be increased to exactly twice its original value, the wavelength changes to λ' , where
- (a) λ' is less than $\lambda/2$
(b) λ' is greater than $\lambda/2$
(c) λ' is greater than $\lambda/2$ but less than λ
(d) λ' is exactly equal to $\lambda/2$
- 68** Light of wavelength 330 nm falling on a piece of metal ejects electrons with sufficient energy which requires voltage V_0 to prevent them from reaching a collector. In the same set up, light of wavelength 220 nm, ejects electrons which require twice the voltage V_0 to stop them in reaching a collector. The numerical value of voltage V_0 is
- (a) $\frac{16}{15}$ V (b) $\frac{15}{16}$ V (c) $\frac{15}{8}$ V (d) $\frac{8}{15}$

- 69** An electron (mass m) with an initial velocity $\mathbf{v} = v_0 \hat{\mathbf{i}}$ ($v_0 > 0$) is in an electric field $\mathbf{E} = -E_0 \hat{\mathbf{i}}$ ($E_0 = \text{constant} > 0$). Its de-Broglie wavelength at time t is given by [NCERT Exemplar]

(a) $\frac{\lambda_0}{\left(1 + \frac{eE_0 t}{m v_0}\right)}$ (b) $\lambda_0 \left(1 + \frac{eE_0 t}{m v_0}\right)$
 (c) λ_0 (d) $\lambda_0 t$

- 70** Light of wavelength $0.6 \mu\text{m}$ from a sodium lamp falls on a photocell and causes the emission of photoelectrons for which the stopping potential is 0.5 V . With light of wavelength $0.4 \mu\text{m}$ from a sodium lamp, stopping potential is 1.5 V . With this data, the value of h/e is

(a) $4 \times 10^{-19} \text{ Vs}$ (b) $0.25 \times 10^{15} \text{ Vs}$
 (c) $4 \times 10^{-15} \text{ Vs}$ (d) $4 \times 10^{-8} \text{ Vs}$

- 71** An electron (mass m) with an initial velocity $\mathbf{v} = v_0 \hat{\mathbf{i}}$ is in an electric field $\mathbf{E} = E_0 \hat{\mathbf{j}}$. If

$\lambda_0 = h / mv_0$, its de-Broglie wavelength at time t is given by [NCERT Exemplar]

(a) λ_0 (b) $\lambda_0 \sqrt{1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}$
 (c) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}}$ (d) $\frac{\lambda_0}{\left(1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}\right)}$

- 72** When photons of energy 4.25 eV strike the surface of a metal A , the ejected photoelectrons have maximum kinetic energy T_A expressed in eV and de-Broglie wavelength λ_A . The maximum kinetic energy of photoelectron liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50) \text{ eV}$. If the de-Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then choose the wrong option.

- (a) The work function of A is 2.25 eV
 (b) The work function of B is 4.20 eV
 (c) $T_A = 2.00 \text{ eV}$
 (d) $T_B = 2.75 \text{ eV}$

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-4) These questions consist of two statements each linked as Assertion and Reason.

While answering these questions you are required to choose any one of the following four responses :

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
 (b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
 (c) If Assertion is true but Reason is false.
 (d) If Assertion is false but Reason is true.

- 1 Assertion** Electromagnetic theory of light failed to explain photoelectric effect.

Reason Photoelectric effect can only be explained by assuming particle nature of light.

- 2 Assertion** Photoelectric emission is an instantaneous process.

Reason There is very small time tag ($\sim 10^{-9} \text{ s}$) between incidence of light and emission of photoelectron.

- 3 Assertion** When wavelength of incident radiation in a photoelectric experiment is decreased, then the stopping potential will decrease.

Reason With decrease in wavelength, energy of incident light increases.

- 4 Assertion** In photoelectric effect, if the intensity of light is doubled, then maximum kinetic energy of photoelectrons will become double.

Reason The maximum kinetic energy of emitted photoelectrons depends on nature of material of the plate.

Statement based questions

- 1** Study the following statements.

- I. Electron can be emitted from a metal surface by heating, under strong electric field and by hammering.
- II. Work function of lithium decreases with the temperature.
- III. Alkali metals like Na, K, etc., show photoelectric effect with ultraviolet light.

Choose the correct statement.

- (a) Both I and II
 (b) Both II and III
 (c) Both I and III
 (d) I, II and III

- 2** According to wave theory, which amongst the following statement(s) is/are incorrect regarding photoelectric effect?

- I. Maximum kinetic energy of the photoelectrons on the surface increases with increase in intensity.
- II. A sufficiently intense beam is able to impart enough energy for photoemission.

III. It takes hours or more for a single electron to come out of the metal.

- (a) Both I and II (b) Only III
(c) Both I and III (d) I, II and III

3 Study the following statements regarding photons.

- I. They are deflected by electric and magnetic fields just like electrons.
II. Wavelength of a photon changes as the media changes.
III. Number of photons are conserved in electron-photon collision.

Choose the incorrect statement.

- (a) Only I (b) Both II and III
(c) Both I and III (d) Only II

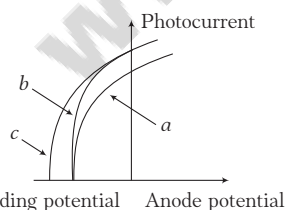
4 Choose the incorrect statement from the following options.

- (a) In Compton effect, the energy of the incident photons is completely absorbed by the electrons, similar to photoelectric effect.
(b) Photocells convert change in intensity of illumination into change in photocurrent.
(c) Inertial mass of a photon is $\frac{h\nu}{c^2}$.
(d) Both (a) and (c)

5 Threshold frequency for a metal is 10^{15} Hz. Light of $\lambda = 4000 \text{ \AA}$ falls on its surface. Which of the following statements is correct?

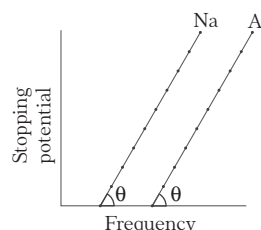
- (a) No photoelectric emission takes place.
(b) Photoelectrons come out with zero speed.
(c) Photoelectrons come out with 10^3 ms^{-1} speed.
(d) Photoelectrons come out with 10^5 ms^{-1} speed.

6 The figure shows a plot of photocurrent *versus* anode potential for a photosensitive surface for three different radiations. Which one of the following is a correct statement?



- (a) Curves *a* and *b* represent incident radiations of different frequencies and different intensities.
(b) Curves *a* and *b* represent incident radiations of same frequency but of different intensities.
(c) Curves *b* and *c* represent incident radiations of different frequencies and different intensities.
(d) Curves *b* and *c* represent incident radiations of same frequency having same intensity.

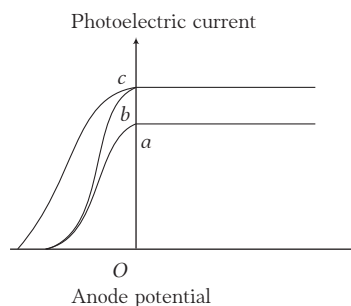
7 On the basis of the figure describing photoelectric effect, which of the following statement is correct?



- (a) Na and Al both have the same threshold frequency.
(b) Maximum kinetic energy for both the metals depend linearly on the frequency.
(c) The stopping potentials are different for Na and Al for the same change in frequency.
(d) Al is better photosensitive material than Na.

Match the columns

1 The adjoining figure shows the variation of photocurrent with anode potential for a photosensitive surface for three different radiations. Let I_a , I_b and I_c be the intensities and f_a , f_b and f_c be the frequencies for the curves *a*, *b* and *c* respectively.



Now, match the Column I with Column II and choose the correct option from the codes given below.

Column I	Column II
(A) $\frac{I_a}{I_b}$	(p) 1
(B) $\frac{I_b}{I_c}$	(q) < 1
(C) $\frac{V_a}{V_c}$	(r) > 1

Codes

A	B	C	A	B	C
(a) r	q	p	(b) q	p	q
(c) p	r	q	(d) p	p	q

- 2 Light rays of intensity $3.3 \times 10^{-3} \text{ W/m}^2$ and wavelength $6000 \text{ \AA}$ falls normally on a photosensitive surface of area 2 cm^2 and work function 2.0 eV .

Assuming that, there is no light loss due to reflection.

Match the Column II with Column I and choose the correct option from the codes given below.

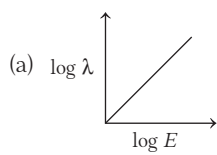
Column I	Column II
A. Energy (in J) of incident photon	(p) 2×10^{12}
B. Work function (in J) of surface	(q) 3.3×10^{-19}
C. Number of photoelectrons emitted per second is	(r) 3.2×10^{-19}

Codes

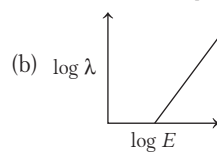
A	B	C	A	B	C
(b) p	r	q	(a) r	p	q
(c) p	q	r	(d) q	r	p

(C) Medical entrances' gallery

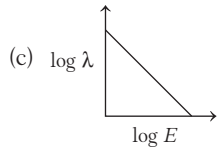
Collection of questions asked in NEET and various medical entrance exams

- 1 Light of frequency 1.5 times the threshold frequency is incident on a photosensitive material. What will be the photoelectric current, if the frequency is halved and intensity is doubled? [NEET 2020]
 (a) Four times (b) One-fourth
 (c) Zero (d) Doubled
- 2 An electron is accelerated from rest through a potential difference of V volt. If the de-Broglie wavelength of the electron is $1.227 \times 10^{-2} \text{ nm}$, the potential difference is [NEET 2020]
 (a) 10^2 V (b) 10^3 V
 (c) 10^4 V (d) 10 V
- 3 Light with an average flux of 20 W/cm^2 falls on a non-reflecting surface at normal incidence having surface area 20 cm^2 . The energy received by the surface during time span of 1 min is [NEET 2020]
 (a) $12 \times 10^3 \text{ J}$ (b) $24 \times 10^3 \text{ J}$
 (c) $48 \times 10^3 \text{ J}$ (d) $10 \times 10^3 \text{ J}$
- 4 The de-Broglie wavelength of an electron moving with kinetic energy of 144 eV is nearly [NEET 2020]
 (a) $102 \times 10^{-3} \text{ nm}$ (b) $102 \times 10^{-4} \text{ nm}$
 (c) $102 \times 10^{-5} \text{ nm}$ (d) $102 \times 10^{-2} \text{ nm}$
- 5 The wave nature of electrons was experimentally verified by [NEET 2020]
 (a) de-Broglie (b) Hertz
 (c) Einstein (d) Davisson and Germer
- 6 The work function of a photosensitive material is 4.0 eV . This longest wavelength of light that can cause photon emission from the substance is (approximately) [NEET (Odisha) 2019]
 (a) 3100 nm (b) 966 nm
 (c) 31 nm (d) 310 nm
- 7 An electron is accelerated through a potential difference of 10000 V . Its de-Broglie wavelength is (nearly) (Take, $m_e = 9 \times 10^{-31} \text{ kg}$) [NEET (National) 2019]
 (a) $12.2 \times 10^{-12} \text{ m}$ (b) $12.2 \times 10^{-14} \text{ m}$
 (c) 12.2 nm (d) $12.2 \times 10^{-13} \text{ m}$
- 8 A proton and an α -particle are accelerated from rest to the same energy. The de-Broglie wavelengths λ_p and λ_α are in the ratio [NEET (Odisha) 2019]
 (a) 2 : 1 (b) 1 : 1 (c) $\sqrt{2} : 1$ (d) 4 : 1
- 9 When the light of frequency $2\nu_0$ (where, ν_0 is threshold frequency), is incident on a metal plate, the maximum velocity of electrons emitted is v_1 . When the frequency of the incident radiation is increased to $5\nu_0$, the maximum velocity of electrons emitted from the same plate is v_2 . The ratio of $v_1 : v_2$ is [NEET 2018]
 (a) 4 : 1 (b) 1 : 4 (c) 1 : 2 (d) 2 : 1
- 10 The graph between the energy $\log E$ of an electron and its de-Broglie wavelength $\log \lambda$ will be [AIIMS 2018]
- 

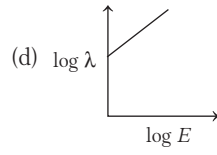
(a) $\log \lambda$ vs $\log E$



(b) $\log \lambda$ vs $\log E$



(c) $\log \lambda$ vs $\log E$



(d) $\log \lambda$ vs $\log E$
- 11 The photoelectric threshold wavelength of silver is $3250 \times 10^{-10} \text{ m}$. The velocity of the electron ejected from a silver surface by ultraviolet light of

wavelength 2536×10^{-10} m is (Take,
 $h = 4.14 \times 10^{-15}$ eV-s and $c = 3 \times 10^8$ ms $^{-1}$)

[NEET 2017]

- (a) $\approx 6 \times 10^5$ ms $^{-1}$ (b) $\approx 0.6 \times 10^8$ ms $^{-1}$
 (c) $\approx 61 \times 10^3$ ms $^{-1}$ (d) $\approx 0.3 \times 10^6$ ms $^{-1}$

- 12** The de-Broglie wavelength of a neutron in thermal equilibrium with heavy water at a temperature T (kelvin) and mass m is [NEET 2017]

- (a) $\frac{h}{\sqrt{mkT}}$ (b) $\frac{h}{\sqrt{3mkT}}$
 (c) $\frac{2h}{\sqrt{3mkT}}$ (d) $\frac{2h}{\sqrt{mkT}}$

- 13** The wavelength λ of a photon and the de-Broglie wavelength of an electron have the same value. Find the ratio of energy of photon to the kinetic energy of electron in terms of mass m , speed of light c and Planck's constant (h). [JIPMER 2017]

- (a) $\lambda mc/h$ (b) hmc/λ
 (c) $2hmc/\lambda$ (d) $2\lambda mc/h$

- 14** When a metallic surface is illuminated with radiation of wavelength λ , the stopping potential is V . If the same surface is illuminated with radiation of wavelength 2λ , the stopping potential is $V/4$. The threshold wavelength for the metallic surface is [NEET 2016]

- (a) 5λ (b) $\frac{5}{2}\lambda$ (c) 3λ (d) 4λ

- 15** An electron of mass m and a photon have same energy E . The ratio of de-Broglie wavelengths associated with them is (c being velocity of light) [NEET 2016]

- (a) $\left(\frac{E}{2m}\right)^{\frac{1}{2}}$ (b) $c(2mE)^{\frac{1}{2}}$
 (c) $\frac{1}{c}\left(\frac{2m}{E}\right)^{\frac{1}{2}}$ (d) $\frac{1}{c}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

- 16** Photons with energy 5 eV are incident on a cathode C in a photoelectric cell. The maximum energy of emitted photoelectrons is 2 eV. When photons of energy 6 eV are incident on C, no photoelectrons will reach the anode A, if the stopping potential of A relative to C is [NEET 2016]

- (a) + 3 V (b) + 4 V (c) - 1 V (d) - 3 V

- 17** Electrons of mass m with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength (λ_0) of the emitted X-ray is [NEET 2016]

- (a) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \frac{2h}{mc}$
 (c) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

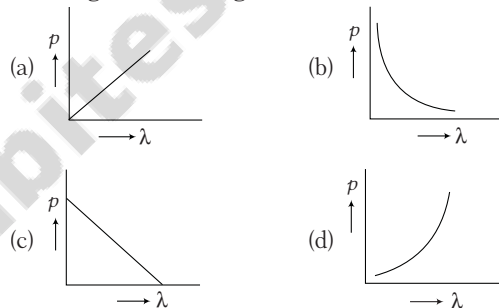
- 18** A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is (c = velocity of light) [CBSE AIPMT 2015]

- (a) $\frac{E}{c}$ (b) $\frac{2E}{c}$ (c) $\frac{2E}{c^2}$ (d) $\frac{E}{c^2}$

- 19** A certain metallic surface is illuminated with monochromatic light of wavelength λ . The stopping potential for photoelectric current for this light is $3V_0$. If the same surface is illuminated with light of wavelength 2λ , the stopping potential is V_0 . The threshold wavelength for this surface for photoelectric effect is [CBSE AIPMT 2015]

- (a) 6λ (b) 4λ (c) $\frac{\lambda}{4}$ (d) $\frac{\lambda}{6}$

- 20** Which of the following figures represent the variation of particle momentum and the associated de-Broglie wavelength? [CBSE AIPMT 2015]



- 21** When a light beam of frequency ν is incident on a metal surface, photoelectrons are emitted. If these electrons describe a circle of radius r in a magnetic field of strength B , then the work function of the metal is [EAMCET 2015]

- (a) $h\nu + \frac{Ber}{2m_e}$ (b) $h\nu - \frac{(Ber)^2}{2m_e}$
 (c) $h\nu + \frac{(Ber)^2}{2m_e}$ (d) $h\nu - \frac{Ber}{2m_e}$

- 22** The work function of metals is in the range of 2 eV to 5 eV. Find which of the following wavelength of light cannot be used for photoelectric effect. (Consider, Planck's constant = 4×10^{-15} eV-s and velocity of light = 3×10^8 ms $^{-1}$) [WB JEE 2015]

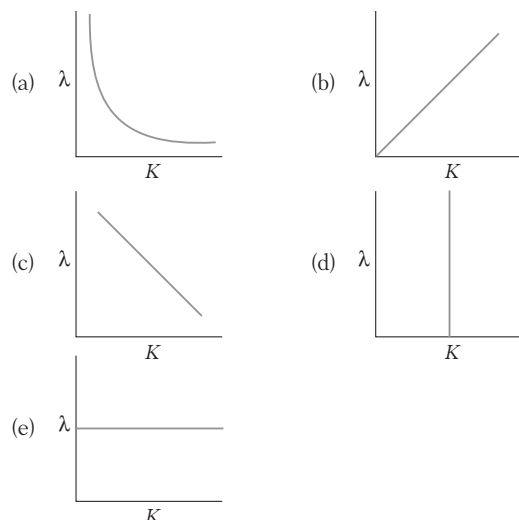
- (a) 510 nm (b) 650 nm
 (c) 400 nm (d) 570 nm

- 23** Consider two particles of different masses. In which of the following situations, the heavier of the two particles will have smaller de-Broglie wavelength? [WB JEE 2015]

- (a) Both have a free fall through the same height
 (b) Both move with the same kinetic energy
 (c) Both move with the same linear momentum
 (d) Both move with the same speed

- 24** The de-Broglie wavelength associated with an electron accelerated by a potential of 64 V is [Manipal 2015]
 (a) 0.153 nm (b) 0.613 nm
 (c) 1.22 nm (d) 0.302 nm
- 25** The electrons are emitted in the photoelectric effect from a metal surface [Manipal 2015]
 (a) only if the temperature of the surface is high
 (b) with a maximum velocity proportional to the frequency of the incident radiation
 (c) only if the frequency of the incident radiation is above a certain threshold value
 (d) at a rate that is independent of the nature of the metal
- 26** If the kinetic energy of the particle is increased to 16 times its previous value, the percentage change in the de-Broglie wavelength of the particle is [CBSE AIPMT 2014]
 (a) 25 (b) 75
 (c) 60 (d) 50
- 27** When the energy of the incident radiation is increased by 20%, the kinetic energy of the photoelectrons emitted from a metal surface increased from 0.5 eV to 0.8 eV. The work function of the metal is [UP CPMT 2014]
 (a) 0.65 eV (b) 1 eV
 (c) 1.3 eV (d) 1.5 eV
- 28** Light of wavelength λ_A and λ_B falls on two identical metal plates A and B, respectively. The maximum kinetic energy of photoelectrons is K_A and K_B respectively, then which one of the following relations is true? ($\lambda_A = 2\lambda_B$) [MHT CET 2014]
 (a) $K_A < \frac{K_B}{2}$ (b) $2K_A = K_B$
 (c) $K_A = 2K_B$ (d) $K_A > 2K_B$
- 29** The maximum kinetic energy of the photoelectrons depends only on [KCET 2014]
 (a) charge (b) frequency
 (c) incident angle (d) pressure
- 30** A light of wavelength 5000 Å falls on a sensitive plate with photoelectric work function 1.90 eV. Kinetic energy of the emitted photoelectrons will be (Take, $h = 6.62 \times 10^{-34}$ Js) [UP CPMT 2014]
 (a) 0.1 eV (b) 2 eV
 (c) 0.58 eV (d) 1.581 eV
- 31** When monochromatic light falls on a photosensitive material, the number of photoelectron emitted per second is n and their maximum kinetic energy is $K_{\max}$. If the intensity of incident light is doubled, then [EAMCET 2014]
 (a) n is doubled but $K_{\max}$ remains same
 (b) $K_{\max}$ is double but n remains same
 (c) Both n and $K_{\max}$ are doubled
 (d) Both n and $K_{\max}$ are halved
- 32** Find the correct statement(s) about photoelectric effect. [WB JEE 2014]
 (a) There is no significant time delay between the absorption of a suitable radiation and the emission of electrons.
 (b) Einstein analysis gives a threshold frequency above which no electron can be emitted.
 (c) The maximum kinetic energy of the emitted photoelectrons is proportional to the frequency of incident radiation.
 (d) The maximum kinetic energy of electrons depends on the intensity of radiation.
- 33** If the wavelength of incident light falling on a photosensitive material decreases, then [Kerala CEE 2014]
 (a) photoelectric current increases
 (b) stopping potential decreases
 (c) stopping potential remains constant
 (d) stopping potential increases
 (e) None of the above
- 34** The de-Broglie wavelength of an electron is the same as that of a 50 keV X-ray photon. The ratio of the energy of the photon to the kinetic energy of the electron is (the energy equivalent of electron mass of 0.5 MeV) [WB JEE 2014]
 (a) 1 : 50 (b) 1 : 20 (c) 20 : 1 (d) 50 : 1
- 35** For photoelectric emission from certain metal, the cut-off frequency is ν . If radiation of frequency 2ν impinges on the metal plate, the maximum possible velocity of the emitted electron will be (m is the electron mass) [NEET 2013]
 (a) $\sqrt{\frac{h\nu}{2m}}$ (b) $\sqrt{\frac{h\nu}{m}}$
 (c) $\sqrt{\frac{2h\nu}{m}}$ (d) $2\sqrt{\frac{h\nu}{m}}$
- 36** The wavelength λ_e of an electron and λ_p of a photon of same energy E are related by [NEET 2013]
 (a) $\lambda_p \propto \lambda_e^2$ (b) $\lambda_p \propto \lambda_e$
 (c) $\lambda_p \propto \sqrt{\lambda_e}$ (d) $\lambda_p \propto \frac{1}{\sqrt{\lambda_e}}$
- 37** Maximum velocity of the photoelectron emitted by a metal is $1.8 \times 10^6 \text{ ms}^{-1}$. Take, the value of specific charge of the electron is $1.8 \times 10^{11} \text{ C kg}^{-1}$, then the stopping potential in volt is [KCET 2013]
 (a) 1 (b) 3
 (c) 9 (d) 6

- 38** Identify the graph depicting the variation of the de-Broglie wavelength λ of an electron with its kinetic energy K . [Kerala CEE 2013]



- 39** Photocells convert [Kerala CEE 2013]

- (a) heat energy into electrical energy
- (b) light energy into mechanical energy
- (c) thermal energy into mechanical energy
- (d) light energy into electrical energy
- (e) electrical energy into light energy

- 40** Maximum kinetic energy of electrons emitted in photoelectric effect increases when [MP PMT 2013]

- (a) intensity of light is increased
- (b) light source is brought nearer the metal
- (c) frequency of light is decreased
- (d) wavelength of light is decreased

- 41** In photoelectric effect, the photoelectric current [UP CPMT 2013]

- (a) depends both on intensity and frequency of incident beam
- (b) does not depend on frequency but depends only on intensity of incident beam
- (c) increases when frequency of incident beam increases
- (d) decreases when frequency of incident beam increases

- 42** A 200 W sodium street lamp emits yellow light of wavelength $0.6 \mu\text{m}$. Assuming it to be 25% efficient in converting electrical energy to light, the number of photons of yellow light it emits per second is [CBSE AIPMT 2012]

- (a) 1.5×10^{20}
- (b) 6×10^{18}
- (c) 62×10^{20}
- (d) 3×10^{19}

- 43** An α -particle moves in a circular path of radius 0.83 cm in the presence of a magnetic field of 0.25 Wb m^{-2} . The de-Broglie wavelength associated with the particle will be [CBSE AIPMT 2012]

- (a) $1 \text{ \AA}$
- (b) $0.1 \text{ \AA}$
- (c) $10 \text{ \AA}$
- (d) $0.01 \text{ \AA}$

- 44** What should be the velocity of an electron, so that its momentum becomes equal to that of a photon of wavelength $5200 \text{ \AA}$? [Manipal 2012]

- (a) 700 ms^{-1}
- (b) 1000 ms^{-1}
- (c) 1400 ms^{-1}
- (d) 2800 ms^{-1}

- 45** The de-Broglie wavelength of an electron moving with a velocity $1.5 \times 10^8 \text{ ms}^{-1}$ is equal to that of a photon. The ratio of the kinetic energy of the electron to the energy of the photon is [AMU 2012]

- (a) $\frac{1}{4}$
- (b) $\frac{1}{2}$
- (c) 2
- (d) 4

- 46** The threshold frequency for a certain photosensitive metal is ν_0 . When it is illuminated by light of frequency $\nu = 2\nu_0$, the maximum velocity of photoelectrons is ν_0 . What will be the maximum velocity of the photoelectrons when the same metal is illuminated by light of frequency $\nu = 5\nu_0$? [AMU 2012]

- (a) $\sqrt{2} \nu_0$
- (b) $2 \nu_0$
- (c) $2\sqrt{2} \nu_0$
- (d) $4 \nu_0$

- 47** The name of ions knocked out from the hot surface is [AFMC 2011]

- (a) nuclei
- (b) neutrons
- (c) electrons
- (d) protons

ANSWERS

CHECK POINT 11.1

1. (c)	2. (a)	3. (d)	4. (b)	5. (b)	6. (a)	7. (c)	8. (c)	9. (b)	10. (d)
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CHECK POINT 11.2

1. (a)	2. (a)	3. (c)	4. (a)	5. (d)	6. (c)	7. (a)	8. (b)
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CHECK POINT 11.3

1. (d)	2. (c)	3. (d)	4. (a)	5. (c)	6. (b)	7. (a)	8. (a)	9. (d)	10. (a)
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(A) Taking it together

1. (d)	2. (a)	3. (b)	4. (a)	5. (c)	6. (a)	7. (b)	8. (c)	9. (b)	10. (a)
11. (a)	12. (d)	13. (c)	14. (a)	15. (a)	16. (b)	17. (d)	18. (b)	19. (b)	20. (a)
21. (c)	22. (b)	23. (c)	24. (c)	25. (c)	26. (b)	27. (a)	28. (d)	29. (c)	30. (a)
31. (b)	32. (d)	33. (b)	34. (c)	35. (b)	36. (a)	37. (c)	38. (c)	39. (d)	40. (b)
41. (b)	42. (b)	43. (b)	44. (a)	45. (c)	46. (b)	47. (a)	48. (c)	49. (d)	50. (b)
51. (d)	52. (d)	53. (c)	54. (a)	55. (b)	56. (a)	57. (c)	58. (b)	59. (a)	60. (b)
61. (b)	62. (b)	63. (b)	64. (a)	65. (c)	66. (d)	67. (c)	68. (c)	69. (a)	70. (c)
71. (c)	72. (d)								

(B) Medical entrance special format questions

• Assertion and reason

1. (a)	2. (a)	3. (d)	4. (d)
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• Statement based questions

1. (b)	2. (d)	3. (c)	4. (a)	5. (a)	6. (b)	7. (b)
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• Match the columns

1. (b)	2. (d)
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(C) Medical entrances' gallery

1. (c)	2. (c)	3. (b)	4. (d)	5. (d)	6. (d)	7. (a)	8. (a)	9. (c)	10. (c)
11. (a)	12. (b)	13. (d)	14. (c)	15. (d)	16. (d)	17. (a)	18. (b)	19. (b)	20. (b)
21. (b)	22. (b)	23. (d)	24. (a)	25. (c)	26. (b)	27. (b)	28. (a)	29. (b)	30. (c)
31. (a)	32. (a)	33. (d)	34. (c)	35. (c)	36. (a)	37. (c)	38. (a)	39. (d)	40. (d)
41. (b)	42. (a)	43. (d)	44. (c)	45. (a)	46. (a)	47. (c)			

Hints & Explanations

• CHECK POINT 11.1

5 (b) $\phi_0 = h\nu = \frac{hc}{\lambda_0}$, where $\phi_0 = 2 \text{ eV}$.

$$\therefore \lambda \text{ (in } \text{\AA}) = \frac{12375}{2} = 6187.5 \approx 6200 \text{ \AA} = 620 \text{ nm}$$

6 (a) As, $K_{\max} = E - W$
 $\Rightarrow K = E - W$... (i)
 and $K' = 2E - W$
 $= 2(E) - (E - K)$ [using Eq. (i)]
 $= E + K = K + W + K = W + 2K$
 $\Rightarrow K' > 2K$ ($\because W > 0$)

7 (c) As, $eV_0 = h\nu - W$

or $V_0 = \frac{h}{e} \nu - \frac{W}{e}$

When frequency is doubled,

$$V_0' = \frac{2h\nu}{e} - \frac{W}{e} = 2\left(V_0 + \frac{W}{e}\right) - \frac{W}{e} = 2V_0 + \frac{W}{e}$$

or $V_0' > 2V_0$

9 (b) For photoemission to take place, $h\nu > \phi_0$.

Here, $E < \phi_0$, so it is not possible.

• CHECK POINT 11.2

2 (a) Momentum,

$$p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{(5000 \times 10^{-10})} \\ = 1.3 \times 10^{-27} \text{ kg}\cdot\text{ms}^{-1}$$

3 (c) Momentum, $p = \frac{E}{c}$

$$\Rightarrow E = pc = 2 \times 10^{-16} \times (3 \times 10^8) = 6 \times 10^{-8} \text{ erg}$$

4 (a) Momentum, $p = \frac{E}{c}$

$$\therefore E = pc \Rightarrow E^2 = p^2 c^2$$

6 (c) Power, $P = \frac{nhc}{\lambda}$

$$\therefore 100 = \frac{n \times 6 \times 10^{-34} \times 3 \times 10^8}{540 \times 10^{-9}} \Rightarrow n = 3 \times 10^{20}$$

7 (a) Pressure = $\frac{2I}{c}$ (For perfectly reflecting surface)

where, c is speed of light.

$$= \frac{2 \times 10^4}{3 \times 10^8} = \frac{2}{3} \times 10^{-4} \text{ N/m}^2 \\ = 0.667 \times 10^{-4} = 6.67 \times 10^{-5} \text{ N/m}^2$$

8 (b) Pressure = $\frac{I_0}{c}$ (For a perfectly absorbing surface)

$$\Rightarrow 4.5 \times 10^{-5} = \frac{I_0}{3 \times 10^8}$$

$$\Rightarrow I_0 = 13.5 \times 10^3 = 1.35 \times 10^4 \approx 1.4 \times 10^4 \text{ W/m}^2$$

• CHECK POINT 11.3

2 (c) As, $\lambda = \frac{h}{mv}$

where, v = velocity of an electron

and λ = de-Broglie wavelength.

Also, $\lambda \propto 1/v$ (de-Broglie frequency)

When v is doubled, λ gets halved and v (de-Broglie frequency) gets doubled.

3 (d) As, $\lambda = \frac{h}{p}$, so for $\lambda_p = \lambda_\alpha$, we get

$$p_p = p_\alpha$$

4 (a) All the three particles have same wavelength, therefore they will same momentum say p .

Energy of photon $E = pc$. Therefore, energy of photon is maximum.

5 (c) $\lambda = \frac{h}{\sqrt{2qVm}}$ or $\lambda \propto \frac{1}{\sqrt{m}}$

As,

$$m_e < m_p$$

$\therefore$

$$\lambda_e > \lambda_p$$

6 (b) Wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$$\therefore \lambda \propto \frac{1}{\sqrt{E}} \quad (h \text{ and } m \text{ are constants})$$

So, when the KE of an electron is increased, the wavelength of the associated wave will decrease.

7 (a) The de-Broglie wavelength, $\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$

Here, $V = 100 \text{ V}$

$$\Rightarrow \lambda = \frac{1.227}{\sqrt{100}} \text{ nm} \approx 0.123 \text{ nm}$$

8 (a) As, $\lambda \propto \frac{1}{\sqrt{T}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}}$

Here, $T_2 = 927^\circ\text{C} = (927 + 273) \text{ K} = 1200 \text{ K}$

$$T_1 = 27^\circ\text{C} = (27 + 273) \text{ K} = 300 \text{ K}$$

$$\lambda_1 = \lambda$$

$$\therefore \frac{\lambda}{\lambda_2} = \sqrt{\frac{1200}{300}} \Rightarrow \lambda_2 = \frac{\lambda}{2}$$

(A) Taking it together

$$2 \text{ (a) As, } \lambda = \frac{h}{p}$$

$$\text{and } \lambda = \sqrt{\frac{h}{2mE}}$$

$$\Rightarrow \lambda \propto \frac{1}{p} \propto \frac{1}{E}$$

So, when wavelength is decreased, both p and E increase.

$$3 \text{ (b) As, } \lambda = \frac{h}{\sqrt{2mK}}$$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$$

Since, mass of α -particle is largest as compare to all the particles given. So, λ is shortest for α -particle.

$$4 \text{ (a) As, } \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2qVm}}$$

$$\text{So, } \lambda \propto \frac{1}{\sqrt{m}}$$

$$\therefore \frac{\lambda_e}{\lambda_p} = \left(\frac{m_p}{m_e} \right)^{1/2}$$

$$5 \text{ (c) } E \text{ (in eV)} = \frac{12375}{\lambda \text{ (in } \text{\AA})}$$

$$= \frac{12375}{7000} = 1.76 \text{ eV}$$

$$6 \text{ (a) As, } p = \frac{h\nu}{c}$$

$$\therefore v = \frac{pc}{h} = \frac{(6.6 \times 10^{-29}) (3 \times 10^8)}{(6.6 \times 10^{-34})}$$

$$= 3 \times 10^{13} \text{ Hz}$$

$$7 \text{ (b) Given, } E = 100 \text{ eV} = (100 \times 1.6 \times 10^{-19}) \text{ J}$$

$$\text{and } h = 6.62 \times 10^{-34} \text{ Js}$$

$$E = h\nu$$

$$\text{or } v = \frac{E}{h} = \frac{100 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}}$$

$$= 0.24 \times 10^{17} = 2.417 \times 10^{16} \text{ Hz}$$

8. (c) According to Einstein's photoelectric equation,

$$KE_{\max} = h\nu - \phi_0$$

Comparing with the equation of straight line

$$y = mx + c$$

We get, slope of graph between KE and $v = h$ (i.e. same for all metals)

9 (b) Work function, $W_0 = h\nu_0$

$$\text{For sodium, } W_{01} = h\nu_{01} = \frac{hc}{\lambda_0}$$

$$\Rightarrow \frac{2.3 \text{ eV}}{h} = \nu_{01} = \frac{c}{\lambda_{01}}$$

For copper, $W_{02} = h\nu_{02}$

$$\Rightarrow \frac{4.5 \text{ eV}}{h} = \nu_{02} = \frac{c}{\lambda_{02}}$$

$$\Rightarrow \frac{\lambda_{01}}{\lambda_{02}} = \frac{ch}{2.3} \times \frac{4.5}{ch} \approx \frac{2}{1} \Rightarrow \lambda_{01} : \lambda_{02} = 2 : 1$$

$$11 \text{ (a) As, } \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\text{So, } \frac{h}{m_1 v_1} = \frac{h}{m_2 v_2}$$

$$\therefore \frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{4}{1} \quad (\because m_\alpha = 4m_p)$$

$$12 \text{ (d) As, } \lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 80 \times 1.6 \times 10^{-19}}}$$

$$= 1.4 \text{ \AA}$$

$$13 \text{ (c) Momentum, } p = \frac{h}{\lambda} \text{ or } E = h\nu = \frac{hc}{\lambda} = pc$$

So, if the energy of photon is increased by a factor of 4, then its momentum will also increase by a factor of 4.

14 (a) Work function, $W_0 = h\nu_0$

where, ν_0 = threshold frequency.

$$\text{So, } W \propto \nu_0$$

Hence, Pt > Al > K

15 (a) de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mkT}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} T}}$$

$$= \frac{6.62 \times 10^{-34}}{2.15 \times 10^{-25} \sqrt{T}} \text{ m}$$

$$= \frac{3.079}{\sqrt{T}} \times 10^{-9} \text{ m} = \frac{30.79}{\sqrt{T}} \text{ \AA} \approx \frac{30.8}{\sqrt{T}} \text{ \AA}$$

$$16 \text{ (b) } E = \frac{12375}{\lambda(\text{\AA})} = \frac{12375}{3500} \text{ eV} = 3.53 \text{ eV}$$

Since, $E > W_B$ but $E < W_A$, therefore photoelectrons will be emitted by metal B.

17 (d) Mass of an electron, $m = 9.11 \times 10^{-31} \text{ kg}$

$$T = 27 + 273 = 300 \text{ K}$$

$$\text{de-Broglie wavelength of electron, } \lambda = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

$$= 6.2 \times 10^{-9} \text{ m}$$

18 (b) Given in the question,

Energy of a photon, $E = 1 \text{ MeV} = 10^6 \text{ eV}$

$$\text{Now, } hc = 1243 \text{ eV-nm}$$

$$\text{Now, } E = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E} = \frac{1243 \text{ eV-nm}}{10^6 \text{ eV}} = 1.2 \times 10^{-3} \text{ nm}$$

19 (b) de-Broglie wavelength, $\lambda = \frac{h}{mv}$

For electron, $\lambda_e = \frac{h}{m_e v}$... (i)

For proton, $\lambda_p = \frac{h}{m_p v}$... (ii)

From Eqs. (i) and (ii), we get

$$\therefore \frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{m_e v}}{\frac{h}{m_p v}}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{1.67 \times 10^{-27}}{9.1 \times 10^{-31}}$$

$$= 0.183516 \times 10^4 \approx 1836$$

20 (a) Wavelength of photon will be greater than that of electron because mass of photon is less than that of electron

$$\Rightarrow \lambda_{ph} > \lambda_{el} \quad \left(\because \lambda \propto \frac{1}{\sqrt{m}} \right)$$

21 (c) Number of photons, $n = \frac{P}{E} = \frac{P}{h\nu} = \frac{P\lambda}{hc}$

$$= \frac{1000 \times 198.6}{6.6 \times 10^{-34} \times 3 \times 10^8} \approx 10^{30}$$

22 (b) $E = \frac{12375}{\lambda \text{ (in Å)}} = \frac{12375}{4360} = 2.84 \text{ eV}$

$$\therefore K_{\max} = E - W = 1.60 \text{ eV}$$

Thus, the retarding potential is 1.60 V.

23 (c) $E_1 = \frac{12375}{3500} \text{ eV} = 3.54 \text{ eV}$

$$E_2 = \frac{12375}{5400} \text{ eV} = 2.29 \text{ eV}$$

Now, $\frac{2}{1} = \frac{3.54 - W}{2.29 - W}$

Solving this equation, we get $W = 1.05 \text{ eV}$.

24 (c) $\frac{E_1}{E_2} = \frac{1}{2}$ (given)

We know that, $\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \lambda \propto \frac{1}{\sqrt{K}}$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{K_2}}{\sqrt{K_1}} \Rightarrow \frac{\lambda_1^2}{\lambda_2^2} = \frac{K_2}{K_1}$$

$$\Rightarrow \frac{\lambda_1^2}{\lambda_2^2} = \frac{2}{1} \Rightarrow \lambda_1 : \lambda_2 = \sqrt{2} : 1$$

25 (c) $P = n \frac{hc}{\lambda} \Rightarrow n \propto \lambda$

$$\therefore \lambda_r > \lambda_b$$

$$\therefore n_r > n_b$$

26 (b) Work function, $\phi_0 = \frac{hc}{\lambda_0}$

$$\Rightarrow \phi_0 \propto \frac{1}{\lambda_0} \Rightarrow \frac{(\phi_0)_1}{(\phi_0)_2} = \frac{(\lambda_0)_2}{(\lambda_0)_1} = \frac{600}{300} = 2 : 1$$

27 (a) The change in wavelength is given by

$$\Delta\lambda = \lambda' - \lambda = \lambda_c (1 - \cos \phi)$$

where, λ' = wavelength of the X-rays scattered

and $\lambda_c = \frac{h}{m_0 c}$ = compton wavelength = $2.426 \times 10^{-12} \text{ m}$

$$\Rightarrow \lambda' = \lambda + \lambda_c (1 - \cos 30^\circ) = 10 \text{ pm} + 0.134 \lambda_c = 10.32 \text{ pm}$$

28 (d) When a beam of electrons of energy E_0 is incident on a metal surface kept in an evacuated chamber, electrons can be emitted with maximum energy E_0 (due to elastic collision) and with any energy less than E_0 , when a part of incident energy of electron is used in liberating the electrons from the surface of metal.

29 (c) By law of conservation of momentum,

$$0 = m_1 v_1 + m_2 v_2$$

$$m_1 v_1 = -m_2 v_2$$

Negative sign indicates that both the particles are moving in opposite directions. Now, de-Broglie wavelengths,

$$\lambda_1 = \frac{h}{m_1 v_1} \text{ and } \lambda_2 = \frac{h}{m_2 v_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2 v_2}{m_1 v_1} = 1$$

30 (a) Mass of electron in motion is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{(0.5c)^2}{c^2}}} = \frac{m_0}{\sqrt{0.75}}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34} \times \sqrt{0.75}}{m_0 v}$$

$$= \frac{6.6 \times 10^{-34} \times \sqrt{0.75}}{9.1 \times 10^{-31} \times 1.5 \times 10^8} = 4.2 \times 10^{-12} \text{ m}$$

31 (b) $K_1 = hf - W$ ($\because W_1 = W, f_1 = f$)

and $K_2 = 2hf - 2W$ ($\because W_2 = 2W, f_2 = 2f$)

$$\therefore \frac{K_1}{K_2} = \frac{1}{2}$$

32 (d) As, $\lambda = \frac{h}{\sqrt{2qVm}}$ or $\lambda \propto \frac{1}{\sqrt{qVm}}$

$$\therefore \frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{q_p V_p m_p}}{\sqrt{q_\alpha V_\alpha m_\alpha}} = \sqrt{\frac{1 \times 100 \times 1}{2 \times 800 \times 4}} = \frac{1}{8}$$

$$\therefore \lambda_\alpha = \frac{\lambda_p}{8} = \frac{\lambda_0}{8}$$

33 (b) de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mE_e}} = \frac{hc}{E_p}$

or $2mE_e = \frac{E_p^2}{c^2}$... (i)

$$\text{But } E_e = \frac{1}{2}mv^2 \text{ or } m = \frac{2E_e}{v^2}$$

From Eq. (i), we get

$$2\left[\frac{2E_e}{v^2}\right]E_e = \frac{E_p^2}{c^2} \text{ or } \frac{4E_e^2}{v^2} = \frac{E_p^2}{c^2}$$

$$\text{or } \frac{E_e^2}{E_p^2} = \frac{v^2}{4c^2} \text{ or } \frac{E_e}{E_p} = \frac{v}{2c}$$

34 (c) From the given graph, we get

$$V_0 = -3.2 \text{ V}$$

$$\text{As, } eV_0 = h\nu - \phi_0$$

$$\Rightarrow \nu = \frac{eV_0 + \phi_0}{h}$$

$$= \frac{-3.2 \times 1.6 \times 10^{-19} + 4 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= 1.939 \times 10^{14} \text{ s}^{-1}$$

35 (b) As, $E = \frac{hc}{\lambda} - \phi_0$... (i)

$$4E = \frac{hc}{\lambda/3} - \phi_0$$
 ... (ii)

$$\text{Solving these equations, we get } \phi_0 = \frac{hc}{3\lambda}$$

36 (a) $eV_s = h\nu - \phi$

$$\therefore V_s = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$$

Comparing with equation of straight line $y = mx + c$

$$\text{Slope (m)} = \tan \theta = \frac{h}{e}$$

37 (c) Maximum kinetic energy,

$$K_{\max} = \frac{hc}{\lambda} - \phi_0 = \frac{6.4 \times 10^{-34} \times 3 \times 10^8}{6400 \times 10^{-10}} - 1.6 \times 10^{-19}$$

$$= 1.4 \times 10^{-19} \text{ J}$$

38 (c) Work function, $\phi_0 = \frac{hc}{\lambda_0}$

$$\therefore \frac{(\phi_0)_T}{(\phi_0)_{Na}} = \frac{\lambda_{Na}}{\lambda_T}$$

$$\text{or } \lambda_T = \frac{\lambda_{Na} \times (\phi_0)_{Na}}{(\phi_0)_T} = \frac{5600 \times 2.3}{4.5} = 2862 \text{ \AA}$$

39 (d) By decreasing the wavelength of incident light, its energy will increase. Thus, stopping potential will also increase.

40 (b) As, $\lambda = \frac{h}{\sqrt{2mE}}$

$$\Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

So, $\frac{\lambda}{\lambda_2} = \sqrt{\frac{M}{m}}$

$$\therefore \lambda_2 = \lambda \sqrt{\left(\frac{m}{M}\right)}$$

41 (b) Intensity of incident light $\propto$ No. of electrons emitted per sec

$$\propto \frac{1}{(\text{distance})^2}$$

$\therefore$ If distance becomes $d/2$, then number of electrons becomes 4 times.

42 (b) Stopping potential, $V_0 = \frac{h}{e}(\nu - \nu_0)$

$$\text{or } (\nu - \nu_0) = \frac{V_0 \times e}{h} = \frac{4.144 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$\nu - 1 \times 10^{15} = 10^{15}$$

$$\text{Frequency} = 10^{15} + 1 \times 10^{15} = 2 \times 10^{15} \text{ Hz}$$

43 (b) Given, $\nu_{\max} = 4 \times 10^8 \text{ cm/s} = 4 \times 10^6 \text{ m/s}$

$$\therefore K_{\max} = \frac{1}{2}mv_{\max}^2 = \frac{1}{2} \times 9 \times 10^{-31} \times (4 \times 10^6)^2$$

$$= 7.2 \times 10^{-18} \text{ J} = 45 \text{ eV}$$

$$\text{Hence, stopping potential, } V_0 = \frac{K_{\max}}{e} = \frac{45 \text{ eV}}{e} = 45 \text{ V}$$

44 (a) Given, $\mathbf{v} = \nu_0 \hat{\mathbf{i}}$ and $\mathbf{B} = B_0 \hat{\mathbf{j}}$

Force on moving electron due to magnetic field,

$$\mathbf{F} = -e(\mathbf{v} \times \mathbf{B}) = -e[\nu_0 \hat{\mathbf{i}} \times B_0 \hat{\mathbf{j}}] \Rightarrow \mathbf{F} = -e\nu_0 B_0 \hat{\mathbf{k}}$$

As this force is perpendicular to $\mathbf{v}$ and $\mathbf{B}$, so the magnitude of $\mathbf{v}$ will not change, i.e. momentum ($= m\nu$) will remain constant in magnitude.

Hence, de-Broglie wavelength, $\lambda = \frac{h}{m\nu}$ remains constant.

45 (c) Wavelength $\propto \frac{1}{\text{Energy}}$. Hence, for ejection of

photoelectrons, wavelength should be smaller than threshold wavelength. As, wavelength of ultraviolet rays ($\sim 400 \text{ nm}$ to 1 nm) is less than that for infrared rays ($\sim 1 \text{ nm}$ to 700 nm). So, emission would be possible only ultraviolet lamp.

46 (b) When the distance is increased, frequency of incident light and hence the stopping potential does not change. But the intensity changes and hence the saturation current decreases nine times. $[\because \text{Intensity of incident radiation} \propto \frac{1}{(\text{distance})^2}]$

47 (a) According to Einstein's equation,

$$h\nu = \phi_0 + K_{\max}$$

where, ϕ_0 is the work function.

$$\Rightarrow h\nu = \phi_0 + eV_0 \Rightarrow V_0 = \frac{h\nu}{e} - \frac{\phi_0}{e}$$

Comparing with equation, $y = mx + c$

$$\phi_0 = -(\text{intercept}) \times e = OB \times e \text{ in eV}$$

48 (c) In Davisson-Germer experiment, the de-Broglie wavelength associated with electron,

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA} \quad \dots(i)$$

where, V is the applied voltage.

If there is a maxima of the diffracted electrons at an angle θ , then

$$2d \sin \theta = \lambda \quad \dots(ii)$$

From Eq. (i), we note that if V is inversely proportional to the wavelength λ .

i.e. V will increase with the decrease in the λ .

From Eq. (ii), we note that, wavelength λ is directly proportional to $\sin \theta$ and hence θ .

So, with the decrease in λ , θ will also decrease.

Thus, when the voltage applied to A is increased. The diffracted beam will have the maxima at a value of θ that will be less than the earlier value.

- 49** (d) The electrons may be accelerated or retarded in electric field.

- 50** (b) Maximum kinetic energy,

$$K_{\max}(\text{in eV}) = \frac{hc}{e} \left[\frac{1}{\lambda} - \frac{1}{\lambda_0} \right] = 12375 \left[\frac{1}{1000} - \frac{1}{2000} \right] = 6.2 \text{ eV}$$

- 51** (d) Velocity of a body falling from a height H is given by

$$v = \sqrt{2gH}$$

We know that, de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}} \Rightarrow \lambda \propto \frac{1}{\sqrt{H}}$$

Here, $\frac{h}{m\sqrt{2g}}$ is a constant (say K).

$$\text{So, } \lambda = K \frac{1}{\sqrt{H}} \Rightarrow \lambda \propto \frac{1}{\sqrt{H}}$$

$$\Rightarrow \lambda \propto H^{-1/2}$$

- 52** (d) We know that, $E = W + \text{KE}$

$$\begin{aligned} \text{KE} &= E - W = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \\ &= hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) = hc \left(\frac{\lambda_0 - \lambda}{\lambda \lambda_0} \right) \end{aligned}$$

- 53** (c) de-Broglie wavelength, $\lambda = \frac{h}{p}$

$$\Rightarrow \lambda - \frac{0.5\lambda}{100} = \frac{h}{p + \Delta p}$$

$$\Rightarrow \frac{199\lambda}{200} = \frac{h}{p + \Delta p} = \frac{199h}{200p}$$

$$\Rightarrow p + \Delta p = \frac{200}{199} p$$

$$\Rightarrow p = 199 \Delta p$$

- 54** (a) Here, $\lambda_1 = \frac{12375}{E_1(\text{eV})} \text{ \AA} = 1000 \text{ \AA}$

$$\therefore E_1 = 12.375 \text{ eV}$$

$$\text{Similarly, } E_2(\text{eV}) = \frac{12375}{\lambda_2(\text{\AA})} \text{ eV} = \frac{12375}{2000} = 6.1875 \text{ eV}$$

$$\text{Now, } E_1 - W_0 = eV_1$$

$$\text{and } E_2 - W_0 = eV_2$$

$$\text{Hence, } 12.375 - W_0 = 7.7 \text{ eV}$$

$$\text{and } 6.1875 - W_0 = eV_2$$

$$\text{Solving, we get } V_2 = 1.5 \text{ V}$$

- 55** (b) According to the question,

$$eV = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(i)$$

$$\text{and } \frac{eV}{3} = hc \left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right) \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$3 = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)}{\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right)}$$

$$\text{or } 3 \left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right) = \frac{1}{\lambda} - \frac{1}{\lambda_0}$$

$$\text{or } \frac{3}{2\lambda} - \frac{1}{\lambda} = \frac{3}{\lambda_0} - \frac{1}{\lambda_0}$$

$$\text{or } \frac{1}{2\lambda} = \frac{2}{\lambda_0}$$

Threshold wavelength for metallic surface, $\lambda_0 = 4\lambda$

- 56** (a) Energy radiated as visible light = $\frac{5}{100} \times 100 = 5 \text{ J/s}$

Let n be the number of photons emitted per second, then

$$nh\nu = E = 5$$

$$\therefore n = \frac{5\lambda}{hc} = \frac{5 \times 5.6 \times 10^{-7}}{(6.62 \times 10^{-34})(3 \times 10^8)} \approx 1.4 \times 10^{19}$$

- 57** (c) Intensity, $I = 10^{-10} \text{ Wbm}^{-2} = 10^{-10} \text{ Js}^{-1} \text{ m}^{-2}$.

Let the number of photons required be n , then

$$I = \frac{nh\nu}{A} = \frac{nh\nu}{10^{-4}} = 10^{-10}$$

$$\begin{aligned} \text{Hence, } n &= \frac{10^{-10} \times 10^{-4}}{h\nu} = 10^{-14} \frac{\lambda}{hc} \\ &= \frac{10^{-14} \times 660 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 3.3 \times 10^4 \end{aligned}$$

- 58** (b) We know that, the relation between λ and K is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Here, for the given value of energy K , $\frac{h}{\sqrt{2K}}$ is a constant.

$$\text{Thus, } \lambda \propto \frac{1}{\sqrt{m}}$$

$$\therefore \lambda_p : \lambda_n : \lambda_e : \lambda_\alpha$$

$$\Rightarrow = \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{m_n}} : \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_\alpha}}$$

Since, $m_p = m_n$, hence $\lambda_p = \lambda_n$
 As, $m_\alpha > m_p$, therefore $\lambda_\alpha < \lambda_p$
 As, $m_e < m_n$, therefore $\lambda_e > \lambda_n$
 Hence, $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$

59 (a) As $\frac{mv^2}{r} = qvB \Rightarrow \frac{mv}{qB} = r$

Also, $mv = p \Rightarrow p = \sqrt{2mK}$
 $\Rightarrow r = \frac{\sqrt{2Km}}{Bq} = \frac{\sqrt{2(E - W_0)m}}{eB}$

60 (b) As, $E - W_0 = \frac{1}{2}mv^2$

First case, $2W_0 - W_0 = \frac{1}{2}mv_1^2$... (i)

Second case, $5W_0 - W_0 = \frac{1}{2}mv_2^2$... (ii)

Therefore, from Eqs. (i) and (ii), we get

$$\frac{2W_0 - W_0}{5W_0 - W_0} = \frac{v_1^2}{v_2^2} \quad \text{or} \quad \frac{v_1}{v_2} = \frac{1}{2}$$

61 (b) Energy corresponding to 2000 Å,

$$E = \frac{12375}{\lambda(\text{in } \text{\AA})} = \frac{12375}{2000} \text{ eV} = 6.2 \text{ eV}$$

Maximum kinetic energy,

$$K = h\nu - h\nu_0$$

$$K = E - W$$

$$= (6.2 - 5.01) \text{ eV} = 1.19 \text{ eV}$$

Now, $K = \frac{1}{2}mv_{\text{max}}^2$

$$\Rightarrow \frac{1}{2} \times 9.1 \times 10^{-31} \times v_{\text{max}}^2 = 1.19 \times 1.6 \times 10^{-19}$$

or $v_{\text{max}}^2 = \frac{1.19 \times 1.6 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}$

or $v_{\text{max}}^2 = 0.418 \times 10^{12} = 41.8 \times 10^{10}$

or $v_{\text{max}} = 6.46 \times 10^5 \text{ m/s}$

62 (b) From equation,

$$eV_0 = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

$$4.8 = \frac{hc}{e} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \quad \dots (i)$$

and $1.6 = \frac{hc}{e} \left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right) \quad \dots (ii)$

From Eqs. (i) and (ii), we get

$$\frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)}{\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0} \right)} = \frac{4.8}{1.6} \Rightarrow \lambda_0 = 4\lambda$$

63 (b) As, $\lambda = \frac{h}{\sqrt{2mK}}$ or $\lambda^2 = \frac{h^2}{2mK}$

$$\therefore K = \frac{h^2}{2m\lambda^2} = \frac{(6.6 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (10 \times 10^{-12})^2} \text{ J}$$

$$= 15.1 \text{ keV} \approx 15 \text{ eV}$$

64 (a) Consider a spherical chamber of radius 20 cm.

Bulb is placed at point S as shown in figure.

Intensity of light at surface,

$$I = \frac{0.6P}{4\pi r^2}$$

Force exerted on the surface, $F = \frac{P}{c}$,

where P is power of bulb.

$$\therefore F = \frac{1100}{3 \times 10^8} = 3.6 \times 10^{-6} \text{ N}$$

Pressure exerted by light $= \frac{I}{c} = \frac{0.6P}{4\pi r^2 c} = \frac{0.6 \times 1100}{4\pi (0.2)^2 \times 3 \times 10^8}$
 $= 4.4 \times 10^{-6} \text{ N/m}^2$

65 (c) Force totally reflecting surface,

$$F = \frac{2P \cos 60^\circ}{c}$$

$$= \frac{2 \times \frac{nhc}{\lambda} \cos 60^\circ}{c} = \frac{10^{19} \times 6.6 \times 10^{-34}}{663 \times 10^{-9}} = 10^{-8} \text{ N}$$

66 (d) As, $\frac{1}{2}mv_{\text{max}}^2 = \frac{hc}{\lambda} - \phi_0$

$$v_{\text{max}} = \sqrt{\frac{2}{m} \left(\frac{hc}{\lambda} - \phi_0 \right)} = v$$

$$v'_{\text{max}} = \sqrt{\frac{2}{m} \left(\frac{hc}{\lambda/4} - \phi_0 \right)}$$

$$= 2 \sqrt{\frac{2}{m} \left(\frac{hc}{\lambda} - \phi_0 \right) + \frac{6}{m} \phi_0}$$

$$> 2v$$

67 (c) Energy of photoelectron,

$$E = \frac{hc}{\lambda} - \phi_0$$

$$\Rightarrow \frac{hc}{\lambda} = E + \phi_0 \quad \dots (i)$$

where ϕ_0 is the work function for the metal surface (constant).

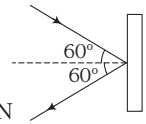
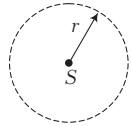
When $E' = 2E$, then

$$2E = \frac{hc}{\lambda'} - \phi_0$$

$$\Rightarrow \frac{hc}{\lambda'} = 2E + \phi_0 \quad \dots (ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\lambda'}{\lambda} = \frac{E + \phi_0}{2E + \phi_0}$$



$$\frac{\lambda'}{\lambda} = \frac{(E + \phi_0)}{2\left(E + \frac{\phi_0}{2}\right)}$$

$$\therefore \frac{\lambda'}{\lambda} > \frac{1}{2} \quad \text{or} \quad \lambda' > \frac{\lambda}{2}$$

or $\lambda > \lambda' > \frac{\lambda}{2}$

68 (c) Let W be the work function of metal, then

$$eV_0 = \frac{hc}{330 \times 10^{-9}} - W \quad \dots(i)$$

$$e(2V_0) = \frac{hc}{220 \times 10^{-9}} - W \quad \dots(ii)$$

Solving these two equations, we get

$$\begin{aligned} V_0 &= \frac{10^9 \times h \times c}{110 \times e \times 6} \\ &= \frac{10^9 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{110 \times 1.6 \times 10^{-19} \times 6} = \frac{15}{8} \text{ V} \end{aligned}$$

69 (a) Initial de-Broglie wavelength of electron,

$$\lambda_0 = \frac{h}{mv_0} \quad \dots(i)$$

Force on electron in electric field,

$$\mathbf{F} = -e\mathbf{E} = -e[-E_0\hat{\mathbf{i}}] = eE_0\hat{\mathbf{i}}$$

$$\text{Acceleration of electron, } a = \frac{\mathbf{F}}{m} = \frac{eE_0\hat{\mathbf{i}}}{m}$$

Velocity of electron after time t ,

$$v = v_0\hat{\mathbf{i}} + \left(\frac{eE_0\hat{\mathbf{i}}}{m}\right)t = \left(v_0 + \frac{eE_0}{m}t\right)\hat{\mathbf{i}} = v_0\left(1 + \frac{eE_0}{mv_0}t\right)\hat{\mathbf{i}}$$

de-Broglie wavelength associated with electron at time t ,

$$\begin{aligned} \lambda &= \frac{h}{mv} \\ \Rightarrow \lambda &= \frac{h}{m\left[v_0\left(1 + \frac{eE_0}{mv_0}t\right)\right]} = \frac{\lambda_0}{\left[1 + \frac{eE_0}{mv_0}t\right]} \quad \left(\because \lambda_0 = \frac{h}{mv_0}\right) \end{aligned}$$

70 (c) As, $eV = \frac{hc}{\lambda} - W$

$$\therefore V = \left(\frac{h}{e}\right) \cdot \frac{c}{\lambda} - \frac{W}{e}$$

$$\text{So, } V_1 = \left(\frac{h}{e}\right) \frac{c}{\lambda_1} - \frac{W}{e} \quad \dots(i)$$

$$V_2 = \left(\frac{h}{e}\right) \frac{c}{\lambda_2} - \frac{W}{e} \quad \dots(ii)$$

Solving these two equations, we get

$$\begin{aligned} \frac{h}{e} &= \frac{\lambda_1\lambda_2(V_1 - V_2)}{c(\lambda_2 - \lambda_1)} \\ &= \frac{(0.6 \times 0.4 \times 10^{-12})(1.0)}{(3 \times 10^8)(0.2 \times 10^{-6})} \\ &= 4 \times 10^{-15} \text{ Vs} \end{aligned}$$

71 (c) Initial de-Broglie wavelength of electron, $\lambda_0 = \frac{h}{mv_0}$

Force on electron in electric field, $\mathbf{F} = -e\mathbf{E} = -eE_0\hat{\mathbf{j}}$

$$\text{Acceleration of electron, } \mathbf{a} = \frac{\mathbf{F}}{m} = -\frac{eE_0\hat{\mathbf{j}}}{m}$$

It is acting along negative Y -axis.

The initial velocity of electron along X -axis, $\mathbf{v}_{x_0} = v_0\hat{\mathbf{i}}$.

Initial velocity of electron along Y -axis, $\mathbf{v}_{y_0} = 0$.

Velocity of electron after time t along X -axis, $\mathbf{v}_x = v_0\hat{\mathbf{i}}$

Velocity of electron after time t along Y -axis,

$$\mathbf{v}_y = 0 + \left(-\frac{eE_0}{m}\hat{\mathbf{j}}\right)t = -\frac{eE_0}{m}t\hat{\mathbf{j}}$$

Magnitude of velocity of electron after time t ,

$$\begin{aligned} \mathbf{v} &= \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + \left(\frac{eE_0}{m}t\right)^2} \\ \Rightarrow &= v_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}} \\ \text{de-Broglie wavelength, } \lambda' &= \frac{h}{mv} \\ \Rightarrow &= \frac{h}{mv_0\sqrt{1 + e^2E_0^2t^2 / (m^2v_0^2)}} = \frac{\lambda_0}{\sqrt{1 + e^2E_0^2t^2 / m^2v_0^2}} \end{aligned}$$

72 (d) We know, $K_{\max} = E - W$

$$\therefore T_A = 4.25 - W_A \quad \dots(i)$$

$$T_B = (T_A - 1.50) = 4.70 - W_B \quad \dots(ii)$$

From these two equations, we have

$$W_B - W_A = 1.95 \text{ eV} \quad \dots(iii)$$

de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2Km}} \quad \text{or} \quad \lambda \propto \frac{1}{\sqrt{K}}$$

$$\therefore \frac{\lambda_B}{\lambda_A} = \sqrt{\frac{K_A}{K_B}} \Rightarrow 2 = \sqrt{\frac{T_A}{T_A - 1.5}}$$

$$\Rightarrow T_A = 2 \text{ eV}, W_A = 2.25 \text{ eV}, W_B = 4.20 \text{ eV}$$

$$\text{and } T_B = 0.5 \text{ eV}$$

(B) Medical entrance special format questions

• Assertion and reason

1 (a) The electromagnetic theory of light failed to explain photoelectric effect. It cannot be explained by assuming wave nature of radiation. If wave nature of radiation is assumed, then it will take quite a long time (about one year) for electron to come out of the metal surface which contradicts the experimental observations. Thus, photoelectric effect can only be explained by assuming particle nature of light.

2 (a) There is a very small time lag between the instants, the photon of light falls on the metal surface and emission of photoelectrons from the metal surface. Thus, photoelectric effect is an instantaneous process.

3 (d) Decreasing the λ of incident photon means energy of incident light is increased, so E_k increases and hence stopping potential increases.

4 (d) The maximum kinetic energy of emitted photoelectrons depends on the radiation source and nature of material of plate but is independent of the intensity.

So, it will remain unchanged.

• Statement based questions

1 (b) ϕ_0 depends upon the properties of the metal.

However, minimum energy required for the electron emission from the metal surface can be provided by

- (i) heating it, *i.e.* thermionic emission.
- (ii) putting it under strong electric field $\approx 10^8$ V/m
- (iii) by bombarding it with high speed electrons
- (iv) by photoelectric effect.

Also, alkali metals like Li, Na, K, Cs, etc., show photoelectric effect with visible light and other high frequency wave like UV-light.

2 (d) Maximum kinetic energy of the photoelectrons is independent of the intensity of incident radiation.

A sufficiently intense beam is not necessary to be able to impart enough energy for photoemission.

Photoemission is an instantaneous process.

3 (c) Photons are electrically neutral. Thus, are not deflected by electric and magnetic fields.

Also, total energy and momentum are conserved in photon electron collision. However, number of photons may not be conserved. As, photons may be absorbed or a new photon may be created in a collision.

4 (a) In Compton effect, scattered photon has less energy than that of incident photon. The energy lost by the photon is taken by the electron as kinetic energy.

Thus, in Compton effect only a part of energy of incident photon is absorbed by the electron but in photoelectric effect, the energy of incident photons is completely absorbed by the electrons.

5 (a) Frequency of light of wavelength ($\lambda = 4000 \text{ \AA}$) is

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{4000 \times 10^{-10}} \\ = 0.75 \times 10^{15} \text{ Hz}$$

This frequency is less than the given threshold frequency. Hence, no photoelectric emission takes place.

7 (b) Stopping potential equals to maximum kinetic energy since, stopping potential is varying linearly with the frequency. Therefore, maximum KE for both the metals also vary nearly with frequency.

• Match the columns

1 (b) In the figure, stopping potential $V_a = V_b$ but V_a or $V_b < V_c$.

Moreover $I_a < I_b$ but $I_b = I_c$.

Hence, $A \rightarrow q$, $B \rightarrow p$, $C \rightarrow q$.

$$\begin{aligned} \text{2 (d) Energy of incident photon} &= \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10}} \\ &= 3.3 \times 10^{-19} \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Work function of surface} &= 2 \text{ eV} = 2 \times 1.6 \times 10^{-19} \\ &= 3.2 \times 10^{-19} \text{ J} \end{aligned}$$

Number of photoelectrons emitted = Number of incident photons per second on $2 \text{ cm}^2 (= 2 \times 10^{-4} \text{ m}^2)$ area

$$n = \frac{\text{Intensity of incident radiations} \times \text{surface area}}{\text{Energy of one photon}}$$

$$n = \frac{2 \times 3.3 \times 10^{-3} \times 10^{-4}}{3.3 \times 10^{-19}} = 2 \times 10^{12} \text{ per second}$$

Hence, $A \rightarrow q$, $B \rightarrow r$, $C \rightarrow p$.

(C) Medical entrances' gallery

1 (c) Initially, frequency of light, $\nu = 1.5\nu_0$

But when the frequency of incident light is halved, then

$$\nu' = \frac{\nu}{2} = 0.75\nu_0. \text{ Thus, it becomes less than the threshold}$$

frequency. In that case, no photoelectric effect takes place.

Hence, photoelectric current becomes zero.

2 (c) Given, $\lambda = 1.227 \times 10^{-2} \text{ nm} = 1.227 \times 10^{-11} \text{ m}$

Potential difference, $V = ?$

Relation for de-Broglie wavelength for moving electron,

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

$$\Rightarrow \sqrt{V} = \frac{12.27}{\lambda} \times 10^{-10} \text{ m} = \frac{12.27 \times 10^{-10}}{1.227 \times 10^{-11}} = 10^2 \text{ m}$$

$$\Rightarrow V = 10^4 \text{ V}$$

3 (b) Given, average flux = 20 W/cm^2

Surface area = 20 cm^2

Time = $1 \text{ min} = 60 \text{ s}$

$$\begin{aligned} \text{For non-reflecting surface, energy received} &= \text{average flux} \\ &\quad \times \text{surface area} \times \text{time} \\ &= 20 \times 20 \times 60 = 24 \times 10^3 \text{ J} \end{aligned}$$

4 (d) Kinetic energy of electron,

$$K = 144 \text{ eV} \Rightarrow eV = 144 \text{ eV} \Rightarrow V = 144 \text{ V}$$

$\therefore$ de-Broglie wavelength,

$$\begin{aligned} \lambda &= \frac{12.27}{\sqrt{V}} \text{ \AA} = \frac{12.27}{\sqrt{144}} \text{ \AA} = \frac{12.27}{12} \text{ \AA} \\ &= 0.102 \text{ \AA} = 1.02 \text{ nm} = 102 \times 10^{-2} \text{ nm} \end{aligned}$$

6 (d) The work function of material is given by

$$\phi = h\nu$$

$$\phi = \frac{hc}{\lambda}$$

$$\dots(i) \left(\because \nu = \frac{c}{\lambda} \right)$$

where, h = Planck's constant = $6.63 \times 10^{-34} \text{ J-s}$,

c = speed of light = $3 \times 10^8 \text{ ms}^{-1}$ and λ = wavelength of light.

Here, $\phi = 4 \text{ eV} = 4 \times 1.6 \times 10^{-19} \text{ J}$

Substituting the given values in Eq. (i), we get

$$4 \times 1.6 \times 10^{-19} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\Rightarrow \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4 \times 1.6 \times 10^{-19}}$$

$$= 3.108 \times 10^{-7} \text{ m} \approx 310 \text{ nm}$$

- 7** (a) Given, potential difference, $V = 10000 \text{ V}$

If electron is accelerated through a potential of V volt, then the wavelength associated with it is given by

$$\lambda = \frac{h}{\sqrt{2eVm_e}} \quad \dots (i)$$

where, h = Planck's constant = $6.63 \times 10^{-34} \text{ J-s}$,

e = electronic charge = $1.6 \times 10^{-19} \text{ C}$

and m_e = mass of electron

$$= 9 \times 10^{-31} \text{ kg}$$

Substituting these values in Eq. (i), we get

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA} = \frac{12.27}{\sqrt{10000}} \times 10^{-10}$$

$$= \frac{12.27 \times 10^{-10}}{100} = 12.2 \times 10^{-12} \text{ m}$$

- 8** (a) The de-Broglie wavelength associated with a charged particle is given by

$$\lambda = \frac{h}{p}$$

where, h = Planck's constant

and p = momentum = $\sqrt{2mKE}$ (here, KE is the kinetic energy of the charged particle).

$$\Rightarrow \lambda = \frac{h}{\sqrt{2m(KE)}}$$

For proton and α -particle, the wavelengths are respectively given as

$$\lambda_p = \frac{h}{\sqrt{2m_p(KE)_p}}$$

$$\text{and } \lambda_\alpha = \frac{h}{\sqrt{2m_\alpha(KE)_\alpha}}$$

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{2m_\alpha(KE)_\alpha}}{\sqrt{2m_p(KE)_p}} \quad \dots (i)$$

Here, $(KE)_\alpha = (KE)_p$ and $m_\alpha = 4m_p$

Substituting these above mentioned relations in Eq. (i), we get

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4m_p}{m_p}} = \frac{2}{1} \text{ or } \lambda_p : \lambda_\alpha = 2 : 1$$

- 9** (c) According to the Einstein's photoelectric equation,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = h\nu - \phi_0$$

$$= h\nu - h\nu_0 \quad \dots (i)$$

where, $K_{\max}$ is the maximum kinetic energy of photoelectrons having maximum velocity $v_{\max}$.

When incident frequency of light, $\nu = 2\nu_0$

Substituting the value of ν in Eq. (i), we get

$$\frac{1}{2}mv_1^2 = h(2\nu_0) - h\nu_0$$

$$= 2h\nu_0 - h\nu_0 = h\nu_0 \quad \dots (ii)$$

If incident frequency of radiation,

$$\nu = 5\nu_0$$

Substituting the value of ν in Eq. (i), we get

$$\frac{1}{2}mv_2^2 = h(5\nu_0) - h\nu_0$$

$$= 5h\nu_0 - h\nu_0 = 4h\nu_0 \quad \dots (iii)$$

On dividing Eq. (ii) by Eq. (iii), we get

$$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{h\nu_0}{4h\nu_0}$$

$$\Rightarrow \frac{v_1^2}{v_2^2} = \frac{1}{4} \text{ or } \frac{v_1}{v_2} = \frac{1}{2}$$

$$\therefore v_1 : v_2 = 1 : 2$$

- 10** (c) As we know that, wavelength of a charged particle,

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m}} \cdot \frac{1}{\sqrt{E}}$$

$$\Rightarrow \log \lambda = \log \left(\frac{h}{\sqrt{2m}} \cdot \frac{1}{\sqrt{E}} \right)$$

$$\Rightarrow \log \lambda = \log \frac{h}{\sqrt{2m}} + \log \frac{1}{E^{1/2}}$$

$$\Rightarrow \log \lambda = \log \left(\frac{h}{\sqrt{2m}} \right) - \frac{1}{2} \log E$$

$$\Rightarrow \log \lambda = -\frac{1}{2} \log E + \log \frac{h}{\sqrt{2m}}$$

Comparing the above equation with the equation of straight line, i.e. $y = mx + c$.

We can conclude that, $\log \lambda$ varies linearly with $\log E$, with slope $-1/2$.

It has been correctly shown in graph (c).

- 11** (a) Given, threshold wavelength,

$$\lambda_0 = 3250 \times 10^{-10} \text{ m}$$

Wavelength of ultraviolet light, $\lambda = 2536 \times 10^{-10} \text{ m}$

Let velocity of ejected electron be v .

Now, applying Einstein's photoelectric equation, we have

$$E = K + \phi_0$$

$$\Rightarrow h\nu = \frac{1}{2}mv^2 + h\nu_0$$

$$\Rightarrow \frac{1}{2}mv^2 = h\nu - h\nu_0 = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

$$\Rightarrow \text{Velocity of electron, } v = \sqrt{\frac{2hc}{m_e} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)}$$

$$\begin{aligned}
 &= \sqrt{\frac{2 \times 4.14 \times 10^{-15} \times 1.6 \times 10^{-19} \times 3 \times 10^8}{9.1 \times 10^{-31} \times 10^{-20}} \left(\frac{3250 - 2536}{3250 \times 2536} \right)} \\
 &\approx 0.6 \times 10^6 \text{ ms}^{-1} \\
 &\approx 6 \times 10^5 \text{ ms}^{-1}
 \end{aligned}$$

- 12** (b) de-Broglie wavelength associated with a moving particle can be given as

$$\lambda = \frac{h}{p}$$

At thermal equilibrium, temperature of neutron and heavy water will be same. This common temperature is given as T .

Also, we know that, kinetic energy of a particle, $\text{KE} = \frac{p^2}{2m}$

where, p = momentum of the particle
and m = mass of the particle.

Kinetic energy of the neutron at T is $\text{KE} = \frac{3}{2} kT$

$\therefore$ de-Broglie wavelength of the neutron,

$$\begin{aligned}
 \lambda &= \frac{h}{p} = \frac{h}{\sqrt{2m(\text{KE})}} \\
 &= \frac{h}{\sqrt{2m \times (3/2) kT}} = \frac{h}{\sqrt{3mkT}}
 \end{aligned}$$

- 13** (d) The de-Broglie wavelength of an electron,

$$\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda} \quad \dots(i)$$

Energy of photon,

$$E_p = \frac{hc}{\lambda} \text{ (since, } \lambda \text{ is same for electron and photon)} \quad \dots(ii)$$

Kinetic energy of photon to kinetic energy of electron

$$\begin{aligned}
 \frac{E_p}{E_e} &= \frac{\frac{hc}{\lambda}}{\left(\frac{1}{2}\right)mv^2} = \frac{2hc}{\lambda mv^2}
 \end{aligned}$$

Substituting the value of v from Eq. (i), we get

$$\frac{E_p}{E_e} = \frac{2hc}{\lambda m \left(\frac{h}{m\lambda}\right)^2} = \frac{2\lambda mc}{h}$$

- 14** (c) In Ist case, when a metallic surface is illuminated with radiation of wavelength λ , the stopping potential is V . So, photoelectric equation can be written as

$$eV = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \quad \dots(i)$$

In IInd case, when the same surface is illuminated with radiation of wavelength 2λ , the stopping potential is $\frac{V}{4}$. So,

photoelectric equation can be written as

$$\begin{aligned}
 \frac{eV}{4} &= \frac{hc}{2\lambda} - \frac{hc}{\lambda_0} \\
 \Rightarrow eV &= \frac{4hc}{2\lambda} - \frac{4hc}{\lambda_0} \quad \dots(ii)
 \end{aligned}$$

From Eqs. (i) and (ii), we get

$$\begin{aligned}
 \Rightarrow \frac{hc}{\lambda} - \frac{hc}{\lambda_0} &= \frac{4hc}{2\lambda} - \frac{4hc}{\lambda_0} \\
 \Rightarrow \frac{1}{\lambda} - \frac{1}{\lambda_0} &= \frac{2}{\lambda} - \frac{4}{\lambda_0} \Rightarrow \lambda_0 = 3\lambda
 \end{aligned}$$

- 15** (d) Since, it is given that electron has mass m .

de-Broglie wavelength for an electron will be given as

$$\lambda_e = \frac{h}{p} \quad \dots(i)$$

where, h = Planck's constant

and p = linear momentum of electron.

As, kinetic energy of electron, $E = \frac{p^2}{2m}$

$$\Rightarrow p = \sqrt{2mE} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\lambda_e = \frac{h}{\sqrt{2mE}} \quad \dots(iii)$$

Energy of a photon can be given as $E = h\nu$

$$\begin{aligned}
 \Rightarrow E &= \frac{hc}{\lambda_p} \\
 \Rightarrow \lambda_p &= \frac{hc}{E} \quad \dots(iv)
 \end{aligned}$$

where, λ_p = de-Broglie wavelength of photon.

Now, divide Eq. (iii) by Eq. (iv), we get

$$\begin{aligned}
 \frac{\lambda_e}{\lambda_p} &= \frac{h}{\sqrt{2mE}} \cdot \frac{E}{hc} \\
 \Rightarrow \frac{\lambda_e}{\lambda_p} &= \frac{1}{c} \cdot \sqrt{\frac{E}{2m}}
 \end{aligned}$$

- 16** (d) Here, we will use Einstein's photoelectric equation.

We know that, $E = (\text{KE})_{\text{max}} + \text{Work function } (\phi)$

where, $\phi = h\nu_0$, $E = h\nu$ and $(\text{KE})_{\text{max}} = \frac{1}{2}mv_0^2$

$$\begin{aligned}
 \Rightarrow (\text{KE})_{\text{max}} &= h\nu - \phi \\
 \Rightarrow 2\text{eV} &= 5\text{eV} - \phi \quad \text{(given)} \\
 \Rightarrow \phi &= 3\text{eV}
 \end{aligned}$$

$$\begin{aligned}
 \text{Thus, } V_{\text{cathode}} - V_{\text{anode}} &= 3\text{V} \\
 \Rightarrow V_{\text{anode}} - V_{\text{cathode}} &= -3\text{V}
 \end{aligned}$$

- 17** (a) Given, mass of electrons = m

and de-Broglie wavelength = λ

$$\text{So, kinetic energy of electron} = \frac{p^2}{2m} = \frac{\left(\frac{h}{\lambda}\right)^2}{2m} = \frac{h^2}{2m\lambda^2}$$

Now, maximum energy of photon can be given by

$$\begin{aligned}
 E &= \frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2} \\
 \Rightarrow \lambda_0 &= \frac{hc \times 2\lambda^2 \cdot m}{h^2} = \frac{2mc\lambda^2}{h}
 \end{aligned}$$

- 18 (b)** The radiation energy is given by $E = \frac{hc}{\lambda}$
 Initial momentum of the radiation is $p_i = \frac{h}{\lambda} = \frac{E}{c}$
 The reflected momentum is $p_r = -\frac{h}{\lambda} = -\frac{E}{c}$
 So, the change in momentum of light is

$$\Delta p_{\text{light}} = p_r - p_i = -\frac{2E}{c}$$

 Thus, the momentum transferred to the surface is $\Delta p_{\text{light}} = \frac{2E}{c}$.

- 19 (b)** From photoelectric equation,
 $h\nu = W + eV$ (where, W = work function)
 So, $\frac{hc}{\lambda} = W + 3eV_0$... (i)
 Also, $\frac{hc}{2\lambda} = W + eV_0 \Rightarrow \frac{hc}{\lambda} = 2W + 2eV_0$... (ii)
 Subtracting Eq. (i) from Eq. (ii), we get
 $0 = W - eV_0 \Rightarrow W = eV_0$
 From Eq. (i), $\frac{hc}{\lambda} = eV_0 + 3eV_0 = 4eV_0$
 The threshold wavelength is given by

$$\lambda_{\text{th}} = \frac{hc}{W} = \frac{4eV_0\lambda}{eV_0} = 4\lambda$$

- 20 (b)** The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} \Rightarrow p\lambda = h$$

This equation is in the form of $yx = c$, which is the equation of a rectangular hyperbola. Hence, the graph given in option (b) is the correct one.

- 21 (b)** When charged particle is moving on a circular path in a uniform magnetic field B , then

$$\frac{mv^2}{r} = qvB \Rightarrow \frac{mv}{r} = qB \Rightarrow v = \frac{qBr}{m}$$

$$\text{Thus, kinetic energy} = \frac{1}{2}mv^2 = \frac{1}{2}m \times \left(\frac{qBr}{m}\right)^2 = \frac{1}{2m}(qBr)^2$$

Now for photoelectric effect, Einstein's equation is

$$h\nu = W + (\text{KE})$$

where,

W = work function

$$\Rightarrow h\nu - \text{KE} = W \Rightarrow W = h\nu - \frac{(qBr)^2}{2m}$$

Here, charged particle is a photoelectron denoted by e

$$\therefore W = h\nu - \frac{(eBr)^2}{2m_e}$$

- 22 (b)** Given that, $2 \text{ eV} \leq \phi \leq 5 \text{ eV}$

Wavelength corresponding to minimum and maximum values of work function are

$$\lambda_{\text{max}} = \frac{hc}{E} = \frac{4 \times 10^{-15} \times 3 \times 10^8}{2}$$

$$= 6 \times 10^{-7} \text{ m} = 600 \text{ nm}$$

$$\text{and } \lambda_{\text{min}} = \frac{hc}{E} = \frac{4 \times 10^{-15} \times 3 \times 10^8}{5} \\ = 2.4 \times 10^{-7} \text{ m} = 240 \text{ nm}$$

Clearly, wavelength range would be

$$240 \text{ nm} \leq \lambda \leq 600 \text{ nm}$$

Thus, wavelength 650 nm is not suitable for photoelectric effect.

- 23 (d)** The de-Broglie wavelength is

$$\lambda = \frac{h}{mv} \Rightarrow m = \frac{h}{\lambda v} \Rightarrow m \propto \frac{1}{\lambda}, \text{ if } \frac{h}{v} = \text{constant}$$

Again, $\frac{h}{v}$ will be constant only when v is constant.

Thus, in present case the de-Broglie wavelength of heavier particle will be smaller, when the velocities of two particles will be same.

- 24 (a)** The de-Broglie wavelength of electron associated with voltage is given by

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm} = \frac{1.227}{\sqrt{64}} \text{ nm} = 0.153 \text{ nm}$$

- 26 (b)** For de-Broglie wavelength,

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad \dots (i)$$

$$\lambda_2 = \frac{h}{\sqrt{2m \cdot 16K}} = \frac{h}{4\sqrt{2mK}} = \frac{\lambda_1}{4} \quad \dots (ii)$$

$$\lambda_2 = 25\% \text{ of } \lambda_1$$

There is 75% change in the wavelength.

- 27 (b)** From Einstein's photoelectric equation, $(\text{KE})_{\text{max}} = h\nu - \phi_0$

$$\text{For first condition, } 0.5 = E - \phi_0 \quad \dots (i)$$

For second condition,

$$0.8 = \left(E + \frac{E \times 20}{100}\right) - \phi_0$$

$$\Rightarrow 0.8 = 1.2E - \phi_0 \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$0.3 = 0.2E$$

$$\Rightarrow E = \frac{3}{2} = 1.5 \text{ eV}$$

$$\text{From Eq. (i), } \phi_0 = E - 0.5 = 1.5 - 0.5 = 1 \text{ eV}$$

- 28 (a)** According to Einstein's photoelectric equation,

$$K_A = \frac{hc}{\lambda_A} - \phi \Rightarrow K_A + \phi = \frac{hc}{\lambda_A}$$

$$K_B = \frac{hc}{\lambda_B} - \phi = \frac{hc}{(\lambda_A/2)} - \phi$$

$$\text{or } K_B = \frac{2hc}{\lambda_A} - \phi = 2(K_A + \phi) - \phi = 2K_A + \phi \Rightarrow K_A < \frac{K_B}{2}$$

- 29 (b)** Above the threshold frequency, the maximum kinetic energy of the emitted photoelectrons depends on the frequency of the incident light, but is independent of charge, pressure, angle of incidence, etc., of the incident light.

- 30** (c) From photoelectric equation, $E_K = E - W$

where, E_K is kinetic energy of emitted photoelectrons, W is the work function and E is the energy supplied.

$$E = h\nu = \frac{hc}{\lambda}$$

where, h is Planck's constant, c is the speed of light and λ is the wavelength.

$$\therefore E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} = 3.96 \times 10^{-19} \text{ J}$$

$$\text{Also, } 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\therefore E = \frac{3.96 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.48 \text{ eV}$$

$$\text{Hence, } E_K = 2.48 - 1.90 = 0.58 \text{ eV}$$

- 31** (a) We know that, $K_{\max} = eV_0$

where, e is the charge in the electron. Since, at the stopping potential the photoelectric current cannot be obtained by increasing the intensity of the incident light. Thus, stopping potential or maximum kinetic energy of the photoelectrons, does not depend upon the intensity of the incident light. If the intensity of light be doubled, then the number of photoelectrons emitted per second is doubled.

So, n is doubled but $K_{\max}$ remains same.

- 33** (d) If the wavelength of incident light falling on a photosensitive material decreases, then stopping potential increases. This is because

$$V_0 = \frac{h}{e} (\nu - \nu_0) \Rightarrow V_0 = \frac{h}{e} \left(\frac{c}{\lambda} - \frac{c}{\lambda_0} \right) \Rightarrow V_0 \propto \frac{1}{\lambda}$$

- 34** (c) de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mK}}$

$$\text{The kinetic energy of the electron, } K_{\text{electron}} = \frac{h^2}{2m \cdot \lambda^2} \quad \dots(i)$$

where, h = Planck's constant, λ = wavelength

$$\text{The photon energy, } E_{\text{photon}} = \frac{ch}{\lambda} \quad \dots(ii)$$

From Eqs. (ii) and (i), we get

$$\begin{aligned} \frac{E_{\text{photon}}}{K_{\text{electron}}} &= \frac{ch/\lambda}{h^2/2m \cdot \lambda^2} = \frac{hc \cdot \lambda^2 2m}{h^2 \cdot \lambda} = \frac{2m\lambda c}{h} \\ &= \frac{2(mc^2)}{hc/\lambda} = \frac{2 \times (5 \times 10^5)}{50 \times 10^3} = 20 \end{aligned}$$

$$F_{\text{photon}} : K_{\text{electron}} = 20 : 1$$

- 35** (c) As, $(KE)_{\max} = \frac{1}{2} mv_{\max}^2 = h\nu \Rightarrow v_{\max}^2 = \frac{2h\nu}{m}$

$$\therefore v_{\max} = \sqrt{\frac{2h\nu}{m}}$$

- 36** (a) We know, for electron, $\lambda_e = \frac{h}{\sqrt{2m_e E}}$

$$\Rightarrow \lambda_e \propto \frac{1}{\sqrt{E}} \quad \dots(i)$$

$$\text{For photon, } \lambda_p = \frac{hc}{E} \Rightarrow \lambda_p \propto \frac{1}{E} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\lambda_p \propto \lambda_e^2$$

- 37** (c) Given, $\nu = 1.8 \times 10^6 \text{ ms}^{-1}$

$$\frac{e}{m} = 1.8 \times 10^{11} \text{ Ckg}^{-1}$$

$$\text{We have, } eV_0 = \frac{1}{2} mv^2 \Rightarrow V_0 \frac{e}{m} = \frac{v^2}{2}$$

$$\Rightarrow V_0 \times 1.8 \times 10^{11} = \frac{1.8 \times 1.8 \times (10^6)^2}{2} \Rightarrow V_0 = 9 \text{ V}$$

- 38** (a) de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} \quad \text{and} \quad \frac{h}{p} \propto \frac{1}{\sqrt{K}}$$

$$\text{So, } \lambda \propto \frac{1}{\sqrt{K}} \text{ or } \lambda \sqrt{K} = \text{constant}$$

Hence, the graph between λ and K will be a curved line.

- 42** (a) Efficient power, $P = \frac{N}{t} \times \frac{hc}{\lambda} = 200 \times 0.25 = 50$

$$\begin{aligned} \frac{N}{t} &= 50 \times \frac{\lambda}{hc} = \frac{50 \times 0.6 \times 10^{-6}}{6.6 \times 10^{-34} \times 3 \times 10^8} \\ &= 1.5 \times 10^{20} \text{ per second} \end{aligned}$$

- 43** (d) We know, $R = \frac{mv}{qB}$ and $\lambda = \frac{h}{mv}$

$$\lambda = \frac{h}{qBR} = \frac{6.6 \times 10^{-34}}{1.6 \times 10^{-19} \times 0.25 \times 0.83 \times 10^{-2}} = 0.01 \text{ \AA}$$

- 44** (c) Momentum, $p = mv = \frac{h}{\lambda}$ or $v = \frac{h}{m\lambda}$

$$\therefore v = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 5.2 \times 10^{-7}}$$

$$\Rightarrow v = \frac{6.62 \times 10^{-4}}{9.1 \times 5.2} \Rightarrow v \approx 1400 \text{ ms}^{-1}$$

- 45** (a) $K_e = \frac{1}{2} mv^2$ and $\lambda = \frac{h}{mv}$

$$K_e = \frac{1}{2} \left(\frac{h}{\lambda v} \right) v^2 = \frac{vh}{2\lambda} \text{ and } E_p = \frac{hc}{\lambda}$$

$$\therefore \frac{K_e}{E_p} = \frac{v}{2c} = \frac{1.5 \times 10^8}{2 \times 3 \times 10^8} = \frac{1}{4}$$

- 46** (a) According to Einstein's photoelectric equation,

$$K_{\max} = h\nu - h\nu_0$$

$$\text{Initially, } \frac{1}{2} mv_0^2 = h(2\nu_0) - (h\nu_0) = h\nu_0 \quad \dots(i)$$

Then, when, $\nu = 5\nu_0$

$$\frac{1}{2} mv^2 = 5h\nu_0 - h\nu_0 = 4h\nu_0$$

$$\Rightarrow \frac{1}{2} mv^2 = 4 \left(\frac{1}{2} mv_0^2 \right) \quad [\because \text{from Eq. (i)}]$$

$$\Rightarrow v^2 = 4v_0^2 \text{ or } v = \sqrt{2} v_0$$

CHAPTER 12

Atoms

The basic unit of matter and defining structure of any element is atom. In 1898, JJ Thomson for the first time proposed the physical structure of an atom and named it as **Plum-Pudding model**. Then, Rutherford performed an experiment on α -particle scattering in early 90's to investigate the atomic structure. But his work was rejected on the basis of classical theory. However, his shortcomings were rectified by Neil Bohr through his atomic model.

In this chapter, we will study Bohr's model, energy levels, hydrogen spectrum and X-rays followed by Rutherford α -particle scattering experiment.

RUTHERFORD'S α -PARTICLE SCATTERING EXPERIMENT

Rutherford with his two assistants, Geiger and Marsden did an experiment in 1911, in which they directed a narrow beam of α -particle onto gold foil of about 10^{-8} m thickness.

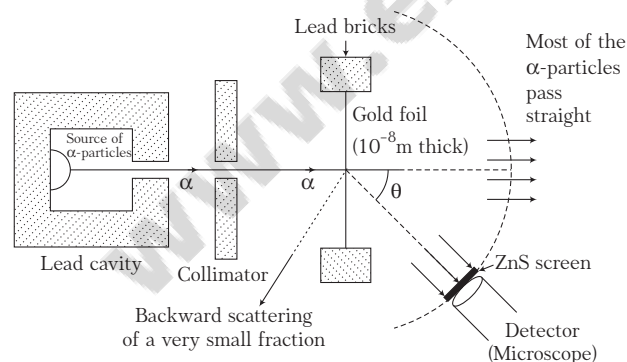


Fig. 12.1 Arrangement for Rutherford's experiment

α -particle were emitted by some radioactive material, $^{214}_{83}\text{Bi}$ kept inside a thick lead box. These α -particles were collimated into narrow beam by their passage through lead bricks. This well collimated beam was then allowed to fall on a thin gold foil. While passing through the gold foil, α -particles scattered through different angles. A zinc sulphide (ZnS) screen and a microscope was placed on the other side of the gold foil. This screen was movable, so as to receive the α -particles, scattered from the gold foil at angles varying from 0° to 180° . When an α -particle strikes the screen, it produces a flash of light which were observed by the microscope.

Inside

1 Rutherford's α -particle scattering experiment

- Rutherford's atomic model

2 Bohr's atomic model

3 Energy of electron in n th orbit

- Energy of atom

4 Hydrogen spectrum

5 X-rays

- Moseley's law for characteristic spectrum
- Absorption of X-rays
- Bragg's law

Observations

Rutherford made following observations from the experiment

- Most of the α -particles pass through the foil straight away undeflected.
- Some of them are deflected through small angles.
- Few α -particles (about 1 in 8000) are deflected through an angle more than 90° .
- A very few α -particles return back, i.e. deflected by 180° . Thus, few α -particles reverse their motion without actually touching the gold foil.
- The scattering of α -particles is proportional to the thickness of target for thin targets.
- The number of α -particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as $\sin^4(\theta/2)$,

$$\text{i.e.} \quad N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

This number $[N(\theta)]$ as a function of θ can be represented by following graph

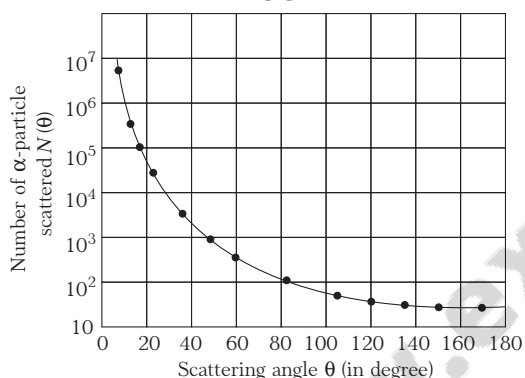


Fig. 12.2 Number of α -particle scattered versus scattering angle θ

Example 12.1 In Rutherford's α -particle experiment with thin gold foil, 8100 scattered α -particles per unit area per minute were observed at an angle of 60° . Find the number of scattered α -particles per unit area per minute at an angle of 120° .

Sol. It is given that, $\theta_1 = 60^\circ$, $\theta_2 = 120^\circ$, $N_1 = 8100$, $N_2 = ?$

Number of α -particles scattered at an angle θ ,

$$\begin{aligned}
 N &\propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} \Rightarrow \frac{N_1}{N_2} = \frac{\left(\sin^4\frac{\theta_2}{2}\right)}{\left(\sin^4\frac{\theta_1}{2}\right)} \\
 \Rightarrow \frac{8100}{N_2} &= \frac{\left(\sin^4\frac{120^\circ}{2}\right)}{\left(\sin^4\frac{60^\circ}{2}\right)} \Rightarrow \frac{8100}{N_2} = \frac{\sin^4 60^\circ}{\sin^4 30^\circ} \\
 \Rightarrow N_2 &= \frac{8100 \times \frac{1}{16}}{\frac{9}{16}} = 900
 \end{aligned}$$

Rutherford's atomic model

The essential features of Rutherford's model of the atom or planetary model of the atom are as follows

- Most of the mass (at least 99.95%) and all of the charge of an atom is concentrated in a very small region of atom called **atomic nucleus**.
- Nucleus is positively charged and its size is of the order of $10^{-15} \text{ m} \approx 1 \text{ fermi}$. The nucleus occupies only about 10^{-12} of the total volume of the atom or less.
- In an atom, maximum space is empty and the electrons revolve around the nucleus in the same way as the planets revolve around the sun (i.e. in elliptical orbits).

Note Electrons are light weight as compared to α -particles, hence much deviation occurs when α -particles are used in scattering experiment. Whereas when α -particle come in contact with nucleus, which is massive, then due to Coulomb's force of repulsion, the path of the α -particles after deflection is hyperbolic but the nucleus remains at rest.

Distance of closest approach

The minimum distance from the nucleus upto which the α -particle approaches is called the distance of closest approach (r_0). At this distance (when particle deflect by 180°), whole of the KE of α -particle converts into electrostatic potential energy,

$$\text{i.e.} \quad \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{(Ze)2e}{r_0}$$

[$\because$ charge on α -particle is $+2e$ and charge on target nucleus is Ze , where Z is atomic number]

$$\Rightarrow r_0 = \frac{1}{4\pi\epsilon_0} \frac{2Ze^2}{\left[\frac{1}{2}mv^2\right]} = \frac{4kZe^2}{mv^2} \quad \left(\because k = \frac{1}{4\pi\epsilon_0}\right)$$

where, m is mass and v is velocity of α -particle.

This distance of closest approach (r_0) can be seen in the figure below

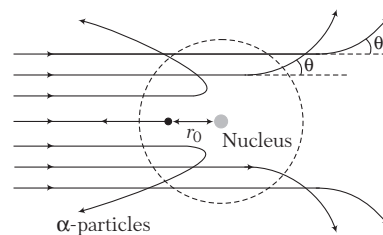


Fig. 12.3 Distance of closest approach (r_0)

Example 12.2 An α -particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. Determine the distance of the closest approach.

Sol. According to law of conservation of energy,

Kinetic energy of α -particle = Potential energy of α -particle
at the distance of closest approach

$$\text{i.e.} \quad \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_0}$$

Given, atomic number of uranium, $Z = 92$

Charge of electron, $e = 1.6 \times 10^{-19} \text{C}$

Charge of α -particle, $q_1 = 2e$

Charge of uranium nucleus, $q_2 = 92e$

$$\therefore 5 \text{ MeV} = \frac{9 \times 10^9 \times (2e) \times (92e)}{r_0} \quad \left(\because \text{given, } \frac{1}{2}mv^2 = 5 \text{ MeV} \right)$$

$$\Rightarrow r_0 = \frac{9 \times 10^9 \times 2 \times 92 \times (1.6 \times 10^{-19})^2}{5 \times 10^6 \times 1.6 \times 10^{-19}} \quad (\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

$$\therefore r_0 = 5.3 \times 10^{-14} \text{ m} \\ = 5.3 \times 10^{-12} \text{ cm}$$

Example 12.3 Atomic number of silver metal is 47. Calculate the speed at which a beam of protons has to be fired at a sheet of silver foil, if the protons were able to approach to within $2.5 \times 10^{-14} \text{ m}$ of the silver nucleus.

Sol. The distance between protons and silver nucleus,

$$r = 2.5 \times 10^{-14} \text{ m}$$

Mass of proton, $m = 1.67 \times 10^{-27} \text{ kg}$

Charge of protons, $q_1 = 1.6 \times 10^{-19} \text{ C}$

Atomic number of silver, $Z = 47$

Charge of silver nucleus, $q_2 = Ze = (47)(1.6 \times 10^{-19})$

By the law of conservation of energy, at distance of closest approach, the kinetic energy of proton must be equal to the potential energy of proton at that point. Therefore, we get

$$K = U \Rightarrow \frac{1}{2}mv^2 = \frac{kq_1 q_2}{r}$$

$$\begin{aligned} \text{Therefore, } v &= \sqrt{\frac{2kq_1 q_2}{mr}} \\ &= \sqrt{\frac{2 \times (9 \times 10^9) (1.6 \times 10^{-19}) (47)(1.6 \times 10^{-19})}{(1.67 \times 10^{-27}) (2.5 \times 10^{-14})}} \\ &= 2.26 \times 10^7 \text{ ms}^{-1} \end{aligned}$$

Impact parameter (b)

Perpendicular distance of the velocity vector of α -particle from the central line of the nucleus of the atom is called impact parameter (as shown in Fig. 12.4).

Mathematically, it is expressed as

$$b = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2 \cot \frac{\theta}{2}}{K}$$

Thus, smaller the impact parameter b , greater the scattering angle θ and greater the kinetic energy of α -particle.

Example 12.4 In Rutherford scattering experiment, what will be the correct angle for α -particles scattering for an impact parameter, $b = 0$?

Sol. We know that, impact parameter, $b \propto \cot \frac{\theta}{2}$

$$\Rightarrow b = K \cot \frac{\theta}{2}$$

where, K is a constant.

Here, if $b = 0$

$$\Rightarrow K \cot \theta/2 = 0 \Rightarrow \cot \theta/2 = 0 \Rightarrow \frac{\theta}{2} = \frac{\pi}{2}$$

$$\text{Hence, } \theta = \pi = 180^\circ$$

α -particle trajectory

The α -particles which pass through the atom at a large distance from the nucleus experience a small electrostatic force of repulsion due to the nucleus and hence, undergo a very small deflection. However, the α -particles which pass through the atom at a smaller distance from the nucleus suffer a large deflection. On the other hand, α -particles which travel towards the nucleus directly, slow down and ultimately come to rest and then after being deflected through 180° retrace their path.

The trajectory of the α -particles at various scattering angles, is shown in the figure below

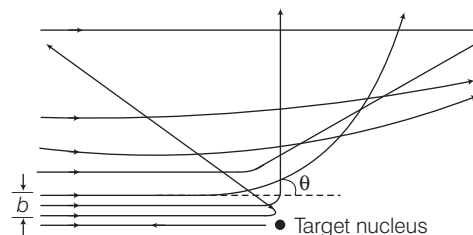


Fig. 12.4 Trajectory of α -particles in the Coulomb field of a target nucleus. The impact parameter b and scattering angle θ are also depicted.

Electron orbits

In the Rutherford's nuclear model of the atom pictures, the atom as an electrically neutral sphere consisting of a very small, massive and positively charged nucleus at the centre surrounded by the revolving electrons in their respective dynamically stable orbits. The electrostatic force of attraction F_e between the revolving electrons and the nucleus provides the requisite centripetal force F_c to keep them in their orbits. Thus, for a dynamically stable orbit in a H-atom,

$$\begin{aligned} F_c &= F_e \\ \Rightarrow \frac{mv^2}{r} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{r^2} \quad (\because Z = 1) \end{aligned}$$

Thus, the relation between the orbit radius and the

electron velocity, $r = \frac{e^2}{4\pi\epsilon_0 mv^2}$.

The kinetic energy K and electrostatic potential energy U of the electron in H-atom are

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\epsilon_0 r} \quad \left(\because mv^2 = \frac{e^2}{4\pi\epsilon_0 r} \right)$$

and $U = -2K = -\frac{e^2}{4\pi\epsilon_0 r}$

(the negative sign in U signifies that the electrostatic force is in the $-r$ direction or attractive in nature.) Thus, the total mechanical energy E of the electron in a H-atom is

$$E = K + U = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r}$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If E were positive, an electron will not follow a closed orbit around the nucleus and it would leave the atom.

Example 12.5 It is found experimentally that 13.6 eV energy is required to separate a hydrogen atom into a proton and an electron. Calculate the velocity of the electron in hydrogen atom.

Sol. Total energy of the electron in hydrogen atom
 $= -13.6 \text{ eV} = -13.6 \times 1.6 \times 10^{-19} \text{ J}$
 $= -2.2 \times 10^{-18} \text{ J}$

Total energy of the electron in hydrogen atom, $E = -\frac{e^2}{8\pi\epsilon_0 r}$

$$\therefore -\frac{e^2}{8\pi\epsilon_0 r} = -2.2 \times 10^{-18}$$

Orbital radius, $r = -\frac{e^2}{8\pi\epsilon_0 (-2.2 \times 10^{-18})}$

$$\Rightarrow r = \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(2)(2.2 \times 10^{-18})} \quad \left(\because \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2} \right)$$

or $r = 5.3 \times 10^{-11} \text{ m}$

The velocity of the revolving electron is given by

$$\Rightarrow v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}}$$

Putting, $m = 9.1 \times 10^{-31} \text{ kg}$ and $r = 5.3 \times 10^{-11} \text{ m}$

We have $v = \frac{1.6 \times 10^{-19}}{\sqrt{\left(\frac{1}{9 \times 10^9}\right) \times 9.1 \times 10^{-31} \times 5.3 \times 10^{-11}}}$

or $v = 2.2 \times 10^6 \text{ m/s}$

Drawbacks of Rutherford's model

Rutherford's model suffers two major drawbacks

(i) Regarding stability of atom

Electrons revolving around the nucleus have centripetal acceleration. According to classical electromagnetic theory, accelerated charged particles must radiate energy in the form of electromagnetic wave.

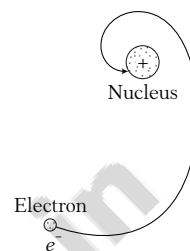


Fig. 12.5 Classically an atomic electron should spiral rapidly into the nucleus as it radiates energy due to its acceleration.

Therefore, the energy of the accelerating electron should, continuously decreases.

Due to this continuous loss of energy of the electrons, the radii of their orbits should be continuously decreasing and ultimately the electrons should fall in the nucleus (as shown in Fig. 12.5). Thus, such an atom cannot remain stable.

(ii) Regarding explanation of line spectrum

Due to continuous decrease in radii of electron's orbit, the frequency of revolution of electron will also change. According to classical theory of electromagnetism, frequency of EM wave emitted by electron is equal to frequency of revolution of electron.

So, due to continuous change in frequency of revolution of electron, it will radiate EM waves of all frequencies, i.e. the spectrum of these waves will be continuous in nature. But, this is not the case, experimentally we get line spectrum. Thus, Rutherford's model was unable to explain the line spectrum.

BOHR'S ATOMIC MODEL

Bohr proposed a model for hydrogen atom which is applicable for some lighter atoms in which a single electron revolves around a stationary nucleus of positive charge which are usually known as **hydrogen like atom**, e.g. ${}_1\text{H}^1$, ${}_1\text{H}^2$, He^+ , Li^{++} , etc.

Bohr's model is based on the following three postulates

- (i) Bohr postulated that an electron in an atom can move around nucleus in certain circular **stable orbits** without emitting radiations.

- (ii) Electrons can revolve only in those orbits in which their angular momentum is an integral multiple of $h/2\pi$, where h is Planck's constant. If the mass of the electron be m and it is revolving with velocity v in an orbit of radius r , then its angular momentum will be mvr .

According to Bohr's postulate, we have

$$\text{angular momentum, } mvr = \frac{nh}{2\pi}$$

where, n is an integer ($n=1, 2, 3, \dots$) and is called the **principal quantum number** of the orbit. This equation is called **Bohr's quantisation condition**.

- (iii) When the atom receives energy from outside, then one (or more) of its outer electrons leaves its orbit and goes to some higher orbit. This state of the atom is called **excited state**.

The electron in the higher orbit stays only for 10^{-8} s and returns back to lower orbit. While returning back, the electron radiates energy in the form of electromagnetic waves.

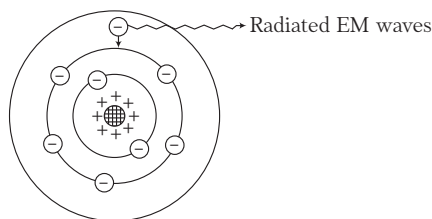


Fig. 12.6 Electron falls down to lower orbit and emits energy

If the energy of electron in the higher orbit be E_2 and that in the lower orbit be E_1 , then the frequency ν of the radiated waves is given by

$$h\nu = E_2 - E_1$$

$$\text{or } \nu = \frac{E_2 - E_1}{h}$$

This equation is called **Bohr's frequency condition**.

Example 12.6 Compute the angular momentum in 4th orbit, if L is the angular momentum of the electron in the 2nd orbit of hydrogen atom.

Sol. We know that, the angular momentum of the electron of hydrogen atom in n th orbit,

$$L_n = \frac{nh}{2\pi} \quad \dots(i)$$

But according to question, the angular momentum in 2nd orbit ($n=2$) is L .

$$\text{So, } L = \frac{2h}{2\pi} = \frac{h}{\pi} \quad \dots(ii)$$

Hence, the angular momentum in 4th orbit is

$$L_4 = \frac{4h}{2\pi} = \frac{2h}{\pi} = 2L \quad [\text{from Eq. (ii)}]$$

Mathematical analysis of Bohr's atomic model of hydrogen like atoms

A hydrogen like atom consists of a tiny positively charged nucleus and an electron revolving in a stable circular orbit around it as shown in the figure below. Let e , m and v be respectively, the charge, mass and velocity of the revolving electron and r be the radius of the orbit. The positive charge of the nucleus is Ze , where Z is the atomic number (in case of hydrogen atom, $Z=1$). As the centripetal force is provided by the electrostatic force of attraction, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{(Ze) \times e}{r^2} \quad \text{or} \quad mv^2 = \frac{Ze^2}{4\pi\epsilon_0 r} \quad \dots(i)$$

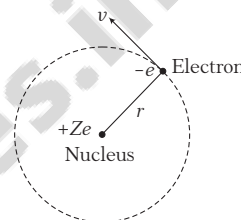


Fig. 12.7 An electron is revolving around a nucleus of hydrogen like atoms

From the first postulate, the angular momentum of the electron,

$$mvr = n \frac{h}{2\pi} \quad \dots(ii)$$

Radius of electron in n th orbit

Squaring Eq. (ii) and dividing by Eq. (i), we get

$$r_n = n^2 \frac{h^2 \epsilon_0}{\pi m Ze^2} \quad (\because r = r_n) \quad \dots(iii)$$

This is the equation for the radii of the permitted orbits.

According to this equation, $r_n \propto n^2$.

Since, $n=1, 2, 3, \dots$, and so on, it follows that the radii of the permitted orbits increase in the ratio $1:4:9:16 \dots$ and so on, from the first orbit.

Bohr's radius

The radius of the first orbit ($n=1$) of H-atom ($Z=1$) will be

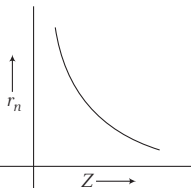
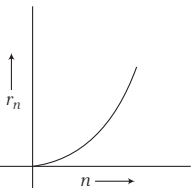
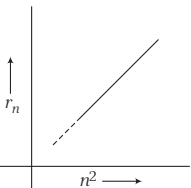
$$r_1 = \frac{h^2 \epsilon_0}{\pi m e^2} \quad \dots(iv)$$

Substituting the value of constants, we get $r_1 = 0.53 \text{ \AA}$. This is called Bohr's radius. From Eqs. (iii) and (iv), we get Radius of n th orbit is,

$$r_n = 0.53 \frac{n^2}{Z} \text{ \AA} = \frac{n^2}{Z} r_1$$

Variations of r_n

Using the relation, $r_n = \frac{n^2}{Z} r_1$, we can plot the following graphs

r_n versus Z	r_n versus n	r_n versus n^2
 Fig. 12.8 r_n versus Z graph Here, $r_n \propto 1/Z$	 Fig. 12.9 r_n versus n graph Here, $\sqrt{r_n} \propto n$	 Fig. 12.10 r_n versus n^2 graph Here, $r_n \propto n^2$
Nature of graph will be rectangular hyperbola.	Nature of graph will be parabola.	Nature of graph will be straight line.

Example 12.7 Using known values for hydrogen atom, calculate radius of third orbit for Li^{2+} .

Sol. As, for Li^{2+} ($Z = 3$),

Further we know that, $r_n = \frac{n^2}{Z} r_1$

Substituting, $n = 3$, $Z = 3$ and $r_1 = 0.529 \text{ \AA}$

We have, $r_3 = \frac{(3)^2}{(3)} (0.529) \text{ \AA} = 1.587 \text{ \AA}$

Speed of electron in n th orbit

From Eq. (ii), we have $v_n = n \frac{h}{2\pi m r}$

Putting the value of r from Eq. (iii), we get

$$v_n = \frac{Ze^2}{2h\epsilon_0} \frac{1}{n} \quad \dots(\text{v})$$

where, principal quantum number,

$n = 1, 2, 3, \dots$, and so on.

Thus, $v_n \propto \frac{1}{n}$

This shows that the velocity of electron is maximum in the lowest orbit ($n = 1$) and goes on decreasing in higher orbits.

The velocity of electron in the first orbit ($n = 1$) of H-atom ($Z = 1$) is

$$v_1 = \frac{e^2}{2h\epsilon_0} = \frac{c}{137} \approx 2.2 \times 10^6 \text{ ms}^{-1} \quad \dots(\text{vi})$$

Thus, from Eqs. (v) and (vi), we get

Velocity of electron in n th orbit is,

$$v_n = 2.2 \times 10^6 \frac{Z}{n} \text{ ms}^{-1} = \frac{Z}{n} v_1$$

Variations of v_n

Using the relation, $v_n = \frac{Z}{n} v_1$, we can plot the following graph

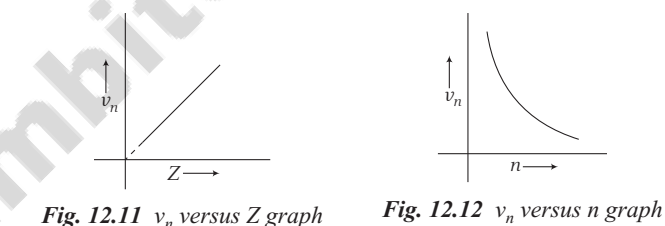


Fig. 12.11 v_n versus Z graph

Here, $v_n \propto Z$

Nature of graph will be straight line.

Fig. 12.12 v_n versus n graph

Here, $v_n \propto 1/n$

Nature of graph will be rectangular hyperbola.

Example 12.8 Using known values for hydrogen atom, calculate speed of electron in fourth orbit of He^+ .

Sol. As, for He^+ ($Z = 2$),

Also, we know that, $v_n = \frac{Z}{n} v_1$

Substituting $n = 4$, $Z = 2$ and $v_1 = 2.2 \times 10^6 \text{ ms}^{-1}$, we get

$$v_4 = \left(\frac{2}{4}\right) (2.2 \times 10^6) = 1.1 \times 10^6 \text{ ms}^{-1}$$

Time period and frequency of revolution of electron in n th orbit

Number of revolutions completed per second by the electron in a stationary orbit, around the nucleus is known as its **frequency**. It is represented by ν .

From $v_n = r\omega_n = r_n(2\pi\nu_n)$ ($\because \omega_n = 2\pi\nu_n$)

$$\therefore \nu_n = \frac{v_n}{2\pi r_n}$$

Putting the value of v_n in above equation, we get

$$v_n = \frac{1}{2\pi r_n} \frac{Ze^2}{2h\epsilon_0} \frac{1}{n} \quad [\text{from Eq. (v)}]$$

$$\Rightarrow v_n = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{nhr_n} \Rightarrow v_n = \frac{kZe^2}{nhr_n} \quad \left(\because k = \frac{1}{4\pi\epsilon_0} \right)$$

$\therefore$ Time period of revolution of electron in n th orbit,

$$T_n = \frac{1}{v_n} = \frac{nhr_n}{kZe^2}$$

Since, $r_n \propto \frac{n^2}{Z}$

$\therefore T_n \propto \frac{n^3}{Z^2}$ and $v_n \propto \frac{Z^2}{n^3}$

Also, $\omega_n = 2\pi v_n \Rightarrow \omega_n \propto \frac{Z^2}{n^3}$

CHECK POINT 12.1

- The radius of an atom is of the order of
(a) 10^{-8} m (b) 10^{-9} m
(c) 10^{-11} m (d) 10^{-10} m
- An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze . Then, the distance of closest approach for the alpha nucleus will be proportional to
(a) v^2 (b) $1/m$ (c) $1/v^4$ (d) $1/Ze$
- The number of α -particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as
(a) $\cos^4\left(\frac{\theta}{2}\right)$ (b) $\sin^4\left(\frac{\theta}{2}\right)$
(c) $\tan^4\left(\frac{\theta}{2}\right)$ (d) $\cot^4\left(\frac{\theta}{2}\right)$
- In Rutherford scattering experiment for scattering angle of 360° , what will be the value of impact parameter?
(a) 0 (b) ∞
(c) 1 (d) Data is insufficient
- Bohr's atom model assumes
(a) the nucleus is of infinite mass and is at rest
(b) electrons in a quantised orbit will not radiate energy
(c) mass of electron remains constant
(d) All the above conditions
- The angular momentum (L) of an electron moving in a stable orbit around nucleus is
(a) half integral multiple of $\frac{h}{2\pi}$
(b) integral multiple of h
(c) integral multiple of $\frac{h}{2\pi}$
(d) half integral multiple of h

Example 12.9 Find the ratio of product of velocity and time period of electron orbiting in 2nd and 3rd stable orbits for an hydrogen like atom.

Sol. We know that, time period $T_n \propto n^3$ and velocity $v_n \propto 1/n$.

$$\Rightarrow \text{Time period } (T_n) \times \text{velocity } (v_n) \propto n^2$$

$$\therefore \frac{T_2 v_2}{T_3 v_3} = \frac{2^2}{3^2} = \frac{4}{9}$$

Example 12.10 In the Bohr model of hydrogen atom, the electron is pictured to rotate in a circular orbit of radius 5×10^{-11} m at a speed 2.2×10^6 ms $^{-1}$. Calculate the current associated with electron motion?

Sol. Frequency of revolution of electron can be written as,

$$v = \frac{v}{2\pi r} = \frac{2.2 \times 10^6}{2\pi (5 \times 10^{-11})} = 7 \times 10^{15} \text{ Hz}$$

$$\therefore \text{Current associated, } I = q/t = qv \quad \left(\because \frac{1}{t} = v \right)$$

$$= (1.6 \times 10^{-19})(7 \times 10^{15}) = 11.2 \times 10^{-4} \text{ A} = 1.12 \text{ mA}$$

- In Bohr's model, the atomic radius of the first orbit is r_0 , then the radius of the third orbit is
(a) $r_0/9$ (b) r_0
(c) $9r_0$ (d) $3r_0$
- In which of the following systems will the radius of the first orbit ($n=1$) be minimum?
(a) Hydrogen atom
(b) Deuterium atom
(c) Singly ionised helium
(d) Doubly ionised lithium
- The ratio of the speed of the electrons in the ground state of hydrogen atom to the speed of light in vacuum is
(a) $1/2$ (b) $2/137$
(c) $1/137$ (d) $1/237$
- Speed (v_n) of an electron in n th orbit *versus* principal quantum number (n) graph is

(a)

(b)

(c)

(d)
- The orbital frequency of an electron in hydrogen like atom is proportional to
(a) n^3 (b) n^{-3}
(c) n (d) n^0

ENERGY OF ELECTRON IN n th ORBIT

The energy E of an electron in an orbit is the sum of kinetic and potential energies.

Kinetic energy

Electron possess kinetic energy because of its motion. Closer orbits have greater kinetic energy than outer ones.

Mathematically, $KE_n = \frac{1}{2}mv_n^2$

Substituting the value of v_n from Eq. (v), we get

i.e.
$$v_n = \frac{Ze^2}{2h\epsilon_0} \left(\frac{1}{n} \right)$$

we get
$$KE_n = \frac{mZ^2e^4}{8\epsilon_0^2n^2h^2}$$

or
$$KE_n = \frac{mZ^2e^4}{8\epsilon_0^2h^2} \left(\frac{1}{n^2} \right) \quad \dots(\text{vii})$$

In simplified form,

$$KE_n = \frac{RhcZ^2}{n^2}$$

where, $R = \frac{me^4}{8\epsilon_0^2ch^3} = \text{Rydberg's constant}$
 $= 1.09 \times 10^7 \text{ m}^{-1}$

Potential energy

The potential energy of the electron in an orbit is due to the electrostatic attraction by nucleus.

Mathematically,
$$U_n = \frac{1}{4\pi\epsilon_0} \frac{(Ze)(-e)}{r_n}$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_n}$$

Substituting the value of r_n from Eq. (iii), we get

i.e.
$$r_n = \frac{n^2h^2\epsilon_0}{\pi mZe^2}$$

we get
$$U_n = \frac{-mZ^2e^4}{4\epsilon_0^2n^2h^2} \quad \dots(\text{viii})$$

In simplified form,

$$U_n = -\frac{2RhcZ^2}{n^2}$$

Total energy

The total energy of the electron,

$$E_n = KE_n + U_n = \frac{Ze^2}{8\pi\epsilon_0r_n} - \frac{Ze^2}{4\pi\epsilon_0r_n} = -\frac{Ze^2}{8\pi\epsilon_0r_n}$$

Substituting for r_n from Eq. (iii), we get

$$E_n = \frac{-mZ^2e^4}{8\epsilon_0^2h^2} \left(\frac{1}{n^2} \right) \quad \dots(\text{ix})$$

This is the expression for the total energy of the electron in the n th orbit

or
$$E_n = \frac{-RhcZ^2}{n^2}$$

Also, from the above equations, we can write

$$U_n = -2KE_n$$

$$\Rightarrow |U_n| = 2|KE_n|$$

$$\therefore E_n = KE_n - 2KE_n = -KE_n$$

Some general points related to the energy of electron

(i) $Rhc = 1 \text{ Rydberg} = 13.6 \text{ eV}$.

(ii) For hydrogen atom $Z=1$, substituting the standard values, we get $E_n = \frac{-13.6}{n^2} \text{ eV}$.

(iii) Negative energy of the electron shows that the electron is bound to the nucleus and is not free to leave it.

(iv) For $n=1$, $E_1 = -13.6 \text{ eV}$. Substituting $n=2, 3, 4, \dots$ etc, we get energies of electron in different orbits.
i.e. $E_2 = -3.40 \text{ eV}$, $E_3 = -1.51 \text{ eV}$... $E_\infty = 0$.

Example 12.11 The ground state energy of hydrogen atom is -13.6 eV . What is the kinetic and potential energies of the electron in this state?

Sol. As, we know that, kinetic energy = $-$ total energy
 $= -(-13.6) \text{ eV} = 13.6 \text{ eV}$
and potential energy = $+2$ (total energy)
 $= 2(-13.6) \text{ eV} = -27.2 \text{ eV}$

Example 12.12 The total energy of electron in the ground state of hydrogen atom is -13.6 eV . Find the kinetic energy of an electron in the first excited state.

Sol. The energy of electron in hydrogen atom when it revolves in n th orbit,

$$E_n = \frac{-13.6}{n^2} \text{ eV}$$

In the ground state ($n=1$),

$$\therefore E = \frac{-13.6}{1^2} = -13.6 \text{ eV}$$

For first excited state ($n=2$), $E = \frac{-13.6}{2^2} = -3.4 \text{ eV}$

So, kinetic energy of electron in the first excited state, i.e. for ($n = 2$), kinetic energy = - total energy

$$K = -E = -(-3.4) = 3.4 \text{ eV}$$

Energy of atom

According to Bohr's theory, in a given stationary orbit, an electron has a definite value of energy. Thus, the energy associated with hydrogen or hydrogen like atom is given by this definite value of energy of the electron in the n th orbit. Therefore, energy of the n th orbit of the hydrogen like atom is

$$E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}$$

where, Z = atomic number of the atom.

The electrons in the atom can jump from one orbit to another when the atom emits or absorbs energy.

During absorption of energy, electron jumps from lower to higher orbit. Similarly, during emission of energy, an electron jumps from higher to lower orbit. When electron jumps from higher to lower or lower to higher orbits respectively, it emits or absorb a photon and the energy of a photon is equal to the difference of energies of two orbits.

Let an electron of the atom jumps from n_1 to n_2 ($n_2 > n_1$).

$$\therefore E_{n_2} - E_{n_1} = -13.6 \frac{Z^2}{n_2^2} - \left(-13.6 \frac{Z^2}{n_1^2} \right)$$

$$\Rightarrow \Delta E = 13.6Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Energy of absorbed photon, $\Delta E = h\nu = \frac{hc}{\lambda}$

where, ν = frequency of absorbed photon,

λ = wavelength of absorbed photon

and c = speed of light.

So, we can write

$$\Delta E = h\nu = \frac{hc}{\lambda} = E_{n_2} - E_{n_1} = \frac{Z^2 me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\Rightarrow \frac{1}{\lambda} = \frac{Z^2 me^4}{8h^3 \epsilon_0^2 c} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\text{or } \boxed{\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)}$$

where, R is Rydberg's constant.

Note The reciprocal of the wavelength is called **wave number**.

$$\Rightarrow \nu = 1/\lambda$$

Energy levels for hydrogen atom

Energy of n th state of an atom is

$$E_n = -13.6 \frac{Z^2}{n^2} \text{ eV} \quad \dots(i)$$

$$\text{For hydrogen } (Z = 1), E_n = -\frac{13.6}{n^2} \text{ eV}$$

The energy of an atom is least, when its electron is revolving in an orbit closest to the nucleus, i.e. for which $n = 1$.

$\therefore E_1 = -13.6 \text{ eV}$, this is the energy of the electron, when it revolves in smallest allowed orbit.

For $n = 2$,

$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV, etc.}$$

For $n = 3, 4, \dots$, and so on, the absolute value of energy E decreases, so the energy is progressively larger in outer orbits as shown in the Fig. 12.13.

The lowest state of an atom is called the **ground state**, this state has lowest energy. The energy of this state for H-atom is -13.6 eV .

At room temperature, most of the hydrogen atoms are in ground state. When these atoms receive energy by a process such as electron collision, the atom may acquire sufficient energy to raise the electron to higher energy states. The atom is then said to be in an **excited state**.

From Eq. (i), for $n = 2$, the energy E_2 is -3.40 eV . It means that, the energy required to excite an electron in hydrogen atom to its first excited state is

$$E_2 - E_1 = -3.40 \text{ eV} - (-13.6) \text{ eV} = 10.2 \text{ eV}.$$

Similarly, $E_3 = -1.51 \text{ eV}$, so $E_3 - E_1 = 12.09 \text{ eV}$ or to excite the hydrogen atom from its ground state ($n = 1$) to second excited state ($n = 3$), 12.09 eV energy is required, and so on.

From these excited states, the electron can then fall back to a state of lower energy, emitting a photon in the process. Thus, as the excitation of hydrogen atom increases (that is as n increases) the value of minimum energy required to free the electron from the excited atom decreases.

The energy level diagram for the stationary state of a hydrogen atom, computed from Eq. (i), is given in Fig. 12.13.

The principal quantum number n labels the stationary states in the ascending order of energy. In this diagram, the highest energy state corresponds to $n = \infty$ in Eq. (i) and has an energy of 0 eV . This is the energy of the atom when the electron is completely removed ($r = \infty$) from the nucleus and is at rest.

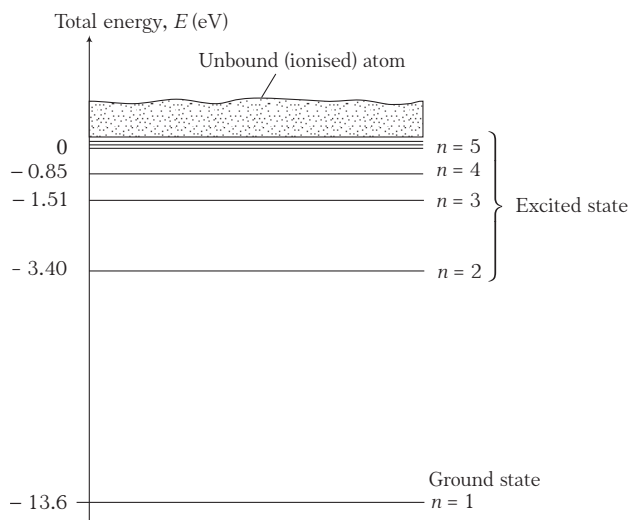


Fig. 12.13 The energy level diagram for the hydrogen atom.

Example 12.13 A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom make a transition from the upper level to the lower level?

Sol. Here, difference of energy,

$$\Delta E = 2.3 \text{ eV} = 2.3 \times 1.6 \times 10^{-19} \text{ J} \quad (\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

Using the relation, $h\nu = \Delta E = E_2 - E_1$

$$\text{or} \quad \nu = \frac{\Delta E}{h} = \frac{2.3 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 0.5559 \times 10^{15} \approx 5.6 \times 10^{14} \text{ Hz}$$

Example 12.14 Determine the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.

Sol. Given, for Li^{++} , $Z = 3$ and as the excitation is from first to third Bohr orbit, so $n_1 = 1$, $n_2 = 3$

Using the relation,

$$\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = (3)^2 R \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = 8R$$

$$\Rightarrow \text{Wavelength, } \lambda = \frac{1}{8R} = \frac{1}{8 \times 1.097 \times 10^7} = 0.114 \times 10^{-7} = 11.4 \text{ nm}$$

Example 12.15 A hydrogen atom initially in the ground level absorbs a photon, which excites it to $n = 4$ level. Determine the wavelength and frequency of photon.

Sol. If $E = h\nu$ be the energy absorbed, then

we know, $h\nu = \Delta E = E_4 - E_1$

Now, using the relation, $E_n = \frac{-13.6}{n^2}$

$$\text{For } n = 4, \quad E_4 = \frac{-13.6}{4^2} = \frac{-13.6}{16} = -0.85 \text{ eV} \quad \dots(i)$$

As, H-atom initially at ground level, for $n = 1$,

$$E_1 = -13.6 \text{ eV} \quad \dots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$h\nu = -0.85 - (-13.6)$$

$$= 12.75 \text{ eV}$$

$$= 12.75 \times 1.6 \times 10^{-19} \text{ J}$$

$$\nu = \frac{12.75 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}}$$

$$= 3.08 \times 10^{15} \text{ Hz}$$

$$(\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J and } h = 6.62 \times 10^{-34} \text{ J-s})$$

Using the relation $c = \nu\lambda$, we get

$$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3.08 \times 10^{15}}$$

$$= 974.02 \times 10^{-10} \text{ m} \approx 974 \text{ \AA}$$

Example 12.16 A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n , a photon of energy 40.8 eV is emitted. Find n , Z and the ground state energy (in eV) for this atom. Also, calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. (Take, ground state energy of hydrogen atom = 13.6 eV)

Sol. Given, $E_{2n} - E_1 = 204 \text{ eV}$

Using the relation $\Delta E = -13.6Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$, we get

$$(13.6)Z^2 \left(1 - \frac{1}{4n^2} \right) = 204 \quad \dots(i)$$

Similarly, $E_{2n} - E_n = 40.8 \text{ eV}$

$$\therefore 13.6 Z^2 \left(\frac{1}{n^2} - \frac{1}{4n^2} \right) = 40.8 \quad \dots(ii)$$

Solving Eqs. (i) and (ii), we get

$$n = 2 \text{ and } Z = 4$$

We know that, in ground state ($n = 1$), $E = -13.6Z^2 \text{ eV}$

$$\text{So, } E_1 = (-13.6) Z^2 \text{ eV}$$

$$= (-13.6) (4)^2 \text{ eV} = -217.6 \text{ eV}$$

During de-excitation, minimum energy emitted is

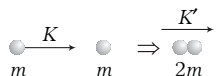
$$E_{\min} = E_{2n} - E_{2n-1} = E_4 - E_3$$

$$= \frac{-217.6}{4^2} - \left(\frac{-217.6}{3^2} \right)$$

$$= 10.58 \text{ eV}$$

Example 12.17 A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state?

Sol. Let K be the kinetic energy of the moving hydrogen atom and K' , the kinetic energy of combined mass after collision.

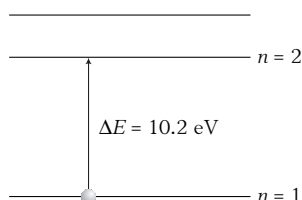


From conservation of linear momentum,

$$p = p' \quad \text{or} \quad \sqrt{2Km} = \sqrt{2K'(2m)} \quad \text{or} \quad K = 2K' \quad \dots(i)$$

From conservation of energy,

$$K = K' + \Delta E \quad \dots(ii)$$



Solving Eqs. (i) and (ii), we get

$$\Delta E = \frac{K}{2}$$

Using the relation,

$$\Delta E = -13.6Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For $n_1 = 1$ and $n_2 = 2$, we get

$$\Delta E = -13.6(1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 10.2 \text{ eV} \quad (\because Z = 1)$$

So, minimum value of ΔE for hydrogen atom is 10.2 eV.

or $\Delta E \geq 10.2 \text{ eV}$

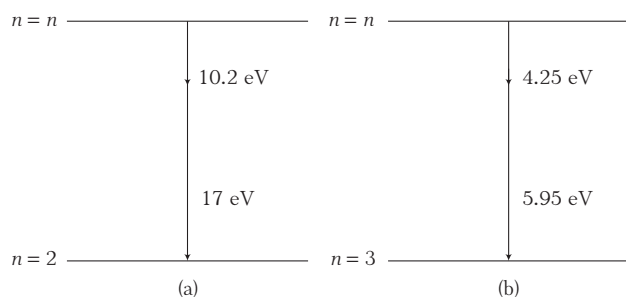
$$\therefore \frac{K}{2} \geq 10.2$$

or $K \geq 20.4 \text{ eV}$

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV.

Example 12.18 A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n . This atom then can make a transition to first excited state by emitting two photons successively of energies 10.2 eV and 17 eV, respectively. Similarly, it make transition to second excited state by emitting two photons successively of energies 4.25 eV and 5.95 eV, respectively. Determine the values of Z and n .

Sol. According to the question,



We know, energy of electron in n th orbit is given by

$$\Delta E = 13.6Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Case I

Energy emitted by photon, $\Delta E = 10.2 + 17 = 27.2 \text{ eV}$

$$\text{So,} \quad \Delta E = 13.6Z^2 \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad (\because n_1 = 2, n_2 = n)$$

$$\Rightarrow 27.2 = 13.6Z^2 \left(\frac{1}{4} - \frac{1}{n^2} \right) \quad \dots(i)$$

Case II

Energy emitted by photons, $\Delta E = 4.25 + 5.95 = 10.2 \text{ eV}$

$$\Delta E = 13.6Z^2 \left(\frac{1}{3^2} - \frac{1}{n^2} \right) \quad (\because n_1 = 3, n_2 = n)$$

$$\Rightarrow 10.2 = 13.6Z^2 \left(\frac{1}{9} - \frac{1}{n^2} \right) \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\begin{aligned} \frac{27.2}{10.2} &= \frac{1/4 - 1/n^2}{1/9 - 1/n^2} \\ \Rightarrow \frac{27.2}{9} - \frac{27.2}{n^2} &= \frac{10.2}{4} - \frac{10.2}{n^2} \\ \Rightarrow \frac{17.2}{n^2} &= \frac{27.9}{9} - \frac{10.2}{4} \\ \Rightarrow n^2 &= 36 \quad \text{or} \quad n = 6 \end{aligned}$$

Substituting the value of n in Eq. (i), we get

$$\begin{aligned} 27.2 &= 13.6Z^2 \left(\frac{1}{4} - \frac{1}{6^2} \right) \\ &= 13.6Z^2 \left(\frac{1}{4} - \frac{1}{36} \right) = 13.6Z^2 \left(\frac{9-1}{36} \right) = 13.6Z^2 \times \frac{2}{9} \\ \Rightarrow Z^2 &= 9 \quad \text{or} \quad Z = 3 \end{aligned}$$

Excitation energy and excitation potential

The energy required to take an electron from its lower energy state to higher energy state is known as **excitation energy**. For example 10.2 eV is excitation energy for an electron to move from $n = 1$ to $n = 2$ in H-atom. Also, 12.1 eV is excitation energy from $n = 1$ to $n = 3$. Similarly, the potential through which an electron should be accelerated to gain higher state is called **excitation potential**. For H-atom, when electron moves from $n = 1$ to $n = 2$ and $n = 1$ to $n = 3$ excitation potentials are 10.2 V and 12.1 V, respectively.

Ionisation energy and ionisation potential

The energy required to ionise an atom is called **ionisation energy**. It is the energy required to make the electron jump from the present orbit to the infinite orbit.

∴ Ionisation energy,

$$E_{\text{ionisation}} = E_{\infty} - E_n = 0 - \left[-13.6 \frac{Z^2}{n^2} \right] = + \frac{13.6Z^2}{n^2} \text{ eV}$$

Similarly, the potential difference through which an electron should be accelerated to get ionised is called **ionisation potential**.

$$\therefore \text{Ionisation potential} = \frac{E_{\text{ionisation}}}{e} = \frac{13.6Z^2}{n^2} \text{ V}$$

Binding energy

The energy required to separate the constituent of a nucleus to infinity is called binding energy. The binding energy of hydrogen atom in ground state is therefore

$$\text{BE} = -(E_1) = -(-13.6 \text{ eV}) = 13.6 \text{ eV}$$

Example 12.19 A doubly ionised lithium atom is hydrogen like with atomic number 3. Find the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. The ionisation energy of the hydrogen atom is 13.6 eV.

Sol. We know, $E_n = -\frac{Z^2}{n^2} (13.6 \text{ eV})$

By putting $Z = 3$ (given), we have

$$E_n = -\frac{122.4}{n^2} \text{ eV}$$

As, excitation is from 1st to 3rd Bohr orbit,

i.e. $n_1 = 1$ to $n_2 = 3$.

$$\Rightarrow E_1 = -\frac{122.4}{(1)^2} = -122.4 \text{ eV}$$

$$\text{and } E_3 = -\frac{122.4}{(3)^2} = -13.6 \text{ eV}$$

$$\therefore \Delta E = E_3 - E_1 = -13.6 - (-122.4) = 108.8 \text{ eV}$$

The corresponding wavelength is

$$\lambda = \frac{12375}{\Delta E (\text{in eV})} = \frac{12375}{108.8} = 113.74 \text{ \AA}$$

Absorption and emission spectrum

An atom can be excited by incident photon or by collision. If an atom absorbs a photon, it goes to higher orbits/state and remains, there for a time of 10^{-8} s and returns back to ground state by different ways.

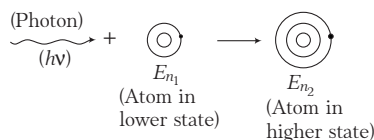


Fig. 12.14 Absorption of photon by an atom

From the above diagram, it is clear that

$$E_{n_1} + h\nu = E_{n_2}$$

Let us now consider an example in which a photon of energy 12.09 eV is incident on hydrogen atom in ground state.

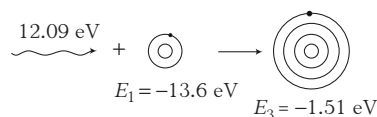


Fig. 12.15 Absorption of photon of energy 12.09 eV by an atom

Energy of the atom after absorption

$$\begin{aligned} &= 12.09 + (-13.6) = -1.51 \text{ eV} \\ &= E_3 \end{aligned}$$

After absorbing photon, the atom goes to state $n = 3$, hence in this absorption spectrum, we have single line as shown below

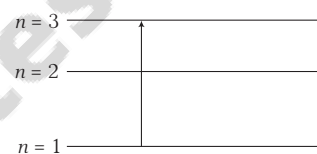


Fig. 12.16 Absorption spectrum

Now, in this state $n = 3$, the atom remains for the time 10^{-8} s and then returns to ground state by emitting photon(s). This atom can come directly to $n = 1$ by emitting one photon or by emitting two photons from $n = 3 \rightarrow 2$, $2 \rightarrow 1$. Therefore, possible transitions are $3 \rightarrow 2$, $2 \rightarrow 1$, $3 \rightarrow 1$, as shown in the figure

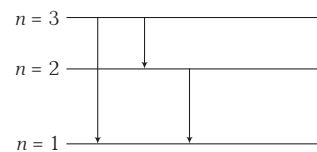


Fig. 12.17 Emission spectrum

From the figure it is clear that, number of emission spectrum equal to two. Out of three transitions, frequency of emitted photon will be maximum corresponding to $3 \rightarrow 1$ (since, $\Delta E = h\nu$), therefore wavelength will be minimum. In the transition $3 \rightarrow 2$, frequency of emitted photon will be minimum and hence wavelength will be maximum. For $n = 3$ to $n = 1$, we have seen that number of lines in emission spectrum $N = \frac{3(3-1)}{2} = 3$

So, for an atom to makes transition from $n = n_1$ to $n = n_2$, number of lines in emission spectrum,

$$N = \frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$$

Putting $n_1 = n$ and $n_2 = 1$, this reduces to $N = \frac{n(n-1)}{2}$

Note Emission/absorption of photons only takes place when the energy of the photon is exactly equal to the difference of energy level.

Example 12.20 Monochromatic radiation of wavelength λ is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of ten different wavelengths. Find the value of λ .

Sol. From n th energy state a total of $\frac{n(n-1)}{2}$ lines are emitted.

$$\therefore \frac{n(n-1)}{2} = 10 \quad \text{or} \quad n = 5$$

Further, $E_n = \frac{E_1}{n^2}$

$$\therefore E_5 = \frac{-13.6}{(5)^2} = -0.54 \text{ eV}$$

The energy needed to take a hydrogen atom from its ground state to $n = 5$ is,

$$\Delta E = E_5 - E_1 = -0.54 - (-13.6) = 13.06 \text{ eV}$$

The corresponding wavelength of light,

$$\lambda = \frac{12375}{\Delta E} \text{ \AA} = \frac{12375}{13.06} = 947.5 \text{ \AA}$$

Example 12.21 Ionisation potential of hydrogen atom is 13.6 V. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy is 12.1 eV. According to Bohr's theory, find the possible spectral lines emitted by hydrogen.

Sol. Ionisation energy corresponding to ionisation potential = -13.6 eV

Energy of incident photon = 12.1 eV

So, the energy of electron in excited state

$$= -13.6 + 12.1 = -1.5 \text{ eV}$$

As, $E_n = -\frac{13.6}{n^2} \text{ eV}$

$$\Rightarrow -1.5 = \frac{-13.6}{n^2} \Rightarrow n^2 = \frac{-13.6}{-1.5} = 9 \Rightarrow n = 3$$

i.e. the energy of electron in excited state corresponds to third orbit.

The possible spectral lines are when electron jumps from orbit 3rd to 2nd, 3rd to 1st and 2nd to 1st. Thus, three spectral lines are emitted.

Example 12.22 An electron is moving in an orbit of a hydrogen atom from which there can be a maximum of six transition. An electron is moving in an orbit of another hydrogen atom from which there can be a maximum of three transition. Find ratio of the velocities of the electron in these two orbits.

Sol. Number of spectral lines obtained due to transition of electrons from n th orbit to lower orbit is given by

$$N = \frac{n(n-1)}{2}$$

Case I $6 = \frac{n_1(n_1-1)}{2}$

$$\Rightarrow n_1^2 - n_1 - 12 = 0$$

$$\Rightarrow (n_1 - 4)(n_1 + 3) = 0 \Rightarrow n_1 = 4$$

because n_1 cannot be negative, i.e. -3.

Case II $3 = \frac{n_2(n_2-1)}{2}$

$$\Rightarrow n_2^2 - n_2 - 6 = 0 \Rightarrow (n_2 + 2)(n_2 - 3) = 0$$

$$\Rightarrow n_2 = 3$$

because n_2 cannot be negative, i.e. -2.

Velocity of electron in hydrogen atom in n th orbit,

$$v_n \propto \frac{1}{n} \Rightarrow \frac{v_n}{v'_n} = \frac{n_2}{n_1} \Rightarrow \frac{v_6}{v_3} = \frac{3}{4}$$

HYDROGEN SPECTRUM

Hydrogen spectrum or the line spectra of hydrogen atom consists of discrete bright lines in a dark background and it is specifically known as **hydrogen emission spectrum**.

There is one more type of hydrogen spectrum that exists where we get dark lines on the bright background, it is known as **absorption spectrum**.

Hydrogen being the simplest atom, has the simplest spectrum. In its observed spectrum, spacing between lines within certain sets of the spectrum decreases in a regular way. Each of these sets is called a **spectral series**. The very first of such series is the Balmer series.

Balmer find an empirical formula by the observation of a small part of this spectrum and it is represented by

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \text{ where } n = 3, 4, 5, \dots$$

where, R is a constant called **Rydberg's constant** and its value is $1.097 \times 10^7 \text{ m}^{-1}$.

Balmer series in the emission spectrum of H-atom is shown below

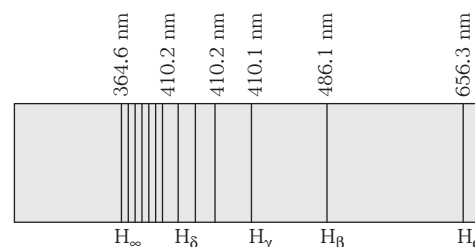


Fig. 12.18 Balmer series in the emission spectrum of hydrogen

The line with the longest wavelength 656.3 nm in the red is called H_{α} and the next line with wavelength 486.1 nm in blue-green is called H_{β} and so on.

Other series of spectra for hydrogen were subsequently discovered and known by the name of their discoverers. The lines of Balmer series are found in the visible part of the spectrum.

Other series were found in the invisible parts of the spectrum. *e.g.* Lyman series in the ultraviolet region and (Paschen, Brackett and Pfund series in the infrared region).

The wavelengths of line in these series can be expressed by the following formulae

(i) For Lyman series

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right), \text{ where } n = 2, 3, 4, \dots$$

(ii) For Balmer series

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \text{ where } n = 3, 4, 5, \dots$$

(iii) For Paschen series

$$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right), \text{ where } n = 4, 5, 6, \dots$$

(iv) For Brackett series

$$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right), \text{ where } n = 5, 6, 7, \dots$$

(v) For Pfund series

$$\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right), \text{ where } n = 6, 7, 8, \dots$$

Explanation The different series of hydrogen spectrum can be explained by Bohr's theory.

According to Bohr's theory, if the ionised state of hydrogen atom be taken as zero energy level (at ∞), then the energies of the different energy levels of the atom can be expressed by the following formula

$$E_n = -\frac{Rhc}{n^2}, \text{ where } n = 1, 2, 3, \dots$$

where, R is Rydberg's constant, h is Planck's constant and the integer n is called principal quantum number.

When the atom gets energy from outside, its electron goes from the lower energy level to some higher energy level.

But it returns from there, within 10^{-8} s, to the lower energy level directly or through other lower energy levels. While returning back, the atom emits photons.

The energy levels of hydrogen atom are shown in the figure

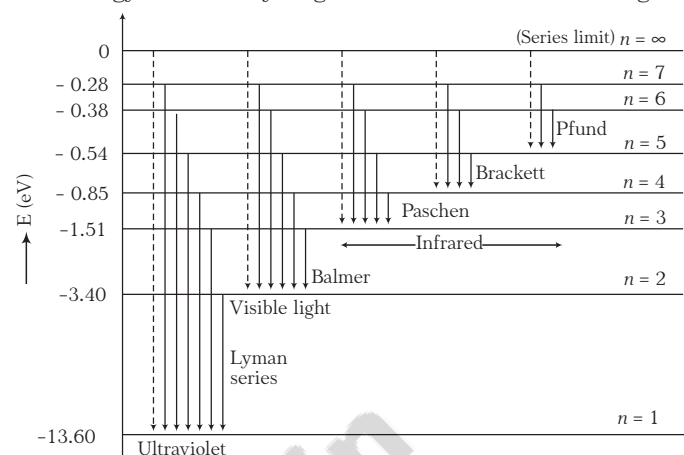


Fig. 12.19 Emission spectrum of H-atom

Series limit

The transitions corresponding to maximum energy or smallest wavelength of a given series of hydrogen spectrum is called series limit.

To calculate wavelength corresponding to series limit, we use $n_2 = \infty$.

For Lyman series, $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{\infty} \right) \Rightarrow \lambda = \frac{1}{R}$

For Balmer series, $\frac{1}{\lambda} = \frac{R}{4}$

In general, series limit $\frac{1}{\lambda} = \frac{R}{n^2}$

where, $n = 1, 2, 3, 4, \dots$

Example 12.23 A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Sol. Energy of electron beam, $E = 12.5$ eV

Planck's constant, $h = 6.62 \times 10^{-34}$ J-s

So, using the relation, $E = \frac{hc}{\lambda}$, we get

$$\begin{aligned} \lambda &= \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{12.5 \times 1.6 \times 10^{-19}} \\ &= 0.993 \times 10^{-7} = 993 \times 10^{-10} \text{ m} \\ &= 993 \text{ \AA} \end{aligned}$$

This wavelength falls in the range of Lyman series (912 Å to 1216 Å), thus we conclude that Lyman series is emitted.

Example 12.24 In H-atom, a transition takes place from $n = 3$ to $n = 2$ orbit. Calculate the wavelength of the emitted photon and will the photon be visible? To which spectral series will this photon belong? (Take, $R = 1.097 \times 10^7 \text{ m}^{-1}$)

Sol. The wavelength of the emitted photon is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

When the transition takes place from $n = 3$ to $n = 2$, then

$$\begin{aligned} \frac{1}{\lambda} &= (1.097 \times 10^7) \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \\ &= 1.097 \times 10^7 \times \frac{5}{36} \end{aligned}$$

$$\therefore \lambda = \frac{36}{1.097 \times 10^7 \times 5} = 6.563 \times 10^{-7} \text{ m} = 6563 \text{ \AA}$$

λ falls in the visible (red) part of the spectrum, hence the photon will be visible. This photon is the first member of the Balmer series.

Example 12.25 What is the shortest wavelength present in the Paschen series of spectral lines?

Sol. For the shortest wavelength of Paschen series of spectral lines, $n_1 = 3$, $n_2 = \infty$. If $\lambda_{\min}$ be the minimum wavelength in this series of spectral lines, then

$$\frac{1}{\lambda_{\min}} = R \left(\frac{1}{(3)^2} - \frac{1}{(\infty)^2} \right) = \frac{R}{9}$$

$$\begin{aligned} \text{or } \lambda_{\min} &= \frac{9}{R} = \frac{9}{1.097 \times 10^7} \quad (\because R = 1.097 \times 10^7 \text{ m}^{-1}) \\ &= 8204.2 \times 10^{-10} \text{ m} = 8204.2 \text{ \AA} \end{aligned}$$

Example 12.26 Find the largest and shortest wavelengths in the Brackett series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

Sol. The wavelength relation for Brackett series is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right), \text{ where } n = 5, 6, \dots$$

The largest wavelength is corresponding to $n = 5$.

$$\therefore \frac{1}{\lambda_{\max}} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{25} \right)$$

$$\text{or } \lambda_{\max} = 40.650 \times 10^{-7} \text{ m} = 40650 \text{ \AA}$$

The shortest wavelength corresponds to $n = \infty$.

$$\therefore \frac{1}{\lambda_{\min}} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{\infty} \right)$$

$$\text{or } \lambda_{\min} = 14.58 \times 10^{-7} \text{ m} = 14585 \text{ \AA}$$

Both of these wavelengths lie in infrared region of electromagnetic spectrum.

Example 12.27 If the series limit wavelength of the Lyman series for hydrogen atom is $912 \text{ \AA}$, then find the series limit wavelength for the Balmer series for the hydrogen atom.

Sol. For series limit wavelength of Lyman series,

$$\begin{aligned} n_1 &= 1, n_2 = \infty \\ \Rightarrow \frac{1}{\lambda} &= R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = R \end{aligned}$$

$$\text{or } \lambda = \frac{1}{R} = 912 \text{ \AA} = 912 \times 10^{-10} \text{ m} \quad \dots(i)$$

For series limit of Balmer series, we can write

$$\begin{aligned} n_1 &= 2, n_2 = \infty \\ \Rightarrow \frac{1}{\lambda} &= R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \\ &= R \left[\frac{1}{(2)^2} - \frac{1}{(\infty)^2} \right] = \frac{R}{4} \end{aligned}$$

$$\begin{aligned} \therefore \lambda &= \frac{4}{R} = 4 \times 912 \times 10^{-10} \text{ m} \quad [\text{from Eq. (i)}] \\ &= 4 \times 912 \text{ \AA} = 3648 \text{ \AA} \end{aligned}$$

Example 12.28 Calculate (i) the wavelength and (ii) the frequency of the H_β line of the Balmer series for hydrogen.

Sol. (i) H_β line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. So, the corresponding wavelength for H_β line is

$$\begin{aligned} \frac{1}{\lambda} &= (1.097 \times 10^7) \left(\frac{1}{2^2} - \frac{1}{4^2} \right) \\ \text{or } \lambda &= 4.9 \times 10^{-7} \text{ m} \end{aligned}$$

$$(ii) \text{ Further, } f = \frac{c}{\lambda} = \frac{3 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14} \text{ Hz}$$

Example 12.29 The wavelength of the first line of Lyman series for hydrogen is identical to that of the second line of Balmer series for some hydrogen like ion Y. Calculate energies of the first three levels of Y.

Sol. Wavelength of the first line of Lyman series for hydrogen atom ($Z = 1$) will be given by the equation,

$$\frac{1}{\lambda_1} = R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3R}{4} \quad \dots(i)$$

The wavelength of second line of Balmer series for hydrogen like ion Y is

$$\frac{1}{\lambda_2} = RZ^2 \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = \frac{3RZ^2}{16} \quad \dots(ii)$$

$$\text{Given that, } \lambda_1 = \lambda_2 \quad \text{or} \quad \frac{1}{\lambda_1} = \frac{1}{\lambda_2}$$

Now, from Eqs. (i) and (ii), we get

$$\frac{3R}{4} = \frac{3RZ^2}{16}$$

$$\therefore Z = 2$$

i.e. Y ion is He^+ .

$\therefore$ The energies of first three levels of Y are

$$E_1 = -\frac{(13.6) Z^2}{12} = -(13.6) 2^2 = -54.4 \text{ eV}$$

$$E_2 = \frac{E_1}{(2)^2} = -3.6 \text{ eV}$$

$$\text{and } E_3 = \frac{E_1}{(3)^2} = -6.04 \text{ eV}$$

Example 12.30 Hydrogen atom in its ground state is excited by a radiation of $\lambda = 990.7 \text{ \AA}$. How many different wavelength are possible in the resulting emission spectrum? Find the longest wavelength amongst these.

Sol. Wavelength of radiation, $\lambda = 990.7 \text{ \AA}$

$$\approx 99 \text{ nm}$$

$$\text{As we know, } \Delta E = \frac{1242 \text{ eV}}{\lambda (\text{in nm})} = \frac{1242}{99} = 12.54 \text{ eV}$$

$$\text{Using the relation, } \Delta E = 13.6 Z^2 \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$$

($\because n = 1, Z = 1$ because H-atom is in ground state)

$$12.54 = 13.6(1)^2 \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$$

$$\Rightarrow n = 3$$

Number of wavelengths in resulting spectrum,

$$\frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

and we also know,

$$\lambda \propto \frac{1}{\Delta E}$$

So, for ΔE to be minimum, it will be in the transition $3 \rightarrow 2$.

$$\text{Using the relation, } \Delta E = 13.6 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$= 13.6(1)^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$= 13.6 \times \frac{5}{36}$$

$$= 1.89 \text{ eV}$$

$$\text{So, the wavelength, } \lambda = \frac{1242}{1.89} = 657.14 \text{ nm}$$

Limitations of Bohr's model

The limitation of Bohr's model are as follows

- Bohr's model is applicable only to a single electron atoms/ions.
- It does not explain the fine structure of spectral lines. Application of external magnetic field (Zeeman effect) and external electric field (Stark effect) to radiating atoms could not be explained.
- Bohr's assumption regarding stationary orbits is against Heisenberg uncertainty principle.
- No explanation is given regarding principle of quantisation of angular momentum.
- This model does not explain why orbits of electrons are taken as circular whereas elliptical orbits are also possible.

de-Broglie explanation of Bohr's second postulate of quantisation

According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie standing waves associated with the revolving electron.

For an electron revolving in n th circular orbit of radius r_n ,

Total distance covered = Circumference of the orbit = $2\pi r_n$

$\therefore$ For the permissible orbit, $2\pi r_n = n\lambda$

According to de-Broglie wavelength, $\lambda = \frac{h}{mv_n}$

where, v_n = speed of electron revolving in n th orbit.

$$\therefore 2\pi r_n = \frac{nh}{mv_n} \text{ or } mv_n r_n = \frac{nh}{2\pi} = n(h/2\pi)$$

i.e. angular momentum of electron revolving in n th orbit must be an integral multiple of $h/2\pi$, which is the quantum condition proposed by Bohr in second postulate.

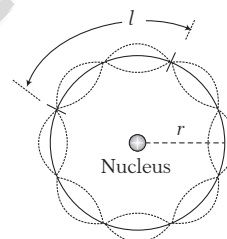


Fig. 12.20 A standing wave is shown on a circular orbit for $n = 4$

Example 12.31 An electron of a hydrogen like atom is in excited state. If total energy of the electron is -4.6 eV , then evaluate

- the kinetic energy and
- the de-Broglie wavelength of the electron.

Sol. Given, total energy of electron, $E = -4.6 \text{ eV}$

(i) Kinetic energy of electron, $K = -(\text{Total energy, } E)$

$$\Rightarrow K = -E = -(-4.6) = 4.6 \text{ eV}$$

(ii) de-Broglie wavelength,

$$\lambda_d = \frac{h}{\sqrt{2mK}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 4.6 \times 1.6 \times 10^{-19}}} = 0.57 \times 10^{-9} \text{ m} = 0.57 \text{ nm}$$

Example 12.32 Let us assume that the de-Broglie wave associated with an electron forms a standing wave between the atoms arranged in a one-dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed, if the distance between the atoms of the array is $3 \text{ \AA}$. A similar standing wave is again formed, if d is increased to $3.5 \text{ \AA}$ but not for any intermediate value of d . Find the energy (in eV) of the electrons and the least value of d for which the standing wave of the type described above can form.

Sol. For standing waves, the separation between consecutive nodes or anti-nodes.

$$d = n \frac{\lambda}{2} \quad \dots(i)$$

where, $n = 1, 2, \dots$

$$\text{For next node, } d' = (n+1) \frac{\lambda}{2} \quad \dots(ii)$$

Subtracting Eq. (ii) from Eq. (i), we get

$$d' - d = \frac{\lambda}{2} \Rightarrow 3.5 - 3 = \frac{\lambda}{2} \Rightarrow \lambda = 1 \text{ \AA}$$

de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mK}}$

$$\Rightarrow K = \frac{h^2}{2m\lambda^2} = \frac{(6.6 \times 10^{-34})^2}{2(9.1 \times 10^{-31}) \times (10^{-10})^2} = 2.415 \times 10^{-17} \text{ J}$$

$$= \frac{2.415 \times 10^{-17}}{1.6 \times 10^{-19}} \quad \left(\because 1 \text{ J} = \frac{1}{1.6 \times 10^{-19}} \text{ eV} \right)$$

$$= 150 \text{ eV}$$

For minimum value of d , $n = 1$

$$\therefore d = n \frac{\lambda}{2} = \frac{\lambda}{2} = \frac{1}{2} = 0.5 \text{ \AA}$$

Example 12.33 Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy becomes -3.02 eV . Now, the photoelectric plate having $\phi = 4.6 \text{ eV}$ is exposed to the emission spectra of this gas. Assuming all the transition to be possible, find the minimum de-Broglie wavelength of ejected photoelectrons.

Sol. Potential energy, $U = -3.02 \text{ eV}$

We know,

$$\text{Total energy (E)} = \frac{\text{Potential energy (U)}}{2} = \frac{-3.02}{2} = -1.51 \text{ eV}$$

$$\text{Using the relation, } E_n = \frac{-13.6}{n^2} \text{ eV}$$

$$\Rightarrow -1.51 = \frac{-13.6}{n^2}$$

$$\Rightarrow n^2 = 9 \Rightarrow n = 3$$

Three transitions are possible, i.e. $3 \rightarrow 2$, $2 \rightarrow 1$, $3 \rightarrow 1$.

The emission will take place for those transition in which $\Delta E > 1$.

$$\text{Further, using the relation, } \Delta E = 13.6Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For $2 \rightarrow 1$,

$$\Delta E = -13.6 \left(\frac{1}{2^2} - \frac{1}{1^2} \right) = 10.2 \text{ eV}$$

For $3 \rightarrow 2$,

$$\Delta E = -13.6 \left(\frac{1}{3^2} - \frac{1}{2^2} \right) = 13.6 \times 5/36 = +1.88 \text{ eV}$$

For $3 \rightarrow 1$,

$$\Delta E = -13.6 \left(\frac{1}{3^2} - \frac{1}{1^2} \right) = +13.6 \times \frac{8}{9} = 12.08 \text{ eV}$$

So, according to the relation, $\Delta E = \phi + K_{\max}$

Minimum kinetic energy of ejected electron is possible in $3 \rightarrow 1$.

where, ϕ = work function.

$$\Rightarrow 12.08 = 4.6 + K_{\max}$$

$$\Rightarrow K_{\max} = 7.48 \text{ eV}$$

$\therefore$ Maximum de-Broglie wavelength,

$$\lambda_d = \frac{h}{\sqrt{2mK_{\max}}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 7.48 \times 1.6 \times 10^{-19}}}$$

$$= 0.45 \text{ nm}$$

CHECK POINT 12.2

1. Kinetic energy of electron in n th orbit is given by

- (a) $\frac{Rhc}{2n^2}$ (b) $\frac{2Rhc}{n}$ (c) $\frac{Rhc}{n}$ (d) $\frac{Rhc}{n^2}$

2. Potential energy (PE_n) and kinetic energy (KE_n) of electron in n th orbit are related as

- (a) $PE_n = KE_n$ (b) $PE_n = -2KE_n$
(c) $PE_n = 2KE_n$ (d) $PE_n = -KE_n$

3. In the n th orbit, the energy of an electron, $E_n = -\frac{13.6}{n^2} \text{ eV}$ for

hydrogen atom. The energy required to take the electron from first orbit to second orbit will be

- (a) 10.2 eV (b) 12.1 eV
(c) 13.6 eV (d) 3.4 eV

4. Ionisation potential (IP) and ionisation energy (IE) are related as

- (a) $IP = (IE)e$ (b) $IP = \frac{IE}{e}$ (c) $IE = \frac{IP}{e^2}$ (d) $IP = \frac{IE}{e^2}$

5. In hydrogen atom, if the difference in the energy of the electron in $n = 2$ and $n = 3$ orbits is E , the ionisation energy of hydrogen atom is

- (a) $13.2E$ (b) $7.2E$
(c) $5.6E$ (d) $3.2E$

6. Atomic hydrogen is excited from the ground state to the n th state. The number of lines in the emission spectrum will be

- (a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$ (c) $\frac{(n-1)^2}{2}$ (d) $\frac{(n+1)^2}{2}$

7. In which of the following transitions will the wavelength be minimum?

- (a) $n = 5$ to $n = 4$ (b) $n = 4$ to $n = 3$
(c) $n = 3$ to $n = 2$ (d) $n = 2$ to $n = 1$

8. Which one of the series of hydrogen spectrum is in the visible region?

- (a) Lyman series (b) Balmer series
(c) Paschen series (d) Brackett series

9. In which of the following systems will the wavelength corresponding to $n = 2$ to $n = 1$ be minimum?
 (a) Hydrogen atom (b) Deuterium atom
 (c) Singly ionised helium (d) Doubly ionised lithium

10. The ratio of the largest to shortest wavelengths in Balmer series of hydrogen spectrum is
 (a) $\frac{25}{9}$ (b) $\frac{17}{6}$ (c) $\frac{9}{5}$ (d) $\frac{5}{4}$

11. Wavelength corresponding to series limit is
 (a) $\frac{1}{\lambda} = \frac{2R}{n}$ (b) $\frac{1}{\lambda} = \frac{R^2}{n^2}$ (c) $\frac{1}{\lambda} = \frac{R}{n^2}$ (d) $\frac{1}{\lambda} = \frac{R}{n}$

12. For the Bohr's first orbit of circumference $2\pi r$, the de-Broglie wavelength of revolving electron will be
 (a) $2\pi r$ (b) πr (c) $\frac{1}{2\pi r}$ (d) $\frac{1}{4\pi r}$

13. Which of the following is incorrect regarding limitations of Bohr's model?
 (a) This model is applicable only to single electron system.
 (b) It does not explain fine structure of spectral lines.
 (c) It's assumption regarding stationary orbits supports Heisenberg uncertainty principle.
 (d) None of the above

X-RAYS

In 1895, Wilhelm Roentgen found that a highly penetrating radiation of unknown nature is produced when fast moving electrons strike a target of high atomic number and high melting point.

These radiations were given a name X-rays as their nature was unknown (in mathematics an unknown quantity is normally designated by X).

Later it was discovered that these are high energy photons (or electromagnetic waves).

Hence, X-rays are a form of electromagnetic radiation with wavelengths ranging from $0.1 \text{ \AA}$ to $100 \text{ \AA}$.

However, the boundaries of this category are not sharp.

The shorter wavelength end overlaps γ -rays and the longer wavelength end overlaps ultraviolet rays, i.e. wavelengths of X-rays are in the order of

$$\lambda_{\gamma\text{-rays}} < \lambda_{\text{X-rays}} < \lambda_{\text{UV-rays}}$$

Production of X-rays

Here, Fig. 12.21 shows a diagram of a highly evacuated glass tube called **filament X-ray tube** or the **collidge tube**.

A cathode (a plate connected to negative terminal of a battery) is heated by a tungsten filament through which an electric current is passed, supplies electrons by thermionic emission.

The filament is coated with oxides of barium or strontium to have an emission of electrons even at low temperature.

The high potential difference V maintained between the cathode and a metallic target accelerate the electrons toward the target.

The target is a material of high atomic weight, high melting point and high thermal conductivity made of tungsten or molybdenum and is embedded in a copper block.

The face of the target is at an angle relative to the electron beam, and the X-rays that leave the target pass through the side of the tube.

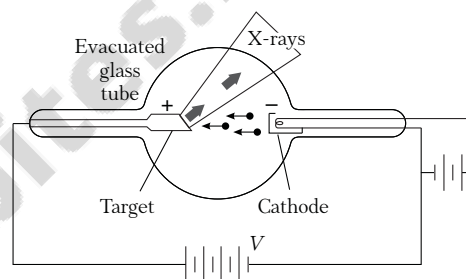


Fig. 12.21 An X-ray tube

Kinetic energy of electron is converted into energy of photon during the production of X-rays. The higher accelerating voltage V , shorter the wavelengths of the X-rays and hence greater will be its penetrating power, due to higher energy.

Properties of X-rays

Properties of X-rays are given below

- (i) X-rays are electromagnetic waves with wavelength range $0.1 \text{ \AA}$ - $100 \text{ \AA}$.
- (ii) These rays are invisible.
- (iii) They travel in straight line with speed of light.
- (iv) They show all important properties of light rays such as reflection, refraction, interference, diffraction and polarisation, etc.
- (v) X-rays are not deflected by electric and magnetic fields.
- (vi) These show continuous spectrum.
- (vii) X-rays carry no charge.
- (viii) Ionisation power of the X-rays is measured in Roentgen.
- (ix) They do not pass through heavy metals and bones.
- (x) They effect the photographic plates.

- (xi) Lead is best absorber of X-rays.
- (xii) Long exposure to X-rays is injurious for human body.
- (xiii) The wavelength of X-rays is very small as compared to the wavelength of light.

Classification of X-rays

X-rays are of two types, continuous X-rays and characteristic X-rays. While the former depends only on the accelerating voltage V , the later depends on the target used.

(i) Continuous X-rays

When accelerated electrons strike with the target material, it loses its KE and finally is brought to rest. A part of this lost KE is converted into a photon, remaining part goes into heating the target. The energy of photon formed may be anything between 0 to many eV, depending on energy lost.

Wavelength is minimum, when energy loss is maximum.

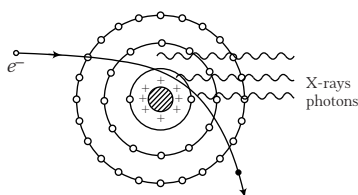


Fig. 12.22 Emission of X-ray photons forming continuous X-ray spectrum

The continuous X-rays (or bremsstrahlung X-rays) produced at a given accelerating potential V vary in wavelength, but none has a wavelength shorter than a certain value $\lambda_{\min}$. This minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy qV or eV of the striking electrons. Thus,

$$\frac{hc}{\lambda_{\min}} = eV$$

$$\text{or} \quad \lambda_{\min} = \frac{hc}{eV}$$

After substituting values of h , c and e , we obtain the following simple formula for $\lambda_{\min}$.

$$\lambda_{\min} \text{ (in } \text{\AA}) = \frac{12375}{V} \quad \dots(i)$$

If V is increased, then $\lambda_{\min}$ decreases. This wavelength is also known as the **cut-off wavelength** or the **threshold wavelength**.

Intensity-wavelength graph

Following figure shows the intensity of X-ray variation with different wavelength for various accelerating voltages applied to X-ray tube.

From the graph, it is clear that the wavelengths at which the intensity is maximum depends on the accelerating voltage, being shorter for higher voltage and *vice-versa*.

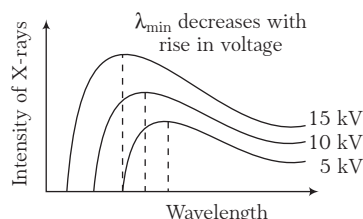


Fig. 12.23 Variation of intensity of X-rays with wavelength

Example 12.34 Find the cut-off wavelength for the continuous X-rays coming from an X-ray tube operating at 40 kV.

Sol. Cut-off wavelength $\lambda_{\min}$ is given by

$$\lambda_{\min} \text{ (in } \text{\AA}) = \frac{12375}{V \text{ (in V)}} = \frac{12375}{40 \times 10^3} = 0.31$$

Example 12.35 An X-ray tube is operated at 30 kV. If a particular electron loses 10% of its kinetic energy to emit an X-ray photon during the collision. Find the wavelength and maximum frequency associated with this photon.

Sol. Given, kinetic energy of electron = 30 keV

According to the question, 10% of this energy is converted to photon which is given by

$$\frac{10}{100} \times 30 = 3 \text{ keV}$$

If V is accelerating voltage, then $eV = 3 \text{ keV} = 3 \times 10^3 \text{ eV}$

$$\Rightarrow V = 3 \times 10^3 \text{ V}$$

$$\therefore \lambda_{\min} \text{ (in } \text{\AA}) = \frac{12375}{V \text{ (in V)}} = \frac{12375}{3 \times 10^3} = 4.12$$

$$\text{Also, } \nu_{\max} = \frac{c}{\lambda_{\min}} = \frac{3 \times 10^8}{(4.12 \times 10^{-10})} = 7.3 \times 10^{17} \text{ Hz}$$

Example 12.36 The operating voltage in an X-ray tube is increased to 3 times the original value when the short wavelength limit shifts by 25 nm. Find the original value of the operating voltage.

Sol. As we know, $\frac{hc}{\lambda} = eV \Rightarrow \lambda \propto \frac{1}{V}$

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{V_2}{V_1}$$

$$\text{According to the question, } \frac{\lambda}{\lambda - 25} = \frac{3V}{V}$$

$$\Rightarrow \lambda = 3\lambda - 3 \times 25$$

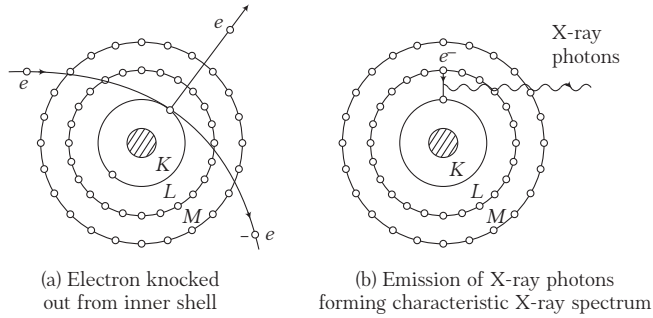
$$\Rightarrow \lambda = 37.5 \text{ nm} = 375 \text{ \AA}$$

$$\therefore E(eV) = \frac{12375}{\lambda \text{ (in } \text{\AA})} = \frac{12375}{375} = 33 \text{ eV}$$

$$\Rightarrow eV_1 = 33 \text{ eV} \quad \text{or} \quad V_1 = 33 \text{ V}$$

(ii) Characteristic X-rays

Some of the fast moving electrons with high velocity penetrate the surface atoms of the target material and knock out the electrons even from the inner most shells of atom. Now, a vacancy is created at that place.



Electrons from higher shell jumps to fill the created vacancy. When the electron jumps from a higher energy level E_2 to lower energy orbit E_1 , it radiates energy $(E_2 - E_1)$. Thus, this energy difference is radiated in the form of X-rays of very small but definite wavelength, which depends upon target material. This X-ray spectrum consists of sharp lines and is called **characteristic X-ray spectrum**.

K, L and M-series If the electron striking the target ejects an electron from the K-shell of the target atom, a vacancy is created in the K-shell. Immediately an electron from one of the outer shell, jumps to K-shell emitting an X-ray photon of energy equal to the energy difference between the two shells.

The X-ray photons emitted due to the jump of electron from the L, M, N shell to the K-shells gives K_α , K_β , K_γ lines, respectively of the K-series of the spectrum.

If the electron striking the target ejects an electron from the L-shell of the target atom, an electron from the M, N, ... shells jumps to the L-shell, so that X-ray photons of lesser energy are emitted. In a similar way, the formation of M-series, N-series, etc., can be explained.

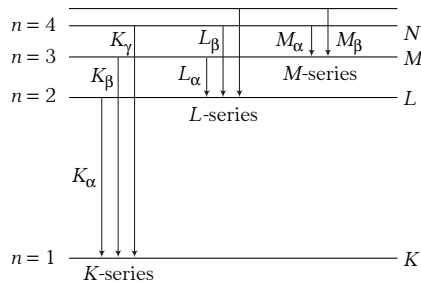


Fig. 12.25 Energy level diagram for K, L and M-series

From the above energy level diagram, we have

$$E_K > E_L > E_M > E_N \dots$$

where $E_K, E_L, E_M, E_N, \dots$ are magnitudes of energies of different levels.

Now, we can write, $\lambda_{K_\alpha} = \frac{hc}{E_K - E_L}$, $\lambda_{K_\beta} = \frac{hc}{E_K - E_M}$

$$\lambda_{L_\alpha} = \frac{hc}{E_L - E_M}, \text{ etc.}$$

Also, the characteristics wavelength depends upon the target material as below

$$\frac{1}{\lambda} = R (Z - b)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

and energy of X-ray radiation as

$$\Delta E = Rhc(Z - b)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\Rightarrow v = Rc (Z - b)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where, Z = atomic number of the target and b = constant.

Note X-rays of high frequency or low wavelength are known as hard X-rays, whereas X-rays of low frequency or high wavelength are known as soft X-rays.

Intensity-wavelength graph

At certain sharply defined wavelengths, the intensity of X-rays is very large as marked K_α , K_β , ..., as shown in figure.

These X-rays are known as **characteristic X-rays**. At other wavelengths, the intensity varies gradually and these X-rays are called **continuous X-rays**.

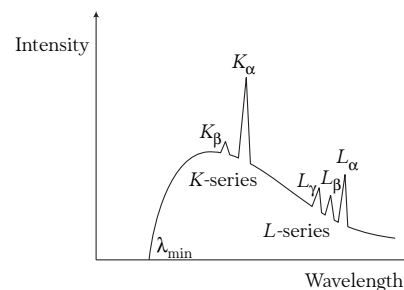


Fig. 12.26 X-ray spectrum

Example 12.37 The wavelength of K_α X-ray for an element is 21.3 pm. It takes 12.5 keV to knock out an electron from the L-shell of the atom of the element. What should be the minimum accelerating voltage across an X-ray tube having the element as target which allows production of K_α X-ray?

Sol. Since, $E_K - E_L = \frac{hc}{\lambda_{K_\alpha}}$

It is given that $\lambda_{K_\alpha} = 21.3 \text{ pm} = 0.0213 \text{ nm}$

$$\text{Also, } E = \frac{1242}{\lambda (\text{in nm})} = \frac{1242}{0.0213} = 58.3 \text{ keV}$$

$$\text{As, } E_K - E_L = E$$

$$\Rightarrow E_K - 12.5 = 58.3$$

$$\Rightarrow E_K = 70.8 \text{ keV}$$

$$\Rightarrow eV' = 70.5 \text{ keV}$$

$$\Rightarrow V' = 70.8 \text{ kV}$$

$\therefore$ Accelerating potential, $V' = 70.8 \text{ kV}$

Example 12.38 The wavelength of K_α line in copper is $1.54 \text{ \AA}$. Find the ionisation energy of K-electron in copper (in joule).

Sol. For K_α line we can write,

$$\frac{1}{\lambda_\alpha} = R(Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = R(Z-1)^2 \times \frac{3}{4} \Rightarrow R(Z-1)^2 = \frac{4}{3\lambda_\alpha}$$

Ionisation energy is given by

$$E = Rhc (Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{\infty} \right) = hc \frac{4}{3\lambda_\alpha}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 4}{3 \times 1.54 \times 10^{-10}} = 1.7 \times 10^{-15} \text{ J}$$

Example 12.39 The wavelength of K_α X-ray line for an element is $0.42 \text{ \AA}$. Find the wavelength of K_β line emitted by the same element.

Sol. The characteristic wavelength for particular element is

$$\text{given by } \frac{1}{\lambda} = R(Z-b)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\text{For } K_\alpha \text{ line, } \frac{1}{\lambda_\alpha} = (Z-b)^2 R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = (Z-b)^2 R \left(\frac{3}{4} \right)$$

$$\text{For } K_\beta \text{ line, } \frac{1}{\lambda_\beta} = (Z-b)^2 R \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = (Z-b)^2 R \left(\frac{8}{9} \right)$$

$$\Rightarrow \frac{\lambda_\beta}{\lambda_\alpha} = \frac{(3/4)}{(8/9)} = \frac{27}{32}$$

It is given that λ_α is $0.42 \text{ \AA}$.

$$\therefore \lambda_\beta = 0.42 \times \frac{27}{32} = 0.35 \text{ \AA}$$

Example 12.40 The energy of an element with a vacancy in K-shell is 35.2 keV , in L-shell is 5.25 keV and in M-shell is 0.55 keV higher than the energy of the atom with no vacancy. Find the frequency of K_α , K_β and L_α X-rays of that element.

Sol. It is given that,

$$E_K = 35.2 \text{ keV}, E_L = 5.25 \text{ keV}, E_M = 0.55 \text{ keV}$$

$$\Rightarrow E_K - E_L = \frac{hc}{\lambda_{K_\alpha}} = h\nu_{K_\alpha}$$

$$\Rightarrow (35.2 - 5.25) \times 10^3 \times 1.6 \times 10^{-19} = 6.6 \times 10^{-34} \nu_{K_\alpha}$$

$$(\because e = 1.6 \times 10^{-19} \text{ C})$$

$$\Rightarrow \nu_{K_\alpha} = \frac{29.95 \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 7.26 \times 10^{18} \text{ Hz}$$

$$\text{Similarly, } E_K - E_M = h\nu_{K_\beta}$$

$$\Rightarrow \nu_{K_\beta} = \frac{(35.2 - 0.55) \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 8.4 \times 10^{18} \text{ Hz}$$

$$\text{Also, } E_L - E_M = h\nu_{L_\alpha}$$

$$\Rightarrow \nu_{L_\alpha} = \frac{(5.25 - 0.55) \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.14 \times 10^{18} \text{ Hz}$$

Moseley's law for characteristic spectrum

Although multielectron atoms cannot be analysed with the Bohr's model. In 1914, Henry GJ Moseley made an effort towards this. Moseley measured the frequencies of characteristic X-rays emitted from a large number of elements and plotted the square root of the frequency $\sqrt{f}$ against the atomic number Z of the element. He discovered that the plot is very close to a straight line. He then concluded that the spectra of different elements are very similar and with increasing atomic number, the spectral lines merely shift towards higher frequencies.

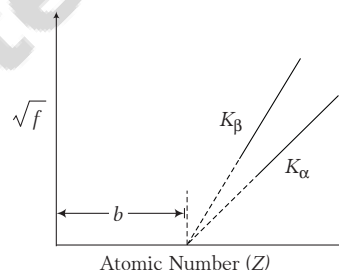


Fig. 12.27 Frequency versus atomic number graph

He also gave the following relation, $\sqrt{f} = a(Z-b)$

where, f = frequency of emitted line,
 Z = atomic number of target material,
 a = proportionality constant,
 b = screening constant

and $(Z-b)$ = effective atomic number.

For K_α X-rays, when an electron from the L shell moves a transition to the vacant K shell, effective charge at centre

$$= (Z-1)e$$

$$\Rightarrow b = 1$$

$$\text{For } K_\alpha \text{ transition, } \Delta E = h\nu = Rhc (Z-b)^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$$\sqrt{\nu} = \sqrt{\frac{3Rc}{4}} (Z-b)$$

From Moseley's law,

$$\sqrt{\nu} = a(Z-b)$$

$$\Rightarrow a = \sqrt{\frac{3Rc}{4}}, b = 1$$

Here, a and b does not depend on the nature of target.

Different values of a and b are as follows

$$b=1 \quad \text{for} \quad K\text{-series}$$

$$b=7.4 \quad \text{for} \quad L\text{-series}$$

$$b=19.2 \quad \text{for} \quad M\text{-series}$$

Moseley's law supported Bohr's theory. It experimentally determined the atomic number (Z) of elements.

If the graph was plotted between $\sqrt{f}$ and the atomic weight, then straight line was not obtained. So, he concluded that the elements in the periodic table should be arranged in the increasing order of atomic number and not of atomic weight. When these changes were made, the anomalies of the Mendeleev's table disappeared.

Example 12.41 Use Moseley's law with $b = 1$, find the frequency of the K_{α} X-rays of La ($Z = 57$), if the frequency of the K_{α} X-rays of Cu ($Z = 29$) is known to be 1.88×10^{18} Hz.

Sol. Using the equation, $\sqrt{f} = a(Z - b)$

For K-series, $b = 1$

$$\Rightarrow \frac{f_{La}}{f_{Cu}} = \left(\frac{Z_{La} - 1}{Z_{Cu} - 1} \right)^2 \quad \text{or} \quad f_{La} = f_{Cu} \left(\frac{Z_{La} - 1}{Z_{Cu} - 1} \right)^2$$

$$= 1.88 \times 10^{18} \left(\frac{57 - 1}{29 - 1} \right)^2 = 7.52 \times 10^{18} \text{ Hz}$$

Example 12.42 The K_{α} X-rays of aluminium ($Z = 13$) and copper ($Z = 29$) have wavelength 887 pm and 140 pm, respectively. Use Moseley's law $\sqrt{f} = a(Z - b)$ to find the wavelength of the K_{α} X-ray of iron ($Z = 26$).

Sol. According of Moseley's law,

$$\sqrt{f} = a(Z - b) \Rightarrow \sqrt{\frac{c}{\lambda}} = a(Z - b)$$

$$\text{For aluminium, } \sqrt{\frac{c}{887 \times 10^{-12}}} = a(13 - b) \quad \dots(i)$$

$$\text{For copper, } \sqrt{\frac{c}{140 \times 10^{-12}}} = a(29 - b) \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we have

$$\sqrt{\frac{887}{140}} = \frac{29 - b}{13 - b} \Rightarrow 2.5(13 - b) = 29 - b \Rightarrow b = 2.3$$

Now, putting the value of b in Eq. (ii), we get

$$\sqrt{\frac{c}{140 \times 10^{-12}}} = a(29 - b) \Rightarrow \sqrt{\frac{3 \times 10^8}{140 \times 10^{-12}}} = a(29 - 2.3)$$

$$\Rightarrow 1.46 \times 10^9 = a(26.7)$$

$$\Rightarrow a = 0.055 \times 10^9 \approx 5 \times 10^7$$

Now, for iron we can write

$$\sqrt{\frac{c}{\lambda}} = a(Z - b) = 5 \times 10^7 (26 - 2.3) = 118.5 \times 10^7$$

$$\Rightarrow \lambda = \frac{3 \times 10^8}{(118.5 \times 10^7)^2} = 2.13 \times 10^{-10} \approx 2 \text{ \AA}$$

Absorption of X-rays

When a beam of X-rays of intensity I_0 passes through a thickness t of any material, they get absorbed as shown in the figure.

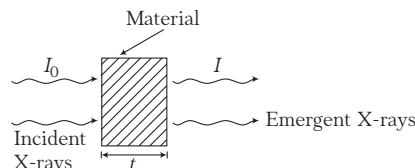


Fig. 12.28 Absorption of X-rays when incident on any material

Thus, the intensity of the emergent X-rays is given by

$$I = I_0 e^{-\mu t}$$

where, I_0 = intensity of incident X-rays,

I = intensity of emergent X-rays

and μ = absorption coefficient of the material.

Example 12.43 If the intensity of an X-ray becomes $\frac{I_0}{3}$ from I_0 after travelling 3.0 cm inside a target, then find its intensity after travelling 9 cm.

Sol. Given that, $I_1 = \frac{I_0}{3}$ at $t = 3.0$ cm, $I_2 = ?$ at $t = 9$ cm

Since, intensity of X-ray after travelling a distance t becomes

$$I = I_0 e^{-\mu t} \Rightarrow I_1 = \frac{I_0}{3} = I_0 e^{-\mu \times 3.0} \Rightarrow e^{-\mu \times 3.0} = \frac{1}{3}$$

Again, we have $I_2 = I_0 e^{-\mu \times 9} = I_0 (e^{-\mu \times 3.0})^2$

$$\Rightarrow I_2 = I_0 \left(\frac{1}{3} \right)^2 = \frac{I_0}{9}$$

Bragg's law

When X-rays are incident on a crystal, these rays may be diffracted by suitable diffracting centres as shown in the figure. However, when the path difference is equal to n (a whole number) times the wavelength, a constructive interference will occur. To obtain a maximum intensity we have

$$\text{Path difference, } 2d \sin \theta = n\lambda$$

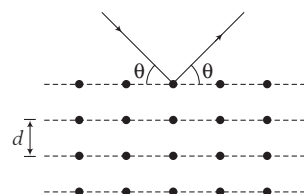


Fig. 12.29 X-rays incident on a crystal

where, $n = 1, 2, 3, \dots$

θ = angle of incidence, d = inter-atomic distance

and λ = wavelength of X-rays.

Example 12.44 The distance between two successive atomic planes of a calcite crystal is 0.3 nm . Find the minimum angle for Bragg scattering of $0.3 \text{ \AA}$ X-rays.

Sol. To obtain maximum intensity, we must have

$$2d \sin \theta = n\lambda$$

Given, $\lambda = 0.3 \text{ \AA} = 0.3 \times 10^{-10} \text{ m}$, $d = 0.3 \text{ nm} = 0.3 \times 10^{-9} \text{ m}$

For θ to be minimum, $n = 1$

$$\Rightarrow \sin \theta = \frac{n\lambda}{2d} = \frac{1 \times 0.3 \times 10^{-10}}{2 \times 0.3 \times 10^{-9}} = 0.05 \text{ rad}$$

$$\Rightarrow \sin \theta \approx \theta = \frac{0.05 \times 180^\circ}{\pi} = 2.86^\circ$$

CHECK POINT 12.3

- X-rays are being produced in a tube operating at 10^5 V . The velocity of X-rays produced is
(a) $3 \times 10^8 \text{ ms}^{-1}$ (b) $2.8 \times 10^8 \text{ ms}^{-1}$
(c) $3.1 \times 10^8 \text{ ms}^{-1}$ (d) $3 \times 10^{10} \text{ ms}^{-1}$
- If V be the accelerating voltage, then the maximum frequency of continuous X-rays is given by
(a) $\frac{eh}{V}$ (b) $\frac{hV}{e}$
(c) $\frac{eV}{h}$ (d) $\frac{h}{eV}$
- The kinetic energy of electrons that strike the target is increased, then the cut-off wavelength of continuous X-ray spectrum
(a) increases (b) decreases
(c) no change (d) cannot be said
- An X-ray tube is operated at 50 kV . The minimum wavelength produced is
(a) $0.5 \text{ \AA}$ (b) $0.75 \text{ \AA}$ (c) $0.25 \text{ \AA}$ (d) $1 \text{ \AA}$
- The energy of a photon of characteristic X-ray from a Coolidge tube comes from
(a) the kinetic energy of the striking electron
(b) the kinetic energy of the free electrons of the target
(c) the kinetic energy of the ions of the target
(d) an atomic transition in the target
- In X-ray spectrum, transition of an electron from an outer shell to an inner shell gives a characteristic X-ray spectral line. If we consider the spectral line K_β , L_β and M_α , then
(a) K_β and L_β have a common inner shell
(b) K_β and L_β have a common outer shell
(c) L_β and M_α have a common outer shell
(d) L_β and M_α have a common inner shell

Uses of X-rays

X-rays are

- used for medical imaging.
- used in treating cancer.
- useful for determining crystal structure by X-ray crystallography.
- useful for airport security.
- used in laboratories for materials characterisation.
- In art, the change occurring in old oil paintings can be examined by X-rays.

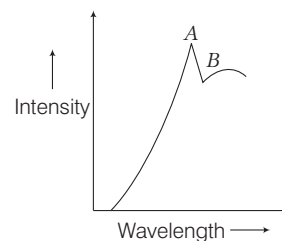
- In X-ray spectrum wavelength λ of line K_α depends on atomic number Z as

- $\lambda \propto Z^2$
- $\lambda \propto (Z-1)^2$
- $\lambda \propto \frac{1}{(Z-1)}$
- $\lambda \propto \frac{1}{(Z-1)^2}$

- For characteristic X-ray, choose the correct option.

- $E(K_\gamma) < E(K_\beta) < E(K_\alpha)$
- $E(K_\alpha) < E(L_\alpha) < E(M_\alpha)$
- $\lambda(K_\gamma) < \lambda(K_\beta) < \lambda(K_\alpha)$
- $\lambda(M_\alpha) < \lambda(L_\alpha) < \lambda(K_\alpha)$

- Sharp peak point A in the following graph represents



- Characteristic X-rays
- Continuous X-rays
- Bremsstrahlung
- Discontinuous spectrum

- Moseley's law for characteristic X-rays is $\sqrt{\nu} = a(Z - b)$. In this

- Both a and b are independent of the material
- a is independent but b depends on the material
- b is independent but a depends on the material
- Both a and b depends on the material

- If the minimum angle for scattering is 3.8° for Bragg scattering of $0.2 \text{ \AA}$ X-rays, then the distance between two successive atomic plane of the crystal is

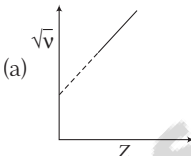
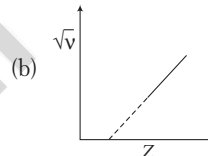
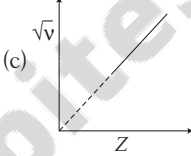
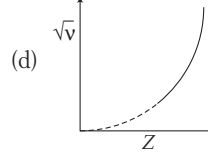
- $1.51 \text{ \AA}$
- $2 \text{ \AA}$
- $2.86 \text{ \AA}$
- $4 \text{ \AA}$

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1 The penetrating power of X-rays increases with the increase in its
 - (a) velocity
 - (b) intensity
 - (c) frequency
 - (d) wavelength
- 2 X-rays are not used for radar purpose because they
 - (a) are not reflected by the target
 - (b) are not electromagnetic waves
 - (c) are completely absorbed by the air
 - (d) sometimes damage the target
- 3 The X-ray beam coming from an X-ray tube will be
 - (a) monochromatic
 - (b) having all wavelengths larger than a certain minimum wavelength
 - (c) having all wavelengths smaller than a certain maximum wavelength
 - (d) having all wavelengths between a minimum and a maximum wavelength
- 4 Consider a photon of continuous X-ray coming from a coolidge tube. Its energy comes from
 - (a) the kinetic energy of the striking electron
 - (b) the kinetic energy of the free electrons of the target
 - (c) the kinetic energy of the ions of the target
 - (d) an atomic transition in the target
- 5 Hydrogen atom does not emit X-rays because
 - (a) its energy levels are too close to each other
 - (b) its energy levels are too far apart
 - (c) it has a very small mass
 - (d) it has a single electron
- 6 A set of atoms in an excited state decays
[NCERT Exemplar]
 - (a) in general to any of the states with lower energy
 - (b) into a lower state only when excited by an external electric field
 - (c) all together simultaneously into a lower state
 - (d) to emit photons only when they collide
- 7 In an X-rays tube, the intensity of the emitted X-rays beam is increased by
 - (a) increasing the filament current
 - (b) decreasing the filament current
 - (c) increasing the target potential
 - (d) decreasing the target potential
- 8 The difference in angular momentum associated with the electron in the two successive orbits of hydrogen atom is
 - (a) $\frac{h}{\pi}$
 - (b) $\frac{h}{2\pi}$
 - (c) $\frac{h}{2}$
 - (d) $(n-1)\frac{h}{2\pi}$
- 9 Which of the following parameters are the same for all hydrogen like atoms and ions in their ground states?
 - (a) Radius of the orbit
 - (b) Speed of the electrons
 - (c) Energy of the atom
 - (d) Orbital angular momentum of the electron
- 10 The second line in Paschen series is obtained when the electron makes transition from
 - (a) fourth orbit to third orbit
 - (b) seventh orbit to third orbit
 - (c) sixth orbit to third orbit
 - (d) fifth orbit to third orbit
- 11 For production of characteristic K_{β} X-rays, the electron transition is
 - (a) $n = 2$ to $n = 1$
 - (b) $n = 3$ to $n = 2$
 - (c) $n = 3$ to $n = 1$
 - (d) $n = 4$ to $n = 2$
- 12 Molybdenum is used as a target element for production of X-rays because it is
 - (a) light and can easily deflect electrons
 - (b) light and can absorb electrons
 - (c) a heavy element with a high melting point
 - (d) an element having high thermal conductivity
- 13 The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because
[NCERT Exemplar]
 - (a) of the electrons not being subject to a central force
 - (b) of the electrons colliding with each other
 - (c) of screening effects
 - (d) the force between the nucleus and an electron will no longer be given by Coulomb's law
- 14 An electron moves in a circular orbit at a distance from a proton with kinetic energy E . To escape to infinity, the minimum energy which must be supplied to the electron is
 - (a) E
 - (b) $2E$
 - (c) $0.5E$
 - (d) $E\sqrt{2}$

- 15** A X-ray tube operates at an accelerating potential of 20 kV. Which of the following wavelengths will be absent in the continuous spectrum of X-rays?
 (a) 12 pm (b) 75 pm (c) 65 pm (d) 95 pm
- 16** The patient is asked to drink BaSO_4 for examining the stomach by X-rays because X-rays are
 (a) reflected by heavy atoms
 (b) refracted by heavy atoms
 (c) less absorbed by heavy atoms
 (d) more absorbed by heavy atoms
- 17** In Rutherford scattering experiment, what will be the ratio of impact parameter for scattering angles $\theta_1 = 90^\circ$ and $\theta_2 = 120^\circ$.
 (a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{3}$
- 18** The minimum energy required to ionise an atom is the energy equal to
 (a) add one electron to the gaseous state of atom
 (b) excite the atom from its ground state to its first excited state
 (c) remove one outermost electron from the gaseous state of atom
 (d) remove one innermost electron from the gaseous state of atom
- 19** The velocity of an electron in the first orbit of H-atom is v . The velocity of an electron in the 2nd orbit of He^+ is
 (a) $2v$ (b) v
 (c) $v/2$ (d) $v/4$
- 20** A metal block is exposed to beams of X-ray of different wavelengths $2\text{\AA}$, $4\text{\AA}$, $6\text{\AA}$ and $8\text{\AA}$. Which of the following wavelength of X-rays penetrates most?
 (a) $2\text{\AA}$ (b) $4\text{\AA}$ (c) $6\text{\AA}$ (d) $8\text{\AA}$
- 21** Hydrogen atom is excited from ground state to another excited state with principal quantum number ($n = 4$). Then, the number of spectral lines in the emission spectra will be
 (a) 2 (b) 3
 (c) 5 (d) 6
- 22** The orbital angular momentum of electron in the n_1 th shell of element of atomic number Z_1 is L_1 and the same in the n_2 th shell of element of atomic number Z_2 is L_2 . If $L_2 > L_1$, then
 (a) $n_2 > n_1$ (b) $Z_2 > Z_1$
 (c) $n_2 Z_2 > n_1 Z_1$ (d) Both (a) and (b)
- 23** An electron jumps from the 4th orbit to 2nd orbit of the hydrogen atom. Given, Rydberg's constant $R = 10^5 \text{ cm}^{-1}$, the frequency (in Hz) of the emitted radiations will be
 (a) $\frac{3}{16} \times 10^5$ (b) $\frac{3}{16} \times 10^{15}$ (c) $\frac{9}{16} \times 10^{15}$ (d) $\frac{3}{4} \times 10^{15}$
- 24** An electron makes a transition from 4th orbit to 2nd orbit of a hydrogen atom. The wave number of the emitted radiations will be ($R = \text{Rydberg's constant}$)
 (a) $\frac{16}{3R}$ (b) $\frac{2R}{16}$ (c) $\frac{3R}{16}$ (d) $\frac{4R}{16}$
- 25** The longest wavelength that can be analysed by a sodium chloride crystal of spacing $d = 2.82 \text{\AA}$ in the second order is
 (a) $2.82 \text{\AA}$ (b) $5.64 \text{\AA}$ (c) $8.46 \text{\AA}$ (d) $11.28 \text{\AA}$
- 26** The graph between the square root of the frequency of a specific line of characteristic spectrum of X-rays and the atomic number of the target will be
- 

- 

- 27** Hard X-rays for the study of fractures in bones should have a minimum wavelength of 10^{-11} m . The accelerating voltage for electrons in X-ray machine should be
 (a) $< 124.2 \text{ kV}$
 (b) $> 124.2 \text{ kV}$
 (c) Between 60 kV and 70 kV
 (d) equal to 100 kV
- 28** If an α -particle of mass m , charge q and velocity v is incident on a nucleus of charge Q and mass M , then the distance of closest approach is
 (a) $\frac{Qq}{4\pi\epsilon_0 m^2}$ (b) $\frac{Qq}{2\pi\epsilon_0 mv^2}$ (c) $\frac{Qq mv^2}{2}$ (d) $\frac{Qq}{mv^2}$
- 29** In a hypothetical Bohr hydrogen, the mass of the electron is doubled. The energy E_0 and radius r_0 of the first orbit will be (a_0 is the Bohr radius)
 (a) $E_0 = -27.2 \text{ eV}$, $r_0 = a_0/2$
 (b) $E_0 = -27.2 \text{ eV}$, $r_0 = a_0$
 (c) $E_0 = -13.6 \text{ eV}$, $r_0 = a_0/2$
 (d) $E_0 = -13.6 \text{ eV}$, $r_0 = a_0$
- 30** Ionisation energy of a hydrogen like ion A is greater than that of another hydrogen like ion B . Let r , u , E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron, respectively. In ground state
 (a) $r_A > r_B$ (b) $u_A > u_B$ (c) $E_A < E_B$ (d) $L_A > L_B$

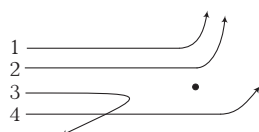
- 31** If the radius of first Bohr's orbit is x , then de-Broglie wavelength of electron in 3rd orbit is nearly

(a) $2\pi x$ (b) $6\pi x$ (c) $9x$ (d) $\frac{x}{3}$

- 32** If the number of scattering particles are 56 for 90° angle, then at an angle 60° , number of scattered particles will be

(a) 224 (b) 256 (c) 98 (d) 108

- 33** The diagram shows the path of four α -particles of the same energy being scattered by the nucleus of an atom simultaneously. Which of these are/is not physically possible?



(a) Both 3 and 4 (b) Both 2 and 3
(c) Both 1 and 4 (d) Only 4

- 34** If ω is the speed of electron in the n th orbit hydrogen atom, then

(a) $\omega \propto n^{1/2}$ (b) $\omega \propto \frac{1}{n}$ (c) $\omega \propto \frac{1}{n^2}$ (d) $\omega \propto \frac{1}{n^3}$

- 35** Hydrogen atom emits blue light when it jumps from $n = 4$ energy level to the $n = 2$ level. Which colour of light would the atom emit when it changes from the $n = 5$ level to the $n = 2$ level?

(a) Red (b) Yellow (c) Green (d) Violet

- 36** Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about

[NCERT Exemplar]

(a) 53 pm (b) 27 pm (c) 18 pm (d) 13 pm

- 37** When an electron in hydrogen atom is excited, from its 4th to 5th stationary orbit, the change in angular momentum of electron is (take, Planck's constant, $h = 6.6 \times 10^{-34}$ J-s)

(a) 4.16×10^{-34} J-s (b) 3.32×10^{-34} J-s
(c) 1.05×10^{-34} J-s (d) 5.25×10^{-34} J-s

- 38** The electron in a hydrogen atom makes a transition from $n = n_1$ to $n = n_2$ state. The time period of the electron in the initial state (n_1) is eight times that in the final state (n_2). The possible values of n_1 and n_2 are

(a) $n_1 = 8, n_2 = 1$ (b) $n_1 = 2, n_2 = 1$
(c) $n_1 = 2, n_2 = 4$ (d) $n_1 = 1, n_2 = 8$

- 39** Two elements A and B with atomic numbers Z_A and Z_B are used to produce characteristic X-rays with frequencies ν_A and ν_B , respectively. If $Z_A : Z_B = 1 : 2$, then $\nu_A : \nu_B$ will be

(a) $1 : \sqrt{2}$ (b) $1 : 8$ (c) $4 : 1$ (d) $1 : 4$

- 40** An electron revolves around a nucleus of charge Ze . In order to excite the electron from the state $n = 2$ to $n = 3$, the energy required is 47.2 eV. Z is equal to

(a) 3 (b) 4 (c) 5 (d) 2

- 41** Excitation energy of a hydrogen like ion in its first excitation state is 40.8 eV. Energy needed to remove the electron from the ion in ground state is

(a) 54.4 eV (b) 13.6 eV (c) 40.8 eV (d) 27.2 eV

- 42** Ionisation potential of hydrogen atom is 13.6 V. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV. The spectral lines emitted by hydrogen atoms according to Bohr's theory will be

(a) one (b) two (c) three (d) four

- 43** The acceleration of electron in the first orbit of hydrogen atom is

(a) $\frac{4\pi^2 m}{h^3}$ (b) $\frac{h^2}{4\pi^2 m r}$ (c) $\frac{h^2}{4\pi^2 m^2 r^3}$ (d) $\frac{m^2 h^2}{4\pi^2 r^3}$

- 44** The ratio of the wavelength for $2 \rightarrow 1$ transition in Li^{++} , He^+ and H is

(a) 1 : 2 : 3 (b) 1 : 4 : 9 (c) 4 : 9 : 36 (d) 3 : 2 : 1

- 45** Which element has a K_α line of wavelength 1.785 Å?

(a) Copper (b) Cobalt (c) Sodium (d) Aluminium

- 46** The wavelength of K_α line for an element of atomic number 43 is λ . Then the wavelength of K_α line for an element of atomic number 29 is

(a) $\frac{43}{29} \lambda$ (b) $\frac{42}{28} \lambda$ (c) $\frac{9}{4} \lambda$ (d) $\frac{4}{9} \lambda$

- 47** The K_α and K_β lines of characteristic X-ray spectrum of molybdenum are 0.76 Å and 0.64 Å, respectively. The wavelength of L_α line is

(a) 1.4 Å (b) 2.4 Å (c) 4.1 Å (d) 3.6 Å

- 48** The magnetic moment (μ) of a revolving electron around the nucleus varies with principal quantum number n as

(a) $\mu \propto n$ (b) $\mu \propto 1/n$ (c) $\mu \propto n^2$ (d) $\mu \propto 1/n^2$

- 49** The wavelength of K_α X-rays for lead isotopes Pb^{208} , Pb^{206} , Pb^{204} are λ_1 , λ_2 and λ_3 , respectively. Then

(a) $\lambda_1 = \lambda_2 > \lambda_3$ (b) $\lambda_1 > \lambda_2 > \lambda_3$
(c) $\lambda_1 < \lambda_2 < \lambda_3$ (d) $\lambda_2 = \sqrt{\lambda_1 \lambda_3}$

- 50** The ratio between acceleration of the electron in singly ionised helium atom and doubly ionised lithium atom (both in ground state) is

(a) $\frac{4}{9}$ (b) $\frac{27}{8}$ (c) $\frac{8}{27}$ (d) $\frac{9}{4}$

- 51** The wavelength of radiation emitted is λ_0 when an electron jumps from the third to the second orbit of hydrogen atom. For the electron jump from the fourth to the second orbit of the hydrogen atom, the wavelength of radiation emitted will be
 (a) $\frac{16}{25} \lambda_0$ (b) $\frac{20}{27} \lambda_0$ (c) $\frac{27}{20} \lambda_0$ (d) $\frac{25}{16} \lambda_0$
- 52** An electron with kinetic energy 5 eV is incident on a H-atom in its ground state. The collision
 (a) must be elastic
 (b) may be partially elastic
 (c) must be completely inelastic
 (d) may be completely inelastic
- 53** When an electron jumps from the orbit $n = 2$ to $n = 4$, then wavelength of the radiations absorbed will be (R is Rydberg's constant)
 (a) $\frac{16}{3R}$ (b) $\frac{16}{5R}$ (c) $\frac{5R}{16}$ (d) $\frac{3R}{16}$
- 54** If ν_1 is the frequency of the series limit of Lyman series, ν_2 is the frequency of the first line of Lyman series and ν_3 is the frequency of the series limit of the Balmer series. Then,
 (a) $\nu_1 - \nu_2 = \nu_3$ (b) $\nu_1 = \nu_2 - \nu_3$
 (c) $\frac{1}{\nu_2} = \frac{1}{\nu_1} + \frac{1}{\nu_3}$ (d) $\frac{1}{\nu_1} = \frac{1}{\nu_2} + \frac{1}{\nu_3}$
- 55** The energy of an electron in excited hydrogen atom is -3.4 eV. Then, according to Bohr's theory, the angular momentum of the electron is
 (a) 2.1×10^{-34} J-s (b) 3×10^{-34} J-s
 (c) 2×10^{-34} J-s (d) 0.5×10^{-34} J-s
- 56** The shortest wavelength which can be obtained in hydrogen spectrum is ($R = 10^7 \text{ m}^{-1}$)
 (a) 1000 Å (b) 800 Å (c) 1300 Å (d) 2100 Å
- 57** The wavelength of the first spectral line of sodium is 5896 Å. The first excitation potential of sodium atom will be (Planck's constant, $h = 6.63 \times 10^{-34}$ J-s)
 (a) 4.2 V (b) 3.5 V
 (c) 2.1 V (d) None of these
- 58** The shortest wavelength in Lyman series is 91.2 nm. The longest wavelength of the series is
 (a) 121.6 nm (b) 182.4 nm
 (c) 243.4 nm (d) 364.8 nm
- 59** Energy of 24.6 eV is required to remove one of the electron from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is
 (a) 38.2 (b) 49.2
 (c) 51.8 (d) 79.0
- 60** An excited hydrogen atom emits a photon of wavelength λ in returning to the ground state. The quantum number n of the excited state is given by (R = Rydberg constant)
 (a) $\sqrt{\lambda R(\lambda R - 1)}$ (b) $\sqrt{\frac{\lambda R}{(\lambda R - 1)}}$
 (c) $\sqrt{\frac{(\lambda R - 1)}{\lambda R}}$ (d) $\sqrt{\frac{1}{\lambda R(\lambda R - 1)}}$
- 61** According to Moseley's law the ratio of the slope of graph between $\sqrt{f}$ and Z for K_β and K_α is
 (a) $\sqrt{\frac{32}{27}}$ (b) $\sqrt{\frac{27}{32}}$ (c) $\sqrt{\frac{5}{36}}$ (d) $\sqrt{\frac{36}{5}}$
- 62** de-Broglie wavelength of an electron in the n th Bohr orbit of hydrogen atom is λ_n and the angular momentum is J_n , then
 (a) $J_n \propto \lambda_n$ (b) $\lambda_n \propto \frac{1}{J_n}$
 (c) $\lambda_n \propto J_n^2$ (d) None of these
- 63** The recoil momentum of H-atom due to the transition of an electron from $n = 4$ state to $n = 1$ state is
 (a) $13.6 \times 10^{-19} \text{ kg ms}^{-1}$ (b) $6.8 \times 10^{-27} \text{ kg ms}^{-1}$
 (c) $12.75 \times 10^{-24} \text{ kg ms}^{-1}$ (d) $9.86 \times 10^{-18} \text{ kg ms}^{-1}$
- 64** The radius of hydrogen atom in its ground state is $5.3 \times 10^{-11} \text{ m}$. After collision with an electron it is found to have a radius of $21.2 \times 10^{-11} \text{ m}$. What is the principal quantum number n of the final state of the atom?
 (a) $n = 4$ (b) $n = 2$
 (c) $n = 16$ (d) $n = 3$
- 65** An α -particle accelerated through V volt is fired towards a nucleus. Its distance of closest approach is r . If a proton accelerated through the same potential is fired towards the same nucleus, the distance of closest approach of proton will be
 (a) r (b) $2r$
 (c) $\frac{r}{2}$ (d) $\frac{r}{4}$
- 66** In a Rutherford scattering experiment, when a projectile of charge Z_1 and mass M_1 approaches a target nucleus of charge Z_2 and mass M_2 , the distance to closest approach is r_0 . The energy of the projectile is
 (a) directly proportional to $M_1 \times M_2$
 (b) directly proportional to $Z_1 Z_2$
 (c) directly proportional to Z_1
 (d) directly proportional to mass M_1

- 67** In the Bohr's model of a hydrogen atom, the centripetal force is furnished by the Coulomb attraction between the proton and the electron. If a_0 is the radius of the ground state orbit, m is the mass, e is the charge on electron and ϵ_0 is the permittivity of free space, the speed of the electron is

(a) $\frac{e}{\sqrt{\epsilon_0 a_0 m}}$ (b) zero (c) $\frac{e}{\sqrt{4\pi\epsilon_0 a_0 m}}$ (d) $\frac{\sqrt{4\epsilon_0 a_0 m}}{e}$

- 68** The first member of the Balmer's series of the hydrogen has a wavelength λ , the wavelength of the second member of its series is

(a) $\frac{27}{20}\lambda$ (b) $\frac{20}{27}\lambda$ (c) $\frac{27}{10}\lambda$ (d) $\frac{10}{9}\lambda$

- 69** In a hydrogen atom, the binding energy of the electron in the ground state is E_1 . Then the frequency of revolution of the electron in the n th orbit is

(a) $\frac{2E_1}{n^3 h}$ (b) $\frac{2E_1 n^3}{h}$ (c) $\sqrt{\frac{2mE_1}{n^3 h}}$ (d) $\frac{E_1 n^2}{h}$

- 70** The electron in a hydrogen atom makes transition from M shell to L -shell. The ratio of magnitude of initial to final acceleration of the electron is

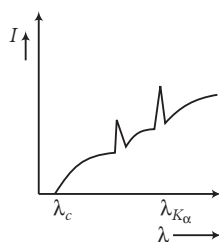
(a) 9 : 4 (b) 81 : 16
(c) 4 : 9 (d) 16 : 81

- 71** For the ground state, the electron in the H-atom has an angular momentum $= h$, according to the simple Bohr model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,

[NCERT Exemplar]

- (a) because Bohr model gives incorrect values of angular momentum
(b) because only one of these would have a minimum energy
(c) angular momentum must be in the direction of spin of electron
(d) because electrons go around only in horizontal orbits

- 72** The intensity of X-rays from a coolidge tube is plotted against wavelength as shown in the figure. The minimum wavelength found is λ_c and the wavelength of the K_α line is λ_{K_α} . As the accelerating voltage is increased



- (a) $(\lambda_{K_\alpha} - \lambda_c)$ increases (b) $(\lambda_{K_\alpha} - \lambda_c)$ decreases
(c) λ_{K_α} increases (d) λ_{K_α} decreases

- 73** A H-atom moving with speed v makes a head on collision with a H-atom at rest. Both atoms are in ground state. The minimum value of velocity v for which one of the atom may excite is ($m_H = 1.67 \times 10^{-27}$ kg)

(a) $6.25 \times 10^4 \text{ ms}^{-1}$ (b) $8 \times 10^4 \text{ ms}^{-1}$
(c) $7.25 \times 10^4 \text{ ms}^{-1}$ (d) $13.6 \times 10^4 \text{ ms}^{-1}$

- 74** In an inelastic collision an electron excites a hydrogen atom from its ground state to a M -shell state. A second electron collides instantaneously with the excited hydrogen atom in the M -shell state and ionizes it. At least how much energy the second electron transfers to the atom in the M -shell state?

(a) + 3.4 eV (b) + 1.51 eV
(c) - 3.4 eV (d) - 1.51 eV

- 75** A hydrogen-like atom emits radiations of frequency 2.7×10^{15} Hz when it makes a transition from $n = 2$ to $n = 1$. The frequency emitted in a transition from $n = 3$ to $n = 1$ will be

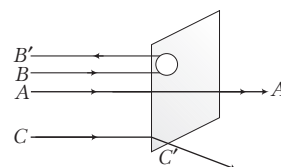
(a) 1.8×10^{15} Hz (b) 3.2×10^{15} Hz
(c) 4.7×10^{15} Hz (d) 6.9×10^{15} Hz

- 76** Two H-atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is

[NCERT Exemplar]

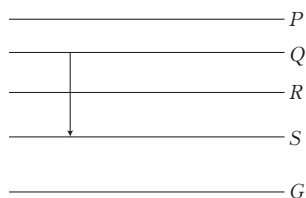
(a) 10.2 eV (b) 20.40 eV (c) 13.6 eV (d) 27.2 eV

- 77** A beam of fast moving α -particles were directed towards a thin film of gold. The parts A' , B' and C' of the transmitted and reflected beams corresponding to the incident parts A , B and C of the beam, are shown in the adjoining diagram. The number of α -particles in



- (a) B' will be minimum and in C' maximum
(b) A' will be maximum and in B' minimum
(c) A' will be minimum and in B' maximum
(d) C' will be minimum and in B' maximum

- 78** Figure shows the energy levels P , Q , R , S and G of an atom where G is the ground state. A red line in the emission spectrum of the atom can be obtained by an energy level change from Q to S . A blue line can be obtained by following energy level change



- (a) P to Q (b) Q to P (c) R to S (d) R to G

- 79** The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is

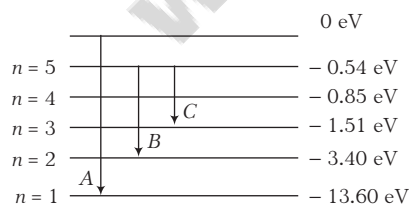
$$B = -\frac{me^4}{8n^2\epsilon_0^2h^2} \quad (m = \text{electron mass})$$

If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$$B = -\frac{Me^4}{8n^2\epsilon_0^2h^2} \quad (M = \text{proton mass})$$

This last expression is not correct, because [NCERT Exemplar]

- (a) n would not be integral
 (b) Bohr-quantisation applies only two electron
 (c) the frame in which the electron is at rest, is non-inertial
 (d) the motion of the proton would not be in circular orbits, even approximately.
- 80** If the intensity of an X-ray becomes $\frac{I_0}{2}$ from I_0 after travelling 2.0 cm inside a target, then its intensity after travelling a distance of 4 cm will be
- (a) $\frac{I_0}{2}$ (b) $\frac{I_0}{4}$ (c) $2I_0$ (d) I_0
- 81** In figure the energy levels of the hydrogen atom have been shown along with some transitions marking A, B, C. The transitions A, B, and C respectively, represents

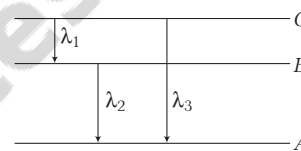


- (a) the first member of the Lyman series, third member of Balmer series and second member of Paschen series
 (b) the ionisation potential of H, second member of Balmer series and third member of Paschen series
 (c) the series limit of Lyman series, second member of Balmer series and second member of Paschen series
 (d) the series limit of Lyman series, third member of Balmer series and second member of Paschen series

- 82** Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atomic model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength of photon in the Balmer series has wavelength λ (given in terms of the Rydberg constant R for hydrogen atom) equal to

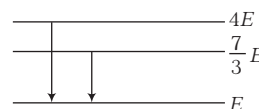
- (a) $\frac{9}{5R}$ (b) $\frac{36}{5R}$ (c) $\frac{18}{5R}$ (d) $\frac{4}{5R}$

- 83** Energy levels A, B and C of a certain atom corresponding to increasing values of energy, i.e. $E_A < E_B < E_C$. If λ_1 , λ_2 and λ_3 are the wavelengths of radiations corresponding to the transitions C to B, B to A and C to A respectively, which of the following option is correct?



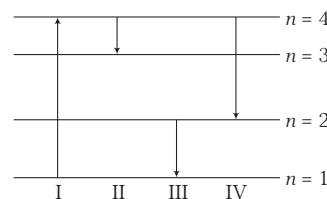
- (a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$
 (c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

- 84** The following diagram indicates the energy levels of a certain atom when the system moves from $4E$ level to E . A photon of wavelength λ_1 is emitted. The wavelength of photon produced during its transition from $7/3 E$ level to E is λ_2 . The ratio λ_1/λ_2 will be



- (a) $\frac{4}{9}$ (b) $\frac{4}{8}$ (c) $\frac{3}{8}$ (d) $\frac{5}{9}$

- 85** The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



- (a) III (b) IV (c) I (d) II

- 86** When the voltage applied to an X-ray tube is increased from $V_1 = 10$ kV to $V_2 = 20$ kV, the wavelength difference between the K_α line and the short wavelength limit of the continuous X-ray

spectrum increases by a factor 3. The atomic number of the element of which the tube anticathode is made will be

- (a) 62 (b) 56
(c) 45 (d) 29

- 87** The ionisation potential of hydrogen atom is -13.6 eV. An electron in the ground state of a hydrogen atom absorbs a photon of energy 12.75 eV. How many different spectral lines can one expect when the electron make a downward transition

- (a) 1 (b) 4
(c) 2 (d) 6

- 88** The distance of closest approach of an α -particle fired towards a nucleus with momentum p , is r . If the momentum of the α -particle is $2p$, then the corresponding distance of closest approach is

- (a) $r/2$ (b) $2r$
(c) $4r$ (d) $r/4$

- 89** The first excited state of hydrogen atom is 10.2 eV above its ground state. The temperature is needed to excite hydrogen atoms to first excited level, is

- (a) 7.9×10^4 K (b) 3.5×10^4 K
(c) 5.8×10^4 K (d) 14×10^4 K

- 90** Hydrogen (${}_1\text{H}^1$), deuterium (${}_1\text{H}^2$), singly ionised helium (${}_2\text{He}^4$)⁺ and doubly ionised lithium (${}_3\text{Li}^6$)⁺⁺

(${}_3\text{Li}^6$)⁺⁺ all have one electron around the nucleus.

Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4

respectively, then approximately which one of the following is correct?

- (a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$
(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

- 91** Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$, where k is a constant and r is the

distance of the electron from the origin. By applying Bohr model to this system, the radius of the n th orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n . Then, which of the following is true?

- (a) T_n independent of $n, r_n \propto n$
(b) $T_n \propto \frac{1}{n}, r_n \propto n$
(c) $T_n \propto \frac{1}{n}, r_n \propto n^2$
(d) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) These questions consist of two statements each linked as Assertion and Reason. While answering of these questions you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are true but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

- 1 Assertion** Angular momentum of single electron in any orbit of hydrogen type atoms is independent of the atomic number of the element.

Reason In ground state, angular momentum is minimum.

- 2 Assertion** The radius of second orbit of hydrogen atom is $2.1\text{\AA}$.

Reason Radius of n th orbit of hydrogen atom is $r_n \propto n^2$.

- 3 Assertion** It is essential that all the lines available in the emission spectrum will also be available in the absorption spectrum.

Reason The spectrum of hydrogen atom may not be absorption spectrum.

- 4 Assertion** Second orbit circumference of hydrogen atom is two times the de-Broglie wavelength of electrons in that orbit.

Reason de-Broglie wavelength of electron in ground state is minimum.

- 5 Assertion** Characteristic X-rays depends only on the accelerating voltage and continuous X-rays depends on the target used.

Reason Lead is the best absorber of X-rays.

Statement based questions

- 1** Study the following statements.

- I. Nature of the graph between the radius of the electron in n th orbit versus n^2 is a straight line.
II. Electrons in the higher orbit stays only for 10^{-8} s and returns back to lower orbit.

III. $1Rhc = 13.6 \text{ eV}$

Choose the correct statement.

- (a) I only (b) II only
(c) II and III only (d) All of these

2 Study the following statements.

- I. Frequencies of the Brackett lines of the H-atom lie in the visible region.
II. Bremsstrahlung is produced when protons are accelerated by the nucleus.
III. There exists a sharp limit on the short wavelength side for each continuous X-ray spectrum.

Choose the correct statement.

- (a) I and II only (b) III only
(c) I only (d) II only

3 Study the following statements.

- I. The characteristic X-ray spectrum depends on the nature of the material of the target.
II. The short wavelength limit of continuous X-ray spectrum varies inversely on the potential difference applied to the X-ray tube.

Choose the correct statement.

- (a) I is true and II is false (b) I is false and II is true
(c) Both I and II are true (d) Both I and II are false

4 The highly excited states ($n \gg 1$) for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n . Then, study the following statements.

- I. The relative change in the radii of two consecutive orbitals does not depend on Z .
II. The relative change in the radii of two consecutive orbitals varies as $1/n$.
III. The relative change in the energy of two consecutive orbitals varies as $1/n^3$.
IV. The relative change in the angular momentum of two consecutive orbitals varies as $1/n$.

Now, choose the correct statement.

- (a) I and III only (b) II and IV only
(c) I, II and IV only (d) III only

5 The Rutherford α -particle experiment shows that most of the α -particles pass through almost unscattered while some are scattered through large angles. Then, which of the following statement is in accordance with the predictions of Rutherford.

- (a) Atom is hollow.
(b) The whole mass of the atom is concentrated in a small centre called nucleus.
(c) Nucleus is positively charged.
(d) All of the above

6 Which of the following statement is not correct in Bohr's model of hydrogen atom?

- (a) The radius of n th orbit is proportional to n^2 .
(b) The total energy of electron in n th orbit is proportional to n .

- (c) The angular momentum of an electron in an orbit is an integral multiple of $h/2\pi$.
(d) The magnitude of the potential energy of an electron in any orbit is greater than its kinetic energy.

7 A particular hydrogen like atom has its ground state binding energy 122.4 eV . It is in ground state. Then, choose the incorrect statement.

- (a) Its atomic number is 3.
(b) An electron of 90 eV can excite it.
(c) An electron of kinetic energy nearly 91.8 eV can be brought to almost rest by this atom.
(d) An electron of kinetic energy 2.6 eV may emerge from the atom when electron of kinetic energy 125 eV collides with this atom.

8 The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true?

- (a) Its kinetic energy increases, and the potential and total energies decrease.
(b) Its kinetic energy decreases but the potential energy increases, and thus the total energy remains the same.
(c) Its kinetic and total energies decrease, and the potential energy increases.
(d) Its kinetic, potential and total energies decrease.

9 Electrons each having the energy 80 keV are incident on the tungsten target of an X-ray tube. The K-shell electrons of tungsten have -72.5 keV energy. Then, which amongst the following X-ray spectrum is emitted by the tube?

- (a) A continuous X-ray spectrum (bremsstrahlung) with a minimum wavelength of $\sim 0.0155 \text{ nm}$.
(b) A continuous X-ray spectrum (bremsstrahlung) with all wavelengths.
(c) Characteristic X-ray spectrum of tungsten.
(d) A continuous X-ray spectrum (bremsstrahlung) with a minimum wavelength of $\sim 0.0155 \text{ nm}$ and the characteristic X-ray spectrum of tungsten.

Match the columns

1 In Column I physical quantities corresponding to hydrogen and hydrogen like atoms are given. In Column II, powers of principal quantum number n are given on which, those physical quantities depend. Match the two columns and choose the correct option from the codes given below.

Column I	Column II
(A) Centripetal acceleration in circular motion of electron	(p) -4
(B) Angular momentum of electron	(q) 1
(C) Moment of inertia of electron about centroidal axis	(r) -3
	(s) 4

Codes

- | | | | | | |
|-------|---|---|-------|---|---|
| A | B | C | A | B | C |
| (a) s | p | q | (b) q | q | s |
| (c) p | q | s | (d) p | p | s |

- 2** Ground state energy of hydrogen atom is E_0 . Match the following two columns and choose the correct option from the codes given below.

Column I	Column II
(A) Electrostatic potential energy in ground state of hydrogen atom	(p) E_0
(B) Total energy in first excited state of He^+ ion	(q) $-E_0$
(C) Kinetic energy of electron in first excited state of He^+ ion	(r) $2E_0$
(D) Kinetic energy of electron in ground state of hydrogen atom	(s) $-2E_0$

Codes

A	B	C	D	A	B	C	D
(a) p	r	s	p	(b) r	p	q	q
(c) p	p	p	r	(d) r	p	s	q

- 3** Match the following two columns and choose the correct option from the codes given below.

Column I	Column II
(A) $\frac{Z^3}{n^5}$	(p) Angular speed
(B) $\frac{Z^2}{n^2}$	(q) Magnetic field at the centre due to revolution of electron
(C) $\frac{Z^2}{n^3}$	(r) Potential energy of an electron in n th orbit
(D) $\frac{Z}{n}$	(s) Speed of electron in n th orbit

Codes

A	B	C	D
(a) s	p	q	r
(b) q	r	p	s
(c) q	r	s	p
(d) q	p	r	s

(C) Medical entrances' gallery

(Collection of questions asked in NEET & various medical entrance exams)

- 1** The total energy of an electron in the n th stationary orbit of the hydrogen atom can be obtained by [NEET 2020]
 (a) $E_n = \frac{13.6}{n^2} \text{ eV}$ (b) $E_n = -\frac{13.6}{n^2} \text{ eV}$
 (c) $E_n = -\frac{1.36}{n^2} \text{ eV}$ (d) $E_n = -13.6 \times n^2 \text{ eV}$
- 2** For which one of the following, Bohr model is not valid? [NEET 2020]
 (a) Singly ionised helium atom (He^+)
 (b) Deuteron atom
 (c) Singly ionised neon atom (Ne^+)
 (d) Hydrogen atom
- 3** The total energy of an electron in an atom in an orbit is -3.4 eV . Its kinetic and potential energies are, respectively [NEET 2019]
 (a) -3.4 eV , -6.8 eV (b) 3.4 eV , -6.8 eV
 (c) 3.4 eV , 3.4 eV (d) -3.4 eV , -3.4 eV
- 4** The radius of the first permitted Bohr orbit for the electron, in a hydrogen atom equals $0.51 \text{ \AA}$ and its ground state energy equals -13.6 eV . If the electron in the hydrogen atom is replaced by muon (μ^-) [Charge same as electron and mass $207 m_e$], the first Bohr radius and ground state energy will be [NEET (Odisha) 2019]
 (a) $0.53 \times 10^{-13} \text{ m}$, -3.6 eV
 (b) $25.6 \times 10^{-13} \text{ m}$, -2.8 eV
 (c) $2.56 \times 10^{-13} \text{ m}$, -2.8 keV
 (d) $2.56 \times 10^{-13} \text{ m}$, -13.6 eV
- 5** 15 eV is given to electron in 4th orbit, then find its final energy when it comes out of H-atom. [AIIMS 2019]
 (a) 14.15 eV (b) 13.6 eV
 (c) 12.08 eV (d) 15.85 eV
- 6** The ratio of kinetic energy to the total energy of an electron in a Bohr orbit of the hydrogen atom is [NEET 2018]
 (a) $2 : -1$ (b) $1 : -1$
 (c) $1 : 1$ (d) $1 : -2$
- 7 Assertion** If electron in an atom were stationary, then they would fall into the nucleus.
Reason Electrostatic force of attraction acts between negatively charged electrons and positive nucleus. [AIIMS 2018]
 (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.
- 8** The ratio of wavelength of the last line of Balmer series and the last line of Lyman series is [NEET 2017]
 (a) 2 (b) 1
 (c) 4 (d) 0.5
- 9** Given the value of Rydberg constant is 10^7 m^{-1} , the wave number of the last line of the Balmer series in hydrogen spectrum will be [NEET 2016]
 (a) $0.5 \times 10^7 \text{ m}^{-1}$ (b) $0.25 \times 10^7 \text{ m}^{-1}$
 (c) $2.5 \times 10^7 \text{ m}^{-1}$ (d) $0.025 \times 10^4 \text{ m}^{-1}$

- 10** When an α -particle of mass m moving with velocity v bombards on a heavy nucleus of charge Ze , its distance of closest approach from the nucleus depends on m as [NEET 2016]
- (a) $\frac{1}{\sqrt{m}}$ (b) $\frac{1}{m^2}$
(c) m (d) $\frac{1}{m}$
- 11** If an electron in a hydrogen atom jumps from the 3rd orbit to the 2nd orbit, it emits a photon of wavelength λ . When it jumps from the 4th orbit to the 3rd orbit, the corresponding wavelength of the photon will be [NEET 2016]
- (a) $\frac{16}{25}\lambda$ (b) $\frac{9}{16}\lambda$
(c) $\frac{20}{7}\lambda$ (d) $\frac{20}{13}\lambda$
- 12** Consider 3rd orbit of He^+ (Helium), using non-relativistic approach, the speed of electron in this orbit will be (take, $K = 9 \times 10^9$ constant, $Z = 2$ and Planck's constant, $h = 6.6 \times 10^{-34}$ J-s) [CBSE AIPMT 2015]
- (a) $2.92 \times 10^6 \text{ ms}^{-1}$ (b) $1.46 \times 10^6 \text{ ms}^{-1}$
(c) $0.73 \times 10^6 \text{ ms}^{-1}$ (d) $3 \times 10^8 \text{ ms}^{-1}$
- 13** Number of spectral lines in hydrogen atom is [Guj. CET 2015]
- (a) 8 (b) 6
(c) 15 (d) ∞
- 14** The excitation potential of hydrogen atom in the first excited state is [CG PMT 2015]
- (a) 13.6 V (b) -13.6 V
(c) 10.2 V (d) -10 V
- 15** Frequency of K_β X-ray of a material, frequency of K_α X-ray and frequency of L_α X-ray are related as [UP CPMT 2015]
- (a) $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$ (b) $\nu_{K_\beta} = \nu_{K_\alpha} - \nu_{L_\alpha}$
(c) $\nu_{K_\beta} = \frac{\nu_{K_\alpha}}{\nu_{L_\alpha}}$ (d) $\nu_{K_\beta}^2 = \frac{\nu_{K_\alpha} \cdot \nu_{L_\alpha}}{2}$
- 16** The ionisation energy of the electron in the hydrogen atom in its ground state is 13.6 eV. The atoms are excited to higher energy levels to emit radiations of 6 wavelengths. Maximum wavelength of emitted radiation corresponds to the transition between [Manipal 2015]
- (a) $n = 3$ to $n = 1$ states (b) $n = 4$ to $n = 3$ states
(c) $n = 3$ to $n = 2$ states (d) $n = 2$ to $n = 1$ states
- 17** A photon of wavelength 300 nm interacts with a stationary hydrogen atom in ground state. During the interaction, whole energy of the photon is transferred to the electron of the atom. State which possibility is correct. (Take, Planck's constant $= 4 \times 10^{-15}$ eVs, velocity of light $= 3 \times 10^8 \text{ ms}^{-1}$, ionisation energy of hydrogen $= 13.6$ eV) [WB JEE 2015]
- (a) Electron will be knocked out of the atom.
(b) Electron will go to any excited state of the atom.
(c) Electron will go only to first excited state of the atom.
(d) Electron will keep orbiting in the ground state of the atom.
- 18** What is the wavelength of light for the least energetic photon emitted in the Lyman series of the hydrogen spectrum? (Take, $hc = 1240$ eV-nm) [KCET 2015]
- (a) 122 nm (b) 82 nm
(c) 150 nm (d) 102 nm
- 19** The wavelength of K_α X-rays produced by an X-ray tube is $0.76 \text{ \AA}$. The atomic number of the anode material of the tube is [CG PMT 2015]
- (a) 20 (b) 60
(c) 41 (d) 80
- 20** Hydrogen atom in ground state is excited by a monochromatic radiation of $\lambda = 975 \text{ \AA}$. Number of spectral lines in the resulting spectrum emitted will be [CBSE AIPMT 2014]
- (a) 3 (b) 2
(c) 6 (d) 10
- 21** The de-Broglie wavelength of an electron in 4th orbit is (where, r = radius of 1st orbit) [MHT CET 2014]
- (a) $2\pi r$ (b) $4\pi r$
(c) $8\pi r$ (d) $16\pi r$
- 22** The ionisation energy of hydrogen is 13.6 eV. The energy of the photon released when an electron jumps from the first excited state ($n = 2$) to the ground state of hydrogen atom is [WB JEE 2014]
- (a) 3.4 eV (b) 4.53 eV
(c) 10.2 eV (d) 13.6 eV
- 23** If an electron in hydrogen atom jumps from an orbit of level $n = 3$ to an orbit of level $n = 2$, emitted radiation has a frequency (R = Rydberg's constant, c = velocity of light) [MHT CET 2014]
- (a) $\frac{3Rc}{27}$ (b) $\frac{Rc}{25}$
(c) $\frac{8Rc}{9}$ (d) $\frac{5Rc}{36}$

- 24** Ratio of longest wavelengths corresponding to Lyman and Balmer series in hydrogen spectrum is [NEET 2013]

- (a) $\frac{5}{27}$ (b) $\frac{3}{23}$
(c) $\frac{7}{29}$ (d) $\frac{9}{31}$

- 25** The Rutherford scattering experiment proves that an atom consists of [J & K CET 2013]

- (a) a sphere of positive charge in which electrons are embedded like seeds of water-melon
(b) a sphere of negative charge in which protons are embedded like seeds of water-melon
(c) a sphere of electron cloud in which the positive charge is placed at the centre of the sphere
(d) a sphere of neutral charge

- 26** According to Bohr model of hydrogen atom, only those orbits are permissible which satisfy the condition [J & K CET 2013]

- (a) $mv = nh$ (b) $\frac{mv^2}{r} = n \left(\frac{h}{2\pi} \right)$
(c) $mvr = n \left(\frac{h}{2\pi} \right)$ (d) $mvr^2 = n \left(\frac{h}{2\pi} \right)$

- 27** In Moseley's law $\sqrt{\nu} = a(Z - b)$, the values of the screening constant for K -series and L -series of X-rays are respectively [EAMCET 2013]

- (a) 1, 7.4 (b) 1, 4
(c) 4, 6 (d) 2, 4

- 28** The K_{α} X-ray of molybdenum has a wavelength of 71×10^{-12} m. If the energy of a molybdenum atom with K -electron removed is 23.32 keV, then the energy of molybdenum atom when an L -electron removed is ($hc = 12.42 \times 10^{-7}$ eV) [EAMCET 2013]

- (a) 17.5 keV (b) 40.82 keV
(c) 23.32 keV (d) 5.82 keV

- 29** Light emitted during the de excitation of electron from $n = 3$ to $n = 2$, when incident on a metal, photoelectrons are just emitted from that metal. In which of the following de excitations photoelectric effect is not possible? [Karnataka CET 2013]

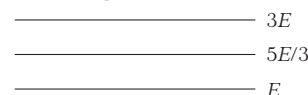
- (a) From $n = 2$ to $n = 1$
(b) From $n = 3$ to $n = 1$
(c) From $n = 5$ to $n = 2$
(d) From $n = 4$ to $n = 3$

- 30** The ionisation energy of an electron in the ground state of helium atom is 24.6 eV. The energy required to remove both the electron is [Karnataka CET 2013]

- (a) 51.8 eV (b) 79 eV
(c) 38.2 eV (d) 49.2 eV

- 31** The figure shows the energy level of certain atom. When the electron de-excites from $3E$ to E , an electromagnetic wave of wavelength λ is emitted. What is the wavelength of the electromagnetic wave emitted when the electron de-excites from $\frac{5E}{3}$ to E ?

[Karnataka CET 2013, UP CPMT 2013]



- (a) 3λ (b) 2λ
(c) 5λ (d) $\frac{3\lambda}{5}$

- 32** Spectrum of X-rays is [MP PMT 2013]

- (a) continuous
(b) linear
(c) continuous and linear
(d) band

- 33** Which series of hydrogen spectrum corresponds to ultraviolet region? [MP PMT 2013]

- (a) Balmer series (b) Brackett series
(c) Paschen series (d) Lyman series

- 34** As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionised Li-atom ($Z = 3$) is [UP CPMT 2013]

- (a) 151 (b) 28.7
(c) 53.9 (d) 122.4

- 35** Electron in hydrogen atom first jumps from third excited state to second excited state and then from second excited to the first excited state. The ratio of the wavelengths $\lambda_1 : \lambda_2$ emitted in the two cases is [CBSE AIPMT 2012]

- (a) 7/5 (b) 27/20
(c) 27/5 (d) 20/7

- 36** An electron of a stationary hydrogen atom passes from the fifth energy level to the ground level. The velocity that the atom acquired as a result of photon emission will be (m is the mass of the electron, R is Rydberg constant and h is Planck's constant) [CBSE AIPMT 2012]

- (a) $\frac{24 hR}{25 m}$ (b) $\frac{25 hR}{24 m}$
(c) $\frac{25 m}{24 h R}$ (d) $\frac{24 m}{25 h R}$

- 37 Assertion** Balmer series lies in the visible region of electromagnetic spectrum.

Reason $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$, where $n = 3, 4, 5, \dots$

[AIIMS 2012]

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
 (b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
 (c) If Assertion is true but Reason is false.
 (d) If both Assertion and Reason are false.
- 38** If the series limit of Lyman series for hydrogen atom is equal to the series limit of Balmer series for a hydrogen like atom, then atomic number of this hydrogen like atom is [BCECE Mains 2012]
 (a) 1 (b) 2
 (c) 3 (d) 4
- 39** In Bohr model of hydrogen atom, the force on the electron depends on the principal quantum number (n) as [BHU 2012]
 (a) independent of n (b) $F \propto \frac{1}{n^5}$
 (c) $F \propto \frac{1}{n^4}$ (d) $F \propto \frac{1}{n^3}$
- 40** The wavelength of K_α line in copper is $1.5 \text{ \AA}$. The ionisation energy of K electron in copper is [Manipal 2012]
 (a) $11.2 \times 10^{-17} \text{ J}$
 (b) $13.2 \times 10^{-16} \text{ J}$
 (c) $1.7 \times 10^{-15} \text{ J}$
 (d) $10 \times 10^{-16} \text{ J}$
- 41** The wavelength of first line of Balmer series is $6563 \text{ \AA}$. The wavelength of first line of Lyman series will be [Manipal 2012]
 (a) $1215.4 \text{ \AA}$ (b) $2500 \text{ \AA}$
 (c) $7500 \text{ \AA}$ (d) $600 \text{ \AA}$
- 42** The electron of a hydrogen atom revolves round the proton in a circular n th orbit of radius

$$r_n = \epsilon_0 n^2 h^2 / (\pi m e^2)$$
 with a speed, $v_n = \frac{e^2}{2\epsilon_0 n h}$.
 The current due to the circulating charge is proportional to [AMU 2012]
 (a) e^2 (b) e^3
 (c) e^5 (d) e^6
- 43** Consider a hydrogen like atom whose energy in n th excited state is given by $E_n = -\frac{13.6Z^2}{n^2}$, when this excited atom makes a transition from excited state to ground state, most energetic photons have energy $E_{\max} = 52.224 \text{ eV}$ and least energetic photons have energy $E_{\min} = 1.224 \text{ eV}$. The atomic number of atom is [JCECE 2011]
 (a) 2 (b) 4
 (c) 5 (d) None of these

ANSWERS

CHECK POINT 12.1

1. (d)	2. (b)	3. (b)	4. (b)	5. (b)	6. (c)	7. (c)	8. (d)	9. (c)	10. (c)
11. (b)									

CHECK POINT 12.2

1. (d)	2. (b)	3. (a)	4. (b)	5. (b)	6. (b)	7. (d)	8. (b)	9. (d)	10. (c)
11. (c)	12. (a)	13. (c)							

CHECK POINT 12.3

1. (a)	2. (c)	3. (b)	4. (c)	5. (d)	6. (c)	7. (d)	8. (c)	9. (a)	10. (a)
11. (a)									

(A) Taking it together

1. (c)	2. (a)	3. (d)	4. (a)	5. (a)	6. (a)	7. (a)	8. (b)	9. (d)	10. (d)
11. (c)	12. (d)	13. (a)	14. (a)	15. (a)	16. (d)	17. (d)	18. (c)	19. (b)	20. (a)
21. (d)	22. (a)	23. (c)	24. (c)	25. (a)	26. (b)	27. (a)	28. (b)	29. (a)	30. (b)
31. (b)	32. (a)	33. (d)	34. (d)	35. (d)	36. (c)	37. (c)	38. (b)	39. (d)	40. (c)
41. (a)	42. (c)	43. (c)	44. (c)	45. (b)	46. (c)	47. (c)	48. (a)	49. (d)	50. (c)
51. (b)	52. (a)	53. (a)	54. (a)	55. (a)	56. (a)	57. (c)	58. (a)	59. (d)	60. (b)
61. (a)	62. (a)	63. (b)	64. (b)	65. (a)	66. (b)	67. (c)	68. (b)	69. (a)	70. (d)
71. (a)	72. (a)	73. (a)	74. (d)	75. (b)	76. (a)	77. (b)	78. (d)	79. (c)	80. (b)
81. (d)	82. (c)	83. (b)	84. (a)	85. (a)	86. (d)	87. (d)	88. (d)	89. (a)	90. (c)
91. (a)									

(B) Medical entrance special format questions

- **Assertion and reason**

1. (b) 2. (a) 3. (d) 4. (b) 5. (d)

- **Statement based questions**

1. (d) 2. (b) 3. (c) 4. (c) 5. (d) 6. (b) 7. (b) 8. (a) 9. (d)

- **Match the columns**

1. (c) 2. (b) 3. (b)

(C) Medical entrances' gallery

1. (b)	2. (c)	3. (b)	4. (c)	5. (a)	6. (b)	7. (a)	8. (c)	9. (b)	10. (d)
11. (c)	12. (b)	13. (d)	14. (c)	15. (a)	16. (b)	17. (d)	18. (a)	19. (c)	20. (a)
21. (c)	22. (c)	23. (d)	24. (a)	25. (c)	26. (c)	27. (a)	28. (b)	29. (d)	30. (b)
31. (a)	32. (a)	33. (d)	34. (d)	35. (d)	36. (a)	37. (a)	38. (b)	39. (c)	40. (b)
41. (a)	42. (c)	43. (a)							

Hints & Explanations

• CHECK POINT 12.1

2 (b) For the distance of closest approach, we can write

$$\frac{1}{2}mv^2 = \frac{k \times (Ze)(2e)}{r_0} \Rightarrow r_0 \propto \frac{1}{m}$$

4 (b) We know that, impact parameter, $b = K \cot \frac{\theta}{2}$

$$\text{Given, } \theta = 360^\circ$$

$$\therefore b = K \times \cot 180^\circ = \infty$$

7 (c) As, $r \propto n^2$

$$\frac{r_3}{r_1} = \frac{3^2}{1^2} = 9$$

$$\Rightarrow r_3 = 9r_1 = 9r_0 \quad (\because r_1 = r_0)$$

8 (d) $r_n \propto \frac{n^2}{Z}$ or $r_n \propto \frac{1}{Z}$ (when n is constant)

Since, Z is minimum for hydrogen atom and maximum for doubly ionised lithium. So, r is minimum for option (d).

9 (c) $v_n = \frac{Ze^2}{2h\epsilon_0} \frac{1}{n}$

The velocity of electron in the first orbit ($n = 1$) of H-atom ($Z = 1$) is

$$v_1 = \frac{e^2}{2h\epsilon_0} = \frac{c}{137}$$

$$\therefore \frac{v_1}{c} = \frac{1}{137}$$

11 (b) Time period of revolution of electron in n th orbit,

$$T_n = \frac{nh r_n}{kZe^2} \quad \left(\text{where, } k = \frac{1}{4\pi\epsilon_0} \right)$$

$$\text{Since, } r_n \propto \frac{n^2}{Z}$$

$$\therefore T_n \propto n^3 \Rightarrow \frac{1}{v_n} \propto n^3 \text{ or } v_n \propto n^{-3}$$

• CHECK POINT 12.2

3 (a) For $n = 2$, using, $E_n = \frac{-13.6\text{eV}}{n^2}$, $E_2 = -\frac{13.6}{(2)^2} = -3.4 \text{ eV}$

$$\text{For } n = 1, E_1 = -13.6 \text{ eV}$$

$$\therefore E_{1 \rightarrow 2} = E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$$

5 (b) $E_3 - E_2 = E$ or $E = -13.6 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = E_1 \left(\frac{1}{9} - \frac{1}{4} \right)$

$$\Rightarrow E = \frac{E_1}{9} - \frac{E_1}{4} \text{ or } E_1 = -7.2E$$

$\therefore$ Ionisation energy of hydrogen atom is $7.2E$.

7 (d) The wavelength will be minimum for which $\left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$ is maximum. Hence, option (d) is correct.

9 (d) Wavelength is minimum for which Z is maximum

$$\left(\text{Since, } \lambda \propto \frac{1}{Z^2} \right)$$

$\therefore$ Here, $Z_{\max} = 3$ for lithium.

$$10 (c) \lambda \propto \frac{1}{\left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)}$$

$$\therefore \frac{\lambda_{\text{largest}}}{\lambda_{\text{shortest}}} = \frac{\left(\frac{1}{4} - \frac{1}{\infty} \right)}{\left(\frac{1}{4} - \frac{1}{9} \right)} = \frac{(1/4)}{(5/36)} = \frac{9}{5}$$

12 (a) According to Bohr's first postulate,

$$mvr = \frac{nh}{2\pi}$$

$$\therefore 2\pi r = n \left(\frac{h}{mv} \right) = n\lambda$$

$$\text{For } n = 1, \lambda = 2\pi r$$

• CHECK POINT 12.3

1 (a) X-rays travel in straight line with speed of light, i.e. $3 \times 10^8 \text{ m/s}$.

2 (c) Energy, $E = eV = h\nu_{\max}$

$$\therefore \nu_{\max} = \frac{eV}{h}$$

4 (c) Minimum wavelength produced,

$$\lambda_{\min} = \frac{12375}{50 \times 10^3} = 0.247 = 0.25 \text{ \AA}$$

7 (d) In X-ray spectrum wavelength λ of K_α line is given by

$$\frac{1}{\lambda_{K_\alpha}} = \frac{3}{4} (Z - 1)^2$$

$$\therefore \lambda_{K_\alpha} \propto \frac{1}{(Z - 1)^2}$$

8 (c) As, $E(K_\gamma) > E(K_\beta) > E(K_\alpha)$

$$\Rightarrow \lambda(K_\gamma) < \lambda(K_\beta) < \lambda(K_\alpha) \quad \left(\because \lambda \propto \frac{1}{E} \right)$$

11 (a) $2d \sin \theta = n\lambda$

For θ to be minimum,

$$n = 1$$

$$\Rightarrow \sin \theta \approx \theta = \frac{n\lambda}{2d}$$

$$\text{or } d = \frac{n\lambda}{2\theta} = \frac{1 \times 0.2 \times 180^\circ}{2 \times 3.8^\circ \times \pi} = 1.51 \text{ \AA}$$

(A) Taking it together

- 2** (a) When X-rays fall on a target, they are absorbed by it.
Hence, they are not used for radar purposes.
- 8** (b) $\Delta L = \frac{nh}{2\pi} - \frac{(n-1)h}{2\pi} = \frac{h}{2\pi}$
- 14** (a) If kinetic energy is E , then total energy will be $-E$, i.e. energy E must be supplied to the electron to escape to infinity.
- 15** (a) As, we know, $\lambda_{\min} (\text{in } \text{\AA}) = \frac{12375}{V} = \frac{12375}{20000} \text{\AA} = 61.87 \text{ pm}$
Since, λ should be greater than $\lambda_{\min}$.
Hence, $\lambda = 12 \text{ pm}$ is absent in spectrum.
- 16.** (d) BaSO_4 is a heavy element and it absorbs X-rays as they cannot pass through heavy metals.
- 17** (d) Impact parameter, $b \propto \cot \frac{\theta}{2} \Rightarrow \frac{b_1}{b_2} = \frac{\cot 45^\circ}{\cot 60^\circ} = \sqrt{3}$
- 18** (c) The minimum energy required to ionise an atom is equal to the energy to excite an electron in the atom from its ground state corresponding to $n = 1$, to its ionisation level corresponding to $n = \infty$. If the amount of energy given to atom is equal to the ionisation energy, then the electron is just able to become a free electron and the atom is about to be ionised.
When this occurs, one outermost electron from the atom is just to be removed from the atom. Much higher energy is necessary to free an innermost electron from the atom because this will occur only after the outermost electrons are all removed.
- 19** (b) $v \propto \frac{Z}{n}$, since Z and n both have become twice. Thus, velocity of electron in second orbit of He^+ will be v .
- 20** (a) Penetrating power is greater for lower wavelength. Hence, from given option $2 \text{\AA}$ wavelength penetrate most.
- 21** (d) If $n = 4$, then spectral lines emitted $= \frac{n(n-1)}{2} = 6$
- 22** (a) $L = \frac{nh}{2\pi}$, it does not depend on Z .
 $\therefore n_2 > n_1$
- 23** (c) $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = R \left(\frac{16-4}{16 \times 4} \right) = \frac{12R}{16 \times 4} = \frac{3}{16} R$
or $\lambda = \frac{16}{3} R$
 $v = \frac{c}{\lambda} = \frac{3 \times 10^8 \times 10^7 \times 3}{16} (\because R = 10^5 \text{ cm}^{-1}, = 10^7 \text{ m}^{-1} \text{ given})$
 $= \frac{9}{16} \times 10^{15} \text{ Hz}$
- 24** (c) Wave number is reciprocal of wavelength,
$$v = \frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] = R \left[\frac{1}{2^2} - \frac{1}{4^2} \right]$$
$$= R \left[\frac{1}{4} - \frac{1}{16} \right] = \frac{3R}{16}$$

- 25** (a) Use Bragg's X-ray diffraction law,
 $n\lambda = 2d \sin \theta$
 $\therefore \lambda = \frac{2d \sin \theta}{n}$
For longest wavelength take $\sin \theta = 1$
 $\therefore \lambda = \frac{2 \times 2.82 \times 1}{2} (\because n = 2, \text{ for second order})$
 $= 2.82 \text{\AA}$
- 26** (b) According to Moseley's law,
 $v \propto (Z - b)^2$ or $\sqrt{v} \propto (Z - b)$
So, option (b) is correct.
- 27** (a) $V_{\max} = \frac{12400 \times 10^{-10}}{10^{-11}} = 124 \text{ kV}$
 $\Rightarrow V < 124 \text{ kV}$
- 28** (b) According to the question,
For the distance of closest approach kinetic energy will totally be converted to potential energy.
Hence, $\frac{1}{2} mv^2 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r_0} \Rightarrow r_0 = \frac{Qq}{2\pi\epsilon_0 mv^2}$
- 29** (a) $E \propto m$ and $r \propto \frac{1}{m}$
i.e. energy will become two times and radius will become half.
Hence, $E_0 = 2(-13.6) = -27.2 \text{ eV}$ and $r_0 = a_0/2$
- 31** (b) $r_3 = (3)^2 \cdot r_1 = 9x$
Now, $3\lambda_3 = (2\pi r_3) = 18\pi x$
 $\therefore \lambda_3 = 6\pi x$
- 32** (a) According to scattering formula,
$$N \propto \frac{1}{\sin^4(\theta/2)} \Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin(\theta_1/2)}{\sin(\theta_2/2)} \right]^4$$
$$\Rightarrow \frac{N_2}{N_1} = \left[\frac{\sin \frac{90^\circ}{2}}{\sin \frac{60^\circ}{2}} \right]^4 = \left[\frac{\sin 45^\circ}{\sin 30^\circ} \right]^4$$
$$\Rightarrow N_2 = (\sqrt{2})^4 \times N_1 = 4 \times 56 = 224$$
- 33** (d) α -particles cannot be attracted by the nucleus.
- 34** (d) $L_n = \frac{nh}{2\pi} \Rightarrow I\omega = \frac{nh}{2\pi} \Rightarrow (mr^2)\omega = \frac{nh}{2\pi}$
As, $r \propto n^2$
 $\therefore \omega \propto \frac{1}{n^3}$
- 35** (d) $\Delta E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$
Also, $\Delta E \propto \frac{1}{\lambda}$
Since, λ is less for transition from $n = 5$ to $n = 2$ than for $n = 4$ to $n = 2$.
From the given option, only $\lambda_{\text{violet}} < \lambda_{\text{blue}}$. Thus, light of violet colour would have been emitted.

- 36** (c) The atomic number of lithium is 3, therefore the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about $\frac{1}{3}$ times to that of Bohr radius.

Therefore, the radius of lithium ion is near $\frac{53}{3} \approx 18$ pm.

- 37** (c) Change in the angular momentum,

$$\Delta L = L_{n_2} - L_{n_1} = \frac{n_2 h}{2\pi} - \frac{n_1 h}{2\pi}$$

$$\Delta L = \frac{h}{2\pi} (n_2 - n_1) = \frac{6.6 \times 10^{-34}}{2 \times 3.14} (5 - 4) = 1.05 \times 10^{-34} \text{ J-s}$$

- 38** (b) In a hydrogen atom the time period is given by $T \propto n^3$

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3 \Rightarrow \frac{8}{1} = \left(\frac{n_1}{n_2}\right)^3 \Rightarrow \frac{n_1}{n_2} = \frac{2}{1}$$

Thus, the values must be $n_1 = 2$ and $n_2 = 1$.

- 39** (d) According to Moseley's law, $\sqrt{\nu} \propto Z$

$$\therefore \frac{v_A}{v_B} = \left(\frac{Z_A}{Z_B}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

- 40** (c) $Z^2 \left[\frac{13.6}{4} - \frac{13.6}{9} \right] = 47.2$

$$\therefore Z = 5$$

- 41** (a) Excitation energy, $\Delta E = E_2 - E_1 = 13.6 Z^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$

$$40.8 = 13.6 Z^2 \times \frac{3}{4}$$

$$\therefore Z = 2$$

So, required energy to remove the electron from ground state

$$= + \frac{13.6 Z^2}{(1)^2} = 13.6 (2)^2 = 54.4 \text{ eV}$$

- 42** (c) Final energy of electron $= -13.6 + 12.1 = -1.51 \text{ eV}$

This energy corresponds to third level, i.e. $n = 3$

Hence, number of spectral lines emitted

$$= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

- 43** (c) As, $mvr = \frac{h}{2\pi}$ (in first orbit)

$$\therefore v = \frac{h}{2\pi mr} \Rightarrow a = \frac{v^2}{r} = \frac{h^2}{4\pi^2 m^2 r^3}$$

- 44** (c) $\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \Rightarrow \lambda \propto \frac{1}{Z^2}$

$$\lambda_{\text{Li}^{++}} : \lambda_{\text{He}^+} : \lambda_{\text{H}} = \frac{1}{3^2} : \frac{1}{2^2} : \frac{1}{1^2} = \frac{1}{9} : \frac{1}{4} : \frac{1}{1} = 4 : 9 : 36$$

- 45** (b) We know that, $\frac{1}{\lambda_{K\alpha}} = R(Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$

$$(Z-1)^2 = \frac{4}{3} \times \frac{1}{\lambda_{K\alpha} R} = 680$$

$$(Z-1) = 26 \quad (\because 680 \approx 26^2)$$

$$Z = 27 \rightarrow \text{Cobalt}$$

- 46** (c) For K_{α} line, $\lambda_{K\alpha} \propto \frac{1}{(Z-1)^2}$

$$\text{So, } \frac{\lambda_2}{\lambda_1} = \left(\frac{Z_1 - 1}{Z_2 - 1} \right)^2 \quad (\because \lambda_1 = \lambda)$$

$$\Rightarrow \frac{\lambda_2}{\lambda} = \left(\frac{43-1}{29-1} \right)^2 = \left(\frac{42}{28} \right)^2$$

$$\Rightarrow \lambda_2 = \frac{9}{4} \lambda$$

- 47** (c) For K_{α} line, $E_2 - E_1 = \frac{12375}{0.76} \text{ eV}$... (i)

$$\text{For } K_{\beta} \text{ line, } E_3 - E_1 = \frac{12375}{0.64} \text{ eV} \quad \dots \text{(ii)}$$

$$\text{For } L_{\alpha} \text{ line, } E_3 - E_2 = \frac{12375}{0.64} - \frac{12375}{0.76}$$

$$\text{or } \lambda_{L_{\alpha}} = \frac{12375}{E_3 - E_2} = 4.1 \text{ \AA}$$

- 48** (a) Magnetic moment μ of a revolving electron is given by

$$\mu = \frac{e}{2m} L = \frac{e}{2m} \cdot \frac{nh}{2\pi} = \left(\frac{eh}{2m} \right) \cdot \frac{n}{2\pi} = \frac{n}{2\pi} \mu_B$$

where, $\mu_B = \frac{eh}{2m}$ is Bohr magneton.

$$\therefore \mu \propto n$$

- 49** (d) Wavelength of K_{α} lines for given isotopes of lead can be given by a general expression,

$$\frac{1}{\lambda} = R(Z-1) \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$

where, R is Rydberg's constant and Z is atomic number of the isotopes.

$$\frac{1}{\lambda_1} = R(82-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$

$$\frac{1}{\lambda_1} = \frac{3R}{4} \times (81)^2$$

$$\text{Similarly, } \frac{1}{\lambda_2} = \frac{1}{\lambda_3} = \frac{3}{4} R \times (81)^2$$

$$\Rightarrow \left(\frac{1}{\lambda_2} \right)^2 = \frac{1}{\lambda_1} \times \frac{1}{\lambda_3}$$

$$\Rightarrow \lambda_2 = \sqrt{\lambda_1 \lambda_3}$$

- 50** (c) $a = \frac{v^2}{r}$ or $a \propto \frac{(Z^2)}{(1/Z)}$

$$\therefore a \propto Z^3$$

For singly ionised helium atom, $Z = 2$

and doubly ionised lithium atom, $Z = 3$

$$\Rightarrow \frac{a_{\text{He}^+}}{a_{\text{Li}^{++}}} = \frac{(2)^3}{(3)^3} = \frac{8}{27}$$

- 51** (b) For transition $n = 3$ to $n = 2$,

$$\frac{1}{\lambda_1} = R \left[\frac{1}{2^2} - \frac{1}{3^2} \right] = \frac{5R}{36} \quad \dots \text{(i)}$$

For transition $n = 4$ to $n = 2$,

$$\frac{1}{\lambda_2} = R \left[\frac{1}{2^2} - \frac{1}{4^2} \right] = \frac{3R}{16} \quad \dots(ii)$$

Now, dividing Eq. (i) by Eq. (ii), we get

$$\frac{\lambda_2}{\lambda_1} = \frac{20}{27} \text{ or } \lambda_2 = \frac{20}{27} \lambda_1 = \frac{20}{27} \lambda_0 \quad (\because \lambda_1 = \lambda_0)$$

52 (a) In hydrogen atom, $E_2 - E_1 = 10.2 \text{ eV}$

Since, $5 \text{ eV} < 10.2 \text{ eV}$

The electron cannot excite the hydrogen atom, therefore the collision must be elastic.

53 (a) Wavelength is given by

$$\begin{aligned} \frac{1}{\lambda} &= R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \\ \Rightarrow \frac{1}{\lambda} &= R \left[\frac{1}{(2)^2} - \frac{1}{(4)^2} \right] \\ \Rightarrow \frac{1}{\lambda} &= R \left(\frac{1}{4} - \frac{1}{16} \right) \Rightarrow \frac{1}{\lambda} = R \left(\frac{4-1}{16} \right) \\ \Rightarrow \frac{1}{\lambda} &= \frac{3R}{16} \Rightarrow \lambda = \frac{16}{3R} \end{aligned}$$

54 (a) We know that, frequency, $\nu = Rc \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

$$\text{This gives, } \nu_1 = Rc \left(1 - \frac{1}{\infty} \right) = Rc$$

$$\nu_2 = Rc \left(1 - \frac{1}{4} \right) = \frac{3}{4} Rc$$

$$\nu_3 = Rc \left(\frac{1}{4} - \frac{1}{\infty} \right) = \frac{Rc}{4} \Rightarrow \nu_1 - \nu_2 = \nu_3$$

55 (a) According to the question,

Energy, $E = -3.4 \text{ eV}$

$$\Rightarrow -\frac{13.6}{n^2} = -3.4$$

$$\Rightarrow n^2 = 4$$

$$\Rightarrow n = 2$$

So, the angular momentum of the electron, $mvr = \frac{nh}{2\pi}$

$$\begin{aligned} L = mvr &= \frac{2 \times 6.626 \times 10^{-34}}{2 \times 3.14} \\ &= \frac{6.626 \times 10^{-34}}{3.14} = 2.1 \times 10^{-34} \text{ J-s} \end{aligned}$$

56. (a) The wavelength can be given as

$$\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \dots(i)$$

For λ to be shortest, $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$ should be maximum.

So, $n_2 = \infty$ and $n_1 = 1$

Now, from Eq. (i), we get

$$\frac{1}{\lambda} = R \left(\frac{1}{1} \right) \Rightarrow \lambda = \frac{1}{R} = \frac{1}{10^7} \text{ m} \approx 1000 \text{ \AA}$$

57 (c) The energy of first excitation of sodium is $E = h\nu = \frac{hc}{\lambda}$

$$\begin{aligned} \therefore E &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5896 \times 10^{-10}} \text{ J} \\ E &= 3.37 \times 10^{-19} \text{ J} \end{aligned}$$

Also, since $1.6 \times 10^{-19} \text{ J} = 1 \text{ eV}$

$$\therefore E = \frac{3.37 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$\Rightarrow E = 2.1 \text{ eV}$$

Hence, corresponding first excitation potential is 2.1 V.

58 (a) The wavelength of lines is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$$

For Lyman series, the shortest wavelength is for $n = \infty$ and longest is for $n = 2$

$$\therefore \frac{1}{\lambda_s} = R \left(\frac{1}{1^2} \right) \quad \dots(i)$$

$$\frac{1}{\lambda_L} = R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3}{4} R \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{\lambda_L}{\lambda_s} = \frac{4}{3}$$

$$\text{Given, } \lambda_s = 91.2 \text{ nm} \Rightarrow \lambda_L = 91.2 \times \frac{4}{3} = 121.6 \text{ nm}$$

59 (d) After removing one electron from helium atom it will become hydrogen like atom.

$$\therefore E = 24.6 + (13.6) (2)^2 = 79 \text{ eV}$$

60 (b) As, $\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$

$$\therefore \frac{R}{n^2} = R - \frac{1}{\lambda} = \frac{\lambda R - 1}{\lambda}$$

$$\text{or } n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

61 (a) As, $\sqrt{f} \propto (Z-1) \sqrt{\frac{1}{n_1^2} - \frac{1}{n_2^2}}$

$$\therefore \text{Slope} \propto \sqrt{\frac{1}{n_1^2} - \frac{1}{n_2^2}}$$

$$\therefore \frac{(\text{Slope})_{K_\beta}}{(\text{Slope})_{K_\alpha}} = \frac{\sqrt{1 - \frac{1}{9}}}{\sqrt{1 - \frac{1}{4}}} = \sqrt{\frac{8}{9} \times \frac{4}{3}} = \sqrt{\frac{32}{27}}$$

62 (a) $\lambda = \frac{h}{p} = \frac{h}{mv}$ or $\lambda \propto \frac{1}{v}$ ($\because v \propto \frac{1}{n}$)

$$\therefore \lambda_n \propto n \quad J_n = n \frac{h}{2\pi}$$

$$\text{i.e. } J_n \propto n$$

$$\text{Hence, } J_n \propto \lambda_n$$

63 (b) $\Delta E_{4 \rightarrow 1} = \left(\frac{13.6}{1^2} - \frac{13.6}{16} \right) \text{ eV} = 12.75 \text{ eV}$

Momentum of hydrogen atom = Momentum of photon

$$= \frac{E}{c} = \frac{12.75 \times 1.6 \times 10^{-19}}{3.0 \times 10^8} = 6.8 \times 10^{-27} \text{ kg}\cdot\text{ms}^{-1}$$

64 (b) Since, $r \propto n^2$ i.e. $\frac{r_f}{r_i} = \left(\frac{n_f}{n_i} \right)^2$

$$\Rightarrow \frac{21.2 \times 10^{-11}}{5.3 \times 10^{-11}} = \left(\frac{n}{1} \right)^2 \Rightarrow n^2 = 4 \Rightarrow n = 2$$

65 (a) For α -particle, at distance of closest approach, then

Decrease in KE = Increase in PE

$$2 \text{ eV} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2e \cdot Ze}{r^2}$$

$$\Rightarrow Ze = 4\pi\epsilon_0 r^2 V \quad \dots(i)$$

For proton, if r_1 be the distance of closest approach

Decrease in KE = Increase in PE

$$\begin{aligned} R_1^2 &= \frac{1}{4\pi\epsilon_0 V} (Ze) \\ &= \frac{1}{4\pi\epsilon_0 V} \cdot 4\pi\epsilon_0 r^2 V \quad [\text{from Eq. (i)}] \end{aligned}$$

$$\Rightarrow r_1^2 = r^2$$

$$\Rightarrow r_1 = r$$

66 (b) The energy of the projectile will be equivalent to potential energy of the charge system,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Z_1 Z_2}{r_0} \Rightarrow \text{Energy, } E \propto Z_1 Z_2$$

68 (b) For the wavelength of Balmer's series,

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where, R is Rydberg's constant.

For Balmer series, $n = 3$ gives the first member of series and $n = 4$ gives the second member of series. Hence,

$$\frac{1}{\lambda_1} = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = R \left(\frac{5}{36} \right) \quad \dots(i)$$

$$\Rightarrow \frac{1}{\lambda_2} = R \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = R \left(\frac{12}{16 \times 4} \right) = \frac{3R}{16} \quad \dots(ii)$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{16}{3} \times \frac{5}{36} = \frac{20}{27}$$

$$\text{Given, } \lambda_1 = \lambda \Rightarrow \lambda_2 = \frac{20}{27} \lambda$$

69 (a) Here, $E_1 = \frac{me^4}{8\epsilon_0^2 h^2} \Rightarrow me^4 = 8\epsilon_0^2 h^2 E_1$

$$\begin{aligned} f_n &= \frac{v_n}{2\pi r_n} = \frac{(e^2 / 2 \epsilon_0 n h)}{(2\pi) \epsilon_0 n^2 h^2 / \pi m e^2} \\ &= \frac{me^4}{4\epsilon_0^2 n^3 h^3} = \frac{8E_1 \epsilon_0^2 h^2}{4\epsilon_0^2 n^3 h^3} = \frac{2E_1}{n^3 h} \end{aligned}$$

70 (d) As, $a = v^2 / r$

$$r \propto n^2 \text{ and } v \propto \frac{1}{n}$$

$$\therefore a \propto \frac{1}{n^4}$$

$$\Rightarrow \frac{a_M}{a_L} = \left(\frac{2}{3} \right)^4 = \frac{16}{81}$$

71 (a) In the simple Bohr model, only the magnitude of angular momentum is kept equal to some integral multiple of $\frac{h}{2\pi}$,

where h is Planck's constant and thus, the Bohr model gives incorrect values of angular momentum.

72 (a) Wavelength $\lambda_{K\alpha}$ is independent of the accelerating voltage (V), while the minimum wavelength λ_c is inversely proportional to V . Therefore, as V increases, $\lambda_{K\alpha}$ remains unchanged whereas λ_c decreases or $\lambda_{K\alpha} - \lambda_c$ will increase.

73 (a) Minimum 10.2 eV energy is required to excite a hydrogen atom. In perfectly inelastic collision maximum energy is lost.

$$\therefore \text{Energy lost} = \frac{1}{2} (2m) \left(\frac{v}{2} \right)^2 = 10.2 \times 1.6 \times 10^{-19}$$

$$\frac{1}{4} mv^2 = 1.632 \times 10^{-18}$$

$$\therefore \text{Velocity, } v = \sqrt{\frac{1.632 \times 4 \times 10^{-18}}{1.67 \times 10^{-27}}} = 6.25 \times 10^4 \text{ ms}^{-1}$$

74 (d) The ground state of hydrogen ($n = 1$) is represented by K , the first excited state ($n = 2$) is represented by L , the second excited state ($n = 3$) is represented by M . So, the energy transferred by the second electron can be given by

$$E = -13.6 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = -13.6 \left(\frac{1}{3^2} - \frac{1}{\infty^2} \right) \approx -1.51 \text{ eV}$$

75 (b) According to Bohr's theory, the wavelength of the radiation

$$\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \Rightarrow \frac{v}{c} = Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$v = c Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \Rightarrow 2.7 \times 10^{15} = c Z^2 R \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$

$$v = c Z^2 R \left(\frac{1}{1^2} - \frac{1}{3^2} \right) \Rightarrow v = \frac{32}{27} \times 2.7 \times 10^{15}$$

$$= 3.2 \times 10^{15} \text{ Hz}$$

76 (a) The total energy associated with the two H-atoms in the ground state collide inelastically $= 2 \times (13.6 \text{ eV}) = 27.2 \text{ eV}$.

The maximum amount by which their combined kinetic energy is reduced when any one of them goes into first excited state after the inelastic collision.

The total energy associated with the two H-atoms after the collision

$$= \left(\frac{13.6}{2^2} \right) + (13.6) = 17.0 \text{ eV}$$

Therefore, maximum loss of their combined kinetic energy

$$= 27.2 - 17.0 = 10.2 \text{ eV}$$

77 (b) Because atom is hollow and whole mass of atom is concentrated in a small centre called nucleus. So, A' will be maximum while B' minimum.

78 (d) If E is the energy radiated in transition, then from figure

$$E_{R \rightarrow G} > E_{Q \rightarrow S} > E_{R \rightarrow S} > E_{Q \rightarrow R} > E_{P \rightarrow Q}$$

For obtaining blue line, energy radiated should be more.

Hence, correct option is (d).

79 (c) When one decides to work in a frame of reference where the electron is at rest, the given expression is not true as it forms the non-inertial frame of reference.

80 (b) Given that, $I_1 = \frac{I_0}{2}$ at $t = 2.0$ cm, $I_2 = ?$ at $t = 4$ cm

Since, intensity of X-ray after travelling a distance t becomes

$$I = I_0 e^{-\mu t}$$

$$\Rightarrow I_1 = \frac{I_0}{2} = I_0 e^{-\mu \times 2.0} \Rightarrow e^{-\mu \times 2.0} = \frac{1}{2}$$

Again, we have

$$I_2 = I_0 e^{-\mu \times 4} = I_0 (e^{-\mu \times 2.0})^2$$

$$\Rightarrow I_2 = I_0 \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

81 (d) The transition marked as A represents series limit of Lyman series, B represents third member of Balmer series and C represents second member of Paschen series.

82 (c) $E \propto m$

$$\therefore \lambda \propto \frac{1}{m}$$

In case of hydrogen atom for longest wavelength of photon in the Balmer series,

$$\frac{1}{\lambda} = R \left(\frac{1}{4} - \frac{1}{9} \right) \text{ or } \lambda = \frac{36}{5R}$$

$$\text{For this atom, } \lambda' = \frac{\lambda}{2} = \frac{18}{5R} \quad (\because m = 2m)$$

83 (b) Let energy corresponding to state A, B and C be E_A , E_B and E_C , respectively.

So, from energy level diagram,

$$(E_C - E_B) + (E_B - E_A) = (E_C - E_A)$$

$$\text{or } \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = \frac{hc}{\lambda_3} \Rightarrow \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

84 (a) For transition from $4E$ to E ,

$$(4E - E) = \frac{hc}{\lambda_1} \Rightarrow \lambda_1 = \frac{3hc}{4E} \quad \dots(i)$$

For transition from $\frac{7}{3}E$ to E ,

$$\left(\frac{7}{3}E - E \right) = \frac{hc}{\lambda_2} \Rightarrow \lambda_2 = \frac{3hc}{4E} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{4}{9}$$

85 (a) Energy of emitted photon, $E = Rhc \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

$$\Rightarrow R_{(4 \rightarrow 3)} = Rhc \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = Rhc \left(\frac{7}{9 \times 16} \right) = 0.05 Rhc$$

$$E_{(4 \rightarrow 2)} = Rhc \left(\frac{1}{2^2} - \frac{1}{4^2} \right) = Rhc \left(\frac{3}{16} \right) = 0.2 Rhc$$

$$E_{(2 \rightarrow 1)} = Rhc \left[\frac{1}{(1)^2} - \frac{1}{(2)^2} \right] = Rhc \left(\frac{3}{4} \right) = 0.75 Rhc$$

$$\text{and } E_{(1 \rightarrow 4)} = Rhc \left[\frac{1}{(4)^2} - \frac{1}{(1)^2} \right] = -\frac{8}{9} Rhc = -0.9 Rhc$$

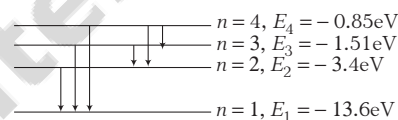
Thus, transition III gives largest amount of energy.

86. (d) $\lambda_\alpha - \lambda'_m = 3(\lambda_\alpha - \lambda_m)$

$$\therefore \frac{12375}{10.2(Z-1)^2} - \frac{12375}{20 \times 10^3} = 3 \left[\frac{12375}{10.2(Z-1)^2} - \frac{12375}{10 \times 10^3} \right]$$

Solving, we get $Z = 29$

87 (d) $E = E_1 / n^2$



Energy levels of H-atom

Energy used for excitation is 12.75 eV,

i.e. $(-13.6 + 12.75)$ eV = -0.85 eV

The photons of energy 12.75 eV can excite the fourth level of

H-atom. As spectral lines emitted is $= \frac{n(n-1)}{2} = \frac{4(3)}{2} = 6$

Therefore, six lines will be emitted.

88 (d) At the distance of closed approach r ,

$$K = \frac{1}{4\pi\epsilon_0} \frac{(2e)(Ze)}{r}$$

$$\Rightarrow r = \frac{2Ze^2}{4\pi\epsilon_0 K}$$

where, Ze = charge of the nucleus,

$2e$ = charge of the alpha particle

and K = kinetic energy of the alpha particle.

$$\therefore K = \frac{p^2}{2m}$$

where, p is the momentum of the α -particle and m is the mass of the electron.

$$\therefore r = \frac{2Ze^2 2m}{4\pi\epsilon_0 p^2} \text{ or } r \propto \frac{1}{p^2}$$

$$\frac{r'}{r} = \left(\frac{p}{p'} \right)^2 = \left(\frac{p}{2p} \right)^2 = \frac{1}{4} \Rightarrow r' = \frac{r}{4}$$

89 (a) According to kinetic interpretation of temperature,

$$KE = \left(\frac{1}{2} mv^2 \right) = \frac{3}{2} kT$$

$$\Rightarrow 10.2 \times 1.6 \times 10^{-19} = \frac{3}{2} \times (1.38 \times 10^{-23}) T$$

$$\Rightarrow T = 7.9 \times 10^4 \text{ K}$$

90 (c) $\frac{1}{\lambda} = RZ^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$

$$\frac{1}{\lambda_1} = R(1)^2(3/4), \frac{1}{\lambda_2} = R(1)^2(3/4)$$

$$\frac{1}{\lambda_3} = R(2)^2(3/4), \frac{1}{\lambda_4} = R(3)^2(3/4)$$

$$\frac{1}{\lambda_1} = \frac{1}{\lambda_2} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4}$$

$$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

So, option (c) is correct.

91 (a) According to Bohr theory,

$$mvr = n \frac{h}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$$

and $\frac{mv^2}{r} \propto \frac{k}{r} \Rightarrow \frac{m}{r} \left(\frac{n^2 h^2}{4\pi^2 m^2 r^2} \right) \propto \frac{k}{r}$

$$\Rightarrow r_n \propto n$$

Kinetic energy of the electron,

$$T = \frac{1}{2} mv^2 = \frac{1}{2} m \left(\frac{n^2 h^2}{4\pi^2 m^2 r^2} \right) \Rightarrow T_n \propto \frac{n^2}{r^2}$$

But as $r \propto n$, therefore $T \propto n^0$

(B) Medical entrance special format questions

• Assertion and reason

1 (b) $L = n \left(\frac{h}{2\pi} \right)$, i.e. $L \propto n$.

In ground state $n = 1$,
so L = minimum.

2 (a) Radius of Bohr's orbit $r \propto n^2$

$$\Rightarrow \frac{r_1}{r_2} = \left(\frac{n_1}{n_2} \right)^2 \Rightarrow \frac{5.29 \times 10^{-11}}{r_2} = \left(\frac{1}{2} \right)^2$$

$$\Rightarrow r_2 = 5.29 \times 10^{-11} \times 4 = 21.16 \times 10^{-11} \text{ m} \\ = 2.116 \times 10^{-10} \text{ m} \\ \approx 2.1 \text{ \AA}$$

4 (b) For second orbit, $mv_2 r_2 = 2 \left(\frac{h}{2\pi} \right)$

$$\therefore 2\pi r_2 = 2 \left(\frac{h}{mv_2} \right) = 2\lambda_2$$

Further, $\lambda = \frac{h}{p}$

Speed or momentum is maximum in ground state. Hence, wavelength is minimum.

5 (d) Continuous X-rays depends only on the accelerating voltage but characteristic X-rays depends on the target used.

Lead has high atomic number and high density, hence it is used to absorb X-rays.

• Statement based questions

1 (d) $r_n = \frac{n^2}{Z} r_1 \Rightarrow r_n \propto n^2$

$$\text{As, } R = 1.09 \times 10^7 \text{ m}^{-1}, h = 6.626 \times 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$$

$$\text{and } c = 3 \times 10^8 \text{ ms}^{-1} \Rightarrow 1/Rhc \approx 13.6 \text{ eV}$$

2 (b) Frequencies of the Brackett lines of the H-atom lie in the infrared region.

Bremsstrahlung is produced when electrons travel through the electric field around a nucleus.

Hence, only III statement is correct.

4 (c) Orbital radius, $r = r_0 \frac{n^2}{Z} \Rightarrow \frac{\Delta r}{r} = \frac{2\Delta n}{n}$

$$\text{For } \Delta n = 1, \frac{\Delta r}{r} \propto \frac{1}{n}$$

$$\text{Angular momentum, } L = \frac{nh}{2\pi} \Rightarrow \frac{\Delta L}{L} = \frac{\Delta n}{n}$$

$$\therefore \frac{\Delta L}{L} \propto \frac{1}{n}$$

$$\text{Total energy, } E_n = -\frac{13.6 Z^2}{n^2}$$

$$\therefore \frac{\Delta E_n}{E_n} = -\frac{2\Delta n}{n} \Rightarrow \frac{\Delta E}{E} \propto \frac{1}{n}$$

Hence, the correct statements are I, II and IV only.

7 (b) $E = -122.4 = -13.6 \frac{Z^2}{n^2}$

$$\Rightarrow Z = 3$$

If, BE + KE = 125

$$\Rightarrow \text{KE} = 2.6 \text{ eV}$$

Since, BE = 122.4 eV > 90 eV.

Thus, it cannot excite an electron.

Hence, only statement in option (b) is incorrect.

8 (a) In H-atom, $E_{\text{total}} = -\frac{13.6}{n^2}$, KE = $\frac{13.6}{n^2}$ and PE = $-\frac{2(13.6)}{n^2}$.

For transition from excited state ($n > 1$) to ground state ($n = 1$),

kinetic energy of electron increases $\left(\frac{13.6}{n^2} \rightarrow 13.6 \right)$ whereas

potential energy and total energy decrease (become more negative).

9 (d) As the energy of the incident electrons is greater than the magnitude of the energy of the K-shell electrons, the target atoms will have vacancies in the K-shell (K-shell electrons will be knocked out). This will cause emission of the entire characteristic spectrum of tungsten. Along with this, continuous X-rays will always be present, with

$$\lambda_c = \frac{1242 \text{ nm-eV}}{80 \times 10^3 \text{ eV}} \approx 0.0155 \text{ nm}$$

• Match the columns

1 (c)

$$(A) a = r \omega^2 \propto (n^2) (n^{-3})^2 \propto n^{-4}$$

$$(B) L = n \frac{h}{2\pi} \text{ or } L \propto n$$

$$(C) I = mr^2 \text{ or } I \propto r^2 \propto (n^2)^2 \propto n^4$$

Hence, A $\rightarrow$ p; B $\rightarrow$ q; C $\rightarrow$ s

2 (b) As, $E \propto \frac{Z^2}{n^2}$

$$\therefore U = 2E_0, K = -E_0$$

Hence, A $\rightarrow$ r; B $\rightarrow$ p; C $\rightarrow$ q; D $\rightarrow$ q

3 (b) Angular speed,

$$\omega = 2\pi n = 2\pi \frac{v}{2\pi r} = \frac{v}{r}$$

$$\text{As, } v \propto \frac{Z}{n} \text{ and } r \propto \frac{n^2}{Z}$$

$$\Rightarrow \omega \propto \frac{Z^2}{n^3}$$

$$E_n \propto \frac{Z^2}{n^2}$$

$$B_{\text{centre}} = \mu_0 \frac{eV}{2\pi r} = \frac{\mu_0 eV}{4\pi r^2} \propto \frac{Z^3}{n^5}$$

Hence, A $\rightarrow$ q; B $\rightarrow$ r; C $\rightarrow$ p; D $\rightarrow$ s

(C) Medical entrances' gallery

1 (b) Total energy of an electron in the n th stationary orbit of hydrogen atom is given by

$$E_n = \frac{-Rhc}{n^2} = \frac{-13.6}{n^2} \text{ eV} \quad (\because 1Rhc = 13.6 \text{ eV})$$

2 (c) Since, Bohr's model is valid for hydrogen and hydrogen atom like He^+ , deuteron, etc. So, it is not valid for singly ionised neon atom (Ne^+).

3 (b) According to Bohr's model, the kinetic energy of electron in term of Rydberg's constant R is given by

$$\text{KE} = \frac{Rhc}{n^2} \quad \dots (i)$$

where, h = Planck's constant, c = speed of light and n = principal quantum number.

Similarly, potential energy is given by

$$\text{PE} = -\frac{2Rhc}{n^2} \quad \dots (ii)$$

$$\therefore \text{Total energy, } E = \text{PE} + \text{KE} = -\frac{Rhc}{n^2} \quad [\text{from Eqs. (i) and (ii)}]$$

$$\therefore \text{KE} = -E \text{ and } \text{PE} = 2E$$

$$\text{Given, } E = -3.4 \text{ eV}$$

$$\therefore \text{KE} = -(-3.4) = 3.4 \text{ eV}$$

$$\text{and } \text{PE} = 2(-3.4) = -6.8 \text{ eV}$$

4 (c) Given, radius of first orbit for electron, $r_1 = 0.51 \text{ \AA}$

Ground state energy of electron, $E_1 = -13.6 \text{ eV}$

Mass of electron = m_e

Mass of muon, $m_\mu = 207m_e$

Mass of nucleus, $M = 1836m_e$

When electron in hydrogen atom is replaced by muon, the reduced mass of muon is

$$m_\mu' = \frac{m_\mu M}{m_\mu + M} \quad \dots (i)$$

Substituting the given values in Eq. (i), we get

$$m_\mu' = \frac{207m_e \times 1836m_e}{207m_e + 1836m_e} \approx 186m_e \quad \dots (ii)$$

The radius of first orbit in hydrogen atom for electron is given by

$$r_1 = \frac{h^2 \epsilon_0}{\pi m_e e^2} \quad \dots (iii)$$

The radius of first orbit for muon is given by

$$r_1' = \frac{h^2 \epsilon_0}{\pi m_\mu' e^2} \quad (\because \text{charge of } \mu = \text{charge of } e^-)$$

$$= \frac{h^2 \epsilon_0}{\pi \times 186m_e e^2} \quad [\text{from Eq. (ii)}]$$

$$= \left(\frac{h^2 \epsilon_0}{\pi m_e e^2} \right) \frac{1}{186} = \frac{r_1}{186} \quad [\text{from Eq. (iii)}]$$

$$= \frac{0.51 \text{ \AA}}{186} \quad (\because r_1 = 0.51 \text{ \AA})$$

$$= 2.74 \times 10^{-13} \text{ m} \quad (\because 1 \text{ \AA} = 10^{-10} \text{ m})$$

The total energy of electron is given by

$$E_n = \frac{-mZ^2 e^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2} \right)$$

$$\Rightarrow E_n \propto m$$

For electron in first orbit of hydrogen atom,

$$E_1 = km_e \quad \dots (iv)$$

$$\text{where, } k = \frac{e^4}{8\epsilon_0^2 h^2} = \text{constant.}$$

For muon in first orbit,

$$E_1' = km_\mu' \quad [\text{from Eq. (ii)}]$$

$$= k \times 186m_e$$

$$= 186km_e \quad [\text{from Eq. (iv)}]$$

$$= 186(-13.6) \text{ eV} \quad (\text{given, } E_1 = -13.6 \text{ eV})$$

$$= -2529.6 \text{ eV} = -2.5 \text{ keV}$$

$\therefore$ These values are closest to that of option (c).

5 (a) Total energy given to electron in 4th orbit, $E = 15 \text{ eV}$

Energy of n th orbit of H-atom,

$$E' = -\frac{13.6}{n^2} \text{ eV}$$

$$\text{So, energy of 4th orbit of H-atom, } E' = \frac{-13.6}{4^2} = -0.85 \text{ eV}$$

$$\therefore \text{Ionisation energy of 4th orbit} = |-E'| = |-0.85 \text{ eV}| = 0.85 \text{ eV}$$

Thus, final energy of the electron, when it comes out of H-atom,

E = Total energy given to electron-Ionisation energy of 4th orbit.

$$= 15 - 0.85 = 14.15 \text{ eV}$$

- 6 (b) Kinetic energy of an electron in a Bohr orbit of a hydrogen atom is given as

$$(KE)_n = \frac{Rhc}{n^2} \quad \dots(i)$$

Total energy of an electron in a Bohr orbit of a hydrogen atom is given as

$$(TE)_n = \frac{-Rhc}{n^2} \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{(KE)_n}{(TE)_n} = \frac{(Rhc/n^2)}{(-Rhc/n^2)}$$

So, $(KE)_n : (TE)_n = 1 : -1$

- 7 (a) In an atom, electron revolves around nucleus. For this, required centripetal forces is provided by electrostatic force of attraction between electron and nucleus. If electrons were stationary, then due to this force, they would fall into the nucleus.

Thus, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

- 8 (c) Wavelength of spectral lines are given by

$$\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For last line of Balmer series, $n_1 = 2$ and $n_2 = \infty$

$$\Rightarrow \frac{1}{\lambda_B} = Z^2 R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right) = \frac{R}{4} \quad (\because Z = 1)$$

Similarly, for last line of Lyman series, $n_1 = 1$ and $n_2 = \infty$

$$\Rightarrow \frac{1}{\lambda_L} = Z^2 R \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right) = R$$

Now, the ratio of the wavelength, $\frac{\lambda_L}{\lambda_B} = \frac{1}{4} \Rightarrow \frac{\lambda_B}{\lambda_L} = 4$

- 9 (b) Given, Rydberg constant, $R = 10^7 \text{ m}^{-1}$

For last line in Balmer series, $n_2 = \infty$, $n_1 = 2$

As we know that,

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\Rightarrow \frac{1}{\lambda} = 10^7 \left(\frac{1}{2^2} - \frac{1}{\infty} \right)$$

$$v = \frac{1}{\lambda} = \frac{10^7}{4} = 0.25 \times 10^7 \text{ m}^{-1}$$

- 10 (d) When an α -particle of mass m moving with velocity v bombards on a heavy nucleus of charge Ze , then there will be no loss of energy, so initial kinetic energy of α -particle = potential energy of α -particle at distance of closest approach.

$$\Rightarrow \frac{1}{2} mv^2 = \frac{2Ze^2}{4\pi\epsilon_0 r_0} \Rightarrow r_0 \propto \frac{1}{m}$$

- 11 (c) When an electron jumps from an excited state, excess energy of electron appears as photon.

From Rydberg's formula,

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{5R}{36}$$

$$\text{Similarly, } \frac{1}{\lambda'} = R \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = \frac{7R}{144}$$

$$\frac{1}{\lambda} / \frac{1}{\lambda'} = \frac{5R/36}{7R/144}$$

$$\Rightarrow \frac{\lambda'}{\lambda} = \frac{5R}{36} \times \frac{144}{7R} = \frac{20}{7} \Rightarrow \lambda' = \frac{20}{7} \lambda$$

- 12 (b) Energy of electron in the 3rd orbit of He^+ is

$$E_3 = -13.6 \times \frac{Z^2}{n^2} \text{ eV} = -13.6 \times \frac{4}{3^2} \text{ eV}$$

$$= -13.6 \times \frac{4}{9} \times 1.6 \times 10^{-19} \text{ J}$$

From Bohr's model, $E_3 = -KE_3 = -\frac{1}{2} m v^2$

$$\Rightarrow -13.6 \times \frac{4}{9} \times 1.6 \times 10^{-19} = -\frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$$

$$\Rightarrow v^2 = \frac{13.6 \times 4 \times 1.6 \times 10^{-19} \times 2}{9.1 \times 10^{-31} \times 9}$$

$$\Rightarrow v^2 = 212 \times 10^{12} \text{ or } v = 1.46 \times 10^6 \text{ ms}^{-1}$$

- 14 (c) Energy of electron in ground state ($n = 1$) of hydrogen atom,

$$E_1 = -13.6 \text{ eV} \quad \left(\because E_n = \frac{-13.6}{n^2} \text{ eV} \right)$$

$\therefore$ Energy of electron in first excited state ($n = 2$) of hydrogen atom,

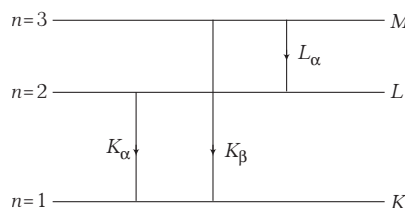
$$E_2 = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

Hence, the excitation energy of hydrogen atom in first excited state,

$$E = E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$$

Therefore, excitation potential is 10.2 V.

- 15 (a) Energy level diagram for K, L series of X-rays is as shown below.



Here, from diagram, $E_{K_\alpha} + E_{L_\alpha} = E_{K_\beta}$

$$\therefore h\nu_{K_\alpha} + h\nu_{L_\alpha} = h\nu_{K_\beta} \Rightarrow \nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$$

- 16 (b) Number of lines emitted = $\frac{n(n-1)}{2}$

where, n = number of orbit from which transition takes place.

$$\Rightarrow 6 = \frac{n(n-1)}{2} \text{ or } n = 4$$

$\therefore$ The wavelength of emitted radiations will be maximum for transition $n = 4$ to $n = 3$.

- 17 (d) The energy of the photon,

$$E = \frac{hc}{\lambda} = \frac{4 \times 10^{-15} \times 3 \times 10^8}{300 \times 10^{-9}} = 4 \text{ eV}$$

The ionisation energy is 13.6 eV which is greater than the energy of photon, so electron cannot be knocked out or into excited state. Thus, it will keep orbiting in ground state of the atom.

- 18 (a) Given, $hc = 1240 \text{ eV-nm}$

For least energetic photon, $E = E_2 - E_1 = 10.2 \text{ eV}$

We know that, $E = \frac{hc}{\lambda}$ or

$$\lambda = \frac{hc}{E}$$

$$\Rightarrow \lambda = \frac{1240}{10.2}$$

$$\lambda = 121.57 \approx 122 \text{ nm}$$

- 19 (c) Given, wavelength, $\lambda_{K\alpha} = 0.76 \text{ \AA} = 0.76 \times 10^{-10} \text{ m}$

Moseley's empirical formula for $K\alpha$, X-rays when adapted to Bohr's model is given by

$$h\nu_{K\alpha} = \frac{3}{4} \times 13.6 (Z - 1)^2 \text{ eV}$$

$$\text{or } \frac{hc}{\lambda_{K\alpha}} = \frac{3}{4} \times 13.6 \times 1.6 \times 10^{-19} (Z - 1)^2 \text{ J}$$

$$(\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

$$\Rightarrow \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{0.76 \times 10^{-10}} = 16.32 \times 10^{-19} (Z - 1)^2$$

$$(\because h = 6.626 \times 10^{-34} \text{ J-s and } c = 3 \times 10^8 \text{ ms}^{-1})$$

$$\Rightarrow 26.15 \times 10^{-16} = 16.32 \times 10^{-19} (Z - 1)^2$$

$$\Rightarrow 1.602 \times 10^3 = (Z - 1)^2$$

$$\Rightarrow 16.02 \times 10^2 = (Z - 1)^2$$

$$\Rightarrow Z - 1 = 40.03$$

$$\therefore Z = 40.03 + 1 \approx 41$$

- 20 (a) Energy provided to the ground state electron

$$= \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10}}$$

$$= \frac{6.6 \times 3}{975} \times 10^{-16}$$

$$= 0.020 \times 10^{-16} = 2 \times 10^{-18} \text{ J}$$

$$= \frac{20 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{20}{1.6} = 12.5 \text{ eV}$$

It means the electron is jump to $n = 3$ from $n = 1$.

The number of possible spectral lines from $n = 3$ to $n = 1$ is 3.

$$\text{i.e. } (n = 3) \longrightarrow (n = 1)$$

$$(n = 3) \longrightarrow (n = 2)$$

$$\text{and } (n = 2) \longrightarrow (n = 1)$$

- 21 (c) de-Broglie wavelength, $\lambda = \frac{h}{p}$... (i)

According to Bohr's quantisation condition,

$$mvr_n = \frac{nh}{2\pi} \Rightarrow pr_n = \frac{nh}{2\pi} \quad \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\lambda = \frac{h \times 2\pi r_n}{nh} \Rightarrow \lambda = \frac{2\pi r_n}{n}$$

$$\text{For fourth orbit } (n = 4) \Rightarrow \lambda = \frac{2\pi r_4}{4} \quad \dots \text{(iii)}$$

$$\text{Moreover, } r \propto n^2 \Rightarrow \frac{r_1}{r_4} = \frac{(1)^2}{(4)^2} \Rightarrow r_4 = 16r$$

Substituting the value of r_4 in Eq. (iii), we get

$$\lambda = \frac{2\pi [16r_1]}{4} = 8\pi r_1 = 8\pi r \quad (\because \text{given, } r_1 = r)$$

- 22 (c) The energy of the photon $= Rhc \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$

$$\Rightarrow 13.6 \left(\frac{1}{1} - \frac{1}{4} \right) = 13.6 \left(\frac{4-1}{4} \right)$$

$$= 13.6 \times \frac{3}{4} = \frac{40.8}{4} = 10.2 \text{ eV}$$

- 23 (d) When an electron jumps from orbit n_i to orbit n_f ($n_i > n_f$), the frequency of emitted photon is given by

$$\nu = cR \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\Rightarrow \nu = cR \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\nu = cR \left(\frac{5}{36} \right) \Rightarrow \nu = \frac{5cR}{36}$$

- 24 (a) As, $\frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

For longest wavelength of Lyman series $n_1 = 1$ and $n_2 = 2$

For longest wavelength of Balmer series $n_1 = 2$ and $n_2 = 3$

$$\frac{1}{\lambda_{\text{Balmer}}} = \frac{R \left(\frac{1}{2^2} - \frac{1}{3^2} \right)}{R \left(\frac{1}{1^2} - \frac{1}{2^2} \right)} = \frac{\frac{5}{36} R}{\frac{3}{4} R}$$

$$\text{So, } \frac{\lambda_{\text{Lyman}}}{\lambda_{\text{Balmer}}} = \frac{5}{27}$$

- 25 (c) Rutherford's model states that an atom is a sphere of diameter about 10^{-10} m with whole of its positive charge concentrated at the centre and electron revolves around this centre.

- 26 (c) According to Bohr's model, electrons can revolve those orbits in which their angular momentum is an integral multiple of $h/2\pi$. i.e.

$$mvr = \frac{nh}{2\pi}$$

27 (a) In Moseley's law $\sqrt{\nu} = a(Z - b)$, the values of the screening constant for K -series and L -series of X-rays are respectively, 1 and 7.4.

28 (b) Given, $E_K = 23.32 \text{ keV}$
 $= 23.32 \times 10^3 \text{ eV}$

$$hc = 12.42 \times 10^{-7} \text{ eV}$$

$$\text{and } \lambda_{K\alpha} = 71 \times 10^{-12} \text{ m}$$

$$\text{Now, } E_L - E_K = \frac{hc}{\lambda_{K\alpha}}$$

$$E_L - 23.32 \times 10^3 = \frac{12.42 \times 10^{-7}}{71 \times 10^{-12}}$$

$$\Rightarrow E_L = 40812.9$$

$$= 40.82 \text{ keV}$$

29 (d) The photoelectric effect is possible when light of sufficient energy incident on metal surface. When electron de-excites from $n = 3$ to $n = 2$, the light produces photoelectric effect. So, for the given options, the photoelectric effect is not possible for that level in which energy of emitted light is less than that for $n = 3$ to $n = 2$. Since, $(h\nu)_{4 \rightarrow 3} < (h\nu)_{3 \rightarrow 2}$

Hence, photoelectric effect is not possible from $n = 4$ to $n = 3$.

30 (b) Ionisation energy in ground state = 24.6 eV

$$\text{Energy required to remove 2nd electron from He}^{2+}$$

$$= Z^2 (13.6) \text{ eV} = (2)^2 (13.6) = 54.4 \text{ eV}$$

$$\text{So, total energy required} = 24.6 + 54.4 = 79 \text{ eV}$$

31 (a) As, electron de-excites from $3E \rightarrow E$

$$\therefore h\nu = 3E - E$$

$$\text{or } \frac{hc}{\lambda} = 2E \quad \dots(i)$$

$$\text{When electron de-excites from } \frac{5E}{3} \rightarrow E$$

$$\frac{hc}{\lambda'} = \frac{5E}{3} - E = \frac{2E}{3} \quad \dots(ii)$$

Substituting the value of E from Eq. (i) in above equation

$$\frac{hc}{\lambda'} = \frac{2}{3} \left(\frac{hc}{2\lambda} \right) \Rightarrow \lambda' = 3\lambda$$

32 (a) Continuous spectrum of X-rays consist of radiations of all possible X-ray wavelengths within a certain range starting from $\lambda_{\min}$ onwards.

34 (d) Minimum energy, $E = 13.6 Z^2 \text{ eV}$
 $= 13.6 (3)^2 = 122.4 \text{ eV}$

35 (d) Here, for wavelength λ_1 , $n_1 = 4$ and $n_2 = 3$
 and for wavelength λ_2 , $n_1 = 3$ and $n_2 = 2$

$$\text{As, we know that, } \frac{hc}{\lambda} = -13.6 \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right]$$

So, for λ_1 ,

$$\Rightarrow \frac{hc}{\lambda_1} = -13.6 \left[\frac{1}{(4)^2} - \frac{1}{(3)^2} \right]$$

$$\Rightarrow \frac{hc}{\lambda_1} = 13.6 \left[\frac{7}{144} \right] \quad \dots(i)$$

Similarly, for λ_2 ,

$$\Rightarrow \frac{hc}{\lambda_2} = -13.6 \left[\frac{1}{(3)^2} - \frac{1}{(2)^2} \right]$$

$$\Rightarrow \frac{hc}{\lambda_2} = 13.6 \left[\frac{5}{36} \right] \quad \dots(ii)$$

Hence, from Eqs. (i) and (ii), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{20}{7}$$

36 (a) Here, $E_5 - E_1 = \frac{hc}{\lambda}$

$$\text{and } \frac{Rhc}{25} - Rhc = \frac{hc}{\lambda}$$

$$\Rightarrow \frac{24}{25} R = \frac{1}{\lambda}$$

$$\text{But } p = \frac{h}{\lambda}$$

$$\text{and } v = \frac{h}{m\lambda} = \frac{24}{25} \frac{Rh}{m}$$

37 (a) The wavelength in Balmer series is given by

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \text{ where } n = 3, 4, 5$$

$$\frac{1}{\lambda_{\max}} = R \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$

$$\Rightarrow \lambda_{\max} = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} = 6563 \text{ \AA}$$

$$\text{and } \frac{1}{\lambda_{\min}} = R \left(\frac{1}{2^2} - \frac{1}{\infty^2} \right)$$

$$\Rightarrow \lambda_{\min} = \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 3646 \text{ \AA}$$

The wavelength 6563 \AA and 3646 \AA lie in visible region.

Therefore, Balmer series lies in visible region.

38 (b) By using, $\frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

$$\text{Lyman series for hydrogen atom, } \frac{1}{\lambda_{\min}} = R \left[\frac{1}{1^2} - \frac{1}{\infty} \right] = R$$

$$\frac{1}{\lambda_{\min}} = R \text{ or } (\lambda_{\min})_H = \frac{1}{R} \quad \dots(i)$$

Balmer series for another atom,

$$\frac{1}{\lambda_{\min}} = RZ^2 \left(\frac{1}{2^2} - \frac{1}{\infty} \right) = \frac{RZ^2}{4}$$

$$\Rightarrow \lambda_{\min} = \frac{4}{RZ^2} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{R} = \frac{4}{RZ^2} \Rightarrow Z = 2$$

39 (c) We have, $F \propto \frac{v^2}{r}$

Also, $v \propto \frac{1}{n}$ and $r \propto n^2$

$\Rightarrow F \propto \frac{1}{n^4}$

40 (b) Required ionisation energy,

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.5 \times 10^{-10}} = 13.2 \times 10^{-16} \text{ J}$$

41 (a) $\frac{\lambda_{\text{Lyman}}}{\lambda_{\text{Balmer}}} = \frac{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{\left(\frac{1}{1^2} - \frac{1}{2^2}\right)} = \frac{5}{27} \quad \left[\because \frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \right]$

$\Rightarrow \lambda_{\text{Lyman}} = \frac{5}{27} \times \lambda_{\text{Balmer}} = \frac{5}{27} \times 6563 = 1215.4 \text{ \AA}$

42 (c) Time period of electron,

$$T = \frac{2\pi r}{v} = \frac{2\pi \times \epsilon_0 n^2 h^2}{\pi m e^2} = \frac{2\epsilon_0 n h}{m e^2}$$

$$T = \frac{4\epsilon_0^2 n^3 h^3}{m e^4}$$

Current, $i = \frac{e}{T} = \frac{e}{\frac{4\epsilon_0^2 n^3 h^3}{m e^4}}$

$\Rightarrow i = \frac{m e^5}{4\epsilon_0^2 n^3 h^3}$

i.e. $i \propto e^5$

43 (a) Maximum energy is liberated for transition $E_n \rightarrow E_1$ and minimum energy for $E_n \rightarrow E_{n-1}$.

Hence, $\frac{E_1}{n^2} - E_1 = 52.224 \text{ eV} \quad \dots(i)$

and $\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224 \text{ eV} \quad \dots(ii)$

Solving Eqs. (i) and (ii), we get

$E_1 = -54.4 \text{ eV}$ and $n = 5$

But $E_1 = -\frac{13.6 Z^2}{1^2}$

$\therefore -54.4 = -\frac{13.6}{1^2} Z^2$

$\Rightarrow Z = 2$

CHAPTER 13

Nuclei

In the last chapter, we learnt that atomic nucleus was discovered in the year 1911 by Rutherford and his associates from the study of large angle scattering of alpha particles from the thin metal foil. In this chapter, we shall study the constituents of the nucleus and how they are held together. We shall also study different properties of nuclei such as size, mass, density and stability of nuclei and associated phenomena such as radioactivity, nuclear fission and nuclear fusion.

NUCLEUS

As per the Rutherford's experiment, it was concluded that the entire positive charge and mass of an atom was present at its very tiny centre known as nucleus.

Composition of nucleus

Nucleus is composed of two types of particles

- (i) **Protons** Positively charged particles
- (ii) **Neutrons** Neutral particles

Mass of neutron is slightly greater than the mass of the proton.

$$\text{Mass of proton, } m_p = 1.6726231 \times 10^{-27} \text{ kg}$$

$$\text{Mass of neutron, } m_n = 1.6749286 \times 10^{-27} \text{ kg}$$

$$m_p \approx 1840 m_e, \text{ where } m_e = \text{mass of electron.}$$

Atomic number

Atomic number of an element is the number of proton present inside the nucleus of a neutral atom. It is also equal to the number of electrons revolving in various orbits around the nucleus of an uncharged atom. It is represented by Z .

$$\text{Atomic number (Z) = Number of proton = Number of electron}$$

Mass number

The total number of neutrons and protons, i.e. number of nucleons present inside the nucleus of an element is called its mass number A . Number of neutrons is represented by N .

$$\text{So, } \text{Mass number (A)} = Z + N$$

A chemical element is represented by ${}_Z(\text{chemical symbol})^A$ or ${}_Z X^A$.

Total charge of the nucleus is equal to the total charge on all protons Z in it, i.e. Ze , where $e = 1.6 \times 10^{-19} \text{ C}$.

Inside

1 Nucleus

- Isotopes, isobars and isotones

2 Mass-energy relation

- Binding energy of nucleus
- Binding energy curve
- Nuclear forces
- Nuclear stability
- Nuclear reaction

3 Nuclear energy

- Nuclear fission
- Nuclear fusion

4 Radioactivity

- Radioactive decay
- Pair production and pair annihilation
- Rutherford and Soddy's law
- Applications of radioactivity

Size of nucleus

From various experiments, it was found that the volume of the nucleus is directly proportional to the number of nucleons (mass number) constituting the nucleus.

Let R is the radius of the nucleus having mass number A , then volume, $V = \frac{4}{3}\pi R^3 \propto A \Rightarrow R \propto A^{1/3}$

$$\Rightarrow \boxed{R = R_0 A^{1/3}}$$

where, $R_0 = 1.2 \times 10^{-15}$ m, is the range of nuclear size.

It is also known as **nuclear unit radius**.

Owing to the small size of the nucleus, fermi (fm) is found to be a convenient unit of length in Nuclear Physics.

It is defined as, 1 fermi (fm) = 10^{-15} m.

Example 13.1 What is the nuclear radius of ^{125}Fe , if that of ^{27}Al is 6.4 fermi?

Sol. Radius of the nucleus, $R = R_0 A^{1/3}$

$$\frac{R_{\text{Fe}}}{R_{\text{Al}}} = \left(\frac{A_{\text{Fe}}}{A_{\text{Al}}}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3}$$

$$\text{Radius, } R_{\text{Fe}} = \frac{5}{3} R_{\text{Al}} = \frac{5}{3} \times 6.4 \approx 10 \text{ fm}$$

Nuclear density

Density of nuclear matter is the ratio of mass of nucleus to its volume.

If m is the average mass of a nucleon ($\approx 1.67 \times 10^{-27}$ kg) and A is the mass number of element, then the mass of nucleus = mA . If R is the nuclear radius, then

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

As, density of nuclear matter = $\frac{\text{mass of nucleus}}{\text{volume of nucleus}}$

$$\therefore \rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} \quad \text{or} \quad \boxed{\rho = \frac{3m}{4\pi R_0^3}}$$

Thus, the density of nucleus is constant and independent of A for all nuclei. Different nuclei are like a drop of liquid of constant density. The density of nuclear matter is approximately $2.3 \times 10^{17} \text{ kg m}^{-3}$. This density is very large as compared to ordinary matter.

Note Number of nucleon per unit volume = $\frac{\text{Density}}{\text{Mass of one nucleon}}$

$$= \frac{2.3 \times 10^{17}}{1.67 \times 10^{-27}} \text{ per m}^3$$

$$\approx 10^{44} \text{ per m}^3$$

Example 13.2 Assuming that protons and neutrons have equal masses, calculate how many times nuclear matter is denser than water. (Take, mass of nucleon = 1.67×10^{-27} kg and $R_0 = 1.2 \times 10^{-15}$ m)

Sol. Density of nucleus (of water),

$$\rho = \frac{3m}{4\pi R_0^3} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times \frac{22}{7} \times (1.2 \times 10^{-15})^3}$$

$$= \frac{7 \times 3 \times 1.67 \times 10^{18}}{88 \times 1.2 \times 1.2 \times 1.2} = 2.307 \times 10^{17} \text{ kg/m}^3$$

Density of water, $\rho' = 10^3 \text{ kg/m}^3$

$$\therefore \frac{\rho}{\rho'} = \frac{2.307 \times 10^{17}}{10^3} = 2.307 \times 10^{14}$$

Nuclear spin

Protons and neutrons inside a nucleus moves in a well defined quantum state and thus they have orbital angular momentum. But, both of them also have internal spin angular momentum.

The total angular momentum of the nucleus is the resultant of all the spin and orbital angular momentum of the individual nucleons. This total angular momentum of the nucleus is known as nuclear spin of that nucleus.

Isotopes, isobars and isotones

Isotopes

The atoms of same elements having the same number of protons but different number of neutrons are called isotopes. In other words, isotopes have same value of atomic number (Z) but different value of mass number (A). Almost every element has isotopes. Because of the same atomic number, isotopes of an element have the same place in the periodic table.

The isotopes of some elements are



In nature, the isotopes of chlorine (${}_{17}\text{Cl}^{35}$ and ${}_{17}\text{Cl}^{37}$) are found in the ratio 75.4% and 24.6%. When chlorine is prepared in laboratory, its atomic mass is found to be

$$M = (35 \times 0.754) + (37 \times 0.246) = 35.5$$

Note Since, the isotopes have the same atomic number, they have the same chemical properties. Their physical properties are different as they have different mass numbers. Two isotopes, thus cannot be separated by chemical method, but they can be separated from the physical methods.

Example 13.3 Two stable isotopes of ${}_3\text{Li}^6$ and ${}_3\text{Li}^7$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic weight of lithium.

Sol. Atomic weight = Average weight of the isotopes

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{7.5 + 92.5}$$

$$= \frac{45.1134 + 648.98}{100} = 6.941 \text{ u}$$

Isobars

The atoms of different elements having the same mass number (A) but different atomic number (Z) are called isobars. They have different places in periodic table. Their chemical as well as physical properties are different.

${}_1\text{H}^3$ and ${}_2\text{He}^3$, ${}_8\text{O}^{17}$ and ${}_9\text{F}^{17}$ are examples of isobars.

Isotones

The atoms of different elements having equal number of neutrons ($A - Z$) but different number of protons are called isotones. ${}_3\text{Li}^7$ & ${}_4\text{Be}^8$ and ${}_1\text{H}^3$ & ${}_2\text{He}^4$ are examples of isotones.

CHECK POINT 13.1

- The mass number of a nucleus is
 - always less than its atomic number
 - always more than its atomic number
 - always equal to its atomic number
 - sometimes more than and sometimes equal to its atomic number
- Outside a nucleus
 - neutron is stable
 - proton and neutron both are stable
 - neutron is unstable
 - Neither neutron nor proton is stable
- Nucleus of an atom whose mass number is 24 consists of
 - 11 electrons, 11 protons and 13 neutrons
 - 11 electrons, 13 protons and 11 neutrons
 - 11 protons and 13 protons
 - 11 protons and 13 electrons
- For a nucleus to be stable, the correct relation between neutron number N and proton number Z for light nuclei is
 - $N > Z$
 - $N = Z$
 - $N < Z$
 - None of these
- In helium nucleus, there are
 - 2 protons and 2 electrons
 - 2 neutrons, 12 protons and 2 electrons
 - 2 protons and 2 neutrons
 - 2 positrons and 2 protons

Example 13.4 Select the pairs of isotones from the following nuclei.
 ${}_{12}\text{Mg}^{24}$, ${}_1\text{H}^3$, ${}_2\text{He}^4$, ${}_{11}\text{Na}^{23}$.

Sol. Isotones have same number of neutrons,

$$N = A - Z$$

- ${}_1\text{H}^3$ and ${}_2\text{He}^4$
 Number of neutrons = $3 - 1 = 1$
 or $4 - 2 = 2$
- ${}_{12}\text{Mg}^{24}$ and ${}_{11}\text{Na}^{23}$
 Number of neutrons = $24 - 12 = 12$
 or $23 - 11 = 12$

Note Nuclei having same mass number A but with proton number (Z) and neutron number ($A - Z$) interchanged are called **mirror nuclei**.

e.g. ${}_1\text{H}^3$ and ${}_2\text{H}^3$, ${}_3\text{Li}^7$ and ${}_4\text{Be}^7$.

Nuclide

A nuclide is a specific nucleus of an atom, which is characterised by its atomic number Z and mass number A .

Note All nuclei with a given Z and N are all together taken as **nuclide**.
 e.g. All ${}_{16}\text{S}$ together taken as one nuclide.

- In ${}_{88}\text{Ra}^{226}$ nucleus, there are
 - 138 protons and 88 neutrons
 - 138 neutrons and 88 protons
 - 226 neutrons and 88 electrons
 - 226 neutrons and 138 electrons
- As compared ${}^{12}\text{C}$ atom, ${}^{14}\text{C}$ atom have
 - two extra protons and two extra electrons
 - two extra protons but no extra electrons
 - two extra neutrons and no extra electrons
 - two extra neutrons and two extra electrons
- The radius of a nucleus of a mass number A is directly proportional to
 - A^3
 - A
 - $A^{2/3}$
 - $A^{1/3}$
- The radius of ${}_{29}\text{Cu}^{64}$ nucleus in fermi is (given, $R_0 = 1.2 \times 10^{-15} \text{ m}$)
 - 4.8
 - 1.2
 - 7.7
 - 9.6
- The three stable isotopes of neon ${}_{10}\text{Ne}^{20}$, ${}_{10}\text{Ne}^{21}$ and ${}_{10}\text{Ne}^{22}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u respectively. The average atomic mass of neon will be
 - 20.17 u
 - 30.17 u
 - 20.98 u
 - 30.98 u

MASS-ENERGY RELATION

According to Einstein, mass and energy are interconvertible. He predicted that, if the energy of a body changes by an amount E , its mass changes by an amount Δm given by the equation, $E = \Delta mc^2$

where, c is the speed of light.

Everyday examples of energy gain are too small to produce detectable changes of mass. But in nuclear physics, this plays an important role. Mass appears as energy and the two can be regarded as equivalent.

Mass defect of nucleus

It has been found that, the rest mass of the nucleus of a stable atom is always less than the sum of the masses of its constituents nucleons (protons and neutrons) in the free state. The difference between the sum of the masses of the nucleons constituting a nucleus and the rest mass of the nucleus is called mass defect (Δm).

Mass defect, $\Delta m = (\text{Mass of protons} + \text{Mass of neutrons}) - \text{Mass of the nucleus}$

Let us consider an atom of element ${}_Z X^A$.

Its atomic number is Z and mass number is A . Thus, its nucleus has Z protons and $(A - Z)$ neutrons. If m_p is the mass of a proton, m_n is the mass of a neutrons and m_N is the mass of the nucleus, then the mass defect,

$$\Delta m = Zm_p + (A - Z)m_n - m_N$$

Atomic mass unit or amu

In nuclear physics, mass is measured in unified atomic mass units abbreviated amu or u as mass of an atom is very small.

(i) The amu is defined as $\frac{1}{12}$ th mass of a ${}_6\text{C}^{12}$ atom.

(ii) 1 amu (or 1u) = 1.66×10^{-27} kg.

(iii) It can be shown using relation ($\Delta E = \Delta mc^2$) that, 1u mass has energy of 931.5 MeV.

$$\begin{aligned} \text{For 1u, } \Delta m &= 1.66 \times 10^{-27} \text{ kg} \\ c &= 3 \times 10^8 \text{ ms}^{-1} \\ \Delta E &= \Delta mc^2 = 1.66 \times 10^{-27} (3 \times 10^8)^2 \\ &= 1.494 \times 10^{-10} \text{ J} \\ &= \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} \text{ eV} = 931.5 \text{ MeV} \end{aligned}$$

$$\therefore \quad \mathbf{1 \text{ u} = 931.5 \text{ MeV}}$$

Example 13.5 Calculate the mass defect of Helium [${}_2\text{He}^4$].

(Take, mass of proton = 1.007276 u,
mass of neutron = 1.008665 u
and mass of ${}_2\text{He}^4$ = 4.001506 u)

Sol. Mass defect = Mass of nucleons – Mass of nucleus

[Here, A (mass number) = 4

and Z (atomic number) = 2

$\therefore$ Number of protons = 2

and number of neutrons = $A - Z = 4 - 2 = 2$]

$$\begin{aligned} \Delta m &= \text{Mass of 2 protons} + \text{Mass of 2 neutrons} - \text{Mass of nucleus} \\ &= [2 \times 1.007276 + 2 \times 1.008665 - 4.001506] \\ &= 0.030376 \text{ u} \end{aligned}$$

Example 13.6 Calculate the energy required to remove the least tightly bound neutron in ${}_{20}\text{Ca}^{40}$. (Given ,

mass of ${}_{20}\text{Ca}^{40}$ = 39.962589 amu,

mass of ${}_{20}\text{Ca}^{39}$ = 38.970691 amu

and mass of neutron = 1.008665 amu)

Sol. Here in order to remove a neutron, energy has to be supplied.

$$\begin{aligned} \text{Mass defect, } \Delta m &= M({}_{20}^{39}\text{Ca}) + M({}_0^1n) - M({}_{20}^{40}\text{Ca}) \\ &= 38.970691 + 1.008665 - 39.962589 \\ &= 0.016767 \text{ amu} \end{aligned}$$

$$\begin{aligned} \text{Equivalent energy} &= 0.016767 \times 931 \quad (\because 1 \text{ amu} = 931 \text{ MeV}) \\ &= 15.6 \text{ MeV} \end{aligned}$$

Example 13.7 A neutron breaks into a proton and an electron.

Calculate the energy produced in this reaction. (Take,

$m_e = 9 \times 10^{-31}$ kg, $m_p = 1.6725 \times 10^{-27}$ kg,

$m_n = 1.6747 \times 10^{-27}$ kg and $c = 3 \times 10^8$ m/s)

Sol. We have, ${}_0^1n \rightarrow {}_1^1p + {}_{-1}^0e$

$$\begin{aligned} \text{Mass defect, } \Delta m &= m_n - m_p - m_e \\ &= [1.6747 \times 10^{-27} - 1.6725 \times 10^{-27} - 0.0009 \times 10^{-27}] \\ &= 1.3 \times 10^{-30} \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{Energy produced} &= \Delta mc^2 = 1.3 \times 10^{-30} \times (3 \times 10^8)^2 \\ &= 11.7 \times 10^{-14} \text{ J} \\ &= \frac{11.7 \times 10^{-14}}{1.6 \times 10^{-13}} \\ &= 0.73 \text{ MeV} \quad (\because 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}) \end{aligned}$$

Packing fraction

It is a ratio, which is mathematically defined as

$$\frac{\text{Actual isotopic mass} - \text{Mass number}}{\text{Mass number}} = \frac{m - A}{A}$$

It may be positive, negative and zero.

Packing fraction of Ca^{40} $\left(\frac{n}{p} = 1\right)$ is zero.

If isotopic mass is less than mass number, packing fraction is negative, indicating some has been converted into energy called **binding energy (BE)**.

In general, lower the packing fraction, greater is the BE per nucleon and hence greater is the stability.

Binding energy of nucleus

The binding energy (E_b) of a nucleus is defined as the minimum energy required to separate its nucleons and place them at rest at infinite distance apart.

Or

The binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If Δm is mass defect, then according to Einstein's mass-energy relation.

$$\text{Binding energy} = \Delta mc^2 = [Zm_p + (A - Z)m_n] - m_N] \cdot c^2$$

This binding energy is expressed in joule, if Δm is measured in kg.

If Δm is measured in amu, then binding energy,

$$E_b = \Delta m \times 931 \text{ MeV}$$

where, $\Delta m = [Zm_p + (A - Z)m_n] - m_N$ amu

Note Binding energy can also be written as

$$E_b = [Zm_H + (A - Z)m_N - m_A]c^2$$

where, m_H is the mass of H-atom and m_A the atomic mass. By using the masses of H-atoms rather than protons, masses of the electrons in the atom cancel out. We do this because it is atomic masses that are measured directly by mass spectrometer. A slight error made by doing so, but that is negligible.

Example 13.8 Find the binding energy of an α -particle from the following data.

(Take, mass of the helium nucleus, $m({}_2\text{He}^4) = 4.001265u$,

mass of proton, $m_p = 1.007277u$,

mass of neutron, $m_n = 1.008666u$

and $1u = 931.4813 \text{ MeV}$)

Sol. α -particle consists of 2 protons and 2 neutrons

$$\begin{aligned} \text{Mass defect, } \Delta m &= [Zm_p + Nm_n - m({}_2\text{He}^4)] \\ &= 2 \times 1.007277 + 2 \times 1.008666 - 4.001265 \\ &= 0.03062u \end{aligned}$$

$$\begin{aligned} \text{Binding energy} &= \Delta m \times 931.4813 \text{ MeV} \\ &= 0.03062 \times 931.48 = 28.52 \text{ MeV} \end{aligned}$$

Binding energy per nucleon

The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

Binding energy per nucleon

$$\begin{aligned} &= \frac{\text{Total binding energy}}{\text{Mass number (i.e. total number of nucleons)}} \\ &= \frac{\Delta m \times 931 \text{ MeV}}{A} \end{aligned}$$

$$\Rightarrow E_{\text{bn}} = \frac{E_b}{A}$$

Binding energy per nucleon $\propto$ stability of nucleus

Example 13.9 Binding energy of ${}_{17}\text{Cl}^{35}$ and ${}_{15}\text{P}^{31}$ are 287.67 MeV and 262.48 MeV. Which of the two nuclei is more stable?

$$\begin{aligned} \text{Sol. Binding energy per nucleon of } {}_{17}\text{Cl}^{35} &= \frac{\text{Binding energy}}{\text{Mass number}} \\ &= \frac{287.67}{35} = 8.22 \text{ MeV} \end{aligned}$$

$$\text{Binding energy per nucleon of } {}_{15}\text{P}^{31} = \frac{262.48}{31} = 8.47 \text{ MeV}$$

BE of ${}_{15}\text{P}^{31}$ is more than BE of ${}_{17}\text{Cl}^{35}$.

Hence, ${}_{15}\text{P}^{31}$ is more stable.

Example 13.10 If the binding energy per nucleon of deuteron is 1.115 MeV, find its mass defect in atomic mass unit.

$$\text{Sol. Binding energy per nucleon, } E_{\text{bn}} = \frac{E_b}{A}$$

For deuteron, atomic mass, $A = 2$

Substituting the values in above equation, we get

$$\therefore 1.115 \text{ MeV} = \frac{E_b}{2} \Rightarrow E_b = 2 \times 1.115 \text{ MeV}$$

Since, $E_b = 931.5 \times \Delta m$

$$\Rightarrow \Delta m = \frac{E_b}{931.5} \Rightarrow \Delta m = \frac{2 \times 1.115}{931.5} = 0.0024u$$

Binding energy curve

It is the graph between binding energy per nucleon and total number of nucleons (i.e. mass number A).

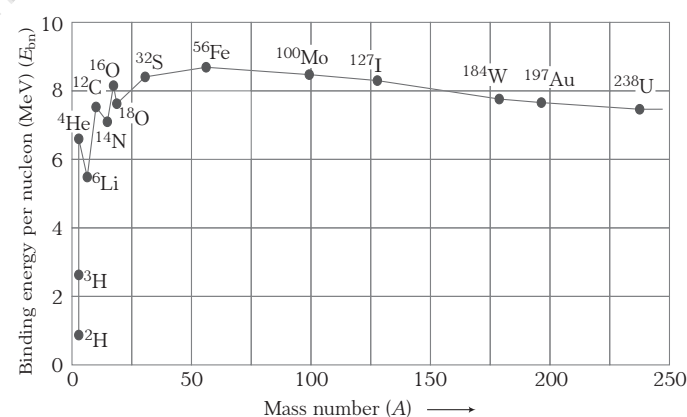


Fig. 13.1 Binding energy per nucleon as a function of mass number

Some features of this curve are given below

(i) The curve has almost a flat maximum roughly from $A = 50$ to $A = 80$ corresponding to an average BE/nucleon of about 8.5 MeV. So, the nuclei having mass number between 50 to 80 are most stable. Iron (Fe^{56}) ($A = 56$), having a BE/nucleon of about 8.8 MeV has maximum stability.

(ii) For nuclei having mass number more than 80, the average binding energy per nucleon decreases slowly and drop to about 7.6 MeV for uranium ($A = 238$).

The lower value of binding energy per nucleon fails to overcome the repulsion among protons in nuclei having large number of protons, *i.e.* the nuclei of heavier atoms, beyond ${}_{83}\text{Bi}^{209}$ are radioactive.

- (iii) For nuclei having mass number below 20, it decreases sharply. For example, for heavy hydrogen (H^2), it is only about 1.1 MeV. This shows that the nuclei having mass number below 20 are comparatively less stable.
- (iv) Below $A = 50$, the curve does not fall continuously, but has subsidiary peaks at ${}_{8}\text{O}^{16}$, ${}_{6}\text{C}^{12}$ and ${}_{2}\text{He}^4$. This shows that these (even-even) nuclei are more stable than their immediate neighbours.

Conclusions from the graph

- (i) The curve shows that very heavy and very light nuclei have a lower average BE/nucleon than the nuclei of intermediate masses. So, if a very heavy nucleus (such as U^{238}) be split into two lighter nuclei near the flat maximum of the curve, the BE/nucleon will increase. Hence, energy will be released in the process. This method of releasing nuclear energy by breaking up a heavy nucleus into two lighter nuclei of comparable masses is called **nuclear fission**. It is the basis of nuclear bombs and nuclear reactors.
- (ii) Alternatively, if two or more very light nuclei (such as H^2) be combined to form a heavier nucleus (such as He^4), the BE/nucleon will again increase and now by a much greater amount than in the fission process. This will result in much larger release of energy. This method of releasing nuclear energy is called **nuclear fusion**. It occurs inside sun and other stars and is the source of their energy.

Nuclear forces

As, we know that nucleus consists of positively charged proton and neutral particle neutron. The protons due to their positive charge repel each other, so there must be a strong force inside the nucleus which overcome the repulsive force of protons and bind them together inside the nucleus. This force is called nuclear force.

Therefore, The forces that holds the nucleons together inside the nucleus of an atom are called nuclear forces.

Some important properties of nuclear forces are given below

- (i) Nuclear forces are charge independent.
- (ii) Nuclear forces are short range forces. These do not exist at large distances greater than 10^{-15}m .
- (iii) These are attractive force and cause stability of the nucleus.
- (iv) Nuclear forces are non-central force.
- (v) Nuclear forces are the strongest force in nature.

- (vi) Nuclear forces depends on the direction of the spins of the nucleons. The force is stronger, if the spin of nucleons is parallel and is weak, if it is anti-parallel.
- (vii) Electrical and gravitational forces follow inverse square law (*i.e.* $F \propto 1/r^2$), but nuclear forces are complicated in nature and does not follow it.

Yukawa's Meson theory of nuclear forces

A Japanese scientist Yukawa in 1935 suggested that the nuclear forces are **exchange forces** which are produced by the exchange of new particles called **π -mesons between nucleons**. These particles were later on actually discovered in cosmic radiation.

There are three types of π -mesons; π^+ , π^- and π^0 . There is a continuous exchange of π -mesons between protons and neutrons due to which they continue to be converted into one another.



The forces between a pair of neutrons or a pair of protons are the result of the exchange of neutral meson (π^0) between them, *i.e.* $p \rightarrow p' + \pi^0$ and $n \rightarrow n' + \pi^0$

Thus exchange of π meson between nucleons keeps the nucleons bound together. It is responsible for the nuclear force.

Nuclear stability

The stability of a nucleus is determined by the value of its binding energy per nucleon. Higher the binding energy of nucleon, more stable is the nucleus. The stability of nucleus is also determined by its neutron to proton ratio. A plot of number of neutrons and number of protons is shown in the figure below.

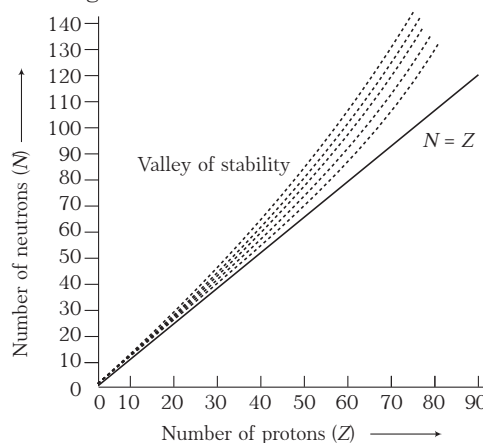


Fig. 13.2 Graphical representation of number of neutrons versus number of protons

In the figure, the solid line shows the nuclei with equal number of protons and neutrons ($N = Z$). Only light nuclei are on this line, *i.e.* they are stable if they contain approximately same number of protons and neutrons.

Heavy nuclei are stable, only when they have more neutrons than protons.

The long narrow region shown in the figure, which contains the cluster of short lines representing stable nuclei is referred to as the valley of stability.

The nuclide ${}^{209}_{83}\text{Bi}$ is the heaviest stable nucleus.

Experimental study shows that the stable nuclei contain even number of protons or neutrons or both.

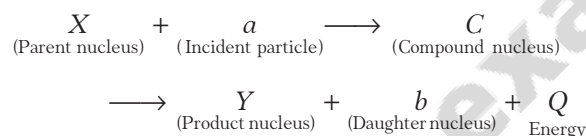
Magic numbers

We know that, the electrons in an atom are grouped in **shells** and **sub-shells**. Atoms with 2, 10, 18, 36, 54 and 86 electrons have all of their shells completely filled. Such atoms are unusually stable and chemically inert.

A similar situation exists with nuclei also. Nuclei having 2, 8, 20, 28, 50, 82 and 126 nucleons of the same kind (either protons or neutrons) are more stable than nuclei of neighbouring mass numbers. These numbers are called as magic numbers.

Nuclear reaction

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction. The general expression for the nuclear reaction is as follows



Here, X & a are the reactants, Y & b are the products and Q is the kinetic energy released in the nuclear reaction.

Conservation laws obeyed by nuclear reaction are

- (i) Mass number and electric charge is conserved.

$$\begin{aligned} \text{i.e.} \quad \Sigma Z_{\text{initial}} &= \Sigma Z_{\text{final}} \\ \Sigma A_{\text{initial}} &= \Sigma A_{\text{final}} \end{aligned}$$

- (ii) Linear momentum and angular momentum are conserved.

- (iii) Total energy of the reaction remains conserved, here the term Q helps to balance the total energy of reaction.

Q-value

Q -value means the difference between the rest mass energy of initial constituents and the rest mass energy of final constituents of a nuclear reaction.

Q-value of nuclear reaction

As considered, $X + a \longrightarrow Y + b + Q$

where, $Q = U_i - U_f$

Here, U_i = rest mass energy of initial constituents

and U_f = rest mass energy of final constituents

$$= (M_X + M_a - M_Y - M_b) \cdot c^2$$

If $Q = +ve$, reaction is exothermic

and $Q = -ve$, reaction is endothermic.

e.g. For α -decay, ${}_Z X^A \rightarrow {}_{Z-2} Y^{A-4} + {}_2\text{He}^4$ (α particle)

So, Q value will be

$$Q = [m({}_Z X^A) - m({}_{Z-2} Y^{A-4}) - m({}_2\text{He}^4)] \cdot c^2$$

which represents the kinetic energy available to the products.

For β^- -decay, ${}_Z X^A \rightarrow {}_{Z+1} Y^A + e^- + \bar{\nu}$

$$U_i = [m({}_Z X^A) - Zm_e] \cdot c^2$$

$$U_f = [m({}_{Z+1} Y^A) - (Z+1)m_e + m_e] \cdot c^2$$

Kinetic energy available to the products,

$$Q = U_i - U_f = [m({}_Z X^A) - m({}_{Z+1} Y^A)] \cdot c^2$$

For β^+ -decay, ${}_Z X^A \rightarrow {}_{Z-1} Y^A + e^+ + \nu$

$$U_i = [m({}_Z X^A) - Zm_e] \cdot c^2$$

$$U_f = [m({}_{Z-1} Y^A) - (Z-1)m_e + m_e] \cdot c^2$$

Kinetic energy available to the products,

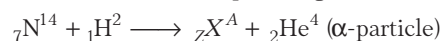
$$Q = [m({}_Z X^A) - m({}_{Z-1} Y^A) - 2m_e] \cdot c^2$$

For electron capture, ${}_Z X^A \rightarrow {}_{Z-1} Y^A$

$$Q = [m({}_Z X^A) - m({}_{Z-1} Y^A)] \cdot c^2$$

Example 13.11 A deuteron strikes ${}^{14}_7\text{N}$ nucleus with the subsequent emission of an α -particle. Find the atomic number, mass number and chemical name of the element so produced.

Sol. The nuclear reaction as per the given data is given as



According to laws of conservation,

- (i) For charge, $\Sigma Z_{\text{initial}} = \Sigma Z_{\text{final}}$

$$\Rightarrow 7 + 1 = Z + 2$$

$$\Rightarrow Z = 6$$

- (ii) For mass number, $\Sigma A_{\text{initial}} = \Sigma A_{\text{final}}$

$$\Rightarrow 14 + 2 = A + 4$$

$$\Rightarrow A = 12$$

$$\Rightarrow {}^{12}_6\text{X}, \text{ hence the element is carbon } {}^{12}_6\text{C}.$$

NUCLEAR ENERGY

The energy released during nuclear reaction is called nuclear energy. Generally, there is loss of mass in nuclear reaction, which results (according to Einstein's equation) into release of energy.

Consequently, if nuclei with less total binding energy transform to nuclei with greater binding energy, there will be a net energy release. This happens when a heavy nucleus decays into two or more intermediate mass fragments (fission) or when light nuclei fuse into a heavier nucleus (fusion).

In a nuclear reaction, the energy released is of the order of MeV. For same quantity of matter, nuclear sources produces a million times more energy than a chemical source.

Two distinct ways of obtaining energy from nucleus are as

- (i) Nuclear fission (ii) Nuclear fusion

Nuclear fission

The process of splitting of a heavy nucleus into two lighter nuclei of comparable masses (after bombardment with a energetic particle) with liberation of energy is called nuclear fission.

Fission reaction resulting from the absorption of neutron is known as **induced fission**.

One of the common example of fission is when uranium isotope ${}_{92}\text{U}^{235}$ bombarded with a neutron breaks into two intermediate mass nuclear fragments.

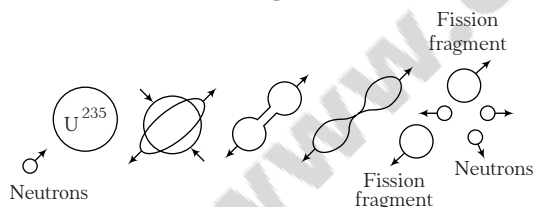


Fig. 13.3 Nuclear fission in U^{235}



This is one of the type into which the fission of ${}_{92}\text{U}^{235}$ takes place, there can be other combinations also.

In general, fission nuclei contains excess of neutrons, therefore are highly unstable. Hence, undergo beta decay until they reaches a stable end product.

The energy released in U^{235} fission is about 200 MeV or 0.8 MeV per nucleon and on an average 2.5 neutrons are liberated. These neutrons are called **fast neutrons** and their energy is about 2 MeV (each). These fast neutrons can escape from the reaction. So, in order to proceed the

reaction, they are needed to slow down. Fission of U^{235} occurs by slow neutrons only of energy about 1 eV or even by thermal neutrons of energy about 0.025 eV.

Note The mass of the compound nucleus must be greater than the sum of masses of fission products.

The $\frac{\text{binding energy}}{A}$ of compound nucleus must be less than that of the fission products. Hence, mass difference, i.e. mass defect, is converted into energy ($E = mc^2$). Thus, tremendous amount of energy is released, first appears as kinetic energy of the fragments and neutrons, then it is transferred to the surrounding appearing as heat.

Chain reaction

In nuclear fission, many neutrons are produced along with the release of large energy. Under favourable conditions, these neutrons can cause further fission of other nuclei, producing large number of neutrons.

Thus, a chain of nuclear fissions is established which continues until the whole of the uranium is consumed.

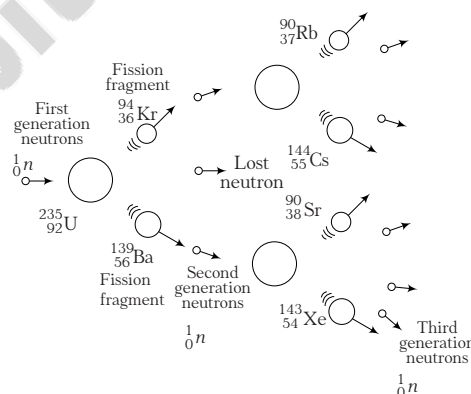


Fig. 13.4 Fission reaction

In the chain reaction, the number of nuclei undergoing fission increases very fast, so the energy produced takes a tremendous magnitude very soon.

Important points related to chain reaction

A chain reaction once started will remain steady, accelerate or retard depends upon neutron reproduction factor or multiplication factor (R) which is defined as

$$R = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

If $R = 1$, then chain reaction will be steady. The size of the fissionable sample used is said to be the **critical size** and its mass, the **critical mass**.

If $R > 1$, then chain reaction grows faster. Size of the material is said to be **super critical**.

If $R < 1$, then chain reaction comes to halt. Size of the material is said as **sub-critical**.

There are two types of chain reaction

- (i) **Controlled chain reaction** The reaction which can be controlled by absorbing suitable number of neutrons at each stage of the reaction, so that all neutrons are absorbed except one, which is available for further fission. This type of reaction is called controlled chain reaction. The rate of this reaction is slow and the energy liberated is always less than the explosive energy. The reproduction factor is $R = 1$. Nuclear reactor works on this principle.
- (ii) **Uncontrolled chain reaction** The reaction which cannot be controlled externally, rather there are more than one neutron taking part in reaction, then the reaction will accelerate at such a rapid rate that the whole material will explode within few micro-seconds, liberating tremendous amount of energy. The reproduction factor is $R > 1$. Atom bomb works on this principle.

Nuclear reactor

A nuclear reactor is a device in which nuclear fission can be carried out through a sustained and controlled chain reaction.

It is also called an **atomic pile**. It is thus a source of controlled energy which is utilised for many useful purposes. Nuclear reactor's energy is used to generate steam, which operates on a turbine and turns an electric generator.

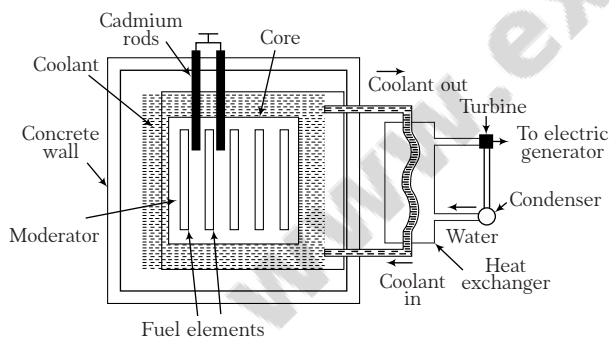


Fig. 13.5 Nuclear reactor

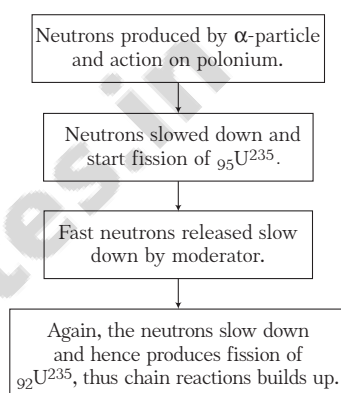
Construction

The key components of the nuclear reactor are as follows

- (i) **Fissionable material (Fuel)** The fissionable material used in the reactor is called the fuel of the reactor. Uranium isotope (U^{235}), thorium isotope (Th^{232}) and plutonium isotopes (Pu^{239} , Pu^{240} and Pu^{241}) are the most commonly used fuel in the reactor.
- (ii) **Moderator** It is used to slow down the fast moving neutrons. Most commonly used moderators are graphite and heavy water (D_2O). Heavy water is the best moderator.

- (iii) **Control rods** Cadmium rods are inserted into the core of the reactor because they can absorb the neutrons. The neutrons available for fission are controlled by moving the cadmium rods in or out of the core of the reactor.
- (iv) **Coolant** It is a liquid used to remove heat from nuclear reactor core and transfer it to electrical generator and environment. Ordinary water under high pressure is used as coolant.
- (v) **Shielding** It is the protective covering made of concrete wall to protect from harmful radiations.

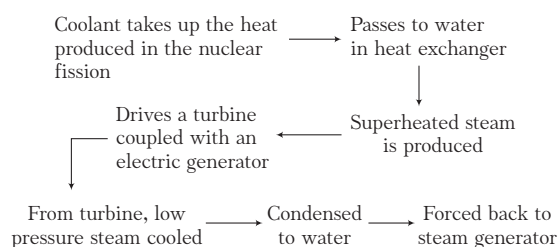
Working



By adjusting **control rods**, the chain reaction can be controlled.

As **water** is used as **coolant** as well as **moderator**.

Hence, the water cycle is as follows

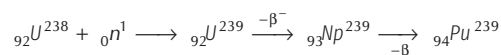


Uses of nuclear reactor

- (i) In electric power generation.
- (ii) To produce radioactive isotopes for their use in medical science, agriculture and industry.
- (iii) In manufacturing of Pu^{239} which is used in atom bomb.

Note Breeder reactor The reactors which can produce fuel more than they use, i.e. produce more fissile material than it consumes. e.g. $^{238}_{92}U$ isotope, when bombarded with fast neutrons,

transformations occurs like

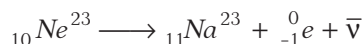


Thus, breeder reactor produces more fissionable nuclei, i.e. more fuel in the form of Pu^{239} from non-fissile uranium.

Atom bomb

In the atomic bomb, an uncontrolled chain reaction occurs in a very short time, when two pieces of uranium-235 are rapidly brought together to form a mass greater than the critical size.

Example 13.12 Neon-23 decays in the following way



Find the minimum and maximum kinetic energy that the beta particle (${}_{-1}^0e$) can have. The atomic masses of ${}_{10}\text{Ne}^{23}$ and ${}_{11}\text{Na}^{23}$ are 22.9945 u and 22.9898 u, respectively.

Sol. Here, atomic masses are given (not the nuclear masses), so we shall use them for calculating the mass defect because mass of electrons get cancelled both sides. Thus, mass defect, $\Delta m = m({}_{10}\text{Ne}^{23}) - m({}_{11}\text{Na}^{23})$

$$= (22.9945 - 22.9898) = 0.0047 \text{ u}$$

Kinetic energy is given by $Q = \Delta m \cdot c^2$

$$\therefore Q = (0.0047 \text{ u}) (931.5 \text{ MeV/u}) = 4.4 \text{ MeV}$$

Hence, the energy of beta particles can range from 0 to 4.4 MeV.

Example 13.13 Find the amount of energy produced in joules due to fission of 1 g of ${}_{92}\text{U}^{235}$, assuming that 0.1% of mass is transformed into energy. Atomic mass of ${}_{92}\text{U}^{235}$ is 235 amu, Avogadro number, $N_A = 6.023 \times 10^{23}$. (Take, the energy released per fission is 200 MeV)

Sol. It is given that, 0.1% of 1 g of ${}_{92}\text{U}^{235}$ is being used. So,

$$\text{mass used, } m = \frac{0.1}{100} \times 1 = 10^{-3} \text{ g}$$

$$\begin{aligned} \text{Number of fissions} &= \frac{\text{Given mass}}{\text{Atomic mass}} \times N_A \\ &= \frac{10^{-3}}{235} \times 6.023 \times 10^{23} = \frac{6.023 \times 10^{20}}{235} \end{aligned}$$

$$\begin{aligned} \text{Energy produced, } E &= \text{Number of fissions} \times \text{Energy released per fission} \\ &= \frac{6.023 \times 10^{20}}{235} \times 200 \times 1.6 \times 10^{-13} = 8.2 \times 10^7 \text{ J} \\ &\quad (\because 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}) \end{aligned}$$

Example 13.14 Find the amount of energy released when 1 atom of Uranium ${}_{92}\text{U}^{235}$ (235.0439 amu) undergoes fission by slow neutron (1.0087 amu) and is splitted into Krypton ${}_{36}\text{Kr}^{92}$ (91.8973 amu) and Barium ${}_{56}\text{Ba}^{141}$ (140.9139 amu) assuming no energy is lost. Hence, find the energy in kWh, when 1 g of it undergoes fission.

Sol. Fission equation, ${}_{92}\text{U}^{235} + {}_0^1n \rightarrow {}_{56}\text{Ba}^{141} + {}_{36}\text{Kr}^{92} + 3({}_0^1n)$

Mass defect,

$$\begin{aligned} \Delta m &= [m({}_{92}\text{U}^{235}) + m_n] - [m({}_{56}\text{Ba}^{141}) + m({}_{36}\text{Kr}^{92}) + 3m_n] \\ &= [235.0439 + 1.0087] - [140.9139 + 91.8973 + 3 \times 1.0087] \end{aligned}$$

$$= 0.2135 \text{ amu}$$

$$\text{Equivalent energy, } E = \Delta mc^2 = \Delta m \times 931 \text{ MeV}$$

$$= 0.2153 \times 931 = 200.4 \text{ MeV}$$

Number of atoms/fission in 1g of atom is

$$n = \frac{N_A}{A} = \frac{1}{235} \times 6.023 \times 10^{23} = 2.56 \times 10^{21}$$

Energy released in fission of 1 g of ${}_{92}\text{U}^{235}$

$$\Rightarrow n \times E = 2.56 \times 10^{21} \times 200.4 \text{ MeV}$$

$$= 2.56 \times 10^{21} \times 200.4 \times 1.6 \times 10^{-13} \text{ J}$$

$$(\because 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J})$$

$$= \frac{2.56 \times 10^{21} \times 200.4 \times 1.6 \times 10^{-13}}{3.6 \times 10^6} \text{ kWh}$$

$$= 2.28 \times 10^4 \text{ kWh}$$

Example 13.15 In a neutron induced fission of ${}_{92}\text{U}^{235}$

nucleus, usable energy of 185 MeV is released. If a ${}_{92}\text{U}^{235}$ reactor is continuously operating it at a power level of 100 MW, how long will it take for 1 kg of uranium to be consumed in this reactor?

Sol. Total number of nuclei in 1 kg of uranium,

$$n = (\text{Number of moles}) \times (\text{Avogadro's number})$$

$$= \left(\frac{1000}{235} \right) (6.02 \times 10^{23}) = 2.56 \times 10^{24}$$

By the fission of one nucleus, 185 MeV energy is released.

Hence total energy released by the fission of 1 kg of uranium is,

$$E = (2.56 \times 10^{24}) (185 \times 1.6 \times 10^{-13}) \text{ J} = 7.58 \times 10^{13} \text{ J}$$

Power of the power plant is given 100 MW or 10^8 J/s .

Therefore total time upto which the power plant can be run

$$\text{by 1 kg uranium is, } t = \frac{7.58 \times 10^{13}}{10^8} = 7.58 \times 10^5 \text{ s}$$

Nuclear fusion

When two light nuclei combine to form a heavier nucleus, then this process is called nuclear fusion.

In order to fuse two nuclei, they must come together in the range of nuclear force and to do this, they must overcome electrical repulsion of their positive charges.

Hence, the kinetic energy required for fusion depends on the Coulomb barrier height $\approx 400 \text{ keV}$ for two protons.

This energy is only achieved at high temperature.

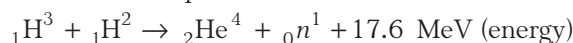
Hence, for fusion, high pressure $\approx 10^{16} \text{ atm}$ and high temperature of the order of 10^7 K to 10^8 K is required and so the reaction is called **thermonuclear reaction**.

e.g. Two deuterons (heavy-hydrogen nuclei) can be fused to form a triton (tritium) according to following reaction



The triton so formed can further fuse with a third

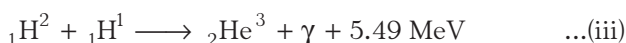
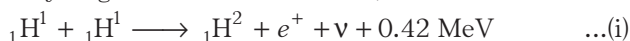
deuteron to form an α -particle.



The net result of the two reactions is the fusion of three deuterons and the formation of α -particle (${}_2\text{He}^4$), a neutron (${}_0n^1$) and a proton (${}_1\text{H}^1$). The total energy released is 21.6 MeV.

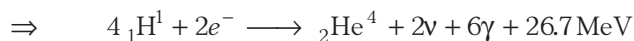
Solar energy is due to fusion in the interior of the sun which has a temperature of $1.5 \times 10^7 \text{ K}$ ($\ll 3 \times 10^8 \text{ K}$). The fusion in star and sun involves protons whose energies are much above the average energy.

The fusion reaction in the sun is a multi-step process in which hydrogen is fused into helium, which is follows as



First three reactions occurs twice to react to fourth reaction.

$$\Rightarrow 2(\text{i}) + 2(\text{ii}) + 2(\text{iii}) + (\text{iv})$$



Thus, four hydrogen atoms combine to form an ${}_2\text{He}^4$ atom with a release of 26.7 MeV of energy.

As the hydrogen in the core gets depleted and becomes helium, the core start to cool. The star begins to collapse under its own gravity, which decreases the temperature of the core.

Note Hydrogen bomb is due to nuclear fusion.

Controlled thermonuclear fusion

The aim of thermonuclear fusion is to generate steady power by heating the nuclear fuel to the temperature in the range of 10^8 K . At these temperature, the fuel is a mixture of positive ions and electrons (plasma). The challenge is to confine this plasma, since no container can stand such a high temperature. Several countries around the world including India are developing techniques for this.

Note If a controlled fusion reaction can be achieved, then an almost unlimited supply of energy will become available from deuterium in the water of oceans.

Example 13.16 The binding energy per nucleon of deuterium and helium atom is 1.1 MeV and 7.0 MeV, respectively. If two deuterium nuclei fuse to form helium atom, find the energy released.

Sol. Given, fusion reaction, ${}_1\text{H}^2 + {}_1\text{H}^2 \longrightarrow {}_2\text{He}^4 + \text{energy}$

Binding energy of a (${}_1\text{H}^2$) deuterium nuclei

$$= 2 \times 1.1 = 2.2 \text{ MeV}$$

Total binding energy of two deuterium nuclei

$$= 2.2 \times 2 = 4.4 \text{ MeV}$$

Binding energy of a (${}_2\text{He}^4$) nuclei = $4 \times 7 = 28 \text{ MeV}$

$$\begin{aligned} \text{So, energy released in fusion} &= \text{Final BE} - \text{Initial BE} \\ &= 28 - 4.4 = 23.6 \text{ MeV} \end{aligned}$$

Example 13.17 In the fusion reaction

${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_2\text{He}^3 + {}_0n^1$, the masses of deuteron, helium and neutron expressed in amu are 2.015, 3.017 and 1.009, respectively. If 1 kg of deuterium undergoes complete fusion, then find the amount of total energy released. (Take, $1 \text{ amu} = 931.5 \text{ MeV}/c^2$)

Sol. Given, fusion reaction, ${}_1\text{H}^2 + {}_1\text{H}^2 \rightarrow {}_2\text{He}^3 + {}_0n^1$

$$\text{Mass defect, } \Delta m = [2 \times m({}_1\text{H}^2) - m({}_2\text{He}^3) + m_n]$$

$$= [(2 \times 2.015) - (3.017 + 1.009)] = 4 \times 10^{-3} \text{ amu}$$

$$\begin{aligned} \text{Equivalent energy, } E &= \Delta mc^2 = \Delta m \times 931.5 \text{ MeV}/c^2 \\ &= 4 \times 10^{-3} \times 931.5 = 3.726 \text{ MeV} \end{aligned}$$

Number of atoms in 1 kg of ${}_1\text{H}^2$

$$\begin{aligned} &= \frac{\text{Given mass}}{\text{Atomic mass}} \times \text{Avogadro number} \\ &= \frac{1000}{2} \times 6.023 \times 10^{23} \end{aligned}$$

In one reaction, two atoms of ${}_1\text{H}^2$ are used.

So, total energy released

$$\begin{aligned} &= \frac{1}{2} \left(\frac{1000}{2} \times 6.023 \times 10^{23} \right) \times 3.726 \times 1.6 \times 10^{-13} \\ &= 8.98 \times 10^{13} \approx 9 \times 10^{13} \text{ J} \end{aligned}$$

Example 13.18 In a nuclear reactor ${}^{235}\text{U}$ undergoes fission releasing energy 100 MeV. The reactor has 20% efficiency and the power produced is 2000 MW. If the reactor is to function for 5 yr, find the total mass of uranium required.

Sol. Given, output power, $P_o = 2000 \text{ MW} = 2 \times 10^9 \text{ W}$

$$\text{Efficiency, } \eta = \frac{\text{Output power } P_o}{\text{Input power } P_i}$$

$$\Rightarrow \frac{20}{100} = \frac{2 \times 10^9}{P_i} \Rightarrow P_i = 10^{10} \text{ W}$$

Consider the mass of uranium be m .

Number of fissions = Number of atoms

$$\begin{aligned} &= \frac{\text{Given mass}}{\text{Atomic mass}} \times \text{Avogadro number} \\ &= \frac{m}{A} \times N_A = \frac{m}{235} \times 6.023 \times 10^{23} \end{aligned}$$

Energy produced, $E = \text{Number of fissions} \times \text{Energy per fission}$

$$= \frac{m}{235} \times 6.023 \times 10^{23} \times 100 \text{ MeV}$$

$$= \frac{m}{235} \times 6.023 \times 10^{23} \times 100 \times 1.6 \times 10^{-13} \text{ J}$$

$$(\because 1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J})$$

$$= m \times 0.041 \times 10^{12} \text{ J} = m \times 4.1 \times 10^{10} \text{ J}$$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} \Rightarrow 10^{10} = \frac{m \times 4.1 \times 10^{10}}{5 \times 365 \times 24 \times 3600}$$

$$m = 3.84 \times 10^7 \text{ g} = 38.4 \times 10^3 \text{ kg}$$

CHECK POINT 13.2

- 1 g of hydrogen is converted into 0.993 g of helium in a thermonuclear reaction. The energy released is
(a) 63×10^7 J (b) 63×10^{10} J (c) 63×10^{14} J (d) 63×10^{20} J
- If an electron and a positron annihilate, then the energy released is
(a) 3.2×10^{-13} J (b) 1.6×10^{-13} J
(c) 4.8×10^{-13} J (d) 6.4×10^{-13} J
- The mass defect in a particular nuclear reaction is 0.3 g. The amount of energy liberated in kilowatt hours is (velocity of light = 3×10^8 ms⁻¹)
(a) 1.5×10^6 (b) 2.5×10^6 (c) 3×10^6 (d) 7.5×10^6
- If m , m_n and m_p are the masses of ${}_Z X^A$ nucleus, neutron and proton respectively, then
(a) $m < (A - Z)m_n + Zm_p$ (b) $m = (A - Z)m_n + Zm_p$
(c) $m = (A - Z)m_p + Zm_n$ (d) $m > (A - Z)m_n + Zm_p$
- The masses of neutron and proton are 1.0087 amu and 1.0073 amu, respectively. If the neutrons and protons combine to form a helium nucleus (alpha particles) of mass 4.0015 amu, the binding energy of the helium nucleus will be (1 amu = 931 MeV)
(a) 28.4 MeV (b) 20.8 MeV (c) 27.3 MeV (d) 14.2 MeV
- In a fission reaction ${}^{236}_{92}\text{U} \rightarrow {}^{117}\text{X} + {}^{117}\text{Y} + n + n$, the binding energy per nucleon of X and Y is 8.5 MeV, whereas of ${}^{236}\text{U}$ is 7.6 MeV. The total energy liberated will be about
(a) 200 keV (b) 2 MeV (c) 200 MeV (d) 2000 MeV
- Fission of nuclei is possible because the binding energy per nucleon in them
(a) increases with mass number at high mass numbers
(b) decreases with mass number at high mass numbers
(c) increases with mass number at low mass number
(d) decreases with mass number at low mass number
- A chain reaction is continuous due to
(a) large mass defect
(b) large energy
(c) production of more neutrons in fission
(d) None of the above
- Which of these is a fusion reaction?
(a) ${}^3\text{H}_1 + {}^2\text{H}_1 \rightarrow {}^4\text{He}_2 + {}^1_0\text{n}$
(b) ${}^{238}_{92}\text{U} \rightarrow {}^{206}_{82}\text{Pb} + 8({}^4_2\text{He}) + 6({}^0_{-1}\text{B}_1)$
(c) ${}^{12}_7\text{C} \rightarrow {}^{12}_6\text{C} + \beta + \gamma$
(d) None of the above
- Solar energy is mainly caused due to
(a) fission of uranium present in the sun
(b) fusion of protons during synthesis of heavier elements
(c) gravitational contraction
(d) burning of hydrogen in the oxygen
- Nuclear fusion is common to the pair
(a) thermonuclear reactor, uranium based nuclear reactor
(b) energy production in sun, uranium based nuclear reactor
(c) energy production in sun, hydrogen bomb
(d) disintegration of heavy nuclei, hydrogen bomb
- Fusion reaction takes place at high temperature because
(a) molecules break up at high temperature
(b) nuclei break up at high temperature
(c) atoms get ionised at high temperature
(d) kinetic energy is high enough to overcome the Coulomb repulsion between nuclei
- Heavy water is used as moderator in a nuclear reactor. The function of the moderator is
(a) to control the energy released in the reactor
(b) to absorb neutrons and stop chain reaction
(c) to cool the reactor factor
(d) to slow down the neutron of thermal energies
- Pick out the correct statement from the following
(a) Energy released per unit mass of the reactant is less in case of fusion reaction
(b) Packing fraction may be positive or may be negative
(c) Pu^{239} is not suitable for a fission reaction
(d) For stable nucleus, the specific binding energy is low

RADIOACTIVITY

Radioactivity was discovered by Henry Becquerel in uranium salt in the year 1896.

The phenomenon of spontaneous emission of radiation by nucleus of some elements is known as **radioactivity**. The elements which show this phenomenon are called **radioactive elements**.

Atoms are radioactive, if their nuclei are unstable and spontaneously (and randomly) emit various (α , β and γ) radiations and decay into stable nucleus.

When naturally occurring nuclei are unstable, then such an unstable nucleus undergoes spontaneous emission of radioactive radiation (α , β and γ) and this process continues until a stable nucleus is obtained. This phenomenon is called **natural radioactivity**. When

some nuclei are made radioactive nuclei in laboratory, then such phenomena is known as **artificial radioactivity**. Some examples of radioactive substances are uranium, radium, thorium, polonium, neptunium, etc.

Generally, three types of radiations are emitted from radioactive nuclei, which α , β and γ radiations.

Properties of radioactive radiations

The properties of radioactive radiations α , β and γ are given below

Properties of α -particles

α -particle carries charge of $+2e$ and mass equals 4 times that of proton, i.e. it is equal to doubly ionised helium atom (${}_2\text{He}^4$).

Some important properties of α -particles are as follows

- (i) α -particles are deflected in electric and magnetic fields. They are positively charged.
- (ii) The velocity of α -particles vary from 0.01-0.1 times c (velocity of light).
- (iii) α -particles have low penetrating power.
- (iv) α -particles have high ionisation power. Their ionising power is 100 times greater than that of β -particles and 10000 times greater than that of γ -particles.
- (v) α -particles produce fluorescence in substances like zinc sulphide and barium platinocyanide. They produce scintillation on fluorescent screen.
- (vi) α -particles feebly affect photographic plate. They also produce heating effect, when stopped and can cause incurable burns on human body.
- (vii) All α -particles coming from a source have same energy.

Properties of β -particles

β -particles carries charge of $-e$ and mass equal to that of electrons, i.e. it is the fast moving electron ($_{-}\beta^0$ or β^-).

- (i) β -particles are deflected by electric and magnetic fields. They are negatively charged. Their deflection is much larger than the deflection of α -particles. This shows that β -particles are much lighter than the α -particles.
- (ii) The velocity of β -particles vary from 0.01 to 0.99 times the velocity of light.
- (iii) The penetrating power of β -particles is about 100 times larger than the penetrating power of α -particles. They can pass through 1mm thick sheet of aluminium.
- (iv) β -particles ionise gases, but their ionising power is much smaller, only (1/100)th of the ionising power of α -particles. As β -particles cannot produce ionisation continuously, their tracks in cloud chamber do not appear to be continuous.
- (v) β -particles produce fluorescence in calcium tungstate, barium platinocyanide and zinc sulphide.
- (vi) β -particles affect photographic plate more than the α -particles.

Properties of γ -rays

γ -rays carries no charge and mass. They are like the photons of light.

- (i) γ -rays are not deflected by electric and magnetic fields. This indicates that they have no charge.

(ii) γ -rays travel with the speed of light ($3 \times 10^8 \text{ ms}^{-1}$).

(iii) The penetrating power of γ -rays is much more than that of α and β -particles. They can pass through 30 cm thick iron sheet.

(iv) γ -rays ionise gases, but their ionisation power is very small compared to that of α and β -particles.

(v) γ -rays produce fluorescence.

(vi) γ -rays affect photographic plate more than β -particles.

(vii) γ -rays are diffracted by crystals in the same way as X-rays.

Note No radioactive substance emits both α and β -particles simultaneously. γ -rays are emitted along with both α and β -particles.

Comparison of the properties of α -particle, β -particle and γ -rays

Property	α -particle	β -particle	γ -rays
Nature	Helium nucleus	Fast moving electrons	Electromagnetic waves
Charge	$+2e$	$-e$	zero
Rest mass	$2.67 \times 10^{-26} \text{ kg}$	$9.1 \times 10^{-31} \text{ kg}$	zero
Speed	1.4×10^7 to $2.2 \times 10^7 \text{ ms}^{-1}$	1 to 99% of c	$c = 3 \times 10^8 \text{ ms}^{-1}$
Ionising power	10^4	10^2	1
Penetrating power	1	10^2	10^4

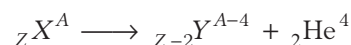
Radioactive decay

A radioactive nucleus (parent nucleus), decays by emission of particles into another nucleus (daughter nucleus), this phenomenon is called radioactive decay.

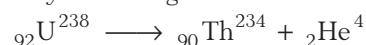
In nature, three types of radioactive decay occurs

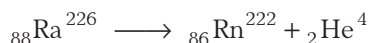
1. Alpha decay

An alpha particle is a helium nucleus. Thus, a nucleus emitting an alpha particle loses two protons and two neutrons. Therefore, the atomic number Z decreases by 2 units, the mass number A decreases by 4 units and the neutron number N decreases by 2. The decay can be written as



where, X is the parent nucleus and Y is the daughter nucleus. For example, ${}_{92} \text{U}^{238}$ and ${}_{88} \text{Ra}^{226}$ are both alpha emitters and decay according to





As a general rule, in any decay sum of mass numbers A and atomic numbers Z must be the same on both sides.

Alpha decay occurs with nuclei that are very heavy, hence unstable. So, emitting alpha particle, mass number decreases and it moves towards stability.

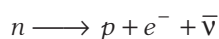
2. Beta decay

Beta decay involves the emission of electrons or positrons. A positron is a form of anti-matter, which has a charge equal to $+e$ and a mass equal to that of electron. The electrons and positron emitted in β -decay do not exist inside the nucleus. They are created at the time of emission, just as photons are created when an atom makes a transition from higher to a lower energy state.

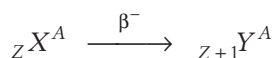
There are three different types of β -decay

- (i) In β^- **decay**, a neutron in the nucleus is transformed into a proton, an electron and an anti-neutrino.

An anti-neutrino is represented by symbol $\bar{\nu}$. It has zero rest mass, is chargeless and its spin quantum number is $\pm \frac{1}{2}$.



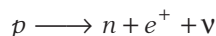
Thus a parent nucleus with atomic number Z and mass number A decays by β^- emission into a daughter nuclei with atomic number $Z + 1$ and the same mass number A .



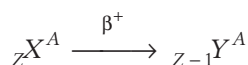
β^- decay occurs in nuclei that have too many neutrons, i.e. N/Z ratio is too large for stability. An example of β^- decay is the decay of carbon 14 into nitrogen,



- (ii) In β^+ **decay**, a proton changes into a neutron with the emission of a positron and a neutrino. Neutrino and anti-neutrino are anti-particles to each other.

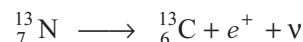


Positron (e^+) emission from a nucleus decreases the atomic number Z by 1 while keeping the same mass number A .

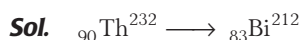


β^+ decay occurs in nuclei that have too many protons, i.e. N/Z ratio is too small for stability.

β^+ decay can occur only when the neutral atomic mass of the original atom (${}_Z X^A$) is at least two electron of mass ($2m_e$) larger than that of the final atom (${}_{Z-1} Y^A$).



Example 13.19 How many α and β^- -particles will be emitted when ${}_{90}\text{Th}^{232}$ changes to ${}_{83}\text{Bi}^{212}$?



Decrease in mass number = $232 - 212 = 20$

Number of α -particles emitted due to the above decrease in mass number = $\frac{20}{4} = 5$

Expected decrease in atomic number due to emission of 5 α -particles = $5 \times 2 = 10$

Expected atomic number of the nucleus formed = $90 - 10 = 80$

But the atomic number of nucleus formed = 83

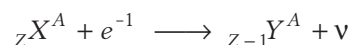
Increase in atomic number = $83 - 80 = 3$

Number of β^- -particles emitted = 3

Thus, 5 α -particles and 3 β^- -particles are emitted.

- (iii) **Electron capture** is competitive with positron emission, since both processes lead to the same nuclear transformation.

This occurs when a parent nucleus captures one of its own orbital electrons and emits a neutrino.



In most cases, it is a K -shell electron, i.e. captured and for this reason the process is referred to as K -capture. One example of capture of an electron by ${}_4\text{Be}^7$ is given as



Note

- (i) Electron capture occurs in those nuclides on which $\frac{N}{Z}$ ratio is too small for stability but β^+ emission is not energetically possible.
- (ii) After an electron capture, a vacancy is created in the atomic shell and hence X-rays are emitted.

3. Gamma decay

In ordinary physical and chemical transformation, the nucleus always remains in its ground state. When a nucleus is placed in an excited state, by a radioactive transformation, it can decay to the ground state by

emission of one or more photons called gamma rays and this process is called gamma decay.

A gamma ray is emitted when an α or a β decay results in a daughter nucleus in an excited state.

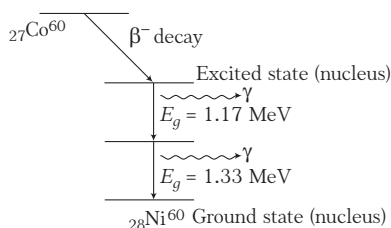
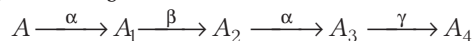


Fig. 13.6 β -decay of ${}_{27}\text{Co}^{60}$ followed by 2 γ decay

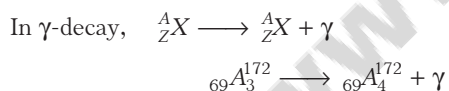
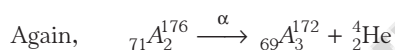
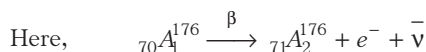
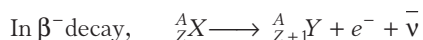
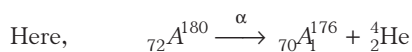
Note In both α and β -decays, the value of Z of a nucleus changes and nucleus of one element becomes the nucleus of a different element, whereas in γ -decay, the element does not change, but the nucleus merely goes from a more excited state to a less excited state.

Example 13.20 A radioactive nucleus undergoes a series of decay according to scheme.

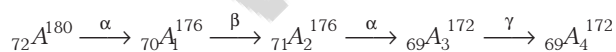


If the mass number and atomic number of A are 180 and 72 respectively, then what are these numbers for A_4 ?

Sol. In α -decay, ${}_Z^AX \longrightarrow {}_{Z-2}^{A-4}Y + {}_2^4\text{He}$



So, we have,



Pair production and pair annihilation

Pair production

When an energetic γ -ray photon falls on a heavy substance, it is absorbed by some nucleus of the substance and its energy gives rise to the production of an electron and a positron.

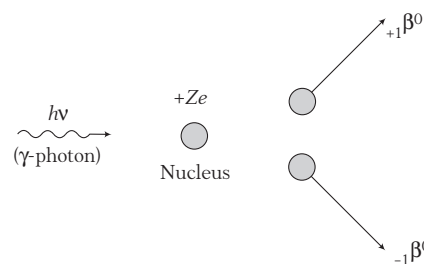


Fig. 13.7 Pair production

This phenomenon is called pair production and may be represented by the equation

$$h\nu \longrightarrow {}_{+1}\beta^0 + {}_{-1}\beta^0$$

From the equation, $E = mc^2$, the least energy of photon required for pair production will be

$$\begin{aligned} E &= 2(9.1 \times 10^{-31} \text{ kg})(3.0 \times 10^8 \text{ ms}^{-1})^2 \\ &= 1.64 \times 10^{-13} \text{ J} = \frac{1.64 \times 10^{-13}}{1.6 \times 10^{-13}} \text{ MeV} \\ &= 1.02 \text{ MeV} \end{aligned}$$

Hence, for pair production, it is essential that the energy of γ -photon must be at least 1.02 MeV.

Note

- If the energy of γ -photon is less than 1.02 MeV, it would cause photoelectric effect or Compton effect on striking the matter (no pair production).
- If the energy of γ -photon is more than 1.02 MeV, then electron and positron are produced and the energy in excess of 1.02 MeV is obtained as the kinetic energy of these particles. This kinetic energy is not distributed equally between these two particles, but the positron gets more than half. This is because (positively charged) nucleus accelerates the escaping (positively charged) positron due to repulsion, but retards the electron due to attraction.

Pair annihilation

The converse phenomenon of pair production is called pair annihilation.

When an electron and a positron come very close to each other, they annihilate each other by combining together and two γ -photons are produced.

This phenomenon can be represented by the following equation

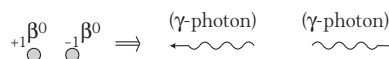
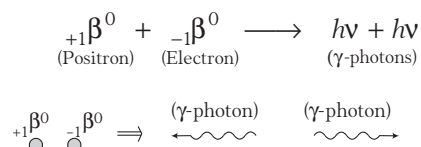


Fig. 13.8 Pair annihilation

Note The two photons (not one) are produced in pair annihilation to conserve both the energy and the momentum.

Rutherford and Soddy's law

According to this law, the rate of decay of radioactive nuclei at any instant is proportional to the number of nuclei present at that instant.

Let N be the number of nuclei present in a radioactive substance at any instant t . Let dN be the number of nuclei that disintegrates in a short interval dt .

Then, the rate of disintegration $\frac{-dN}{dt}$ is proportional to N

$$\text{i.e.} \quad -\frac{dN}{dt} \propto N$$

or

$$-\frac{dN}{dt} = \lambda N$$

where, λ is a constant for the given substance and is called **decay constant** (or disintegration constant or radioactive constant or transformation constant) and the negative sign indicates that radioactive nuclei are decreasing with time. For a given element, the value of λ is constant but for different elements, it is different. From the above

equation, we have $\frac{dN}{N} = -\lambda dt$.

Integrating above equation, we get

$$\int_0^N \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\log_e N = -\lambda t + C \quad \dots(i)$$

where, C is the integration constant. To determine C , we apply the initial conditions. Suppose, there were N_0 nuclei in the beginning, i.e. $N = N_0$ at $t = 0$.

Then,

$$\log_e N_0 = C$$

Substituting this value of C in Eq.(i), we have

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e N - \log_e N_0 = -\lambda t$$

$$\text{or} \quad \log_e \frac{N}{N_0} = -\lambda t$$

$$\Rightarrow \quad \frac{N}{N_0} = e^{-\lambda t}$$

$$\Rightarrow \quad \boxed{N = N_0 e^{-\lambda t}} \quad \dots(ii)$$

where, N_0 and N are numbers of nuclei in a radioactive substance at time $t = 0$ and after time t , respectively. According to this equation, the decay of a radioactive substance is exponential, i.e. the decay is rapid in the beginning and then its rate decreases continuously.

It means that a radioactive substance will take infinite time in decaying completely.

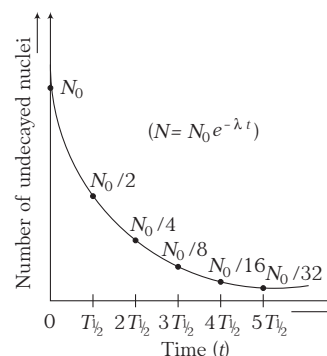


Fig. 13.9 Exponential decay of a radioactive substance

Note

(i) Number of nuclei decayed after time t ,

$$N_1 = N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0 (1 - e^{-\lambda t})$$

The corresponding graph is as shown in figure

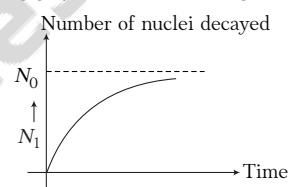


Fig. 13.10

(ii) Probability of a nucleus to survive in time t is

$$P(\text{survival}) = \frac{N}{N_0} = e^{-\lambda t}$$

(iii) Probability of a nucleus to disintegrate in time t ,

$$P(\text{disintegration}) = 1 - P(\text{survival})$$

$$= 1 - e^{-\lambda t}$$

Example 13.21 At time $t = 0$, number of nuclei of a radioactive substance are 100. At $t = 1$ s, these numbers become 90. Find the number of nuclei at $t = 2$ s.

Sol. In 1 s, 90% of the nuclei have remained undecayed, so in another 1 s, 90% of 90, i.e. 81 nuclei will remain undecayed.

Activity of radioactive substance

The rate of decay of a radioactive substance is called the activity (R) of the substance.

$$\text{i.e.} \quad R = \frac{-dN}{dt}$$

Also, according to Rutherford and Soddy law,

$$\frac{-dN}{dt} \propto N \quad \text{or} \quad -\frac{dN}{dt} = \lambda N$$

Thus,

$$R = -\frac{dN}{dt} \quad \text{or} \quad R = \lambda N$$

or

$$R = \lambda N_0 e^{-\lambda t} \quad [\text{from Eq. (ii)}]$$

or

$$\boxed{R = R_0 e^{-\lambda t}} \quad \dots(iii)$$

where, $R_0 = \lambda N_0$ is the activity of the radioactive substance at time $t = 0$. The activity *versus* time graph is shown in Fig. 13.11.

Thus, the number of nuclei and hence the activity of the radioactive substance decrease exponentially with time.

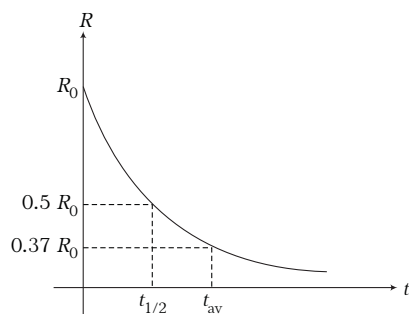


Fig. 13.11

Units of activity The SI unit for the decay rate is the becquerel (Bq), but the curie (Ci) and rutherford (Rd) are often used in practice.

$$1 \text{ Bq} = 1 \text{ decay/s}, \quad 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

$$\text{and} \quad 1 \text{ Rd} = 10^6 \text{ Bq}$$

Note Activity per unit mass or per unit volume is known as **specific activity**.

Half-life

The time interval in which the mass of a radioactive substance (the number of its nuclei) is reduced to half of its initial value is called half-life of that substance. It is represented by $T_{1/2}$.

From the equation, $N = N_0 e^{-\lambda t}$

$$\text{At half-life, } t = T_{1/2}, \quad N = \frac{N_0}{2}$$

$$\therefore \quad \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\Rightarrow \quad e^{\lambda T_{1/2}} = 2$$

Taking log on both sides, we get

$$\begin{aligned} \lambda T_{1/2} &= \log_e 2 \Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} \\ &= \frac{2.303 \log_{10} 2}{\lambda} = \frac{0.3010 \times 2.303}{\lambda} \end{aligned}$$

($\because \log_{10} 2 = 0.3010$)

$$T_{1/2} = \frac{0.6931}{\lambda}$$

After n half-life, the number of nuclei left undecayed is given by $t = n T_{1/2}$, $N = N_0 \left(\frac{1}{2}\right)^n$ and $R = R_0 \left(\frac{1}{2}\right)^n$.

Note The half-life rule is applicable to any fraction. e.g.

$$N_0 \xrightarrow{T_{1/3}} \frac{N_0}{3} \xrightarrow{T_{1/3}} \frac{N_0}{3^2} \xrightarrow{T_{1/3}} \frac{N_0}{3^3} \rightarrow \dots$$

Example 13.22 There is a stream of neutrons with a kinetic energy of 0.0327 eV . If the half-life of neutrons be 700 s , what fraction of neutrons will decay before they travel a distance of 10 m ? (Take, mass of neutron $= 1.675 \times 10^{-27} \text{ kg}$)

Sol. Kinetic energy of neutrons is given to be 0.0327 eV . From the relation $K = \frac{1}{2} m v^2$, we can find the speed of the neutrons.

$$\begin{aligned} \therefore \quad K &= \frac{1}{2} m v^2 \quad \text{or} \quad v = \sqrt{\frac{2K}{m}} \\ &= \sqrt{\frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}} = 2.5 \times 10^3 \text{ ms}^{-1} \end{aligned}$$

Time taken by the neutrons to travel a distance of 10 m with this speed, $t = \frac{10}{2.5 \times 10^3} = 4.0 \times 10^{-3} \text{ s}$

Fraction of neutrons decayed in the given time interval,

$$f = \frac{N_0 (1 - e^{-\lambda t})}{N_0} = (1 - e^{-\lambda t})$$

Given, $T_{1/2} = 700 \text{ s}$

$$T_{1/2} \text{ (half-life)} = \frac{0.693}{\lambda} \quad (\text{where, } \lambda = \text{decay constant})$$

$$\Rightarrow \quad \lambda = 0.693 / T_{1/2} = \frac{0.693}{700}$$

$$\Rightarrow \quad f = 1 - e^{-\left(\frac{0.693}{700}\right)(4.0 \times 10^{-3})} = 3.96 \times 10^{-6}$$

Example 13.23 The disintegration rate of a certain radioactive sample at any instant is 4750 disintegrations per minute. Five minutes later, the rate becomes 2700 per minute. Calculate

(i) decay constant (ii) and half-life of the sample.

Sol. (i) From the relation,

$$R = R_0 e^{-\lambda t} \text{ (here, } R = \text{activity of sample)}$$

Substituting the values, we have $2700 = 4750 e^{-5\lambda}$

$$\therefore \quad 5\lambda = \ln \left(\frac{4750}{2700} \right) = 0.56$$

$$\therefore \text{Decay constant, } \lambda = 0.113 \text{ min}^{-1}$$

$$\text{(ii) Half-life of the sample, } t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.113} = 6.132 \text{ min}$$

Example 13.24 At time $t = 0$, activity of a radioactive substance is 1600 Bq and at $t = 8 \text{ s}$, activity remains 100 Bq . Find the activity at $t = 2 \text{ s}$.

Sol. Activity of the sample after n half-life, $R = R_0 \left(\frac{1}{2}\right)^n$

$$\text{In } 8 \text{ s, } \frac{R}{R_0} = \frac{100}{1600} = \frac{1}{16} \Rightarrow R = \frac{R_0}{16}$$

$$\therefore \frac{R_0}{16} = R_0 \left(\frac{1}{2}\right)^n \text{ or } \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$$

or number of half-lives, $n = 4$

Four half-lives are equivalent to 8 s. Hence, 2 s is equal to one half-life. So in one half-life, activity will remain half of 1600 Bq, i.e. 800 Bq.

Example 13.25 A ${}^7\text{Li}$ target is bombarded with a proton beam current of 10^{-4} A for 1 h to produce ${}^7\text{Be}$ of activity 2.8×10^8 disintegrations per second. Assuming that, one ${}^7\text{Be}$ radioactive nuclei is produced by bombarding 1000 protons, determine its half-life.

Sol. Charge, $q = It = 10^{-4} \times 1 \times 3600 = 0.36$ C

$$\text{Number of protons} = \frac{q}{e} = \frac{0.36}{1.6 \times 10^{-19}} = 2.25 \times 10^{18}$$

$$\text{Number of } {}^7\text{Be nuclei} = \frac{2.25 \times 10^{18}}{1000} = 2.25 \times 10^{15}$$

$$\text{Activity, } R = \frac{\ln 2}{T_{1/2}} N, \text{ where } R = 2.8 \times 10^8 \text{ dps (given)}$$

$$\begin{aligned} \text{Half-life of substance, } T_{1/2} &= \frac{N \ln 2}{R} = \frac{2.25 \times 10^{15} \times 0.693}{2.8 \times 10^8} \\ &= 0.556 \times 10^7 \text{ s} \end{aligned}$$

Example 13.26 Determine the amount of ${}_{84}\text{Po}^{210}$ (polonium) necessary to provide a source of α -particles of 5 millicurie strength, if half-life of polonium is 138 days. (Given, 1 curie = 3.7×10^{10} disintegrations/s)

Sol. From the relation $R = \lambda N$, we have

$$N = \frac{R}{\lambda} \quad (\text{here, } R = \text{activity of sample})$$

$$\text{where, } R = 5 \times 10^{-3} \times 3.7 \times 10^{10} \text{ dps}$$

$$\text{and } t_{1/2} = 138 \text{ days} = 138 \times 24 \times 3600 \text{ s}$$

$$\therefore N = \frac{(5 \times 10^{-3}) (3.7 \times 10^{10})}{0.693 / (138 \times 24 \times 3600)} \quad \left(\because \lambda = \frac{0.693}{t_{1/2}} \right)$$

$$= 3.18 \times 10^{15}$$

$$\text{Number of moles required, } n = \frac{N}{N_A} = \frac{3.18 \times 10^{15}}{6.02 \times 10^{23}} = 5.28 \times 10^{-9}$$

where, N_A = Avogadro's number.

$\therefore$ Amount of polonium required for the desired activity,

$$\begin{aligned} m &= \text{number of moles} \times \text{atomic mass} = (5.28 \times 10^{-9}) (210) \text{ g} \\ &= 1.11 \times 10^{-6} \text{ g} \end{aligned}$$

Example 13.27

(i) Calculate the activity of 2 mg sample ${}_{28}\text{Sr}^{90}$ whose half-life period is 28 yr.

(ii) An experiment is performed to determine the half-life of a radioactive substance which emits one beta particle for each decay process. Observations show that an average of 8.4 β -particles are emitted each second by 2.5 mg of the substance. The atomic weight of the substance is 230. Calculate the half-life of the substance.

Sol. (i) Number of atoms, $N = \frac{\text{Mass}}{\text{Atomic weight}} \times N_A$;

(where N_A is Avogadro's number)

$$\begin{aligned} &= \frac{2 \times 10^{-3}}{90} \times 6.023 \times 10^{23} \\ &= 13.38 \times 10^{18} \end{aligned}$$

$$\text{Decay constant, } \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{28} / \text{yr}$$

$$\begin{aligned} \text{Activity, } R &= \lambda N = \frac{0.693}{28} \times 13.38 \times 10^{18} \\ &= 0.331 \times 10^{18} / \text{yr} \\ &= \frac{0.331 \times 10^{18}}{365 \times 24 \times 3600} = 10^{10} \text{ dps} \end{aligned}$$

(ii) Half-life of the substance,

$$N = \frac{2.5 \times 10^{-3}}{230} \times 6.023 \times 10^{23} = 6.5 \times 10^{18}$$

$$\text{Activity, } R = 8.4 / \text{s} = \lambda N = \frac{\ln 2}{T_{1/2}} N$$

$$\Rightarrow T_{1/2} = \frac{\ln 2}{R} N$$

Half-life of the substance,

$$\begin{aligned} T_{1/2} &= \frac{0.693 \times 6.5 \times 10^{18}}{8.4} \\ &= 0.54 \times 10^{18} \text{ s} \\ &= 1.7 \times 10^{10} \text{ yr} \end{aligned}$$

Example 13.28 In an experiment on two radioactive isotopes of an element (which do not decay into each other), their number of atoms ratio at a given instant was found to be 3. The rapidly decaying isotope has larger mass and an activity of $1.0 \mu\text{Ci}$ initially. The half-lives of the two isotopes are known to be 12 h and 16 h. What would be the number of atoms ratio after two days?

Sol. We know that, $t = nt_{1/2}$

Here, $t = 2$ days

(given)

and after n half-life, number of nuclei left undecayed,

$$N = N_0 \left(\frac{1}{2}\right)^n$$

For first isotope,

$$n_1 = \frac{24 \times 2}{12} \Rightarrow n_1 = \frac{48}{12} = 4 \quad \left(\because n = \frac{t}{t_{1/2}} \right)$$

$\therefore$ After 4 half-lives of first isotope, number of atoms,

$$N_A = (N_A)_0 \left(\frac{1}{2}\right)^{n_1} = (N_A)_0 \left(\frac{1}{2}\right)^4$$

For second isotope,

$$n_2 = \frac{24 \times 2}{16}$$

$$\Rightarrow n_2 = \frac{48}{16} = 3$$

∴ After 3 half-lives of second isotope, number of atoms,

$$N_B = (N_B)_0 \left(\frac{1}{2}\right)^{n_2} = (N_B)_0 \left(\frac{1}{2}\right)^3$$

∴ Ratio of number of atoms of two isotope,

$$\frac{N_A}{N_B} = \left(\frac{N_A}{N_B}\right)_0 \left(\frac{1}{2}\right) = \frac{3}{2} \quad \left[\because \left(\frac{N_A}{N_B}\right) = 3 \right]$$

Average or mean life of radioactive substance

The average of the lives of all the nuclei in a radioactive substance is called the mean life or average life of that substance. It is denoted by τ .

$$t_{av} = \tau = \frac{\int_0^\infty t dN}{\int_0^\infty dN} = \frac{\int_0^\infty t \lambda N_0 e^{-\lambda t} dt}{N_0}$$

It can be shown that, the mean life of a radioactive substance is equal to the reciprocal decay constant.

$$i.e. \quad \tau = \frac{1}{\lambda}$$

Also, $\lambda = \frac{0.6931}{T_{1/2}}$, where $T_{1/2}$ is the half-life of the substance.

$$\therefore \quad \tau = \frac{T_{1/2}}{0.6931} = 1.443 T_{1/2}$$

Thus, mean life is longer than half-life. The reason is that the last few nuclei of the radioactive substance have very long lives.

Example 13.29 At a given instant, there are 25% undecayed radioactive nuclei in a sample. After 10 s, the number of undecayed nuclei reduces to 12.5%. Calculate the mean life of the nuclei.

Sol. The number of undecayed nuclei reduces from 25% to 12.5% in 10 s. It means that, the half-life of the sample is 10 s.

$$i.e. \quad T_{1/2} = 10 \text{ s}$$

But, $T_{1/2} = 0.6931 / \lambda$, where λ is decay constant.

$$\text{Thus,} \quad \lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{10}$$

$$\text{The mean life of the sample, } \tau = 1/\lambda = \frac{10}{0.6931} = 14.428 \text{ s}$$

Example 13.30 1 g of a radioactive substance disintegrates at the rate of 3.7×10^{10} dps. The atomic mass of the substance is 226. Calculate its mean life.

Sol. We know that, number of nuclei present, $N = \frac{m}{M} N_A$

where, m = given mass, M = atomic mass
and N_A = Avogadro's number.

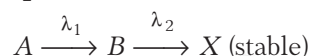
$$\text{Activity, } R = \lambda N = \frac{1}{\tau} \frac{m N_A}{M}$$

where, τ = mean life.

$$\begin{aligned} \text{Mean life of substance, } \tau &= \frac{m N_A}{M R} = \frac{1 \times 6.023 \times 10^{23}}{226 \times 3.7 \times 10^{10}} \text{ s} \\ &= 7.2 \times 10^{10} \text{ s} \end{aligned}$$

Successive disintegration

Let A be a radioactive nuclei (decay constant λ_1) disintegrates into another radioactive nuclei B (decay constant λ_2), then



$$\text{At } t = 0, \quad N_0; \quad 0; \quad 0$$

$$\text{At } t = t, \quad N; \quad N'; \quad N''$$

$$\frac{dN}{dt} = -\lambda_1 N$$

$$\frac{dN'}{dt} = \lambda_1 N - \lambda_2 N'$$

$$\frac{dN''}{dt} = -\lambda_2 N'$$

(i) If $(t_{1/2})_1 \gg (t_{1/2})_2$ or $\lambda_1 \ll \lambda_2$

$$\lambda_1 N = \lambda_2 N' \quad (\text{Secular equilibrium})$$

(ii) If $(t_{1/2})_1 > (t_{1/2})_2$ or $\lambda_1 < \lambda_2$

$$\lambda_1 N = (\lambda_2 - \lambda_1) N' \quad (\text{Transient equilibrium})$$

Note Parallel radioactive disintegration

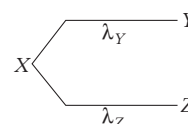


Fig. 13.12

$$\lambda_X = \lambda_Y + \lambda_Z$$

where, λ_X = decay constant of X ,

λ_Y = decay constant of Y

and λ_Z = decay constant of Z .

Example 13.31 In the chemical analysis of a rock, the mass ratio of two radioactive isotopes is found to be 100 : 1. The mean lives of the two isotopes are 4×10^9 yr and 2×10^9 yr, respectively. If it is assumed that at the time of formation, the atoms of both the isotopes were in equal proportion, then calculate the age of the rock. Ratio of the atomic weights of the two isotopes is 1.02 : 1.

Sol. At the time of observation ($t = t$),

$$\frac{m_1}{m_2} = \frac{100}{1} \quad (\text{given})$$

$$\text{Further it is given that, } \frac{A_1}{A_2} = \frac{1.02}{1}$$

$$\begin{aligned} \text{Number of atoms, } N &= \frac{m}{A} \\ \therefore \frac{N_1}{N_2} &= \frac{m_1}{m_2} \times \frac{A_2}{A_1} = \frac{100}{1.02} \quad \dots(i) \end{aligned}$$

Let N_0 be the number of atoms of both the isotopes at the time of formation, then

$$\frac{N_1}{N_2} = \frac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}} = e^{(\lambda_2 - \lambda_1)t} \quad \dots(ii)$$

Equating Eqs. (i) and (ii), we get

$$e^{(\lambda_2 - \lambda_1)t} = \frac{100}{1.02}$$

$$\text{or } (\lambda_2 - \lambda_1)t = \ln(100) - \ln(1.02)$$

Substituting the values, we get

$$\begin{aligned} t &= \frac{\ln(100) - \ln(1.02)}{\left(\frac{1}{2 \times 10^9} - \frac{1}{4 \times 10^9}\right)} \quad (\because \text{mean life, } \tau = 1/\lambda) \\ t &= 1.834 \times 10^{10} \text{ yr} \end{aligned}$$

Example 13.32 Uranium ores on the earth at the present time typically have a composition consisting of 99.3% of the isotope ${}_{92}\text{U}^{238}$ and 0.7% of the isotope ${}_{92}\text{U}^{235}$. The half lives of these isotopes are 4.47×10^9 yr and 7.04×10^8 yr, respectively. If these isotopes were equally abundant when the earth was formed, estimate the age of the earth.

Sol. Let N_0 be number of atoms of each isotope at the time of formation of the earth ($t = 0$) and N_1 & N_2 the number of atoms-at present ($t = t$). Then, according to law of radioactivity,

$$N_1 = N_0 e^{-\lambda_1 t} \quad \dots(i)$$

$$\text{and } N_2 = N_0 e^{-\lambda_2 t} \quad \dots(ii)$$

$$\therefore \frac{N_1}{N_2} = e^{(\lambda_2 - \lambda_1)t} \quad \dots(iii)$$

$$\text{Further it is given that, } \frac{N_1}{N_2} = \frac{99.3}{0.7} \quad \dots(iv)$$

Equating Eqs. (iii) and (iv) and taking log on both sides, we have

$$(\lambda_2 - \lambda_1)t = \ln\left(\frac{99.3}{0.7}\right)$$

$$\therefore t = \left(\frac{1}{\lambda_2 - \lambda_1}\right) \ln\left(\frac{99.3}{0.7}\right)$$

where λ_1 and λ_2 are the decay constants.

Substituting the values, we get

$$\begin{aligned} t &= \frac{1}{\frac{0.693}{7.04 \times 10^8} - \frac{0.693}{4.47 \times 10^9}} \ln\left(\frac{99.3}{0.7}\right) \\ &\quad \left(\because t_{1/2} = \frac{0.693}{\lambda} \Rightarrow \lambda = \frac{0.693}{t_{1/2}}\right) \end{aligned}$$

$$\therefore \text{Age of the earth, } t = 5.97 \times 10^9 \text{ yr}$$

Example 13.33 The nuclei of two radioactive isotopes of same substance A^{236} and A^{234} are present in the ratio of 4 : 1 in an ore obtained from some other planet. Their half-lives are 30 min and 60 min, respectively. Both isotopes are alpha emitters and the activity of the isotope with half-life 30 min is 10^6 dps. Calculate after how much time their activities will become identical. Also, calculate the time required to bring the ratio of their atoms to 1:1.

Sol. We have, $\left(\frac{N_1}{N_2}\right)_0 = \frac{4}{1}$

Half-life of two isotopes,

$$(T_{1/2})_1 = 30 \text{ min and } (T_{1/2})_2 = 60 \text{ min}$$

Activity of first isotope, $(R_1)_0 = 10^6$ dps

We know that, activity of radioactive substance, $R_0 = \lambda N_0$ and decay constant, $\lambda = \frac{0.693}{T_{1/2}}$, where $T_{1/2}$ = half-life.

$$\begin{aligned} \therefore \frac{(R_1)_0}{(R_2)_0} &= \frac{\lambda_1(N_1)_0}{\lambda_2(N_2)_0} = \frac{(T_{1/2})_2}{(T_{1/2})_1} \left(\frac{N_1}{N_2}\right)_0 \\ \frac{10^6}{(R_2)_0} &= \frac{60}{30} \times \frac{4}{1} \Rightarrow (R_2)_0 = \frac{1}{8} \times 10^6 \text{ dps} \end{aligned}$$

Let their activities become equal after time t ,

$$\Rightarrow R_1 = R_2$$

We know that, after n half-life, $R = R_0 \left(\frac{1}{2}\right)^n$

$$\Rightarrow (R_1)_0 \left(\frac{1}{2}\right)^{n_1} = (R_2)_0 \left(\frac{1}{2}\right)^{n_2} \text{ and } t = n T_{1/2}$$

$$\text{We have, } n_1 = \frac{t}{(T_{1/2})_1} = \frac{t}{30} \text{ and } n_2 = \frac{t}{(T_{1/2})_2} = \frac{t}{60}$$

$$\therefore 10^6 \left(\frac{1}{2}\right)^{\frac{t}{30}} = \frac{1}{8} \times 10^6 \left(\frac{1}{2}\right)^{\frac{t}{60}}$$

$$\left(\frac{1}{2}\right)^{\frac{t}{30} - \frac{t}{60}} = \frac{1}{8} = \left(\frac{1}{2}\right)^3 \Rightarrow \left(\frac{1}{2}\right)^{\frac{t}{60}} = \left(\frac{1}{2}\right)^3 \Rightarrow \frac{t}{60} = 3$$

$$\Rightarrow \text{Time, } t = 180 \text{ min}$$

According to question, when $N_1 = N_2$

After n half-life, number of nuclei left undecayed,

$$N = N_0 \left(\frac{1}{2}\right)^n$$

$$\Rightarrow (N_1)_0 \left(\frac{1}{2}\right)^{t/30} = (N_2)_0 \left(\frac{1}{2}\right)^{t/60}$$

$$\text{or } \left(\frac{1}{2}\right)^{t/60} = \left(\frac{N_2}{N_1}\right)_0 = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

$$= \frac{t}{60} = 2$$

$$\Rightarrow \text{Time, } t = 120 \text{ min}$$

Example 13.34 The mean lives of a radioactive substance are 1600 yr and 400 yr for α -emission and β -emission, respectively. Find out the time during which 7/8th of a sample will decay, if it is decaying both by α -emission and β -emission simultaneously.

Sol. For α - emission and β -emission respectively, according to parallel disintegration $\lambda = \lambda_\alpha + \lambda_\beta$

$$= \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1600} + \frac{1}{400} = \frac{5}{1600} \text{ yr}^{-1}$$

$$\left(\because \text{mean life} = \frac{1}{\text{decay constant}} \right)$$

If (7/8)th sample will decay, i.e. remaining (1/8)th.

$$\text{After } n \text{ half-life, } N = N_0 \left(\frac{1}{2} \right)^n \Rightarrow \frac{N_0}{8} = N_0 \left(\frac{1}{2} \right)^n \Rightarrow n = 3$$

$$\text{Time during (7/8)th sample will decay, } t = n T_{1/2} = n \frac{\ln 2}{\lambda}$$

$$= 3 \times \frac{0.693}{5 / 1600} \approx 665 \text{ yr}$$

Example 13.35 U^{238} is found to be in secular equilibrium with Ra^{226} in its ore. If chemical analysis shows 1 nuclei of Ra^{226} per 3.6×10^6 nuclei of U^{238} , find the half-life of U^{238} . (Given, the half-life of $Ra^{226} = 1500 \text{ yr}$)

Sol. According to secular equilibrium,

$$\lambda_U N_U = \lambda_{Ra} N_{Ra}$$

where, λ_U = decay constant of uranium,
 λ_{Ra} = decay constant of Ra,
 N_U = number of nuclei of U
and N_{Ra} = number of nuclei of Ra.

We know that,

$$\text{Decay constant } (\lambda) = \frac{\ln 2}{\text{Half-life } (t_{1/2})}$$

$$\Rightarrow \frac{\ln 2}{(t_{1/2})_U} N_U = \frac{\ln 2}{(t_{1/2})_{Ra}} N_{Ra} \Rightarrow \frac{3.6 \times 10^6}{(t_{1/2})_U} = \frac{1}{1500}$$

$$\Rightarrow (t_{1/2})_U = 3.6 \times 10^6 \times 1500$$

$$= 5400 \times 10^6 = 5.4 \times 10^9 \text{ yr}$$

Example 13.36 A radioactive element decays by β -emission. A detector records n -beta particles in 2 s and in next 2 s, it records 0.45n beta particles. Find mean life.

Sol. Let N_0 specifies initial number of nuclei at time $t = 0$,

where N = number of undecayed nuclei in time t .

According to the law of radioactivity, $N = N_0 e^{-\lambda t}$

The number of decayed nuclei in time t ,

$$N' = N_0 - N = N_0(1 - e^{-\lambda t}) = n \quad \dots(i)$$

The number of undecayed nuclei in next time t ,

$$N'_0 = N e^{-\lambda t} = N_0 e^{-\lambda t} \cdot e^{-\lambda t} = N_0 e^{-2\lambda t}$$

Number of decayed nuclei in next time t ,

$$N'' = N - N'_0 = N_0 e^{-\lambda t} - N_0 e^{-2\lambda t}$$

$$= N_0 e^{-\lambda t} (1 - e^{-\lambda t}) = 0.45 n \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$e^{-\lambda t} = 0.45 = \frac{9}{20}$$

$$\Rightarrow e^{\lambda t} = \frac{20}{9} \Rightarrow \lambda t = \ln \left(\frac{20}{9} \right)$$

As, time = 2 s (given)

$$\Rightarrow 2 \times \lambda = \ln \left(\frac{20}{9} \right) \Rightarrow \lambda = \frac{1}{2} \ln \left(\frac{20}{9} \right)$$

$$\text{So, mean life, } \tau = \frac{1}{\lambda} = \frac{2}{\ln \left(\frac{20}{9} \right)}$$

Example 13.37 A small quantity of a solution containing ^{24}Na radio nuclide (half-life 15 h) of activity 1.0 micro - curie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 h shows an activity of 360 disintegration per minute. Determine the total volume of blood in the body of the person, assume that the radioactive is mixed uniformly in the blood of the person.

Sol. Half-life, $t_{1/2} = 15 \text{ h}$

Initial activity, $R_0 = 1 \mu\text{Ci} = 10^{-6} \times 3.7 \times 10^{10} = 3.7 \times 10^4 \text{ dps}$

At $t = 5 \text{ h}$, activity of sample, $R = \frac{360}{60} \text{ dps}$,

volume of blood, $V = 1 \text{ cm}^3$

Let initial volume of blood = $V_0 \text{ cm}^3$

According to specific activity,

$$\frac{R}{V} = \frac{R_0}{V_0} e^{-\lambda t} = \frac{R_0}{V_0} e^{\left(\frac{0.693}{t_{1/2}} t \right)} \quad \left(\because \lambda = \frac{0.693}{t_{1/2}} \right)$$

$$\frac{360/60}{1} = \frac{3.7 \times 10^4}{V_0} e^{\left(\frac{0.693}{15} \times 5 \right)}$$

$$6 = \frac{3.7 \times 10^4}{V_0 \times e^{0.231}}$$

Volume of the blood, $V_0 = 4894.72 \text{ cm}^3 = 4.894 \text{ L}$

Example 13.38 A 25 g piece of charcoal is found in some ruins of an ancient city. The sample shows a C^{14} activity of 250 decay/min. How long has the tree from which this charcoal came from been dead? Given, the half-life of C^{14} is 5730 yr. The ratio of C^{14} and C^{12} in the living sample is 1.3×10^{-12} .

Sol. The decay constant λ of C^{14} will be

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{(5730)(365 \times 24 \times 60 \times 60)}$$

$$= 3.84 \times 10^{-12} \text{ s}^{-1}$$

The number of C^{14} nuclei can be calculated in two steps.
First, the number of C^{12} nuclei in 25.0 g of carbon,

$$N(C^{12}) = \frac{m}{M} N_A = (6.02 \times 10^{23}) \left(\frac{25}{12} \right) = 1.25 \times 10^{24}$$

The ratio of C^{14} to C^{12} in the live sample was 1.3×10^{-12} , we see that number of C^{14} nuclei in 25.0 g before decay was,

$$\begin{aligned} N_0(C^{14}) &= (1.3 \times 10^{-12})(1.25 \times 10^{24}) \\ &= 1.6 \times 10^{12} \end{aligned}$$

Hence, the initial activity of the sample was

$$\begin{aligned} R_0 &= \lambda N_0 = (3.84 \times 10^{-12})(1.6 \times 10^{12}) \\ &= 6.14 \text{ decay/s} \\ &\approx 370 \text{ decay/min} \end{aligned}$$

Activity at the time of observation is given as 250 decay/min.

From the equation, $R = R_0 e^{-\lambda t}$

$$\text{We have, } \lambda t = \ln \left(\frac{R_0}{R} \right) \quad \text{or} \quad t = \frac{1}{\lambda} \ln \left(\frac{R_0}{R} \right)$$

$$\text{Time, } t = \frac{1}{3.84 \times 10^{-12}} \ln \left(\frac{370}{250} \right) = 1.0 \times 10^{11} \text{ s} = 3200 \text{ yr}$$

Applications of radioactivity

Applications of radioactivity are given below

- Na-24 is used for testing blood circulation.
- Radio mercury-203 is used for detecting brain tumor.
- Phosphorous-31 is used for skin diseases.
- C^{14} is used for determining age of archaeological sample.
- Cobalt-60 is used for cancer treatment.

CHECK POINT 13.3

- Alpha rays emitted from a radioactive substance are
 - negatively charged particles
 - ionised hydrogen nuclei
 - doubly ionised helium atom
 - uncharged particles having the mass equal to proton
- Pick out the correct statement from the following.
 - α -particles are not deflected by electric and magnetic fields.
 - β -particle has less penetrating power than α -particle.
 - γ -rays has the least ionising power than α and β -particles.
 - γ -rays has the least penetrating power than α and β -rays.
- In the reaction, identify X , ${}_7N^{14} + \alpha \longrightarrow {}_8X^{17} + {}_1P^1$
 - An oxygen nucleus with mass number 17
 - An oxygen nucleus with mass number 16
 - A nitrogen nucleus with mass number 17
 - A nitrogen nucleus with mass number 16
- β -decay means emission of electrons from
 - innermost electron orbit
 - a stable nucleus
 - outermost electrons orbit
 - radioactive nucleus
- During a negative beta decay,
 - an atomic electron is ejected
 - an electron which is already present with in the nucleus is ejected
 - a neutron in the nucleus decays emitting an electron
 - a part of the binding energy is converted into electron
- A radioactive nucleus ${}_{92}X^{235}$ decays to ${}_{91}Y^{231}$. Which of the following particles are emitted?
 - One alpha and one electron
 - Two deuterons and one positron
 - One alpha and one proton
 - One proton and four neutrons
- Half-life of radioactive element depend upon
 - amount of element present
 - temperature
 - pressure
 - nature of element
- The half-life of a radioactive element which has only (1/32)nd of its original mass left after a lapse of 60 days is
 - 12 days
 - 32 days
 - 60 days
 - 64 days
- A radioactive substance emits n beta particles in the first 2 s and 0.5 n beta particles in the next 2 s. The half life of the sample is
 - 4 s
 - 2 s
 - $\frac{1}{2}$ s
 - $\frac{3}{4}$ s
- The half-life ($T_{1/2}$) and the disintegration constant (λ) of a radioactive substance are related as
 - $\lambda T_{1/2} = 1$
 - $\lambda T_{1/2} = 0.693$
 - $\frac{T_{1/2}}{\lambda} = 0.693$
 - $\frac{\lambda}{T_{1/2}} = 0.693$
- The half-life of polonium is 140 days. After how many days, 16 g polonium will decay to 1 g (or 15 g will decay)?
 - 700 days
 - 280 days
 - 560 days
 - 420 days
- Certain radioactive substance reduces to 25% of its value in 16 days. Its half-life is
 - 32 days
 - 8 days
 - 64 days
 - 28 days
- C^{14} has half-life of 5700 yr. At the end of 11400 yr, the actual amount left is
 - 0.5 of original amount
 - 0.25 of original amount
 - 0.125 of original amount
 - 0.0625 of original amount
- If the half-life of a radioactive sample is 10 h, then its mean life is
 - 14.4 h
 - 7.2 h
 - 20 h
 - 6.93 h
- Mean life of a radioactive sample is 100 s, then its half-life (in min) is
 - 0.693
 - 1
 - 10^{-4}
 - 1.155

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- Following process is known as, $h\nu \rightarrow e^+ + e^-$
 - pair production
 - photoelectric effect
 - Compton effect
 - Zeeman effect
- The operation of a nuclear reactor is said to be critical, if the multiplication factor (k) has a value
 - 1
 - 1.5
 - 2.1
 - 2.5
- Which of the following isotopes is used for the treatment of cancer?
 - K^{40}
 - Co^{60}
 - Sr^{90}
 - I^{131}
- When a nucleus in an atom undergoes a radioactive decay, then the electronic energy levels of the atom

[NCERT Exemplar]

 - do not change for any type of radioactivity
 - change for α and β -radioactivity but not for γ -radioactivity
 - change for α -radioactivity but not for others
 - change for β -radioactivity but not for others
- Heavy stable nuclei have more neutrons than protons. This is because of the fact that

[NCERT Exemplar]

 - neutrons are heavier than protons
 - electrostatic force between protons are repulsive
 - neutrons decay into protons through beta decay
 - nuclear forces between neutrons are weaker than that between protons
- In one average life
 - half the active nuclei decay
 - less than half the active nuclei decay
 - more than half the active nuclei decay
 - All the nuclei decay
- When the number of nucleons in nuclei increase, then the binding energy per nucleon
 - increases continuously with mass number
 - decreases continuously with mass number
 - remains constant with mass number
 - first increases and then decreases with increase of mass number
- The binding energy per nucleon is maximum in the case of
 - ${}^4_2\text{He}$
 - ${}^{56}_{26}\text{Fe}$
 - ${}^{141}_{56}\text{Ba}$
 - ${}^{235}_{92}\text{U}$
- Which of the following process will result into fission reaction?
 - ${}_{92}\text{U}^{235}$ is bombarded with fast moving neutrons
 - ${}_{92}\text{U}^{235}$ is bombarded with thermal neutrons
 - U^{238} is bombarded with slow moving neutrons
 - U^{235} being unstable breaks into smaller fragments
- Suppose, we consider a large number of containers each containing initially 10000 atoms of a radioactive material with a half-life of 1 yr. After 1 yr,

[NCERT Exemplar]

 - all the containers will have 5000 atoms of the material
 - all the containers will contain the same number of atoms of the material but that number will only be approximately 5000
 - the containers will in general have different numbers of the atoms of the material but their average will be close to 5000
 - None of the containers can have more than 5000 atoms
- The principle of controlled chain reaction is used in
 - atomic energy reactor
 - atom bomb
 - the core of sun
 - artificial radioactivity
- In an atomic bomb, the energy is released due to
 - chain reaction of neutrons and ${}_{92}\text{U}^{235}$
 - chain reaction of neutrons and ${}_{92}\text{U}^{238}$
 - chain reaction of neutrons and ${}_{92}\text{U}^{240}$
 - chain reaction of neutrons and ${}_{92}\text{U}^{236}$
- Boron rods are used in nuclear reactors because
 - boron can absorb neutrons
 - strength is given to the plant
 - boron speeds up neutrons
 - the reactors look good
- The decay constant of radioactive element is $1.5 \times 10^{-9} \text{ s}^{-1}$. Its mean life (in s) will be
 - 1.5×10^9
 - 4.62×10^8
 - 6.67×10^8
 - 10.35×10^8
- A nuclear reaction along with the masses of the particle taking part in it is as follows

A	+	B	$\rightarrow$	C	+	D	+ $Q \text{ MeV}$
1.002		1.004		1.001		1.003	
amu		amu		amu		amu	

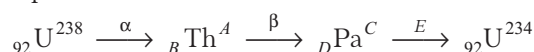
The energy Q liberated in the reaction is

 - 1.234 MeV
 - 0.931 MeV
 - 0.465 MeV
 - 1.863 MeV

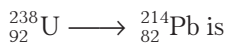
- 16** The curve of binding energy per nucleon as a function of atomic mass number has a sharp peak for helium nucleus.

This implies that helium

- (a) can easily be broken up
(b) is very stable
(c) can be used as fissionable material
(d) is radioactive
- 17** In the given nuclear reaction A, B, C, D and E represents



- (a) $A = 234, B = 90, C = 234, D = 91$ and $E = \beta$
(b) $A = 234, B = 90, C = 238, D = 94$ and $E = \alpha$
(c) $A = 238, B = 93, C = 234, D = 91$ and $E = \beta$
(d) $A = 234, B = 90, C = 234, D = 93$ and $E = \alpha$
- 18** An atom of mass number 15 and atomic number 7, captures an α -particle and then emits a proton. The mass number and atomic number of the resulting product will respectively be
(a) 14 and 2 (b) 15 and 3 (c) 16 and 4 (d) 18 and 8
- 19** The binding energies of the nuclei A and B are E_a and E_b , respectively. Three atoms of the element B fuse to give one atom of element A and an energy Q is released. Then, E_a, E_b and Q are related as
(a) $E_a - 3E_b = Q$ (b) $3E_b - E_a = Q$
(c) $E_a + 3E_b = Q$ (d) $E_b + 3E_a = Q$
- 20** The total number of α and β particles emitted in the nuclear reaction



- (a) 6 (b) 7 (c) 8 (d) 10

- 21** 90% of a radioactive sample is left undecayed after time t has elapsed. What percentage of the initial sample will decay in a total time $2t$?
(a) 20% (b) 19% (c) 40% (d) 38%
- 22** M_x and M_y denote the atomic masses of the parent and the daughter nuclei respectively, in radioactive decay. The Q -value for a β^- decay is Q_1 and that for a β^+ decay is Q_2 . If m_e denotes the mass of an electron, then which of the following statements is correct?

[NCERT Exemplar]

- (a) $Q_1 = (M_x - M_y) c^2$ and $Q_2 = [M_x - M_y - 2m_e] c^2$
(b) $Q_1 = (M_x - M_y) c^2$ and $Q_2 = (M_x - M_y) c^2$
(c) $Q_1 = (M_x - M_y - 2m_e) c^2$ and $Q_2 = (M_x - M_y + 2m_e) c^2$
(d) $Q_1 = (M_x - M_y + 2m_e) c^2$ and $Q_2 = (M_x - m_y + 2m_e) c^2$
- 23** Half-life of a radioactive substance is 20 min. The time between 20% and 80% decay will be
(a) 20 min (b) 40 min (c) 30 min (d) 25 min

- 24** A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is

- (a) 128 h (b) 24 h
(c) 6 h (d) 12 h

- 25** The decay constant of a radioactive substance is λ . Its half-life and mean life respectively, are

- (a) $\frac{1}{\lambda}$ and $\log_e 2$ (b) $\frac{\log_e 2}{\lambda}$ and $\frac{1}{\lambda}$
(c) $2 \log_e 2$ and $\frac{1}{\lambda}$ (d) $\frac{1}{\lambda}$ and $\frac{1}{\lambda}$

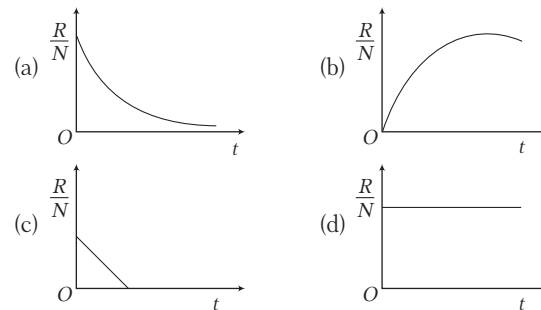
- 26** A radioactive substance disintegrates $1/64$ of initial value in 60 s. The half-life of this substance is

- (a) 5 s (b) 10 s
(c) 30 s (d) 20 s

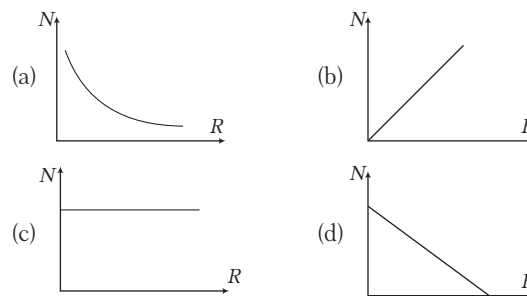
- 27** In the reaction ${}_1^2\text{H} + {}_1^3\text{H} \rightarrow {}_2^4\text{He} + {}_0^1n$, if the binding energies of ${}_1^2\text{H}, {}_1^3\text{H}$ and ${}_2^4\text{He}$ are respectively, a, b and c (in MeV), then the energy (in MeV) released in this reaction is

- (a) $c + a - b$ (b) $c - a - b$
(c) $a + b + c$ (d) $a + b - c$

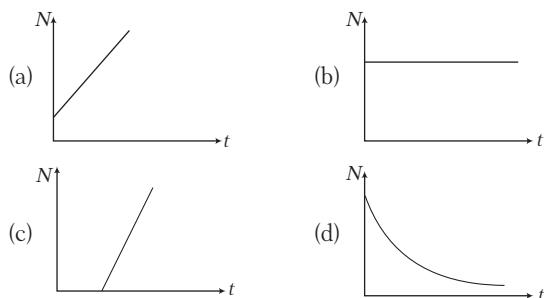
- 28** A radioactive sample has N_0 active atoms at $t = 0$. If the rate of disintegration at any time is R and the number of atoms is N , then the ratio R/N varies with time as



- 29** The plot of the number (N) of decayed atoms versus activity (R) of a radioactive substance is



- 30** The graph between the instantaneous concentration (N) of a radioactive element and time (t) is



- 31** Two radioactive materials X_1 and X_2 have decay constants 10λ and λ , respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time

(a) $\frac{1}{10\lambda}$ (b) $\frac{1}{11\lambda}$ (c) $\frac{11}{10\lambda}$ (d) $\frac{1}{9\lambda}$

- 32** Two radioactive nuclei A and B have disintegration constants λ_A and λ_B and initially N_A and N_B number of nuclei of them are taken, then the time after which their undisintegrated nuclei are same, is

(a) $\frac{\lambda_A \lambda_B}{(\lambda_A - \lambda_B)} \ln \left(\frac{N_B}{N_A} \right)$ (b) $\frac{1}{(\lambda_A + \lambda_B)} \ln \left(\frac{N_B}{N_A} \right)$
 (c) $\frac{1}{(\lambda_B - \lambda_A)} \ln \left(\frac{N_B}{N_A} \right)$ (d) $\frac{1}{(\lambda_A - \lambda_B)} \ln \left(\frac{N_B}{N_A} \right)$

- 33** What is the binding energy per nucleon of ${}^{12}_6\text{C}$ nucleus?

Given, mass of C^{12} (m_C) = 12.000 u,

Mass of proton (m_p) = 1.0078 u,

Mass of neutron (m_n) = 1.0087 u

and $1 \text{ amu} = 931.4 \frac{\text{MeV}}{c^2}$.

- (a) 5.26 MeV (b) 6.2 MeV
 (c) 4.65 MeV (d) 7.68 MeV

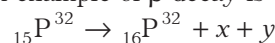
- 34** A radioactive isotope X with a half-life of 1.37×10^9 yr decays to Y which is stable. A sample of rock from the moon was found to contain both the elements X and Y which were in the ratio of 1 : 7. The age of the rock is

- (a) 1.96×10^8 yr (b) 3.85×10^9 yr
 (c) 4.11×10^9 yr (d) 9.59×10^9 yr

- 35** If the energy released in the fission of one nucleus is 200 MeV, then the number of nuclei required per second in a power plant of 16 kW will be

- (a) 0.5×10^{14} (b) 0.5×10^{22}
 (c) 5×10^{12} (d) 5×10^{14}

- 36** A common example of β -decay is



Then, x and y stand for

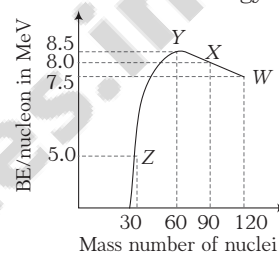
- (a) electron and neutrino (b) positron and neutrino
 (c) electron and antineutrino (d) positron and antineutrino

- 37** A nucleus with $Z = 92$ emits the following in a sequence

$\alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. The Z of the resulting nucleus is

- (a) 74 (b) 76 (c) 78 (d) 82

- 38** Binding energy per nucleon *versus* mass number curve for nuclei is shown in the figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is



- (a) $Y \rightarrow 2Z$ (b) $W \rightarrow X + Y$ (c) $W \rightarrow 2Y$ (d) $X \rightarrow Y + Z$

- 39** The gravitational force between a H-atom and another particle of mass m will be given by Newton's law

$$F = G \frac{M \cdot m}{r^2}, \text{ where } r \text{ is in m and}$$

[NCERT Exemplar]

- (a) $M = m_{\text{proton}} + m_{\text{electron}}$
 (b) $M = m_{\text{proton}} + m_{\text{electron}} - \frac{B}{c^2}$ ($B = 13.6 \text{ eV}$)
 (c) M is not relate to the mass of the hydrogen atom
 (d) $M = m_{\text{proton}} + m_{\text{electron}} - \frac{|V|}{c^2}$ ($|V|$ = magnitude of the potential energy of electron in the H-atom)

- 40** In a sample of radioactive material, what fraction of the initial number of active nuclei will remain undisintegrated after half of the half-life of the sample?

- (a) $\sqrt{2} - 1$ (b) $1/\sqrt{2}$ (c) $1/2\sqrt{2}$ (d) $1/4$

- 41** In a radioactive sample, the fraction of initial number of radioactive nuclei, which remains undecayed after n mean lives is

- (a) $\frac{1}{e^n}$ (b) e^n (c) $1 - \frac{1}{e^n}$ (d) $\left(\frac{1}{e-1} \right)^n$

- 42** At any instant, the ratio of the amount of radioactive substance is 2 : 1. If their half-lives be respectively, 12 h and 16 h, then after 2 days, what will be the ratio of the substances?

- (a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 4

- 43** A radioactive isotope X with half-life 1.5×10^9 yr decays into a stable nucleus Y . A rock sample contains both elements X and Y in the ratio 1: 15. The age of the rock is
 (a) 3.0×10^9 yr (b) 4.5×10^9 yr
 (c) 6.0×10^9 yr (d) 9.0×10^9 yr
- 44** The half-life of a sample of a radioactive substance is 1 h. If 8×10^{10} atoms are present at $t = 0$, then the number of atoms decayed in the duration $t = 2$ h to $t = 4$ h will be
 (a) 2×10^{10} (b) 1.5×10^{10} (c) zero (d) infinity
- 45** Half-life of a radioactive substance is 20 min. Ratio between points of time when it is 33% disintegrated and 67% disintegrated is approximately
 (a) 33/67 (b) 29/80 (c) 67/33 (d) 80/29
- 46** A and B are two radioactive substances whose half-lives are 1 yr and 2 yr, respectively. Initially, 10 g of A and 1 g of B is taken. The time (approximate) after which they will have same quantity remaining is
 (a) 6.64 yr (b) 5 yr (c) 3.2 yr (d) 7 yr
- 47** The decay constant of a radioactive element is defined as the reciprocal of the time interval after which the number of atoms of the radioactive element falls to nearly
 (a) 50% of its original number
 (b) 36.8% of its original number
 (c) 63.2% of its original number
 (d) 75% of its original number
- 48** A radioactive material has mean lives of 1620 yr and 520 yr for α and β -emission, respectively. The material decays by simultaneous α and β -emission. The time in which $(1/4)$ th of the material remains intact is
 (a) 4675 yr (b) 720 yr (c) 545 yr (d) 324 yr
- 49** A radioactive nucleus is being produced at a constant rate α per second and its decay constant is λ . If N_0 are the number of nuclei at time $t = 0$, then maximum number of nuclei possible are
 (a) $\frac{\alpha}{\lambda}$ (b) $N_0 + \frac{\alpha}{\lambda}$ (c) N_0 (d) $\frac{\lambda}{\alpha} + N_0$
- 50** After three average life times of a radioactive sample, the amount of substance remains $x\%$ of the original amount. Then, approximate value of x is
 (a) 14 (b) 37 (c) 7 (d) 5
- 51** An archaeologist analyses the wood in a prehistoric structure and finds that C^{14} (half-life = 5700yr) to C^{12} ratio is only one-third of that found in the cells buried plants. The age of the wood is about
 (a) 5700 yr (b) 2850 yr (c) 11400 yr (d) 22800 yr
- 52** A radioactive element A of decay constant λ_A decays into another radioactive element B of decay constant λ_B . Initially, the number of active nuclei of A was N_0 and B was absent in the sample. The maximum number of active nuclei of B is found at $t = 2 \ln 2 / \lambda_A$. The maximum number of active nuclei of B is
 (a) $\frac{N_0}{4}$ (b) $\frac{\lambda_A}{\lambda_B} N_0 e^{-\lambda_B t}$
 (c) $\frac{\lambda_A}{\lambda_B} \frac{N_0}{4}$ (d) None of these
- 53** Two identical samples (same material and same amount) P and Q of a radioactive substance having mean life T are observed to have activities R_P and R_Q respectively, at the time of observation. If P is older than Q , then the difference in their age is
 (a) $T \ln \left(\frac{R_P}{R_Q} \right)$ (b) $T \ln \left(\frac{R_Q}{R_P} \right)$
 (c) $T \left(\frac{R_P}{R_Q} \right)$ (d) $T \left(\frac{R_Q}{R_P} \right)$
- 54** The activity of a sample of radioactive material is R_1 at time t_1 and R_2 at time t_2 ($t_2 > t_1$). Its mean life is T , then
 (a) $R_1 t_1 = R_2 t_2$ (b) $\frac{R_1 - R_2}{t_2 - t_1} = \text{constant}$
 (c) $R_2 = R_1 e^{(t_1 - t_2)/T}$ (d) $R_2 = R_1 e^{(t_1/t_2)/T}$
- 55** For a substance, the average life for α -emission is 1620 yr and for β -emission is 405 yr. After how much time the $\frac{1}{4}$ of the material remains after α and β -emission?
 (a) 1500 yr (b) 300 yr
 (c) 449 yr (d) 810 yr
- 56** Two radioactive nuclei P and Q , in a given sample decay into a stable nucleus R . At time $t = 0$, number of P species are $4 N_0$ and that of Q are N_0 . Half-life of P (for conversion to R) is 1 min, where as that of Q is 2 min. Initially, there are no nuclei of R present in the sample. When number of nuclei of P and Q are equal, then the number of nuclei of R present in the sample would be
 (a) $\frac{5N_0}{2}$ (b) $2N_0$ (c) $3N_0$ (d) $\frac{9N_0}{2}$
- 57** The half-life of a radioactive substance is 40 yr. How long will it take to reduce to one-fourth of its original amount and what is the value of decay constant?
 (a) 40 yr, 0.9173/yr (b) 90 yr, 9.017/yr
 (c) 80 yr, 0.0173/yr (d) None of these

- 58** Starting with a sample of pure ^{66}Cu , $\frac{7}{8}$ of it decays into Zn in 15 min. The corresponding half-life is
 (a) 5 min (b) $7\frac{1}{2}$ min (c) 10 min (d) 15 min
- 59** A stationary radioactive nucleus of mass 210 units disintegrates into an alpha particle of mass 4 units and residual nucleus of mass 206 units. If the kinetic energy of the alpha particle is E , then the kinetic energy of the residual nucleus is
 (a) $\left(\frac{2}{105}\right)E$ (b) $\left(\frac{2}{103}\right)E$ (c) $\left(\frac{103}{105}\right)E$ (d) $\left(\frac{103}{2}\right)E$
- 60** A bone containing 200 g carbon-14 has a β -decay rate of 375 decay/min. Calculate the time that has elapsed, since the death of the living one. Given the rate of decay for the living organism = 15 decay per min per gram of carbon and half-life of carbon-14 = 5730 yr.
 (a) 22920 yr (b) 11460 yr
 (c) 17190 yr (d) None of these
- 61** Assuming that about 200 MeV energy is released per fission of $_{92}\text{U}^{235}$ nuclei. What would be the mass of U^{235} consumed per day in the fission of reactor of power 1 MW approximately?
 (a) 10 kg (b) 1 kg (c) 1 g (d) 10 g
- 62** The activity of a radioactive sample is measured as N_0 counts per minute at $t = 0$ and $\frac{N_0}{e}$ counts per minute at $t = 5$ min. The time (in minutes) at which the activity reduces to half its value is
 (a) $5 \log_e 2$ (b) $\log_e \frac{2}{5}$ (c) $\frac{5}{\log_e 2}$ (d) $5 \log_{10} 2$
- 63** In the nuclear fusion reaction $^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + ^1_0\text{n}$, given that the repulsive potential energy between the two nuclei is -7.7×10^{-14} J. The temperature at which the gases must be heated to initiate the reaction is of the order of [Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K]
 (a) 10^9K (b) 10^7K (c) 10^5K (d) 10^3K
- 64** The half-life of radioactive radon is 3.8 days. The time at the end of which $\frac{1}{20}$ th of the radon sample will remain undecayed is (given, $\log_{10} e = 0.4343$)
 (a) 3.8 days (b) 16.5 days
 (c) 33 days (d) 76 days
- 65** The half-life of radioactive polonium (Po) is 138.6 days. For ten lakh polonium atoms, the number of disintegrations in 24 h is
 (a) 2000 (b) 3000
 (c) 4000 (d) 5000
- 66** If half-life of a substance is 3.8 days and its quantity is 10.38 g. Then, substance quantity remaining left after 19 days will be
 (a) 0.151 g (b) 0.32 g
 (c) 1.51 g (d) 0.16 g
- 67** Two radioactive substance A and B have decay constants 5λ and λ , respectively. At $t = 0$, they have the same number of nuclei. The ratio of number of nuclei of A to those of B will be $\left(\frac{1}{e}\right)^2$ after a time interval of
 (a) $\frac{1}{4\lambda}$ (b) 4λ (c) 2λ (d) $\frac{1}{2\lambda}$
- 68** A star initially has 10^{40} deuterons. It produces energy via the processes $^2_1\text{H} + ^2_1\text{H} \rightarrow ^3_1\text{H} + p$ and $^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + n$, where the masses of the nuclei are $m(^2_1\text{H}) = 2.014$ amu, $m(p) = 1.007$ amu, $m(n) = 1.008$ amu and $m(^4_2\text{He}) = 4.001$ amu. If the average power radiated by the star is 10^{16} W, then the deuteron supply of the star is exhausted in a time of the order of
 (a) 10^{18} s (b) 10^{28} s (c) 10^{12} s (d) 10^{16} s
- 69** The sun radiates energy in all directions. The average radiation received on the earth surface from the sun 1.4 kW/m^2 . The average earth-sun distance 15×10^{11} m. The mass lost by the sun per day is (1 day = 86400 s)
 (a) 4.4×10^9 kg (b) 7.6×10^{14} kg
 (c) 3.8×10^{12} kg (d) 3.8×10^{14} kg
- 70** If 10% of a radioactive material decays in 5 days, then the amount of original material left after 20 days is approximately
 (a) 60% (b) 65%
 (c) 70% (d) 75%
- 71** A small quantity of solution containing Na^{24} radio nuclide of activity 1 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 h shows an activity of 296 disintegration per minute. What will be the total value of the blood in the body of the person? Assume that, the radioactive solution mixes uniformly in the blood of the person. (Take, 1 curie = 3.7×10^{10} disintegration per second and $e^{-\lambda t} = 0.7927$, where λ = disintegration constant)
 (a) 5.94 L (b) 2 L
 (c) 317 L (d) 1 L

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) These questions consist of two statements each linked as Assertion and Reason. While answering of these questions you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.
- (b) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
- (c) If Assertion is true but Reason is false.
- (d) If Assertion is false but Reason is true.

1 Assertion Density of all the nuclei is same.

Reason Radius of nucleus is directly proportional to the cube root of mass number.

2 Assertion In one half-life of a radioactive substance more number of nuclei are decayed than in one average life.

Reason Average life = Half-life/ $\ln(2)$

3 Assertion Two different radioactive substances have initially same number of nuclei. Their decay constants are λ_1 and λ_2 ($\lambda_2 < \lambda_1$), then initially first radioactive substance decays at faster rate.

Reason Half-life of first radioactive substance is less.

4 Assertion Positive value of packing fraction implies a large value of binding energy.

Reason The ratio of the difference between the actual isotopic mass of nucleus and the mass number of the nucleus to that of mass number is called packing fraction.

5 Assertion If high pressure is applied on a radioactive substance, then rate of radioactivity will increase.

Reason Radioactivity is a nuclear process.

2 Identify the correct statement given below.

- (a) Alpha rays have the highest penetrating power.
- (b) Beta rays have the highest penetrating power.
- (c) Gamma rays have the highest penetrating power.
- (d) All three kinds of rays have the same penetrating power.

3 Which of the following is/are correct statement about binding energy in a nucleus?

- I. It is the energy required to break a nucleus into its constituent nucleons.
 - II. It is the energy made available when free nucleons combine to form a nucleus.
 - III. It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus.
- (a) Only I (b) Only II
(c) Only III (d) I, II and III

4 Tritium is an isotope of hydrogen whose nucleus triton contains 2 neutrons and 1 proton. Free neutrons decay into $p + e^- + \bar{\nu}$. If one of the neutrons in triton decays, then it would transform into He^3 nucleus. This does not happen. Choose the correct statement which give the cause of it.

[NCERT Exemplar]

- (a) Triton energy is less than that of a He^3 nucleus.
- (b) The electron created in the beta decay process cannot remain in the nucleus.
- (c) Both the neutrons in triton have to decay simultaneously resulting in a nucleus with 3 protons, which is not a He^3 nucleus.
- (d) Free neutrons decay due to external perturbations which is absent in triton nucleus.

Match the columns

1 From a radioactive substance, x numbers of α -particles and y numbers of β -particles are emitted. As a result, atomic number decreases by n and mass number by m . Then, match the Column I with Column II and choose the correct option from the codes given below.

Column I	Column II
(A) $x = 1$ and $y = 2$	(p) $n = 2, m = 8$
(B) $x = 2$ and $y = 1$	(q) $n = 0, m = 4$
(C) $x = 2$ and $y = 2$	(r) $n = 2, m = 4$
(D) $x = 1$ and $y = 0$	(s) None

Statement based questions

- 1** I. In any nuclear process energy is released, if total binding energy of daughter nuclei is more than the total binding energy of parent nuclei.
II. Binding energy per nucleon is of the order of MeV. Choose the correct statement from above.
- (a) Only I
 - (b) Only II
 - (c) Both I and II
 - (d) None of the above

Codes

A	B	C	D
(a) r	s	q	p
(b) r	s	p	q
(c) q	s	p	r
(d) q	p	s	r

- 2 Match the statement in Column I to the appropriate statement(s) from Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Nuclear fusion	(p) Converts some matter into energy
(B) Nuclear fission	(q) Generally possible for nuclei with low atomic number
(C) β -decay	(r) Generally possible for nuclei with higher atomic number
(D) Exothermic nuclear reaction	(s) Essentially proceeds by weak nuclear forces

Codes

A	B	C	D	A	B	C	D
(a) p,q	q,r	s	q,r	(b) p,q	p,r	s	p,q,r
(c) r,p	p,r	p	s,q,r	(d) r,p	q,r	p	p,q

- 3 Match the statement in Column I to the appropriate statement(s) from Column II and choose the correct option from the codes given below.

Column I	Column II
(A) Stability of nucleus decided by	(p) Negative
(B) Four radioactive substance spontaneously decays because its	(q) Binding energy per nucleon is minimum
(C) For the stable orbit or bound orbit, total energy	(r) Neutron-proton ratio
(D) Stopping potential	(s) Packing fraction

Codes

A	B	C	D	A	B	C	D
(a) r	q	q	p	(b) r,s	q	p	p
(c) s	q	p	s	(d) s,r	s	p	s

- 4 Match Column I of the nuclear processes with Column II containing parent nucleus and one of the end products of each process. Choose the correct option from the codes given below.

Column I	Column II
(A) Alpha decay	(p) $^{15}_8\text{O} \rightarrow ^{15}_7\text{N} + \dots$
(B) β^+ decay	(q) $^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + \dots$
(C) Fission	(r) $^{185}_{83}\text{Bi} \rightarrow ^{184}_{82}\text{Pb} + \dots$
(D) Proton emission	(s) $^{239}_{94}\text{Pu} \rightarrow ^{140}_{57}\text{La} + \dots$

Codes

A	B	C	D	A	B	C	D
(a) r	q	p	s	(b) q	p	q	r
(c) q	s	r	p	(d) q	p	s	r

(C) Medical entrances' gallery

Collection of questions asked in NEET & various medical entrance exams

- The energy equivalent of 0.5 g of a substance is
 (a) 4.5×10^{13} J (b) 1.5×10^{13} J [NEET 2020]
 (c) 0.5×10^{13} J (d) 4.5×10^{16} J
- When a uranium isotope $^{235}_{92}\text{U}$ is bombarded with a neutron, it generates $^{89}_{36}\text{Kr}$, three neutrons and
 (a) $^{91}_{40}\text{Zr}$ (b) $^{101}_{36}\text{Kr}$ [NEET 2020]
 (c) $^{103}_{36}\text{Kr}$ (d) $^{144}_{56}\text{Ba}$
- What happens to the mass number and atomic number of an element when it emits γ -radiation? [NEET 2020]
 (a) Mass number decreases by four and atomic number decreases by two.
 (b) Mass number and atomic number remain unchanged.
 (c) Mass number remains unchanged, while atomic number decreases by one.
 (d) Mass number increases by four and atomic number increases by two.
- The half-life of a radioactive sample undergoing α -decay is 1.4×10^{17} s. If the number of nuclei in the sample is 2.0×10^{21} , the activity of the sample is nearly [NEET 2020]
 (a) 10^4 Bq (b) 10^5 Bq (c) 10^6 Bq (d) 10^3 Bq
- α -particle consists of [NEET 2019]
 (a) 2 electrons, 2 protons and 2 neutrons
 (b) 2 electrons and 4 protons only
 (c) 2 protons only
 (d) 2 protons and 2 neutrons only
- The rate of radioactive disintegration at an instant, for a radioactive sample of half-life 2.2×10^9 s is 10^{10} s^{-1} . The number of radioactive atoms in that sample at that instant is [NEET (Odisha) 2019]
 (a) 3.17×10^{20} (b) 3.17×10^{17}
 (c) 3.17×10^{18} (d) 3.17×10^{19}

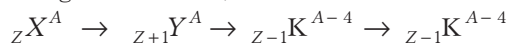
- 7** If half-life of an element is 69.3 h, then how much of its per cent will decay in 10th to 11th hours? (Initial activity = $50 \mu\text{Ci}$) [AIIMS 2019]
 (a) 1% (b) 2% (c) 3% (d) 4%
- 8** Ba-122 has half-life of 2 min. Experiment has to be done using Ba-122 and it takes 10 min to set up the experiment. If initially 80 g of Ba-122 was taken, how much Ba was left when experiment was started? [JIPMER 2019]
 (a) 2.5 g (b) 5 g (c) 10 g (d) 20 g
- 9** For a radioactive material, half-life is 10 min. If initially there are 600 number of nuclei, the time taken for disintegration of 450 nuclei is [NEET 2018]
 (a) 30 min (b) 10 min (c) 20 min (d) 15 min
- 10** The half-life a radioactive substance is 20 min. The approximate time interval ($t_2 - t_1$) between the time t_2 , when $\frac{2}{3}$ of it has decayed and time t_1 , when $\frac{1}{3}$ of it had decayed is [AIIMS 2018]
 (a) 14 min (b) 20 min (c) 28 min (d) 7 min
- 11 Assertion** Radioactive nuclei emits β -particles.
Reason Electron exist inside the nucleus. [AIIMS 2018]
 (a) Both Assertion and Reason are correct and Reason is correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.
- 12** In the fusion reaction, the masses of deuteron, helium and neutron expressed in amu are 2.015, 3.017 and 1.009, respectively. If 1 kg of deuterium undergoes complete fusion, then find the amount of total energy released. (1 amu = 931.5 MeV) [JIPMER 2018]
 (a) $9 \times 10^{13} \text{ J}$ (b) $20 \times 10^5 \text{ J}$ (c) 4×10^{22} (d) 5×10^{15}
- 13** Radioactive material (A) has decay constant 8λ and material (B) has decay constant λ . Initially, they have same number of nuclei. After what time, the ratio of number of nuclei of material B to that A will be I/e ? [NEET 2017]
 (a) $\frac{1}{\lambda}$ (b) $\frac{1}{7\lambda}$ (c) $\frac{1}{8\lambda}$ (d) $\frac{1}{9\lambda}$
- 14 Assertion** In α -decay, atomic number of daughter nucleus reduces by a unit from the parents nucleus.
Reason An α -particle carriers four units of mass. [AIIMS 2017]
 (a) Both Assertion and Reason are correct and Reason is correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Assertion is incorrect but Reason is correct.
- 15** A nuclear explosive is designed to deliver 1MW power in the form of heat energy. If the explosion is designed with nuclear fuel consisting of U^{235} to run a reactor at this power level for one year, then the amount of fuel needed is (Take, energy per fission = 200 MeV) [AIIMS 2017]
 (a) 1 kg (b) 0.01 kg
 (c) 3.84 kg (d) 0.384 kg
- 16** Radioactive decay will occur as follows
 ${}_{86}\text{Rn}^{220} \longrightarrow {}_{84}\text{Po}^{216} + {}_2\text{He}^4$, half-life = 55 s
 ${}_{84}\text{Po}^{216} \longrightarrow {}_{82}\text{Pb}^{212} + {}_2\text{He}^4$, half-life = 0.66 s
 ${}_{82}\text{Pb}^{212} \longrightarrow {}_{82}\text{B}^{212} + \lambda^0 \text{e}$, half-life = 10.6 h
 If a certain mass of radon (Rn = 220) is allowed to decay, then after 5 min the element with greater mass will be [JIPMER 2017]
 (a) radon (b) lead
 (c) polonium (d) bismuth
- 17** The half-life of a radioactive substance is 30 min. The time (in min) taken between 40% decay and 85% decay of the same radioactive substance is [NEET 2016]
 (a) 15 (b) 30 (c) 45 (d) 60
- 18** If radius of the ${}_{13}^{27}\text{Al}$ nucleus is taken to be R_{Al} , then the radius of ${}_{53}^{125}\text{Te}$ nucleus is nearly [CBSE AIPMT 2015]
 (a) $\left(\frac{53}{13}\right)^{1/3} R_{\text{Al}}$ (b) $\frac{5}{3} R_{\text{Al}}$ (c) $\frac{3}{5} R_{\text{Al}}$ (d) $\left(\frac{13}{53}\right)^{1/3} R_{\text{Al}}$
- 19** Two spherical nuclei have mass numbers 216 and 64 with their radii R_1 and R_2 , respectively. The ratio, $\frac{R_1}{R_2}$ is equal to [AIIMS 2015]
 (a) 3 : 2 (b) 1 : 3
 (c) 1 : 2 (d) 2 : 3
- 20** The kinetic energy of α -particle emitted in the α -decay of ${}_{88}\text{Ra}^{226}$ is [Take, $m(\text{Ra}) = 226.02540\text{u}$, $m(\text{Rn}) = 222.01750\text{u}$ and $m(\text{He}) = 4.002603\text{u}$] [UP CPMT 2015]
 (a) 5.201 MeV (b) 3.301 MeV
 (c) 6.023 MeV (d) 4.871 MeV
- 21** In a nuclear reactor, the number of U^{235} nuclei undergoing fissions per second is 4×10^{20} . If the energy released per fission is 250 MeV, then the total energy released in 10 h is (1 eV = $1.6 \times 10^{-19} \text{ J}$) [EAMCET 2015]
 (a) $576 \times 10^6 \text{ J}$ (b) $576 \times 10^{12} \text{ J}$
 (c) $576 \times 10^{15} \text{ J}$ (d) $576 \times 10^{18} \text{ J}$

- 22** Nuclear fusion is not found in [Kerala CEE 2015]
 (a) thermonuclear reactor (b) hydrogen bomb
 (c) energy production in sun (d) atom bomb
 (e) energy production in stars
- 23** The approximate ratio of nuclear mass densities of $^{197}_{79}\text{Au}$ and $^{107}_{47}\text{Ag}$ nuclei is [Kerala CEE 2015]
 (a) 197 : 107 (b) 47 : 79
 (c) 79 : 47 (d) 1 : 1
 (e) 107 : 197
- 24** A radioactive element X disintegrates successively

$$X \xrightarrow{\beta^-} X_1 \xrightarrow{\alpha} X_2 \xrightarrow{\beta^-} X_3 \xrightarrow{\alpha} X_4$$

 If atomic number and atomic mass number of X are respectively, 72 and 180, then what are the corresponding values for X_4 ? [Guj. CET 2015]
 (a) 69, 172 (b) 69, 176
 (c) 71, 176 (d) 70, 172
- 25** The energy released by the fission of one uranium atom is 200 MeV. The number of fission per second required to produce 6.4W power is [Guj. CET 2015]
 (a) 2×10^{11} (b) 10^{11}
 (c) 10^{10} (d) 2×10^{10}
- 26** By the successive disintegration of $^{238}_{92}\text{U}$, the final product obtained is $^{206}_{82}\text{Pb}$, then how many number of α and β -particles are emitted? [Guj. CET 2015]
 (a) 6 and 8 (b) 8 and 6
 (c) 12 and 6 (d) 8 and 12
- 27** If 20% of a radioactive element decays in 10 days, then amount of original material left after 30 days is (approx) [UP CPMT 2015]
 (a) 78 % (b) 62 %
 (c) 51 % (d) 48 %
- 28** A nucleus at rest splits into two nuclear parts having radii in the ratio 1 : 2. Their velocities are in the ratio [KCET 2015]
 (a) 4 : 1 (b) 8 : 1 (c) 2 : 1 (d) 6 : 1
- 29** What is the energy released by fission of 1 g of U^{235} ? (Assume 200 MeV energy is liberated on fission of 1 nucleus) [CG PMT 2015]
 (a) 2.26×10^4 kWh (b) 1.2×10^4 kWh
 (c) 4.5×10^2 kWh (d) 2.5×10^2 kWh
- 30** The half-life of radium is 1620 yr and its atomic weight is 226 kg per kilo mol. The number of atoms that will decay from its 1g sample per second will be (Take, Avogadro's number, $N_A = 6.023 \times 10^{23}$ atom/mol) [Manipal 2015]
 (a) 3.61×10^{10} (b) 311×10^{15}
 (c) 3.6×10^{12} (d) 311×10^{15}
- 31** The relationship between decay constant λ and half-life T of a radioactive substance is [UP PMT 2015]
 (a) $\lambda = \frac{\log_{10} 2}{T}$ (b) $\lambda = \frac{\log_e 2}{T}$
 (c) $\lambda = \frac{\log_2 10}{T}$ (d) $\lambda = \frac{\log_2 e}{T}$
- 32** The binding energy per nucleon of ^7_3Li and ^4_2He nuclei are 5.60 MeV and 7.06 MeV, respectively. In the nuclear reaction $^7_3\text{Li} + ^1_1\text{H} \rightarrow ^4_2\text{He} + ^4_2\text{He} + Q$, the value of energy Q released is [CBSE AIPMT 2014]
 (a) 19.6 MeV (b) -2.4 MeV
 (c) 8.4 MeV (d) 17.3 MeV
- 33** A radio isotope X with a half-life 1.4×10^9 yr decays to Y , which is stable. A sample of the rock from a cave was found to contain X and Y in the ratio 1 : 7. The age of the rock is [CBSE AIPMT 2014]
 (a) 1.96×10^9 yr (b) 3.92×10^9 yr
 (c) 4.20×10^9 yr (d) 8.40×10^9 yr
- 34** For the stability of any nucleus, [KCET 2014]
 (a) binding energy per nucleon will be more
 (b) binding energy per nucleon will be less
 (c) number of electrons will be more
 (d) None of the above
- 35** A uranium nucleus $^{238}_{92}\text{U}$ emits an α -particle and a β -particle in succession. The atomic number and mass number of the final nucleus will be [Kerala CEE 2014]
 (a) 90, 233 (b) 90, 238
 (c) 91, 238 (d) 91, 234
 (e) 93, 238
- 36** After 300 days, the activity of a radioactive sample is 5000 dps (disintegration per second). The activity becomes 2500 dps after another 150 days. The initial activity of the sample (in dps) is [Kerala CEE 2014]
 (a) 20000 (b) 10000
 (c) 7000 (d) 25000
 (e) 15000
- 37** For the radioactive nuclei that undergo either α or β decay, which one of the following cannot occur? [WB JEE 2014]
 (a) Isobar of original nucleus is produced
 (b) Isotope of the original nucleus is produced
 (c) Nuclei with higher atomic number than that of the original nucleus is produced
 (d) Nuclei with lower atomic number than that of the original nucleus is produced

- 38 In a given reaction,



The radioactive radiations are emitted in the sequence of

[UK PMT 2014]

- (a) α, β, γ (b) γ, α, β (c) β, α, γ (d) γ, β, α
- 39 The control rods used in a nuclear reactor can be made up of
- [Kerala CEE 2014]
- (a) graphite (b) cadmium
(c) uranium (d) barium
(e) lead
- 40 The fusion reaction in the sun is a multi-step process in which the
- [Kerala CEE 2014]
- (a) helium is burned into deuterons
(b) helium is burned into hydrogen
(c) deuteron is burned into hydrogen
(d) hydrogen is burned into helium
(e) helium is burned into neutrons
- 41 The nuclear fusion reaction between deuterium and tritium takes place
- [EAMCET 2014]
- (a) at low temperature and low pressure
(b) at very high temperature and very high pressure
(c) when the temperature is near absolute zero
(d) at ordinary temperature and pressure
- 42 The half-life of a radioactive isotope X is 20 yr. It decays to another element Y which is stable. The two elements X and Y were found to be in the ratio 1: 7 in a sample of a given rock. The age of the rock is estimated to be
- [NEET 2013]
- (a) 40 yr (b) 60 yr
(c) 80 yr (d) 100 yr
- 43 A certain mass of hydrogen is changed to helium by the process of fusion. The mass defect in fusion reaction is 0.02866 u. The energy liberated per fusion is (Take, $1 \text{ u} = 931 \text{ MeV}$)
- [NEET 2013, EAMCET 2013]
- (a) 2.67 MeV (b) 26.7 MeV
(c) 6.675 MeV (d) 13.35 MeV
- 44 A U^{235} reactor generates power at a rate of P producing 2×10^{18} fission per second. The energy released per fission is 185 MeV. The value of P is
- [EAMCET 2013]
- (a) 59.2 MW (b) 370×10^{18} MW
(c) 0.59 MW (d) 370 MW
- 45 The purpose of using heavy water in nuclear reactor is
- [EAMCET 2013]
- (a) to increase the energy released in nuclear fission
(b) to cool the reactor to room temperature
(c) to make the dynamo blades to work well
(d) to decrease the energy of fast neutrons to the thermal energy

- 46 The ratio of volumes of nuclei (assumed to be in spherical shape) with respective mass numbers 8 and 64 is
- [Kerala CET 2013]

(a) 0.5 (b) 2 (c) 0.125 (d) 0.25

- 47 Radioactivity of a sample at T_1 time is R_1 and at time T_2 is R_2 . If half-life of sample is T , then in time $(T_2 - T_1)$, the number of decayed atoms is proportional to
- [MP PMT 2013]

(a) $R_1 T_2 - R_2 T_1$ (b) $(R_1 - R_2) T$
(c) $\frac{(R_1 - R_2)}{T}$ (d) $R_1 - R_2$

- 48 Which pair is isotonic?
- [MP PMT 2013]

(a) ${}_7\text{N}^{13}, {}_7\text{N}^{14}$ (b) ${}_6\text{C}^{14}, {}_6\text{C}^{12}$
(c) ${}_6\text{C}^{14}, {}_7\text{N}^{14}$ (d) ${}_7\text{N}^{13}, {}_6\text{C}^{12}$

- 49 The phenomenon of radioactivity
- [UP CPMT 2013]

(a) increases on applied pressure
(b) is exothermic change which increases or decreases with temperature
(c) is nuclear process which does not depend on external forces
(d) None of the above

- 50 An atom bomb weighing 1 kg explodes releasing 9×10^{13} J of energy. What percentage of mass is converted into energy?
- [UP CPMT 2013]

(a) 0.1% (b) 1% (c) 2% (d) 10%

- 51 Pick out the correct statements from the following.

I. Electron emission during β -decay is always accompanied by neutrons.
II. Nuclear force is charge independent.
III. Fusion is the chief source of stellar energy.

[Karnataka CET 2013]

(a) Both I and II (b) Both I and III
(c) Only I (d) Both II and III

- 52 A nucleus ${}_Z X^A$ emits an α -particle with velocity v .

The recoil speed of the daughter nucleus is

[Karnataka CET 2013]

(a) $\frac{A-4}{4v}$ (b) $\frac{4v}{A-4}$ (c) v (d) $\frac{v}{4}$

- 53 A radioactive substance emits 100 beta particles in the first 2 s and 50 beta particles in the next 2s. The mean life of the sample is
- [Karnataka CET 2013]

(a) 4 s (b) 2 s
(c) $\frac{2}{0.693}$ s (d) 2×0.693 s

- 54 When alpha particle captures an electron, then it becomes a
- [Kerala CET 2013]

(a) helium atom (b) hydrogen atom
(c) helium ion (d) hydrogen ion
(e) $\alpha\beta$ -particle

- 55** A mixture consists of two radioactive materials A_1 and A_2 with half-lives of 20 s and 10 s, respectively. Initially the mixture has 40 g of A_1 and 160 g of A_2 . The amount of the two in the mixture will become equal after [CBSE AIPMT 2012]
 (a) 60 s (b) 80 s (c) 20 s (d) 40 s
- 56** If $t_{1/2}$ is the half-life of a substance, then $t_{3/4}$ is the time in which [CBSE AIPMT 2012]
 (a) substance decays $1/2$
 (b) substance decays $(3/4)$ th
 (c) substance remains $1/2$
 (d) substance remains $(3/4)$ th
- 57** The half-life of radioactive element is 600 yr. The fraction of sample that would remain after 3000 yr is [AIIMS 2012, Manipal 2012]
 (a) $1/2$ (b) $1/16$ (c) $1/8$ (d) $1/32$
- 58** If a nucleus ${}_Z X^A$ emits 9 α and 5 β -particles, then the ratio of total protons and neutrons in the final nucleus is [BCECE Mains 2012]
 (a) $\frac{(Z-13)}{(A-36)}$ (b) $\frac{(Z-13)}{(A-Z-13)}$
 (c) $\frac{(Z-18)}{(A-36)}$ (d) $\frac{(Z-13)}{(A-Z-23)}$
- 59** If the half-life of a material is 10 yr, then in what time, it becomes $\frac{1}{4}$ th part of the initial amount? [BCECE Mains 2012]
 (a) 15 yr (b) 20 yr (c) 25 yr (d) 30 yr
- 60** What is the Q -value of the reaction?
 ${}^1\text{H} + {}^7\text{Li} \longrightarrow {}^4\text{He} + {}^4\text{He}$
 The atomic masses of ${}^1\text{H}$, ${}^4\text{He}$ and ${}^7\text{Li}$ are 1.0078254u, 4.0026034u and 7.016004u, respectively. [Manipal 2012]
 (a) 17.35 MeV (b) 18.06 MeV
 (c) 177.35 MeV (d) 170.35 MeV
- 61** A nucleus of mass number 220, initially at rest, emits an α -particle. If the Q -value of the reaction is 5.5 MeV, then the energy of the emitted α -particle will be [AMU 2012]
 (a) 4.8 MeV (b) 5.4 MeV
 (c) 5.5 MeV (d) 6.8 MeV
- 62** A radioactive substance emits n beta particles in the first 2 s and $0.5n$ beta particles in the next 2 s. The mean life of the sample is [AMU 2012]
 (a) 4 s (b) 2 s
 (c) $\frac{2}{\ln 2}$ s (d) $2(\ln 2)$ s
- 63** A radioactive nucleus of mass M emits a photon of frequency ν and the nucleus recoils. The recoil energy will be [CBSE AIPMT 2011]
 (a) $h^2\nu^2/2Mc^2$ (b) zero
 (c) $h\nu$ (d) $Mc^2 - h\nu$
- 64** A luminous body radiates energy at a rate 3.6×10^{28} J/s, then loss of the mass of the body per second is [OJEE 2011]
 (a) 2×10^3 kg/s (b) 3×10^4 kg/s
 (c) 4×10^{11} kg/s (d) 5×10^5 kg/s
- 65** The disintegration rate of a certain radioactive sample at any instant is 5400 disintegrations per minute and 5 min later, the rate becomes 600 disintegrations per minute. Half-life of sample is [EAMCET 2011]
 (a) $\frac{\ln 3}{\ln 2}$ (b) $\frac{\ln 3}{\ln 5}$
 (c) $\frac{\ln 32}{\ln 9}$ (d) $\frac{\ln 9}{\ln 32}$

ANSWERS

CHECK POINT 13.1

1. (d)	2. (c)	3. (a)	4. (b)	5. (c)	6. (b)	7. (c)	8. (d)	9. (a)	10. (a)
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CHECK POINT 13.2

1. (b)	2. (b)	3. (d)	4. (a)	5. (a)	6. (c)	7. (b)	8. (c)	9. (a)	10. (b)
11. (c)	12. (d)	13. (d)	14. (b)						

CHECK POINT 13.3

1. (c)	2. (c)	3. (a)	4. (d)	5. (c)	6. (a)	7. (d)	8. (a)	9. (b)	10. (b)
11. (c)	12. (b)	13. (b)	14. (a)	15. (d)					

(A) Taking it together

1. (a)	2. (a)	3. (b)	4. (b)	5. (b)	6. (c)	7. (d)	8. (b)	9. (b)	10. (c)
11. (a)	12. (a)	13. (a)	14. (c)	15. (d)	16. (b)	17. (a)	18. (d)	19. (a)	20. (c)
21. (b)	22. (a)	23. (b)	24. (d)	25. (b)	26. (b)	27. (b)	28. (d)	29. (d)	30. (d)
31. (d)	32. (c)	33. (d)	34. (c)	35. (d)	36. (c)	37. (c)	38. (c)	39. (b)	40. (b)
41. (a)	42. (a)	43. (c)	44. (b)	45. (b)	46. (a)	47. (b)	48. (c)	49. (a)	50. (d)
51. (c)	52. (c)	53. (b)	54. (c)	55. (c)	56. (d)	57. (c)	58. (a)	59. (b)	60. (c)
61. (c)	62. (a)	63. (a)	64. (b)	65. (d)	66. (b)	67. (d)	68. (c)	69. (d)	70. (b)
71. (a)									

(B) Medical entrance special format questions

• Assertion and reason

1. (a)	2. (d)	3. (a)	4. (d)	5. (d)
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• Statement based questions

1. (c)	2. (c)	3. (d)	4. (a)
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• Match the columns

1. (c)	2. (b)	3. (b)	4. (d)
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(C) Medical entrances' gallery

1. (a)	2. (d)	3. (b)	4. (a)	5. (d)	6. (d)	7. (a)	8. (a)	9. (c)	10. (b)
11. (c)	12. (a)	13. (b)	14. (d)	15. (d)	16. (b)	17. (d)	18. (b)	19. (a)	20. (d)
21. (b)	22. (d)	23. (d)	24. (d)	25. (a)	26. (b)	27. (d)	28. (b)	29. (a)	30. (a)
31. (b)	32. (d)	33. (c)	34. (a)	35. (d)	36. (a)	37. (b)	38. (c)	39. (b)	40. (d)
41. (b)	42. (b)	43. (c)	44. (a)	45. (d)	46. (c)	47. (b)	48. (d)	49. (c)	50. (a)
51. (d)	52. (b)	53. (c)	54. (c)	55. (d)	56. (b)	57. (d)	58. (d)	59. (b)	60. (a)
61. (b)	62. (c)	63. (a)	64. (c)	65. (c)					

Hints & Explanations

• CHECK POINT 13.1

- 1 (d) The mass number of a nucleus is more than its atomic number, but in hydrogen, atomic number and mass number are equal.
- 4 (b) For stability, in case of lighter nuclei $\frac{N}{Z} = 1$ and for heavier nuclei $\frac{N}{Z} > 1$.

5 (c) Helium nucleus = ${}^4_2\text{He}$

Number of protons = $Z = 2$

Number of neutrons = $A - Z = 4 - 2 = 2$

6 (b) ${}_Z X^A = {}_{88}\text{Ra}^{226}$

Number of protons, $Z = 88$

Number of neutrons, $N = A - Z = 226 - 88 = 138$

7 (c) For ${}_6\text{C}^{12}$

Proton = 6, electron = 6 and neutron = 6

For ${}_6\text{C}^{14}$

Proton = 6, electron = 6 and neutron = 8

9 (a) Radius of nucleus, $R = R_0 A^{1/3}$.

where, R_0 is a constant and A is the mass number.

$$R_{\text{Cu}} = (1.2 \times 10^{-15})(64)^{1/3} = (1.2 \times 10^{-15}) \times (4^3)^{1/3} \\ = 4.8 \times 10^{-15} \text{ m} = 4.8 \text{ fm}$$

10 (a) The masses of three isotopes are 19.99 u, 20.99 u and 21.99 u.

Their relative abundances are 90.51%, 0.27% and 9.22%.

$\therefore$ Average atomic mass of neon,

$$m = \frac{90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 21.99}{(90.51 + 0.27 + 9.22)} \\ = \frac{2017.7}{100} = 20.17 \text{ u}$$

• CHECK POINT 13.2

1 (b) Mass defect, $\Delta m = 1 - 0.993 = 0.007 \text{ g}$

$\therefore$ Energy released, $E = (\Delta m)c^2$

$$= (0.007 \times 10^{-3}) \times (3 \times 10^8)^2 = 63 \times 10^{10} \text{ J}$$

2 (b) Mass of electron = Mass of positron = $9.1 \times 10^{-31} \text{ kg}$

Energy released, $E = (2m) \cdot c^2$

$$= 2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 = 1.6 \times 10^{-13} \text{ J}$$

3 (d) $E = \Delta mc^2$; $\Delta m = 0.3 \text{ g} = 0.3 \times 10^{-3} \text{ kg}$ (given)

$$\Rightarrow \text{Amount of energy } E = \frac{0.3}{1000} \times (3 \times 10^8)^2 = 2.7 \times 10^{13} \text{ J}$$

$$= \frac{2.7 \times 10^{13}}{3.6 \times 10^6} = 7.5 \times 10^6 \text{ kWh}$$

$$(\because 1 \text{ kWh} = 3.6 \times 10^6 \text{ J})$$

4 (a) The mass of nucleus formed is always less than the sum of the masses of the constituent proton and neutrons.

$$\text{i.e.} \quad m < (A - Z)m_n + Zm_p$$

5 (a) $\text{BE} = \Delta mc^2 = \Delta m \times 931 \text{ MeV}$

$$\Rightarrow \text{BE} = [Zm_p + (A - Z)m_n - m_N] \times 931 \\ = [2(1.0087 + 1.0073) - 4.0015] \times 931 \\ = 28.4 \text{ MeV}$$

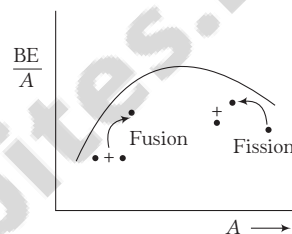
6 (c) Energy released = Products energy - Reactants energy

$$= 8.5 \times 234 - 7.6 \times 236$$

$$= 195.4 \text{ MeV}$$

$$\approx 200 \text{ MeV}$$

7 (b)

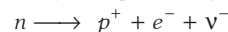


9 (a) In fusion reaction, two lighter nuclei combine to form a heavy nucleus.

• CHECK POINT 13.3

2 (c) γ -rays, ionise gases but their ionisation power is very small compared to that of α and β -particles.

5 (c) Negative β -decay is expressed by the equation,



6 (a) ${}_{92}\text{X}^{235} \xrightarrow{\alpha} {}_{90}\text{Z}^{231} \xrightarrow{\beta^-} {}_{91}\text{Y}^{231}$

Therefore, one alpha and one electron are emitted.

7 (d) Half-life of an element does not depend upon present amount of element, temperature and pressure. It depends upon the nature of the element.

8 (a) We know, after n half-life, undecayed nuclei left are

$$N = N_0 \left(\frac{1}{2} \right)^n; t = n t_{1/2}$$

$$\text{We have, } \frac{N_0}{32} = N_0 \left(\frac{1}{2} \right)^{60/t_{1/2}}$$

$$\Rightarrow 5 = \frac{60}{t_{1/2}}$$

$$\Rightarrow t_{1/2} = 12 \text{ days}$$

$\therefore$ Half-life of a radioactive element, $t_{1/2} = 12 \text{ days}$

9 (b) As, in 2 s, the substance is reduced to half of this previous value.

$\therefore$ The half-life of radioactive substance, $t_{1/2} = 2 \text{ s}$

- 10 (b) We have, $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow 2 = e^{\lambda T_{1/2}}$$

By taking log both sides, we get

$$\ln 2 = \lambda T_{1/2}$$

$$\lambda T_{1/2} = 0.693$$

- 11 (c) We have, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/140}$ $\left[\because N = N_0 \left(\frac{1}{2}\right)^n \text{ and } t = nt_{1/2} \right]$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{t/140} \Rightarrow \frac{t}{140} = 4$$

$$\Rightarrow t = 560 \text{ days}$$

- 12 (b) We have, $N = N_0 \left(\frac{1}{2}\right)^{t/T} \Rightarrow \frac{N_0}{4} = N_0 \left(\frac{1}{2}\right)^{16/T} \Rightarrow T = 8 \text{ days}$

- 13 (b) Here, $N = N_0 \times \left(\frac{1}{2}\right)^{11400/5700}$
 $(\because T_{1/2} = 5700 \text{ yr and } T = 11400 \text{ yr})$
 $= N_0 \left(\frac{1}{2}\right)^2 = 0.25 N_0$

- 14 (a) Mean life = $\frac{\text{Half-life}}{0.6931} = \frac{10}{0.6931} = 14.4 \text{ h}$

- 15 (d) Mean life, $\tau = 1/\lambda = 100 \text{ s}$

$$\text{Half-life} = \frac{0.693}{\lambda} = \frac{0.693 \times 100}{60} = 1.155 \text{ min}$$

(A) Taking it together

- 1 (a) When an energetic γ -ray photon falls on a heavy substance, it is absorbed by some nucleus of the substance and an electron and positron are emitted. This phenomenon is called pair production.
- 4 (b) β -particle carries one unit of negative charge, an α -particle carries 2 units of positive charge and γ -particle carries no charge, therefore electronic energy levels of the atom change for α and β -radioactivity but not for γ -radioactivity.
- 5 (b) Heavy stable nuclei have more neutrons than protons. This is because electrostatic force between protons is repulsive which may reduce stability.
- 6 (c) The average life of a radioactive element is 1.44 times, the half life of the element. So, in one average life more than half the active nuclei decay.
- 10 (c) Radioactivity is a process due to which a radioactive material spontaneously decays. In half-life ($t = 1 \text{ yr}$) of the material, on the average half the number of atoms will decay. Therefore, the containers will in general have different number of atoms of the material but their average will be approx 5000.
- 12 (a) To sustain a chain reaction in atom bomb one of the neutrons emitted in the fission of ^{235}U must be captured by another ^{235}U nucleus and causes its fission.
- 14 (c) Mean life, $\tau = \frac{1}{\lambda} = \frac{1}{15 \times 10^{-9}} = 6.67 \times 10^8 \text{ s}$

- 15 (d) $Q = (1.002 + 1.004 - 1.001 - 1.003) (931.5) \text{ MeV}$
 $= 1.863 \text{ MeV}$

- 16 (b) The elements high on the BE *versus* mass number plot are very tightly bound and hence are stable. And the elements those are lower on this plot, are less tightly bound and hence are unstable.
 Since, helium nucleus shown a peak on this plot, so it is very stable.

- 17 (a) ${}_{92}\text{U}^{238} \xrightarrow{\alpha} {}_{90}\text{Th}^{234} \xrightarrow{\beta} {}_{91}\text{Pa}^{234} \xrightarrow{\beta} {}_{92}\text{U}^{234}$

So, $A = 234, B = 90, C = 234, D = 91$ and $E = \beta$

- 18 (d) ${}_{7}\text{X}^{15} + \alpha\text{-particle} \longrightarrow {}_{9}\text{Y}^{19} \longrightarrow {}_{8}\text{Z}^{18}$

- 19 (a) Three atoms of B $\longrightarrow$ one atom of A

$$\therefore Q = E_a - 3E_b$$

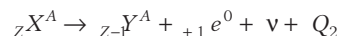
- 20 (c) ${}_{92}\text{U}^{238} \longrightarrow {}_{82}\text{Pb}^{214} + 6 {}_2^4\text{He} + 2 {}_0^1\text{e}$

The total number of α and β -particles emitted in the nuclear reaction is 8.

- 21 (b) $100 \xrightarrow{t} 90\% \xrightarrow{t} 81\%$

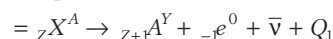
81% is undecayed. Therefore, 19% is decayed.

- 22 (a) Let the nucleus is ${}_Z\text{X}^A \cdot \beta^+$ decay is represented as



$$\begin{aligned} \therefore Q_2 &= [m_n({}_Z\text{X}^A) - m_n({}_{Z-1}\text{Y}^A) - m_e] c^2 \\ &= [m_n({}_Z\text{X}^A) + Zm_e - m_n({}_{Z-1}\text{Y}^A) \\ &\quad - (Z-1)m_e - 2m_e] c^2 \\ &= [m({}_Z\text{X}^A) - m({}_{Z-1}\text{Y}^A) - 2m_e] c^2 = (M_x - M_y - 2m_e) c^2 \end{aligned}$$

β^- decay is represented as



$$\begin{aligned} Q_1 &= [m_n({}_Z\text{X}^A) - m_n({}_{Z+1}\text{Y}^A) - m_e] c^2 \\ &= [m_n({}_Z\text{X}^A) + Zm_e - m_n({}_{Z+1}\text{Y}^A) - (Z+1)m_e] c^2 \\ &= [m({}_Z\text{X}^A) - m({}_{Z+1}\text{Y}^A) - m_e] c^2 = (M_x - M_y) c^2 \end{aligned}$$

- 23 (b) Here, half-life of radioactive substance, $T_{1/2} = 20 \text{ min}$

$$\text{We have, } N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$\text{So, for 20\% decay, } \frac{N}{N_0} = \frac{80}{100} = \left(\frac{1}{2}\right)^{t_1/20} \quad \dots(i)$$

$$\text{For 80\% decay, } \frac{N}{N_0} = \frac{20}{100} = \left(\frac{1}{2}\right)^{t_2/20} \quad \dots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\left(\frac{1}{2}\right)^{\left(\frac{t_2 - t_1}{20}\right)} = \frac{1}{4} \quad \text{or} \quad \left(\frac{1}{2}\right)^{\left(\frac{t_2 - t_1}{20}\right)} = \left(\frac{1}{2}\right)^2$$

On comparing the exponents,

$$\frac{t_2 - t_1}{20} = 2 \Rightarrow t_2 - t_1 = 40 \text{ min}$$

$\therefore$ The time between 20% and 80% decay $t_2 - t_1 = 40 \text{ min}$

24 (d) After n number of half-life, $N = N_0 \left(\frac{1}{2}\right)^n$

$$1 = 64 \left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^n$$

$$n = 6, \text{ i.e. 6 half-lives}$$

$$\therefore \text{ Minimum time, } t = 6 \times 2 = 12 \text{ h}$$

26 (b) We have, $\frac{1}{64} = \left(\frac{1}{2}\right)^n$

$$n = 6$$

6 half-lives are equal to 60 s.

$\therefore$ 1 half-life = 10 s

27 (b) During fusion, BE of daughter nucleus is always greater than the total energy of the parent nuclei, so energy released

$$= c - (a + b) = c - a - b$$

28 (d) Rate, $R = \lambda N$

$$\therefore \frac{R}{N} = \lambda \text{ (constant)}$$

So, graph between R/N and t be a straight line parallel to the time axis.

29 (d) We have, $N = N_0 e^{-\lambda t}$ and $R = R_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t}$ ($\because R_0 = \lambda N_0$)

$$\therefore N_{\text{decayed}} = N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0 \left(1 - e^{-\lambda t}\right)$$

This is the equation of straight line with negative slope.

30 (d) By using, $N = N_0 e^{-\lambda t}$

It shows that N decreases exponentially with time.

31 (d) We have, $\frac{1}{e} = \frac{N_0 e^{-10\lambda t}}{N_0 e^{-\lambda t}} = e^{-9\lambda t}$ or $t = \frac{1}{9\lambda} \left(\because \text{given, } \frac{N_1}{N_2} = \frac{1}{e}\right)$

32 (c) We have, $N_A e^{-\lambda_A t} = N_B e^{-\lambda_B t}$

$$\therefore e^{(\lambda_B - \lambda_A)t} = \frac{N_B}{N_A}$$

$$\therefore \text{ Time, } t = \frac{1}{\lambda_B - \lambda_A} \ln \left(\frac{N_B}{N_A}\right)$$

33 (d) Binding energy per nucleon

$$= \frac{[m_p Z + m_n (A - Z)] - M}{\text{Mass number (A)}} \times 931.4$$

$$= \left(\frac{6 \times 1.0078 + 6 \times 1.0087 - 12}{12}\right) \times 931.4$$

$$= 7.68 \text{ MeV}$$

34 (c) If in the rock there is no Y element, then the time taken by element X to reduce to $\left(\frac{1}{8}\right)$ th the initial value will be equal to

$$\frac{1}{8} = \left(\frac{1}{2}\right)^n \text{ or } n = 3$$

Therefore, from the beginning three half-life time is spent.

Hence, the age of the rock is

$$= 3 \times 1.37 \times 10^9 = 4.11 \times 10^9 \text{ yr}$$

35 (d) Energy released in the fission of one nucleus = 200 MeV

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11} \text{ J}$$

$$P = 16 \text{ kW} = 16 \times 10^3 \text{ W}$$

Now, number of nuclei required per second

$$n = \frac{P}{E} = \frac{16 \times 10^3}{3.2 \times 10^{-11}} = 5 \times 10^{14}$$

36 (c) In β^- decay, a neutron in the nucleus is transformed into a proton, an electron and an anti-neutrino.

$$n \rightarrow p + e^- + \bar{\nu}$$

So, x and y stand for electron and anti-neutrino.

37 (c) $Z_{\text{resulting nucleus}} = 92 - 8 \times 2 + 4 \times 1 - 2 \times 1 = 78$

38 (c) Energy is released in a process when total binding energy (BE) of the nucleus is increased or we can say when total BE of products is more than the reactants. By calculation, we can see that only in case of option (c), this happens.

Given, $W \rightarrow 2Y$

BE of reactants = $120 \times 7.5 = 900 \text{ MeV}$

and BE of products = $2 \times 60 \times 8.5 = 1020 \text{ MeV}$

i.e. BE of products > BE of reactants.

Hence, energy is released in the process of option (c).

In option (b), process is not possible as mass number is not conserved.

39 (b) Given, $F = \frac{GMm}{r^2}$; where

M = effective mass of hydrogen atom

= mass of electron + mass of proton - (B/c^2)

where, B is BE of hydrogen atom = 13.6 eV

40 (b) We have, $\frac{N}{N_0} = e^{-\lambda t} = e^{-\left(\frac{\ln 2}{t_{1/2}}\right)\left(\frac{t}{2}\right)} = e^{-\left(\frac{\ln 2}{2}\right)} = \frac{1}{\sqrt{2}}$

$$\left(\because \lambda = \frac{\ln 2}{t_{1/2}}; t = nt_{1/2}, \text{ here } n = \frac{1}{2}\right)$$

41 (a) $\frac{N}{N_0} = e^{-\lambda \tau} = e^{-\lambda \left(\frac{n}{\lambda}\right)} = \frac{1}{e^n} \quad \left(\because \text{mean life, } \tau = \frac{1}{\lambda}\right)$

42 (a) Number of half-lives in two days for substances 1 and 2 respectively are $n_1 = \frac{2 \times 24}{12} = 4$ and $n_2 = \frac{2 \times 24}{16} = 3$

By using, $N = N_0 \left(\frac{1}{2}\right)^n$

$$\frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2} \times \frac{\left(\frac{1}{2}\right)^{n_1}}{\left(\frac{1}{2}\right)^{n_2}}$$

Ratio, $\frac{N_1}{N_2} = \frac{2}{1} \times \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^3} = \frac{1}{2}$

43 (c) Initially whole isotope was X , ($\because X:Y = 1:15$)

i.e. $N_0 = 1 + 15 = 16$ and after decaying into Y

We have, $N = N_0 e^{-\lambda t} \Rightarrow 1 = 16 e^{-\frac{0.693}{1.5 \times 10^9} \times t}$

$$\Rightarrow \ln\left(\frac{1}{16}\right) = -\frac{0.693}{1.5 \times 10^9} \times t$$

$\Rightarrow$ The age of the rock, $t = 6.0 \times 10^9$ yr

44 (b) As, $N = N_0 \left(\frac{1}{2}\right)^{t/T}$

Number of atoms at $t = 2$ h,

$$N_1 = 8 \times 10^{10} \left(\frac{1}{2}\right)^{\frac{2}{1}} = 2 \times 10^{10}$$

Number of atoms at $t = 4$ h,

$$N_2 = 8 \times 10^{10} \left(\frac{1}{2}\right)^{\frac{4}{1}} = \frac{1}{2} \times 10^{10}$$

$\therefore$ Number of atoms decayed in given duration

$$= \left(2 - \frac{1}{2}\right) \times 10^{10} = 1.5 \times 10^{10}$$

45 (b) Decay constant, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{20} = 0.03465$

Now, time of decay, $t = \frac{2.303}{\lambda} \log \frac{N_0}{N}$

$$\Rightarrow t_1 = \frac{2.303}{0.03465} \log \frac{100}{67} = 11.6 \text{ min}$$

$$\text{and } t_2 = \frac{2.303}{0.03465} \log \frac{100}{33} = 32 \text{ min}$$

So, ratio between points of time

$$= \frac{11.6}{32} = \frac{29}{80}$$

46 (a) We have, $N = N_0 \left(\frac{1}{2}\right)^{t/T}$

$$\therefore N_A = 10 \left(\frac{1}{2}\right)^{t/1} \Rightarrow N_B = 1 \left(\frac{1}{2}\right)^{t/2}$$

Given, $N_A = N_B$

$$\text{So, } 10 \left(\frac{1}{2}\right)^t = \left(\frac{1}{2}\right)^{t/2}$$

$$\text{So, } 10 = \left(\frac{1}{2}\right)^{-t/2} \text{ or } 10 = 2^{t/2}$$

$$\therefore \log_{10} 10 = \frac{t}{2} \log_{10} 2$$

$$1 = \frac{t}{2} \times 0.3010$$

$\therefore$ Time, $t = 6.64$ yr

47 (b) Number of atoms remains undecayed, $N = N_0 e^{-\lambda t}$

$$\text{As, } \lambda = \frac{1}{t} \text{ or } t = \frac{1}{\lambda} \quad (\text{given})$$

$$\Rightarrow N = N_0 e^{-\lambda(t/\lambda)} = \frac{N_0}{e} = 36.8\% \text{ of } N_0$$

48 (c) $\lambda = (\lambda_1 + \lambda_2) = \frac{1}{1620} + \frac{1}{520} = 2.54 \times 10^{-3} \text{ yr}^{-1}$

$$\therefore t_{1/2} = \frac{\ln(2)}{\lambda}$$

$$= 272.8 \text{ yr}$$

(1/4) th of the material remains intact after 2 half-lives.

$$\therefore t = 2t_{1/2} = 2 \times 272.8 \approx 545 \text{ yr}$$

49 (a) Maximum number of nuclei will be present, when rate of decay = rate of formation, $\lambda N = \alpha$

Maximum number of nuclei, $N = \alpha / \lambda$

50 (d) The value of $x = \frac{N}{N_0} \times 100 = \left\{ e^{-\lambda \left(\frac{3}{\lambda}\right)} \right\} \times 100$
 $\left(\because t = nT_{1/2} \text{ and } T_{1/2} = \frac{1}{\lambda} \right)$

$$= 5\%$$

51 (c) $\frac{N}{N_0} = \frac{1}{4} = \left(\frac{1}{2}\right)^{t/5700}$

$$\therefore \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/5700}$$

$$\therefore \frac{t}{5700} = 2$$

$\therefore$ The age of the wood, $t = 11400$ yr

52 (c) We have, $\lambda_B N_B = \lambda_A N_A$

$$\therefore N_B = \frac{\lambda_A}{\lambda_B} \cdot N_A$$

The given time is equivalent to two half-lives of A . Hence,

$$N_A = \frac{N_0}{4} \quad \left[\because N = N_0 \left(\frac{1}{2}\right)^2 \right]$$

$$\therefore N_B = \frac{\lambda_A}{\lambda_B} \left(\frac{N_0}{4}\right)$$

53 (b) Activity, $R = R_0 e^{-\lambda t}$

$$\therefore \frac{R_P}{R_Q} = \frac{R_0 e^{-\lambda t_1}}{R_0 e^{-\lambda t_2}}$$

$$\therefore e^{\lambda(t_1 - t_2)} = \frac{R_Q}{R_P}$$

$$\therefore \frac{t_1 - t_2}{T} = \ln \frac{R_Q}{R_P} \quad \left(\because \lambda = \frac{1}{T} \right)$$

$$\therefore \text{The difference in age, } (t_1 - t_2) = T \ln \left(\frac{R_Q}{R_P} \right)$$

54 (c) Activity, $R = R_0 e^{-\lambda t}$;

$$\therefore R_1 = R_0 e^{-t_1/T} \quad \left(\because \lambda = \frac{1}{T} \right)$$

$$\text{and } R_2 = R_0 e^{-t_2/T}$$

$$\therefore R_2 = R_1 e^{t_1/T} \cdot e^{-t_2/T}$$

$$\Rightarrow R_2 = R_1 e^{\frac{(t_1 - t_2)}{T}}$$

- 55 (c) $\lambda_\alpha = \frac{1}{1620}$ per year and $\lambda_\beta = \frac{1}{405}$ per year and it is given

that the fraction of the material remained $\frac{N}{N_0} = \frac{1}{4}$

Total decay constant, $\lambda = \lambda_\alpha + \lambda_\beta = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$ per yr

We know that, $N = N_0 e^{-\lambda t}$

$$\Rightarrow t = \frac{1}{\lambda} \log \frac{N_0}{N} \Rightarrow t = \frac{1}{\lambda} \log_e 4 = \frac{2}{\lambda} \log_e 2$$

$$= 324 \times 2 \times 0.693 = 449.2 \approx 449 \text{ yr}$$

- 56 (d) Initially $P \rightarrow 4N_0$; $Q \rightarrow N_0$

Half-life, $T_P = 1 \text{ min}$; $T_Q = 2 \text{ min}$

Let after time t , number of nuclei of P and Q are equal

$$\text{i.e. } \frac{4N_0}{2^{t/1}} = \frac{N_0}{2^{t/2}} \quad \left(\because N = \frac{N_0}{2^{t/T_{1/2}}} \right)$$

$$\text{or } \frac{4}{2^{t/2}} = 1 \text{ or } t = 4 \text{ min}$$

So, at $t = 4 \text{ min}$

$$N_P = \frac{(4N_0)}{2^{4/1}} = \frac{N_0}{4}$$

$$\text{and at } t = 4 \text{ min, } N_Q = \frac{N_0}{2^{4/2}} = \frac{N_0}{4}$$

So, number of nuclei of R in the sample

$$= \left(4N_0 - \frac{N_0}{4} \right) + \left(N_0 - \frac{N_0}{4} \right) = \frac{9N_0}{2}$$

- 57 (c) To reduce one-fourth the time taken is

$$t = 2(T_{1/2}) = 2 \times 40 = 80 \text{ yr}$$

$$\text{So, decay constant, } \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{40} = 0.0173 \text{ yr}$$

- 58 (a) According to the question, (7/8) part decays, i.e. remaining part is (1/8).

$$\text{We have, } N = N_0 \left(\frac{1}{2} \right)^{\frac{t}{T_{1/2}}} \Rightarrow \frac{1}{8} = \left(\frac{1}{2} \right)^{\frac{15}{T_{1/2}}} \quad \left(\because \frac{N}{N_0} = \frac{1}{8} \right)$$

$$\Rightarrow \text{Half-life, } T_{1/2} = 5 \text{ min}$$

- 59 (b) Momentum remains constant.

$$\therefore K = \frac{p^2}{2m} \text{ or } K \propto \frac{1}{m}$$

$$\Rightarrow \frac{K_n}{K_\alpha} = \frac{m_\alpha}{m_n} = \frac{4}{206} = \frac{2}{103}$$

$\therefore$ Kinetic energy of the residual nucleus

$$K_n = \left(\frac{2}{103} \right) K_\alpha = \left(\frac{2}{103} \right) E$$

- 60 (c) $R_i = (15 \times 200) = 3000 \text{ decay/min}$

$$R = 375 \text{ decay/min} = \frac{R_i}{8}$$

$$\text{As, } R = R_i \left(\frac{1}{2} \right)^n$$

$$\therefore \text{Time, } t = 3 (\text{half-lives}) = 3 \times 5730 = 17190 \text{ yr}$$

- 61 (c) As, $n = \frac{m}{A} N_A \Rightarrow E = \left(\frac{m}{A} N_A \times 200 \times 1.6 \times 10^{-13} \right) \text{ J}$

For 1 day = 24×3600

$$\text{Power} = \frac{E}{t} = \frac{m N_A \times 3.2 \times 10^{-11}}{235 \times 24 \times 3600} = 10^6 \quad (\because \text{given, } P = 10^6 \text{ W})$$

$$\therefore \text{Mass, } m = \frac{1 \times 10^6 \times 24 \times 3600 \times 235}{3.2 \times 10^{-11} \times 6.02 \times 10^{23}} \approx 1 \text{ g}$$

- 62 (a) We have, $N = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{N_0}{e} = N_0 e^{-\lambda(5)} \Rightarrow \lambda = \frac{1}{5}$$

$$\text{Now, } \frac{N_0}{2} = N_0 e^{-\lambda(t)} \Rightarrow t = \frac{1}{\lambda} \log_e 2 = 5 \log_e 2$$

- 63 (a) Kinetic energy of the molecules of a gas at a temperature T is $\frac{3}{2} kT$.

$$\therefore \text{To initiate the reaction } \frac{3}{2} kT = 7.7 \times 10^{-14} \text{ J}$$

$$\Rightarrow \frac{3}{2} \times 1.38 \times 10^{-23} T = 7.7 \times 10^{-14}$$

$$\Rightarrow T = 3.7 \times 10^9 \text{ K}$$

Thus, the order of temperature is 10^9 K .

- 64 (b) By the formula $N = N_0 e^{-\lambda t}$

$$\text{Given, } \frac{N}{N_0} = \frac{1}{20} \text{ and } \lambda = \frac{0.6931}{3.8} \Rightarrow 20 = e^{\frac{0.6931 \times t}{3.8}}$$

$$(\because t_{1/2} = 3.8 \text{ days})$$

On taking log both sides, we get

$$\text{or } \log 20 = \frac{0.6931 \times t}{3.8} \log_{10} e$$

$$\text{or } 1.3010 = \frac{0.6931 \times t \times 0.4343}{3.8}$$

$$\Rightarrow t = 16.5 \text{ days}$$

- 65 (d) We have, $n = \frac{24}{24 \times 138.6} = \frac{1}{138.6}$

$$\text{Now, } \frac{N}{N_0} = \left(\frac{1}{2} \right)^n = \left(\frac{1}{2} \right)^{1/138.6}$$

$$\Rightarrow N = 1000000 \left(\frac{1}{2} \right)^{1/138.6} = 995011$$

So, number of disintegration

$$= 1000000 - 995011$$

$$= 4989 \approx 5000$$

- 66 (b) Number of half-lives, $n = \frac{19}{3.8} = 5$

$$\text{Now, } \frac{N}{N_0} = \left(\frac{1}{2} \right)^n$$

$$\Rightarrow \frac{N}{10.38} = \left(\frac{1}{2} \right)^5$$

$$\Rightarrow N = 10.38 \times \left(\frac{1}{2} \right)^5 = 0.32 \text{ g}$$

- 67 (d) Number of nuclei remained after time t can be written as

$$N = N_0 e^{-\lambda t}$$

where, N_0 is initial number of nuclei of both the substances

$$N_1 = N_0 e^{-5\lambda t} \quad \dots(i)$$

and

$$N_2 = N_0 e^{-\lambda t} \quad \dots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{N_1}{N_2} = e^{(-5\lambda + \lambda)t} = e^{-4\lambda t} = \frac{1}{e^{4\lambda t}}$$

But, we have given, $\frac{N_1}{N_2} = \left(\frac{1}{e}\right)^2 = \frac{1}{e^2}$

Comparing the powers, we get

$$2 = 4\lambda t \Rightarrow t = \frac{2}{4\lambda} = \frac{1}{2\lambda}$$

- 68 (c) Net process is, $3 {}_1\text{H}^2 \rightarrow {}_2\text{He}^4 + p + n$

$$\begin{aligned} \text{Mass defect, } \Delta m &= (3 \times 2.014) - 4.001 - 1.007 - 1.008 \\ &= 0.026 \text{ amu} \end{aligned}$$

Energy released by the fusion of three deuterons

$$\begin{aligned} &= \Delta m \times 931.48 = 0.026 \times 931.48 \text{ MeV} \\ &= 0.026 \times 931.48 \times 1.6 \times 10^{-13} \text{ J} \\ &= 3.87 \times 10^{-12} \text{ J} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total energy released} &= \frac{10^{40}}{3} \times 3.87 \times 10^{-12} \text{ J} \\ &= 1.29 \times 10^{28} \text{ J} \end{aligned}$$

$$\text{Total time} = \frac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12} \text{ s} \left(\because \text{power} = \frac{\text{energy}}{\text{time}} \right)$$

$\therefore$ The order of time is 10^{12} s

- 69 (d) Energy received = 1.4 kW/m^2

$$\Rightarrow E_r = 1.4 \text{ kJ/sm}^2 = \frac{1.4 \text{ kJ}}{\frac{1}{86400} \text{ day m}^2} = \frac{1.4 \times 86400 \text{ kJ}}{\text{day m}^2}$$

Total energy radiated/day

$$E = \frac{4\pi R^2 \times E_r}{t} = \frac{4\pi \times (1.5 \times 10^{11})^2 \times 1.4 \times 86400}{1} \frac{\text{kJ}}{\text{day}}$$

$\therefore$ We have, $E = mc^2$

$$\begin{aligned} \Rightarrow m &= \frac{E}{c^2} = \frac{4\pi (1.5 \times 10^{11})^2 \times 1.4 \times 86400}{(3 \times 10^8)^2} \times 10^3 \\ &= 3.8 \times 10^{14} \text{ kg} \end{aligned}$$

- 70 (b) We have $N = N_0 e^{-\lambda t}$ ($\because N = N_0 - 0.1N_0$)

$$\therefore 0.9N_0 = N_0 e^{-\lambda \times 5} \Rightarrow 5\lambda = \log_e \frac{1}{0.9} \quad \dots(i)$$

$$\text{and } xN_0 = N_0 e^{-\lambda \times 20} \Rightarrow 20\lambda = \log_e \left(\frac{1}{x} \right) \quad \dots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{1}{4} = \frac{\log_e(1/0.9)}{\log_e(1/x)} = \frac{\log_{10}(1/0.9)}{\log_{10}(1/x)} = \frac{\log_{10} 0.9}{\log_{10} x}$$

$$\Rightarrow \log_{10} x = 4 \log_{10} 0.9$$

$$\Rightarrow x = 0.656 = 65.6\% \approx 65\%$$

- 71 (a) R_0 = Initial activity = 1 microcurie = 3.7×10^4 dps

r = Activity in 1 cm^3 of blood (at $t = 5 \text{ h}$)

$$= \frac{296}{60} = 4.93 \text{ dps cm}^{-3}$$

R = Activity of whole blood (at time $t = 5 \text{ h}$)

$$\begin{aligned} \text{Total volume should be } V &= \frac{R}{r} = \frac{R_0 e^{-\lambda t}}{r} = \frac{3.7 \times 10^4 \times 0.7927}{4.93} \\ &= 5.94 \times 10^3 \text{ cm}^3 = 5.94 \text{ L} \end{aligned}$$

(B) Medical entrance special format questions

• Assertion and reason

- 2 (d) Average life is greater than half-life. Hence, more nuclei decay in one average life.

3 (a) Half-life, $t_{1/2} = \frac{0.693}{\lambda} \Rightarrow t_{1/2} \propto 1/\lambda$

$$\text{As, } \lambda_2 < \lambda_1 \Rightarrow (t_{1/2})_1 < (t_{1/2})_2$$

4 (d) The packing fraction is given as $P = \frac{m - A}{A}$

Positive value of P does not mean that Δm is large and hence the binding energy $[E = (\Delta m) c^2]$.

- 5 (d) Rate of a nuclear process cannot be altered by altering pressure or temperature.

• Statement based questions

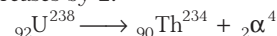
- 4 (a) Tritium (${}_1\text{H}^3$) contains 1 proton and 2 neutrons. A neutron decays as, $n \rightarrow p + e^- + \bar{\nu}$, the nucleus may have 2 protons and one neutron, i.e. tritium will transform into a He^3 (2 protons and 1 neutron).

Triton energy is less than that of a He^3 nucleus, i.e. transformation is not allowed energetically.

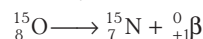
• Match the columns

- 1 (c) By the emission of an α -particle, atomic number decreases by 2 and mass number by 4. But by the emission of a β -particle, atomic number increases by 1 while mass number remains unchanged.
- 3 (b) (i) Stability of nucleus is decided by neutron-proton ratio, packing fraction and binding energy per nucleon.
(ii) For radioactive substance binding energy per nucleon is minimum. So, they are unstable.
(iii) For bound orbit, total energy is always negative.
(iv) Stopping potential is the particular negative potential when no electron reaches the plate.

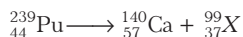
- 4 (d) In α -decay, mass number decreases by 4 and atomic number decreases by 2.



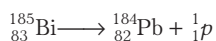
In β^+ decay, mass number remains unchanged while atomic number decreases by 1.



In fission, parent nucleus break into almost two equal fragments.



In proton emission, both mass number and atomic number decreases by 1.



(C) Medical entrances' gallery

- 1 (a) Given, $m = 0.5 \text{ g} = 0.5 \times 10^{-3} \text{ kg}$

Relation for energy equivalent of mass is $E = mc^2$,

where c is speed of light.

$$= 0.5 \times 10^{-3} \times (3 \times 10^8)^2 \quad (\because c = 3 \times 10^8)$$

$$= 4.5 \times 10^{13} \text{ J}$$

- 2 (d) Here, ${}_{92}^{235}\text{U} + {}_0^1\text{n} \longrightarrow {}_{36}^{89}\text{Kr} + {}_Z^AX + 3({}_0^1\text{n})$

According to the law of conservation,

(Atomic number)_{reactant} = (Atomic number)_{product}

$$92 + 0 = 36 + Z$$

$$\Rightarrow Z = 92 - 36 = 56$$

Similarly,

(Atomic masses)_{reactant} = (Atomic masses)_{product}

$$235 + 1 = 89 + A + 3 \times 1$$

$$\Rightarrow A = 144$$

$\therefore$ Other element is ${}_{56}^{144}\text{Ba}$.

- 3 (b) When an atom emits γ -radiation from its nucleus, then there is no change in its atomic number and mass number.

- 4 (a) Given, $T_{1/2} = 1.4 \times 10^{17} \text{ s}$

Number of nuclei in the sample, $N = 2.0 \times 10^{21}$

$\therefore$ Activity of the sample = λN

$$= \frac{0.693}{T_{1/2}} \times 2.0 \times 10^{21}$$

$$= \frac{0.693}{1.4 \times 10^{17}} \times 2.0 \times 10^{21} = 0.99 \times 10^4$$

$$\approx 1 \times 10^4 \text{ Bq} \approx 10^4 \text{ Bq}$$

- 5 (d) α -particles are doubly ionised helium nucleus (He^{2+}) which are emitted in any radioactive process. So, they have 2 protons, 2 neutrons in its nucleus and no electron.

- 6 (d) Given, half-life, $T_{1/2} = 2.2 \times 10^9 \text{ s}$

Rate of disintegration, $R = 10^{10} \text{ s}^{-1}$

If N be the number of nuclei present, then the rate of

disintegration is $\frac{dN}{dt} = \lambda N$ (λ = decay constant)

$$\Rightarrow R = \lambda N \text{ or } N = \frac{R}{\lambda} \quad \dots(i)$$

Also, the half-life is given by

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$\Rightarrow \lambda = \frac{0.693}{T_{1/2}} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$N = \frac{R}{0.693} \times T_{1/2} \\ = \frac{10^{10} \times 2.2 \times 10^9}{0.693} = 3.17 \times 10^{19}$$

- 7 (a) Given, half-life of an element,

$$T_{1/2} = 69.3 \text{ h}$$

Let initial amount (at $t = 0$) of radioactive element is N_0 .

Active nuclei at $t = 10 \text{ h}$,

$$N_1 = N_0 e^{-10\lambda}$$

Active nuclei at $t = 11 \text{ h}$, $N_2 = N_0 e^{-11\lambda}$

% decay in 10th to 11th hours

$$= \frac{N_1 - N_2}{N_1} \times 100$$

$$= \frac{N_0 e^{-10\lambda} - N_0 e^{-11\lambda}}{N_0 e^{-10\lambda}} \times 100$$

$$= (1 - e^{-\lambda}) \times 100$$

$$= \left[1 - e^{-\frac{0.693}{69.3}} \right] \times 100 \quad \left(\because \text{where, } \lambda = \frac{0.693}{T_{1/2}} \right)$$

$$= [1 - e^{-0.01}] \times 100$$

$$= [1 - 0.99] \times 100$$

$$= 0.01 \times 100 = 1\%$$

- 8 (a) As, $T_{1/2} = 2 \text{ min}$, hence $10 \text{ min} = 5 \text{ half-lives}$

We know that, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

where, N = remaining of nuclei,

N_0 = initial number of nuclei

and n = number of half-lives.

$$\Rightarrow N = \frac{N_0}{2^n} = \frac{80}{2^5} = \frac{80}{32} = 2.5 \text{ g}$$

- 9 (c) After n half-life, the number of nuclei left undecayed is given as

$$N = N_0 (1/2)^n$$

where, $n = \frac{t}{T_{1/2}}$

Here, initially number of nuclei, $N_0 = 600$. Disintegrated number of nuclei in time $t = N' = 450$

$\therefore$ Number of nuclei left undecayed,

$$N = N_0 - N'$$

$$N = 600 - 450 = 150$$

As, we know, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$

Substituting the given values, we get

$$\frac{150}{600} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \text{ or } \frac{1}{4} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$\text{or } \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/10} \Rightarrow 2 = \frac{t}{10} \Rightarrow t = 20 \text{ min}$$

10 (b) $N_1 = N_0 - \frac{1}{3}N_0 = \frac{2}{3}N_0$

and $N_2 = N_0 - \frac{2}{3}N_0 = \frac{1}{3}N_0$

$$\therefore \frac{N_2}{N_1} = \frac{1}{2} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \Rightarrow t/T_{1/2} = 1$$

$$\Rightarrow t = 20 \text{ min}$$

where, $t = t_2 - t_1$.

Therefore, the time interval $(t_2 - t_1) = 20 \text{ min}$.

11 (c) For a β -decay, ${}_0^1n \longrightarrow {}_1^1p + {}_{-1}^0e$

where, ${}_{-1}^0e$ is an electron emitted in the transformation of a neutron into proton in the radioactive nucleus. But electron does not exist inside the nucleus.

12 (a) $\Delta m = 2(2.015) - (3.017 + 1.009)$

$$= 0.004 \text{ amu}$$

$$\therefore \text{Energy released} = (0.004 \times 931.5) \text{ MeV}$$

$$= 3.726 \text{ MeV}$$

Energy released per deuterium

$$= \frac{3.726}{2} = 1.863 \text{ MeV}$$

Number of deuterons in 1 kg

$$= \frac{6.02 \times 10^{26}}{2} = 3.01 \times 10^{26}$$

$\therefore$ Energy released per kg of deuterium fusion

$$= (3.01 \times 10^{26} \times 1.863)$$

$$= 5.6 \times 10^{26} \text{ MeV}$$

$$= 9 \times 10^{13} \text{ J}$$

13 (b) Let initial number of nuclei in A and B is N_0 . Number of nuclei of A after time t is

$$N_A = N_0 e^{-8\lambda t} \quad \dots(i)$$

Similarly, number of nuclei of B after time t is

$$N_B = N_0 e^{-\lambda t} \quad \dots(ii)$$

It is given that, $\frac{N_A}{N_B} = \frac{1}{e}$ ($\because N_B > N_A$)

$$\text{Now, from Eqs. (i) and (ii), } \frac{e^{-8\lambda t}}{e^{-\lambda t}} = \frac{1}{e}$$

Rearranging, we get

$$e^{-1} = e^{-7\lambda t} \Rightarrow 7\lambda t = 1$$

$$\text{Time, } t = \frac{1}{7\lambda}$$

14 (d) We know that, α -particle carries two units of positive charge and four units of mass. In α -decays, charge or atomic number of daughter nucleus decreased by two units. Also, mass number reduces by four units.

15 (d) Energy released per fission,

$$E = 200 \text{ MeV} = 200 \times 1.6 \times 10^{-13}$$

$$= 3.2 \times 10^{-11} \text{ J}$$

Total energy required to run a 1 MW reaction for one year

$$= 10^6 \text{ J s}^{-1} \times (365 \times 24 \times 60 \times 60)$$

$$= 3.15 \times 10^{13} \text{ J}$$

Since, 1 fission (atom of U^{235}) produces $3.2 \times 10^{-11} \text{ J}$ of energy.

So, total number of U^{235} atoms required is

$$\frac{3.15 \times 10^{13}}{3.2 \times 10^{-11}} = 9.84 \times 10^{23}$$

$\therefore$ Mass of U^{235} containing 9.84×10^{23} atom,

$$M = \frac{235}{6.02 \times 10^{23}} \times 9.84 \times 10^{23}$$

$$= 384 \text{ g} = 0.384 \text{ kg}$$

16 (b) Let M_0 be initial mass of Rn - 220.

Number of half-lives of Rn in 5 min,

$$= \frac{5 \text{ min}}{55 \text{ s}} = \frac{5 \times 60}{55} = 5.5$$

$$\therefore \text{Mass of Rn-220 (left)} = \left(\frac{1}{2}\right)^{5.5} M_0 = \frac{M_0}{45}$$

$\therefore$ The mass converted in Po-216 is $(44/45)M_0$. The number of half-lives of Po-216 in 5 min

$$= \frac{5 \text{ min}}{0.66} = \frac{300}{0.66} = 455$$

$$\therefore \text{Mass of Po-216 (left)} = \left(\frac{1}{2}\right)^{455} \times \frac{44}{55} M_0 \rightarrow 0$$

This means that Po -216 formed will soon decay to lead.

Hence, after 5 min lead will have maximum mass nearly

$$\left(\frac{44}{45}\right)M_0.$$

17 (d) We have, $\frac{N}{N_0} \left(\frac{1}{2}\right)^n$, where n is the number of half lives.

Half-life of a radioactive substance, $T_{1/2} \propto \log \left(\frac{N_0}{N}\right)$

$$\text{Given, } N_1 = 0.6N_0 \quad (\because 40\% \text{ decay})$$

$$N_2 = 0.15N_0 \quad (\because 85\% \text{ decay})$$

Putting these in the formula,

$$\frac{N_2}{N_1} = \frac{0.15N_0}{0.6N_0} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

So, two half-life periods has passed.

$$\text{Thus, time taken} = 2 \times t_{1/2} = 2 \times 30 = 60 \text{ min}$$

18 (b) Radius of the nucleus is given by

$$R = R_0 A^{1/3} \Rightarrow R \propto A^{1/3}$$

$$\therefore \frac{R_{\text{Al}}}{R_{\text{Te}}} = \left(\frac{A_{\text{Al}}}{A_{\text{Te}}}\right)^{1/3} = \left(\frac{27}{125}\right)^{1/3} = \frac{3}{5}$$

$$R_{\text{Te}} = \frac{5}{3} R_{\text{Al}}$$

- 19 (a)** Radius of nuclei having mass number A is determined as

$$R = R_0 A^{1/3} \quad (\text{where, } R_0 = \text{constant})$$

where, $R_0 = 1.2 \times 10^{-15} \text{ m}$

This implies, $\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{216}{64}\right)^{1/3} = \frac{6}{4} \Rightarrow R_1 : R_2 = 3 : 2$

- 20 (d)** The reaction should be ${}_{88}\text{Ra}^{226} \longrightarrow {}_{86}\text{Rn}^{222} + {}_2\text{He}^4$

$$Q\text{-value} = \left[\begin{array}{l} (\text{mass number of Ra}) \\ - (\text{mass number of Rn}) \\ - m(\text{He}^4) \end{array} \right] \times 931.5 \text{ MeV}$$

$$= 0.005297 \times 9315 = 4.934 \text{ MeV}$$

Energy corresponding mass defect = $\frac{m(\text{Rn})}{m(\text{Ra})} \times Q\text{-value}$
 $\approx 4.871 \text{ MeV}$

- 21 (b)** The fission per second = 4×10^{20}

So, the energy released per second

$$= (4 \times 10^{20} \times 250) \text{ MeV}$$

$$= (4 \times 10^{20} \times 250 \times 10^6 \times 1.6 \times 10^{-19}) \text{ J}$$

Therefore, energy released in 10 h = $36 \times 10^3 \text{ s}$,

$$E = (36 \times 10^3 \times 4 \times 10^{20} \times 250 \times 10^6 \times 1.6 \times 10^{-19}) \text{ J}$$

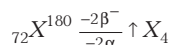
$$= (36 \times 4 \times 250 \times 1.6 \times 10^{10}) \text{ J} = (57600 \times 10^{10}) \text{ J}$$

$$= 576 \times 10^{12} \text{ J}$$

- 22 (d)** Nuclear fusion is not found in atom bomb because it works on the phenomenon of nuclear fission.

- 23 (d)** Nuclear mass density is independent of mass number, therefore ratio of nuclear mass densities of Au^{197} and Ag^{107} is 1: 1.

- 24 (d)** The disintegration can be shown as below



Loss in atomic number = $72 + 2 - 4 = 70$

Loss in mass number = $180 - 8 = 172$

So, final nucleus is ${}_{70}\text{X}_4^{172}$.

- 25 (a)** Let there is n number of fission per second produces a power of 6.4 W, then

$$n \times 200 \times 10^6 \times 1.6 \times 10^{-19} = 6.4$$

$$\therefore n = \frac{6.4}{200 \times 10^{-13} \times 1.6} = \frac{4}{2 \times 10^{-11}} = 2 \times 10^{11}$$

- 26 (b)** The number of α -particles, n_1

$$= \frac{\text{change in mass number}}{4} = \frac{238 - 206}{4} \Rightarrow n_1 = \frac{32}{4} = 8$$

Now, number of β -particles, $n_2 = 82 - (92 - 2n_1)$

$$= 82 - (92 - 2 \times 8) = 82 - 76 = 6$$

- 27 (d)** Amount of element left undecayed after 10 days

$$= 1 - \frac{20}{100} = \frac{8}{10}$$

Now, $N = \frac{8}{10} N_0$ and $t = 10$ days

We have, $N = N_0 e^{-\lambda t}$

$$\frac{8}{10} N_0 = N_0 e^{-\lambda t} \Rightarrow e^{\lambda t} = \frac{10}{8} \Rightarrow \lambda = \frac{\log_e (10/8)}{10}$$

$$\therefore \lambda = 0.022$$

If $T_{1/2}$ is half-life, $T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.022} = 31.5 \text{ days}$

Again, $N = N_0 \left(\frac{1}{2}\right)^n = N_0 \left(\frac{1}{2}\right)^{31.5}$

$$\Rightarrow \% \text{ of } N / N_0 = 48.3\% \approx 48\%$$

- 28 (b)** Given, $\frac{R_1}{R_2} = \frac{1}{2}$

According to conservation of linear momentum, we have

$$m_1 v_1 = m_2 v_2$$

$$\Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1} = \left(\frac{R_2}{R_1}\right)^3 \quad (\because m \propto v \propto R^3)$$

i.e. $\frac{v_1}{v_2} = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$

- 29 (a)** Energy released in one fission of ${}_{92}^{235}\text{U}$ nucleus = 200 MeV

Mass of uranium = 1g

We know that, 235 g of ${}^{235}\text{U}$ has 6.023×10^{23} atoms or nuclei.

$\therefore$ Energy released in fission of 1 g of U^{235} ,

$$E = \frac{6.023 \times 10^{23} \times 1 \times 200}{235} = 5.1 \times 10^{23} \text{ MeV}$$

$$= (5.1 \times 10^{23} \times 1.6 \times 10^{-13}) \text{ J}$$

$$= \frac{5.1 \times 10^{23} \times 1.6 \times 10^{-13}}{3.6 \times 10^6} \text{ kWh} = 2.26 \times 10^4 \text{ kWh}$$

- 30 (a)** We have, $\frac{dN}{dt} = \lambda N$

and $\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{1620 \times 365 \times 24 \times 60 \times 60}$

Total number of atoms, $N = \frac{6.023 \times 10^{23}}{226}$

$$\therefore \frac{dN}{dt} = \frac{0.6931 \times 6.023 \times 10^{23}}{1620 \times 365 \times 24 \times 60 \times 60 \times 226} = 3.61 \times 10^{10}$$

- 31 (b)** The relation between decay constant λ and half-life T can be given as

$$T = \frac{0.693}{\lambda}$$

$$\therefore \lambda = \frac{\log_e 2}{T}$$

- 32 (d)** The binding energy for ${}_1\text{H}^1$ is around zero and also not given in the question, so we can ignore it.

$$Q = 2(4 \times 7.06) - 7 \times (5.60)$$

$$= (8 \times 7.06) - (7 \times 5.60)$$

$$= 56.48 - 39.2$$

$$= 17.28 \text{ MeV}$$

$$\approx 17.3 \text{ MeV}$$

- 33** (c) According to the question, ratio of $X : Y$ is given = 1 : 7

$$\Rightarrow \frac{m_X}{m_Y} = \frac{1}{7} \Rightarrow 7m_X = m_Y$$

Let the initial total mass is m .

$$\Rightarrow m_X + m_Y = m \Rightarrow \frac{m_Y}{7} + m_Y = m$$

$$\Rightarrow \frac{8m_Y}{7} = m \Rightarrow m_Y = \frac{7}{8}m$$

Only $\frac{1}{8}$ part remains left undisintegrated.

Number of half-lives involve are three as

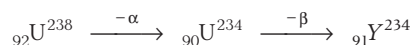
$$1 \xrightarrow{T_{1/2}} \frac{1}{2} \xrightarrow{T_{1/2}} \frac{1}{4} \xrightarrow{T_{1/2}} \frac{1}{8}$$

So, time taken to become $\frac{1}{8}$ unstable part

$$= 3 \times T_{1/2} = 3 \times 1.4 \times 10^9 = 4.20 \times 10^9 \text{ yr}$$

- 34** (a) The binding energy of a nucleus is the energy required to take its nucleons away from one another. It is generally expressed as binding energy per nucleon. It is a measure of the stability of the nucleus. Higher the binding energy per nucleon, more stable is the nucleus.

- 35** (d) Simply, we can analyse from the equation,



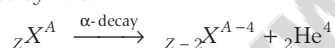
- 36** (a) As, activity dropped from 5000 dps to 2500 dps in 150 days means, $T_{1/2} = 150$ days

$$\Rightarrow 300 \text{ days} = 2 T_{1/2}$$

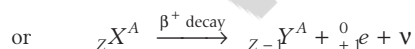
$$\text{So, initial activity} = 2^2 \times 5000 = 20000 \text{ dps} \quad (\because N_0 = N2^n)$$

- 38** (c) Suppose a given nucleus is ${}_Z X^A$.

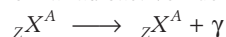
In α -decay, the mass number of the product nucleus is four less than that of decaying nucleus, while the atomic number decreases by two.



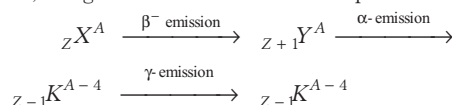
In β -decay, the mass number of product nucleus remains same but atomic number increases or decreases by one.



Gamma decay is the phenomenon of emission of gamma rays photon from a radioactive nucleus.



Hence, the given reaction will have sequence as follows



- 39** (b) The control rods used in a nuclear reactor can be made up of cadmium. As, they absorb fast moving neutrons.

- 40** (d) The fusion reaction in the sun is a multi-step process in which the hydrogen is burned into helium. Because in fusion, lighter nuclei combine to form heavier nucleus.

- 41** (b) The nuclear fusion is a very difficult process. Since, the nuclei to be fused are positively charged, they would repel one another strongly. Hence, they must be brought very close together not only by very high pressure but also with high kinetic energy (≈ 0.1 MeV). For this, a very high temperature of the order of 10^8 K is required. Such high temperature are available in the sun and stars. On the earth, they may be produced by exploding a nuclear fission bomb.

- 42** (b) Ratio of X and $Y = 1 : 7$

$$\frac{m_X}{m_Y} = \frac{1}{7} \Rightarrow m_Y = 7m_X$$

Let the initial total mass = $m \Rightarrow m_X + m_Y = m$

$$\Rightarrow \frac{m_Y}{7} + m_Y = m$$

$$\frac{8m_Y}{7} = m \Rightarrow m_Y = \frac{7}{8}m$$

Only $\left(\frac{1}{8}\right)$ th part remains undisintegrated, number of half-lives

$$\text{involve are three as } 1 \xrightarrow{T_{1/2}} \frac{1}{2} \xrightarrow{T_{1/2}} \frac{1}{4} \xrightarrow{T_{1/2}} \frac{1}{8}$$

So, total time taken to become $\frac{1}{8}$ unstable part

$$= 3T_{1/2} = 3 \times 20 \text{ yr} = 60 \text{ yr}$$

- 43** (c) $4\text{H}_1^1 \longrightarrow {}_2\text{He}^4 + 2\beta^+ + 2\gamma$

Here, $\Delta m = 0.02866\text{u}$

$$\therefore \text{Energy liberated} = \Delta m \times 931\text{MeV}$$

$$= 0.02866 \times 931 = 26.7\text{MeV}$$

$$\therefore \text{Energy liberated per fusion (i.e. } n = 4) = \frac{26.7}{4}\text{MeV}$$

$$= 6.675\text{MeV}$$

- 44** (a) Power, $P = \frac{nE}{t}$

Given, $n = 2 \times 10^{18}$ fission per second and $E = 185$ MeV

$$\text{Here, } P = \frac{2 \times 10^{18} \times 185 \times 10^6 \text{ eV}}{1}$$

$$\Rightarrow = 2 \times 10^{18} \times 185 \times 10^6 \times 16 \times 10^{-19} = 59.2\text{MW}$$

- 46** (c) Nucleus of mass number A has a radius, $R = R_0 A^{1/3}$

where, $R_0 = 1.2 \times 10^{-15}\text{m}$

$$\therefore \frac{R_8}{R_{64}} = \left[\frac{8}{64} \right]^{1/3} = \left[\frac{1}{8} \right]^{1/3}$$

$$\therefore \text{Ratio of volume} = \frac{1}{8} = 0.125$$

- 47** (b) Let $R_1 = N_1 \lambda$ and $R_2 = N_2 \lambda$

$$\text{Mean life, } T = \frac{1}{\lambda}$$

$$R_1 - R_2 = (N_1 - N_2) \lambda = (N_1 - N_2) \frac{1}{T}$$

$$\begin{aligned} \text{So, number of atoms disintegrated in } (T_2 - T_1) \text{ is} \\ = N_1 - N_2 = (R_1 - R_2) T \end{aligned}$$

48 (d) Since, $N = 13 - 7 = 6$, $C = 12 - 6 = 6$

50 (a) From, $E = (\Delta m)c^2$

$$\Delta m = \frac{E}{c^2} = \frac{9 \times 10^{13}}{(3 \times 10^8)^2} = 10^{-3} \text{ kg}$$

$$\therefore \frac{\Delta m}{m} \times 100 = \frac{10^{-3}}{1} \times 100 = 0.1 \%$$

51 (d) We know that, nuclear forces are charge independent and the chief source of stellar energy is fusion.

β -decay is accompanied by both proton and neutron.

52 (b) According to the question, ${}_Z X^A \longrightarrow {}_{Z-2} Y^{A-4} + {}_2 \text{He}^4$

Let the recoil speed of daughter nucleus is v' . By the law of conservation of momentum, we have

$$\text{Initial momentum} = \text{Final momentum} \Rightarrow 0 = (A-4)v' + 4v$$

$$\text{or } (A-4)v' = -4v \text{ or } v' = -\frac{4v}{(A-4)}$$

Here, negative sign shows that the recoil speed is in opposite direction to that of α -particles.

53 (c) Given, in 1st two seconds 100 β particles and in next two seconds 50 β particles are emitted.

$$\therefore T_{1/2} = 2 \text{ s}$$

$$\text{Hence, mean life, } T = \frac{t_{1/2}}{0.693} = \frac{2}{0.693} \text{ s}$$

55 (d) Given, $N_A = N_B$

$$N_0^A \left(\frac{1}{2}\right)^{t/20} = N_0^B \left(\frac{1}{2}\right)^{t/10}$$

$$40 \left(\frac{1}{2}\right)^{t/20} = 160 \left(\frac{1}{2}\right)^{t/10} \Rightarrow t = 40 \text{ s}$$

56 (b) $t_{1/2}$ is given as the half-life of the substance, i.e. substance decays half, so $t_{3/4}$ is the time in which substance decays $\frac{3}{4}$ th.

57 (d) We know that, $n = \frac{t}{T}$

where, T = half-life.

$$\text{Given, } t = 3000 \text{ yr}$$

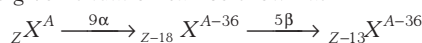
$$T = 600 \text{ yr}$$

$$n = \frac{3000}{600} = 5$$

$$\text{Then, } \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

58 (d) The given situation can be shown as



$\therefore$ Number of protons, $P = (Z-13)$ and number of neutrons,

$$N = (A-36) - (Z-13) = (A-Z-23)$$

$$\therefore \frac{P}{N} = \frac{(Z-13)}{(A-Z-23)}$$

59 (b) We know that, $N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$

$$\frac{1}{4} = \left(\frac{1}{2}\right) \Rightarrow \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^2 \Rightarrow t = 20 \text{ yr}$$

60 (a) The total mass of the initial particles,

$$m_i = 1.007825 + 7.016004 = 8.023829 \text{ u}$$

and the total mass of final particles,

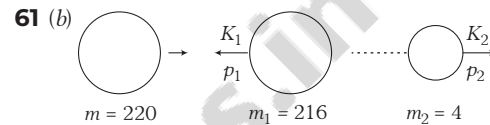
$$m_f = 2 \times 4.002603 = 8.005206 \text{ u}$$

Difference between initial and final mass of particles,

$$\Delta m = m_i - m_f = 8.023829 - 8.005206 = 0.018623 \text{ u}$$

The Q -value is given by

$$Q = (\Delta m) c^2 = 0.018623 \times 931.5 = 17.35 \text{ MeV}$$



Q -value of the reaction is 5.5 MeV.

$$\text{i.e. } K_1 + K_2 = 5.5 \text{ MeV} \quad \dots(i)$$

By conservation of linear momentum, $p_1 = p_2$

$$\Rightarrow \sqrt{2(216) K_1} = \sqrt{2(4) K_2}$$

$$K_2 = 54 K_1 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

The energy of emitted α -particle, $K_2 = 5.4 \text{ MeV}$

62 (c) We have, $\lambda = \frac{0.693}{T_{1/2}}$

$$\text{Mean life, } \tau = \frac{1}{\lambda} = \frac{t_{1/2}}{0.693} = \frac{2}{2.303 \log 2} = \frac{2}{\ln 2} \text{ s}$$

63 (a) Momentum of a photon, $p = \frac{h\nu}{c}$

$$\text{Hence, recoil energy, } E = \frac{p^2}{2M} = \frac{\left(\frac{h\nu}{c}\right)^2}{2M} = \frac{h^2 \nu^2}{2Mc^2}$$

64 (c) Energy and mass are related by the relation, $E = mc^2$

Differentiating, we get

$$\frac{dE}{dt} = m \times [0] + c^2 \frac{dm}{dt}$$

$$\Rightarrow 3.6 \times 10^{28} = 9 \times 10^{16} \frac{dm}{dt} \Rightarrow \frac{dm}{dt} = 4 \times 10^{11} \text{ kg/s}$$

65 (c) From the relation, $R = R_0 e^{-\lambda t}$

(where, R = activity of sample)

Given, $R_0 = 5400 \text{ dis/min}$

$$R = 600 \text{ dis/min and } t = 5 \text{ min}$$

$$\therefore 600 = 5400 e^{-5\lambda}$$

$$\frac{1}{9} = e^{-5\lambda} \Rightarrow \lambda = \frac{\ln 9}{5}$$

$$\therefore \text{Half-life of the sample, } T_{1/2} = \frac{\ln 2}{\lambda} = \frac{5 \ln 2}{\ln 9} = \frac{\ln 2^5}{\ln 9} = \frac{\ln 32}{\ln 9}$$

Solids and Semiconductor Devices

Solid is a state of matter which has a definite shape and a definite volume. Most of the solids can be classified in three types as per their electrical conductivity, *i.e.* conductors, insulators and semiconductors.

Conductors are those materials through which electric charge can flow easily, while insulators are those through which electric charge is difficult to flow. There are, however, certain solids whose electrical conductivity is intermediate between conductors and insulators. They are called **semiconductors**. Silicon and germanium are a few examples of semiconductors.

In this chapter, we will discuss basic concepts of semiconductor physics and some semiconductor devices like junction diodes (two electrode device) and bi-polar junction transistor (three electrode device). Then, we will explore the basics of logic gates.

Classification of conductors, semiconductors and insulators

On the basis of the relative values of electrical conductivity (σ) or resistivity ($\rho = 1/\sigma$), the solids are broadly classified as

- (i) **Conductors or Metals** They possess very low resistivity (or high conductivity).

$$\rho \sim 10^{-2} - 10^{-8} \Omega\text{m}$$

$$\sigma \sim 10^2 - 10^8 \text{ Sm}^{-1}$$

- (ii) **Semiconductors** They have resistivity or conductivity intermediate to metals and insulators.

$$\rho \sim 10^{-5} - 10^6 \Omega\text{m}$$

$$\sigma \sim 10^5 - 10^{-6} \text{ Sm}^{-1}$$

Inside

1 Energy bands in solids

- Energy band formation in solids
- Classification of solids on the basis of energy bands

2 Types of semiconductors

- Electrical conduction through semiconductors
- Effect of temperature on conductivity of semiconductor

3 p-n junction

- Semiconductor diode or p-n junction diode
- p-n junction diode as a rectifier
- Special types of p-n junction diode

4 Junction transistors

- Transistor circuit configurations
- Transistor as an amplifier
- Transistor as an oscillator
- Transistor as a switch

5 Analog and digital circuits

- Binary system
- Decimal and binary number system

6 Logic gates

- Logic system
- Combination of logic gates
- NAND and NOR gates as digital building blocks

- (iii) **Insulators** They have high resistivity (or low conductivity).

$$\rho \sim 10^{11} - 10^{19} \Omega \text{m}$$

$$\sigma \sim 10^{-11} - 10^{-19} \text{Sm}^{-1}$$

The values of ρ and σ given above are indicative of magnitude and could well go outside the ranges as well.

ENERGY BANDS IN SOLIDS

According to Bohr model, the energy of electrons in an isolated atom is decided by the orbit in which it revolves. When two atoms comes close to each other to form solid, their outer orbits of electrons come very close or even overlap. Thus, the motion of electron in a solid is different from that in isolated atom. In a crystal, each electron has a unique position, *i.e.* no two electrons can have the same position. So, each electron will have a different energy level and these different energy levels with continuous energy variation are called energy bands.

Valence band and conduction band

The energy band which includes the energy levels of valence electrons is called **valence band** and the energy band above it, is called **conduction band**.

In isolated condition, *i.e.* with no external energy, all the valence electrons will reside in valence band. Normally, conduction band is empty but when it overlaps on valence band, electrons can move freely into it.

Energy band gap

If there is some energy gap between the conduction band and the valence band, then electrons in the valence band will remain bound and no free electrons will be available in conduction band.

The energy band gap is also defined as the difference between the highest valence band energy (E_V) and lowest conduction band energy (E_C).

i.e.

$$E_g = E_C - E_V$$

The energy gap for different materials is different.

Note If λ is the wavelength of radiation used in shifting the electron from valence band to conduction band, then energy band gap (E_g) is

$$E_g = h\nu = hc/\lambda$$

where, h is called Planck's constant and c is the velocity of light.

Forbidden bands or Forbidden energy gap

The various energy bands in a solid may or may not overlap depending upon the structure of solid. If they do not overlap, then the intervals between them represent energies which the electrons in the solid cannot have.

These intervals are called **forbidden bands** (as shown in the figure). It is also called **forbidden energy gap** because free electrons cannot exist in this gap.

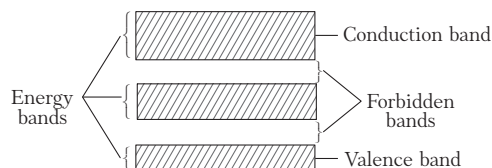


Fig. 14.1 Forbidden bands in a solid

If however, the adjacent energy bands in a solid overlap, the electrons have a continuous distribution of allowed energies as shown in the figure.

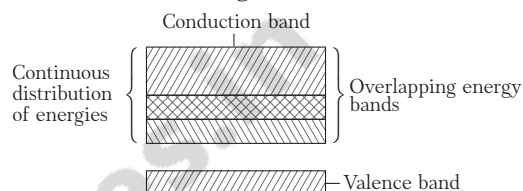


Fig. 14.2 Overlapping of energy bands

Note As temperature increases, forbidden energy gap decreases.

Fermi energy

It is the maximum possible energy possessed by free electrons of a material at absolute zero temperature (*i.e.* 0 K). The value of fermi energy is different for different materials.

Example 14.1 The maximum wavelength at which solid begin to absorb energy is 10000 Å. Calculate the energy gap of a solid (in eV).

Sol. The energy band gap is given by $E_g = h\nu = \frac{hc}{\lambda}$

where, h = Planck's constant, c = speed of light and λ = wavelength at which solid absorbs energy.

On putting the values of h , c and λ , we get

$$E_g = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{(10000 \times 10^{-10})} = 1.98 \times 10^{-19} \text{ J} = \frac{1.98 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 1.24 \text{ eV}$$

Energy band formation in solids

The electronic configuration of a solid like sodium atom which has 11 electrons is $1s^2, 2s^2, 2p^6$ and $3s^1$. The levels $1s$, $2s$ and $2p$ are completely filled. The level $3s$ is half filled and the levels above $3s$ are empty.

Consider a group of N sodium atoms all in ground state separated from each other by large distances such as in sodium vapour. There are total $11N$ electrons. Each atom has two energy states in $1s$ energy level, so there are $2N$ identical energy states labelled as $1s$ and all of them are

filled from $2N$ electrons. Similarly, energy level $2p$ has $6N$ identical energy states which are also completely filled. In $3s$ energy levels, N of the $2N$ states are filled by the electrons and the remaining N states are empty.

These ideas are shown in the table given below

Energy level	Total available energy states	Total occupied states
1s	$2N$	$2N$
2s	$2N$	$2N$
2p	$6N$	$6N$
3s	$2N$	N
3p	$6N$	0

In the above discussion, we have assumed that N sodium atoms are widely spread and hence the electrons of one atom do not interact with others.

As a result, energy states of different states (*e.g.* 1s) are identical. When atoms are drawn closer to one another, electron of one atom starts interacting with the electrons of the neighbouring atoms of the same energy states.

e.g. 1s electrons of one atom interact with 1s electrons of the other.

Due to interaction of electrons, the energy states are not identical, but a sort of energy band is formed.

Thus, the collection of these closely spaced energy levels are called an **energy band**.

These bands are shown in figure given below

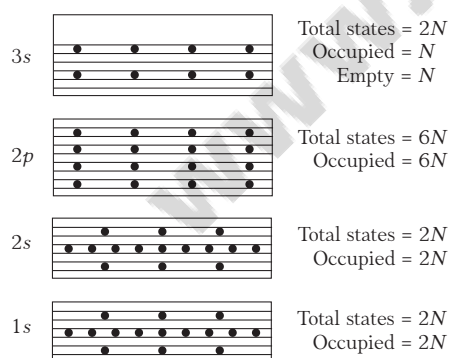


Fig. 14.3 Formation of energy bands in solids

Classification of solids on the basis of energy bands

Depending on whether the energy band gap is zero, large or small, the solids may be classified into conductors, insulators and semiconductors, as explained below

Conductors (Metals)

In case of metals, either the conduction band is partially filled and valence band is partially empty or conduction band and valence band overlap to each other.

In case of overlapping, electrons from valence band can easily move into conduction band, thus large number of electrons are available for conduction. In case, when valence band is empty, electrons from its lower level can move to higher level making conduction possible.

This is the reason why resistance of metals is low or the conductivity is high.

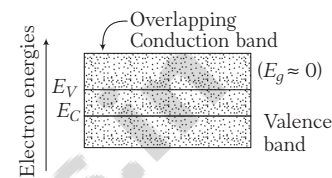


Fig. 14.4 Energy band gap for conductors

Insulators

In insulators, the valence band is completely filled whereas the conduction band is completely empty.

As there is no electron in conduction band, so no electrical conduction is possible. The energy gap between conduction band and valence band is so large ($E_g > 3$ eV) that no electron in valence band can be provided so much energy from any external source that it can jump this energy gap.

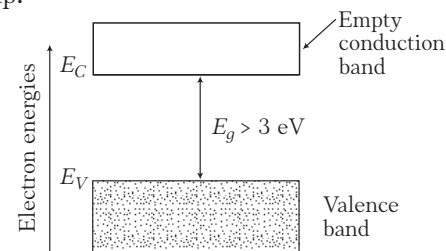


Fig. 14.5 Energy band gap for insulators

Semiconductors

The energy band structure of a semiconductor is shown in figure. It is similar to that of an insulator but with a comparatively small energy gap ($E_g < 3$ eV). At absolute zero temperature, the conduction band of semiconductors is totally empty and valence band is completely filled.

Therefore, they behave as insulators at low temperatures.

However, at room temperature, some electrons in the valence band acquire thermal energy greater than energy band gap and jump over to the conduction band where they are free to move under the influence of a small electric field and acquire small conductivity.

Hence, the resistance of semiconductor is not as high as that of insulators.

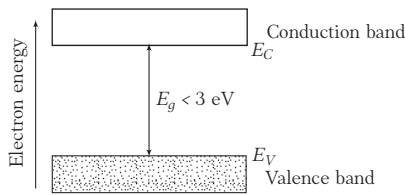


Fig. 14.6 Energy band gap for semiconductors

TYPES OF SEMICONDUCTORS

As discussed above, in semiconductors the conduction band and the valence band are separated by a relatively small energy gap. For silicon, this gap is 1.1 eV and for germanium it is 0.7 eV. However at 0 K, they behave as an insulator. On the basis of purity, semiconductors are divided in two group

1. Intrinsic semiconductors

A pure (free from impurity) semiconductor which has a valency 4 is called an intrinsic semiconductor. Pure germanium and silicon in their natural state are intrinsic semiconductors.

Si and Ge both have 4 valence electrons. In crystalline structure, each Si or Ge atom share each of its four valence electrons with four nearest neighbouring atoms and forms covalent bond.

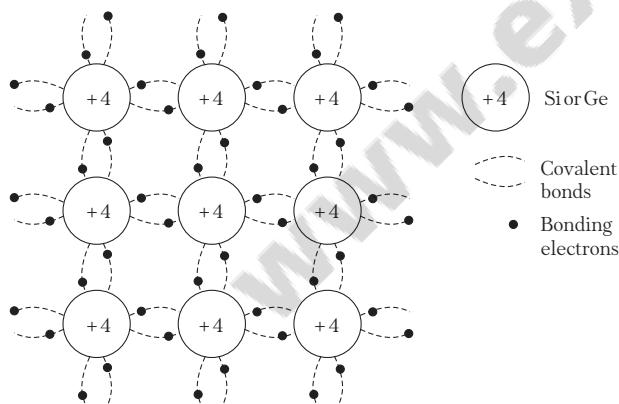


Fig. 14.7 Schematic two-dimensional representation of Si or Ge structure showing covalent bonds at low temperature (all bonds intact). +4 symbol indicates inner cores of Si or Ge.

At temperature close to zero, all shared electrons are tightly bound and so no free electrons are available to conduct electricity through the crystal.

At room temperature, however a few of the covalent bonds are broken due to thermal agitation and thus some of the valence electrons become free. Thus, we can say that a valence electrons is shifted to conduction band

leaving a vacancy of electron in valence band. This vacancy is known as **hole** and carries a positive charge e .

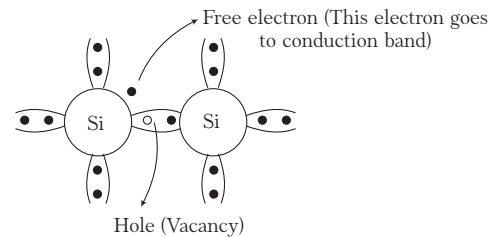


Fig. 14.8 Generation of hole due to thermal agitation

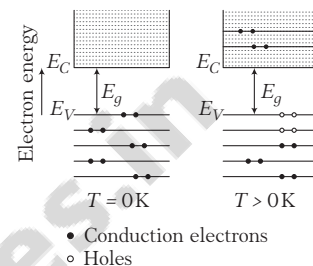


Fig. 14.9 Energy band diagram for an intrinsic semiconductor at $T = 0 \text{ K}$ and $T > 0 \text{ K}$

In intrinsic (or pure) semiconductors, the number of free electrons per unit volume (n_e) is equal to the number of holes per unit volume (n_h) and it is also equal to the number of intrinsic charge carriers per unit volume (n_i).

Mathematically, we have the relation

$$n_e = n_h = n_i$$

Here, (n_i) is also called **intrinsic carrier concentration**.

At equilibrium,

$$n_e n_h = n_i^2$$

This is known as **mass-action law**.

Under the action of an electric field, holes move towards negative potential giving hole current, I_h and electron move towards positive potential giving electron current I_e . The total current is the sum of the electron current I_e and the hole current I_h .

i.e.

$$I = I_e + I_h$$

Example 14.2 In an intrinsic (pure) semiconductor, the number of conduction electrons is 7×10^{19} per cubic metre. Find the total number of current carriers (electrons and holes) in the same semiconductor of size $1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ mm}$.

Sol. In an intrinsic semiconductor, $n_e = n_h$

where, n_e = number of conduction electrons per unit volume and n_h = number of holes per unit volume.

Given, $n_e = 7 \times 10^{19} \text{ per m}^3$

$$\therefore n_h = n_e = 7 \times 10^{19} \text{ per m}^3$$

So, total current carrier density

$$n_e + n_h = 7 \times 10^{19} + 7 \times 10^{19} = 14 \times 10^{19} \text{ per m}^3$$

Now, total number of current carrier
 = Number density $\times$ volume
 = $(14 \times 10^{19}) \times (10^{-2} \times 10^{-2} \times 10^{-3})$
 = 1.4×10^{13}

Example 14.3 Pure Si at 300 K has equal electron (n_e) and hole (n_h) concentration of $1.5 \times 10^{16} \text{ m}^{-3}$. By some means hole concentration increases to $3 \times 10^{22} \text{ m}^{-3}$.

Now, calculate n_e in the Si.

Sol. Given, $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$, $n_h = 3 \times 10^{22} \text{ m}^{-3}$

For the semiconductor in thermal equilibrium,

$n_e n_h = n_i^2$ (Law of mass action)

$$\Rightarrow n_e = \frac{n_i^2}{n_h} = \frac{(1.5 \times 10^{16})^2}{3 \times 10^{22}} = 7.5 \times 10^9 \text{ m}^{-3}$$

Example 14.4 The number of conduction electrons (in per cubic metre) present in a pure semiconductor is 6.5×10^{19} . Calculate the number of holes in a sample having dimensions $1 \text{ cm} \times 1 \text{ cm} \times 2 \text{ mm}$.

Sol. Here, number of conduction electrons, $n_e = 6.5 \times 10^{19} \text{ m}^{-3}$

Volume of the sample, $V = 1 \text{ cm} \times 1 \text{ cm} \times 2 \text{ mm} = 2 \times 10^{-7} \text{ m}^3$

Number of holes in the sample (n_h) = Number of electrons in the sample (n_e)

$$\begin{aligned} &= n_e \times V \\ &= 6.5 \times 10^{19} \times 2 \times 10^{-7} \\ &= 1.3 \times 10^{13} \end{aligned}$$

2. Extrinsic semiconductors

The conductivity of an intrinsic semiconductor is very poor (unless the temperature is very high). At ordinary temperature, only one covalent bond breaks in 10^9 atoms of Ge. It means that, only 1 atom in 10^9 atoms is available for conduction. Conductivity of an intrinsic (pure) semiconductor is significantly increased, if some pentavalent or trivalent impurity is mixed with it. Such impure semiconductors are called **extrinsic or doped semiconductors**.

Note When some desirable impurity is added to intrinsic semiconductor deliberately, then this process is called **doping** and the impurity atoms are called **dopants**.

There are two types of dopants used in doping in the tetravalent Si or Ge

- Trivalent (valency 3) atoms: *e.g.* Indium (In), Boron (B), Aluminium (Al), etc.
- Pentavalent (valency 5) atoms: *e.g.* Arsenic (As), Antimony (Sb), phosphorus (P), etc.

Depending upon the nature of impurity added in intrinsic semiconductor, the extrinsic semiconductors are of two types

- p*-type semiconductor
- n*-type semiconductor

p-type semiconductors

It is obtained when a trivalent impurity (*e.g.* boron, aluminium, gallium or indium) is added to a germanium or silicon crystal.

The impurity atom has one valence electron less than Si or Ge, therefore its three valence electrons form covalent bonds with neighbouring three Ge (or Si) atoms while the fourth valence electron of Ge (or Si) is not able to form the bond.

So, the bond between the fourth neighbouring Ge (or Si) and the trivalent impurity has a vacancy or hole as shown in Fig. 14.10(a).

This vacancy can be filled by an electron in outer orbit in the atom (it may jump to fill the vacancy), leaving a vacancy or hole at its own site.

With this electron, the trivalent impurity atom becomes negatively charged.

So, the dopant atom of *p*-type semiconductor can be considered as core of the negative charge along with its associated hole.

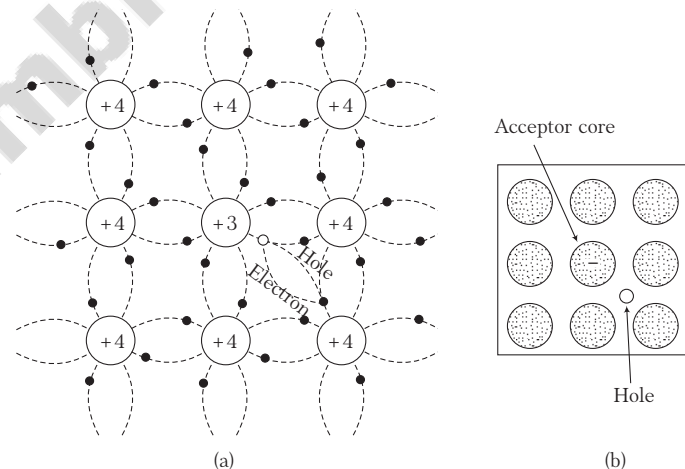


Fig. 14.10 (a) Trivalent acceptor atom (In, Al, B, etc.) doped in tetravalent Si or Ge lattice giving *p*-type semiconductor.

(b) Commonly used schematic representation of *p*-type material which shows only the fixed core of the substituent acceptor with one effective additional negative charge and its associated hole.

The trivalent impurity atoms are called **acceptor atoms** because they create holes which accept electrons.

Following points are worth noting regarding to *p*-type semiconductors

- Holes are the majority charge carriers and electrons are minority charge carriers in case of *p*-type semiconductors or number of holes are much greater than the number of electrons. *i.e.* $n_h \gg n_e$; $i_h > i_e$.
- p*-type semiconductor is electrically neutral.

(c) Conductivity, $\sigma = n_h \mu_h e$

(d) *p*-type semiconductor can be shown as

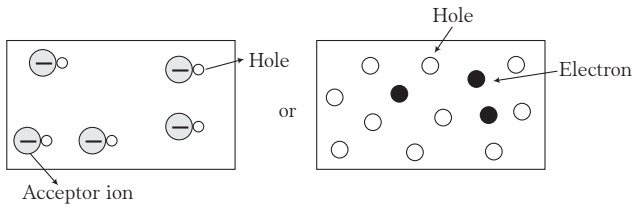


Fig. 14.11 Representation of *p*-type semiconductor

n-type semiconductors

It is obtained, when a pentavalent impurity atom (*e.g.* antimony, phosphorus or arsenic) is added to a Ge (or Si) crystal. Here, the impurity atom has one valence electron more than Si or Ge, therefore four of the five valence electrons of the impurity atom form covalent bonds with four neighbouring Ge (or Si) atoms and the fifth valence electron becomes free to move inside the crystal lattice. Thus, by doping pentavalent impurity, number of free electrons increases.

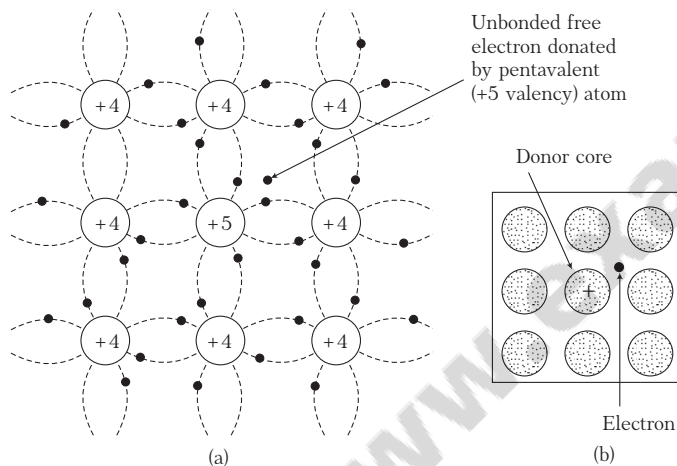


Fig. 14.12 (a) Pentavalent donor atom (*As, Sb, P, etc.*) doped in tetravalent Si or Ge giving *n*-type semiconductor and (b) Commonly used schematic representation of *n*-type material which shows only the fixed cores of the substituent donors with one additional effective positive charge and its associated extra electron.

The impurity (pentavalent) atoms are called **donor atoms** because they donate conduction electrons to the crystal. The number of electrons made available for conduction (by dopant atoms) depend strongly upon the dopant level and is independent of increase in temperature.

Following points are noteworthy regarding *n*-type semiconductors

- (a) Electrons are the majority charge carriers and holes are minority charge carriers or number of electrons are much greater than the number of holes. *i.e.*

$$n_e \gg n_h; i_e \gg i_h$$

- (b) *n*-type semiconductor is also electrically neutral.

(c) Conductivity, $\sigma = n_e \mu_e e$

(d) *n*-type semiconductor can be shown as

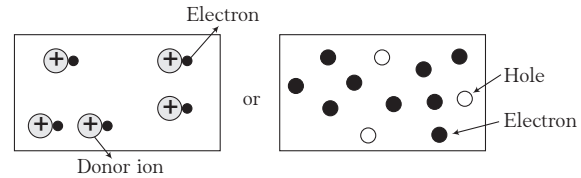


Fig. 14.13 Representation of *n*-type semiconductor

Energy band in extrinsic semiconductors

Energy band structure of semiconductor is affected due to doping. In extrinsic semiconductors, additional energy states due to donor impurities (donor energy level E_D) and acceptor impurities (acceptor energy level E_A) also exist. In the energy band diagram of *n*-type semiconductor, the donor energy level E_D is slightly below the bottom E_C of conduction band and the electrons from this level move into conduction band with very small supply of energy. So the conduction band have most of the electrons coming from donor impurities.

In *p*-type semiconductors, the acceptor energy level E_A is slightly above the top energy level E_V of the valence band. With very small supply of energy, an electron from the valence band can jump to the energy level E_A and ionise the acceptor negatively. At room temperature, most of the acceptor atoms get ionised leaving hole in valence band. Thus, at room temperature, the density of holes in valence band is mainly due to impurity.

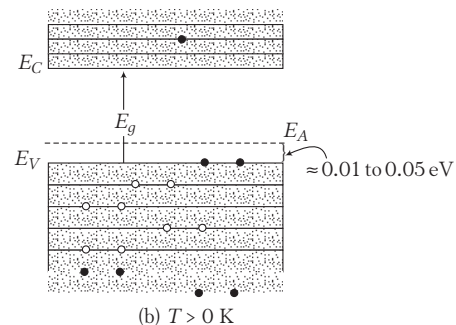
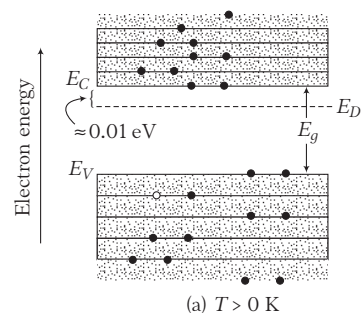


Fig. 14.14 Energy bands of (a) *n*-type semiconductor at $T > 0$ K and (b) *p*-type semiconductor at $T > 0$ K

Electrical conduction through semiconductors

When a battery is connected across a semiconductor (whether intrinsic or extrinsic), a potential difference is developed across its ends. Due to the potential difference, an electric field is produced inside the semiconductors. A current (although very small) starts flowing through the semiconductor. This current may be due to the motion of (i) free electrons (i_e) and (ii) holes (i_h). Electrons move in opposite direction of electric field, while holes move in the same direction.

The motion of holes towards right (in the figure) take place because electrons from right hand side come to fill this hole creating a new hole in their own position. Thus, we can say that holes are moving from left to right. So, current in a semiconductor can be written as

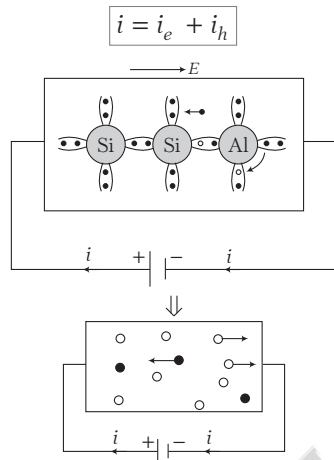


Fig. 14.15 Conduction through semiconductors

But it should be noted that, mobility of holes is less than the mobility of electrons.

Conductivity of semiconductors is given by

$$\sigma = n_e e \mu_e + n_h e \mu_h$$

where, n_e and n_h are densities of conducting electrons and holes, respectively and μ_e and μ_h are their respective mobilities.

where, $\mu_e = \frac{v_e}{E}$, $\mu_h = \frac{v_h}{E}$, here v_e and v_h are drift velocities.

Effect of temperature on conductivity of semiconductor

The conductivity of a semiconductor increases (or the resistivity decreases) with rise in temperature. This is because, as the temperature rises, more and more of the covalent bonds in the crystal lattice break, thus creating

more and more charge carriers (electron-hole pairs). This results in increasing conductivity.

The temperature dependence of charge carrier density is given by

$$n_i \propto e^{-E_g/2kT}$$

where, n_i is number of electron-hole pair, E_g is the energy gap of the semiconductor and k is Boltzmann's constant. Thus, the charge carrier density and hence the conductivity of the semiconductor increases exponentially with rise in temperature.

Example 14.5 A p-type semiconductor has acceptor level 50 meV above the valence band. Find the maximum wavelength of light that can create a hole.

Sol. To create a hole, an electron of valence band has to be excited into one of the acceptor levels, which are 50 meV above the valence band. Hence, a minimum of 50 meV energy is needed to create a hole.

$$\begin{aligned} \text{As, } E_{\min} &= \frac{hc}{\lambda_{\max}} \\ \therefore \lambda_{\max} &= \frac{hc}{E_{\min}} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{50 \times 1.6 \times 10^{-22}} \\ &= 2.47 \times 10^{-5} \text{ m} \end{aligned}$$

Example 14.6 A Ge specimen is doped with Al. The concentration of acceptor atoms is $\sim 10^{21}$ atoms/m³. Given that, the intrinsic concentration of electron-hole pairs is $\sim 10^{19}$ /m³, then find the concentration of electrons in the specimen.

Sol. According to law of mass action, $n_i^2 = n_h n_e$

Here, n_i = intrinsic concentration of electron hole pair = 10^{19} /m³

and n_h = concentration of acceptor atoms = 10^{21} atoms/m³.

$$\therefore n_e = \frac{n_i^2}{n_h} = \frac{(10^{19})^2}{(10^{21})} = 10^{17} \text{ /m}^3$$

Example 14.7 A semiconductor has equal electron-hole concentration of $6 \times 10^8 \text{ m}^{-3}$. On doping with certain impurity, electron concentration increases to $9 \times 10^{12} \text{ m}^{-3}$.

(i) Identify the new semiconductor obtained after doping.

(ii) Calculate the new hole concentration.

(iii) How does the energy gap vary with doping?

Sol. (i) As, the electron concentration increases after doping, so the new semiconductor obtained is of n-type.

(ii) As, $n_e n_h = n_i^2$ (Law of mass action)

Here, $n_i = 6 \times 10^8 \text{ m}^{-3}$, $n_e = 9 \times 10^{12} \text{ m}^{-3}$

$$\therefore n_h = \frac{n_i^2}{n_e} = \frac{(6 \times 10^8)^2}{9 \times 10^{12}} = 4 \times 10^4 \text{ m}^{-3}$$

(iii) The energy gap decreases with doping.

Example 14.8 Suppose a pure Si crystal has 5×10^{28} atoms m^{-3} . It is doped by 1 ppm concentration of pentavalent As. Calculate the number of electrons and holes. (Given that, $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$)

Sol. Here, $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$

Doping concentration of pentavalent As atoms
 $= 1 \text{ ppm} = 1 \text{ part per million}$

$\therefore$ Number density of pentavalent as atoms,

$$N_D = \frac{5 \times 10^{28}}{10^6} = 5 \times 10^{22} \text{ atom m}^{-3}$$

Now, the thermally generated electrons ($n_i \propto 10^{16} \text{ m}^{-3}$) are negligibly small as compared to those produced by doping, so

$$n_e \approx N_D = 5 \times 10^{22} \text{ m}^{-3}$$

Also, $n_e n_h = n_i^2$ (Law of mass action)

$$\therefore n_h = \frac{n_i^2}{n_e} = \frac{1.5 \times 10^{16} \times 1.5 \times 10^{16}}{5 \times 10^{22}} = 4.5 \times 10^9 \text{ m}^{-3}$$

Example 14.9 What will be the conductivity of pure silicon crystal at 300K temperature? If electron hole pairs per cm^3 is 1.072×10^{10} at this temperature and mobilities are $\mu_h = 1350 \text{ cm}^2/\text{V-s}$ and $\mu_e = 480 \text{ cm}^2/\text{V-s}$.

Sol. Conductivity of pure silicon,

$$\sigma = n_i e \mu_e + n_i e \mu_h = n_i e (\mu_e + \mu_h)$$

where, n_i is the total number of charge carrier, μ_e and μ_h are the mobility of electrons and holes, respectively.

Here, $n_i = 1.072 \times 10^{10} \text{ per cm}^3$

$$\mu_h = 1350 \text{ cm}^2/\text{V-s}$$

$$\mu_e = 480 \text{ cm}^2/\text{V-s}$$

$$\Rightarrow \sigma = (1.072 \times 10^{10})(1.6 \times 10^{-19})(1350 + 480) \\ = 3.14 \times 10^{-6} \text{ mho/cm}$$

Example 14.10 An intrinsic germanium semiconductor is to be made n-type semiconductor of conductivity 6 mho/cm. Calculate the number density of donor atoms required, if the mobility of electrons in n-type semiconductor is $3850 \text{ cm}^2/\text{V-s}$.

Sol. In n-type semiconductor, $n_e \gg n_h$

Let number density of donor atoms, $N_0 = n_e$

Conductivity of semiconductor is given by $\sigma = n_e e \mu_e$

On putting the values, we get

$$6 \times 10^2 = n_e \times 1.6 \times 10^{-19} \times 3850 \times 10^{-4}$$

$$\Rightarrow n_e = 9.7 \times 10^{21} / \text{m}^3$$

Example 14.11 The concentration of hole-electron pairs in pure germanium at $T = 300 \text{ K}$ is 7×10^{15} per cubic metre. Antimony is doped into germanium in a proportion of 1 atom per 10^7 Ge atoms. Assuming that, half of the impurity atoms contribute electron in the conduction band, calculate the factor by which the number of charge carriers increases due to doping. The number of germanium atoms per cubic metre is 5×10^{28} .

Sol. In pure semiconductor, electron-hole pair

$$\text{concentration, } n_h = n_e = 7 \times 10^{15} \text{ m}^{-3}$$

Total charge carrier,

$$n_{\text{total initial}} = n_h + n_e = 7 \times 10^{15} + 7 \times 10^{15} \\ = 14 \times 10^{15}$$

After doping, donor impurity,

$$N_D = \frac{5 \times 10^{28}}{10^7} = 5 \times 10^{21}$$

and

$$n'_e = \frac{N_D}{2} = 2.5 \times 10^{21}$$

So,

$$n_{\text{final}} = n_h + n'_e$$

$\Rightarrow$

$$n_{\text{final}} \approx n'_e \approx 2.5 \times 10^{21} \quad (\because n'_e \gg n_h)$$

$$\text{Factor} = \frac{n_{\text{final}} - n_{\text{initial}}}{n_{\text{initial}}} \\ = \frac{2.5 \times 10^{21} - 14 \times 10^{15}}{14 \times 10^{15}} \\ \approx \frac{2.5 \times 10^{21}}{14 \times 10^{15}} = 1.8 \times 10^5$$

Example 14.12 The number of silicon atoms per m^3 is 5×10^{28} . This is doped simultaneously with 5×10^{22} atoms per m^3 of arsenic and 5×10^{20} per m^3 atoms of indium. Calculate the number of electrons and holes. Given that, $n_i = 1.5 \times 10^{16} / \text{m}^3$. Is the material n-type or p-type?

Sol. We know that for each atom doped of arsenic, one free electron is received. Similarly, for each atom doped of indium, a vacancy is created.

So, the number of free electrons introduced by pentavalent impurity added, $n_e = N_{\text{As}} = 5 \times 10^{22} \text{ m}^{-3}$... (i)

The number of holes introduced by trivalent impurity added,

$$n_h = N_{\text{In}} = 5 \times 10^{20} \text{ m}^{-3}$$

$$\text{Now, } n_e - n_h = 5 \times 10^{22} - 5 \times 10^{20} \\ = 4.95 \times 10^{22} \quad \dots \text{(ii)}$$

$$\text{So, } (n_e + n_h)^2 = (n_e - n_h)^2 + 4n_e n_h \\ n_e + n_h = \sqrt{(4.95 \times 10^{22})^2 + 4(1.5 \times 10^{16})^2} \quad \dots \text{(iii)} \\ (\because n_e n_h = n_i^2)$$

Adding Eqs. (iii) and (ii), we get

$$2n_e = 4.95 \times 10^{22} + \sqrt{(4.95 \times 10^{22})^2 + 4(1.5 \times 10^{16})^2} \\ n_e = \frac{1}{2} [4.95 \times 10^{22} + \sqrt{(4.95 \times 10^{22})^2}] \\ = 4.95 \times 10^{22} / \text{m}^3$$

$$\text{Now, } n_i^2 = n_h \times n_e$$

$$\text{or } n_h = \frac{n_i^2}{n_e} = \frac{(1.5 \times 10^{16})^2}{4.95 \times 10^{22}}$$

$$= 4.54 \times 10^9 / \text{m}^3$$

As, number of electrons $n_e (= 4.95 \times 10^{22})$ is greater than number of holes $n_h (= 4.5 \times 10^9)$.

So, the material is n -type semiconductor.

Example 14.13 Find the resistivity of a sample in which 10^{19} atoms of phosphorous are added per m^3 . Take the resistivity of pure silicon as $3000 \Omega\text{-m}$ and the mobilities of electrons and holes as $0.15 \text{ m}^2/\text{V-s}$ and $0.030 \text{ m}^2/\text{V-s}$, respectively.

Sol. The conductivity of a pure silicon is given by

$$\sigma_i = n_i e (\mu_e + \mu_h)$$

Also, the resistivity is reciprocal of conductivity.

$$\therefore \rho_i = \frac{1}{\sigma_i} = \frac{1}{n_i e (\mu_e + \mu_h)}$$

$$\begin{aligned} \Rightarrow n_i &= \frac{1}{\rho_i e (\mu_e + \mu_h)} \\ &= \frac{1}{3000 \times 1.6 \times 10^{-19} (0.15 + 0.030)} \\ &= 1.157 \times 10^{16} / \text{m}^3 \end{aligned}$$

When 10^{19} atoms of phosphorus (donor atoms) are added per m^3 , we have

$$n_e \gg n_i$$

$$\text{or } n_e \gg n_h$$

$$\Rightarrow n_e = 10^{19}$$

$$\begin{aligned} \therefore \rho &= \frac{1}{n_e e \mu_e} \\ &= \frac{1}{10^{19} \times 1.6 \times 10^{-19} \times 0.15} \\ &= 4.17 \Omega\text{-m} \end{aligned}$$

Example 14.14 A pure germanium plate of area $3.5 \times 10^{-4} \text{ m}^2$ and of thickness $1.5 \times 10^{-3} \text{ m}$ is connected across a battery of potential 5 V . Find the amount of current produced at room temperature in the germanium sample. Also, find the amount of heat generated in the plate in 120 s . Given that, the concentration of carriers in germanium at room temperature is 1.6×10^6 per cubic metre. The mobilities of electrons and holes are $0.4 \text{ m}^2/\text{V-s}$ and $0.2 \text{ m}^2/\text{V-s}$, respectively.

Sol. Conductivity of semiconductor is given by

$$\sigma = n_e e \mu_e + n_h e \mu_h$$

Since semiconductor is intrinsic, so $n_e = n_h = n_i$

$$\begin{aligned} \text{Conductivity, } \sigma &= n_i e (\mu_e + \mu_h) \\ &= 1.6 \times 10^6 \times 1.6 \times 10^{-19} (0.4 + 0.2) \\ &= 1.536 \times 10^{-13} \Omega^{-1}\text{m}^{-1} \end{aligned}$$

Current flowing, $I = JA$

$$\text{where, } J = \text{current density} = \sigma E = \sigma \left(\frac{V}{d} \right)$$

$$\begin{aligned} \therefore I &= JA = \sigma \left(\frac{V}{d} \right) \cdot A \\ &= 1.536 \times 10^{-13} \left(\frac{5}{1.5 \times 10^{-3}} \right) (3.5 \times 10^{-4}) \\ &= 1.79 \times 10^{-13} \text{ A} \end{aligned}$$

$$\begin{aligned} \text{Heat produced, } H &= VIt = 5 \times 1.79 \times 10^{-13} \times 120 \\ &= 10.74 \times 10^{-11} \text{ J} \end{aligned}$$

Example 14.15 The energy gap of pure Si is 1.12 eV . The mobilities of electrons and holes are respectively $0.140 \text{ m}^2/\text{V-s}$ and $0.050 \text{ m}^2/\text{V-s}$. They are independent of temperature. The intrinsic carrier concentration is given by

$$n_i = n_0 e^{-E_g/2kT}$$

where, n_0 is a constant, E_g is the gap width and k is the Boltzmann's constant whose value is $1.38 \times 10^{-23} \text{ JK}^{-1}$.

Find the ratio of the electrical conductivities of Si at 400 K and 200 K .

Sol. Conductivity of semiconductor is given by

$$\sigma = n_e e \mu_e + n_h e \mu_h$$

Here, $n_e = n_h = n_i$ (pure semiconductor)

$$\sigma = n_i e (\mu_e + \mu_h) = e (\mu_e + \mu_h) n_i$$

$$= e (\mu_e + \mu_h) n_0 e^{-E_g/2kT}$$

$$\begin{aligned} \therefore \frac{\sigma_{400}}{\sigma_{200}} &= \frac{e^{\frac{-E_g}{2k \times 400}}}{e^{\frac{-E_g}{2k \times 200}}} = e^{E_g/800k} \\ &= e^{1.12 \times 1.6 \times 10^{-19} / 800 \times 1.38 \times 10^{-23}} = e^{16.23} \end{aligned}$$

CHECK POINT 14.1

- The expected energy of the electrons at absolute zero is called
 - fermi energy
 - emission energy
 - work function
 - potential energy
- The conduction band in a solid is partially filled at 0 K. The solid sample is a
 - conductor
 - semiconductor
 - insulator
 - None of these
- The valence band and conduction band of a solid overlap at low temperature, the solid may be
 - a metal
 - a semiconductor
 - an insulator
 - None of these
- In an insulator, forbidden energy gap between the valence band and conduction band is of the order of
 - 1 MeV
 - 0.1 MeV
 - 1 eV
 - 5 eV
- The forbidden energy band gap in conductors, semiconductors and insulators are E_{g1} , E_{g2} and E_{g3} respectively. The relations among them is
 - $E_{g1} = E_{g2} = E_{g3}$
 - $E_{g1} < E_{g2} < E_{g3}$
 - $E_{g1} > E_{g2} > E_{g3}$
 - $E_{g1} < E_{g2} > E_{g3}$
- Doping of intrinsic semiconductor is done
 - to neutralise charge carries
 - to increase the concentration of majority charge carriers
 - to make it neutral before disposal
 - to carry out further purification
- The majority charge carriers in p -type semiconductor are
 - electrons
 - protons
 - holes
 - neutrons
- A p -type semiconductor can be obtained by adding
 - arsenic to pure silicon
 - gallium to pure silicon
 - antimony to pure germanium
 - phosphorous to pure germanium
- To obtain p -type semiconductor, we need to dope pure Si with
 - aluminium
 - phosphorous
 - oxygen
 - germanium
- In p -type semiconductor, the majority and minority charge carriers are respectively.
 - Protons and electrons
 - Electrons and protons
 - Electrons and holes
 - Holes and electrons
- The charge on a hole is equal to the charge of
 - zero
 - proton
 - neutron
 - electron
- If n_e and n_h represent the number of free electrons density and holes density respectively in a semiconducting material, then for n -type semiconducting material
 - $n_e \ll n_h$
 - $n_e \gg n_h$
 - $n_e = n_h$
 - $n_e = n_h = 0$
- Which statement is correct?
 - n -type germanium is negatively charged and p -type germanium is positively charged.
 - Both n -type and p -type germanium are neutral.
 - n -type germanium is positively charged and p -type germanium is negatively charged.
 - Both n -type and p -type germanium are negatively charged.
- When a semiconductor is heated, its resistance
 - decreases
 - increases
 - remains unchanged
 - Nothing is definite
- Which of the following has negative temperature coefficient of resistance?
 - Copper
 - Aluminium
 - Iron
 - Germanium

p-n JUNCTION

A p -type or n -type silicon crystal can be made by adding appropriate impurity as discussed above. Separately p - and n -type semiconductors have limited use. When the two types are combined, a number of new characteristic appears, which make the combination a very useful device. So, when a p -type semiconductor is suitably joined to an n -type semiconductor, then resulting arrangement is called p - n junction.

A p - n junction is formed either when a p -type semiconductor is grown on n -type semiconductor or n -type is grown on p -type semiconductor as given below

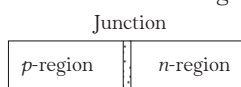


Fig. 14.16 A p - n junction

Formation of depletion region in p - n junction

In an n -type semiconductor, the concentration of electrons is more than the concentration of holes. Similarly, in a p -type semiconductor, the concentration of holes is more than the concentration of electrons.

During formation of p - n junction and due to the concentration gradient across p and n -sides, holes diffuse from p -side to n -side ($p \rightarrow n$) and electrons diffuse from n -side to p -side ($n \rightarrow p$).

The diffused charge carriers combine with their counterparts in the immediate vicinity of the junction and neutralise each other.

Thus, near the junction positive charge is built on n -side and negative charge on p -side as shown in the figure

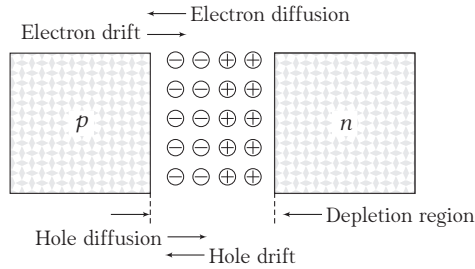


Fig. 14.17 p - n junction formation process

This set up a potential difference across the junction and an internal electric field E_i directed from n -side to p -side. The equilibrium is established when the field E_i becomes strong enough to stop further diffusion of the majority charge carriers (however it helps the minority charge carriers to diffuse across the junction).

The region on either side of the junction which becomes depleted (free) from the mobile charge carriers is called **depletion region** or **depletion layer**. The width of depletion region is of the order of 10^{-6} m or $1\ \mu\text{m}$.

The potential difference developed across the depletion region is called the **potential barrier**. Potential barrier depends on dopant concentration in the semiconductor and temperature of the junction.

On the average, the potential barrier in p - n junction is ~ 0.5 V and is given by $E_B = \frac{V_B}{d} \Rightarrow V_B = E_B d$

where, E_B = barrier electric field
and d = width of depletion region.

For Ge, $V_B = 0.3$ V and for silicon $V_B = 0.7$ V.

Given below are the graphs of potential *versus* distance, charge density *versus* distance and electric field *versus* distance across the depletion region.

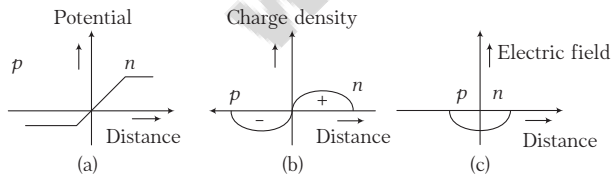
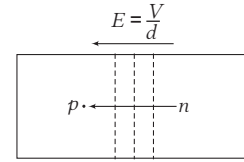


Fig. 14.18 (a) Potential *versus* distance, (b) charge density *versus* distance, (c) Electric field *versus* distance across the depletion region

Example 14.16 A potential barrier of 0.5 V exists across a p - n junction (i) If the depletion region is 5×10^{-7} m wide, what is the intensity of the electric field in the region (ii) An electron with speed $5 \times 10^5\ \text{ms}^{-1}$ approaches the p - n junction from the n -side, with what speed will it enter the p -side?

Sol. (i) Width of depletion layer, $d = 5 \times 10^{-7}$ m

$$\text{Electric field, } E_B = \frac{V}{d} = \frac{0.5}{5 \times 10^{-7}} = 10^6\ \text{Vm}^{-1}$$



(ii) According to work-energy theorem,

$$\frac{1}{2} m_e v_i^2 = eV + \frac{1}{2} m_e v_f^2$$

$$\begin{aligned} \text{Speed, } v_f &= \sqrt{\frac{m_e v_i^2 - 2eV}{m_e}} \\ &= \sqrt{\frac{9 \times 10^{-31} \times (5 \times 10^5)^2 - 2 \times 1.6 \times 10^{-19} \times (0.5)}{9 \times 10^{-31}}} \\ &= 2.7 \times 10^5\ \text{ms}^{-1} \end{aligned}$$

Current flow across a p - n junction

There are two types of current flow across a p - n junction, which are discussed below

1. Diffusion current

Because of concentration difference, holes try to diffuse from the p -side to the n -side and electrons from n -side to p -side at the p - n junction. However, only those holes/electrons cross the junction which have high kinetic energy. The diffusion give rise to a current from p -side to n -side called diffusion current.

2. Drift current

Because of thermal collisions, electron-hole pairs are created at every part of a p - n junction. However, if an electron-hole pair is created in the depletion region, the electron is pushed by the electric field towards the n -side and the hole towards the p -side.

This gives rise to a current from n -side to p -side called the drift current.

Thus, we have, diffusion current,

$$I_{df} \longrightarrow \text{from } p\text{-side to } n\text{-side}$$

and drift current, $I_{dr} \longrightarrow \text{from } n\text{-side to } p\text{-side}$

In steady state, $I_{df} = I_{dr}$ or $I_{\text{net}} = 0$

Semiconductor diode or p - n junction diode

A semiconductor diode [Fig. (a)] is basically a p - n junction with metallic contacts provided at the ends for the application of an external voltage. It is a two terminal

device. A p - n junction diode is represented by the symbol as shown in Fig. (b).

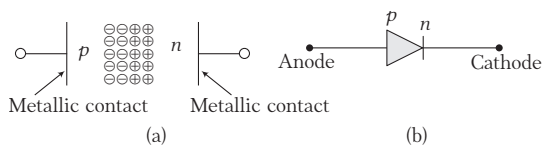


Fig. 14.19 (a) Semiconductor diode
(b) Symbol for p - n junction diode

The direction of arrow indicates the conventional direction of current.

Biasing of junction diode

Biasing is the method of connecting external battery or emf source to a p - n junction diode. The junction diode can be connected to an external battery in two ways, called **forward biasing** and **reverse biasing** of the junction.

1. Forward biasing

A junction is said to be forward biased when the positive terminal of the external battery is connected to the p -side and negative terminal to the n -side of the diode.

Flow of current in forward biasing

In this situation, the forward voltage opposes the potential barrier, due to which the potential barrier decreases and hence width of depletion layer decreases.

Under the effect of external electric field, holes in the p -region and electrons in n -region, both move towards the junction.

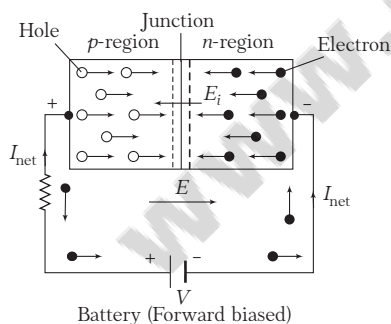


Fig. 14.20 Forward biased p - n junction diode

These holes and electrons mutually combine just near the junction and cease to exist. For each electron-hole combination, a covalent bond breaks up in the p -region near the positive terminal of the battery.

Of the hole and electron so produced, the hole moves towards the junction, while the electron enters the positive terminal of the battery through the connecting wire.

Just at this moment, an electron is released from the negative terminal of the battery which enters the n -region to replace the electron lost by combining with a hole at the junction.

Thus, a current called **forward current**, is constituted by the motion of majority charge carriers across the junction. In forward bias, the junction diode offers low resistance (ideally zero). Also, $I_{df} > I_{dr}$ or I_{net} is from p -side to n -side.

2. Reverse biasing

A junction diode is said to be reverse biased when the positive terminal of the external battery is connected to the n -side and negative terminal to the p -side of the diode.

Flow of current in reverse biasing

In this situation, the reverse voltage supports the potential barrier, due to which the potential barrier increases and hence width of depletion layer increases.

Under the effect of external electric field, holes in the p -region and electrons in the n -region are pushed away from the junction, i.e. they cannot be combined at the junction. So, there is almost no flow of current due to majority charge carriers.

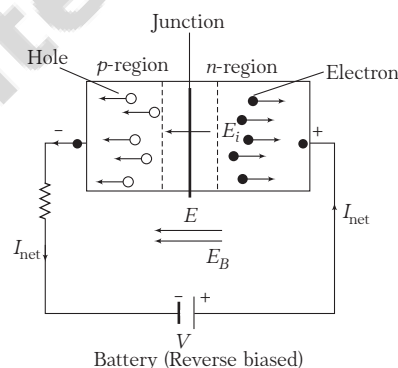
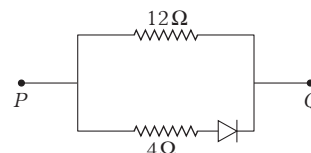


Fig. 14.21 Reverse biased p - n junction diode

However, a very small current due to minority charge carriers, flows across the junction. This current is called **reverse saturation current**. Here, $I_{dr} > I_{df}$ or I_{net} is from n -side to p -side.

Note The p - n junction allows a much larger current flow in forward biasing than in reverse biasing. This is crudely, the basis of the action of a p - n junction as a rectifier.

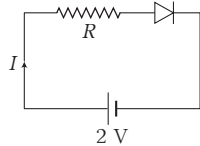
Example 14.17 Find the net resistance of the network shown in the figure between the points P and Q , if (i) $V_P > V_Q$, (ii) $V_P < V_Q$.



Sol. (i) When $V_P > V_Q$, the diode is in forward biased. Let, the resistance of diode will be taken as zero. Hence, $4\ \Omega$ resistance is ineffective. So, the net resistance is $\frac{4 \times 12}{4 + 12} = 3\ \Omega$.

- (ii) When $V_P < V_Q$, the diode is reverse biased. Let, there will be no current in the diode branch, hence $4\ \Omega$ resistance is ineffective. So, the net resistance is $12\ \Omega$.

Example 14.18 The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all current and a maximum power rating of 200 mW . What should be the value of the resistor R , connected in series with the diode, for obtaining maximum current?

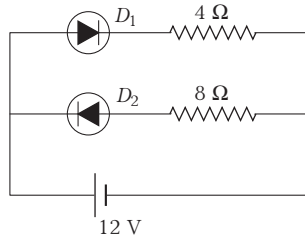


Sol. Current through diode (or circuit), $I = \frac{\text{Power}}{\text{Voltage}}$

$$\therefore I = \frac{200 \times 10^{-3} \text{ W}}{0.5 \text{ V}} = 0.4 \text{ A}$$

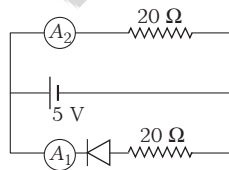
$$\text{and resistance, } R = \frac{\text{Net voltage}}{\text{Current}} = \frac{2 - 0.5}{0.4} = 3.75\ \Omega$$

Example 14.19 Find current passing through $4\ \Omega$ and $8\ \Omega$ resistance in the circuit shown in figure.

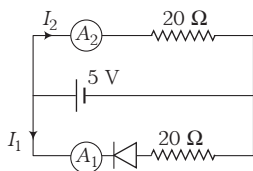


Sol. In the given circuit, diode D_1 is in forward biased and D_2 is in reverse biased. Hence, D_1 will conduct but D_2 will not. Therefore, current through $8\ \Omega$ resistance will be zero while through $4\ \Omega$ resistance will be $\frac{12}{4} = 3\text{ A}$.

Example 14.20 Consider the circuit diagram shown alongside. Two ammeters A_1 and A_2 are connected across a diode and resistor, respectively. Find the amount of current flowing through these two ammeters. Neglect the resistances of the ammeters.



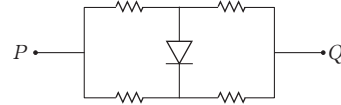
Sol. Let the current I_1 flows across diode and I_2 flows across A_2 as shown below



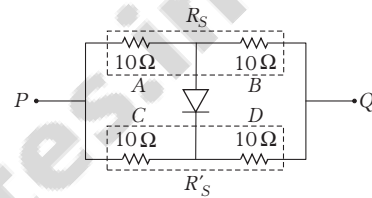
Diode is in reverse biased condition, so it will not conduct. Hence, ammeter A_1 will not show any reading. Now, the value of I_2 will be

$$I_2 = \frac{V}{R} = \frac{5}{20} = 0.25 \text{ A}$$

Example 14.21 Find the net resistance between two points P and Q , if the value of each resistance shown in the figure is $10\ \Omega$.



Sol. The given circuit is in the form of a Wheatstone bridge,



$$\text{i.e. } \frac{A}{B} = \frac{C}{D}$$

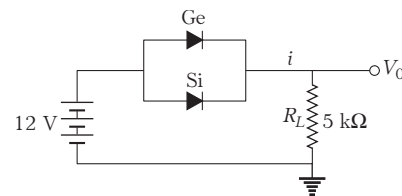
Hence no current will flow through diode, i.e. it will not offer any resistance. Hence, net resistance will be

$$R_N = \frac{R_S \times R'_S}{R_S + R'_S}$$

where R_S and $R'_S = (10 + 10) = 20\ \Omega$

$$\Rightarrow R_N = 10\ \Omega$$

Example 14.22 (i) Calculate the value of V_0 and i , if the silicon and germanium diode start conducting at 0.7 V and 0.3 V , respectively.



(ii) If the Ge diode connection is now reversed, what will be the new values of V_0 and i ?

Sol. (i) Ge diode will start conducting before the silicon diode does so. The effective forward voltage across Ge diode is $(12 - 0.3)\text{ V} = 11.7\text{ V}$. This will appear as the output voltage across the load, i.e.

$$V_o = 11.7 \text{ V}$$

The current through R_L ,

$$i = \frac{11.7}{5 \times 10^3} \text{ A} \\ = 2.34 \text{ mA}$$

- (ii) On reversing the connection of Ge diode, it will be reverse biased and conduct no current. Only Si diode will conduct. Therefore,

$$V_o = (12 - 0.7) \text{ V} = 11.3 \text{ V}$$

$$\text{and current, } i = \frac{11.3}{5 \times 10^3} \text{ A} = 2.26 \text{ mA}$$

Voltage-current characteristic curve of a p - n junction diode

The voltage-current characteristic curves of a p - n junction diode are graphs showing the change in current flowing through the junction with change in a applied voltage, when the junction is in forward biased and in reverse biased.

1. Forward biased characteristics

When the diode is in forward biased, *i.e.* p -side is kept at higher potential, the current in the diode changes with the voltage applied across the diode.

A graph is plotted between voltage and current. The curve so obtained is the forward characteristic of the diode. At the start when applied voltage is low, the current through diode is almost zero. It is because of potential barrier, which opposes the applied voltage.

Till the applied voltage exceeds the potential barrier, the current increases very slowly with increase in applied voltage (OA portion of the graph).

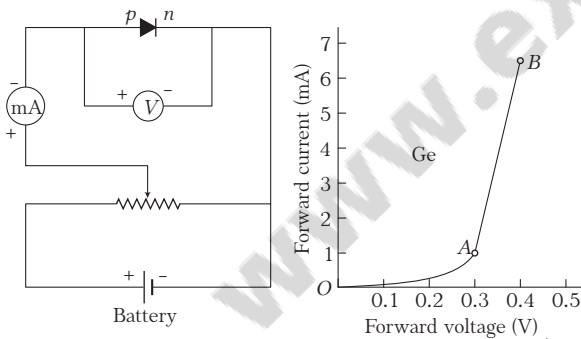


Fig. 14.22 Forward biased characteristics of a diode

After this voltage, the diode current increases rapidly (AB portion of the graph), even for very small increase in the diode voltage. In this situation, the diode behaves like a conductor. The forward voltage beyond which the current through the junction starts increasing rapidly with voltage is called the **threshold voltage** or **cut-off voltage** or **knee voltage**.

If line AB is extended back, it cuts the voltage axis at potential barrier voltage.

The value of the cut in voltage is about 0.3 V for a germanium diode and 0.7 V for a silicon diode.

2. Reverse biased characteristics

The circuit diagram for studying reverse biased characteristics is shown in the figure

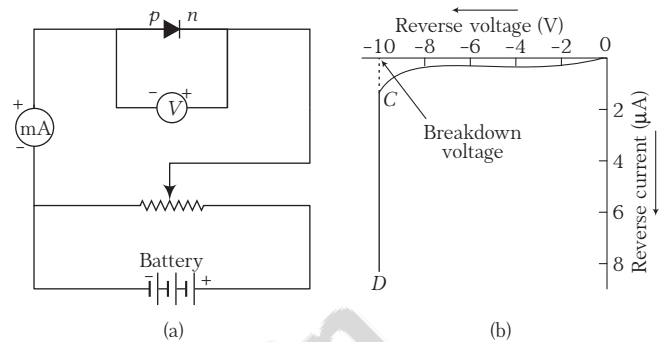


Fig. 14.23 Reverse biased characteristics of a diode

In reverse biased, the applied voltage supports the flow of minority charge carriers across the junction. So, a very small current flows across the junction due to minority charge carriers.

Motion of minority charge carriers is also supported by internal potential barrier, so all the minority carriers cross over the junction.

Therefore, the small reverse current remains almost constant over a sufficiently long range of reverse bias voltage, increasing very little with increasing voltage (OC portion of the graph).

This reverse current is voltage independent upto certain voltage known as **breakdown voltage** and this voltage independent current is called **reverse saturation current**.

This reverse saturation current depends only on temperature of the junction. If it is found that for every 1° rise in temperature, the reverse saturation current increases by about 7%. So for every 10°C rise, the I_0 becomes nearly double.

If the reverse biased voltage is too high, then p - n junction breaks.

It is of following two types

(a) Zener breakdown

When reverse bias voltage is increased, the electric field across the junction also increases. At some stage, the electric field becomes so high, that it breaks the covalent bonds creating electron-hole pairs. Thus, a large number of carriers are generated. So, this causes a large current to flow. This mechanism is known as Zener breakdown.

(b) Avalanche breakdown

At high reverse voltage, due to high electric field, the minority charge carriers, while crossing the junction acquire very high velocities. These by collision break down the covalent bonds generating more carriers. A

chain reaction is established giving rise to high current. This mechanism is called Avalanche breakdown.

Note Zener breakdown occurs in a highly doped p - n junction whereas Avalanche breakdown occurs in a lightly doped p - n junction.

Dynamic resistance

The complete V - I characteristic of a junction diode is shown in figure. It is clear from the graph that, both the forward-reverse bias characteristics of the p - n junction do not obey Ohm's law. So, the resistance offered by junction diode will depend on the applied voltage.

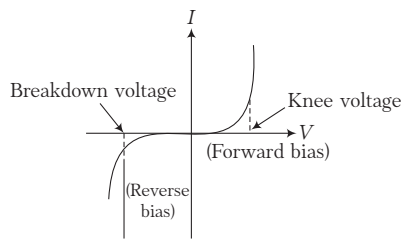


Fig. 14.24 Complete V - I characteristic of a junction diode

Here, we will take the dynamic resistance which is defined as the ratio of small change in voltage to the small change in current produced. It is also called dynamic resistance of the junction diode and is denoted by r_d .

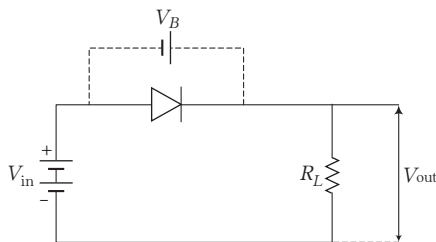
Thus,

$$r_d = \frac{\Delta V}{\Delta I}$$

The region of the characteristic curve, where dynamic resistance is almost independent of the applied voltage is called the **linear region of junction diode**.

Example 14.23 A silicon diode is connected to a load resistance R_L as shown in the figure. Let $V_{in} = 10$ V and $R_L = 10$ k Ω .

- (i) If barrier voltage, $V_B = 0.7$ V, then calculate
 (a) output voltage across R_L ,
 (b) current in diode
 (c) and forward resistance.



(ii) If diode is assumed ideal, then what will be

(a) output voltage

(b) and output current in diode?

Sol. (i) (a) As, $V_{out} = V_{in} - V_B$
 where, V_B = barrier voltage
 $\therefore V_{out} = 10 - 0.7 = 9.3$ V

(b) As diode is in forward biased state, so it will conduct

$$\therefore I = \frac{V_{out}}{R_L} = \frac{9.3}{10 \times 10^3} = 0.9 \text{ mA}$$

(c) Forward resistance,

$$r_f = \frac{V_B}{I} = \frac{0.7}{0.9 \times 10^{-3}} = 777 \Omega$$

(ii) For ideal diode, $r_f = 0$, $V_B = 0$

(a) $V_{out} = V_{in} = 10$ V

$$(b) I = \frac{V_{out}}{R_L} = \frac{10}{10 \times 10^3} = 1 \text{ mA}$$

p - n Junction diode as a rectifier

A rectifier is a device which converts an alternating current (or voltage) into a direct (or unidirectional) current (or voltage).

Principle A p - n junction diode can work as an excellent rectifier because it permits current in one direction only.

It offers a low resistance for the current to flow when it is forward biased, but a very high resistance when reverse biased. Thus, it allows current to flow through it in only one direction and acts as a rectifier.

The junction diode can be used either as a half-wave rectifier or as a full-wave rectifier.

1. p - n junction diode as half-wave rectifier

A simple rectifier circuit called the half-wave rectifier, using only one diode is shown in figure. The AC voltage to be rectified is connected to the primary coil of a step-down transformer. Secondary coil is connected to the diode through resistor R_L across which, output is obtained.

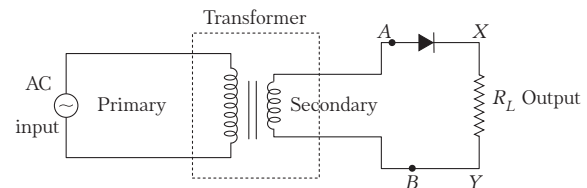


Fig. 14.25 Circuit diagram of a half-wave rectifier

Working

During positive half cycle of the input AC (i.e. $V_A > 0$), the p - n junction is in forward biased. Thus, the resistance in p - n junction becomes low and current flows. Hence, we get output in the load.

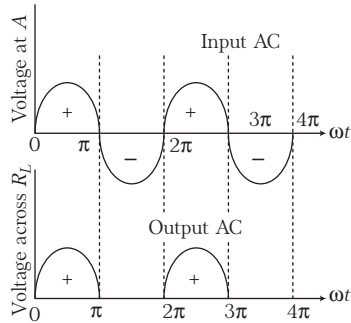


Fig. 14.26 Input and output waveform

During negative half cycle of the input AC (i.e. $V_A < 0$), the p - n junction is in reverse biased. Thus, the resistance of p - n junction is high and current does not flow. Hence, no output is obtained across the load.

Important points related to half-wave rectifier

(i) **Average output** in one cycle

$$I_{DC} = \frac{I_0}{\pi} \text{ and } V_{DC} = \frac{V_0}{\pi}; I_0 = \frac{V_0}{r_f + R_L},$$

$$V_{DC} = \frac{V_0}{\pi \left(1 + \frac{r_f}{R_L} \right)}$$

(r_f = forward biased resistance)
(and R_L = load resistance)

(ii) **rms output** $I_{rms} = \frac{I_0}{2}$, $V_{rms} = \frac{V_0}{2}$

(iii) The ratio of the effective alternating component of the output voltage (or current) to the DC component is known as **ripple factor**.

$$r = \frac{I_{AC}}{I_{DC}} = \left[\left(\frac{I_{rms}}{I_{DC}} \right)^2 - 1 \right]^{1/2} = 1.21$$

The smaller the ripple factor, more effective will be rectifier.

(iv) **Peak inverse voltage (PIV)** The maximum reverse biased voltage that can be applied before commencement of breakdown region is called the peak inverse voltage. When diode is not conducting, PIV across it = V_0 .

(v) **Efficiency** It is given by

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{40.6\%}{1 + r_f/R_L}$$

If $R_L \gg r_f$, then $\eta = 40.6\%$ and if $R_L = r_f$, then $\eta = 20.3\%$.

(vi) The ratio of i_{rms} and i_{DC} is known as **form factor**.

$$\text{Form factor} = \frac{I_{rms}}{I_{DC}} = \frac{\pi}{2} = 1.57$$

(vii) The ripple frequency (ω) for half wave rectifier is same as that of AC.

Example 14.24 The applied input AC to a half wave rectifier is 60 W and the DC output is 20 W. Find the rectification efficiency. Also, calculate the value of power efficiency.

Sol. Rectification efficiency is the ratio of DC output power to the AC input power, i.e.

$$\eta = \frac{\text{DC output power}}{\text{AC input power}} \times 100$$

Here, DC output power = 20 W

AC input power = 60 W

$$\therefore \text{Rectification efficiency, } \eta = \frac{20}{60} \times 100 = 33.3\%$$

Also, power efficiency is given by

$$\begin{aligned} \text{PE} &= \frac{\text{DC output power}}{\text{AC input power for half cycle}} \times 100\% \\ &= \frac{20}{30} \times 100\% = 66.67\% \end{aligned}$$

Example 14.25 A semiconductor diode is used as a half-wave rectifier having internal resistance 200 Ω . The voltage applied is given by $V = 50 \sin \omega t$ volt and load resistance is 650 Ω . Calculate (i) maximum output current, (ii) DC output current, (iii) DC output power (iv) and DC output voltage.

Sol. Here, $V = (50 \sin \omega t)$ volt

Comparing it with general equation $V = V_0 \sin \omega t$, we get

$$V_0 = 50 \text{ volt, } R_L = 650 \Omega, r_f = 200 \Omega$$

(i) Maximum output current, $I_0 = \frac{V_0}{r_f + R_L}$

$$= \frac{50}{(200 + 650)} = 58 \text{ mA}$$

(ii) DC output current, $I_{DC} = \frac{I_0}{\pi} = \frac{58}{\pi} = 18.5 \text{ mA}$

(iii) DC output power,

$$= I_{DC}^2 \cdot R_L = (18.5 \times 10^{-3})^2 \cdot 650 = 0.22 \text{ W}$$

(iv) DC output voltage,

$$= I_{DC} \cdot R_L = 18.5 \times 10^{-3} \times 650 = 12.02 \text{ W}$$

2. *p-n* junction diode as full wave rectifier

The full wave rectifiers are of two types

- (i) Centre tap full wave rectifier
- (ii) Full wave bridge rectifier

These are described in detail below

(i) Centre tap full wave rectifier

Figure shows a circuit which is used in full wave rectification. Two diodes are used for this purpose.

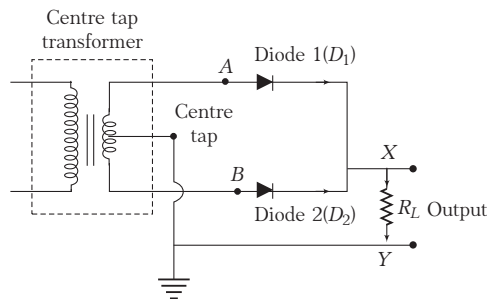


Fig. 14.27 Circuit diagram of full wave rectifier

Working

During the positive half cycle of the input AC, the diode D_1 is in forward biased and the diode D_2 is in reverse biased. The forward current flows through diode D_1 .

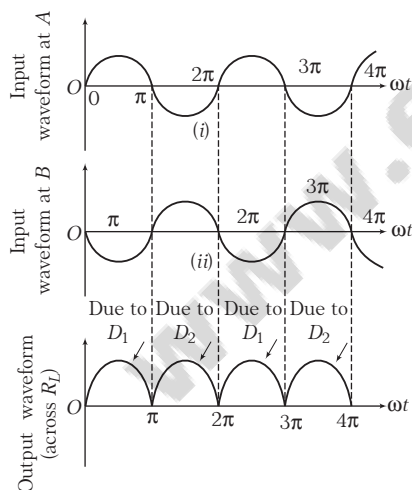


Fig. 14.28 Input and output waveforms of a full-wave rectifier

During the negative half cycle, the diode D_1 is reverse biased and diode D_2 is forward biased. Thus, current flows through diode D_2 . Thus, we find that during both the halves, current flows in the same direction.

(ii) Full wave bridge rectifier

Another full wave rectifier called the bridge rectifier uses four diodes as shown in figure

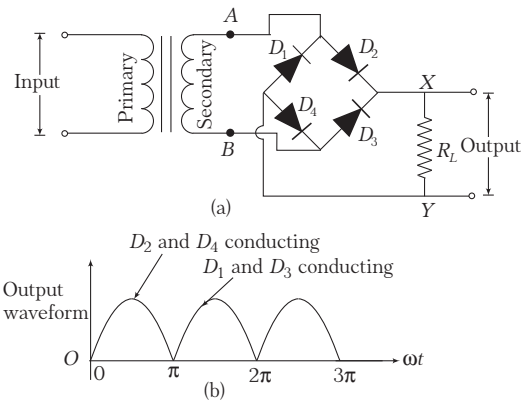


Fig. 14.29 (a) Bridge rectifier (b) and output waveforms for a bridge rectifier

For one-half cycle, diodes D_1 and D_3 are forward biased and D_2 and D_4 are reverse biased. So, D_1 and D_3 conduct but D_2 and D_4 don't. Current through R_L flows from X to Y. In another half cycle, D_2 and D_4 are forward biased and D_1 and D_3 are reverse biased. So, in this half cycle, D_2 and D_4 conduct but D_1 and D_3 do not. Current again flows from X to Y through R_L . Thus, we see that current through R_L always flows in one direction from X to Y.

Important points related to full wave rectifier

- (i) Output voltage is obtained across the load resistance R_L . It is not constant but pulsating in nature.
- (ii) **Average output**

$$V_{av} = \frac{2V_0}{\pi}, I_{av} = \frac{2I_0}{\pi}, V_{DC} = \frac{2V_0}{\pi \left(1 + \frac{r_f}{R_L}\right)}$$
 where, r_f = forward biased resistance
 and R_L = load resistance.
- (iii) **rms output** $V_{rms} = \frac{V_0}{\sqrt{2}}, I_{rms} = \frac{I_0}{\sqrt{2}}$
- (iv) **Ripple factor** $r = 0.48 = 48\%$
- (v) **Ripple frequency** The ripple frequency of full wave rectifier
 $= 2 \times \text{frequency of input AC}$
- (vi) **Peak inverse voltage (PIV)** Its value is $2V_0$.

- (vii) **Efficiency** $\eta_{\%} = \frac{81.2}{1 + \frac{r_f}{R_L}}$ for $r_f \ll R_L$, $\eta = 81.2\%$
- (viii) **Form factor**, $f = \frac{I_{rms}}{I_{DC}} = \frac{\pi}{2\sqrt{2}}$

Note

- (i) The output signal frequency in full wave rectification is double that of input signal frequency but in half-wave rectification, the input and output signal frequency is same.
- (ii) From rectifier, we get pulsating DC, which consists of AC ripples. To remove these unwanted ripples, filter circuits are used.

Example 14.26 In a full wave rectifier circuit operating from 100 Hz mains frequency, what is the fundamental frequency in the ripple?

Sol. In full wave rectification, output signal (ripple) frequency is double that of input frequency. So, output frequency is 200 Hz.

Example 14.27 In a centre tap full wave rectifier, the value of the load resistance is $2\text{ k}\Omega$. The voltage applied across the half of the secondary winding is given by $V = 220 \sin 314t$. Assume that, the each diode has a forward bias dynamic resistance of $20\ \Omega$. Calculate (i) the peak value of current, (ii) the DC value of current (iii) and the rms value of current.

Sol. Comparing $V = 220 \sin 314t$ with general equation

$$V = V_0 \sin \omega t, \text{ we get}$$

$$V_0 = 220\text{ V and } \omega = 314\text{ rads}^{-1}$$

Also, it is given that $R_L = 2\text{ k}\Omega = 2000\ \Omega$, $r_f = 20\ \Omega$

(i) Peak value of current,

$$I_0 = \frac{V_0}{(r_f + R_L)} = \frac{220}{20 + 2000} \approx 109\text{ mA}$$

(ii) DC value of current,

$$I_{\text{DC}} = \frac{2I_0}{\pi} = \frac{2 \times 109}{3.14} = 69.4\text{ mA}$$

(iii) rms value of current, $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = 0.707 \times 109$
 $= 77.06\text{ mA}$

Special types of p - n junction diode

The junction diodes are of many types

1. Light emitting diode (LED)

It is an important light source used in optical communication and it converts electrical energy into light energy. It is a forward biased p - n junction which emits light spontaneously.

(i) Symbol

It is represented by the symbol shown below. In actual, it has two leads, the shorter lead represents n or cathode side while the longer lead represents p or anode side.

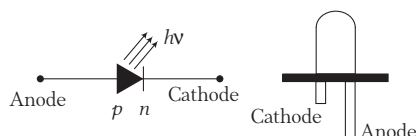


Fig. 14.30 LED symbol

(ii) Working

When p - n junction is forward biased, electrons and holes move towards opposite sides of junction through it. Therefore, there are excess minority carriers on the either

side of the junction boundary, which recombines with majority carriers near the junction.

On recombination of electron and hole, the energy is given out in the form of heat and light.

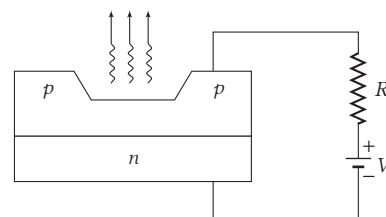


Fig. 14.31 A forward biased LED

(iii) V - I characteristics

V - I characteristics of LED are given below, which is similar to that of a simple junction diode. But the threshold voltages are much higher and slightly different for each colour. The reverse breakdown voltages of LEDs are very low. The colour of light emitted by a given LED, depends on its band gap energy. The photon emitted by an LED is of energy equal to or slightly less than the band gap energy. Forward current conducted by the junction determines the intensity of light emitted by LED.

A low voltage DC supply is required to operate an LED.

Current drawn by LED is of the order of milliamperes. So, in practice, a resistor of suitable value is joined in series with the LED to limit the current upto the safe value required.

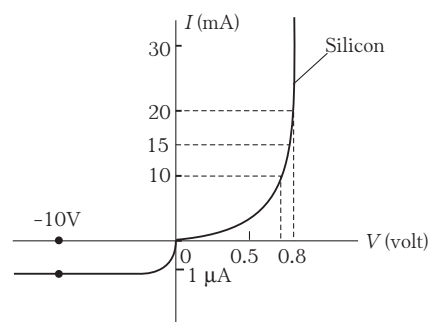


Fig. 14.32 V - I characteristics of LED

(iv) Advantages

- (a) Fast action and no warm up time is required.
- (b) The bandwidth of emitted light is $100\text{ \AA}$ to $500\text{ \AA}$, so its nearly (not exactly) monochromatic.
- (c) Long-life and ruggedness.
- (d) Low operational voltage and less power consumed.
- (e) Fast ON-OFF switching capability.

Example 14.28 A voltage drop of $2V$ occurs across a light emitting diode (LED) and a current of $10\ \mu A$ is passed through it when it is operated with a $6\ V$ battery having a limiting resistor R . What is the value of R ?

Sol. Current, $I = \frac{V}{R}$ or $R = \frac{V}{I}$

$$\text{Resistance of limiting resistor, } R = \frac{(6 - 2)V}{10 \times 10^{-6} A} = 400\ k\Omega$$

2. Photodiode

A photodiode is a special type of junction diode used for detecting optical signals. It is a reverse biased p - n junction made from a photosensitive material. In photodiode, current carriers are generated by photons through photo-excitation.

(i) Symbol

The symbol of a photodiode is

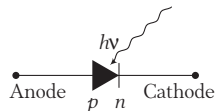


Fig. 14.33 Symbol of a photodiode

(ii) Construction

A photodiode is fabricated with a transparent cover to allow light to fall on the diode and is operated under reverse bias.

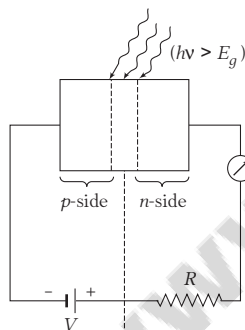


Fig. 14.34 A reverse biased photodiode illuminated with light

(iii) Working

When the photodiode is illuminated with light (photons), having energy greater than the energy gap of the semiconductor, then electron-hole pairs are generated due to the absorption of photons. These charge carriers contribute to the reverse current. They increase the conductivity of the semiconductor.

Larger the intensity of the incident light, larger will be the conductivity of the semiconductor. The value of reverse saturation current increases with the increase in intensity of incident light.

Hence, a measurement of the change in the reverse saturation current on illumination can give the values of light intensity.

(iv) V - I Characteristics

Its V - I characteristics are shown in the figure given below. We observe from the figure that, current in photodiode changes with the change in light intensity (I) when reverse bias is applied.

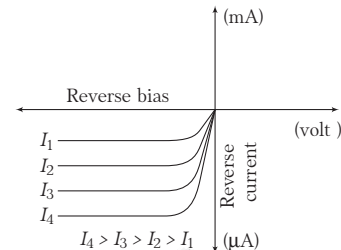


Fig. 14.35 V - I characteristics of photodiode at different intensities

(v) Uses

The p - n photodiode can operate at frequencies of the order of 1MHz , in contrast to the slow and bulk type photoconductor which is limited to a frequency range from 100 to 5000 Hz . Hence, the photodiodes are extensively used in high speed reading of computer punched cards, light-detection systems, light-operated switches, counting of objects interrupting a light-beam, etc.

Example 14.29 A p - n photodiode is made of a material with a band gap of 1.5 eV . What is the minimum wavelength of radiation that can be absorbed by the material?

Sol Use, energy, $E = h\nu = \frac{hc}{\lambda}$

The minimum wavelength of radiation,

$$\begin{aligned}\lambda &= \frac{hc}{E} = \frac{(6.4 \times 10^{-34}) \times (3 \times 10^8)}{1.5 \times 1.6 \times 10^{-19}} \\ &= 8 \times 10^{-7}\text{ m} \\ &= 800\text{ nm}\end{aligned}$$

3. Solar cell

Solar cell is a p - n junction diode, which converts solar energy into electrical energy. It is based on photovoltaic effect.

(i) Symbol

Its symbol is

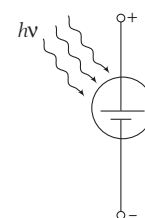


Fig. 14.36 Symbol of a solar cell

(ii) Construction

It consists of a silicon or gallium-arsenide p - n junction diode packed in a can with glass window on the top. Here, a thin layer of n -type is grown on a p -type semiconductor. The top of the n -layer has few finger electrodes for the light to reach it. The bottom of the p -layer is provided with a current collecting electrode.

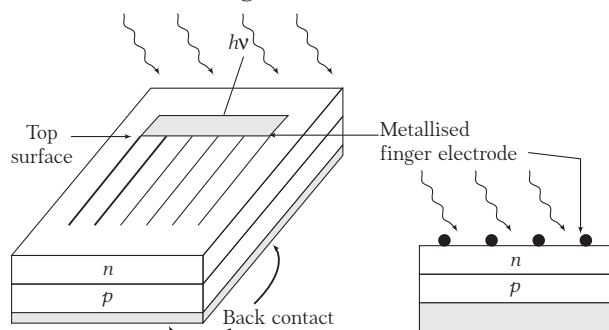


Fig. 14.37 p - n junction of solar cell

(iii) Working

When photons of light (of energy $h\nu > E_g$) falls at the junction, electron-hole pairs are generated near the junction and they move in opposite directions due to junction field.

They will be collected at the two sides of the junction, giving rise to a photovoltage between the top and bottom metal electrodes. The top metal contact acts as negative electrode and bottom metal contact acts as positive electrode. When an external load is connected across metal electrodes, a photocurrent flows.

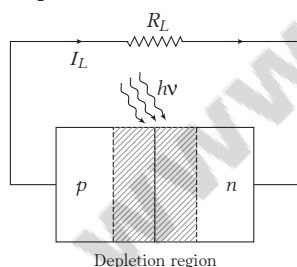


Fig. 14.38 Flow of photocurrent in a solar cell

(iv) I - V characteristics

The I - V characteristics of solar cell are shown in the figure. We can see in the figure that, it is drawn in the fourth quadrant of the coordinate axes, because a solar cell does not draw current but supplies the same to the load.

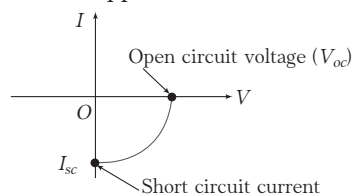


Fig. 14.39 I - V characteristic of a solar cell

(v) Uses

Solar cells are used in space vehicles for charging batteries in day time and then taking electrical power from them during night.

4. Zener diode

It is a reverse biased heavily doped p - n junction diode. It is operated in breakdown region.

Zener diode is designed to operate in the reverse breakdown voltage continuously without being damaged.

This can be achieved by changing the thickness of the depletion layer to which the voltage is applied. Current through the Zener diode is controlled by an external resistance. Zener diode is represented by the symbol shown below



Fig. 14.40 Symbol of Zener diode

(i) V - I Characteristics

The V - I characteristics of Zener diode is shown below and we observe that when the applied reverse voltage (V) reaches the breakdown voltage (V_Z) of the Zener diode, there is a large change in the current.

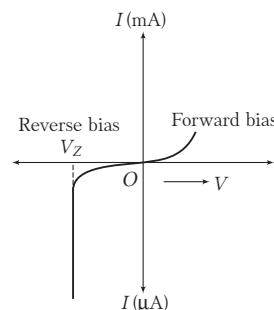


Fig. 14.41 V - I characteristics of a Zener diode

But after the breakdown voltage V_Z , a large change in the current can be produced by almost insignificant change in the reverse bias voltage.

(ii) Uses

Though zener diode finds numerous applications in electronics but mainly it is used as a voltage stabilizer (voltage regulator) and as a fixed reference voltage for calibrating voltmeters.

(iii) Zener diode as a voltage regulator

This is the most important application of a Zener diode.

(a) Principle

When the applied reverse voltage V reaches the breakdown voltage V_Z of the Zener diode, there is a large

change in the current. So, after the breakdown voltage V_Z , a large change in the current can be produced by almost insignificant change in the reverse bias voltage, *i.e.* Zener voltage remains constant even though the current through the Zener diode varies over a wide range.

Zener diode is joined in reverse bias to the fluctuating DC input voltage through a resistance R .

The constant output voltage is taken across a load resistance connected in parallel with Zener diode.

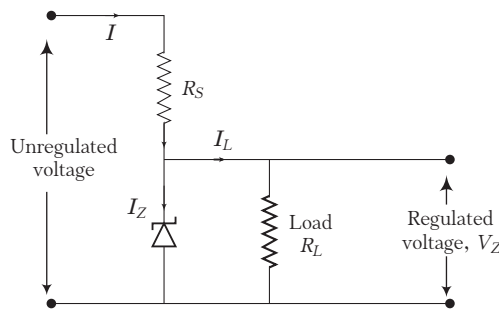


Fig. 14.42 Circuit diagram of Zener diode as a voltage regulator

(b) Working

Here, when input DC voltage increases beyond a certain limit, the current through the circuit rises sharply, causing a sufficient increase in the voltage drop across the resistor R_S . Thus, the voltage across the Zener diode remains constant and also the output voltage remains constant at V_Z . When the input DC voltage decreases, the current through the circuit goes down sharply causing sufficient decrease in the voltage drop across the resistance. Thus, the voltage across the Zener diode remains constant and also the output voltage across R_L remains constant at V_Z .

Hence, the output voltage remains constant in both conditions. Figure shows the graph of output voltage V_o versus input voltage V_{in} of a Zener diode. Clearly, the output voltage remains constant after the reverse breakdown voltage V_Z .

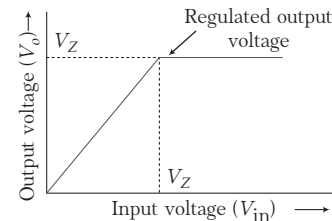
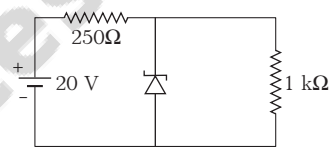


Fig. 14.43 Graph of V_o versus V_{in} for a Zener diode

Example 14.30 The breakdown voltage of a Zener diode is 12 V. It is used in a voltage regulator circuit shown in figure. Find the current through the diode.



Sol. Current through 250 Ω resistor,

$$I_1 = \frac{\text{Net voltage}}{\text{Resistance}} = \frac{(20 - 12)}{250} = 32 \times 10^{-3} \text{ A} = 32 \text{ mA}$$

Current through 1 kΩ resistor,

$$I_2 = \frac{12}{1 \times 10^3} = 12 \times 10^{-3} \text{ A} = 12 \text{ mA}$$

So, current through the Zener diode,

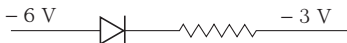
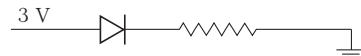
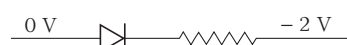
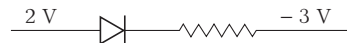
$$I_Z = I_1 - I_2 = 32 \text{ mA} - 12 \text{ mA} = 20 \text{ mA}$$

CHECK POINT 14.2

- In the depletion region of an unbiased p - n junction diode, there are
 - only electrons
 - only holes
 - Both electrons and holes
 - only fixed ions
- The cause of the potential barrier in a p - n diode is
 - depletion of positive charges near the junction
 - concentration of positive charges near the junction
 - depletion of negative charge near the junction
 - concentration of positive and negative charges near the junction.
- Barrier potential of a p - n junction diode does not depend on
 - temperature
 - forward bias
 - doping density
 - diode design
- What control the conduction of p - n junction?
 - Majority carriers
 - Minority carriers
 - Holes
 - Electron
- In the forward bias arrangement of a p - n junction diode,
 - the n -end is connected to the positive terminal of the battery
 - the p -end is connected to the positive terminal of the battery
 - the direction of current is from n -end to p -end in the diode
 - the p -end is connected to the negative terminal of battery
- In forward bias, the width of potential barrier in a p - n junction diode
 - increases
 - decreases
 - remains content
 - first increases then decreases
- On adjusting the p - n junction diode in forward biased,
 - depletion layer increases
 - resistance increases
 - Both decreases
 - None of the above
- If the two ends p and n of a p - n junction diode are joined by a wire, then
 - there will not be a steady current in the circuit
 - there will be a steady current from n -side to p -side
 - there will be a steady current from p -side to n -side
 - there may or may not be a current depending upon the resistance of the connecting wire

9. The reverse biasing in a p - n junction diode
 (a) decreases the width of potential barrier
 (b) increases the width of potential barrier
 (c) increases the number of minority charge carriers
 (d) increases the number of majority charge carriers

10. A reverse-biased diode is

- (a) 
 (b) 
 (c) 
 (d) 

11. On increasing the reverse bias to a large value in a p - n junction diode, current
 (a) increases slowly
 (b) remains fixed
 (c) suddenly increases
 (d) decreases slowly

12. In p - n junction, avalanche current flows in circuit when biasing is

- (a) forward (b) reverse
 (c) zero (d) excess

13. The p - n junction diode is used as

- (a) an amplifier (b) a rectifier
 (c) an oscillator (d) a modulator

14. In a p - n junction photocell, the value of photo-electromotive force produced by monochromatic light is proportional to

- (a) the voltage applied at the p - n junction
 (b) the barrier voltage at the p - n junction
 (c) the intensity of the light falling on the cell
 (d) the frequency of the light falling on the cell

15. What is the order of the reverse saturation current before breakdown in a Zener diode?

- (a) Ampere
 (b) Milliampere
 (c) It depends on the applied voltage
 (d) Microampere

JUNCTION TRANSISTORS

A junction transistor is a three terminal device which is formed by sandwiching a thin wafer of one type of semiconductors between two layers of another type.

The n - p - n transistor has a p -type wafer between two n -type layers. Similarly, the p - n - p transistor has a n -type wafer between two p -type layers.

Type of junction transistors

There are two types of junction transistors

n - p - n transistor

A n - p - n transistor is formed by sandwiching a thin layer of p -type semiconductor between two n -type semiconductors as shown in figure.

Arrow indicates the direction of conventional current. In n - p - n transistor, it will always point outwards, i.e. towards the emitter.

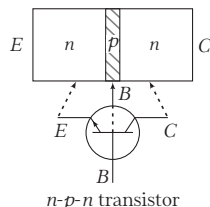


Fig. 14.44 n - p - n transistor and its circuit symbol

p - n - p transistor

A p - n - p transistor is formed by sandwiching a thin layer of n -type semiconductor between two p -type semiconductors as shown in figure.

The arrowhead in the symbol points inwards, i.e. towards the base.

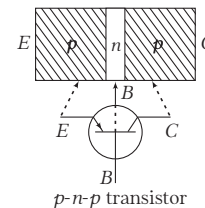


Fig. 14.45 p - n - p transistor and its circuit symbol

Terminals of a transistor

A transistor is basically a three terminal device. Terminals come out from the emitter, base and the collector for external connections.

Emitter (E)

It is heavily doped and of moderate size as it emits the majority charge carriers into the base by which current flows in the transistor.

Base (B)

The base region is very lightly doped and thin. Most of the charge carriers emitted by the emitter pass through it to the collector.

Collector (C)

The doping level of collector is neither very high nor very low, however it lies between the doping levels of emitter and base. The size of the collector region is larger than the other two regions because it collects the charge carriers from the base.

Important points related to transistor

- (i) In normal operation of a transistor, the emitter-base junction is always forward biased and collector-base junction is reverse biased.
- (ii) The arrow on the emitter-base line shows the direction of current between emitter and base.
- (iii) In a $n-p-n$ transistor, there are a large number of conduction electrons in the emitter and a large number of holes in the base. Similarly, in $p-n-p$ transistor, there are a large number of holes in the emitter and large number of electrons in the base.
- (iv) If the junction is forward biased in $n-p-n$ transistor, the electrons will diffuse from emitter to the base and holes will diffuse from the base to the emitter.
- (v) The direction of electric current at this junction is therefore from the base to the emitter in $n-p-n$ transistor.
- (vi) In the connecting wires, electrons are always responsible for current flow, but inside the transistor holes are mainly responsible for current flow in $p-n-p$ transistor and electrons in case of $n-p-n$ transistor.

Biasing of transistor

Biasing of a transistor is defined as the process of applying external voltages or potential difference to it.

The biasing of the two $p-n$ junction in a transistor, i.e. emitter junction and collector junction depends on the four modes, i.e. active mode, saturation mode, cut-off mode and inverse mode.

Active mode is also known as **linear mode operation**. A transistor is mostly used in the active region of operation, i.e. emitter-base junction is in forward biased and collector-base junction is in reverse biased.

In **saturation mode**, maximum collector current flows and transistor acts as a closed switch from collector to emitter terminal. In **cut-off mode**, the circuit behaves as open switch, where only leakage current flows.

The emitter and collector are interchanged in inverse mode.

Different modes of operation of a transistor is given below

Operating mode	Emitter-base bias	Collector-base bias
Active	Forward	Reverse
Saturation	Forward	Forward
Cut off	Reverse	Reverse
Inverse	Reverse	Forward

Working of a transistor

1. $p-n-p$ transistor

From figure, it is clear that emitter-base junction is forward biased and collector-base junction is reverse biased.

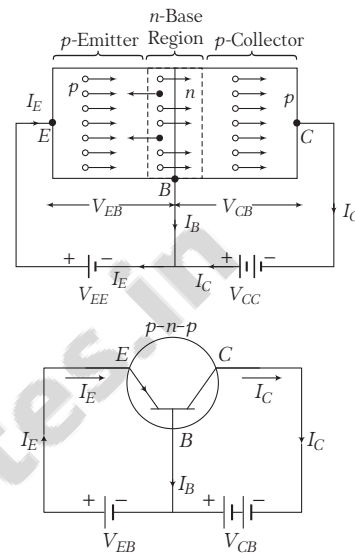


Fig. 14.46 Action of $p-n-p$ transistor and its biasing

The resistance of emitter-base junction is very low. So, the voltage of V_{EB} is quite small whereas the resistance of collector-base junction is very high, so the voltage of V_{CB} is very high. The forward biasing of the emitter-base circuit repels the holes of emitter towards the base and electrons of base towards the emitter. As the base is very thin and lightly doped, most of the holes ($> 95\%$) entering it pass on to collector, while a very few of them ($< 5\%$) recombine with the electrons of the base region. As soon as a hole combines with an electron, an electron from the negative terminal of the battery V_{EB} enters the base. This sets up a small base current I_B .

Each hole entering the collector region combines with an electron from the negative terminal of the battery V_{CB} and gets neutralised. This creates collector current I_C . Both the base current I_B and collector current I_C combine to form emitter current I_E .

$\therefore$

$$I_E = I_B + I_C$$

Thus inside the $p-n-p$ transistor, the current conduction is due to holes while electrons are the charge carriers in the external circuit.

2. $n-p-n$ transistor

In this transistor, the emitter-base junction is forward biased and its resistance is very low. So, the voltage of V_{EB} is quite small.

The collector-base junction is reverse biased. The resistance of this junction is very high, so the voltage of V_{CB} is quite large.

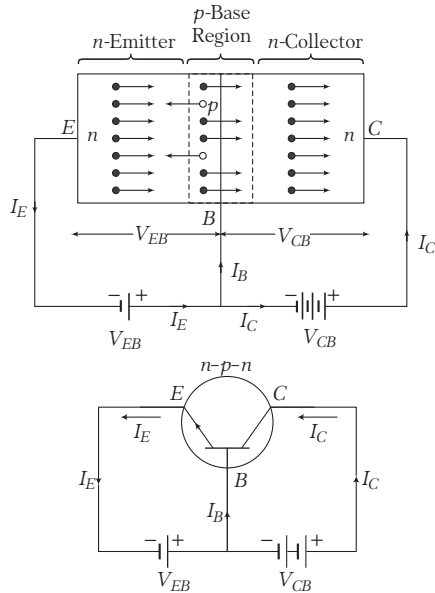


Fig. 14.47 Action of n - p - n transistor and its biasing

The forward bias of the emitter-base circuit repels the electrons of emitter towards the base, setting up emitter current I_E . As the base is very thin and lightly doped, a very few electrons ($< 5\%$) from the emitter combine with the holes of base, giving rise to base current I_B and the remaining electrons ($> 95\%$) are pulled by the collector which is at high positive potential. The electrons are finally collected by the positive terminal of battery V_{CB} , giving rise to collector current I_C .

As soon as an electron from the emitter combines with a hole in the base region, an electron leaves the negative terminal of the battery V_{EB} and at the same time, the positive terminal of battery V_{EB} receives an electron from the base. This sets a base current I_B . Similarly, corresponding to each electron that goes from collector to positive terminal of V_{CB} , an electron enters the emitter from negative terminal of V_{CB} .

Hence, emitter current = base current + collector current
or $I_E = I_B + I_C$ ($I_B \ll I_C$)

Here, I_B is a small fraction of I_C depending on the shape of transistor, thickness of base, doping levels, bias voltages, etc.

Example 14.31 In n - p - n transistor circuit, the collector current is 10 mA. If 95% of the electrons emitted reach the collector, what is the base current?

Sol. $I_C = 95\% I_E$
 $\Rightarrow I_C = 0.95 I_E$
 $\Rightarrow I_E = \frac{I_C}{0.95} = \frac{100}{95} \times 10 \text{ mA} = 10.53 \text{ mA}$

Now, $I_E = I_C + I_B$
 $\therefore$ Base current, $I_B = I_E - I_C$
 $= 10.53 - 10$
 $= 0.53 \text{ mA}$

Transistor circuit configurations

A transistor is a three terminal device. One terminal out of the three serves as a reference point for the entire circuit.

This terminal should be common to the input and output circuits and is connected to ground.

So, a transistor can be used in the following three configurations

1. Common emitter (CE) configuration
2. Common base (CB) configuration
3. Common collector (CC) configuration

The figure given below shows the three types of circuit arrangements for an n - p - n transistor

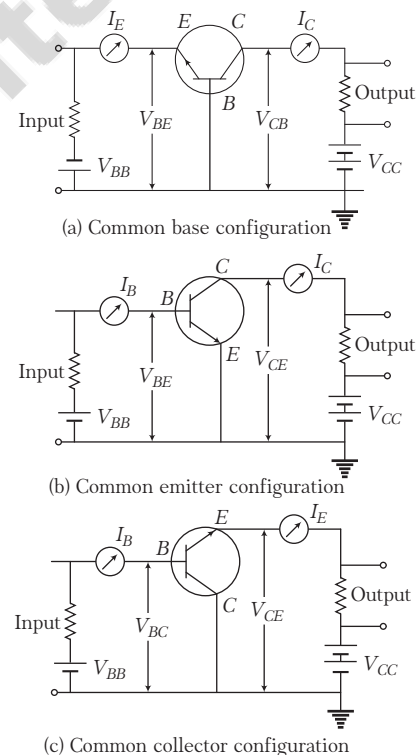


Fig. 14.48 Configurations for n - p - n transistor

Common emitter characteristics

The graphs drawn between voltages and currents, when emitter of a transistor is common to input and output circuits are known as CE characteristics of a transistor.

The circuit diagram for studying CE characteristics of a n - p - n transistor is given below.

Here, base-emitter circuit is forward biased with battery V_{BB} and emitter-collector circuit is reverse biased with battery V_{CC} .

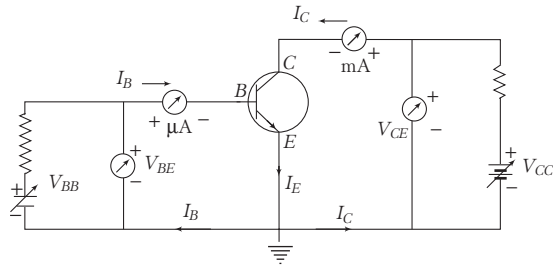


Fig. 14.49

Three types of characteristic curves are obtained

1. Emitter or Input characteristics

A graphical relation between the base-emitter voltage (V_{BE}) and the base-emitter current (I_{BE}) by keeping collector-emitter voltage (V_{CE}) constant is called input characteristics of the transistor. Adjust collector-emitter voltage at a suitable high value V_{CE} (say = +10V).

It is necessary so as to make the base-collector junction reverse biased.

Now, with the help of rheostat, gradually increase the value of base-emitter voltage V_{BE} in small steps and note the corresponding values of base current I_B . Similarly, take the readings for $V_{CE} = 4$ V.

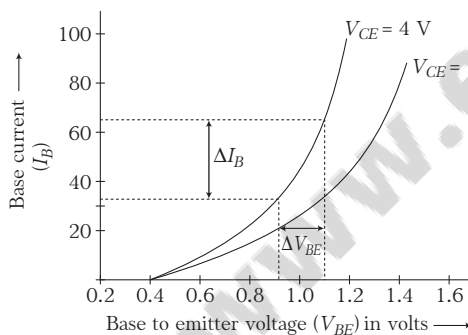


Fig. 14.50 Input characteristics of a CE n-p-n transistor

Observations from the graph

From the graph, it can be concluded that

- For a given collector voltage, the base current increases rapidly with increase in emitter-base voltage. It means that, input resistance is very small.
- For a higher negative collector voltage, the base current increases more rapidly with the collector voltage.

Input resistance

It is defined as the ratio of small change in base-emitter voltage ΔV_{BE} to the resulting change in the base current

ΔI_B at constant collector-emitter voltage V_{CE} . It is reciprocal of slope of I_B - V_{BE} curve.

$$\text{Input resistance, } R_i = \left(\frac{\Delta V_{BE}}{\Delta I_B} \right)_{V_{CE} = \text{constant}}$$

2. Collector or output characteristics

A graphical relation between the collector-emitter voltage V_{CE} and collector current I_C by keeping base current constant is called output characteristics of the transistor.

To study the output characteristics of transistor, we keep value of base current I_B fixed (say at 10 μA) with the help of V_{BE} . Now, gradually change the value of V_{CE} and note the values of collector current I_C . Plot I_C - V_{CE} graph. Repeat the process for different constant values of I_B .

The value of V_{CE} upto which the I_C changes with V_{CE} is called **knee voltage**. The transistors are operated in the region above knee voltage. The output characteristics are shown as

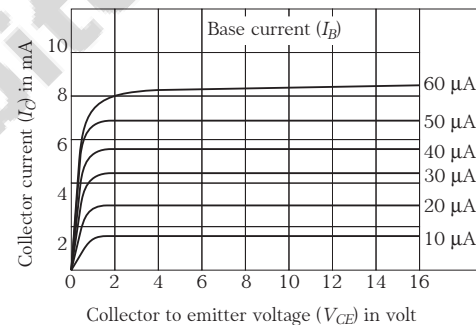


Fig. 14.51 Output characteristics of a CE n-p-n transistor

Observation from the graph

From the graph, it can be concluded

- For a given value of base current, the collector current is not zero even when collector voltage is zero.
- For a given base current, there is a rapid increase in collector current for a small increase of collector voltage. It is the region of low collector resistance, transistor is never operated in this region.
- For a given base current, the collector current become saturated for a certain collector voltage. Beyond which there is no increase in collector current for further increase in collector voltage. It is the region of high collector resistance.

Output resistance

From the output characteristics, we define output resistance of transistor as the ratio of change in collector-emitter voltage to the resulting change in collector current at constant base current. Thus,

Output resistance, $r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C} \right)_{I_B = \text{constant}}$
 = Reciprocal of slope of I_C - V_{CE} curve

3. Transfer characteristics

The graph between the collector current I_C and the base current I_B at constant values of collector-emitter voltage V_{CE} .

The transfer characteristic of a transistor is almost a straight line.

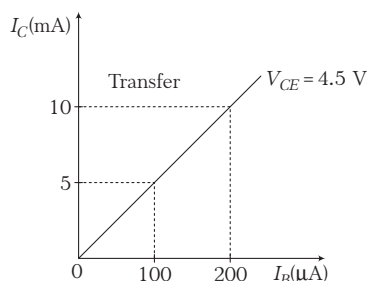


Fig. 14.52 Transfer characteristics of CE n-p-n transistor

Current amplification factors for different configurations of a transistor

There are two types of current amplification factors in a transistor

Common base current amplification factor

It is the ratio of small change in the collector current to the small change in the emitter current at constant collector base voltage. It is denoted by α and is given by

$$\alpha = \left[\frac{\Delta I_C}{\Delta I_E} \right]_{V_{CB} = \text{constant}}$$

Common emitter current amplification factor

It is the ratio of small change in the collector current to the small change in the base current at constant collector-emitter voltage. It is denoted by β and is given by

$$\beta = \left[\frac{\Delta I_C}{\Delta I_B} \right]_{V_{CE} = \text{constant}}$$

Relation between α and β

$$\alpha = \frac{\beta}{1 + \beta} \quad \text{and} \quad \beta = \frac{\alpha}{1 - \alpha}$$

Note

- (i) α and β are independent of current, if the emitter-base junction is in forward biased and the collector-base junction is in reverse biased.
- (ii) The value of current gain β is from 15 to 50 which is much greater than α .

Example 14.32 The current gain for common base transistor is 0.98. Find the current amplification factor, if the transistor is used in common emitter configuration.

Sol. It is given that, $\alpha = 0.98$

$$\text{As we know, } \beta = \frac{\alpha}{1 - \alpha}$$

where, α = current amplification factor for common base transistor and

β = current amplification factor for common emitter transistor.

$$\Rightarrow \beta = \frac{0.98}{(1 - 0.98)} = 49$$

Example 14.33 The current factor of a transistor in a common base arrangement is 0.98. Find the change in collector current corresponding to a change of 5.0 mA in emitter current. What would be the change in base current?

Sol. Given, $\alpha = 0.98$ and $\Delta I_E = 5.0$ mA

$$\text{From the definition, } \alpha = \frac{\Delta I_C}{\Delta I_E}$$

Change in collector current,

$$\Delta I_C = (\alpha)(\Delta I_E) = (0.98)(5.0) \text{ mA} = 4.9 \text{ mA}$$

Further, change in base current,

$$\Delta I_B = \Delta I_E - \Delta I_C = 5 - 4.9 = 0.1 \text{ mA}$$

Example 14.34 The current amplification factor for common emitter arrangement is 59. If the emitter current is 5 mA, find the value of collector current.

Sol. Current amplification factor for common emitter transistor, $\beta = 59$

Emitter current, $I_E = 5$ mA

$$\text{As, } \alpha = \frac{\beta}{1 + \beta}$$

$$\text{Also, } \alpha = \frac{I_C}{I_E}$$

$$\therefore \frac{I_C}{I_E} = \frac{\beta}{1 + \beta} \Rightarrow I_C = \left(\frac{\beta}{1 + \beta} \right) I_E$$

$$I_C = \left(\frac{59}{1 + 59} \right) \times 5 = 4.92 \text{ mA}$$

Example 14.35 In a n - p - n transistor (in common emitter mode), 10^{10} electrons enter the emitter in 10^{-6} s. Only 2% of the electrons are lost in the base. Calculate the base current and the current amplification factor. Charge on electron is 1.6×10^{-19} C.

Sol. Current, $i_E = \frac{q}{t} = \frac{10^{10} \times 1.6 \times 10^{-19}}{10^{-6}} = 1.6 \text{ mA}$

Base current, $i_B = 2\% \text{ of } i_E = \left(\frac{2}{100}\right) \times 1.6 \text{ mA} = 0.032 \text{ mA}$

$\therefore i_C = i_E - i_B = 1.568 \text{ mA}$

Therefore, current amplification factor,

$$\beta = \frac{i_C}{i_B} = \frac{1.568}{0.032} = 49$$

Transistor as an amplifier

An amplifier is a device which is used for increasing the amplitude of input signal. A transistor can be used for amplifying a weak signal. For a transistor to be operated as amplifier, three different basic circuits can be used. These are common base, common emitter and common collector.

Principle

In a transistor, there are two p - n junctions; one is forward biased (low resistance) and other is reverse biased (high resistance). The weak input signal is applied across the forward biased junction and output signal is taken across the reverse biased junction. Since, the input and output currents are almost equal, the output signal appears with a much higher voltage, because of high output resistance.

The emitter is always forward biased whereas the collector is always reverse biased no matter which configuration has to be used.

Common emitter amplifier using a n - p - n transistor

The circuit diagram for n - p - n transistor as an amplifier is shown in the figure given below

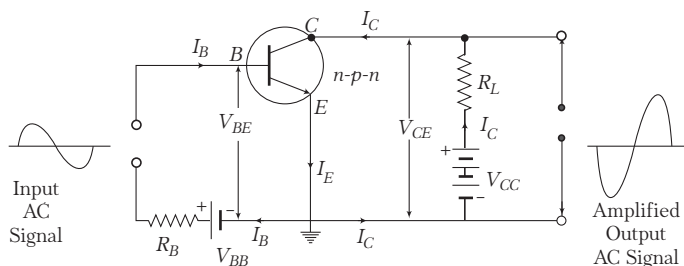


Fig. 14.53 Circuit diagram of transistor as an amplifier

The emitter-base circuit is in forward biased by a low voltage battery V_{BB} , that means the resistance of input circuit is small. The base emitter voltage is given by $V_{BE} = V_{BB} - I_B R_B$.

The collector-emitter circuit is reverse biased by a high voltage battery V_{CC} , that means the resistance of output circuit is high. R_L is a load resistance connected in collector-emitter circuit. The weak input AC signal is applied across the base-emitter circuit and amplified output signal is obtained across collector-emitter circuit. When no AC voltage is applied to the input circuit, we have

$$I_E = I_B + I_C \quad \dots(i)$$

Due to collector current I_C , the voltage drop across load resistance R_L is $I_C R_L$. Therefore, the collector-emitter voltage V_{CE} is given by

$$V_{CE} = V_{CC} - I_C R_L \quad \dots(ii)$$

When the input AC voltage signal is applied across base-emitter circuit, it changes base-emitter voltage and hence, emitter current I_E changes, which in turn changes the collector current I_C . So, the collector-emitter voltage V_{CE} varies in accordance with Eq. (ii). This variation in V_{CE} appears as an amplified output.

Note The output voltage signal is 180° out of phase with the input voltage signal in the common emitter amplifier.

Gains in common emitter amplifier

DC current gain

It is defined as the ratio of the collector current to the base current ($V_{CE} = \text{constant}$) and is denoted by β_{DC} . Thus,

$$\beta_{DC} = \left(\frac{I_C}{I_B}\right)_{V_{CE} = \text{constant}}$$

AC current gain

It is defined as the ratio of the small change in the collector current to the small change in the base current at a constant collector-to-emitter voltage and is denoted by β_{AC} .

$$\text{Thus, } \beta_{AC} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE} = \text{constant}}$$

Note Since, I_C increases with I_B almost linearly and $I_C = 0$ when $I_B = 0$, therefore the values of both β_{AC} and β_{DC} are nearly equal.

AC voltage gain

It is defined as the ratio of the change in the output voltage to the change in the input voltage and is denoted by A_V . Suppose, on applying an AC input voltage signal, the input base current changes by ΔI_B and correspondingly the output collector current changes by ΔI_C . If R_{in} and R_{out} are the resistances of the input and the output circuits respectively, then

$$A_V = \frac{\Delta I_C \times R_{out}}{\Delta I_B \times R_{in}} = \frac{\Delta I_C}{\Delta I_B} \times \frac{R_{out}}{R_{in}}$$

Note The voltage gain in common-emitter amplifier is large compared to that in common base amplifier.

Now, $\Delta I_C / \Delta I_B$ is the AC current gain β_{AC} and R_{out} / R_{in} is the resistance gain, then

$$A_V = \beta_{AC} \times \text{Resistance gain}$$

i.e. Voltage gain = Current gain $\times$ Resistance gain

AC power gain

It is defined as the ratio of the change in the output power to the change in the input power. Since,

$$\text{Power} = \text{Current} \times \text{Voltage}$$

We have, AC power gain = AC current gain $\times$ AC voltage gain

$$= \beta_{AC} \times A_V = \beta_{AC} \times (\beta_{AC} \times \text{Resistance gain})$$

$$= \beta_{AC}^2 \times \text{Resistance gain}$$

Note As $\beta_{AC}^2 \gg \alpha_{AC}^2$, the AC power gain of a common emitter amplifier is much larger than that of a common base amplifier.

Transconductance (g_m)

It is defined as the ratio of the change in the collector current to the change in the base-to-emitter voltage at constant collector-to-emitter voltage and is denoted by g_m .

$$\text{Thus, } g_m = \left(\frac{\Delta I_C}{\Delta V_{BE}} \right)_{V_{CE} = \text{constant}}$$

We can express it in terms of β_{AC} ,

$$g_m = \frac{\Delta I_C}{\Delta I_B} \times \frac{\Delta I_B}{\Delta V_i} \quad (\because V_{BE} = V_i)$$

$$\text{Now, } \frac{\Delta I_C}{\Delta I_B} = \beta_{AC} \text{ and } \frac{\Delta V_i}{\Delta I_B} = R_{in} \quad (\text{a resistance of the input circuit})$$

$$\therefore g_m = \frac{\beta_{AC}}{R_{in}}$$

The unit of g_m is Ω^{-1} (ohm^{-1}) or S (siemen).

Note Common emitter amplifier using n-p-n transistor is same as common emitter amplifier using p-n-p transistor.

Important points in common base amplifier

- (i) The output voltage signal is in phase with the input voltage signal.
- (ii) The common base amplifier is used to amplify high (radio)-frequency signals and to match a very low source impedance ($\sim 20 \Omega$) to a high load impedance ($\sim 100 \text{ k}\Omega$).

Example 14.36 In a transistor connected in common emitter mode $R_0 = 4 \text{ k}\Omega$, $R_i = 1 \text{ k}\Omega$, $i_c = 1 \text{ mA}$ and $i_b = 20 \mu\text{A}$. Find the voltage gain.

$$\text{Sol. Current gain, } \beta = \frac{i_c}{i_b} = \frac{1 \times 10^{-3}}{20 \times 10^{-6}} = 50$$

$$\therefore \text{Voltage gain, } A_V = \beta \left(\frac{R_0}{R_i} \right) = (50) \left(\frac{4}{1} \right) = 200$$

Example 14.37 A load of $3 \text{ k}\Omega$ is connected in the collector branch of an amplifier circuit using a transistor in common emitter mode. The current gain $\beta = 40$. The input resistance of the transistor is $0.40 \text{ k}\Omega$. If the input current is changed by $40 \mu\text{A}$, then

(i) calculate the change in output voltage.

(ii) also find the change in the input voltage.

(iii) calculate the power gain.

Sol. Given, load resistance, $R_L = 3 \text{ k}\Omega = 3 \times 10^3 \Omega$, $\beta = 40$,

$$r_i = 0.40 \text{ k}\Omega = 0.4 \times 10^3 \Omega$$

$$\text{Base current, } \Delta I_B = 40 \mu\text{A} = 40 \times 10^{-6} \text{ A}$$

(i) Output voltage,

$$V_o = \Delta I_C R_L = \beta \Delta I_B R_L = 40 \times 40 \times 10^{-6} \times 3 \times 10^3 = 4.8 \text{ V}$$

(ii) $\Delta V_{BE} = \Delta I_B r_i = 40 \times 10^{-6} \times 0.4 \times 10^3 = 16 \times 10^{-3} \text{ V}$

(iii) Power gain, $A_P = \frac{\beta^2 R_L}{r_i} = (40)^2 \left(\frac{3 \times 10^3}{0.4 \times 10^3} \right) = 12000$

Example 14.38 A transistor is used in common emitter mode in an amplifier circuit. When a signal of 40 mV is added to the base-emitter voltage, the base current changes by $40 \mu\text{A}$ and the collector current changes by 4 mA . The load resistance is $5 \text{ k}\Omega$. Calculate

(i) β ,

(ii) the input resistance R_i ,

(iii) the transconductance g_m

(iv) and the voltage gain.

Sol. (i) Given, $\Delta i_b = 40 \mu\text{A}$ and $\Delta i_c = 4 \text{ mA}$

$$\beta = \frac{\Delta i_c}{\Delta i_b} = \frac{4 \times 10^{-3}}{40 \times 10^{-6}} = 100$$

(ii) $\Delta V_i = (\Delta i_b) \times R_i$

$$\therefore R_i = \frac{\Delta V_i}{\Delta i_b} = \frac{40 \times 10^{-3}}{40 \times 10^{-6}}$$

$$= 1000 \Omega = 1 \text{ k}\Omega$$

(iii) Transconductance, $g_m = \frac{\Delta i_c}{\Delta V_i} = \frac{4 \times 10^{-3}}{40 \times 10^{-3}} = 0.1 \Omega^{-1}$

(iv) Voltage gain, $A_V = \beta \left(\frac{R_{out}}{R_{in}} \right) = (100) \left(\frac{5}{1} \right) = 500$

Example 14.39 In a common emitter amplifier, the load resistance of the output circuit is 500 times the resistance of the input circuit. If $\alpha = 0.98$, then find the (i) voltage gain and (ii) power gain.

Sol. Given, $\alpha = 0.98$ and $\frac{R_{out}}{R_{in}} = 500$, $\beta = \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} = 49$

$$(i) \text{ Voltage gain} = (\beta) \left(\frac{R_{out}}{R_{in}} \right) = (49)(500) = 24500$$

$$(ii) \text{ Power gain} = (\beta^2) \left(\frac{R_{out}}{R_{in}} \right) = (49)^2(500) = 1200500$$

Example 14.40 A transistor is connected in common emitter (CE) configuration. The collector supply is 8 V and the voltage drop across a resistor of 800Ω in the collector circuit is 0.5 V. If the current gain factor α is 0.96, find the base current.

Sol. Given, $V_{CC} = 8 \text{ V}$, $V_0 = 0.5 \text{ V}$, $R_L = 800 \Omega$

and $\alpha = 0.96$

$$\text{Current gain, } \beta = \frac{\alpha}{1 - \alpha} = \frac{0.96}{1 - 0.96} = 24$$

We have, $V_0 = \Delta I_C R_L$

$$\therefore 0.5 = \Delta I_C \times 800$$

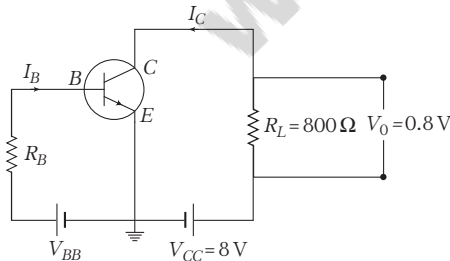
$$\text{Collector current, } \Delta I_C = \frac{0.5}{800} \text{ A}$$

$$\text{Current amplification factor, } \beta = \frac{\Delta I_C}{\Delta I_B}$$

$$\text{Base current, } \Delta I_B = \frac{\Delta I_C}{\beta} = \frac{0.5/800}{24} = 26 \times 10^{-6} \text{ A} = 26 \mu\text{A}$$

Example 14.41 An n-p-n transistor is connected in common emitter configuration in which collector supply is 8 V and the voltage drop across the load resistance of 800Ω connected in the collector circuit is 0.8 V. If current gain factor is (25/26), determine the collector-emitter voltage and base current. If the internal resistance of the transistor is 200Ω , calculate the voltage gain and power gain.

Sol.



We have, $V_0 = \Delta I_C R_L$; $0.8 = \Delta I_C \times 800$

$$\Rightarrow \Delta I_C = 10^{-3} \text{ A}$$

$$V_{CC} = V_{CE} + \Delta I_C R_L$$

$$\Rightarrow 8 = V_{CE} + 10^{-3} \times 800$$

$$\text{Collector emitter voltage, } V_{CE} = 8 - 0.8 = 7.2 \text{ V}$$

$$\text{Current gain factor, } \alpha = \frac{25}{26}$$

$$\Rightarrow \beta = \frac{\alpha}{1 - \alpha} = \frac{25/26}{1 - 25/26} = 25$$

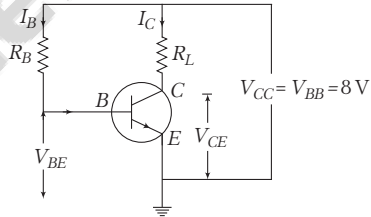
$$\text{Base current, } \Delta I_B = \frac{\Delta I_C}{\beta} = \frac{10^{-3}}{25} = 4 \times 10^{-5} \text{ A} = 40 \mu\text{A}$$

$$\text{Voltage gain, } A_V = \beta \frac{R_L}{r_i} = 25 \times \frac{800}{200} = 100$$

$$\text{Power gain, } A_P = \beta^2 \frac{R_L}{r_i} = \beta A_V = 25 \times 100 = 2500$$

Example 14.42 An n-p-n transistor in a common-emitter mode is used as a simple voltage amplifier with a collector current of 4 mA. The terminal of 8 V battery is connected to the collector through a load resistance R_L and to the base through a resistance R_B . The collector-emitter voltage $V_{CE} = 4 \text{ V}$, base-emitter voltage $V_{BE} = 0.6 \text{ V}$ and base current amplification factor $\beta_{DC} = 100$. Calculate the values of R_L and R_B .

Sol.



Given, $V_{CE} = 4 \text{ V}$, $V_{BE} = 0.6 \text{ V}$, $\beta = 100$

and $\Delta I_C = 4 \text{ mA} = 4 \times 10^{-3} \text{ A}$

we have, $V_{CE} + \Delta I_C R_L = V_{CC}$

$$4 \times 10^{-3} R_L = 8 - 4 = 4$$

Load resistance, $R_L = 1000 \Omega$

$$\text{Base current, } I_B = I_C / \beta = \frac{4 \times 10^{-3}}{100} = 4 \times 10^{-5} \text{ A}$$

We have, $V_{BB} = I_B R_B + V_{BE}$

$$8 = I_B R_B + 0.6$$

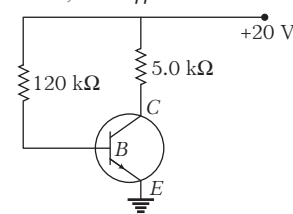
$$\therefore I_B R_B = 7.4 \text{ V}$$

$$4 \times 10^{-5} R_B = 7.4$$

Resistance connected to the base,

$$R_B = \frac{7.4}{4} \times 10^5 = 1.85 \times 10^5 \Omega = 185 \text{ k}\Omega$$

Example 14.43 In the following circuit, the value of β is 200. Find I_B , V_{CE} , V_{BE} and V_{BC} , when $I_C = 2.5 \text{ mA}$. Find whether transistor is in active, cut-off or saturation state.



Sol. Here, $\beta = \frac{I_C}{I_B}$

$$\Rightarrow I_B = \frac{I_C}{\beta} = \frac{2.5}{200} = 0.0125 \text{ mA}$$

As we know,

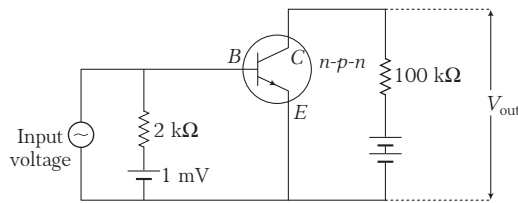
$$V_{CE} = V_{CC} - I_C R_C = 20 - (2.5 \times 10^{-3}) \times (5 \times 10^3) = 7.5 \text{ V}$$

$$\text{Also, } V_{BE} = 20 - I_B R_B = 20 - (0.0125 \times 10^{-3}) \times (120 \times 10^3) = 18.5 \text{ V}$$

$$\therefore V_{BC} = (V_{BE} - V_{CE}) = (18.5 - 7.5) = 11 \text{ V}$$

The transistor is in active state because *EB* region is forward biased and *CB* region is reverse biased.

Example 14.44 In the following common emitter configuration, a *n-p-n* transistor with current gain $\beta = 95$ is used. What is the output voltage of the amplifier?



Sol. Input current, $I_{in} = \frac{V_{in}}{R_{in}} = \frac{1 \text{ mV}}{2 \text{ k}\Omega} = 0.5 \times 10^{-6} \text{ A}$

$$\therefore \beta = \frac{I_{out}}{I_{in}} \quad (\because R_{BE} \approx 0)$$

$\therefore$ Output current,

$$I_{out} = \beta \cdot I_{in} = (95 \times 0.5 \times 10^{-6}) \text{ A} = 0.47 \times 10^{-4} \text{ A}$$

$$\therefore \text{Output voltage, } V_{out} = I_{out} \cdot R_{out} = (0.47 \times 10^{-4} \text{ A})(100 \text{ k}\Omega) = 4.7 \text{ mV}$$

Example 14.45 A *p-n-p* transistor is used in common emitter mode in an amplifier circuit. A change of $45 \mu\text{A}$ in the base current brings a change of 3 mA in collector current and 0.05 V in base-emitter voltage.

Find the

- input resistance r_i and
- the base current amplification factor (β).
- If a load of $7 \text{ k}\Omega$ is used, then also find the voltage gain of the amplifier.

Sol. Given,

$$\Delta I_B = 45 \mu\text{A} = 45 \times 10^{-6} \text{ A}, \Delta I_C = 3 \text{ mA} = 3 \times 10^{-3} \text{ A}$$

$$\Delta V_{BE} = 0.05 \text{ V and } R_L = 7 \text{ k}\Omega = 7 \times 10^3 \Omega$$

(i) Input resistance,

$$r_i = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{0.05}{45 \times 10^{-6}} = 11 \times 10^3 \Omega$$

(ii) Base current amplification factor,

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{3 \times 10^{-3}}{45 \times 10^{-6}} = 66.67$$

(iii) Voltage gain of the amplifier,

$$A_V = \beta \frac{R_L}{r_i} = \frac{66.67 \times 7 \times 10^3}{11 \times 10^3} = 424.27$$

Note Kirchhoff's laws can be applied in a transistor circuit in the similar manner as is done in normal circuits.

Transistor as an oscillator

An oscillator is an electronic device which produces electric oscillation of constant frequency and amplitude, without any externally applied input signal. It converts DC energy obtained from a battery into AC energy.

1. Principle of an oscillator

Figure shows the block diagram of an oscillator. An oscillator may be regarded as amplifier which provides its own input signal.

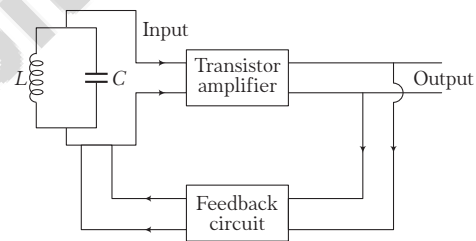


Fig. 14.54 Principle for an oscillator

Essential parts of a transistor oscillator are

- Tank circuit** It is a parallel combination of an inductance L and a capacitance C . The frequency of electric oscillations in the tank circuit is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

- Transistor amplifier** The amplifier receives the oscillations from the tank circuit and amplifies it.
- Feedback circuit** It returns a part of the output power of the transistor amplifier to the tank circuit and produces undamped oscillations. This process is called **positive feedback**.

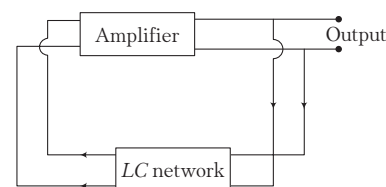


Fig. 14.55 Feedback circuit

2. Construction

A basic circuit using a common emitter $n-p-n$ transistor as an oscillator is given below. A tank circuit ($L-C$ circuit) is connected in the emitter-base circuit. A small coil L' called **feedback or tickler** coil is connected in the emitter-collector circuit. The coil L' is inductively coupled with the coil L of the tank circuit.

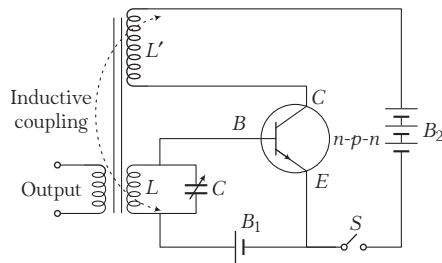


Fig. 14.56 Transistor as an oscillator

3. Working

When the switch S is closed, electrical oscillations are developed in the tank circuit. The circuit containing $n-p-n$ transistor amplifies these oscillations. A part of the amplified signal in the collector circuit is fed back in the base circuit by the coupling between L and L' . This cycle repeats again and again to give electric oscillations of constant amplitude and of constant frequency

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The oscillations of the desired frequency can be obtained by changing the value of capacitance C of the variable capacitor. The oscillations can be transferred to an external circuit by mutual induction in coil connected in that circuit.

4. Need for positive feedback

The oscillations are damped due to the presence of some inherent electrical resistance in the circuit. Hence, the amplitude of oscillations decreases rapidly and ultimately stops. To have oscillations of constant amplitude, a positive feedback is necessary from the output circuit to the input circuit so that the losses in the circuit can be compensated.

Note

(i) Gain of the amplifier is given by

$$A = \frac{V_o}{V_i} \quad (\text{Gain without feedback})$$

(ii) $A = \frac{V_o}{V_i + mV_o}$ (Gain with positive feedback)

where, m = feedback fraction.

Transistor as a switch

Transistor has an ability to turn things ON and OFF, so the most common use of transistor in an electronic circuit is as a simple switch.

A transistor can be used as a switch because its collector current is directly controlled by the base current.

To understand it properly, let us consider a basic circuit using a common emitter $n-p-n$ transistor as a switch which is given below

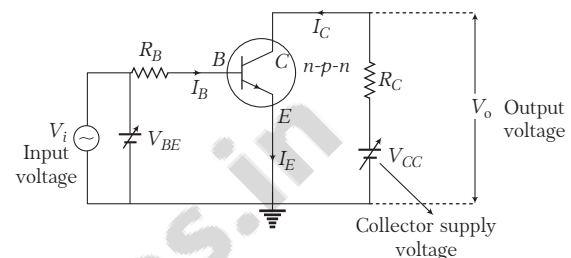


Fig. 14.57 Common emitter $n-p-n$ transistor as a switch

Apply Kirchhoff's voltage rule to the input and output sides of this circuit, we get

$$V_i = I_B \cdot R_B + V_{BE} \quad (V_i = \text{DC input voltage})$$

$$\text{and } V_o = V_{CC} - I_C \cdot R_C \quad (V_o = \text{DC output voltage})$$

In case of silicon transistor, if $V_i < 0.6 \text{ V}$, I_B will be zero, hence I_C will be zero and transistor will operate in cut-off state, and $V_o = V_{CC}$.

When $V_i > 0.6 \text{ V}$, some I_B flows, so some I_C flows and transistor will now operate in active state and hence output V_o decreases as the term $I_C R_C$ increases. With increase in V_i , the I_C increases almost linearly and hence V_o decreases linearly till its value becomes less than about 1 volt.

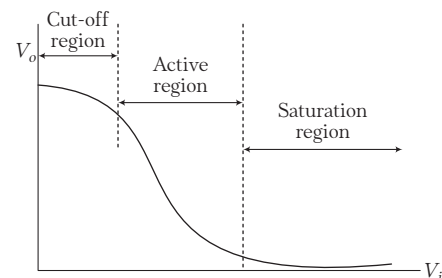


Fig. 14.58 Transfer characteristic

Beyond this, the change becomes non-linear and transistor now goes into the saturation state. When V_i is further increased, V_o is found to decrease towards zero (however, it may never become to zero).

Now the operation of transistor as a switch can be understood in the following way. When V_i is low (in cut-off region), V_o is high. If V_i is high (in saturation region), then V_o is low, almost zero.

If we define low and high states as below and above certain voltage levels corresponding to cut-off and saturation of the transistor, then we can say that a low input switches the transistor OFF and high input switches it ON. Alternatively, we can say that, a low input to the transistor gives a high output and a high input gives a low output. The switching circuits are designed in such a way that the transistor does not remain in active state.

CHECK POINT 14.3

- In a $p-n-p$ transistor, the base is the n -region. Its width relative to the p -region is
 - smaller
 - larger
 - same
 - not related
- The heavily and lightly doped region of a bipolar junction transistor are respectively.
 - base and emitter
 - base and collector
 - collector and base
 - emitter and base
- The concentrations of impurities in a transistor are
 - equal for the emitter, base and collector region
 - least for the emitter region
 - largest for the emitter region
 - largest for the base region
- An $n-p-n$ transistor conducts when
 - both collector and emitter are positive with respect to the base
 - collector is positive and emitter is negative with respect to the base
 - collector is positive and emitter is at same potential as the base
 - both collector and emitter are negative with respect to the base.
- When $p-n-p$ transistor is used as an amplifier
 - electrons move from base to emitter
 - holes move from emitter to base
 - electrons move from base to emitter
 - holes move from base to emitter
- When $n-p-n$ transistor is used as an amplifier
 - electrons move from base to collector
 - holes move from emitter to base
 - electron move from collector to base
 - holes move from base to emitter
- For a transistor, in a common emitter arrangement, the alternating current gain β is given by
 - $\beta = \left(\frac{\Delta I_C}{\Delta I_B} \right)_{V_C}$
 - $\beta = \left(\frac{\Delta I_B}{\Delta I_C} \right)_{V_C}$
 - $\beta = \left(\frac{\Delta I_C}{\Delta I_E} \right)_{V_C}$
 - $\beta = \left(\frac{\Delta I_E}{\Delta I_C} \right)_{V_C}$
- In the CB mode of a transistor when the collector voltage is changed by 0.5 V. The collector current changes by 0.05 mA. The output resistance will be
 - 10 k Ω
 - 20 k Ω
 - 5 k Ω
 - 2.5 k Ω
- While a collector to emitter voltage is constant in a transistor, the collector current changes by 8.2 mA when the emitter current changes by 8.3 mA. The value of forward current ratio β is
 - 82
 - 83
 - 8.2
 - 8.3
- In a common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
 - 50
 - 51
 - 48
 - 49
- Consider an $n-p-n$ transistor amplifier in common emitter configuration. The current gain of the transistor is 100. If the collector current changes by 1 mA, what will be the change in emitter current?
 - 1.1 mA
 - 1.01 mA
 - 0.01 mA
 - 10 mA
- A common emitter amplifier gives an output of 3 V for an input of 0.01 V. If β of the transistor is 100 and the input resistance is 1 k Ω , then the collector resistance is
 - 1 k Ω
 - 3 k Ω
 - 30 k Ω
 - 30 Ω
- An amplifier has a voltage gain $A_V = 1000$. The voltage gain (in dB) is
 - 30
 - 60
 - 3
 - 20
- Which of the following is used to produce radio waves of constant amplitude?
 - Oscillator
 - Diode
 - Rectifier
 - Amplifier
- An oscillator is nothing but an amplifier with
 - positive feedback
 - large gain
 - no feedback
 - negative feedback

ANALOG AND DIGITAL CIRCUITS

All electronic circuits are broadly divided into two categories, which are discussed below

1. Analog circuits

A continuous time-varying voltage or current signal is called analog signal.

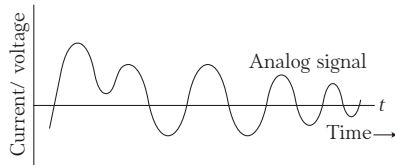


Fig. 14.59 Analog signal

The electronic circuits which process analog signals are called analog circuits.

The devices like amplifiers, radio, television, oscillators etc. make use of analog signals.

2. Digital circuits

A signal that can take only two discrete values of current or voltage is called digital signal.

A digital signal can take only two values, i.e. 0 and 1, which are marked as low and high values respectively.

In the square wave shown in Fig 14.60, a signal of 0 V represents binary 0 and a signal of 5V represents binary 1.

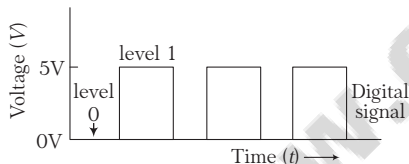


Fig. 14.60 Digital signal

The electrical circuits which process digital signals are called digital circuits. *e.g.* Pocket calculators, burglar alarms, modern computers, etc.

Advantages of digital circuits

- (i) It has high accuracy and precision.
- (ii) It is easier to design.
- (iii) Information can be stored easily.
- (iv) Digital circuits are less affected by the noise.
- (v) They typically use less bandwidth.
- (vi) It enables long distance transmission.

Binary System

There are number of questions which have only two answers Yes or No. A statement can be either True or False. A switch can be either ON or OFF.

These values may be represented by two symbols 0 and 1. In a number system in which, we have only two digits is called a binary system.

In binary system, usually we write 1 for positive response (*e.g.* when a switch is ON) and 0 for negative (when switch is OFF).

Decimal and binary number systems

1. Decimal number system

In decimal number system, ten digits are used, i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The base of the decimal number system is 10.

The place values are found by raising the base 10 to the power of the place.

Also, powers are numbered to the left of the decimal point starting with 0 and to the right of the decimal point starting with -1.

$$\begin{aligned} \text{e.g. } 2876 &= 2000 + 800 + 70 + 6 \\ &\quad \text{MSD} \qquad \qquad \text{LSD} \\ &= 2 \times 10^3 + 8 \times 10^2 + 7 \times 10^1 + 6 \times 10^0 \end{aligned}$$

where, LSD = least significant digit
and MSD = most significant digit.

2. Binary number system

In binary number system, only two digits 0 and 1 are used. In binary system, the base is 2. The two binary digits 0 and 1 are called **bits** and a group of bits is known as a **byte**. [1 byte = 8 bits]

In a binary number, the place value of each bit corresponds to some power of 2. *e.g.*

$$\begin{aligned} 1101.011 &= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} \\ &\qquad \qquad \qquad + 1 \times 2^{-2} + 1 \times 2^{-3} \\ &= 8 + 4 + 0 + 1 + 0 + 0.25 + 0.125 = 13.375 \end{aligned}$$

3. Conversion of a decimal number into its equivalent binary number

A decimal number can be converted into binary number by using divide by 2 rule. We go on dividing the given decimal number by 2, until the quotient is zero and write down the remainder after each division. These remainders when taken in reverse order form the required binary number. Let us convert 23 into its binary equivalent.

Divide by 2	23	Remainder
Divide by 2	11	1 LSD
Divide by 2	5	1
Divide by 2	2	1
Divide by 2	1	0
	0	1 MSD

The remainders in reverse order are 10111. Hence, the binary equivalent of 23 is 10111. Symbolically, we write

$$(23)_{10} = (10111)_2$$

The conversion of a decimal fraction into binary fraction is performed by multiply by 2 rule. Let us convert 0.125 into its binary equivalent.

	Binary
$0.125 \times 2 = 0.250$	0
$0.250 \times 2 = 0.500$	0
$0.500 \times 2 = 1.000$	1
$0.000 \times 2 = 0.000$	0

The recovered binaries taken from top to bottom form the required binary fraction. The binary equivalent of 0.125 is therefore written as 0.0010.

Note If a decimal number contains both the integral and the fractional part, such as 23.125, then after determining the binary equivalent of each part separately, the two parts are combined. Thus, binary equivalent of 23.125 is 10111.0010.

Example 14.46 Convert the decimal number 10.625 into its binary equivalent.

Sol. Integral part is 10. It can be converted into its binary part by using divide by 2 rule.

Divide by 2	10	Remainder
Divide by 2	5	0
Divide by 2	2	1
Divide by 2	1	0
	0	1

Thus, $(10)_{10} = (1010)_2$

Fractional part is 0.625. It can be converted into its binary part by using multiply by 2 rule.

	Binary
$0.625 \times 2 = 1.250$	1
$0.250 \times 2 = 0.500$	0
$0.500 \times 2 = 1.000$	1
$0.000 \times 2 = 0.000$	0

Thus, $(0.625)_{10} = (0.1010)_2$
 $\therefore (10.625)_{10} = (1010.1010)_2$

4. Conversion of a binary number into its equivalent decimal number

In binary number system, base is 2. Therefore, the least significant digit in the binary number is the coefficient of 2 with power zero. As, we move towards the left side of LSD, the power of 2 goes on increasing.

e.g. $(1001101)_2$

↓	↓
MSD	LSD

$$= 1 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 77$$

Note that, while finding the decimal equivalent of a fractional binary number, the multipliers will be $2^{-1}, 2^{-2}, 2^{-3} \dots$ starting from MSD, till LSD is reached.

e.g. $(0.1011)_2$

↓	↓
MSD	LSD

$$= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$$

$$= \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16} = 0.6875$$

$$(0.1011)_2 = (0.6875)_{10}$$

Example 14.47 Convert the binary number 101011.1101 into equivalent decimal number.

Sol. $101011.1101 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$

$$= 32 + 0 + 8 + 0 + 2 + 1 + 0.5 + 0.25 + 0 + 0.0625 = 43.8125$$

$$\therefore (101011.1101)_2 = (43.8125)_{10}$$

5. Sum of binary numbers

The sum of digits 0 and 1 with themselves and with each other are given by the following laws

- (i) $0 + 0 = 0$ (ii) $1 + 0 = 1$
 (iii) $0 + 1 = 1$ (iv) $1 + 1 = 10$

Now, let us take few examples of different types. Through these examples, we can easily understand the method of adding two binary numbers.

Example 14.48 Add the binary numbers 101 and 11.

Sol. We first write these numbers in columns as below

$$\begin{array}{r} 101 \\ + 11 \\ \hline 1000 \end{array}$$

Rule Starting from right, in the first column, we get $1 + 1 = 10$. The 0 is written below this column and 1 is carried in the next column. In the second column, $1 + (0 + 1) = 1 + 1 = 10$. From the value, 0 is written below this column and 1 is carried in the third column. Now, in the third column $1 + 1 = 10$ is obtained. Thus, $101 + 11 = 1000$.

Example 14.49 Add the binary numbers 101 and 110.

Sol. Rewriting the numbers in columns as below

$$\begin{array}{r} 101 \\ + 110 \\ \hline 1011 \end{array}$$

The sum is in accordance with the four laws given in the beginning. Thus, $101 + 110 = 1011$.

Example 14.50 Add the binary numbers 1101 and 1110.

Sol. Writing the numbers in columns, we get

$$\begin{array}{r} 1101 \\ + 1110 \\ \hline 11011 \end{array}$$

Here, in the last column we get $1 + (1 + 1)$ which we wrote 11. This is because, $1 + (1 + 1) = 1 + 10 \Rightarrow 10$

$$\begin{array}{r} + 1 \\ \hline 11 \end{array}$$

Example 14.51 Add the binary numbers 111, 101 and 110.

Sol. For adding more than two binary numbers the above method is repeated again and again. If we want to add 111, 101 and 110. First we will add 111 and 101 by the method discussed above, then add 110 with the sum of 111 and 101.

$$\begin{array}{r} 111 \\ 101 \\ \hline 1100 \end{array}$$

Now, add 1100 and 110

$$\begin{array}{r} 1100 \\ + 110 \\ \hline 10010 \end{array}$$

6. Difference of binary numbers

The four laws of difference (in binary system) are as below

(i) $0 - 0 = 0$ (ii) $1 - 0 = 1$ (iii) $1 - 1 = 0$ (iv) $10 - 1 = 1$

Now, let us take an example in support of the above four laws.

Example 14.52 Subtract the binary number 100 from the binary number 101.

Sol. Let us write the given numbers in columns as below

$$\begin{array}{r} 101 \\ - 100 \\ \hline 001 \end{array}$$

Thus, $101 - 100 = 001$ or simply 1.

Example 14.53 Subtract the binary number 1110 from the binary number 11011.

Sol. Rewriting the given numbers in columns as below.

$$\begin{array}{r} 11011 \\ - 1110 \\ \hline 1101 \end{array}$$

In this example, first and second columns were subtracted in accordance with the four laws given in the starting. In the third column, for subtracting 1 from 0 we carry 1 from the fourth column. Thus, $10 - 1 = 1$ is obtained. In the fourth column, for subtracting 1 from 0 (which remains 0 after

giving 1 to the third column) we carry 1 from the fifth column. Thus, $10 - 1 = 1$ is obtained.

Thus, $11011 - 1110 = 1101$

LOGIC GATES

Logic gates are the building blocks of digital circuit that makes use of diodes and transistors to perform switching function. It is used for performing a particular logical operation. In other words, a gate is a digital circuit that follows certain logical relationship between the input and output voltages.

A logic gate has one output but one or more inputs. There are three basic gates named as OR gate, AND gate and NOT gate.

Each basic logic gate is indicated by a logic symbol and its function is defined and described either by a truth table or by a boolean expression.

1. Truth table

It is a table that shows all possible input combinations and the corresponding output combinations for a logic gate. To understand the concept of truth table, let us take an example.

A bulb is connected to an AC source via two switches S_1 and S_2 .

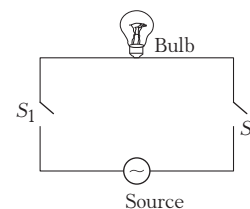


Fig. 14.61 A bulb is connected via two switches

In binary system, we will write 0, if the switch (or bulb) is OFF and write 1 if it is ON. Further, let us write

A for state of switch S_1 ,

B for state of switch S_2

and

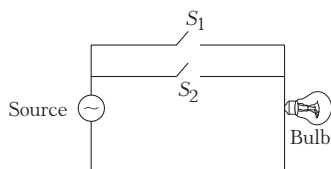
C for state of the bulb.

Now, let us make a table (called truth table).

Switch S_1	Switch S_2	Bulb	A	B	C
OFF	ON	OFF	0	1	0
ON	OFF	OFF	1	0	0
OFF	OFF	OFF	0	0	0
ON	ON	ON	1	1	1

Here, A and B represents the input combination and C represents the output for this particular circuit. This table is called **truth table** and is used to define or describe the function of a particular logic gate. Now, let us discuss one example based on this

Example 14.54 Make a truth table corresponding to the circuit shown in figure.



Sol. Here, the two switches can be operated in different combinations. Note that the two switches are connected in parallel. So, if one switch is OFF, the bulb will still glow. We will write 0, if the switch is OFF and write 1 if it is ON. Further, A stands for state of switch S_1 and B stands for state of switch S_2 and C for state of bulb. Now, let us make a truth table based on this.

Switch S_1	Switch S_2	Bulb	A	B	C
ON	OFF	ON	1	0	1
OFF	ON	ON	0	1	1
OFF	OFF	OFF	0	0	0
ON	ON	ON	1	1	1

2. Boolean expression

Boolean algebra is a different kind of algebra in which only two states of variables (0 and 1) called **boolean variables** are allowed. Hence, to describe the functioning of a logic gate in terms of an equation, we use boolean expression. It is defined as the expression showing the combination of two boolean variables resulting into a new boolean variable known as boolean expression.

Some useful postulates and laws of boolean algebra

(i) Boolean postulates

- (a) $0 + A = A$ (b) $1 + A = 1$
 (c) $0 \cdot A = 0$ (d) $1 \cdot A = A$
 (e) $A + \bar{A} = 1$

(ii) Identity law

- (a) $A + A = A$ (b) $A \cdot A = A$

(iii) Negation law $\bar{\bar{A}} = A$

(iv) De-Morgan's theorem

It states that, the complement of the whole sum is equal to the product of individual complements and *vice-versa*. i.e.

- (a) $\overline{A + B} = \bar{A} \cdot \bar{B}$ (b) $\overline{A \cdot B} = \bar{A} + \bar{B}$

De-Morgan's theorem also states that,

(a) $\overline{\bar{A} + \bar{B}} = \overline{\bar{A}} \cdot \overline{\bar{B}} = A \cdot B$

(b) $\overline{\bar{A} \cdot \bar{B}} = \overline{\bar{A}} + \overline{\bar{B}} = A + B$

(v) Commutative laws

- (a) $A + B = B + A$ (b) $A \cdot B = B \cdot A$

(vi) Associative laws

(a) $A + (B + C) = (A + B) + C$

(b) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

(vii) Distributive laws

(a) $A \cdot (B + C) = A \cdot B + A \cdot C$

(b) $(A + B) \cdot (A + C) = A + BC$

(viii) Absorption laws

(a) $A + A \cdot B = A$

(b) $A \cdot (A + B) = A$

(c) $A \cdot (A + \bar{A}) = A$

(d) $\bar{A} \cdot (A + B) = \bar{A} \cdot B$

(ix) Boolean identities

(a) $A + \bar{A}B = A + B$

(b) $A(\bar{A} + B) = AB$

(c) $A + BC = (A + B)(A + C)$

Logic system

Let us now study the three types of logic gates in detail

1. OR gate

An OR gate has two or more inputs and one output. In this gate, if any of the input or all the inputs are 1, then output is 1.

OR operation Consider the circuit given below

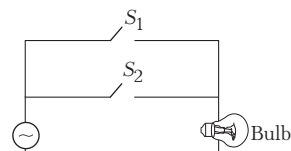


Fig. 14.62 Circuit of OR operation

The two switches are connected in parallel, so when S_1 or S_2 is closed, the bulb is ON, i.e. it glows. So, it represents OR operation.

Boolean expression

Let the two input of OR gate be A and B respectively and its output be Y .

$$Y = A + B$$

Logic symbol

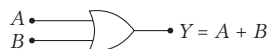


Fig. 14.63 Logic symbol of OR gate

Truth table

Input		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

OR Gate using diodes

As shown in figure, a two input OR gate can be realised by using two ideal diodes D_1 and D_2 and a resistor R . The negative terminal of the battery is grounded (i.e. it is at zero volt) and corresponds to the 0 state, and the positive terminal (which is at say 5V) corresponds to the 1 state.

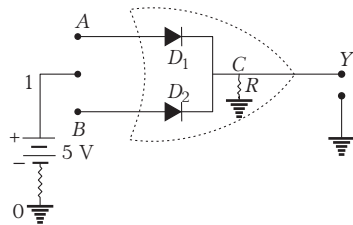


Fig. 14.64 Realisation of OR gate using two p-n junction diodes

The following four cases are possible

- When $A = 0$ and $B = 0$, both the diodes are connected to earth (0V). They do not conduct. Output across R is zero, i.e. $Y = 0$.
 - When $A = 0$ and $B = 1$, D_1 is connected to earth. It does not conduct and D_2 is connected to 5V, it gets forward biased and conducts. Voltage drop across D_2 is zero and the full voltage of 5V appears across R , so $Y = 1$.
 - When $A = 1$ and $B = 0$, D_1 gets forward biased and D_2 (reverse biased) does not conduct. Voltage drop across R is again 5V, so $Y = 1$.
 - When $A = 1$ and $B = 1$, both D_1 and D_2 get forward biased and conduct current. But D_1 and D_2 are in parallel. Voltage drop across R is still 5V, so $Y = 1$.
- Hence, the circuit shown in figure above satisfies the truth-table of OR gate.

Input and output waveforms for an OR gate

The output of an OR gate is high when either A or B or both the inputs are high. Therefore, the output waveform Y will be as shown in Fig. 14.65.

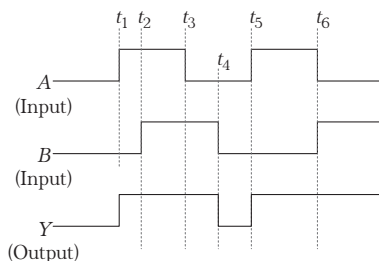


Fig. 14.65 Input and output waveforms of an OR gate

2. AND gate

It has two or more inputs and one output. In AND gate, output is 1 only when all the inputs are 1.

AND operation

Consider the circuit given below

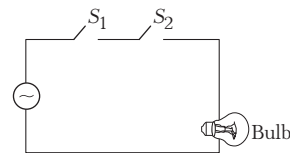


Fig. 14.66 Circuit of AND operation

Now, the two switches are connected in series. If any one switch is open, the current will not flow and hence bulb will not glow. So, when S_1 and S_2 are closed, bulb is ON, i.e. it glow. So, it represents AND operation.

Boolean expression

Let the inputs of AND gate be A and B respectively and its output be Y .

$$Y = A \cdot B$$

Logic symbol

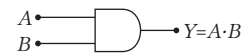


Fig. 14.67 Logic symbol of AND gate

Truth table

Input		Output
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

AND gate using diodes

As shown in figure, two inputs AND gate can be realised by using two ideal junction diodes D_1 and D_2 . Here the resistance R is kept permanently connected to the positive terminal of 5V battery.

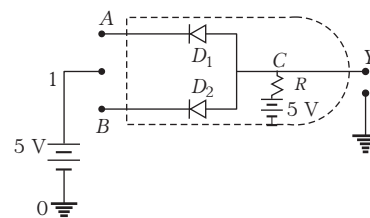


Fig. 14.68 Realisation of an AND gate

The following four cases are possible

- When $A = 0$ and $B = 0$, the input terminals A and B are earthed (0V). The two diodes get forward biased and conduct current. But both diodes are shorted. The point Y also gets earthed through the shorted diodes, hence output $Y = 0$.
- When $A = 0$ and $B = 1$, diode D_1 is forward biased but shorted. Diode D_2 is not forward biased and hence does not conduct. Hence, output $Y = 0$.
- When $A = 1$ and $B = 0$, D_1 does not conduct, D_2 is forward biased but shorted. Hence, output $Y = 0$.
- When $A = 1$ and $B = 1$, both D_1 and D_2 do not conduct as they are not forward biased. The output voltage is equal to the battery voltage of 5V. Hence, $Y = 1$.

Hence, the circuit shown in figure above satisfies the truth table of AND gate.

Input and output waveforms for an AND gate

The output of an AND gate is high when both the inputs are high, otherwise the output is low. Therefore, the output waveform Y for AND gate will be as shown in Fig 14.69.

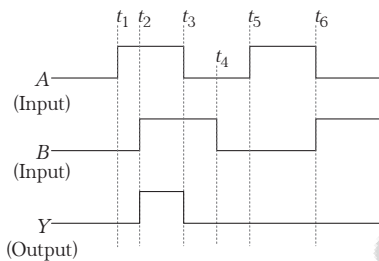


Fig. 14.69 Input and output waveforms of an AND gate

3. NOT gate

It has only one input and one output.

It gives an inverted version of its input, i.e. if input is 1, then output is 0 and vice-versa.

NOT operation

Consider the circuit given below

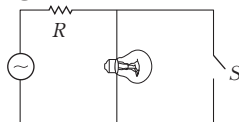


Fig. 14.70 Circuit of NOT operation

Here, only one switch is used and it is connected in parallel with bulb. So, when S is open, bulb is ON, i.e. it glows. It represents inverted or NOT operation.

Boolean expression

Let the input of NOT gate be A and its output be Y .

$$Y = \bar{A}$$

Logic symbol

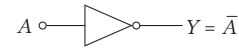


Fig. 14.71 Logic symbol of NOT gate

Truth table

Input	Output
A	$Y = \bar{A}$
0	1
1	0

NOT gate using transistor

As shown in figure, a NOT gate can be obtained by using an $n-p-n$ transistor.

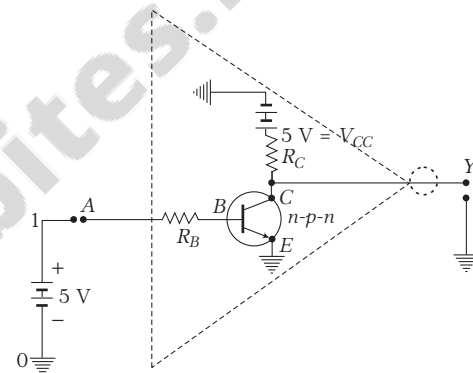


Fig. 14.72 Realisation of NOT gate

The base resistor R_B and the collector resistance R_C are so chosen that when a voltage of 5V is applied at the base of the transistor, a large collector current flows, the voltage at Y drops and the base-collector junction is forward biased.

The following two cases are possible

- When input $A = 0$, the base of the transistor is earthed, the base-emitter junction is not forward biased and the collector-base junction is reverse biased.

Hence, the base current and the collector current are both zero. The transistor is in the cut-off mode. No voltage drop occurs across R_C .

The voltage at Y is 5 V, hence output $Y = 1$.

- When input $A = 1$, the input terminal A is at 5 V, both emitter and collector are forward biased. A large collector current flows.

The transistor is in the saturation mode. The voltage drop across R_C is almost 5 V, hence output $Y = 0$.

Thus, the circuit of given figure satisfies the truth table of a NOT gate.

Input and output waveforms for NOT gate

The output of a NOT gate is high when the input is low and *vice-versa*. Therefore, the output waveform Y for NOT gate is as shown in Fig 14.73

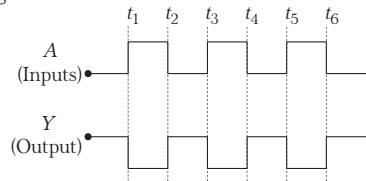


Fig. 14.73 Input and output waveforms of NOT gate

Combination of logic gates

Various combinations of three basic gates (*i.e.* OR, AND and NOT) give rise to complicated digital circuits. Let us now discuss a few combinations of these basic gates using their symbols.

1. NAND gate

It has two or more inputs and one output. This is an AND gate followed by a NOT gate. In this gate, if all the inputs are 1, then output will be 0. Let the two inputs of NAND gate be A and B , respectively and its output be Y .

Boolean expression

$$Y = \overline{A \cdot B} \quad \text{or} \quad Y = \overline{AB}$$

Logic symbol

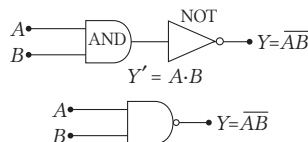


Fig. 14.74 Logic symbol of NAND gate

Truth table

Input		Output	
A	B	$Y' = A \cdot B$	$Y = \overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Input and output waveforms for NAND gate

The output waveform for NAND gate will be as shown in Fig. 14.75.

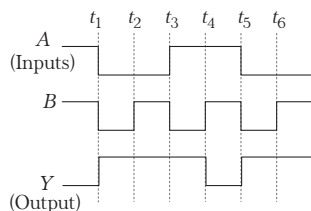


Fig. 14.75 Input and output waveforms of NAND gate

2. NOR gate

It has two or more inputs and one output. When a NOT operation is applied after OR gate, then it is called NOR gate.

In this gate, if all the inputs are 0, then only output will be 1, otherwise it is 0. Let the two inputs be A and B respectively and its output be Y .

Boolean expression

$$Y = \overline{A + B}$$

Logic symbol

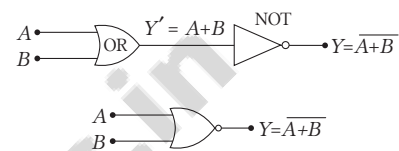


Fig. 14.76 Logic symbol of NOR gate

Truth table

Input		Output	
A	B	$Y' = A + B$	$Y = \overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Input and output waveforms of NOR gate

The output waveform for NOR gate will be as shown in Fig. 14.77

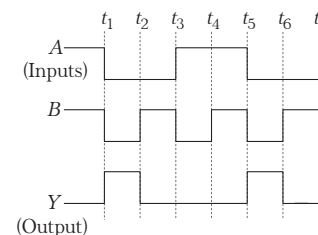


Fig. 14.77 Input and output waveforms of NOR gate

3. XOR gate

It is also called the **exclusive OR function**. It is a function of two logical variables A and B which evaluates to 1, if one of two variables is 0 and the other is 1. The output is zero, if both the variables are 0 or 1.

Boolean expression

$$Y = \overline{A} \oplus B = A \text{ XOR } B \quad \text{or} \quad Y = A \oplus B \quad \text{or} \quad Y = \overline{AB} + A\overline{B}$$

Logic symbol

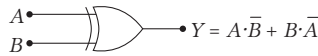


Fig. 14.78 Logic symbol of XOR gate

Truth table

Input		Output				
A	B	$\bar{A}$	$\bar{B}$	$A \cdot \bar{B}$	$\bar{A} \cdot B$	$A = A \cdot \bar{B} + \bar{A} \cdot B$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

Input and output waveforms of XOR gate

The output waveform for XOR gate will be as shown in Fig. 14.79.

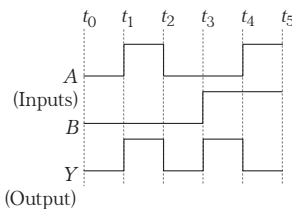


Fig. 14.79 Input and output waveforms of XOR gate

4. XNOR gate

It is also called **exclusive NOR function**. It is a function of two logical variables A and B which evaluates to 0, if one of two variable is 0 and the other is 1. The output is one, if both the variables are 0 or 1.

Boolean expression

$$Y = \overline{A \oplus B} = A \odot B \quad \text{or} \quad Y = \overline{A \cdot B} + AB$$

Logic symbol

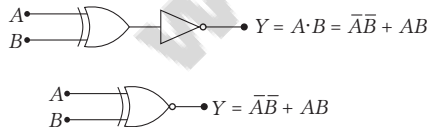


Fig. 14.80 Logic symbol of XNOR gate

Truth table

Input		Output
A	B	$Y = \overline{A \cdot B} + AB$
0	0	1
0	1	0
1	0	0
1	1	1

Input and output waveforms of XNOR gate

The output waveform Y for XNOR gate is as shown in Fig. 14.81

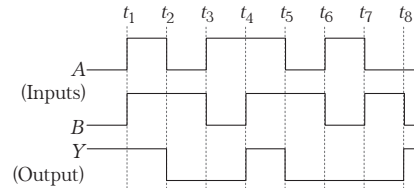


Fig. 14.81 Input and output waveforms of XNOR gate

NAND and NOR gates as digital building blocks

The repeated use of the OR, AND or the NOT gates alone cannot give a different gate. But the repeated use of the NAND or the NOR gates alone can give all basic gates like OR, AND and NOT gates. Hence, the NAND and the NOR gates are also called **universal gates**. In digital circuits, these gates serve as digital building blocks. Now, let us derive the basic logic gates one by one from NAND and NOR gates

1. Logic gates using NAND gate

Using NAND gate we can realise different gates as follows

NOT gate from NAND gate

To obtain NOT gate from NAND gate, the two inputs of the NAND gate are joined together.

Boolean expression

The boolean expression for a NAND gate is given by

$$Y = \overline{A \cdot B}$$

When

$$B = A,$$

$$Y = \overline{A \cdot A}$$

$$= \bar{A}$$

$$(\because A \cdot A = A)$$

It is the boolean expression for NOT gate.

Logic symbol



Fig. 14.82 Realisation of NOT gate using NAND gate

Truth table

Input		Output
A = B		$Y = \bar{A}$
0		1
1		0

AND gate from NAND gate

To obtain AND gate from NAND gate, one NOT gate (made from NAND gate) is used. The output of the NAND gate is given to the input of the NOT gate. The logic circuit so obtained will work as AND gate.

Boolean expression

The output of the NAND gate is given by

$$Y' = \overline{AB}$$

This output will form the input of the NOT gate.

∴ Final output is given by

$$\begin{aligned} Y &= \overline{Y'} \\ &= \overline{\overline{AB}} \\ &= A \cdot B \text{ (boolean expression for AND gate)} \end{aligned}$$

Logic symbol

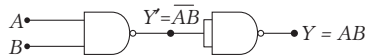


Fig. 14.83 Realisation of AND gate using NAND gate

Truth table

A	B	Y'	Y = AB
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

OR gate from NAND gate

To obtain OR gate from NAND gate, two NOT gates are used. The outputs of two NOT gate (obtain from NAND gate) is given to the inputs of the NAND gate. The logic circuit so obtained will work as OR gate.

Boolean expression

The boolean expression for a NAND gate having $\overline{A}$ and $\overline{B}$ as its two inputs is

$$\begin{aligned} Y &= \overline{(\overline{A} \cdot \overline{B})} \\ &= \overline{\overline{A}} + \overline{\overline{B}} \quad \text{(applying De Morgan's theorem)} \\ &= A + B \quad \text{(boolean expression for OR gate)} \end{aligned}$$

Logic symbol

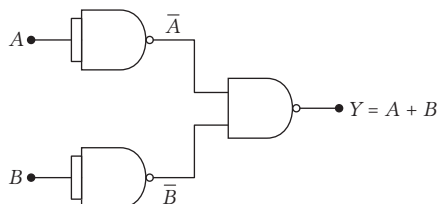


Fig. 14.84 Logic symbol of OR gate obtained from NAND gate

Truth table

Input		Output		
A	B	$\overline{A}$	$\overline{B}$	$Y = A + B$
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

2. Logic gates using NOR gate

Using NOR gate we can realise different gates as follows

NOT gate from NOR gate

To obtain NOT gate from NOR gate, two inputs of NOR gate are joined together.

Boolean expression

The boolean expression for NOR gate is given by

$$Y = \overline{A + B}$$

Putting $B = A$, $Y = \overline{A + A} = \overline{A}$ ($\because A + A = A$)

This is same as boolean expression for NOT gate.

Logic symbol



Fig. 14.85 Realisation of NOT gate using NOR gate

Truth table

A	$Y = \overline{A}$
0	1
1	0

AND gate from NOR gate

To obtain AND gate from NOR gate, two NOT gates (made from NOR gates) are connected to a NOR gate. The logic circuit so obtained will work as AND gate.

Boolean expression

The boolean expression for a NOR gate having $\overline{A}$ and $\overline{B}$ as two inputs is given by

$$\begin{aligned} Y &= \overline{(\overline{A} + \overline{B})} \\ &= \overline{\overline{A}} \cdot \overline{\overline{B}} \quad \text{(applying De Morgan's theorem)} \\ &= A \cdot B \quad \text{(boolean expression for AND gate)} \end{aligned}$$

Logic symbol

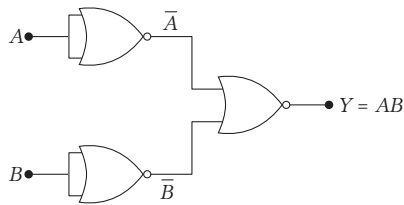


Fig. 14.86 Realisation of AND gate using NOR gate

Truth table

A	B	$\bar{A}$	$\bar{B}$	$Y = A \cdot B$
0	0	1	1	0
1	0	0	1	0
0	1	1	0	0
1	1	0	0	1

OR gate from NOR gate

To obtain OR gate from NOR gate, we connect NOR gate to the NOT gate (made from NOR gate). The logic circuit obtained will work as OR gate.

Boolean expression

The output of the NOR gate is given by $Y' = \overline{A + B}$

This output of the NOR gate forms the input of the NOT gate.

$\therefore$ The final output is given by

$$Y = \overline{Y'} = \overline{\overline{A + B}} = A + B \quad (\text{boolean expression for OR gate})$$

Logic symbol

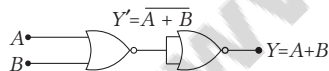
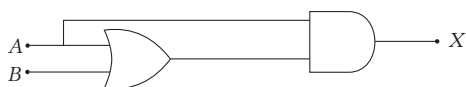


Fig. 14.87 Realisation of OR gate using NOR gate

Truth table

A	B	Y'	Y = A + B
0	0	1	0
1	0	0	1
0	1	0	1
1	1	0	1

Example 14.55 Draw the truth table for the function X of A and B for the following logic gate.



Sol. The output X in terms of the inputs A and B can be written as

$$X = A \cdot (A + B).$$

Let us make the truth table corresponding to this function

Truth table

Input		Output	
A	B	$A + B$	$X = A \cdot (A + B)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Example 14.56 Write the truth table for the following circuit. Name the equivalent gate that this circuit represents.



Sol. The given combination consists of NOR gate and NOT gate, so equivalent gate is OR gate.

Truth table

A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

From the truth table, it is clear that the output is 1 only when atleast one of the inputs is at the high state (i.e. 1).

Example 14.57 Let $X = A \cdot \overline{BC}$. Evaluate X for

- $A = 1, B = 0, C = 1$,
- $A = B = C = 1$
- and $A = B = C = 0$.

Sol. (i) When $A = 1, B = 0$ and $C = 1$

$$BC = 0$$

$$\therefore \overline{BC} = 1 \text{ or } \overline{ABC} = 1$$

(ii) When $A = B = C = 1$

$$\text{Then, } BC = 1 \text{ or } \overline{BC} = 0 \therefore \overline{ABC} = 0$$

(iii) When $A = B = C = 0$

$$\text{Then, } BC = 0$$

$$\therefore \overline{BC} = 1 \text{ or } \overline{ABC} = 0$$

Example 14.58 Write the truth table for the logical function $D = (A \text{ OR } B) \text{ AND } B$.

Sol. $A \text{ OR } B$ is a logical function, say it is equal to X , i.e.

$$X = A \text{ OR } B = A + B$$

Now,

$$D = X \text{ AND } B = X \cdot B$$

The corresponding truth table is as under

A	B	$X = A \text{ OR } B$	$D = (A \text{ OR } B) \text{ AND } B$
1	0	1	0
0	1	1	1
0	0	0	0
1	1	1	1

Note The given function can also be written as $D = (A + B) \cdot B$

Example 14.59 Write down truth tables for

- (i) $X = \bar{A} \cdot \bar{B} + \bar{A} \cdot B$ (ii) $X = (A + \bar{B}) + \bar{A} \cdot \bar{B}$

Sol. (i) The corresponding truth table is as under

A	B	$\bar{A}$	$\bar{B}$	$\bar{A} \cdot \bar{B}$	$\bar{A} \cdot B$	$X = \bar{A} \cdot \bar{B} + \bar{A} \cdot B$
0	0	1	1	1	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	0
1	1	0	0	0	0	0

(ii) The corresponding truth table is as under

A	B	$\bar{B}$	$A + \bar{B}$	$A \cdot B$	$\bar{A} \cdot \bar{B}$	$X = (A + \bar{B}) + (\bar{A} \cdot \bar{B})$
0	0	1	1	0	1	1
0	1	0	0	0	1	1
1	0	1	1	0	1	1
1	1	0	1	1	0	1

Example 14.60 Let $Y = \bar{A}\bar{B}C + \bar{B}\bar{C}A + \bar{C}\bar{A}B$. Find the output Y, if following inputs are given

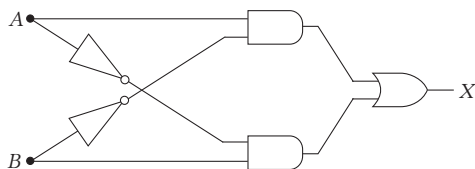
- (i) $A = 1, B = 0, C = 1$ (ii) $A = B = C = 1$
(iii) $A = B = C = 0$

Sol.

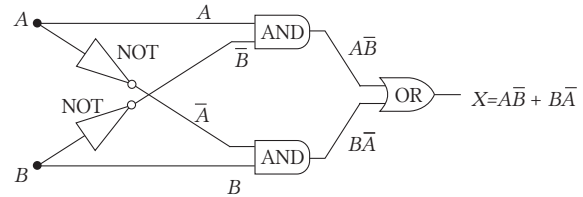
A	B	C	$\bar{A}\bar{B}C$	$\bar{B}\bar{C}A$	$\bar{C}\bar{A}B$	Y
0	0	0	1	1	1	0
1	1	1	0	0	0	0
1	0	1	1	1	0	1

- (i) $Y = 1$ (ii) $Y = 0$ (iii) $Y = 0$

Example 14.61 Write the truth table for the circuit given in figure.



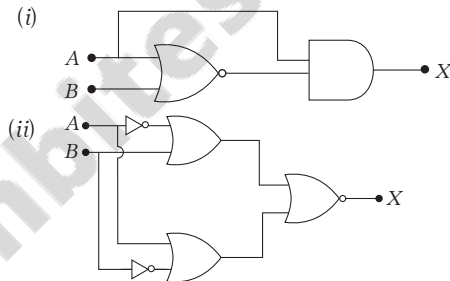
Sol. The circuit can be redrawn as



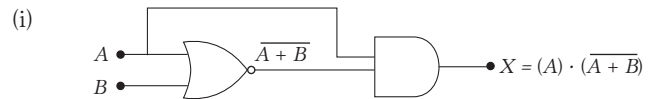
The corresponding truth table is shown below

A	B	$\bar{A}$	$\bar{B}$	$A\bar{B}$	$B\bar{A}$	$A\bar{B} + B\bar{A} = X$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

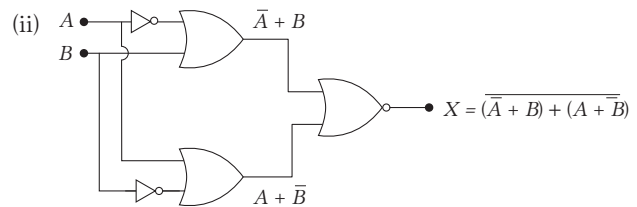
Example 14.62 Construct truth table for the function X of A and B as shown in the figure.



Sol. First write the boolean expression for the figure and then construct truth table.



A	B	$\bar{A} + \bar{B}$	$X = A \cdot (\bar{A} + \bar{B})$
0	0	1	0
0	1	0	0
1	0	0	0
1	1	0	0



A	B	$\bar{A}$	$\bar{B}$	$\bar{A} + \bar{B}$	$A + \bar{B}$	$X = (\bar{A} + \bar{B}) + (A + \bar{B})$
0	0	1	1	1	1	0
0	1	1	0	1	0	0
1	0	0	1	0	1	0
1	1	0	0	1	1	0

CHECK POINT 14.4

- The binary number 10111 is equivalent to the decimal number
(a) 19 (b) 31 (c) 23 (d) 22
- Sum of the two binary numbers $(100010)_2$ and $(11011)_2$ is
(a) $(111101)_2$ (b) $(111111)_2$
(c) $(101111)_2$ (d) $(111001)_2$
- The output of OR gate is 1
(a) if both inputs are zero
(b) if either or both inputs are 1
(c) only if both input are 1
(d) if either input is zero
- If A and B are two inputs in AND gate, then AND gate has an output of 1 when the values of A and B are
(a) $A = 0, B = 0$ (b) $A = 1, B = 1$
(c) $A = 1, B = 0$ (d) $A = 0, B = 1$

- A gate has the following truth table

P	Q	R
1	1	1
1	0	0
0	1	0
0	0	0

The gate is

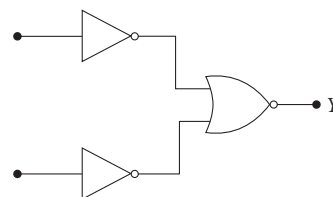
- NOR
 - OR
 - NAND
 - AND
- If A and B are two inputs in AND gate, then AND gate has an output of 0 when the value of A and B are
(a) $A = 0, B = 0$ (b) $A = 0, B = 1$
(c) $A = 1, B = 0$ (d) All are correct
 - A gate in which all the inputs must be low to get a high output is called
(a) NAND gate (b) an inverter
(c) NOR gate (d) AND gate
 - The output of a NAND gate is 0,
(a) if both inputs are 0
(b) if one input is 0 and the other input is 1
(c) if both inputs are 1
(d) either if both inputs are 1 or if one of the inputs is 1 and the other 0

- The truth table given below is for

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(A and B are the inputs, Y is the output)

- NOR
 - AND
 - XOR
 - NAND
- The boolean equation of NOR gate is
(a) $Y = A + B$ (b) $Y = \overline{A + B}$
(c) $Y = A \cdot B$ (d) $Y = \overline{A \cdot B}$
 - Any digital circuit can be realised by repetitive use of only
(a) NOT gates (b) OR gates
(c) AND gates (d) NOR gates
 - The logic gates behind NOR gate is that it gives
(a) high output when both the inputs are low
(b) low output when both the inputs are low
(c) high output when both the inputs are high
(d) None of the above
 - The logic gates having output 1 for the inputs of 1 and 0 are
(a) AND and OR (b) OR and NOR
(c) NAND and NOR (d) NAND and OR
 - If the two inputs of a NAND gate are shorted, the gate is equivalent to
(a) XOR (b) OR
(c) NOR (d) NOT
 - Which logic gate is represented by the following combination of logic gates?



- OR
- NAND
- AND
- NOR

Chapter Exercises

(A) Taking it together

Assorted questions of the chapter for advanced level practice

- 1** Number of electrons in the valence shell of a semiconductor is
(a) 1 (b) 2 (c) 3 (d) 4
- 2** Hole is [NCERT Exemplar]
(a) an anti-particle of electron
(b) a vacancy created when an electron leaves a covalent bond
(c) absence of free electrons
(d) an artificially created particle
- 3** A piece of copper and the other of germanium are cooled from the room temperature to 80 K, then which of the following would be a correct statement?
(a) Resistance of each increases.
(b) Resistance of each decreases.
(c) Resistance of copper increases while that of germanium decreases.
(d) Resistance of copper decreases while that of germanium increases.
- 4** The valence band and conduction band of a solid, overlap at low temperature, the solid may be
(a) a metal (b) a semiconductor
(c) an insulator (d) None of these
- 5** In a semiconductor, the separation between conduction band and valence band is of the order of
(a) 100 eV (b) 10 eV
(c) 1 eV (d) zero
- 6** In a good conductor, the energy gap between the conduction band and the valence band is
(a) infinite (b) wide (c) narrow (d) zero
- 7** In semiconductors, at room temperature the
(a) valence band is partially empty and the conduction band is partially filled
(b) valence band is completely filled and the conduction band is partially filled
(c) valence band is completely filled
(d) conduction band is completely empty
- 8** A piece of semiconductor is connected in series in an electric circuit. On increasing the temperature, the current in the circuit will
(a) decrease (b) remain unchanged
(c) increase (d) stop flowing
- 9** The forbidden energy gap in the energy bands of germanium at room temperature is about
(a) 1.1 eV (b) 0.1 eV
(c) 0.67 eV (d) 6.7 eV
- 10** The energy band gap of Si is
(a) 0.70 eV
(b) 1.1 eV
(c) between 0.70 eV to 1.1 eV
(d) 5 eV
- 11** What enables Ge to behave as semiconductor even though all electrons in the valence band form covalent bonds? It is due to the small width of
(a) valence band (b) conduction band
(c) forbidden energy gap (d) None of these
- 12** E_g for silicon is 1.12 eV and that for germanium is 0.72 eV. Therefore, it can be concluded that
(a) more number of electron-hole pairs will be generated in silicon than in germanium at room temperature
(b) less number of electron-hole pairs will be generated in silicon than in germanium at room temperature
(c) equal number of electron-hole pairs will be generated in both at lower temperatures
(d) equal number of electron-hole pairs will be generated in both at higher temperatures
- 13** When the electrical conductivity of a semiconductor is due to the breaking of its covalent bonds, then the semiconductor is said to be
(a) donor (b) acceptor
(c) intrinsic (d) extrinsic
- 14** The majority charge carriers in *p*-type semiconductor are
(a) electrons (b) protons
(c) holes (d) neutrons
- 15** A *p-n* junction has a thickness of the order of
(a) 1 cm (b) 1 mm
(c) 10^{-6} m (d) 10^{-12} cm
- 16** A *p*-type semiconductor can be obtained by adding
(a) arsenic to pure silicon
(b) gallium to pure silicon
(c) antimony to pure germanium
(d) phosphorous to pure germanium

17 p -type semiconductor is formed when

- I. As impurity is mixed in Si
- II. Al impurity is mixed in Si
- III. B impurity is mixed in Ge
- IV. P impurity is mixed in Ge

- (a) Both I and III
- (b) Both I and IV
- (c) Both II and III
- (d) Both II and IV

18 At room temperature, a p -type semiconductor has

- (a) large number of holes and few electrons
- (b) large number of free electrons and few holes
- (c) equal number of free electrons and holes
- (d) no electrons or holes

19 Which of the following statements is not true?

- (a) The resistance of intrinsic semiconductor decreases with increase of temperature.
- (b) Doping pure Si with trivalent impurities give p -type semiconductors.
- (c) The majority carriers in n -type semiconductors are holes.
- (d) A p - n junction can act as a semiconductor diode.

20 n -type semiconductor is formed

- (a) when a germanium crystal is doped with an impurity containing three valence electrons
- (b) from pure germanium
- (c) from pure silicon
- (d) when a germanium crystal are doped with an impurity containing five valence electrons

21 An n -type semiconductor is

- (a) negatively charged
- (b) positively charged
- (c) neutral
- (d) negatively or positively charged depending upon the amount of impurity

22 Electrical conductivity of intrinsic and p -type semiconductor increases with increase in

- (a) pressure
- (b) volume
- (c) density
- (d) temperature

23 The valency of the impurity atom that is to be added to germanium crystal, so as to make it a n -type semiconductor, is

- (a) 6
- (b) 5
- (c) 4
- (d) 3

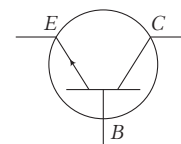
24 An n -type and a p -type silicon can be obtained by doping pure silicon with

- (a) sodium and magnesium respectively
- (b) phosphorus and boron respectively
- (c) boron and phosphorus respectively
- (d) indium and sodium respectively

25 The potential barrier for silicon diode is approximately

- (a) 0.2 V
- (b) 0.6 V
- (c) 1.1 V
- (d) 1.4 V

26 The symbol given in figure represents



- (a) n - p - n transistor
- (b) p - n - p transistor
- (c) forward biased p - n junction diode
- (d) reverse biased n - p junction diode

27 The part of a transistor which is heavily doped to produce a large number of majority carriers, is

- (a) base
- (b) emitter
- (c) collector
- (d) None of these

28 In comparison to half-wave rectifier, full wave rectifier gives lower

- (a) efficiency
- (b) average DC
- (c) average output voltage
- (d) None of these

29 Zener diode is used as

- (a) half-wave rectifier
- (b) full wave rectifier
- (c) AC voltage stabiliser
- (d) DC voltage stabiliser

30 Serious drawback of the semiconductor device is that they

- (a) cannot be used with high voltage
- (b) pollute the environment
- (c) are costly
- (d) do not last for long time

31 Least doped region in a transistor is

- (a) either emitter or collector
- (b) base
- (c) emitter
- (d) collector

32 Intrinsic semiconductor is electrically neutral.

Extrinsic semiconductor having large number of current carriers would be

- (a) positively charged
- (b) negatively charged
- (c) positively charged or negatively charged depending upon the type of impurity that has been added
- (d) electrically neutral

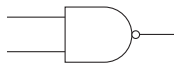
33 When a forward bias is applied to p - n junction, then it

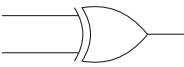
- (a) raises the potential barrier
- (b) reduces the majority carrier current to zero
- (c) lowers the potential barrier
- (d) None of the above

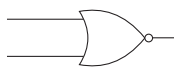
34 The conductivity of a semiconductor increases with increase in temperature, because [NCERT Exemplar]

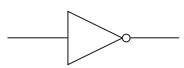
- (a) number density of free current carries increases
- (b) relaxation time increases
- (c) both number density of carries and relaxation time increase
- (d) number density of carriers increases, relaxation time decreases but effect of decrease in relaxation time is much less than increase in number density

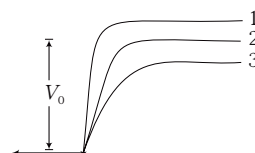
- 35** The drift current in a p - n junction is
 (a) from the n -side to the p -side
 (b) from the p -side to the n -side
 (c) from the n -side to the p -side, if the junction is forward biased and in the opposite direction, if it is reverse biased
 (d) from the p -side to the n -side, if the junction is forward biased and in the opposite direction, if it is reverse biased
- 36** The diffusion current in a p - n junction is
 (a) from the n -side to the p -side
 (b) from the p -side to the n -side
 (c) from the n -side to the p -side, if the junction is forward biased and in the opposite direction, if it is reverse biased
 (d) from the p -side to the n -side, if the junction is forward biased and in the opposite direction, if it is reverse biased
- 37** Diffusion current in a p - n junction is greater than the drift current in magnitude,
 (a) if the junction is forward biased
 (b) if the junction is reverse biased
 (c) if the junction is unbiased
 (d) None of the above
- 38** In the forward bias arrangement of a p - n junction diode, the
 (a) n -end is connected to the positive terminal of the battery
 (b) p -end is connected to the positive terminal of the battery
 (c) direction of current is from n -end to p -end in the diode
 (d) p -end is connected to the negative terminal of battery
- 39** Potential barrier developed in a junction diode opposes
 (a) minority carrier in both regions only
 (b) majority carrier only
 (c) electrons in n -region
 (d) holes in p -region
- 40** If n_e and v_d be the number of electrons and drift velocity in a semiconductor. When the temperature is increased,
 (a) n_e increases and v_d decreases
 (b) n_e decreases and v_d increases
 (c) Both n_e and v_d increases
 (d) Both n_e and v_d decreases
- 41** The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon p - n junctions are
 (a) drift in forward bias, diffusion in reverse bias
 (b) diffusion in forward bias, drift in reverse bias
 (c) diffusion in both forward and reverse bias
 (d) drift in both forward and reverse bias
- 42** In p - n junction, avalanche current flows in circuit when biasing is
 (a) forward (b) reverse (c) zero (d) excess
- 43** In an unbiased p - n junction, holes diffuse from the p -region to n -region because
 (a) free electrons in the n -region attract them
 (b) they move across the junction by the potential difference
 (c) hole concentration in p -region is more as compared to n -region
 (d) All of the above
- 44** In a transistor,
 (a) both the emitter and the collector are equally doped
 (b) base is more heavily doped than the collector
 (c) collector is more heavily doped than the emitter
 (d) the base is made very thin and is lightly doped
- 45** A transistor has three impurity regions. All the three regions have different doping levels. In order of increasing doping level, the regions are
 (a) emitter, base and collector
 (b) collector, base and emitter
 (c) base, emitter and collector
 (d) base, collector and emitter
- 46** n - p - n transistor is more useful than a p - n - p transistor because
 (a) n - p - n transistor offers less resistance to the flow of current
 (b) charge carriers in n - p - n move more easily than the charge carriers in p - n - p
 (c) p - n - p transistor is very costly compared to an n - p - n transistor
 (d) None of the above
- 47** In case of n - p - n transistor, the collector current is always less than the emitter current because
 (a) collector side is reverse biased and emitter side forward biased
 (b) few electrons are lost in the base and only the remaining ones reach the collector
 (c) collector side is forward biased and emitter side is reverse biased
 (d) collector being reverse biased attracts less electrons
- 48** In a p - n - p transistor with normal bias
 (a) only holes cross the collector junction
 (b) only majority carriers cross the collector junction
 (c) the collector junction has a low resistance
 (d) the emitter-base junction is forward biased and the collector-base junction is reverse biased
- 49** The transistors provide good power amplification when they are used in
 (a) common collector configuration
 (b) common emitter configuration
 (c) common base configuration
 (d) None of the above
- 50** When the p -end of the p - n junction is connected to the negative terminal of the battery and the n -end to the positive terminal of the battery, then the p - n junction behaves like
 (a) a conductor (b) an insulator
 (c) a super conductor (d) a semiconductor

- 51** Select the correct statement.
 (a) In a full wave rectifier, two diodes work alternately.
 (b) In a full wave rectifier, two diodes work simultaneously.
 (c) The efficiency of full wave and half wave rectifiers is same.
 (d) The full wave rectifier is bi-directional.
- 52** Temperature coefficient of resistance of semiconductor is
 (a) zero (b) constant (c) positive (d) negative
- 53** A semiconducting device is connected in a series circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, then the current drops to almost zero, the device may be
 (a) extrinsic semiconductor (b) intrinsic semiconductor
 (c) p - n junction (d) p - n - p transistor
- 54** GaAs (with a band gap = 1.5 eV) as an LED can emit
 (a) blue light (b) green light
 (c) infrared rays (d) X-rays
- 55** If no external voltage is applied across p - n junction, then there would be
 (a) no electric field across the junction
 (b) an electric field pointing from n -type to p -type side across the junction
 (c) an electric field pointing from p -type to n -type side across the junction
 (d) a temporary electric field during formation of p - n junction that would subsequently disappear
- 56** Carbon, silicon and germanium have four valence electrons each. These are characterised by valence and conduction bands separated by energy band gap respectively equal to $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$. Which of the following statement(s) is/are true?
 (a) $(E_g)_{Si} < (E_g)_{Ge} < (E_g)_C$
 (b) $(E_g)_C < (E_g)_{Ge} < (E_g)_{Si}$
 (c) $(E_g)_C < (E_g)_{Si} < (E_g)_{Ge}$
 (d) $(E_g)_{Ge} < (E_g)_{Si} < (E_g)_C$
- 57** When a diode is heavily doped, then
 (a) the Zener voltage will be low
 (b) the avalanche voltage will be high
 (c) the depletion region will be thin
 (d) the leakage current will be low
- 58** For a transistor amplifier, the voltage gain
 (a) remains constant for all frequencies
 (b) is high at high and low frequencies and constant in the middle frequency range
 (c) is low at high and low frequencies and constant at mid frequencies
 (d) None of the above
- 59** The electrical circuit used to get smooth DC output from a rectifier circuit is called
 (a) oscillator (b) filter
 (c) amplifier (d) logic gates
- 60** To use a transistor as an amplifier,
 (a) Both junctions are forward biased
 (b) Both junctions are reverse biased
 (c) the emitter-base junction is forward biased and the collector-base junction is reverse biased
 (d) no biasing voltages are required
- 61** In the case of constants α and β of a transistor
 (a) $\alpha = \beta$ (b) $\beta < 1$, $\alpha > 1$ (c) $\alpha\beta = 1$ (d) $\beta > 1$, $\alpha < 1$
- 62** The base of transistor is made very thin compared to emitter and collector. By doing this, which one of the following is achieved?
 (a) High emitter current (b) Low power gain
 (c) Low emitter current (d) High power gain
- 63** In an n - p - n transistor, the collector current is 10 mA. If 90% of the electrons emitted reach the collector, then the emitter current will be about
 (a) 9 mA (b) 11 mA
 (c) 1 mA (d) 0.1 mA
- 64** In a transistor, α is related to β by the relation
 (a) $\beta = \frac{\alpha + 1}{\alpha}$ (b) $\beta = \frac{\alpha - 1}{\alpha}$
 (c) $\beta = \frac{\alpha}{1 - \alpha}$ (d) $\beta = \frac{\alpha}{\alpha + 1}$
- 65** The maximum efficiency of full wave rectifier is
 (a) 100% (b) 25.20%
 (c) 40.2% (d) 81.2%
- 66** Given below are symbols for some logic gates.
- 
I.


II.

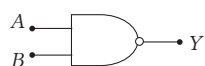

III.


IV.
- The XOR gate and NOR gates respectively are
 (a) I and II (b) II and III
 (c) III and IV (d) I and IV
- 67** In the Boolean algebra, which of the following is not equal to A ?
 (a) $A \cdot A$ (b) $\overline{A + A}$
 (c) $\overline{A} \cdot A$ (d) $\overline{A + \overline{A}}$
- 68** In given figure, V_0 is the potential barrier across a p - n junction, when no battery is connected across the junction [NCERT Exemplar]

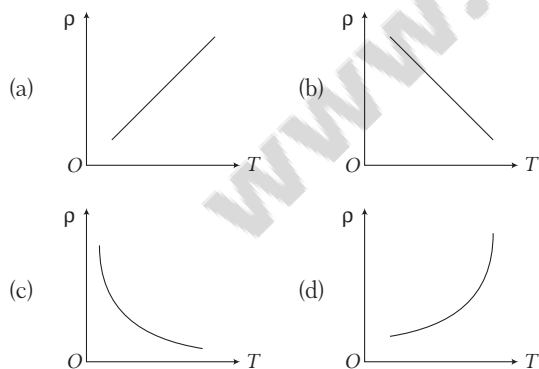


- (a) 1 and 3 both correspond to forward bias of junction
 (b) 3 corresponds to forward bias of junction and 1 corresponds to reverse bias of junction
 (c) 1 corresponds to forward bias and 3 corresponds to reverse bias of junction
 (d) 3 and 1 both correspond to reverse bias of junction
- 69** If a full wave rectifier, circuit is operating in 50 Hz mains, then the fundamental frequency in the ripple will be
 (a) 50 Hz (b) 70.7 Hz (c) 100 Hz (d) 25 Hz
- 70** With an AC input from 50 Hz power line, the ripple frequency is
 (a) 50 Hz in the DC output of half-wave as well as full wave rectifier
 (b) 100 Hz in the DC output of half-wave as well as full wave rectifier
 (c) 50 Hz in the DC output of half-wave and 100 Hz in DC output of full wave rectifier
 (d) 100 Hz in the DC output of half-wave and 50 Hz in the DC output of full wave rectifier

71 The given symbol represents



- (a) NAND gate (b) OR gate
 (c) AND gate (d) NOR gate
- 72** The boolean expression of NOR gate is
 (a) $Y = A + B$ (b) $Y = \overline{A + B}$
 (c) $Y = A \cdot B$ (d) $Y = \overline{A \cdot B}$
- 73** The temperature (T) dependence on resistivity (ρ) of a semiconductor is represented by



- 74** Under which of the following conditions, does an avalanche breakdown in a semiconductor diode occur?
 (a) When potential barrier is reduced to zero
 (b) When reverse bias exceeds a certain value
 (c) When forward bias exceeds a certain value
 (d) When forward current exceeds a certain value
- 75** The width of depletion region in a $p-n$ junction diode is 500 nm and an intense electric field of

$6 \times 10^5 \text{ Vm}^{-1}$ is also found to exist. The height of potential barrier is

- (a) 0.30 V (b) 0.40 V (c) 3 V (d) 4 V

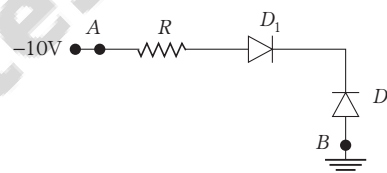
76 Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are $0.36 \text{ m}^2/\text{Vs}$ and $0.17 \text{ m}^2/\text{Vs}$. The electron and hole densities are each equal to $2.5 \times 10^{19} \text{ m}^{-3}$. The electrical conductivity of germanium is

- (a) 0.47 Sm^{-1} (b) 5.18 Sm^{-1}
 (c) 2.12 Sm^{-1} (d) 1.09 Sm^{-1}

77 In a semiconductor, the concentration of electrons is $8 \times 10^{14} / \text{cm}^3$ and that of the holes is $5 \times 10^{12} / \text{cm}^3$. The semiconductor is

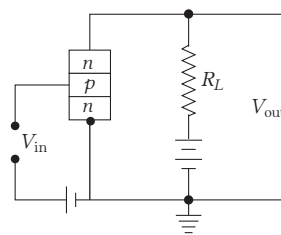
- (a) p -type (b) n -type
 (c) Intrinsic (d) $p-n-p$ -type

78 In figure given below, assuming the diodes to be ideal [NCERT Exemplar]



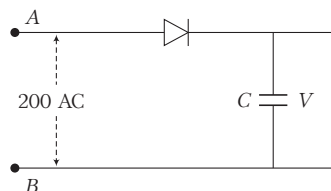
- (a) D_1 is forward biased and D_2 is reverse biased and hence current flows from A to B
 (b) D_2 is forward biased and D_1 is reverse biased and hence no current flows from B to A and *vice-versa*
 (c) D_1 and D_2 are both forward biased and hence current flows from A to B
 (d) D_1 and D_2 are both reverse biased and hence no current flows from A to B and *vice-versa*

79 An $n-p-n$ transistor circuit is arranged as shown in figure. It is

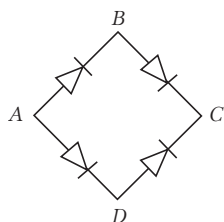


- (a) a common base amplifier circuit
 (b) a common emitter amplifier circuit
 (c) a common collector amplifier circuit
 (d) None of the above
- 80** A Si specimen is made into p -type semiconductor by doping on an average one indium atom per 6×10^7 silicon atoms. If the number density of atoms in Si be $6 \times 10^{28} \text{ m}^{-3}$, then what is the indium atom per cm^3 ?
 (a) 10^{12} (b) 10^{15} (c) 10^{18} (d) 10^{20}

- 81** A 220 V AC supply is connected between points A and B (figure). What will be the potential difference V across the capacitor? [NCERT Exemplar]



- (a) 220V (b) 110 V (c) 0 V (d) $220\sqrt{2}$ V
- 82** In the diagram, the input is across the terminals A and C and the output is across B and D. Then, the output is



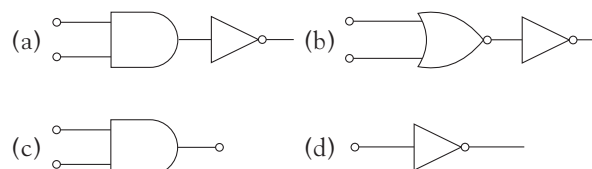
- (a) zero (b) same as the input
(c) full wave rectifier (d) half wave rectifier
- 83** When forward bias is applied to a p - n junction, then what happens to the potential barrier V_B and the width of charge depleted region x ?
- (a) V_B increases, x decreases (b) V_B decreases, x increases
(c) V_B increases, x increases (d) V_B decreases, x decreases

- 84** The following truth table corresponds to the logic gate

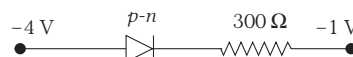
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

- (a) NAND (b) OR
(c) AND (d) XOR
- 85** The transfer ratio β of a transistor is 50. The input resistance of the transistor when used in the common emitter configuration is $1 \text{ k}\Omega$. The peak value of the collector AC current for a peak value of AC input voltage of 0.01 V is
- (a) $100 \mu\text{A}$ (b) $0.01 \mu\text{A}$
(c) $0.25 \mu\text{A}$ (d) $500 \mu\text{A}$
- 86** The voltage gain of an amplifier with 9% negative feedback is 10. The voltage gain without feedback will be
- (a) 90 (b) 10
(c) 1.25 (d) 100

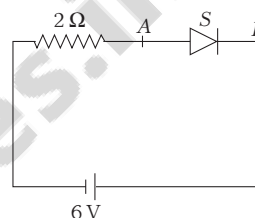
- 87** Which represents NAND gate?



- 88** What is the current in the circuit shown below?



- (a) Zero (b) 10^{-2} A
(c) 1 A (d) 0.10 A
- 89** The diode shown in the circuit is a silicon diode. The potential difference between the points A and B will be



- (a) 6 V (b) 0.6 V
(c) 0.7 V (d) zero
- 90** The value of DC voltage in half-wave rectifier in converting AC voltage $V = 100 \sin(314t)$ into DC is
- (a) 100 V (b) 50 V
(c) 30.3 V (d) 0 V

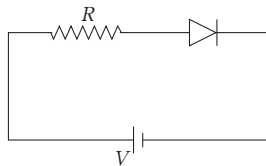
- 91** Below we give four entries for the truth table of two point input OR gate. Which two are wrong?



	A	B	Y
i	1	0	1
ii	1	1	0
iii	0	0	1
iv	0	1	1

- (a) i, ii (b) ii, iii (c) iii, iv (d) iv, i
- 92** Input signal to a common emitter amplifier having a voltage gain of 1000 is given by $V_i = (0.004 \text{ V}) \sin(\omega t + \pi/2)$. The corresponding output signal is
- (a) $(40 \text{ V}) \sin(\omega t + \pi/2)$ (b) $(0.004 \text{ V}) \cos(\omega t + \pi/2)$
(c) $(4 \text{ V}) \cos(\omega t - \pi/2)$ (d) $(4 \text{ V}) \sin(\omega t - \pi/2)$
- 93** In a common base transistor circuit, the current gain is 0.98. On changing the emitter current by 5 mA, the change in collector current is
- (a) 0.196 mA (b) 2.45 mA
(c) 4.9 mA (d) 5.1 mA

- 94** The forbidden energy gap in Ge is 0.72 eV. Given, $hc = 12400 \text{ eV-Å}$. The maximum wavelength of radiation that will generate electron-hole pair is
(a) 172220 Å (b) 172.2 Å (c) 17222 Å (d) 1722 Å
- 95** When the emitter current of a transistor is changed by 1 mA, then its collector current changes by 0.990 mA. In the common base circuit, current gain for the transistor is
(a) 0.099 (b) 1.01 (c) 1.001 (d) 0.990
- 96** What will be the input of A and B for the boolean expression $\overline{(A+B)} \cdot \overline{(A \cdot B)} = 1$?
(a) 0, 0 (b) 0, 1 (c) 1, 0 (d) 1, 1
- 97** In a transistor, the current amplification factor α is 0.9. The transistor is connected in common base configuration. The change in collector current when emitter current changes by 4 mA is
(a) 4 mA (b) 12 mA (c) 24 mA (d) 3.6 mA
- 98** For the given circuit of p - n junction diode, which of the following statement is correct?

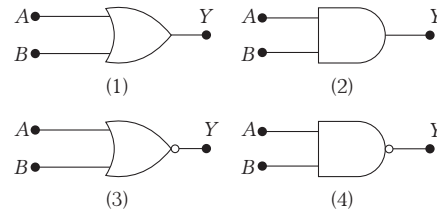


- (a) In forward biasing, the voltage across R is V .
(b) In forward biasing, the voltage across R is $2V$.
(c) In reverse biasing, the voltage across R is V .
(d) In reverse biasing, the voltage across R is $2V$.
- 99** A p -type semiconductor has acceptor level 57 meV above the valence band. The maximum wavelength of light required to create hole is
(a) 57 Å (b) 57×10^{-3} Å
(c) 217100 Å (d) 11.61×10^{-33} Å
- 100** The given truth table is of

A	X
0	1
1	0

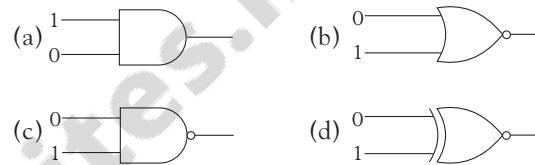
- (a) OR gate (b) AND gate
(c) NOT gate (d) None of these
- 101** The below truth table corresponds to
- | A | B | Y |
|-----|-----|-----|
| 0 | 0 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |
- (a) NAND gate (b) XOR gate
(c) OR gate (d) NOR gate

- 102** Given below are four logic gate symbols (figure). Those for OR, NOR and NAND are respectively

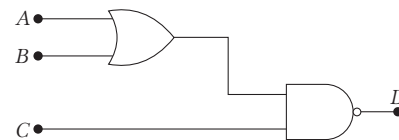


- (a) 1, 4, 3 (b) 4, 1, 2 (c) 1, 3, 4 (d) 4, 2, 1
- 103** The decimal equivalent of the binary number $(11010.101)_2$ is
(a) 9.625 (b) 25.265 (c) 26.625 (d) 26.265

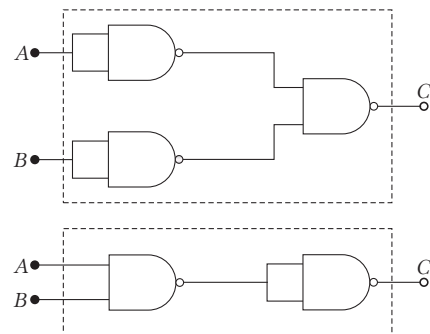
- 104** Which of the following gates will have an output of 1?



- 105** When the inputs of two input logic gates are 0 and 0, then the output is 1. When the inputs are 1 and 0, then the output is zero. The logic gate is of the type
(a) XOR (b) NAND (c) NOR (d) OR
- 106** For the given combination of gates, if the logic states of inputs A , B and C are as follows $A = B = C = 0$ and $A = B = 1$, $C = 0$, then the logic states of output D are

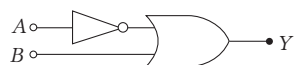


- (a) 0, 0 (b) 0, 1 (c) 1, 0 (d) 1, 1
- 107** The combination of NAND gates shown here are equivalent to an



- (a) OR gate and an AND gate, respectively
(b) AND gate and a NOT gate, respectively
(c) AND gate and an OR gate, respectively
(d) OR gate and a NOT gate, respectively

- 108** The boolean expression for the circuit given in the figure is



- (a) $Y = A + \bar{B}$ (b) $Y = \overline{A + B}$
(c) $Y = \bar{A} + B$ (d) $Y = A + B$

- 109** In the circuit given below, the value of the current is

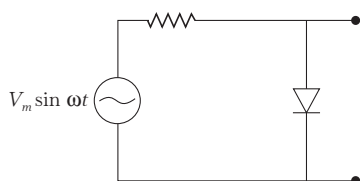


- (a) zero (b) 10^{-2} A (c) 10^2 A (d) 10^{-3} A

- 110** A potential barrier of 0.50 V exists across a p - n junction. If the depletion region is 5.0×10^{-7} m wide, then the intensity of the electric field in this region is

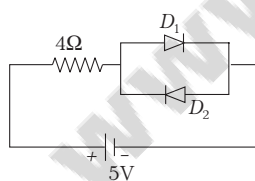
- (a) 1×10^6 Vm $^{-1}$ (b) 1×10^5 Vm $^{-1}$
(c) 2×10^5 Vm $^{-1}$ (d) 2×10^6 Vm $^{-1}$

- 111** The output of the given circuit in figure given below, is [NCERT Exemplar]



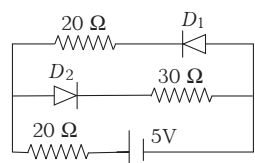
- (a) would be zero at all times
(b) would be like a half wave rectifier with positive cycles in output
(c) would be like a half wave rectifier with negative cycles in output
(d) would be like that of a full wave rectifier

- 112** In the network shown,



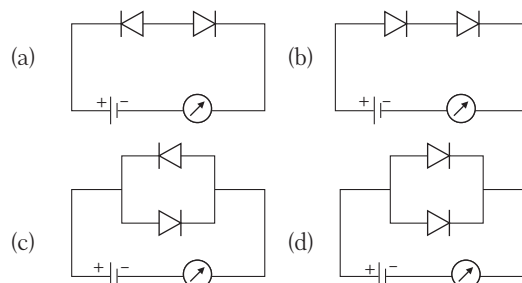
- (a) the potential difference across D_2 is 5 V
(b) current through resistor equals 2.25 A
(c) current through diode D_1 is 1.25 A
(d) current through diode D_2 is 1.25 A

- 113** The current in the circuit will be

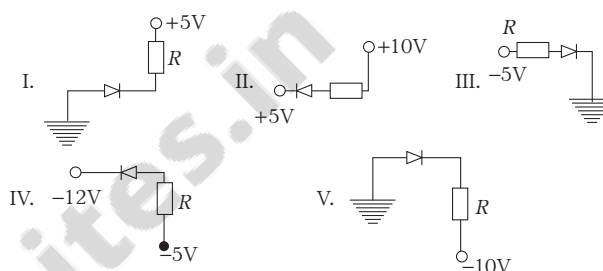


- (a) $(5/40)$ A (b) $(5/50)$ A (c) $(5/10)$ A (d) $(5/20)$ A

- 114** Which circuit will not show current in ammeter?

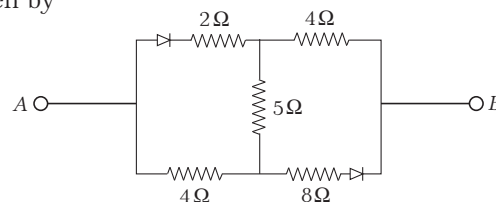


- 115** In the given figure, which of the diodes are forward biased?



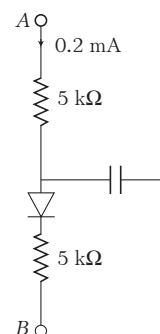
- (a) I, II, III (b) II, IV, V
(c) I, III, IV (d) II, III, IV

- 116** The equivalent resistance of the circuit across AB is given by



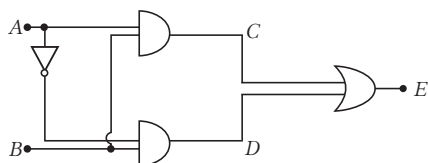
- (a) 4 Ω (b) 13 Ω
(c) 4 Ω or 13 Ω (d) 4 Ω or zero

- 117** In the circuit shown in figure given below, if the diode forward voltage drop is 0.3 V, the voltage difference between A and B is [NCERT Exemplar]



- (a) 1.3 V (b) 2.3 V
(c) 0 (d) 0.5 V

118 Truth table for the given circuit is [NCERT Exemplar]



(a)

A	B	E
0	0	1
0	1	0
1	0	1
1	1	0

(b)

A	B	E
0	0	1
0	1	0
1	0	0
1	1	0

(c)

A	B	E
0	0	0
0	1	1
1	0	0
1	1	1

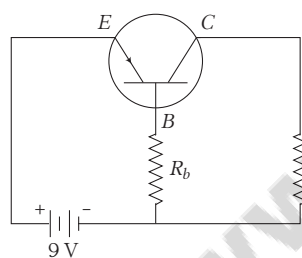
(d)

A	B	E
0	0	0
0	1	1
1	0	1
1	1	0

119 The input resistance of a common emitter amplifier is $2\text{ k}\Omega$ and AC current gain is 20. If the load resistor used is $5\text{ k}\Omega$, then calculate the transconductance of the transistor used.

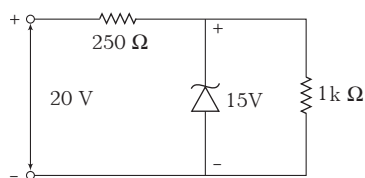
- (a) $0.01\ \Omega^{-1}$ (b) $0.03\ \Omega^{-1}$
 (c) $0.04\ \Omega^{-1}$ (d) $0.07\ \Omega^{-1}$

120 In a transistor circuit shown, the base current is $35\ \mu\text{A}$. The value of the resistor R_b is



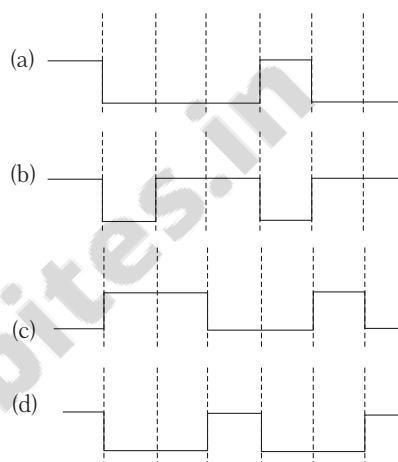
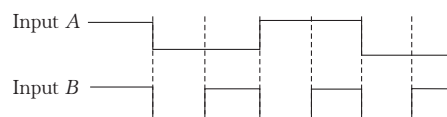
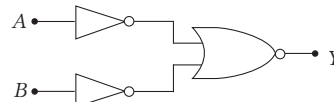
- (a) $123.5\ \text{k}\Omega$
 (b) $257\ \text{k}\Omega$
 (c) $380.05\ \text{k}\Omega$
 (d) None of the above

121 A Zener diode, having breakdown voltage equal to 15 V , is used in a voltage regulator circuit shown in figure. The current through the diode is

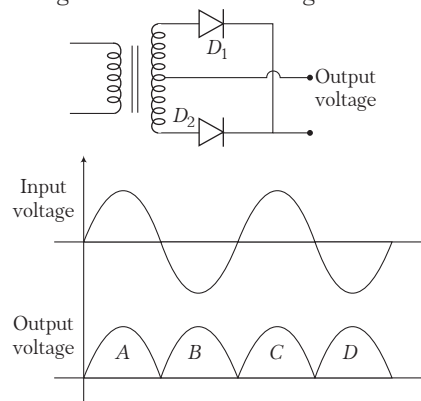


- (a) $20\ \text{mA}$ (b) $5\ \text{mA}$
 (c) $10\ \text{mA}$ (d) $15\ \text{mA}$

122 The logic circuit shown below has the input waveforms A and B as shown. Pick out the correct output waveform.



123 A full wave rectifier circuit along with the input and output voltages is shown in the figure



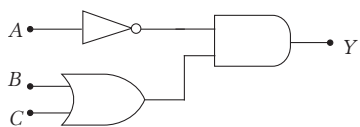
The contribution to output voltage from D_2 is

- (a) A, C (b) B, D
 (c) B, C (d) A, D

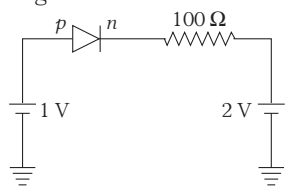
124 A researcher wants an alarm to sound when the temperature of air in his controlled research chamber rises above 40°C or falls below 20°C . The alarm can be triggered by the output of a

- (a) NOT gate (b) AND gate
 (c) NAND gate (d) OR gate

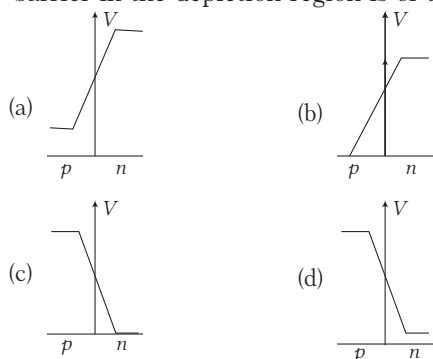
- 125 The boolean expression for the circuit given in figure is



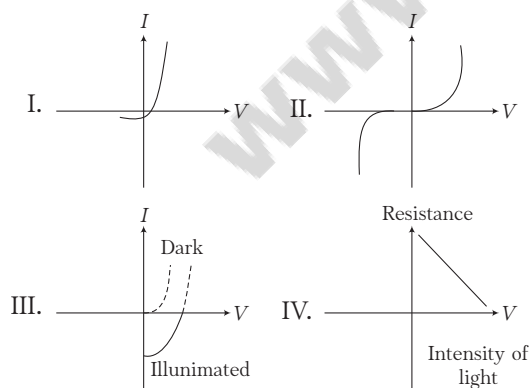
- (a) $Y = \bar{A} \cdot B + C$ (b) $Y = \bar{A} \cdot (\bar{B} + \bar{C})$
 (c) $Y = \bar{A} \cdot (B + \bar{C})$ (d) $Y = \bar{A} \cdot (B + C)$
- 126 The current through an ideal p - n junction shown in the circuit diagram will be



- (a) zero (b) 1 mA (c) 10 mA (d) 30 mA
- 127 In a forwards biased p - n -junction diode, the potential barrier in the depletion region is of the form

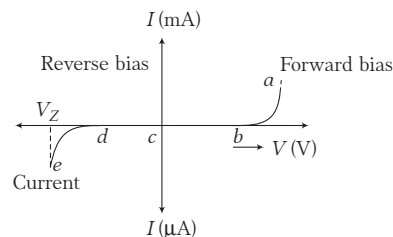


- 128 Identify the semiconductor devices whose characteristics are given below, in the order I, II, III, IV.

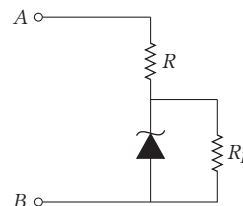


- (a) Zener diode, Simple diode, Light dependent resistance, Solar cell
 (b) Solar cell, Light dependent resistance, Zener diode, Simple diode
 (c) Zener diode, Solar cell, Simple diode, Light dependent resistance
 (d) Simple diode, Zener diode, Solar cell, Light dependent resistance

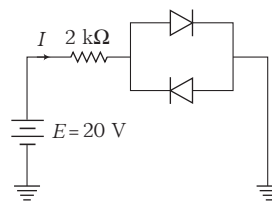
- 129 The graph given below represents the I - V characteristics of a Zener diode. Which part of the characteristics curve is most relevant for its operation as a voltage regulator?



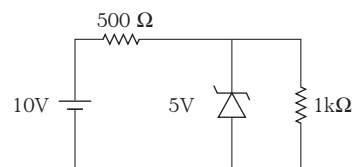
- (a) ab (b) bc (c) cd (d) de
- 130 A diode having potential difference 0.5 V across its junction which does not depend on current, is connected in series with resistance of $20\ \Omega$ across source. If 0.1 A passes through resistance, then what is the voltage of the source?
- (a) 1.5 V (b) 2.0 V (c) 2.5 V (d) 5 V
- 131 If the voltage between the terminals A and B is 17 V and Zener breakdown voltage is 9 V, then the potential across R is



- (a) 6 V (b) 8 V (c) 9 V (d) 17 V
- 132 Assuming the diodes to be of silicon with forward resistance zero, the current I in the following circuit is

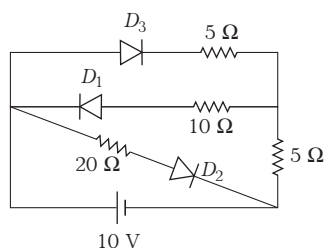


- (a) zero (b) 9.65 mA (c) 10 mA (d) 10.36 mA
- 133 In the following circuit, the current flowing through $1\text{ k}\Omega$ resistor is



- (a) zero (b) 5 mA
 (c) 10 mA (d) 15 mA

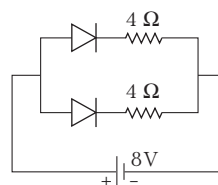
134 In the given circuit



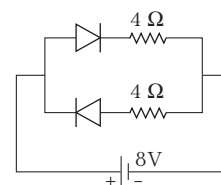
The current through the battery is

- (a) 0.5 A (b) 1 A (c) 1.5 A (d) 2 A

135 Currents flowing in each of the circuits A and B respectively are



(Circuit A)



(Circuit B)

- (a) 1 A, 2 A (b) 2 A, 1 A
(c) 4 A, 2 A (d) 2 A, 4 A

(B) Medical entrance special format questions

Assertion and reason

Directions (Q. Nos. 1-5) These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose any one of the following four responses.

- (a) If both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.

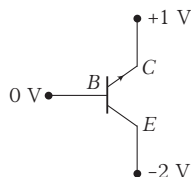
1 Assertion If the temperature of a semiconductor is increased, then its resistance decreases.

Reason The energy gap between conduction band and valence band is very small.

2 Assertion The number of electrons in a *p*-type silicon semiconductor is less than the number of electrons in a pure silicon semiconductor at room temperature.

Reason It is due to law of mass action.

3 Assertion When *p-n* junction is forward biased, then motion of charge carriers at junction is due to diffusion. In reverse biasing, the cause of motion of charge is drifting.



Reason In the following circuit, emitter is reverse biased and collector is forward biased.

4 Assertion NAND or NOR gates are called digital building blocks.

Reason The repeated use of NAND (or NOR) gates can produce all the basic or complicated gates.

5 Assertion The logic gate NOT cannot be built using diode.

Reason The output voltage and the input voltage of the diode have 180° phase difference.

Statement based questions

1 For transistor action, which of the following statements are correct?

- (a) Base, emitter and collector regions should have similar size and doping concentrations.
(b) The base region must be very thick and moderately doped.
(c) The emitter junction is forward biased and collector junction is reverse biased.
(d) Both the emitter junction as well as the collector junction are forward biased.

2 In an *n*-type silicon, which of the following statement is true?

- (a) Electrons are majority carriers and trivalent atoms are the dopants.
(b) Electrons are minority carriers and pentavalent atoms are the dopants.
(c) Holes are minority carriers and pentavalent atoms are the dopants.
(d) Holes are majority carriers and trivalent atoms are the dopants.

3 I. The resistivity of a semiconductor increases with temperature.

II. The atoms of a semiconductor vibrate with larger amplitudes at higher temperatures thereby increasing its resistivity.

Choose the correct statement from the given options.

- (a) Only I (b) Only II
(c) Both I and II (d) None of these

4 I. A rectifier converts an alternating current into a direct current.

II. A *p-n* junction diode can work as a rectifier.

- III. Zener diode works on a principle of breakdown voltage.
 IV. Current increases suddenly after breakdown voltage.

Which one of the following is correct?

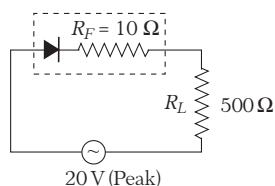
- (a) I, II and IV (b) Both I and IV
 (c) I, III and IV (d) All of these

5 Read the following statements about transistor and choose the correct statements(s).

- I. A transistor amplifier in common emitter configuration has a low input impedance.
 II. The base to emitter region is forward biased.
 (a) Only I (b) Only II
 (c) Both I and II (d) None of these

Match the columns

- 1 In the circuit shown, the barrier voltage of diode is 0.7 V. Match the physical quantities given in Column I to the results given in Column II and mark the correct option from the codes given below.



Column I	Column II
(A) Peak current (in mA) in diode	(p) 37.8
(B) Peak voltage (in volts) at the ends of load	(q) 40.0
(C) Peak current (in mA), if diode is ideal	(r) 20.0
(D) Peak voltage (in volts) at the ends of load, if diode is ideal	(s) 18.9
	(t) Zero

Codes

- A B C D
 (a) p q s r
 (b) q r p r
 (c) p r s q
 (d) p s q r

- 2 Match the symbol of diode given in Column I to the result given in Column II and mark the correct option from the codes given below.

Column I	Column II
(A)	(p) Light emitting diode

Column I	Column II
(B)	(q) Zener diode
(C)	(r) Photodiode
(D)	(s) Solar cells

Codes

- A B C D A B C D
 (a) p r s q (b) q r p s
 (c) q p r s (d) q r s p

- 3 Match the boolean expression of gate given in Column I to the gate given in Column II and mark the correct option from the codes given below.

Column I	Column II
(A) $Y = A + B$	(p) NAND gate
(B) $Y = A \cdot B$	(q) NOT gate
(C) $Y = \bar{A}$	(r) AND gate
(D) $Y = \overline{A \cdot B}$	(s) OR gate

Codes

- A B C D A B C D
 (a) s r p q (b) s r q p
 (c) s q p r (d) q p r s

- 4 Match the symbol of gate given in Column I to the gate given in Column II and mark the correct option from the codes given below.

Column I	Column II
(A)	(p) NOT
(B)	(q) OR
(C)	(r) NAND
(D)	(s) NOR

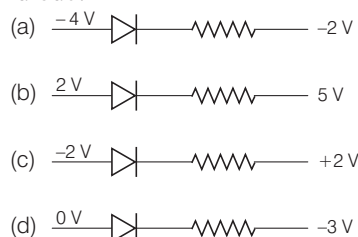
Codes

- A B C D A B C D
 (a) r p q s (b) r q p s
 (c) s p r q (d) s p q r

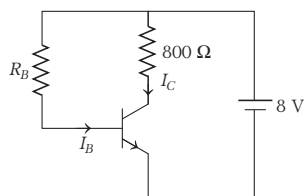
(C) Medical entrances' gallery

Collection of questions asked in NEET and various medical entrance exams

- 1** Out of the following, which one is a forward biased diode? [NEET 2020]



- 2** A n - p - n transistor is connected in common emitter configuration (see figure) in which collector voltage drop across load resistance $800\ \Omega$ connected to the collector circuit is 0.8 V . The collector current is [NEET 2020]



- (a) 2 mA (b) 0.1 mA (c) 1 mA (d) 0.2 mA
- 3** Which of the following gate is called universal gate? [NEET 2020]
- (a) OR gate (b) AND gate (c) NAND gate (d) NOT gate
- 4** An intrinsic semiconductor is converted into n -type extrinsic semiconductor by doping it with [NEET 2020]
- (a) phosphorous (b) aluminium
(c) silver (d) germanium

- 5** The increase in the width of the depletion region in a p - n junction diode is due to [NEET 2020]
- (a) reverse bias only
(b) both forward bias and reverse bias
(c) increase in forward current
(d) forward bias only

- 6** The solids which have the negative temperature coefficient of resistance are [NEET 2020]
- (a) insulator only
(b) semiconductors only
(c) insulators and semiconductors
(d) metals

- 7** For transistor action, which of the following statements is correct? [NEET 2020]
- (a) Base, emitter and collector regions should have same size.
(b) Both emitter junction as well as the collector junction are forward biased.
(c) The base region must be very thin and lightly doped.
(d) Base, emitter and collector regions should have same doping concentrations.

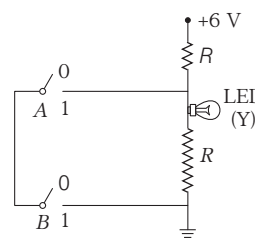
- 8** For a p -type semiconductor, which of the following statements is true? [NEET 2019]

- (a) Holes are the majority carriers and trivalent atoms are the dopants.
(b) Holes are the majority carriers and pentavalent atoms are the dopants.
(c) Electrons are the majority carriers and pentavalent atoms are the dopants.
(d) Electrons are the majority carriers and trivalent atoms are the dopants.

- 9** An LED is constructed from a p - n junction diode using GaAsP and the energy gap is 1.9 eV . The wavelength of the light emitted will be equal to [NEET (Odisha) 2019]

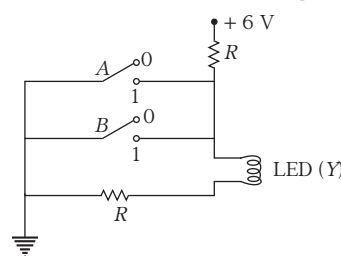
- (a) $10.4 \times 10^{-26}\text{ m}$ (b) 654 nm
(c) $654\text{ \AA}$ (d) $654 \times 10^{-11}\text{ m}$

- 10** The correct boolean operation represented by the circuit diagram drawn is [NEET 2019]



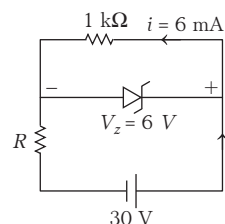
- (a) OR (b) NAND (c) NOR (d) AND

- 11** The circuit diagram shown here corresponds to the logic gate [NEET (Odisha) 2019]



- (a) NOR (b) AND (c) OR (d) NAND

- 12** If voltage across a zener diode is 6 V , then find out the value of maximum resistance in this condition. [AIIMS 2019]



- (a) $2\text{ k}\Omega$ (b) $2\text{ k}\Omega$ (c) $5\text{ k}\Omega$ (d) $4\text{ k}\Omega$

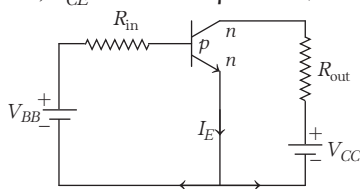
- 13 Assertion** Photodiode and solar cell works on same mechanism.

Reason Area is large for solar cell. [AIIMS 2019]

- (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
 (b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
 (c) Assertion is correct but Reason is incorrect.
 (d) Both Assertion and Reason are incorrect.

- 14** The given transistor operates in saturation region, then what should be the value of V_{BB} ?

(Take, $R_{out} = 200 \Omega$, $R_{in} = 100 \text{ k}\Omega$, $V_{CC} = 3 \text{ V}$, $V_{BE} = 0.7 \text{ V}$, $V_{CE} = 0 \text{ V}$ and $\beta = 200$) [AIIMS 2019]

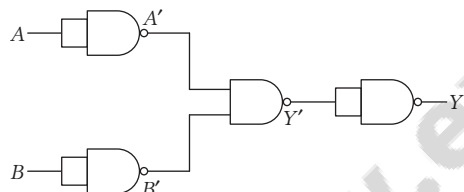


- (a) 4.1 V (b) 7.5 V (c) 8.2 V (d) 6.8 V

- 15** For CE configuration n - p - n transistor, which of the following statement is correct? [JIPMER 2019]

- (a) $I_C = I_E + I_B$ (b) $I_B = I_E + I_C$
 (c) $I_E = I_C + I_B$ (d) All of these

- 16** Following circuit will act as [JIPMER 2019]

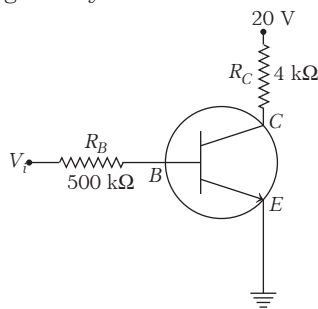


- (a) NOR gate (b) NAND gate (c) AND gate (d) OR gate

- 17** In a p - n junction diode, change in temperature due to heating [NEET 2018]

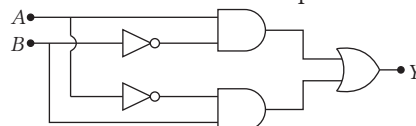
- (a) does not affect resistance of p - n junction
 (b) affects only forward resistance
 (c) affects only reverse resistance
 (d) affects the overall V - I characteristics of p - n junction

- 18** In the circuit shown in the figure, the input voltage V_i is 20 V, $V_{BE} = 0$ and $V_{CE} = 0$. The values of I_B , I_C and β are given by [NEET 2018]



- (a) $I_B = 20 \mu\text{A}$, $I_C = 5 \text{ mA}$ and $\beta = 250$
 (b) $I_B = 25 \mu\text{A}$, $I_C = 5 \text{ mA}$ and $\beta = 200$
 (c) $I_B = 40 \mu\text{A}$, $I_C = 10 \text{ mA}$ and $\beta = 250$
 (d) $I_B = 40 \mu\text{A}$, $I_C = 5 \text{ mA}$ and $\beta = 125$

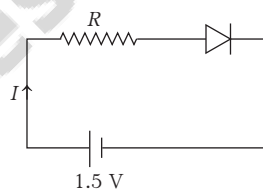
- 19** In the combination of the following gates, the output Y can be written in terms of inputs A and B as



[NEET 2018]

- (a) $\overline{A \cdot B} + A \cdot B$ (b) $A \cdot \overline{B} + \overline{A} \cdot B$
 (c) $\overline{A \cdot B}$ (d) $\overline{A + B}$

- 20** The diode used at a constant potential drop of 0.5 V at all currents and maximum power rating of 100 mW. What resistance must be connected in series to diode, so that current in circuit is maximum? [AIIMS 2018]



- (a) 200 Ω (b) 6.67 Ω (c) 5 Ω (d) 15 Ω

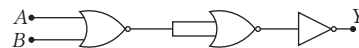
- 21** Which one of the following represents forward bias diode? [NEET 2017]

- (a) $\frac{0\text{V}}{-} \rightarrow \text{diode} \leftarrow \frac{-2\text{V}}{R}$ (c) $\frac{-2\text{V}}{-} \rightarrow \text{diode} \leftarrow \frac{+2\text{V}}{R}$
 (b) $\frac{-4\text{V}}{-} \rightarrow \text{diode} \leftarrow \frac{-3\text{V}}{R}$ (d) $\frac{3\text{V}}{-} \rightarrow \text{diode} \leftarrow \frac{5\text{V}}{R}$

- 22** In a common emitter transistor amplifier, the audio signal voltage across the collector is 3 V. The resistance of collector is 3 k Ω . If current gain is 100 and the base resistance is 2 k Ω , the voltage and power gain of the amplifier is [NEET 2017]

- (a) 200 and 1000 (b) 15 and 200
 (c) 150 and 15000 (d) 20 and 2000

- 23** The given electrical network is equivalent to [NEET 2017]



- (a) AND gate (b) OR gate (c) NOR gate (d) NOT gate

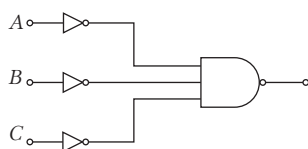
- 24** A specimen of silicon is to be made p -type semiconductor, for this one atom of indium, on an average, is doped in 5×10^7 silicon atoms. If the number density of silicon is $5 \times 10^{28} \text{ atom/m}^3$, then the number of acceptor atoms per cm^3 will be [AIIMS 2017]

- (a) 2.5×10^{30} (b) 1.0×10^{13}
 (c) 1.0×10^{15} (d) 2.5×10^{36}

- 25** The current gain of a transistor in common emitter mode is 49. The change in collector current and emitter current corresponding to change in base current by $5.0 \mu\text{A}$, will be [AIIMS 2017]

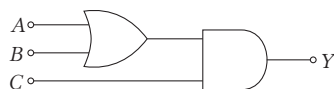
(a) $245 \mu\text{A}$ and $250 \mu\text{A}$
 (b) $240 \mu\text{A}$ and $235 \mu\text{A}$
 (c) $260 \mu\text{A}$ and $255 \mu\text{A}$
 (d) None of the above

- 26** A proper combination of 3 NOT and 1 NAND gates is shown in figure. If $A = 0$, $B = 1$, $C = 1$, then the output of this combination is [AIIMS 2017]



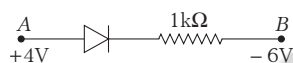
(a) 1
 (b) 0
 (c) Not predictable
 (d) None of these

- 27** To get output 1 for the following circuit, the correct choice for the input is [NEET 2016]



(a) $A = 0, B = 0, C = 0$
 (b) $A = 1, B = 1, C = 0$
 (c) $A = 1, B = 0, C = 1$
 (d) $A = 0, B = 1, C = 0$

- 28** Consider the junction diode as ideal. The value of current flowing through AB is [NEET 2016]



(a) 10^{-2} A
 (b) 10^{-1} A
 (c) 10^{-3} A
 (d) 0 A

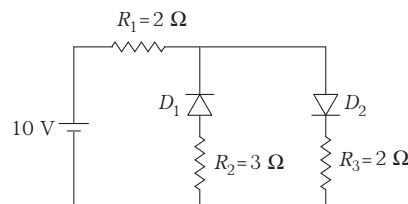
- 29** A $n-p-n$ transistor is connected in common emitter configuration in a given amplifier. A load resistance of 800Ω is connected in the collector circuit and the voltage drop across it is 0.8V . If the current amplification factor is 0.96 and the input resistance of the circuits is 192Ω , then the voltage gain and the power gain of the amplifier will respectively be [NEET 2016]

(a) 3.69, 3.84
 (b) 4, 4
 (c) 4, 3.69
 (d) 4, 3.84

- 30** For CE transistor amplifier, the audio signal voltage across the collector resistance of $2 \text{ k}\Omega$ is 4V . If the current amplification factor of the transistor is 100 and the base resistance is $1 \text{ k}\Omega$, then the input signal voltage is [NEET 2016]

(a) 10 mV
 (b) 20 mV
 (c) 30 mV
 (d) 15 mV

- 31** The given circuit has two ideal diodes connected as shown in the figure below. The current flowing through the resistance R_1 will be [NEET 2016]



(a) 2.5 A
 (b) 10.0 A
 (c) 1.43 A
 (d) 3.13 A

- 32** What is the output Y in the following circuit, when all the three inputs A, B, C are first 0 and then 1?



(a) 0, 1
 (b) 0, 0
 (c) 1, 0
 (d) 1, 1 [NEET 2016]

- 33** The boolean expression $P + \bar{P}Q$, where P and Q are the inputs of the logic circuit, represents [AIIMS 2015]

(a) AND gate
 (b) NAND gate
 (c) NOT gate
 (d) OR gate

- 34** A semiconductor is having electron and hole mobilities μ_n and μ_p , respectively.

If its intrinsic carrier density is n_i , then what will be the value of hole concentration P for which the conductivity will be minimum at a given temperature? [AIIMS 2015]

(a) $n_i \sqrt{\frac{\mu_n}{\mu_p}}$
 (b) $n_h \sqrt{\frac{\mu_n}{\mu_p}}$
 (c) $n_i \sqrt{\frac{\mu_p}{\mu_n}}$
 (d) $n_h \sqrt{\frac{\mu_p}{\mu_n}}$

- 35** When an input signal 1 is applied to a NOT gate, then its output is [UK PMT 2015]

(a) 1
 (b) 0
 (c) either 0 or 1
 (d) any positive value

- 36** The impurity atoms with which pure silicon may be doped to make it a p -type semiconductor are those of [UK PMT 2015]

(a) phosphorous
 (b) antimony
 (c) boron
 (d) iron

- 37** Let n_p and n_e be the number of holes and conduction electrons respectively, in an intrinsic semiconductor, then [UK PMT 2015]

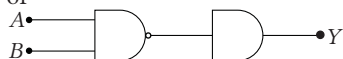
(a) $n_p > n_e$
 (b) $n_p = n_e$
 (c) $n_p < n_e$
 (d) None of these

- 38** Zener diode is used for [UK PMT 2015]

(a) rectification
 (b) amplification
 (c) filtration
 (d) stabilisation

- 39** In a transistor, the value of β is 100, then the value of α will be [UK PMT 2015]
 (a) 0.01 (b) 0.1 (c) 0.99 (d) 1

- 40** The arrangement shown in figure performs the logic function of [UK PMT 2015]



- (a) AND gate (b) NAND gate
 (c) OR gate (d) XOR gate
- 41** A change of 0.04 V takes place between the base and the emitter when an input signal is connected to the CE transistor amplifier. As a result, $20\mu\text{A}$ change take place in the base current and a change of 2mA takes place in the collector current. Find the input resistance and AC current gain. [Guj. CET 2015]
 (a) 1 k Ω , 100 (b) 2 k Ω , 100
 (c) 2 k Ω , 200 (d) 1 k Ω , 200

- 42** In a common emitter transistor amplifier, the voltage gain is [UK PMT 2015]
 (a) $\alpha \frac{R_{AC}}{R_{in}}$ (b) $\alpha \frac{R_{in}}{R_{AC}}$ (c) $\beta \frac{R_{AC}}{R_{in}}$ (d) $\beta \frac{R_{in}}{R_{AC}}$

- 43** In a $n-p-n$ transistor about 10^{10} electrons enter the emitter in $2\mu\text{s}$, when it is connected to a battery. Then, $I_E = \dots \mu\text{A}$. [Guj. CET 2015]
 (a) 400 (b) 200 (c) 800 (d) 1600

- 44** Which gate can be obtained by shorting both the input terminals of a NOR gate? [Guj. CET 2015]
 (a) NOT (b) OR (c) AND (d) NAND

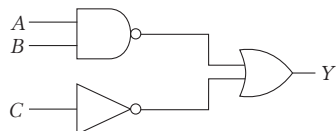
- 45** An LED is constructed from a $p-n$ junction based on a certain semi-conducting material whose energy gap is 1.9 eV. Then, the wavelength of the emitted light is [KCET 2015]
 (a) 6.5×10^{-7} m (b) 2.9×10^{-9} m
 (c) 9.1×10^{-5} m (d) 1.6×10^{-8} m

- 46** The equivalent circuit is



- The circuit represents [Manipal 2015]
 (a) NAND gate (b) OR gate
 (c) AND gate (d) NOR gate

- 47** The inputs to the digital circuit are as shown below. The output Y is [WB JEE 2015]



- (a) $A + B + \bar{C}$ (b) $(A + B) \bar{C}$
 (c) $\bar{A} + \bar{B} + \bar{C}$ (d) $\bar{A} + \bar{B} + C$

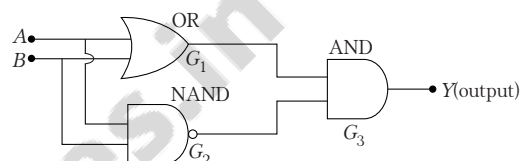
- 48** The p -type semiconductor is [UP CPMT 2015]

- (a) positively charged
 (b) negatively charged
 (c) electrically neutral
 (d) uncharged at 0 K or -273°C but charged at temperature higher than 0 K

- 49** The current gain of a transistor in common base configuration is 0.96. Corresponding to the change in emitter current is 10 mA, the change in base current would be [UP CPMT 2015]

- (a) 0.03 mA (b) 0.2 mA
 (c) 0.4 mA (d) 0.6 mA

- 50** The following logic gate circuit is equivalent to an [UP CPMT 2015]

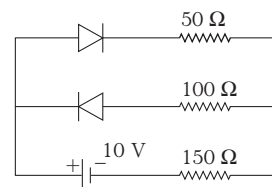


- (a) NAND (b) OR
 (c) XOR (d) NOT

- 51** A common emitter amplifier is designed with $n-p-n$ transistor ($\alpha = 0.99$). The input impedance is 1k Ω and load is 10k Ω . The voltage gain will be [Manipal 2015]

- (a) 9900 (b) 99
 (c) 9.9 (d) 990

- 52** Assume that, each diode as shown in the figure has a forward bias resistance of 50 Ω and an infinite reverse bias resistance. The current through the resistance 150 Ω is [WB JEE 2015]



- (a) 0.66 A (b) 0.05 A
 (c) zero (d) 0.04 A

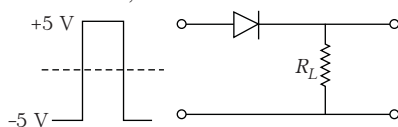
- 53** The input characteristics of a transistor in CE mode is the graph obtained by plotting [KCET 2015]

- (a) I_B against I_C at constant V_{CE}
 (b) I_B against V_{BE} at constant V_{CE}
 (c) I_B against I_C at constant V_{BE}
 (d) I_B against V_{CE} at constant V_{BE}

- 54** In a semiconducting material, the mobilities of electrons and holes are μ_e and μ_h , respectively. Which of the following is true? [Manipal 2015]

- (a) $\mu_e < \mu_h$ (b) $\mu_e = \mu_h$
 (c) $\mu_e < 0, \mu_h > 0$ (d) $\mu_e > \mu_h$

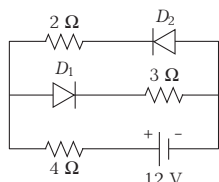
- 55 If in a p - n junction, a square input signal of 10 V is applied as shown,



then the output across R_L will be [CG PMT 2015]

- (a) (b)
(c) (d)

- 56 The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit? [KCET 2015]



- (a) 2.31 A (b) 1.71 A (c) 1.33 A (d) 2 A

- 57 The conductivity in the intrinsic semiconductor does not depend on [CG PMT 2015]

- (a) band gap (b) temperature
(c) fermi energy (d) effective mass of charge carriers

- 58 The barrier potential of a p - n junction depends on

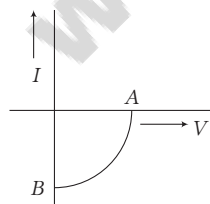
- I. type of semiconductor material
II. amount of doping
III. temperature

Which one of the following is correct?

[CBSE AIPMT 2014]

- (a) Both I and II (b) Only II
(c) Both II and III (d) All of these

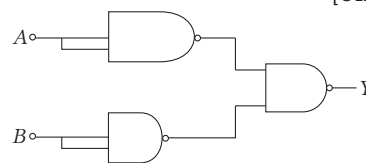
- 59 The given graph represents V - I characteristic for a semiconductor device. [CBSE AIPMT 2014]



Which of the following statement is correct?

- (a) It is V - I characteristic for solar cell, where point A represents open circuit voltage and point B short circuit current
(b) It is for a solar cell and points A and B represent open circuit voltage and current, respectively
(c) It is for a photodiode and points A and B represent open circuit voltage and current, respectively
(d) It is for a LED and points A and B represents open circuit voltage and short circuit current, respectively

- 60 The combination of gates shown below yields is [UK PMT 2014]



- (a) NAND gate (b) NOR gate
(c) XOR gate (d) OR gate

- 61 In insulators (CB is Conduction Band and VB is Valence Band) [MHT CET 2014]

- (a) VB is partially filled with electrons
(b) CB is partially filled with electrons
(c) CB is empty and VB is filled with electrons
(d) CB is filled with electrons and VB is empty

- 62 Identify the wrong statement. [Kerala CEE 2014]

- (a) In conductors, the valence and conduction bands overlap.
(b) Substances with energy gap of the order of 10 eV are insulators.
(c) The resistivity of semiconductors is lower than metals.
(d) The conductivity of metals is high.

- 63 If the band gap between valence band and conduction band in a material is 5.0 eV, then the material is [WB JEE 2014]

- (a) semiconductor (b) good conductor
(c) superconductor (d) insulator

- 64 In n -type semiconductor, electrons are majority charge carriers but it does not show any negative charge. The reason is [KCET 2014]

- (a) electrons are stationary
(b) electrons neutralise with holes
(c) mobility of electrons is extremely small
(d) atom is electrically neutral

- 65 Identify the wrong statement with reference to solar cell. [Kerala CEE 2014]

- (a) It is a p - n junction diode with no external bias.
(b) It uses materials of high optical absorption.
(c) It uses materials with band gap of 5 eV.
(d) It converts light energy into electrical energy.

- 66 In the circuit shown below, assume the diode to be ideal. When V_i increases from 2 V to 6 V, then the change in the current is (in mA) [WB JEE 2014]



- (a) zero (b) 20
(c) 80/3 (d) 40

- 67 Zener diode is used for [UK PMT 2014, Haryana PMT, CG PMT 2010]

- (a) amplification
(b) rectification
(c) voltage regulation
(d) produce oscillation in an oscillator

- 68** In common base circuit of a transistor, current amplification factor is 0.95. Calculate the emitter current, if base current is 0.2 mA. [MHT CET 2014]

(a) 2 mA (b) 4 mA
(c) 6 mA (d) 8 mA

- 69** A tuned amplifier circuit is used to generate a carrier frequency of 2 MHz for the amplitude modulation. The value of $\sqrt{LC}$ is [KCET 2014]

(a) $\frac{1}{2\pi \times 10^6}$ (b) $\frac{1}{2 \times 10^6}$ (c) $\frac{1}{3\pi \times 10^6}$ (d) $\frac{1}{4\pi \times 10^6}$

- 70** If α (current gain) of transistor is 0.98, then what is the value of β (current gain) of the transistor? [KCET 2014]

(a) 0.49 (b) 49
(c) 4.9 (d) 5

- 71** In a transistor output characteristics commonly used in common emitter configuration, the base current I_B , the collector current I_C and the collector-emitter voltage V_{CE} have values of the following orders of magnitude in the active region [WB JEE 2014]

(a) I_B and I_C both are in μA and V_{CE} in V
(b) I_B is in μA , I_C is in mA and V_{CE} in V
(c) I_B is in mA, I_C is in μA and V_{CE} in mV
(d) I_B is in mA, I_C is in mA and V_{CE} in mV

- 72** For the action of a Common Base (CB) transistor (E = emitter, B = base, C = collector), the required CB, EB junction bias conditions are [EAMCET 2014]

(a) Both EB and CB junction forward bias
(b) Both EB and CB junction reverse bias
(c) EB junction forward bias, CB junction reverse bias
(d) EB junction reverse bias, CB junction forward bias

- 73** The minimum number of NAND gates used to construct an OR gate is [Kerala CEE 2014]

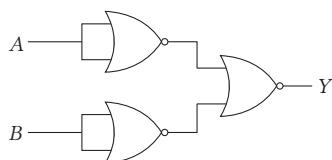
(a) 4 (b) 6 (c) 5 (d) 3
(e) 2

- 74** The output Y of the logic circuit given below is [J&K CET 2014]



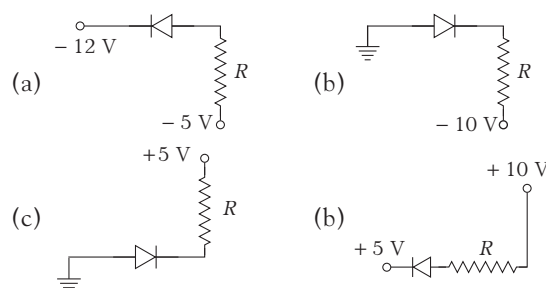
(a) $\bar{A} + B$ (b) $\bar{A}$ (c) $(\bar{A} + B) \cdot \bar{A}$ (d) $(\bar{A} + B) \cdot A$

- 75** For the given digital circuit, identify the logic gate. [KCET 2014]



(a) OR gate (b) NOR gate (c) NAND gate (d) AND gate

- 76** Of the diodes shown in the following diagrams, which one is reverse biased? [UK PMT 2014]



- 77** The truth tables of logic gates G_1 , G_2 , G_3 and G_4 are given here. Identify them correctly. [EAMCET 2014]

G_1			G_2		
Inputs		Output	Inputs		Output
A	B	Y	A	B	Y
0	0	0	0	0	0
0	1	1	0	1	0
1	0	1	1	0	0
1	1	1	1	1	1

G_3			G_4		
Inputs		Output	Inputs		Output
A	B	Y	A	B	Y
0	0	1	0	0	1
0	1	0	0	1	1
1	0	0	1	0	1
1	1	0	1	1	0

(a) G_1 -OR, G_2 -AND, G_3 -NOR, G_4 -NAND
(b) G_1 -OR, G_2 -NOR, G_3 -AND, G_4 -NAND
(c) G_1 -AND, G_2 -OR, G_3 -NAND, G_4 -NOR
(d) G_1 -OR, G_2 -NOR, G_3 -NAND, G_4 -AND

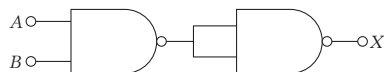
- 78** In a n -type semiconductor, which of the following statement is true? [NEET 2013]

(a) Electrons are majority carriers and trivalent atoms are dopants.
(b) Electrons are minority carriers and pentavalent atoms are dopants.
(c) Holes are minority carriers and pentavalent atoms are dopants.
(d) Holes are majority carriers and trivalent atoms are dopants.

- 79** In a common emitter (CE) amplifier having a voltage gain G , the transistor used has transconductance 0.03 mho and current gain 25. If the above transistor is replaced with another one with transconductance 0.02 mho and current gain 20, then the voltage gain will become [NEET 2013]

(a) $\frac{2}{3} G$ (b) $1.5 G$ (c) $\frac{1}{3} G$ (d) $\frac{5}{4} G$

- 80 The output X of the logic circuit shown in figure will be



[NEET 2013]

- (a) $X = \overline{A} \cdot \overline{B}$ (b) $X = \overline{A} \cdot B$
 (c) $X = A \cdot B$ (d) $X = A + B$
- 81 The output of an AND gate is connected to both the inputs of a NOR gate, then this circuit will act as a [Kerala CET 2013]
- (a) OR gate (b) NOR gate
 (c) AND gate (d) NAND gate
 (e) NOT gate

- 82 The necessary condition in making of a junction transistor (E -emitter, B -base and C -collector) [EAMCET 2013]

- (a) E and B are lightly doped and C is heavily doped
 (b) E is heavily doped, B is thin and lightly doped and C is moderately doped
 (c) E and C are lightly doped and B is thick and heavily doped
 (d) E and B are heavily doped and C is lightly doped

- 83 A $p-n-p$ transistor is used in common emitter mode in an amplifier circuit. When base current is changed by an amount ΔI_B , then the collector current changes by 4 mA. If the current amplification factor is 60, then the value of ΔI_B is [EAMCET 2013]
- (a) 15 μ A (b) 240 mA (c) 66.6 μ A (d) 60 μ A

- 84 In p -type semiconductor, the acceptor level lies [Kerala CET 2013]
- (a) near the conduction band
 (b) halfway between conduction and valence bands
 (c) within conduction band
 (d) near the valence band
 (e) within the valence band

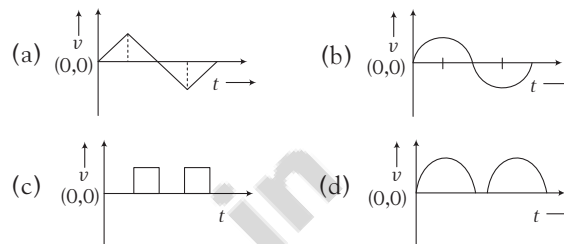
- 85 If the feedback voltage is increased in a negative feedback amplifier, then [Kerala CET 2013]
- (a) both gain and distortion decrease
 (b) the distortion increase
 (c) the gain decrease and distortion increase
 (d) the gain increase
 (e) both gain and distortion increase

- 86 When a $p-n$ junction is reverse biased, then the current through the junction is mainly due to [MP PMT 2013]
- (a) Only diffusion of charges
 (b) Only drift of charges
 (c) Both drift and diffusion of charges
 (d) Neither drift nor diffusion of charges

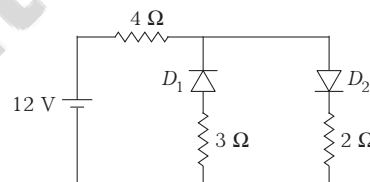
- 87 What is the value of $\overline{A} + A$ in the Boolean algebra? [UP CPMT 2013]
- (a) A (b) 0 (c) 1 (d) $\overline{A}$

- 88 The depletion layer in the $p-n$ junction region is caused by [UP CPMT 2013]
- (a) drift of holes
 (b) drift of electrons
 (c) diffusion of carriers
 (d) migration of impurity ions

- 89 Out of the following curves, which one represents a digital signal? [UP CPMT 2013]

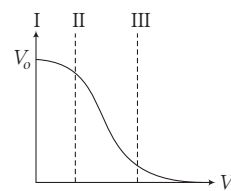


- 90 The circuit has two oppositely connected ideal diodes in parallel as shown in figure. What is the current flowing in the circuit? [UP CPMT 2013]



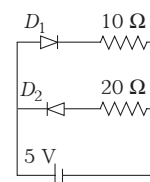
- (a) 1.33 A (b) 1.71 A
 (c) 2.00 A (d) 2.31 A

- 91 Transfer characteristic [output voltage (V_o) versus input voltage (V_i)] for a base biased transistor in CE configuration is as shown in the figure. For using transistor as a switch, it is used [CBSE AIPMT (Screening) 2012; BCECE 2012]



- (a) Only in region III (b) Both in regions I and III
 (c) Only in region II (d) Only in region I

- 92 Find the current in the circuit. [CBSE AIPMT (Screening) 2012]

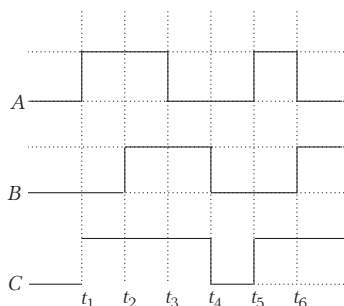


- (a) 0.75 A (b) Zero
 (c) 0.25 A (d) 0.5 A

- 93** In a CE transistor amplifier, the audio signal voltage across the collector resistance of $2\text{ k}\Omega$ is 2 V . If the base resistance is $1\text{ k}\Omega$ and the current amplification of the transistor is 100, then the input signal voltage is
[CBSE AIPMT (Screening) 2012]

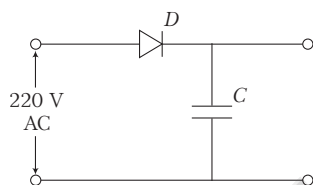
(a) 0.1 V (b) 1.0 V (c) 1 mV (d) 10 mV

- 94** The figure shows a logic circuit with two inputs A and B and the output C . The voltage waveforms across A , B and C are as given. The logic circuit gate is
[CBSE AIPMT (Screening) 2012]



(a) OR gate (b) NOR gate
(c) AND gate (d) NAND gate

- 95** A diode is connected to 220 V (rms) AC in series with a capacitor as shown in figure. The voltage across the capacitor is
[JCECE 2012]

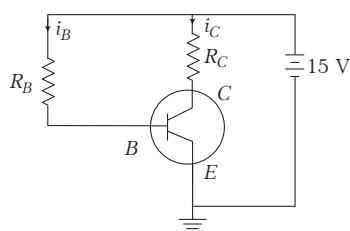


(a) 220 V (b) 110 V
(c) 311.1 V (d) $\frac{220}{\sqrt{2}}\text{ V}$

- 96** Which one of the following statements is not correct in case of a semiconductor?
[BCECE (Mains) 2012]

(a) Temperature coefficient of resistance is negative.
(b) Doping increases conductivity.
(c) At absolute zero temperature, it behaves like an insulator.
(d) Resistivity is in between that of a conductor and insulator.

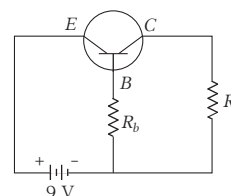
- 97** If for the following common emitter circuit, $\beta=100$, $V_{CE}=7\text{ V}$, V_{BE} is negligible and $R_C=2\text{ k}\Omega$, then $i_B=?$



[BCECE (Mains) 2012]

(a) 0.04 mA (b) 0.03 mA
(c) 0.02 mA (d) 0.01 mA

- 98** In a transistor circuit shown in figure, if the base current is $35\text{ }\mu\text{A}$, then the value of resistor R_b is
[BCECE (Mains) 2012]

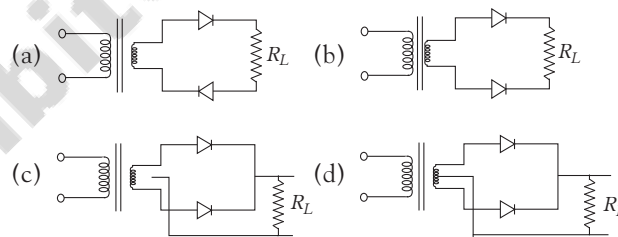


(a) $257\text{ k}\Omega$ (b) $123.5\text{ k}\Omega$
(c) $380.05\text{ k}\Omega$ (d) None of these

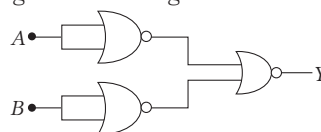
- 99** In intrinsic semiconductor, at room temperature number of electrons and holes are
[Manipal 2012]

(a) equal (b) zero
(c) unequal (d) infinite

- 100** Which of the following is not a rectifier circuit?
[AMU 2012]



- 101** The following network of gates
[AMU 2012]



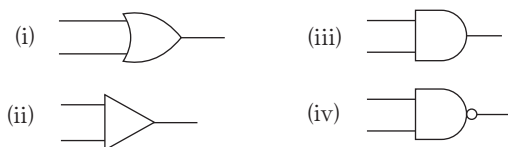
is equivalent to

(a) $A \text{ AND } B$
(b) $A \text{ OR } B$
(c) $A \text{ XOR } B$
(d) $A \text{ NOT } B$

- 102** If a small amount of antimony is added to germanium crystal,
[CBSE AIPMT 2011]

(a) the antimony becomes an acceptor atom
(b) there will be more free electrons than holes in the semiconductor
(c) its resistance is increased
(d) it becomes a p -type semiconductor

- 103** Symbolic representation of four logic gates are shown as [CBSE AIPMT 2011]



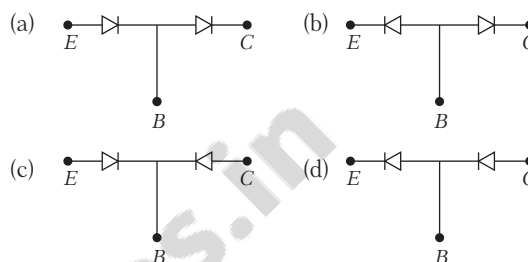
Pick out which ones are for AND, NAND and NOT gates, respectively.

- (a) (iii), (ii) and (i)
 (b) (iii), (ii) and (iv)
 (c) (ii), (iv) and (iii)
 (d) (ii), (iii) and (iv)
- 104** A transistor is operated in common emitter configuration at $V_C = 2$ V such that a change in the base current from $100\mu\text{A}$ to $300\mu\text{A}$ produces a change in the collector current from 10 mA to 20 mA.

The current gain is [CBSE AIPMT 2011]
 (a) 75 (b) 100 (c) 25 (d) 50

- 105** The device that can act as a complete electronic circuit is [CBSE AIPMT 2011]
 (a) junction diode (b) integrated circuit
 (c) junction transistor (d) zener diode

- 106** An n - p - n transistor can be considered to be equivalent to two diodes connected. Which of the following figures is the correct one? [KCET 2011]



ANSWERS

CHECK POINT 14.1

1. (a)	2. (a)	3. (a)	4. (d)	5. (b)	6. (b)	7. (c)	8. (b)	9. (a)	10. (d)
11. (b)	12. (b)	13. (b)	14. (a)	15. (d)					

CHECK POINT 14.2

1. (d)	2. (d)	3. (d)	4. (a)	5. (b)	6. (b)	7. (c)	8. (a)	9. (b)	10. (a)
11. (c)	12. (b)	13. (b)	14. (c)	15. (d)					

CHECK POINT 14.3

1. (a)	2. (d)	3. (c)	4. (b)	5. (b)	6. (a)	7. (a)	8. (a)	9. (a)	10. (d)
11. (b)	12. (b)	13. (a)	14. (a)	15. (a)					

CHECK POINT 14.4

1. (c)	2. (a)	3. (b)	4. (b)	5. (d)	6. (d)	7. (b)	8. (c)	9. (d)	10. (b)
11. (d)	12. (a)	13. (d)	14. (d)	15. (c)					

(A) Taking it together

1. (d)	2. (b)	3. (d)	4. (a)	5. (c)	6. (d)	7. (a)	8. (c)	9. (c)	10. (b)
11. (c)	12. (b)	13. (c)	14. (c)	15. (c)	16. (b)	17. (c)	18. (a)	19. (c)	20. (d)
21. (c)	22. (d)	23. (b)	24. (b)	25. (b)	26. (a)	27. (b)	28. (d)	29. (c)	30. (a)
31. (b)	32. (d)	33. (c)	34. (d)	35. (a)	36. (b)	37. (a)	38. (b)	39. (b)	40. (a)
41. (b)	42. (b)	43. (c)	44. (d)	45. (d)	46. (b)	47. (b)	48. (d)	49. (b)	50. (b)
51. (a)	52. (d)	53. (c)	54. (c)	55. (b)	56. (d)	57. (c)	58. (c)	59. (b)	60. (c)
61. (d)	62. (d)	63. (b)	64. (c)	65. (d)	66. (b)	67. (c)	68. (b)	69. (c)	70. (c)
71. (a)	72. (b)	73. (c)	74. (b)	75. (a)	76. (c)	77. (b)	78. (b)	79. (b)	80. (b)
81. (d)	82. (a)	83. (d)	84. (b)	85. (d)	86. (d)	87. (a)	88. (a)	89. (a)	90. (c)

91. (b)	92. (d)	93. (c)	94. (c)	95. (d)	96. (a)	97. (d)	98. (a)	99. (c)	100. (c)
101. (a)	102. (c)	103. (c)	104. (c)	105. (c)	106. (d)	107. (a)	108. (c)	109. (b)	110. (a)
111. (c)	112. (c)	113. (b)	114. (a)	115. (b)	116. (c)	117. (b)	118. (c)	119. (a)	120. (b)
121. (b)	122. (a)	123. (b)	124. (d)	125. (d)	126. (a)	127. (b)	128. (d)	129. (d)	130. (c)
131. (b)	132. (c)	133. (b)	134. (c)	135. (c)					

(B) Medical entrance special format questions**• Assertion and reason**

1. (a) 2. (a) 3. (b) 4. (a) 5. (c)

• Statement based questions

1. (c) 2. (c) 3. (d) 4. (d) 5. (c)

• Match the columns

1. (d) 2. (c) 3. (b) 4. (a)

(C) Medical entrances' gallery

1. (d)	2. (c)	3. (c)	4. (a)	5. (a)	6. (c)	7. (c)	8. (a)	9. (b)	10. (b)
11. (a)	12. (d)	13. (b)	14. (c)	15. (c)	16. (a)	17. (d)	18. (d)	19. (b)	20. (c)
21. (a)	22. (c)	23. (c)	24. (c)	25. (a)	26. (a)	27. (c)	28. (a)	29. (d)	30. (b)
31. (a)	32. (c)	33. (d)	34. (a)	35. (b)	36. (c)	37. (b)	38. (d)	39. (c)	40. (b)
41. (b)	42. (c)	43. (c)	44. (a)	45. (a)	46. (d)	47. (c)	48. (c)	49. (c)	50. (c)
51. (d)	52. (b)	53. (b)	54. (d)	55. (d)	56. (b)	57. (c)	58. (d)	59. (a)	60. (d)
61. (c)	62. (c)	63. (d)	64. (d)	65. (c)	66. (b)	67. (c)	68. (b)	69. (d)	70. (b)
71. (b)	72. (c)	73. (d)	74. (b)	75. (d)	76. (c)	77. (a)	78. (c)	79. (a)	80. (c)
81. (d)	82. (b)	83. (c)	84. (d)	85. (a)	86. (b)	87. (c)	88. (c)	89. (c)	90. (c)
91. (b)	92. (d)	93. (d)	94. (a)	95. (c)	96. (d)	97. (a)	98. (a)	99. (a)	100. (a)
101. (a)	102. (b)	103. (c)	104. (d)	105. (b)	106. (b)				

Hints & Explanations

• CHECK POINT 14.1

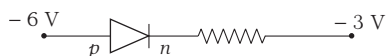
- 4 (d) In insulators, the forbidden energy gap is largest and it is of the order of 5 eV to 15 eV.
- 5 (b) In insulators, the forbidden energy band gap is very large, in case of semiconductor, it is moderate and in conductors, the energy gap is zero because in conductor, valence band and conduction band overlap to each other.
- 9 (a) Aluminium is trivalent impurity which is doped with pure semiconductor to form *p*-type semiconductor.
- 11 (b) The charge on hole is positive which is equivalent to charge on proton.
- 12 (b) For *n*-type semiconducting material, number of free electrons is very large than number of holes, i.e. $n_e \gg n_h$.
- 15 (d) Temperature coefficient of semiconductor is negative. Copper, aluminium, iron are conductors, while germanium is semiconductor.

• CHECK POINT 14.2

- 10 (a) As, $-6\text{ V} < -3\text{ V}$

i.e. *p* is at low potential than *n*.

∴ Diode is reverse biased.



- 11 (c) When a large reverse voltage is applied across a *p-n* junction diode, a huge current flows in the reverse direction suddenly. This is called breakdown of *p-n* junction diode.
- 12 (b) At a particular reverse voltage in *p-n* junction avalanche breakdown occurs, hence a huge current flows in reverse direction known as avalanche current.
- 13 (b) The *p-n* junction is used to convert AC into DC (rectifier).
- 14 (c) When a light falls on the junction, new hole-electron pairs are created. Numbers of produced electron-hole pairs depend upon numbers of photons. So photo-emf or current is proportional to intensity of light.

• CHECK POINT 14.3

- 2 (d) In a bipolar junction transistor, emitter is heavily doped, base is lightly doped and collector is moderately doped.
- 5 (b) In *p-n-p* transistor, emitter-base is forward biased, hence holes move from emitter to base.
- 6 (a) When *n-p-n* transistor is used as an amplifier, majority charge carrier, i.e. electrons of *n*-type emitter move from emitter to base end and then base to collector.
- 8 (a) Here, $\Delta V_C = 0.5\text{ V}$, $\Delta I_C = 0.05\text{ mA}$

Output resistance is given by

$$R_{\text{out}} = \frac{\Delta V_C}{\Delta I_C} = \frac{0.5}{0.05 \times 10^{-3}} = 10^4 \Omega = 10\text{ k}\Omega$$

$$9 (a) \beta = \left(\frac{\Delta I_C}{\Delta I_B} \right)_{V_{CE}} = \frac{8.2}{(8.3 - 8.2)} = 82$$

$$10 (d) \alpha = \frac{i_C}{i_E} = \frac{5.488}{5.60} = 0.98; \beta = \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} = 49$$

$$11 (b) \text{ Current gain, } \beta = \frac{\Delta I_C}{\Delta I_B} \Rightarrow \Delta I_B = \frac{1 \times 10^{-3}}{100} = 10^{-5}\text{ A} = 0.01\text{ mA}$$

By using $\Delta i_E = \Delta i_B + \Delta i_C$; $\Delta i_E = 0.01 + 1 = 1.01\text{ mA}$

$$12 (b) \frac{V_{\text{out}}}{V_{\text{in}}} = \beta \times \frac{R_{\text{out}}}{R_{\text{in}}} \Rightarrow R_{\text{out}} = \frac{V_{\text{out}}}{V_{\text{in}}} \times R_{\text{in}} \times \frac{1}{\beta} = \frac{3}{0.01} \times 1000 \times \frac{1}{100} = 3\text{ k}\Omega$$

$$13 (a) \text{ Voltage gain} = 10 \log_{10} 1000\text{ dB} = 30\text{ dB}$$

- 15 (a) In oscillator, a portion of the output power is returned back to the input in phase with the starting power. This process is termed as positive feedback.

• CHECK POINT 14.4

$$1 (c) 10111 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 16 + 0 + 4 + 2 + 1 = 23$$

$$2 (a) \begin{array}{r} 100010 \\ + 11011 \\ \hline 111101 \end{array}$$

$$\therefore (100010)_2 + (11011)_2 = (111101)_2$$

- 5 (d) The boolean expression for AND gate is $R = P \cdot Q$

$$R = P \cdot Q \Rightarrow 1 \cdot 1 = 1$$

$$1 \cdot 0 = 0; 0 \cdot 1 = 0 \text{ and } 0 \cdot 0 = 0$$

- 6 (d) For AND gate, if output is 0, then one or both the inputs must be 0.

- 8 (c) If inputs are *A* and *B*, then output of NAND gate is $Y = \overline{AB}$.

$$\text{If } A = B = 1, \text{ then } Y = \overline{1 \cdot 1} = \overline{1} = 0.$$

- 9 (d) The given truth table is for NAND gate, since $Y = \overline{A \cdot B}$.

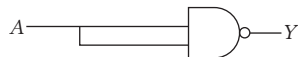
- 11 (d) Any digital circuit can be realised by repetitive use of NOR gate or NAND gate. NOR gate and NAND gate are called universal gates because by using these gates we can realise other basic gates like OR, AND and NOT gates.

- 13 (d)

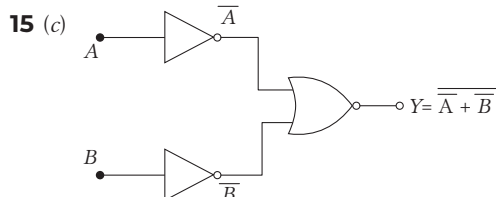
Input		Output			
		OR	AND	NOR	NAND
0	0	0	0	1	1
0	1	1	0	0	1
1	0	1	0	0	1
1	1	1	1	0	0

From the above truth tables it is clear that the logic gates giving output '1' for the inputs 1 and 0 are NAND and OR gates.

- 14 (d) When the two inputs of NAND gates are shorted,



$Y = \overline{A} = \overline{A + A}$ which is equivalent to a NOT gate.



Hence, output $Y = \overline{A + B}$

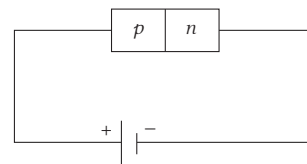
According to de-Morgan's theorem,

$$Y = \overline{A + B} = \overline{A} \cdot \overline{B} = A \cdot B$$

This is the output equation of AND gate.

(A) Taking it together

- 2 (b) The concept of hole describes the lack of an electron at a position where one could exist in an atom or atomic lattice. If an electron is excited into a higher state, it leaves a hole in its old state.
- Thus, hole can be defined as a vacancy created when an electron leaves a covalent bond.
- 3 (d) Resistance of conductor (Cu) decreases with decrease in temperature while that of semiconductor (Ge) increases with decrease in temperature.
- 8 (c) On increasing the temperature, the current in the circuit will increase because with rise in temperature, resistance of semiconductor decreases, hence overall resistance of the circuit decreases, which in turn increases the current in the circuit.
- 14 (c) The majority charge carriers in *p*-type semiconductor are holes, while electrons are minority charge carriers.
- 16 (b) *p*-type semiconductor can be made by doping a silicon atom in the crystal lattice with a gallium (a trivalent impurity).
- 17 (c) For making *p*-type semiconductor, we add trivalent impurity. So, B impurity is mixed in Ge and Al impurity is mixed in Si.
- 18 (a) At room temperature, a *p*-type semiconductor has large number of holes and few electrons, as holes are majority charge carriers.
- 23 (b) Pentavalent impurity atom is to be added to Ge crystal, so as to make a *n*-type semiconductor.
- 31 (b) In a transistor, emitter is heavily doped, base is least doped while collector is moderately doped.
- 32 (d) Extrinsic semiconductors are also electrically neutral.
- 34 (d) The conductivity of a semiconductor increases with increase in temperature, because the number density of current carriers increases. Here, due to high temperature, the relaxation time also decreases but effect of decrease in relaxation time is much less than increase in number density.
- 38 (b) In the forward bias arrangement of a *p-n* junction diode, the *p*-end is connected to the positive terminal of the battery and *n*-end with negative terminal of battery.



- 40 (a) As, $i = n_e A v_d$

$$\text{So, } v_d = \frac{i}{n_e A}$$

So, when temperature is increased, n_e increases and v_d decreases.

- 41 (b) In forward biasing, the diffusion current increases and drift current remains constant, so net current is due to diffusion. In reverse biasing diffusion is not possible, so net current (very small) is due to the drift.
- 42 (b) At a specific reverse voltage (breakdown voltage) in *p-n* junction, a huge current flows in reverse direction known as avalanche current.
- 52 (d) Resistance of semiconductor decreases with increase in temperature. Hence, its temperature coefficient is negative.
- 56 (d) $(E_g)_{\text{Ge}} < (E_g)_{\text{Si}} < (E_g)_{\text{C}}$
 $[\because (E_g)_{\text{Ge}} = 0.7 \text{ eV}, (E_g)_{\text{Si}} = 1.1 \text{ eV}, (E_g)_{\text{C}} = 5.5 \text{ eV}]$
- 59 (b) From rectifier we get pulsating DC, which consists of AC ripples, to remove these unwanted ripples, we use filter circuits.
- 61 (d) α is current gain in case of common base amplifier while β is current gain in case of common emitter amplifier. As, $\beta = \frac{\alpha}{1 - \alpha}$. Since $\alpha < 1$, hence $\beta > 1$. Hence, α is always less than 1 and β is greater than 1.
- 63 (b) Collector current, $I_C = 10 \text{ mA}$
 Since, $I_C = 90\%$ of I_E
 $\Rightarrow I_C = \frac{90}{100} \times I_E$
 $I_E = \frac{100}{90} \times I_C = \frac{100}{90} \times 10 = \frac{100}{9} = 11.11 \text{ mA}$
- 65 (d) For full wave rectifier, $\eta = \frac{81.2}{1 + \frac{r_f}{R_L}}$
 $\Rightarrow n_{\text{max}} = 81.2\% \quad (\because r_f < R_L)$
- 67 (c) Here, $A \cdot A = A$, $A + A = A$
 and $\overline{A + A} = \overline{A \cdot A} = A$
 But $\overline{A} \cdot A = 0$
- 68 (b) When *p-n* junction is forward biased, it opposes the potential barrier at junction but when *p-n* junction is reverse biased, it supports the potential barrier at junction, resulting in increase in potential barrier across the junction.

69 (c) For full wave rectifier, the fundamental frequency in ripple is twice that of input frequency.

72 (b) A NOR gate is combination of NOT and OR gate. The boolean expression of NOR gate is $C = \overline{A + B}$.

73 (c) With the rise in temperature, resistivity of semiconductors decreases exponentially.

75 (a) Given, $E = 6 \times 10^5 \text{ Vm}^{-1}$

$$d = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$$

$$V = ?$$

$$\therefore \text{Potential barrier, } V = Ed = (6 \times 10^5)(500 \times 10^{-9}) = 0.30 \text{ V}$$

76 (c) The electrical conductivity of germanium,

$$\begin{aligned} \sigma &= \frac{1}{\rho} = e(\mu_e n_e + \mu_h n_h) \\ &= (1.6 \times 10^{-19}) [0.36 + 0.17] (2.5 \times 10^{19}) \\ & \quad (\because n_e = n_h = 2.5 \times 10^{19} \text{ m}^{-3}) \\ &= 2.12 \text{ Sm}^{-1} \end{aligned}$$

77 (b) As, $n_e > n_h$

So, it is an n -type semiconductor.

78 (b) In the given circuit, p -side of p - n junction diode D_1 is connected to lower voltage and n -side of D_1 to higher voltage. Thus D_1 is in reverse biased.

The p -side of p - n junction of D_2 is at higher potential and n -side of D_2 is at lower potential. Therefore, D_2 is in forward biased.

Hence, no current flows from B to A .

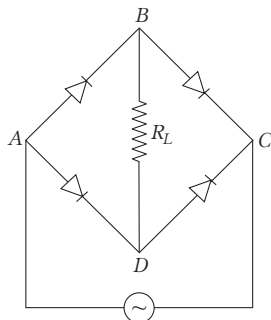
80 (b) In 1 cm^3 of Si, there will be total $\frac{6 \times 10^{28}}{10^6} = 6 \times 10^{22}$ atoms

Per 6×10^7 silicon atoms, there is one indium atom. Therefore,

$$\text{total number of indium atoms will be } \frac{6 \times 10^{22}}{6 \times 10^7} = 10^{15}$$

81 (d) As p - n junction conducts during positive half cycle only, the diode connected here will work in positive half cycle. Potential difference across C = peak voltage of the given AC voltage $= V_0 = V_{\text{rms}} \sqrt{2} = 220\sqrt{2} \text{ V}$.

82 (a) During the first half cycle when $V_A > V_C$, all the four diodes are forward biased. Hence, no current will flow through R_L . During second half cycle when $V_C > V_A$, all the four diodes are reverse biased. Again, no current will flow through R_L .



83 (d) In forward biasing, both potential barrier V_B and the width of charge depleted region x decreases.

85 (d) Given, $\beta = \frac{\Delta i_c}{\Delta i_b} = 50$, $R_i = 1 \text{ k}\Omega = 1 \times 10^3 \Omega$, $\Delta V_i = 0.01 \text{ V}$

$$\therefore \Delta i_{in} = \Delta i_b = \frac{\Delta V_i}{R_i} = \frac{0.01}{10^3} = 10^{-5} \text{ A}$$

$$\therefore \Delta i_c = \beta \Delta i_b = 50 \times 10^{-5} \text{ A} = 500 \mu\text{A}$$

$$\textbf{86} \text{ (d) } A = \frac{V_o}{V_i - mV_o}$$

$$\text{Here, } V_o = A(V_i - 0.09V_o)$$

$$\text{Since, } \frac{V_o}{V_i} = 10 \Rightarrow V_i = \frac{V_o}{10}$$

$$\therefore V_o = A \left(\frac{V_o}{10} - 0.09V_o \right)$$

$$\Rightarrow A = 100$$

87 (a) In figure (a), first gate is AND gate and second gate is NOT gate. So, it is NAND gate.

88 (a) Here, diode is in reverse biased condition. In reverse biasing, it acts as open circuit, hence no current flows.

89 (a) In the given circuit, diode is in reverse biasing, so it acts as open circuit. Hence, potential difference between A and B is 6 V .

92 (d) There is a phase difference of 180° in case of CE amplifier. Also, voltage gain, $A_v = \frac{V_o}{V_i}$, so

$$V_o = A_v \cdot V_i = 1000 \times 0.004 = 4 \text{ V}$$

$$\text{Hence, } V_o = (4 \text{ V}) \sin(\omega t - \pi/2)$$

$$\textbf{93} \text{ (c) } \alpha = \frac{\Delta i_c}{\Delta i_e}$$

$$\therefore \Delta i_c = \alpha \Delta i_e = (0.98)(5) = 4.9 \text{ mA}$$

$$\textbf{94} \text{ (c) Energy gap, } E_g = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E_g} = \frac{12400 \text{ eV} \cdot \text{\AA}}{0.72 \text{ eV}} = 17222 \text{ \AA}$$

95 (d) Given, $\Delta i_c = 0.990 \text{ mA}$

$$\therefore \alpha = \frac{\Delta i_c}{\Delta i_e} = \frac{0.990}{1} = 0.990$$

96 (a) The given boolean expression can be written as

$$\begin{aligned} Y &= (A + B) \cdot (\overline{AB}) \\ &= (\overline{A} \cdot \overline{B}) \cdot (\overline{A} + \overline{B}) = (\overline{A} \cdot \overline{A}) \cdot \overline{B} + \overline{A}(\overline{B} \cdot \overline{B}) \\ &= \overline{A} \cdot \overline{B} + \overline{A} \cdot \overline{B} = \overline{A} \cdot \overline{B} \end{aligned}$$

So, the truth table is

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

Hence, to get $Y = 1$, $A = 0$ and $B = 0$.

- 97 (d) Given, $\alpha = 0.9$ and $\Delta I_E = 4 \text{ mA}$

$$\therefore \beta = \frac{\alpha}{1-\alpha} \Rightarrow \beta = 9$$

$$\therefore \Delta I_C = 9\Delta I_B$$

$$\Rightarrow \Delta I_E - \Delta I_B = 9\Delta I_B$$

$$\Rightarrow \Delta I_B = \frac{\Delta I_E}{10} = \frac{4}{10}$$

$$\Rightarrow I_B = 0.4 \text{ mA}$$

$$\text{So, } \Delta I_C = 9 \times 0.4 = 3.6 \text{ mA}$$

- 98 (a) In case of forward biasing, resistance of p - n junction diode is zero. So, whole voltage appears across the resistance.

99 (c) $\lambda \text{ (in } \text{\AA}) = \frac{12375}{E \text{ (in eV)}} = \frac{12375}{57 \times 10^{-3}} \text{\AA} \approx 217100 \text{\AA}$

- 100 (c) In the truth table, output $X = \bar{A}$. So, this truth table is of NOT gate.

103 (c) $1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} = 26.625$

- 104 (c) For option (c), it is a NAND gate, its output $= \bar{0} \cdot 1 = \bar{0} = 1$

- 106 (d) The output D for the given combination,

$$D = \overline{(A+B) \cdot C} = \overline{(A+B)} + \bar{C}$$

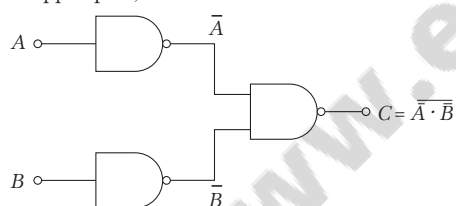
If $A = B = C = 0$, then

$$D = \overline{(0+0)} + \bar{0} = \bar{0} + \bar{0} = 1 + 1 = 1$$

If $A = B = 1$ and $C = 0$, then

$$D = \overline{(1+1)} + \bar{0} = \bar{1} + \bar{0} = 0 + 1 = 1$$

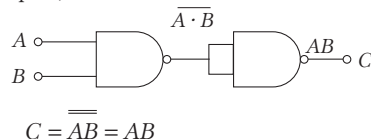
- 107 (a) For upper part,



Here, $C = \overline{\bar{A} \cdot \bar{B}} = \overline{\bar{A}} + \overline{\bar{B}} = A + B$ (from de-Morgan's theorem)

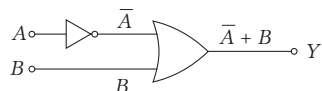
Here, output C is equivalent to OR gate.

For lower part,



In this case, output C is equivalent to AND gate.

- 108 (c)



- 109 (b) Diode is in forward biased condition. So, current flow

$$i = \frac{(4-1)}{300} = 10^{-2} \text{ A}$$

- 110 (a) Intensity of electric field,

$$E = \frac{V}{d} = \frac{0.50}{5 \times 10^{-7}} = 1.0 \times 10^6 \text{ Vm}^{-1}$$

- 111 (c) Due to forward biasing during positive half cycle of input AC voltage, the resistance of p - n junction is low. The current in the circuit is maximum. In this situation, a maximum potential difference will appear across resistance connected in series of circuit. This result into zero output voltage across p - n junction.

Similarly during negative half cycle of AC voltage, the p - n junction is reverse biased. The resistance of p - n junction becomes high which will be more than resistance in series. That is why, there will be voltage across p - n junction with negative cycle in output.

- 112 (c) D_1 is forward biased and D_2 reverse biased.

Therefore, current through the resistance and D_1 will be equal

$$\text{and which is equal to } \frac{5}{4} = 1.25 \text{ A}$$

- 113 (b) D_1 is reverse biased. Therefore, no current will flow through 20Ω resistance.

As, 30Ω and 20Ω resistance are in series, so total current will be $(5/50) \text{ A}$.

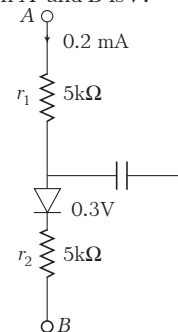
- 114 (a) First diode is in reverse biasing, it acts as open circuit, hence no current flows.

- 115 (b) For forward biasing, p -side is at higher potential and n -side is at lower potential, so diode in figures II, IV and V are forward biased.

- 116 (c) If $V_A > V_B$, then both the junction diodes are in forward biased and the given circuit diagram becomes a balanced Wheatstone bridge. The equivalent resistance in this case becomes 4Ω .

If $V_A < V_B$, then the diodes are reverse biased. In that case, 4Ω , 5Ω and 4Ω are in series, i.e. $R_{\text{net}} = 13 \Omega$.

- 117 (b) Consider the figure given here, suppose the potential difference between A and B is V .



$$\text{Then, } V - 0.3 = [(r_1 + r_2) 10^3] \times (0.2 \times 10^{-3}) \quad (\because V = ir)$$

$$= [(5 + 5) 10^3] \times (0.2 \times 10^{-3})$$

$$= 10 \times 10^3 \times 0.2 \times 10^{-3} = 2$$

$$\Rightarrow V = 2 + 0.3 = 2.3 \text{ V}$$

- 118 (c) Here, $C = A \cdot B$ and $D = \bar{A} \cdot \bar{B}$

$$E = C + D = (A \cdot B) + (\bar{A} \cdot \bar{B})$$

Explanation The truth table of this arrangement of gates can be given by

A	B	$\bar{A}$	$C = A \cdot B$	$d = \bar{A} \cdot B$	$E = (C + D)$
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	0	1

119 (a) Transconductance, $g_m = \frac{\beta_{AC}}{R_{in}}$

where, β_{AC} = current gain
and R_{in} = input resistance.

$$\therefore g_m = \frac{20}{2 \times 10^3} = 0.01 \Omega^{-1}$$

120 (b) Given, $V = 9V$

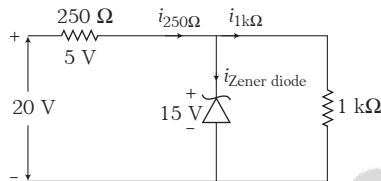
$$i_b = 35 \mu A = 35 \times 10^{-6} A$$

$$R_b = ?$$

We know that, $V = i_b R_b$

$$\Rightarrow R_b = \frac{V}{i_b} = \frac{9}{35 \times 10^{-6}} = 257 \text{ k}\Omega$$

121. (b) Voltage across Zener diode is constant,



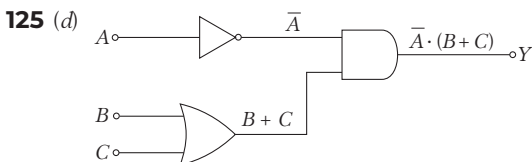
$$i_{1k\Omega} = \frac{15}{1 \times 10^3} = 15 \text{ mA}$$

$$i_{250\Omega} = \frac{(20 - 15)}{250} = \frac{5}{250} = \frac{20}{1000} = 20 \text{ mA}$$

$$\therefore i_{Zener \text{ diode}} = (20 - 15) = 5 \text{ mA}$$

122 (a) $Y = \bar{A} + \bar{B} = A \cdot B$, i.e. it is a AND gate.

123 (b) In the positive half cycle of input AC signal diode, D_1 is in forward biased and D_2 is in reverse biased, so in the output voltage signal, A and C are due to diode D_1 . In negative half cycle of input AC signal, diode D_2 is in forwards biased, hence it conducts, so output signals B and D are due to diode D_2 .

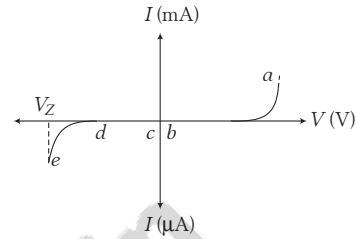


126 (a) The diode is in reverse biasing, so no current flows through it.

127 (b) Potential across the p - n junction varies symmetrically linear, having p side negative and n side positive.

We know that in a forward biased p - n junction diode, the repulsion of holes and electrons takes place which decreases width of potential barrier by striking the combination of holes and electrons. We also know that the options (c) and (d) show the potential barrier in the reverse bias, whereas in options (a) and (b) show the potential barrier in forward bias. Moreover, the width of depletion layer in option (b) is less than shown in option (a). Thus, the option (b) is correct.

129 (d)



If the reverse bias voltage is greater than the V_Z , then there is breakdown condition. In breakdown region, i.e. $V_i > V_Z$ and for a long range of load (R_L), the voltage is constant.

130 (c) Voltage of the source,

$$V = V_D + iR = 0.5 + 0.1 \times 20 = 2.5 \text{ V}$$

131. (b) The potential across $R = 17V - 9V = 8V$

132 (c) Total resistance in the circuit, $R = 2 \text{ k}\Omega$ and $E = 20 \text{ V}$

$$\therefore \text{The current, } I = \frac{E}{R} = \frac{20}{2000} = 10 \text{ mA}$$

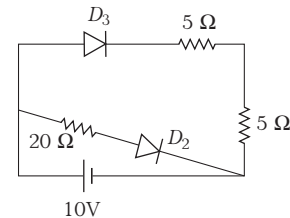
133 (b) In the given circuit, the Zener diode is used as a voltage regulating device.

Hence, the voltage across $1 \text{ k}\Omega$ is 5 V .

Current flowing through $1 \text{ k}\Omega$ resistor,

$$I = \frac{5}{1 \times 10^3} = 5 \times 10^{-3} \text{ A} = 5 \text{ mA}$$

134 (c) In the given circuit, diode D_1 is reverse biased, so it will not conduct. Diodes D_2 and D_3 are forward biased, so they will conduct. The corresponding equivalent circuit is as shown in the figure



The equivalent resistance of the circuit is

$$R_{eq} = \frac{(5 + 5) \times 20}{(5 + 5) + 20} = \frac{10 \times 20}{10 + 20} = \frac{200}{30} = \frac{20}{3} \Omega$$

$$\text{Current through the battery, } I = \frac{10 \text{ V}}{\frac{20}{3} \Omega} = 1.5 \text{ A}$$

135 (c) In circuit A, both the p - n junction diodes are in forward biased.

Total resistance R is given by

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{4} \Rightarrow \frac{1}{R} = \frac{2}{4} \Rightarrow R = 2 \Omega$$

According to Ohm's law

$$\begin{aligned} V &= I_A R \\ \Rightarrow 8 &= I_A \times 2 \\ \Rightarrow I_A &= 4 \text{ A} \end{aligned}$$

In circuit B, lower p - n junction diode is in reverse biased. Hence, no current will flow in it but upper diode is forward biased, so current can flow through it.

$$V = I_B R \Rightarrow 8 = I_B \times 4 \Rightarrow I_B = 2 \text{ A}.$$

(B) Medical entrance special format questions

• Assertion and reason

- (a) In semiconductors, the energy gap between conduction band and valence band is small ($\approx 1 \text{ eV}$). Due to temperature rise, electron in the valence band gain thermal energy and may jump across the small energy gap, (to the conduction band). Thus, conductivity increases and hence resistance decreases.
- (a) According to law of mass action, $n_i^2 = n_e n_h$. In intrinsic semiconductors $n_i = n_e = n_h$ and for p -type semiconductor n_e would be less than n_i , since n_h is necessarily more than n_i .
- (b) In forward biasing of p - n junction, current flows due to diffusion of majority charge carriers. While in reverse biasing, current flows due to drifting of minority charge carriers. The circuit given in the reason is a p - n - p transistor having emitter more negative w.r.t. base, so it is reverse biased and collector is more positive w.r.t. base, so it is forward biased.
- (a) These gates are called digital building blocks because using these gates (either NAND or NOR) we can compile all other gates also (like OR, AND, NOT, XOR).
- (c) In diode, the output is in same phase with the input, therefore it cannot be used to built NOT gate.

• Statement based questions

- (c) In a transistor, the base region must be very thin and lightly doped. For the normal action of transistor emitter junction is in forward biased and collector junction is in reverse biased.
- (d) Resistivity of a semiconductor decreases with the temperature. The atoms of a semiconductor vibrate with larger amplitudes at higher temperature thereby increasing its conductivity not resistivity.
- (c) Input impedance of common emitter configuration

$$= \left| \frac{\Delta V_{BE}}{\Delta i_B} \right|_{V_{CE} = \text{constant}}$$

where, ΔV_{BE} = voltage across base and emitter (base-emitter region is forward biased)

Δi_B = base current which is order of few microampere.

Thus, input impedance of common emitter is low.

• Match the columns

- (d) Diode gets forward biased during positive half cycle of alternating source and current flows. Let the barrier voltage of diode is V_B and its forward resistance be R_F and load resistance be R_L , then peak voltage of applied signal is

$$\begin{aligned} V_0 &= V_B + i_0 (R_F + R_L) \\ \therefore i_0 &= \frac{V_0 - V_B}{R_F + R_L} = \frac{20 - 0.7}{10 + 500} = 37.8 \times 10^{-3} \text{ A} = 37.8 \text{ mA} \end{aligned}$$

Peak voltage at the ends of the load R_L ,

$$V_L = i_0 \times R_L = 37.8 \times 10^{-3} \times 500 = 18.9 \text{ V}$$

For ideal diode, $V_B = 0$ and $R_F = 0$

$$\therefore i_0 = \frac{V_0}{R_L} = \frac{20}{500} = 0.04 \text{ A} = 40 \text{ mA}$$

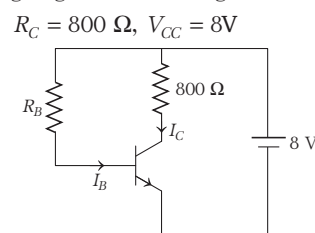
$$\text{and } V_L = i_0 R_L = 0.04 \times 500 = 20 \text{ V}$$

Hence, A $\rightarrow$ p, B $\rightarrow$ s, C $\rightarrow$ q, D $\rightarrow$ r.

(C) Medical entrances' gallery

- (d) A p - n junction diode is in forward biased when p -side is connected with more positive potential than n -side. Since, $0 \text{ V} > -3 \text{ V}$. Hence in option (d), diode circuit is in forward biased.

- (c) According to given circuit diagram



Voltage drop across R_C ,

$$\begin{aligned} V_C &= 0.8 \text{ V} \Rightarrow I_C R_C = 0.8 \\ I_C &= \frac{0.8}{R_C} = \frac{0.8}{800} = 10^{-3} \text{ A} = 1 \text{ mA} \end{aligned}$$

- (c) NAND and NOR gate are called universal gate because all type of logic gates and boolean expressions can be realised with the help of NAND and NOR gate.
- (a) When a pentavalent (phosphorous) impurities is doped with intrinsic semiconductor (Ge, Si), then n -type semiconductor is formed.
- (a) Under reverse bias condition, the holes of p -side are attracted towards the negative terminal of the battery and electrons of the n -side are attracted towards the positive terminal of the battery. This increases the width of the depletion layer. However, in the case of forward biasing, the width of the depletion layer decreases.
- (c) Insulators and semiconductors are those solids, which have negative temperature coefficient of resistance. As, when temperature increases number of free electrons increases in both insulator and semiconductor.

7 (c) In case of transistor action, base region is thin and lightly doped, while emitter region is heavily doped and collector region is moderately doped. Also, the size of collector is slightly more than the size of emitter.

8 (a) *p*-type semiconductors are obtained when a trivalent impurity, *e.g.* boron, aluminium, gallium or indium) is added to an intrinsic semiconductor (*e.g.* germanium or silicon).

In other words, the dopants in *p*-type semiconductor is trivalent atom. Thus, this addition creates deficiencies of valence electron which are most commonly known as holes. These are the majority charge carriers in this type of semiconductor.

However, in *n*-type semiconductors, the dopants are pentavalent impurities. Also, the majority charge carriers in *n*-type semiconductor are electrons.

9 (b) The energy of light of wavelength λ is given by

$$E = h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E} \quad \dots(i)$$

Here, h = Planck's constant = 6.63×10^{-34} J-s

c = speed of light = 3×10^8 m/s

E = energy gap = $1.9 \text{ eV} = 1.9 \times 1.6 \times 10^{-19} \text{ J}$

Substituting the given values in Eq. (i), we get

$$\Rightarrow \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.9 \times 1.6 \times 10^{-19}}$$

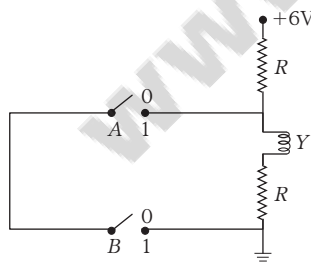
$$= 6.54 \times 10^{-7} \text{ m}$$

$$\approx 654 \text{ nm}$$

Thus, the wavelength of light emitted from LED will be 654 nm.

10 (b) The LED will glow when the current flows through it, *i.e.* when the voltage across it is high.

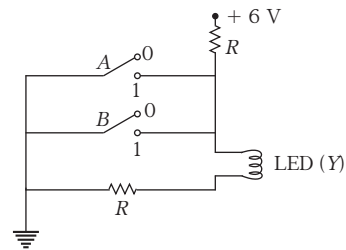
The truth table can be formed from this



A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

The output Y is same as that come from NAND gate.

11 (a) From the circuit diagram given below, it can be seen that the current will flow to ground if any of the switch is closed. Also, the LED will only glow when current flows through it, *i.e.* both the switches should be open.



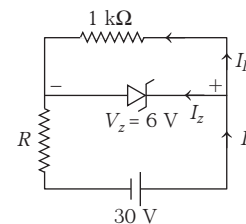
Thus, the truth table for the circuit diagram can be formed as

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

The output (Y) is equivalent to that of NOR gate.

12 (d) Given, voltage across Zener diode, $V_Z = 6 \text{ V}$

Since, Zener diode provide stabilised supply of 6 V.



In this case, the value of R should be chosen such that negligible current flows through the Zener diode.

$$\therefore I = I_Z + I_L = 0 + I_L$$

$$I = \frac{30}{R + 10^3}$$

But $I = 6 \text{ mA} = 6 \times 10^{-3} \text{ A}$

$$\therefore 6 \times 10^{-3} = \frac{30}{R + 10^3} \Rightarrow R = 4 \times 10^3 \Omega = 4 \text{ k}\Omega$$

13 (b) A *p-n* junction photodiode can be operated under photovoltaic conditions similar to that of a solar cell. The current-voltage characteristics of a photodiode and solar cell under illumination are also similar.

When light energy falls on the solar cell, then it converts solar energy into electrical energy.

The area of solar cell should be larger, so that it could absorbed more amount of sunlight and hence more amount of electrical energy is obtained.

Hence, Assertion and Reason both are correct but Reason is not the correct explanation of Assertion.

- 14 (c) Given, $R_{out} = 200 \Omega$, $R_{in} = 100 \text{ k}\Omega = 10^5 \Omega$

$$V_{CC} = 3 \text{ V}, V_{BE} = 0.7 \text{ V}, V_{CE} = 0 \text{ V}, \beta = 200$$

Applying KVL at output side,

$$V_{CE} = V_{CC} - I_C R_{out}$$

$$0 = 3 - I_C \times 200$$

$$\Rightarrow I_C = 15 \text{ mA}$$

$$\therefore I_B = \frac{I_C}{\beta} = \frac{15 \times 10^{-3}}{200} = 75 \times 10^{-6} \text{ A}$$

$$\therefore I_B = 75 \mu\text{A}$$

Again, applying KVL at input side,

$$V_{BE} = V_{BB} - I_B R_{in}$$

$$\Rightarrow V_{BB} = V_{BE} + I_B R_{in} = 0.7 + 75 \times 10^{-6} \times 10^5$$

$$= 0.7 + 7.5 = 8.2 \text{ V}$$

- 15 (c) In n - p - n transistor under CE configuration, emitter current I_E is equal to sum of collector current I_C and base current I_B , i.e.

$$I_E = I_C + I_B$$

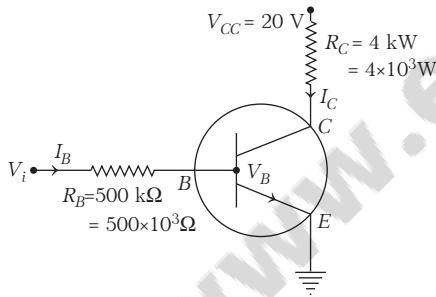
- 16 (a) According to give logic diagram,

$$Y' = (A' B')' = (A')' + (B')' = A + B$$

$$\therefore Y = Y' = (A + B)' = \text{output of NOR gate}$$

- 17 (d) Due to increase in temperature because of heating, thermal collision increases. Thus, net electron-hole pairs increase. This leads to increase in the current in diode and overall resistance of the diode changes. This in turn changes both the forward biasing and the reverse biasing. Thus, the overall I - V characteristics of p - n junction diode gets affected.

- 18 (d) Given, $V_{BE} = 0 \text{ V}$, $V_{CE} = 0 \text{ V}$ and $V_i = 20 \text{ V}$



Applying Kirchhoff's law to the base-emitter loop, we get

$$V_i = I_B R_B + V_{BE}$$

Substituting the values, we get

$$20 = I_B \times (500 \times 10^3) + 0$$

$$\Rightarrow I_B = \frac{20}{500 \times 10^3} = 0.04 \times 10^{-3}$$

$$= 40 \times 10^{-6} = 40 \mu\text{A} \quad \dots(i)$$

Similarly, $V_{CC} = I_C R_C + V_{CE}$

Substituting the given values, we get

$$20 = I_C \times (4 \times 10^3) + 0$$

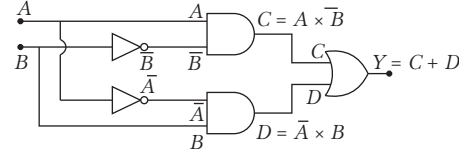
$$\Rightarrow I_C = \frac{20}{4 \times 10^3} = 5 \times 10^{-3} = 5 \text{ mA} \quad \dots(ii)$$

Current gain is given as $\beta = \frac{I_C}{I_B}$

Substituting the value of I_B and I_C from Eqs. (i) and (ii), we get

$$\Rightarrow \beta = \frac{5 \times 10^{-3}}{40 \times 10^{-6}} = 0.125 \times 10^3 = 125$$

- 19 (b) According to the question, the figure of combination of gates in terms of inputs and outputs can be given as



$$\text{Thus, } Y = C + D = A \cdot \bar{B} + \bar{A} \cdot B$$

- 20 (c) Current passing in the circuit,

$$I = \frac{P}{V} = \frac{100 \times 10^{-3}}{0.5} = 0.2 \text{ A}$$

Value of connected series resistance,

$$R = \frac{1.5 - 0.5}{0.2} \Rightarrow R = \frac{1}{0.2}$$

$$\therefore R = 5 \Omega$$

- 21 (a) In the forward biasing of p - n junction, p -side of junction diode is connected to higher potential and n -side of junction diode is connected to lower potential. As $0 \text{ V} > -2 \text{ V}$, hence option (a) is the correct answer.

- 22 (c) Collector current, $i_C = \frac{V}{R} = \frac{3}{3 \times 10^3} = 10^{-3} \text{ A}$

$$\text{Now, base current, } i_B = \frac{i_C}{\beta} = \frac{10^{-3}}{100} = 10^{-5} \text{ A}$$

As, voltage, $V_{in} = i_B R_B$

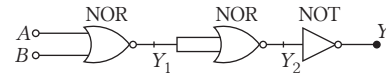
$$\therefore V_{in} = 10^{-5} \times 2 \times 10^3 = 2 \times 10^{-2} \text{ V}$$

$$\text{So, voltage gain, } A_V = \frac{V_{out}}{V_{in}} = \frac{3}{2 \times 10^{-2}} = 150$$

Power gain = $A_V \times \beta$

$$= 150 \times 100 = 15000$$

- 23 (c) Truth table for given network is



A	B	Y ₁	Y ₂	Y
0	0	1	0	1
1	0	0	1	0
0	1	0	1	0
1	1	0	1	0

Output Y of network matches with that of NOR gate.

- 24 (c) Given, one indium atom to be doped in 5×10^7 silicon atoms.

Number density of silicon

$$= 5 \times 10^{28} \text{ atom m}^{-3}$$

$$\approx 5 \times 10^{22} \text{ atom cm}^{-3}$$

$$\begin{aligned} \text{Number of acceptor atom cm}^{-3} \\ = \frac{5 \times 10^{22}}{5 \times 10^7} = 1 \times 10^{15} \text{ cm}^{-3} \end{aligned}$$

Hence, number of acceptor atom/cm³ is $1 \times 10^{15} \text{ cm}^{-3}$.

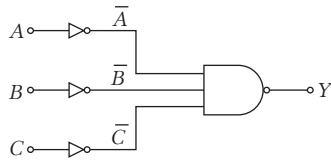
- 25** (a) Given, $\beta = 49$ and $\Delta I_B = 5 \mu\text{A}$

Current gain, $\beta = \frac{\Delta I_C}{\Delta I_B}$, where ΔI_C is change in collector

current and ΔI_B is change in base current.

$$\begin{aligned} \Delta I_C &= \beta \Delta I_B = 49 \times 5 = 245 \mu\text{A} \\ \Delta I_E &= \Delta I_B + \Delta I_C = (245 + 5) \mu\text{A} \\ &= 250 \mu\text{A} \end{aligned}$$

- 26** (a) The simplified circuit is shown in figure below



So, output $Y = \overline{A \cdot B \cdot C} = \overline{A} \cdot \overline{B} \cdot \overline{C} = A + B + C$

If $A = 0, B = 1, C = 1$

$$\therefore Y = 0 + 1 + 1 = 0 + 1 = 1$$

- 27** (c) The resultant boolean expression of the logic circuit will be

$$Y = (A + B) \cdot C$$

Now, let us try with inputs A, B and C given in the options and let's see, which one of them will give output 1 at Y .

If $A = 0, B = 0, C = 0$, then

$$Y = (0 + 0) \cdot 0 \Rightarrow Y = 0$$

If $A = 1, B = 1, C = 0$, then

$$Y = (1 + 1) \cdot 0 \Rightarrow Y = 1 \cdot 0 \Rightarrow Y = 0$$

If $A = 1, B = 0, C = 1$, then

$$Y = (1 + 0) \cdot 1 \Rightarrow Y = 1 \cdot 1 \Rightarrow Y = 1$$

If $A = 0, B = 1, C = 0$, then

$$Y = (0 + 1) \cdot 0 \Rightarrow Y = 1 \cdot 0 \Rightarrow Y = 0$$

So, we have seen that among the given options, only option (c) is the correct choice, i.e.

Output $Y = 1$ only when inputs $A = 1, B = 0$ and $C = 1$.

- 28** (a) Let us assume that current through the diode is I .

From the given condition,

$$\therefore I = \frac{V_A - V_B}{R} = \frac{4 - (-6)}{1 \text{ k}\Omega} = \frac{10}{1 \times 10^3} = 10^{-2} \text{ A}$$

- 29** (d) Given, resistance across load, $R_L = 800 \Omega$

Voltage drop across load, $V_L = 0.8 \text{ V}$

Input resistance of circuit, $R_i = 192 \Omega$

$$\text{Collector current is given by } I_C = \frac{V_L}{R_L} = \frac{0.8}{800} = \frac{8}{8000} = 1 \text{ mA}$$

$$\therefore \text{Current amplification} = \frac{\text{Output current}}{\text{Input current}}$$

$$= \frac{I_C}{I_B} = 0.96 \Rightarrow I_B = \frac{1 \text{ mA}}{0.96}$$

$$\therefore \text{Voltage gain, } A_V = \frac{V_L}{V_{in}} = \frac{V_L}{I_B R_i} = \frac{0.8 \times 0.96}{10^{-3} \times 192} = 4$$

$$\Rightarrow A_V = 4$$

$$\text{and power gain, } A_P = \frac{I_C^2 R_L}{I_B^2 R_i} = \left(\frac{I_C}{I_B} \right)^2 \cdot \frac{R_L}{R_i} = (0.96)^2 \times \frac{800}{192}$$

$$\therefore A_P = 3.84$$

- 30** (b) Given, collector resistance, $R_{out} = 2 \text{ k}\Omega$

Current amplification factor, $\beta = 100$

Base resistance, $R_{in} = 1 \text{ k}\Omega$

Output signal voltage = 4 V

Putting all the values in given equation, we get

$$A_V = \beta \frac{R_{out}}{R_{in}} = 100 \times \frac{2 \text{ k}\Omega}{1 \text{ k}\Omega} \Rightarrow A_V = 200$$

$$\text{Now, } A_V = \frac{(V_{out})_{AC}}{(V_{in})_{AC}} = 200 \Rightarrow (V_{in})_{AC} = \frac{4}{200} = 20 \text{ mV}$$

- 31** (a) We know that a diode only conducts in forward biased condition. In the given circuit, the diode D_1 will be in reverse bias, so it will block the current and diode D_2 will be in forward bias, so it will pass the current.

$$\text{Thus, } i = \frac{V}{R_1 + R_3} = \frac{10}{2 + 2} = 2.5 \text{ A}$$

- 32** (c) Output of the given circuit is given by

$$Y = \overline{(AB)(C)}$$

$$\text{When } A = B = C = 0, Y_1 = \overline{(0)(0)(0)} = \overline{0} = 1$$

$$\text{When } A = B = C = 1, Y_2 = \overline{(1)(1)(1)} = 0$$

- 33** (d) If Y represents the given Boolean expression $P + \overline{P}Q$, then

$$\begin{aligned} Y &= P + \overline{P}Q = P(1) + \overline{P}Q \\ &= P(1 + Q) + \overline{P}Q \quad (\because 1 = 1 + X) \\ &= P + PQ + \overline{P}Q \\ &= P + Q(P + \overline{P}) \\ &= P + Q \quad (\because \overline{X} + X = 1) \\ &= \text{output of OR gate} \end{aligned}$$

Hence, given expression represents OR gate.

- 34** (a) The overall conductivity of a semiconductor,

$$\sigma = n_e e \mu_e + n_p e \mu_p$$

$$\sigma = e [\mu_e n_e + n_p \mu_p]$$

Also, $n_e n_p = n_i^2$ [for an intrinsic semiconductor]

$$\Rightarrow n_e = \frac{n_i^2}{n_p}$$

$$\text{Now, } \sigma = e \left[\frac{n_i^2}{n_p} \mu_n + \mu_p n_p \right]$$

$$\text{On differentiating, } \frac{d\sigma}{dn_p} = e \left[-\frac{n_i^2}{n_p^2} \mu_n + \mu_p \right] = 0$$

$$\Rightarrow \mu_p = \frac{n_i^2}{n_p^2} \mu_n \Rightarrow n_p = n_i \sqrt{\frac{\mu_n}{\mu_p}}$$

35 (b) When input signal 1 is applied to a NOT gate, then its output will be opposite to that of the input, i.e. 0.

37 (b) For an intrinsic or pure semiconductor, the number densities of electron and hole are the same, i.e.

$$n_p = n_e$$

38 (d) The common application of Zener diode is stabilisation or voltage regulation.

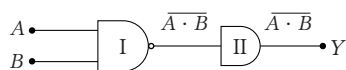
39 (c) We know that, the relation between α and β is given as

$$\beta = \frac{\alpha}{1-\alpha} \Rightarrow 100 = \frac{\alpha}{1-\alpha}$$

$$\Rightarrow 100 - 100\alpha = \alpha$$

$$\therefore \alpha = \frac{100}{101} = 0.99$$

40 (b) Given arrangement is



Gate I $\rightarrow$ NAND gate

Gate II $\rightarrow$ AND gate

Above arrangement represents NAND gate.

41 (b) Given, $\Delta I_B = 20 \mu\text{A} = 20 \times 10^{-6} \text{ A}$

$$\Delta I_C = 2 \text{ mA} = 2 \times 10^{-3} \text{ A}$$

and $\Delta V_{BE} = 0.04 \text{ V}$

$$\therefore \Delta V_{BE} = R_{BE} \times \Delta I_B$$

$$\Rightarrow 0.04 = R_{BE} \times 20 \times 10^{-6} \Omega \Rightarrow R_{BE} = 2 \text{ k}\Omega$$

$$\text{Now, current gain} = \frac{\Delta I_C}{\Delta I_B} = \frac{2 \times 10^{-3} \Omega}{20 \times 10^{-6} \Omega} = \frac{1}{10} \times 10^3 = 100$$

43 (c) The current, $I = \frac{\Delta Q}{\Delta t} = \frac{10^{10} \times 1.6 \times 10^{-19}}{2 \times 10^{-6}}$

$$\Rightarrow I = 10^{10} \times 10^{-19} \times 10^6 \times 0.8 = 0.8 \times 10^{-3} \\ = 800 \times 10^{-6} \text{ A} = 800 \mu\text{A}$$

44 (a) The output of OR gate is given by $A+B=Y$



The output of NOR gate

$$\overline{A+B} = Y$$

When, $A=B$, then $\overline{A+A} = \overline{A} = \text{NOT gate}$

45 (a) Given, energy gap, $E_g = 1.9 \text{ eV} = 1.9 \times 1.6 \times 10^{-19} \text{ V}$

$$\text{We know that, } E_g = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_g}$$

Wavelength of the emitted light,

$$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.9 \times 1.6 \times 10^{-19}}$$

$$\lambda = 6.5 \times 10^{-7} \text{ m}$$

46 (d) Output of NOR gate, $Y' = \overline{A+B}$

Output of NAND gate,

$$Y'' = \overline{(A+B) \cdot (A+B)} = \overline{(A+B)} \cdot \overline{(A+B)} \\ = (\overline{A+B}) (\overline{A+B}) = \overline{A+B}$$

Output of NOT gate,

$$Y''' = \overline{\overline{A+B}}$$

$\therefore$ The boolean expression is for NOR gate.

47 (c) The output of digital circuit can be given as $Y = \overline{AB} + \overline{C}$.

It will be same as $Y = \overline{A} + \overline{B} + \overline{C}$.

48 (c) Each and every extrinsic semiconductor is electrically neutral whether it is p -type or n -type.

49 (c) We know that, $\alpha = \frac{\Delta I_C}{\Delta I_E}$

$$\Rightarrow \Delta I_C = \alpha \Delta I_E = 0.96 \times 10 = 9.6 \text{ mA}$$

The change in base current, $\Delta I_B = \Delta I_E - \Delta I_C$

$$= 10 \text{ mA} - 9.6 \text{ mA} = 0.4 \text{ mA}$$

50 (c) The output of $G_1 = A+B$

Output of $G_2 = \overline{A \cdot B}$

and output of $G_3 = (A+B) \overline{(A \cdot B)} = \overline{AB} + \overline{BA}$

This output is same as the output of an XOR gate.

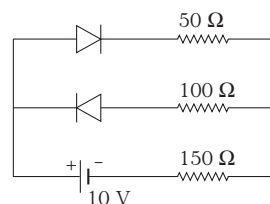
51 (d) Voltage gain = $\beta \times$ Resistance gain

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.99}{(1-0.99)} = 99$$

$$\text{Resistance gain} = \frac{\text{Input impedance}}{\text{Load impedance}} = \frac{10 \times 10^3}{10^3} = 10$$

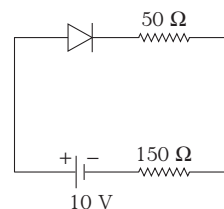
$$\text{Voltage gain} = \beta \times \text{Resistance gain} = 99 \times 10 = 990$$

52 (b) The circuit is



As the lower diode attached to 100Ω resistance is in reverse biased, so it is non-conducting.

Now, the circuit can be redrawn as

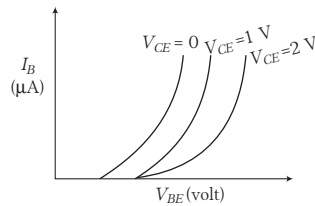


$$\therefore \text{Current through the circuit, } I = \frac{V}{R} = \frac{10}{200} = 0.05 \text{ A}$$

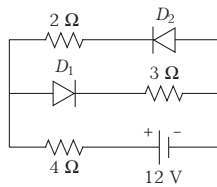
- 53** (b) Input characteristics curve is drawn between base current I_B and emitter-base voltage V_{BE} at constant collector-emitter voltage V_{CE} .

Dynamic input resistance,

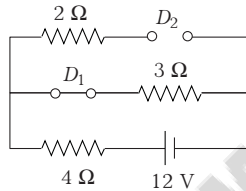
$$R_i = \left(\frac{\Delta V_{BE}}{\Delta I_B} \right)_{V_{CE} = \text{constant}}$$



- 54** (d) The mobility of electron is greater than holes because electron require less energy to move.
- 55** (d) As the diode is forward biased, so it takes only positive value. Hence, option (d) is correct.
- 56** (b)



In the given circuit, D_1 is forward biased and D_2 is reverse biased, so D_1 works like a closed switch and D_2 works like an open circuit, so circuit becomes



Now, current in the circuit,

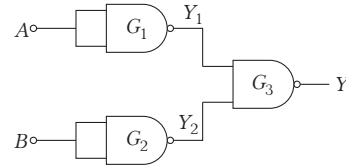
$$I = \frac{E}{R_S} = \frac{12}{4 + 3}$$

$$\Rightarrow I = \frac{12}{7}$$

$$\Rightarrow I = 1.71 \text{ A}$$

- 57** (c) The conductivity of the intrinsic semiconductor is independent of fermi energy. It depends upon band gap, temperature as well as effective mass of charge carriers.
- 58** (d) Barrier potential should depend on all the three points given. Barrier potential depends on the material used to make p - n junction diode (whether it is Si or Ge). It depends on amount of doping due to which the number of majority carriers will change. It should also depend on temperature due to number of minority carriers will change.
- 59** (a) V - I characteristic of a solar cell is shown in the question, where A represents open circuit voltage and B shows short circuit current.

- 60** (d) The combination of three NAND gates are shown in the diagram



$$Y_1 = \text{Output due to NAND gate 1} = \overline{A \cdot A} = \overline{A}$$

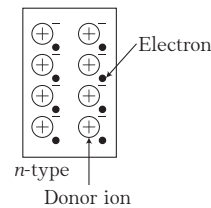
$$Y_2 = \text{Output due to NAND gate 2} = \overline{B \cdot B} = \overline{B}$$

Output due to NAND gate 3

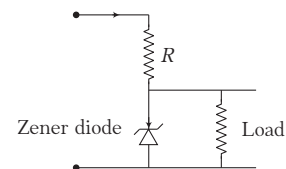
$$Y = \overline{Y_1 \cdot Y_2} = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A + B}} = A + B$$

This is OR gate. Hence, the combination is equivalent to an OR gate.

- 62** (c) The resistivity of semiconductors is greater than metals.
- 64** (d) The n -type semiconductor region has (negative) electrons as majority charge carriers and an equal number of fixed positively charged donor ions. Again, the material as a whole is neutral. That is a reason, that atom is electrically neutral.



- 65** (c) The wrong statement with reference to a solar cell is that, it uses materials with band gap of 5 eV. Because, for solar cell, band gap is less than 3 eV.
- 66** (b) Initial current, $I_{in} = 0$
- $$V_i = 6\text{V} \quad 150\ \Omega \quad \text{Diode} \quad +3\text{V}$$
- $$\text{Final current, } I_f = \frac{(6 - 3)}{150} = 0.02\text{A}$$
- So, change in current, $\Delta I = I_f - I_{in} = 0.02 - 0$
- $$= 0.02\text{A} = 20\text{mA}$$
- 67** (c) When the applied reverse voltage V reaches the breakdown voltage V_Z of the Zener diode, then there is a large change in the current. So, after the breakdown voltage V_Z , a large change in the current can be produced by almost in significant change in the reverse bias voltage.



As Zener voltage remains constant even though the current through the Zener diode varies over a wide range, hence it is used to obtain regulated voltage output.

- 68 (b) Given, $\alpha = 0.95$, $I_b = 0.2$ mA

$$\begin{aligned} \text{As, } \alpha &= \frac{I_C}{I_E} = \frac{I_E - I_B}{I_E} = 1 - \frac{I_B}{I_E} \\ \Rightarrow \frac{I_B}{I_E} &= 1 - \alpha = 1 - 0.95 = 0.05 \\ \Rightarrow I_E &= \frac{I_B}{0.05} = \frac{0.2}{0.05} \\ \Rightarrow I_E &= 4 \text{ mA} \end{aligned}$$

- 69 (d) For transistor as an oscillator, $f = \frac{1}{2\pi\sqrt{LC}}$

where, $f = 2$ MHz $= 2 \times 10^6$ Hz and $\sqrt{LC} = ?$

$$\Rightarrow \sqrt{LC} = \frac{1}{2\pi f} = \frac{1}{2\pi \times 2 \times 10^6} = \frac{1}{4\pi \times 10^6}$$

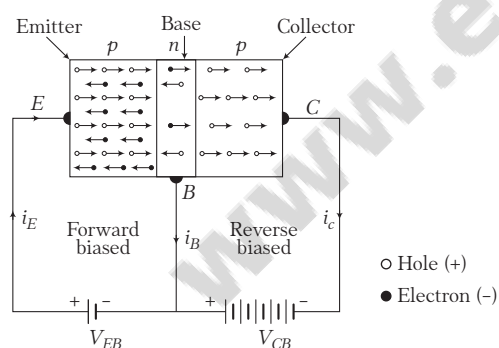
- 70 (b) α and β are the AC current gains of a transistor in the common base configuration and in the common emitter configuration respectively and are related as

$$\begin{aligned} \beta &= \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} \quad (\because \alpha = 0.98) \\ &= \frac{0.98}{0.02} = \frac{98}{2} = 49 \end{aligned}$$

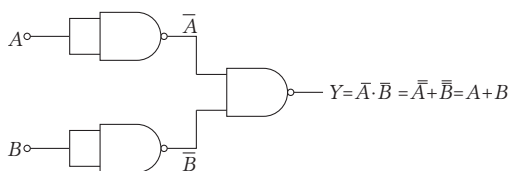
- 71 (b) As I_B is in μA , I_C is in mA and V_{CE} in V.

- 72 (c) A common base circuit of a p - n - p transistor is shown in figure. The emitter-base (p - n) junction on the left is given a small forward bias (fraction of a volt) by an emitter-base battery V_{EB} , while the base collector (n - p) junction is given a large reverse bias (a few volt) by another battery V_{CB} .

So, the required solution is emitter-base (EB) junction forward bias and collector-base (CB) junction reverse bias.

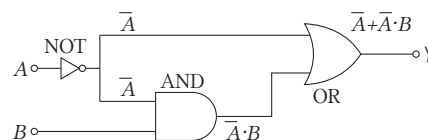


- 73 (d) If the two inputs of a NAND gate are joined together, it works as a NOT gate. Now, if the inputs A and B are inverted by using two NOT gates (obtained from two NAND gates) and the resulting output $\bar{A}$ and $\bar{B}$ are fed to a third NAND gate,



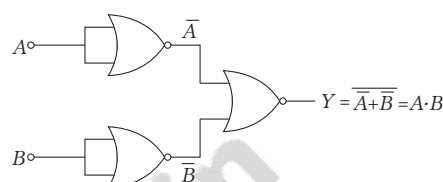
So, three NAND gates are required to make an OR gate.

- 74 (b) By absorption law, output $Y = \bar{A} + \bar{A} \cdot B$



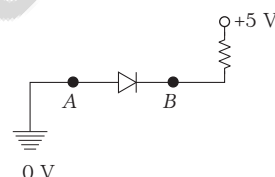
$$\text{or } Y = \bar{A} \cdot B + \bar{A} = \bar{A} \cdot (B + 1) = \bar{A} \cdot 1 = \bar{A}$$

- 75 (d) According to the logical relationship,
Output, $Y = \overline{\bar{A} + \bar{B}} = A \cdot B$



So, it is AND gate.

- 76 (c) A diode is symbolically shown in the adjacent diagram. The diode is said to be forward biased, when $V_A > V_B$. The diode is said to be reverse biased, when $V_A < V_B$.



In the option (b), $5 \text{ V} > 0 \text{ V}$

Hence, $V_B > V_A$

So, the diode is reverse biased.

- 77 (a) The OR gate is a device that has two (or more) input variables A and B and one output variable Y and follows the boolean expression, $A + B = Y$, read as ' A OR B equals Y '.

Truth table for given gate G_1 combination

Input		Output
A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

Similarly, the AND gate is also has two inputs and one output and follows boolean expression $A \cdot B = Y$, read as ' A AND B equals Y '.

Truth table for given gate G_2 combination

Input		Output
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

The NOR gate is a combination of OR and NOT gates.

The boolean expression for the NOR gate is

$$\overline{A + B} = Y.$$

Truth table for given gate G_3 combination

Input		Output	
A	B	$Y' = A + B$	$Y = \overline{A + B} = \overline{Y'}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

The NAND gate is a combination of AND and NOT gates.

The boolean expression for the NAND gate is

$$\overline{A \cdot B} = Y$$

Truth table for given gate G_4 combination

Input		Output	
A	B	$Y' = A \cdot B$	$Y = \overline{A \cdot B} = \overline{Y'}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

So, the required solution is

$G_1 \rightarrow$ OR gate, $G_2 \rightarrow$ AND gate, $G_3 \rightarrow$ NOR gate and

$G_4 \rightarrow$ NAND gate.

- 78 (c)** The n -type semiconductor can be produced by doping an impurity atom of valency 5, i.e. pentavalent atoms.

Here, majority carriers are electron and minority carriers are holes.

- 79 (a)** As, $G = \left(\frac{\beta}{R_i}\right) R_L \Rightarrow G = g_m R_L \quad \left(\because g_m = \frac{\beta}{R_i}\right)$

$$\Rightarrow G \propto g_m$$

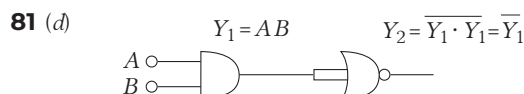
$$\therefore \frac{G_2}{G_1} = \frac{g_{m2}}{g_{m1}}$$

$$\Rightarrow G_2 = \frac{0.02}{0.03} \times G$$

$$\therefore \text{Voltage gain, } G_2 = \frac{2}{3} G$$

- 80 (c)** $X = \overline{AB} = A \cdot B$ (i.e. AND gate)

If the output X of NAND gate is connected to the input of NOT gate (made from NAND gate by joining two inputs) from the given figure, then we get an AND gate.



Truth table for the circuit

A	B	$Y_1 = AB$	$Y_2 = \overline{Y_1}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

It represents a NAND gate.

- 82 (b)** For making a junction transistor, the emitter E is heavily doped, the base B is thin and lightly doped and the collector C is moderately doped semiconductor materials are join together.

- 83 (c)** $\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE} = \text{constant}}$

$$60 = \frac{4 \times 10^{-3}}{\Delta I_B}$$

$$\Rightarrow \Delta I_B = \frac{4 \times 10^{-3}}{60} = 66.6 \mu\text{A}$$

- 87 (c)** $\overline{A} + A$ should always be equal to 1. As, addition of inputs like 1 to 0 or 0 to 1 always gives 1.

- 88 (c)** The depletion layer in the p - n junction region is caused due to the diffusion of carriers on either side of the junction.

- 89 (c)** A digital signal has only two values of voltage variation with time. Its value is either zero or maximum (1).

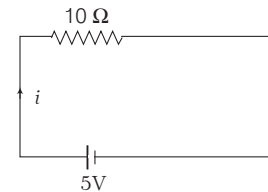
- 90 (c)** The diode D_1 is in reversed biased and D_2 is in forward biased. The resistance of D_1 in becomes infinite and of D_2 is zero. Therefore, current in the circuit,

$$I = \frac{\text{emf}}{\text{total resistance}} = \frac{12}{4 + 2} = 2 \text{ A}$$

- 91 (b)** For using transistor as a switch, it is used in cut-off state and saturation state only.

- 92 (d)** Here in circuit D_1 is forward bias and D_2 is reverse bias.

Current will flow only in diode D_1



The current supplied by the battery, $i = \frac{5}{10} = 0.5 \text{ A}$

- 93 (d)** Current amplification factor, $\beta = \frac{\Delta I_C}{\Delta I_B}$

$$\text{Collector current, } \Delta I_C = \frac{2\text{V}}{2 \times 10^3 \Omega} = 1 \times 10^{-3} \text{ A}$$

Base current,

$$\Delta I_B = \frac{V_B}{R_B} = \frac{V_B}{1 \times 10^3} = V_B \times 10^{-3}$$

Given, $\beta = 100$

Now, $100 = \frac{10^{-3}}{V_B \times 10^{-3}}$

$\Rightarrow V_B = \frac{1}{100} \text{ V} = 10 \text{ mV}$

- 94 (a) From the given waveforms, the following truth table can be made

Input		Output
A	B	C
0	0	0
1	0	1
1	1	1
0	1	1

This truth table is similar to OR gate, so logic gate is OR gate.

Note The logic of the OR gate is, if any of the input is one, then output is one.

95 (c) As, $V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \Rightarrow 220 = \frac{V_0}{\sqrt{2}}$

$\Rightarrow V_0 = 311.1 \text{ V}$

- 96 (d) Resistivity of a semiconductor is $10^{-5} - 10^6 \Omega\text{-m}$ while that of insulator is $10^{11} - 10^{19} \Omega\text{-m}$ and conductor is $10^{-2} - 10^{-8} \Omega\text{-m}$.

97 (a) As, $V = V_{CE} + i_C R_C$

$\Rightarrow 15 = 7 + i_C \times 2 \times 10^3$

$\Rightarrow i_C = \frac{8}{2 \times 10^3} = 4 \text{ mA}$

$\therefore \beta = \frac{i_C}{i_B}$

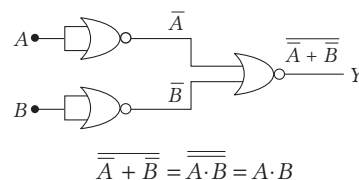
or $i_B = \frac{i_C}{\beta} = \frac{4}{100} = 0.04 \text{ mA}$

98 (a) By using, $V_b = i_b R_b$

$\Rightarrow R_b = \frac{V_b}{i_b} = \frac{9}{35 \times 10^{-6}} = 257 \text{ k}\Omega$

- 99 (a) At room temperature, due to thermal vibrations, the few bonds of intrinsic semiconductor are broken and producing equal number of electrons and holes in the semiconductor.

101 (a)



$\overline{A + B} = \overline{A} \cdot \overline{B} = A \cdot B$

This boolean expression represents AND gate, i.e.



- 102 (b) When a small amount of antimony is added to germanium crystal, the crystal becomes *n*-type semiconductor, because antimony is a pentavalent substrate and act as donor atom. So, there will be more free electrons than hole in the semiconductor.

103 (c) Symbols given in the problem are

- (i) OR gate (ii) AND gate
(iii) NOT gate (iv) NAND gate

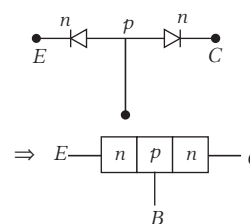
104 (d) Current gain,

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{(20 - 10) \text{ mA}}{(300 - 100) \mu\text{A}}$$

$$= \frac{10 \times 10^{-3}}{200 \times 10^{-6}} = 50$$

- 105 (b) Integrated circuits are miniature electronic circuit produced within a single crystal of a semiconductors such as silicon. They contain a million or so transistors and resistors or capacitors. They are widely used in memory circuits, micro computers, pocket calculators and electronic watches, on account of their low cost and reliability into specific regions of the semiconductor crystals.

- 106 (b) Figure in option (b) shows the equivalent diagram of *n-p-n* transistor.



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